CSE 674 Advanced Data Structures

Trees

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Contents

We will discuss

1. Trees
2. Trees as data structures
3. Tree Traversals
4. Binary Trees
5. Binary Search Trees

Trees

1. Intuitive description: A branch structure between nodes
2. Many variations of trees (sometimes distinctions between them are subtle)
   Reading: CLRS Appendix B.5
3. Free Trees: connected undirected graphs with no cycles
4. In computer science, we like to represent hierarchical structures via *rooted* trees

Rooted Trees

1. Define via recursion

   Definition a tree is a finite set T of one or more nodes where one element, called the root of T, is distinguished. The remaining nodes in the tree form m disjoint subset $(T_1, \ldots, T_m)$ $(m \geq 0)$ where each subset (referred as subtree) is itself a tree.

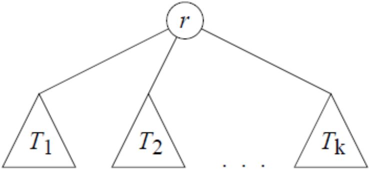
2. Some authors allow the possibly of an *empty* tree with no node.
3. If there is an order among the T_i's, then we will call T an ordered tree.

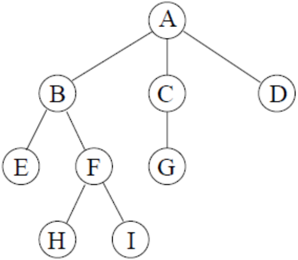
Some Definitions

For an *ordered tree*, we have the following definitions:

1. The *degree of a node n*:
   it is the number of children of n. Note that a leaf node has degree 0
2. A *path* between two nodes n and m:
   it is a sequence of nodes $n = u_1, u_2 \ldots, u_{k-1} u_k = m$ where u_i is a parent of u_{i+1} and in this case, the length of the path is $k - 1$.
3. The *depth of a node n*:
   it is the length of the *unique path* from the root to that node.
4. The *height* of a node n:
   it is the length of the longest path from n to a leaf.
5. The *level* of a node is its depth in the tree.

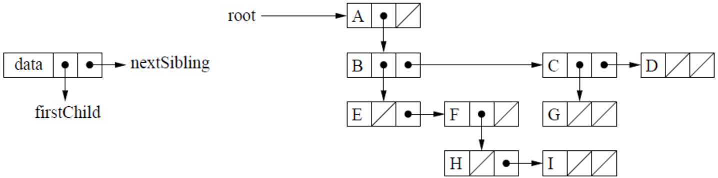
Trees as Data Structures: I





Trees as Data Structures: II

One possible way to implement a rooted tree is:



Question Can you suggest another data structure representation ?

Binary Trees

Definition a binary tree is a structure defined on a finite set of nodes that either

- Contains no nodes, or
- is composed of three disjoint sets of nodes:
  a root node, a binary tree called its left subtree, and a binary tree called its right subtree

Note A heap is a special type of binary trees

Question Suggest a data structure to represent a binary tree

Basic operations on Trees

Many basic operations can be casted as a traversal of nodes in the tree

Examples: Let T be a tree with root r and subtrees $T_1, \ldots, T_m$ ($m \geq 0$)

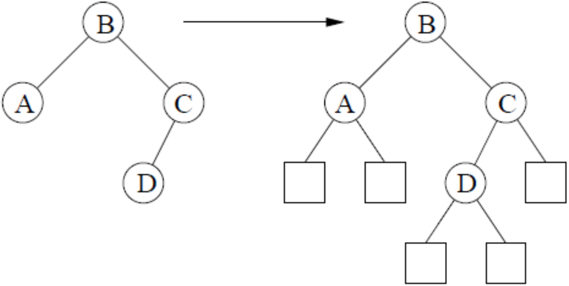
1. Pre-order traversal: visit the root first, then recursively apply pre-order traversal to $T_1, \ldots, T_k$
2. In-order traversal (for binary trees): apply in-order to T_L, visit r and then apply in-order to T_R
3. Post-order traversal: apply post-order traversal to $T_1, \ldots, T_k$ and then visit the root

More on Binary Trees I

1. Extended Binary Trees: replace each null pointer (missing child) with a special leaf node (external node)
2. Threaded Binary Trees: use the null pointer as follows:
   - If a left child pointer of a node n points to null, reset it s.t. it will point to the inorder predecessor of n
   - If a right child pointer of a node n points to null, reset it s.t. it will point to the inorder sucessor of n

   Why are these good ideas ?

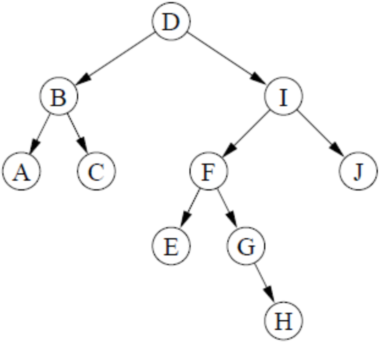
More on Binary Trees II: Extended Binary Trees



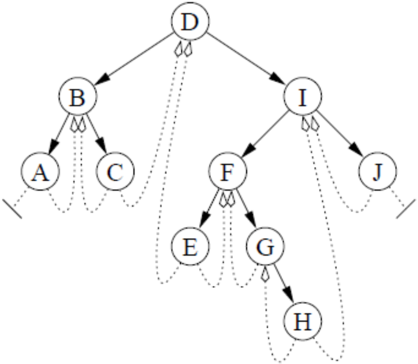
Binary tree and extended binary tree

Note: An extended binary trees with n internal nodes has $n+1$ external nodes.

More on Binary Trees III: Threaded Binary Trees



Binary tree



Binary tree with threads

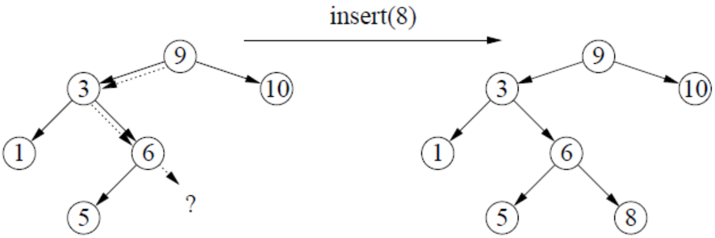
Binary Search Trees

1. Question: What are the drawbacks on using binary search via sequential allocation ?
2. Does using a tree structure helps ?

Operations on Binary Search Trees I

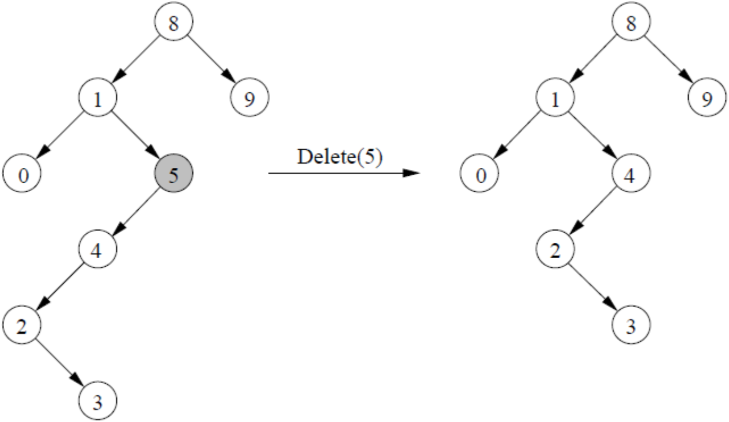
We will discuss how to implement the insert, delete and search operations in class via diagrams.

Insertion:



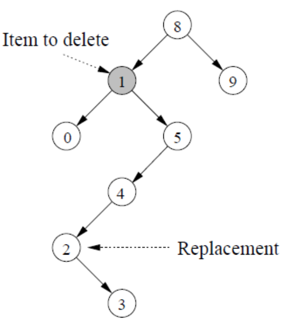
Operations on Binary Search Trees II

Deletion: Case 1



Operations on Binary Search Trees III

Deletion: Case 2



Find the replacement

Delete the replacement

Fill in the hole