

## 02: ordered containers #1

Pointers | Singly Linked Lists | Doubly Linked Lists | Vector Data Structure  
Amortized Complexity | List vs. Vector Performance | Binary Search | HW01

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### Pointers

```
1: // pointers.cpp - download here
2:
3: #include <iostream>
4:
5: struct SingleNode {
6:     SingleNode * m_next;
7:     long long m_value;
8: };
9:
10: void printAddrValues(SingleNode * node, long long addr, const char * title){
11:     std::cout << "=====" << std::endl;
12:     std::cout << "= " << title << " addrs/values: " << std::endl;
13:     std::cout << "=====" << std::endl;
14:     std::cout << "addr(" << title << "): " << std::hex << addr << std::endl;
15:     std::cout << "value(" << title << "): " << std::hex << node << std::endl;
16:     std::cout << "addr(" << title << ".m_next): " << std::hex << &(node->m_next) << std::endl;
17:     std::cout << "value(" << title << ".m_next): " << std::hex << node->m_next << std::endl;
18:     std::cout << "addr(" << title << ".m_value): " << std::hex << &(node->m_value) << std::endl;
19:     std::cout << "value(" << title << ".m_value): " << std::hex << node->m_value << std::endl;
20:     std::cout << std::endl;
21: }
22:
23: int main(int argc, char *argv[]){
24:
25:     std::cout << "=====" << std::endl;
26:     std::cout << "= sizes: " << std::endl;
27:     std::cout << "=====" << std::endl;
28:     std::cout << "sizeof(SingleNode): " << sizeof(SingleNode) << std::endl;
29:     std::cout << "sizeof(SingleNode *): " << sizeof(SingleNode *) << std::endl;
30:     std::cout << "sizeof(long long): " << sizeof(long long) << std::endl;
31:     std::cout << std::endl;
32:
33:     SingleNode * head = new SingleNode();
34:     printAddrValues(head, (long long) &head, "head");
35:
36:     SingleNode * one = new SingleNode();
37:     one->m_value = 1;
38:
39:     SingleNode * two = new SingleNode();
40:     two->m_value = 2;
41:
42:     SingleNode * three = new SingleNode();
43:     three->m_value = 3;
44:
45:     head->m_next = one;
46:     one->m_next = two;
47:     two->m_next = three;
48:     printAddrValues(head, (long long) &head, "head");
49:     printAddrValues(one, (long long) &one, "one");
50:     printAddrValues(two, (long long) &two, "two");
51:     printAddrValues(three, (long long) &three, "three");
52:
53:     return 0;
54: }
```

```
=====
= sizes:
=====
sizeof(SingleNode): 16
sizeof(SingleNode *): 8
sizeof(long long): 8
```

```
=====
= head addrs/values:
=====
addr(head): 7fff16456ff0
value(head): 0x1c20010
addr(head.m_next): 0x1c20010
```

```

value(head.m_next): 0
addr(head.m_value): 0x1c20018
value(head.m_value): 0

```

```

=====
= head adrs/values:
=====

```

```

addr(head): 7fff16456ff0
value(head): 0x1c20010
addr(head.m_next): 0x1c20010
value(head.m_next): 0x1c20030
addr(head.m_value): 0x1c20018
value(head.m_value): 0

```

```

=====
= one adrs/values:
=====

```

```

addr(one): 7fff16456ff8
value(one): 0x1c20030
addr(one.m_next): 0x1c20030
value(one.m_next): 0x1c20050
addr(one.m_value): 0x1c20038
value(one.m_value): 1

```

```

=====
= two adrs/values:
=====

```

```

addr(two): 7fff16457000
value(two): 0x1c20050
addr(two.m_next): 0x1c20050
value(two.m_next): 0x1c20070
addr(two.m_value): 0x1c20058
value(two.m_value): 2

```

```

=====
= three adrs/values:
=====

```

```

addr(three): 7fff16457008
value(three): 0x1c20070
addr(three.m_next): 0x1c20070
value(three.m_next): 0
addr(three.m_value): 0x1c20078
value(three.m_value): 3

```

addr	stack value	label
0x7fff16456ff0:	0x1c20010	head
0x7fff16456ff8:	0x1c20030	one
0x7fff16457000:	0x1c20050	two
0x7fff16457008:	0x1c20070	three

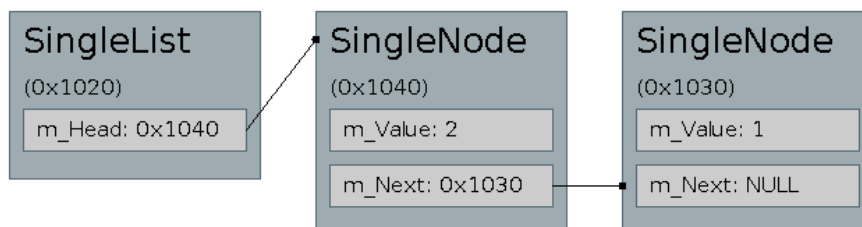
addr	heap value	label
0x1c20010:	0x1c20030	head.m_next
0x1c20018:	0	head.m_value
0x1c20020:		
0x1c20028:		
0x1c20030:	0x1c20050	one.m_next
0x1c20038:	1	one.m_value
0x1c20040:		
0x1c20048:		
0x1c20050:	0x1c20070	two.m_next
0x1c20058:	2	two.m_value
0x1c20060:		
0x1c20068:		
0x1c20070:	0x0000000	three.m_next
0x1c20078:	3	three.m_value

## Singly Linked Lists

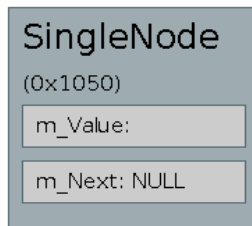
An item can be inserted in the middle of a singly linked list without moving all the remaining elements over. This is required in a vector.

### insertion at the head

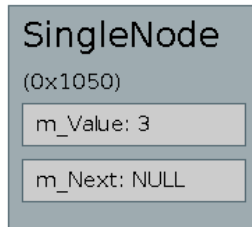
1. start:



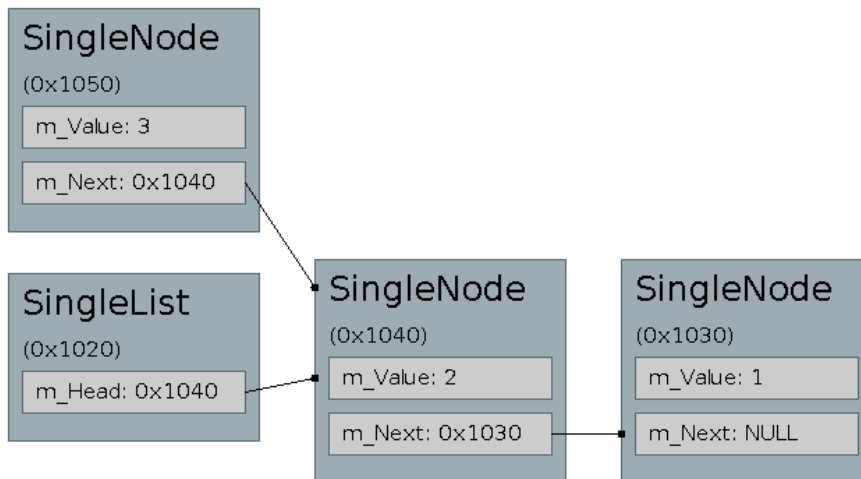
2. create a new\_node:



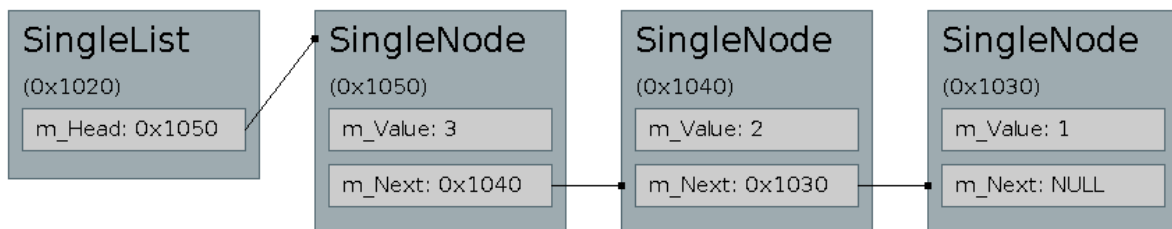
3. set the value of the new\_node:



4. the new\_node's next\_nointer is set point to where the list's head\_pointer currently points to



5. The list's head\_pointer is set to point to the new\_node

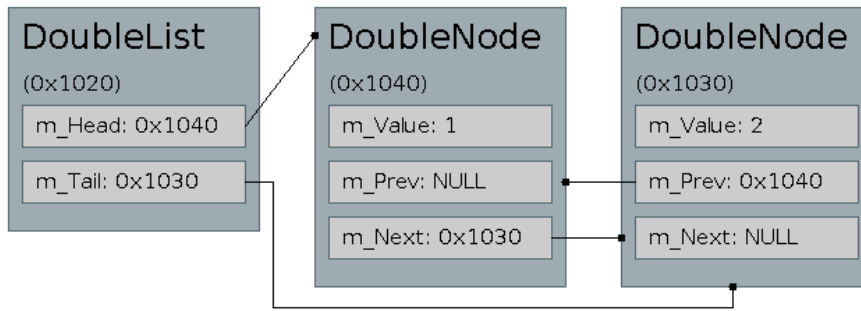


## Doubly Linked Lists

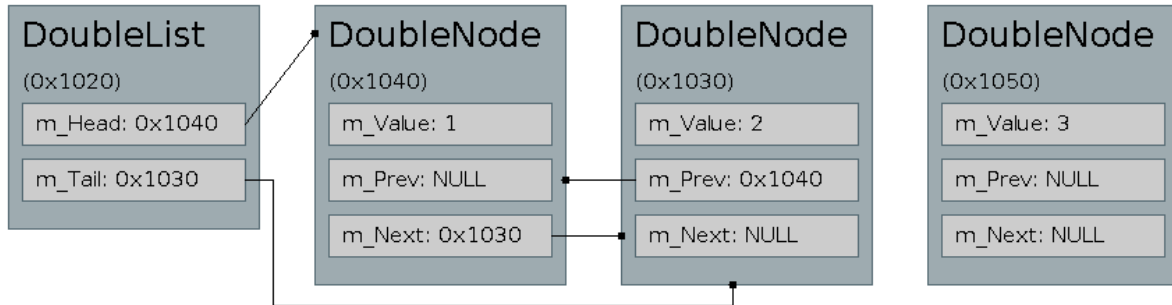
In a singly linked list you can only iterate in one direction. In a doubly linked list you can iterate going both forwards and backwards

### adding a node at the end

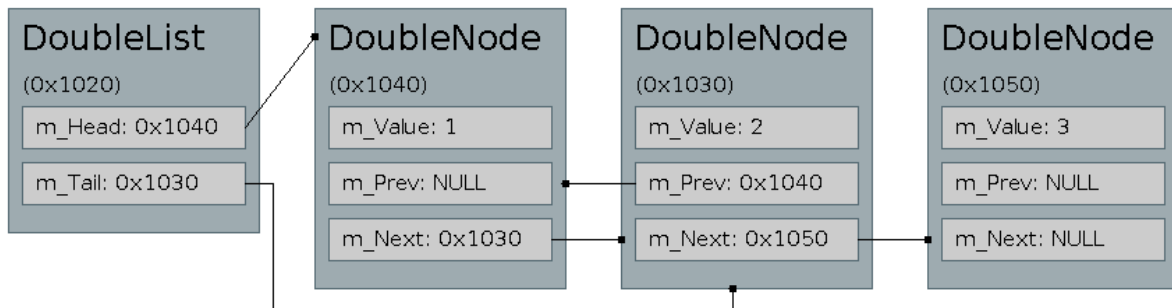
1. start:



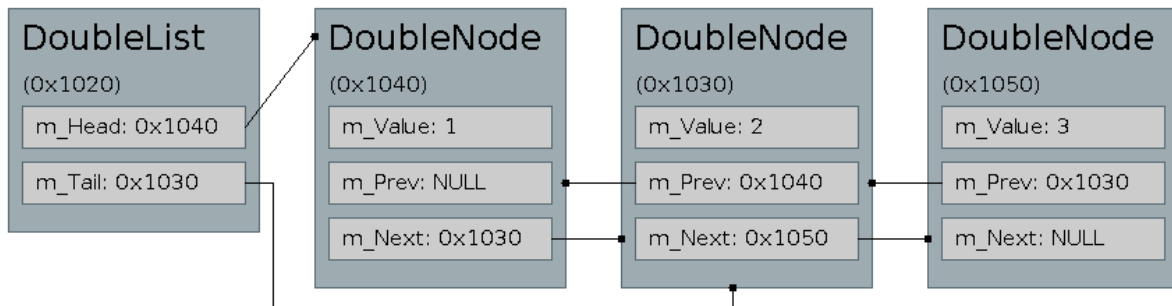
2. create a new node and initialize the value



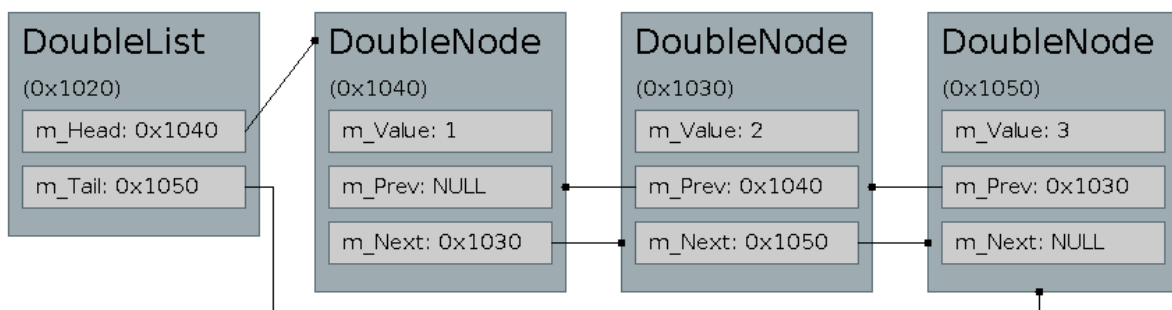
3. set the next of the tail to the new\_node



4. set the prev of the new\_node to the tail



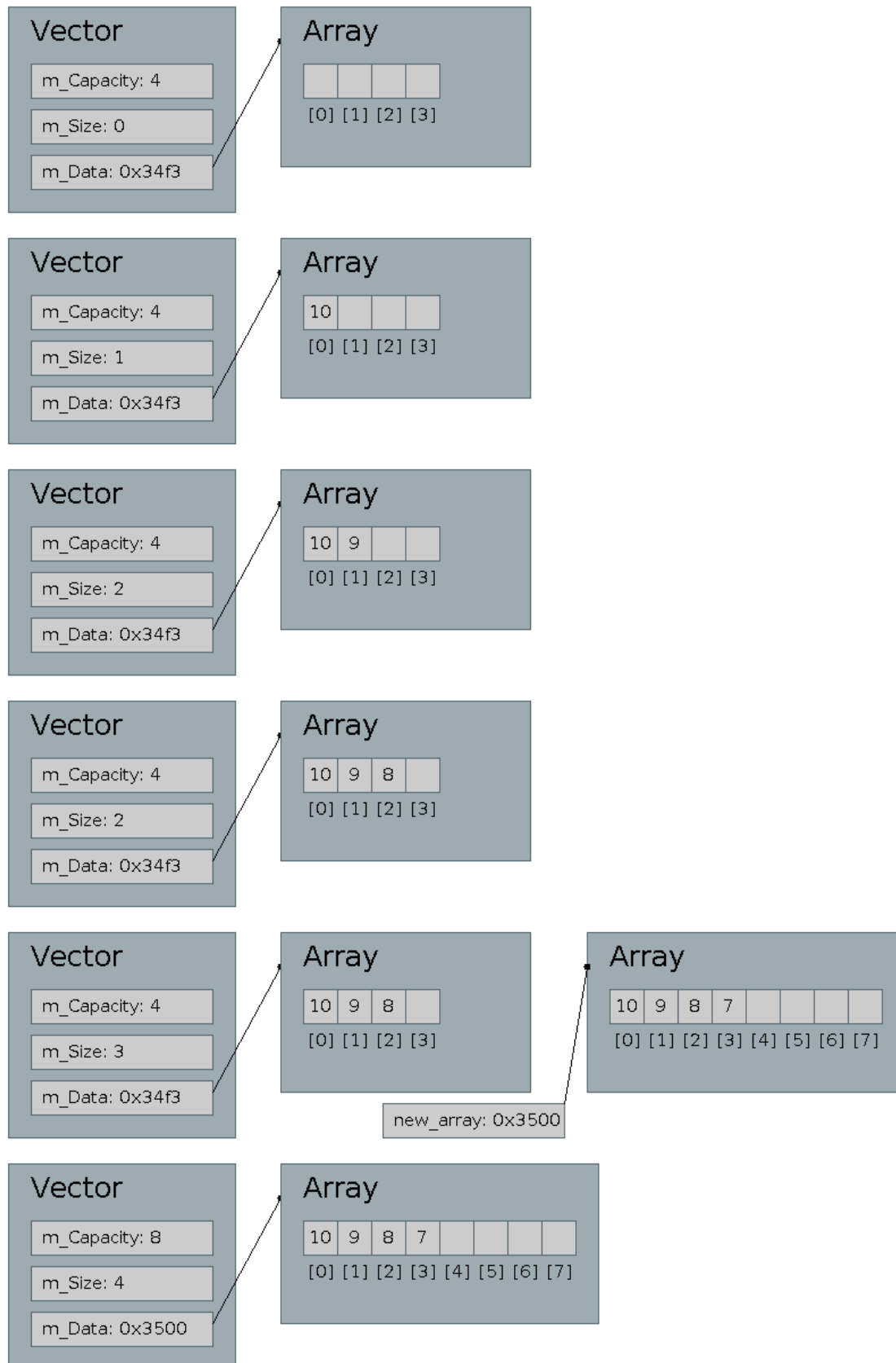
5. set the tail to the new\_node



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## vector

A vector is an array that can automatically grow or shrink.



# Amortized Complexity

This is the aggregate method.

- In a vector data structure adding an element to the back has a  $\Omega(1)$  and  $O(\text{size\_vector})$ .
- We want to have a complexity that is more tightly bounded
- Amortized Complexity attempts to do this

Cost of insertion into vector															
1	1	1	4												
1	1	1	8												
1	1	1	1	1	1	1	16								
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	32

The starting capacity is 4 and we double the capacity when full.

- $T(n) = n + (n/2) - 1 + T(n/2)$
- $T(4) = 4 + 3$

Solving for  $T(n) / n$

- $T(n) = (n + (n/2) - 1) + T(n/2)$
- $T(n) = (n + (n/2) - 1) + (n/2 + (n/4) - 1) + T(n/4)$
- $T(n) = (n + (n/2) - 1) + (n/2 + (n/4) - 1) + (n/4 + (n/8) - 1) + \dots + T(n/n/4)$
- $T(n) = (n + (n/2) - 1) + (n/2 + (n/4) - 1) + (n/4 + (n/8) - 1) + \dots + T(4)$
- $T(n) = (n + (n/2) - 1) + (n/2 + (n/4) - 1) + (n/4 + (n/8) - 1) + \dots + 4 + 3$
- $T(n) = \text{sumof: } (n + n/2 + n/4 + \dots +) \text{ for } \log_2(n-2) \text{ times} + \text{sumof: } (n/2 + n/4 + n/8 + \dots +) \text{ for } \log_2(n-2) \text{ times} + \text{sumof: } (-1) \text{ for } \log_2(n-2) \text{ times}$
- $T(n) = n + \text{sumof}(1) \text{ for } \log_2(n-3) \text{ times} + n/16 + cn$
- $T(n) = c1*n + c2*\log_2(n)$
- $T(n) / n = c1 * n/n + c2*\log_2(n)/n$
- $T(n) / n = O(1)$

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## List vs. Vector Performance

64 bit system:

- linked list value is 4 bytes
- linked list next is 8 bytes

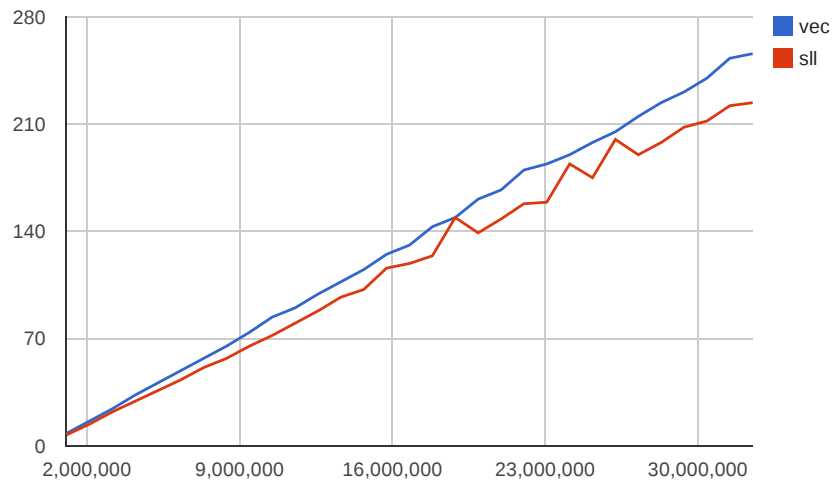
Common cache line size: 64 bytes

- Linked list fits  $64/12=5$  items in cache before fetching new line
- Array holds  $64/4=16$  items in cache before fetching new line

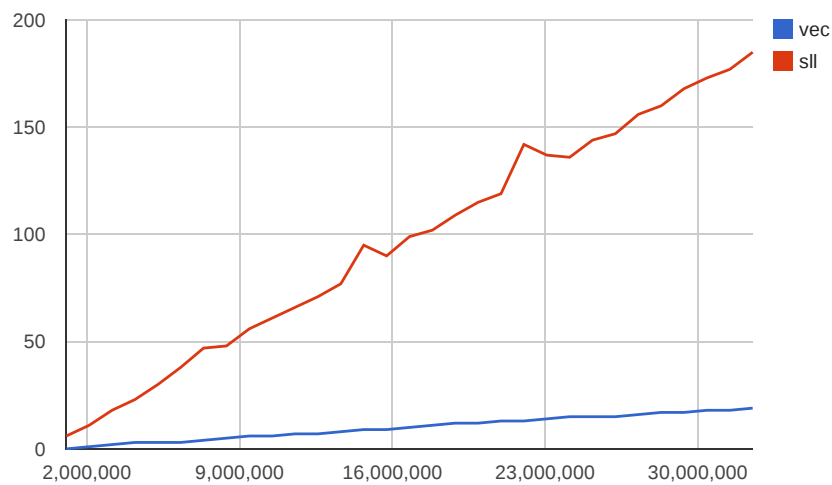
### Insertion Performance

1 600

### Iter Performance: no optimization



### Iter Performance: g++ -O3



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## Binary Search

Binary Search requires that the collection has random access and is sorted. It has  $O(\lg n)$  time complexity.

```
1: // binarySearch.cpp - download here
2:
3: int binarySearch(int * array, int n, int key){
4:     int low = 0;
5:     int mid;
6:     int high = n-1;
7: }
```

```

8: while(low <= high){
9:     mid = (low + high) / 2;
10:    if(key < array[mid]){
11:        high = mid - 1;
12:    } else if(array[mid] < key){
13:        low = mid + 1;
14:    } else {
15:        return mid;
16:    }
17: }
18: return -1;
19: }

```

**Question:** Does the array contain 16?

1. **start:** [low = 0, high = 19, n = 20]

0	1	3	5	6	8	10	12	13	15	16	18	19	20	23	28	30	31	32	40
[00]	[01]	[02]	[03]	[04]	[05]	[06]	[07]	[08]	[09]	[10]	[11]	[12]	[13]	[14]	[15]	[16]	[17]	[18]	[19]

2. **iter1:**  $(low + high) / 2 = (0 + 19) / 2 = 9$

0	1	3	5	6	8	10	12	13	15	16	18	19	20	23	28	30	31	32	40
[00]	[01]	[02]	[03]	[04]	[05]	[06]	[07]	[08]	[09]	[10]	[11]	[12]	[13]	[14]	[15]	[16]	[17]	[18]	[19]

3. **iter1:** At mid (9), is 16 < 15? No. Go right. [low = 10, high = 19]

0	1	3	5	6	8	10	12	13	15	16	18	19	20	23	28	30	31	32	40
[00]	[01]	[02]	[03]	[04]	[05]	[06]	[07]	[08]	[09]	[10]	[11]	[12]	[13]	[14]	[15]	[16]	[17]	[18]	[19]

4. **iter2:**  $(low + high) / 2 = (10 + 19) / 2 = 14$

0	1	3	5	6	8	10	12	13	15	16	18	19	20	23	28	30	31	32	40
[00]	[01]	[02]	[03]	[04]	[05]	[06]	[07]	[08]	[09]	[10]	[11]	[12]	[13]	[14]	[15]	[16]	[17]	[18]	[19]

5. **iter2:** At mid (14), is 16 < 23? Yes. Go left. [low = 10, high = 13]

0	1	3	5	6	8	10	12	13	15	16	18	19	20	23	28	30	31	32	40
[00]	[01]	[02]	[03]	[04]	[05]	[06]	[07]	[08]	[09]	[10]	[11]	[12]	[13]	[14]	[15]	[16]	[17]	[18]	[19]

6. **iter3:**  $(low + high) / 2 = (10 + 13) / 2 = 11$

0	1	3	5	6	8	10	12	13	15	16	18	19	20	23	28	30	31	32	40
[00]	[01]	[02]	[03]	[04]	[05]	[06]	[07]	[08]	[09]	[10]	[11]	[12]	[13]	[14]	[15]	[16]	[17]	[18]	[19]

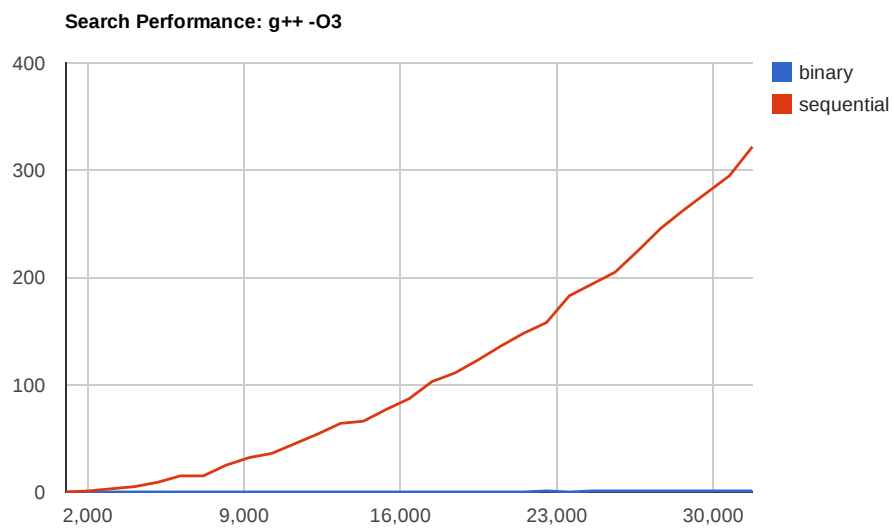
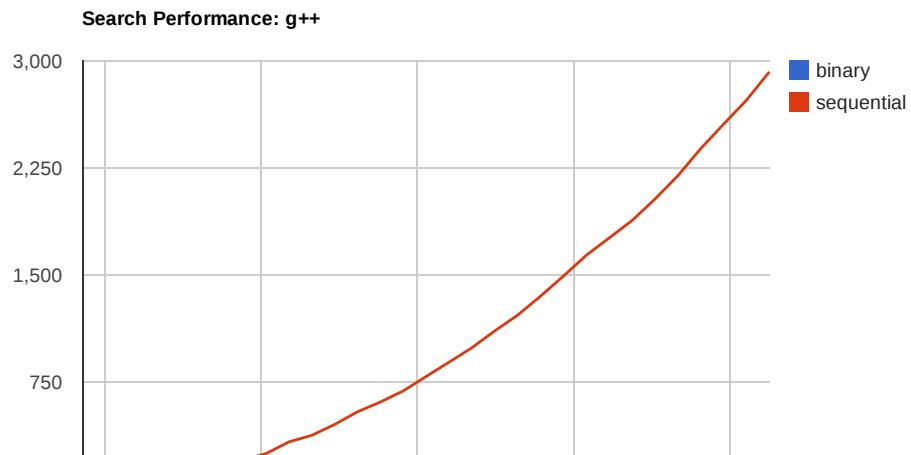
7. **iter3:** At mid (11), is 16 < 18? Yes. Go left. [low = 10, high = 10]

0	1	3	5	6	8	10	12	13	15	16	18	19	20	23	28	30	31	32	40
[00]	[01]	[02]	[03]	[04]	[05]	[06]	[07]	[08]	[09]	[10]	[11]	[12]	[13]	[14]	[15]	[16]	[17]	[18]	[19]

8. **iter4:**  $(low + high) / 2 = (10 + 10) / 2 = 10$ . At mid (10), 16 == 16. Found result.

0	1	3	5	6	8	10	12	13	15	16	18	19	20	23	28	30	31	32	40
[00]	[01]	[02]	[03]	[04]	[05]	[06]	[07]	[08]	[09]	[10]	[11]	[12]	[13]	[14]	[15]	[16]	[17]	[18]	[19]





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## References

1. Adam Drozdek. "Data Structures and Algorithms in C++"
  2. [Google Chart Tools](#)
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