The Heap Data Structure
It uses in sorting & developing a priority queue

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Heap and Complete Binary Tree

- Storing a complete binary trees within an array
- Max/Min Heap property

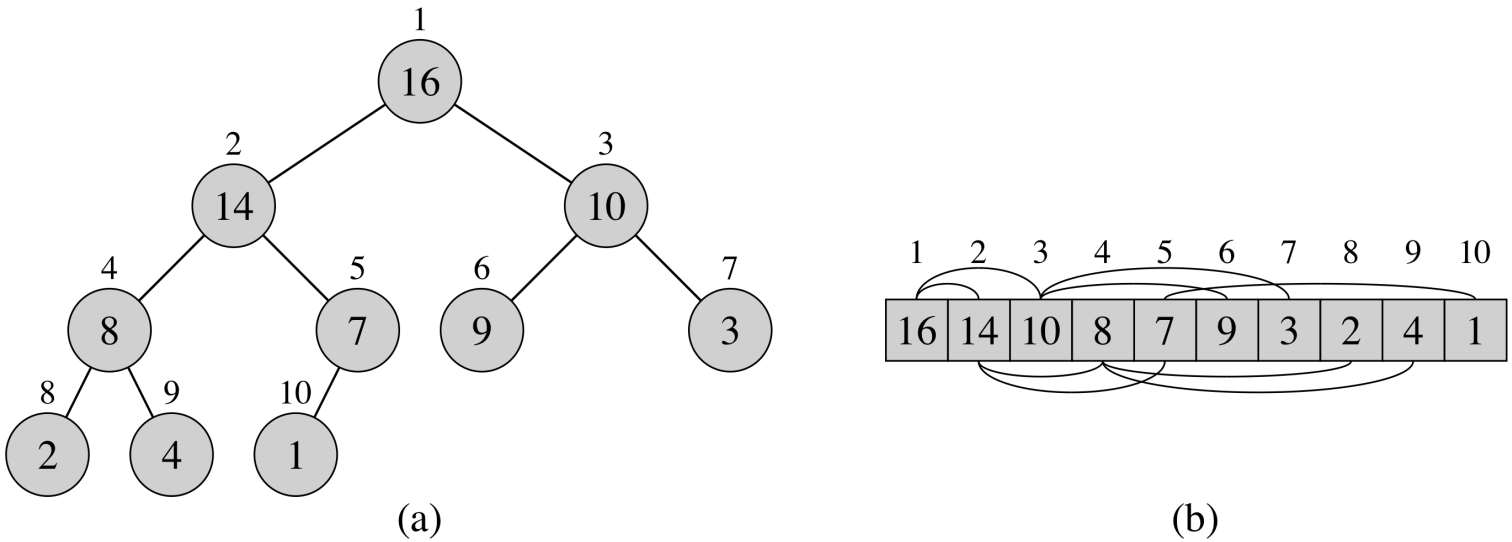

Figure : Heap: two views

Heapify: The idea

Question: How to build a Heap ?

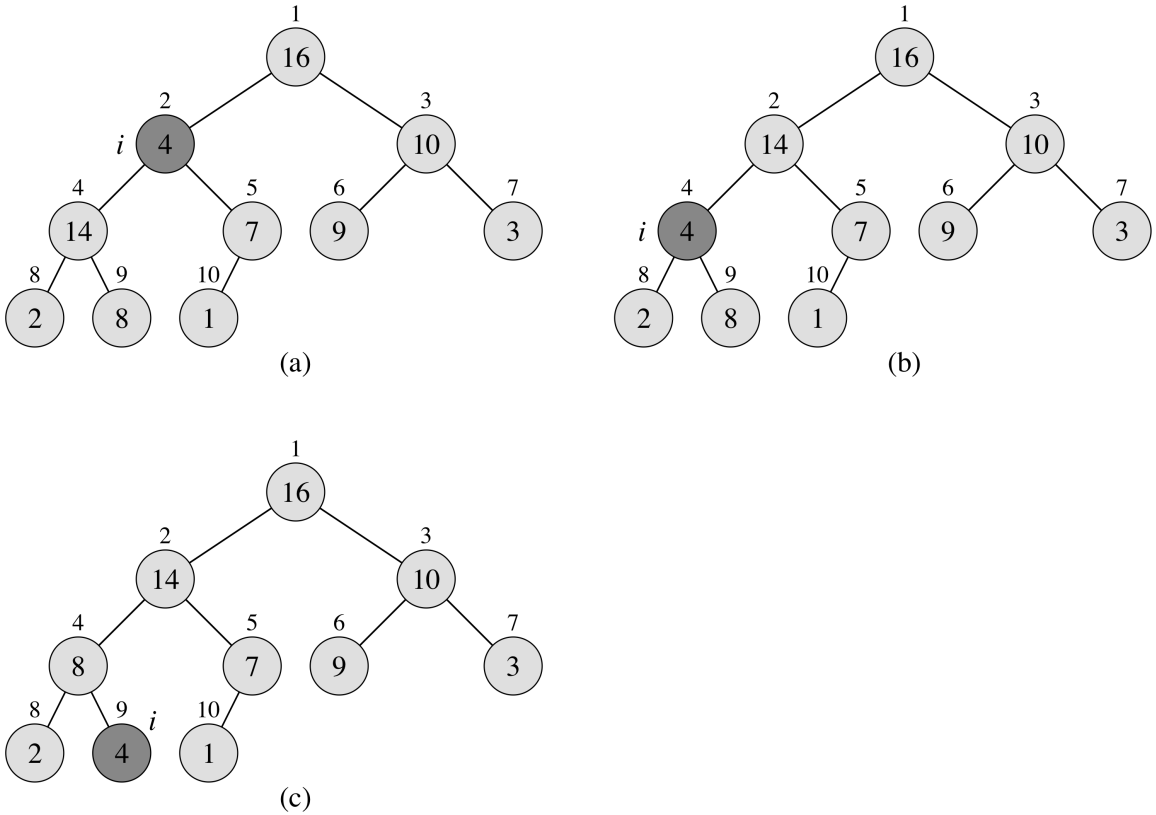

Figure : Running `MAX-HEAPIFY`

MAX-HEAPIFY algorithm

```
MAX-HEAPIFY(A, i, n)
  l = LEFT(i)
  r = RIGHT(i)
  if l ≤ n and A[l] > A[i]
      largest = l
  else largest = i
  if r ≤ n and A[r] > A[largest]
      largest = r
  if largest ≠ i
      exchange A[i] with A[largest]
      MAX-HEAPIFY(A, largest, n)
```

Figure : Pseudocode for `Max-Heapify`

Building a Heap: Main Ideas I

1. What are the array indices for leaves (resp. non-leaves) ?
2. Works from bottom and goes up. Why ?

Building a Heap: Main Ideas II

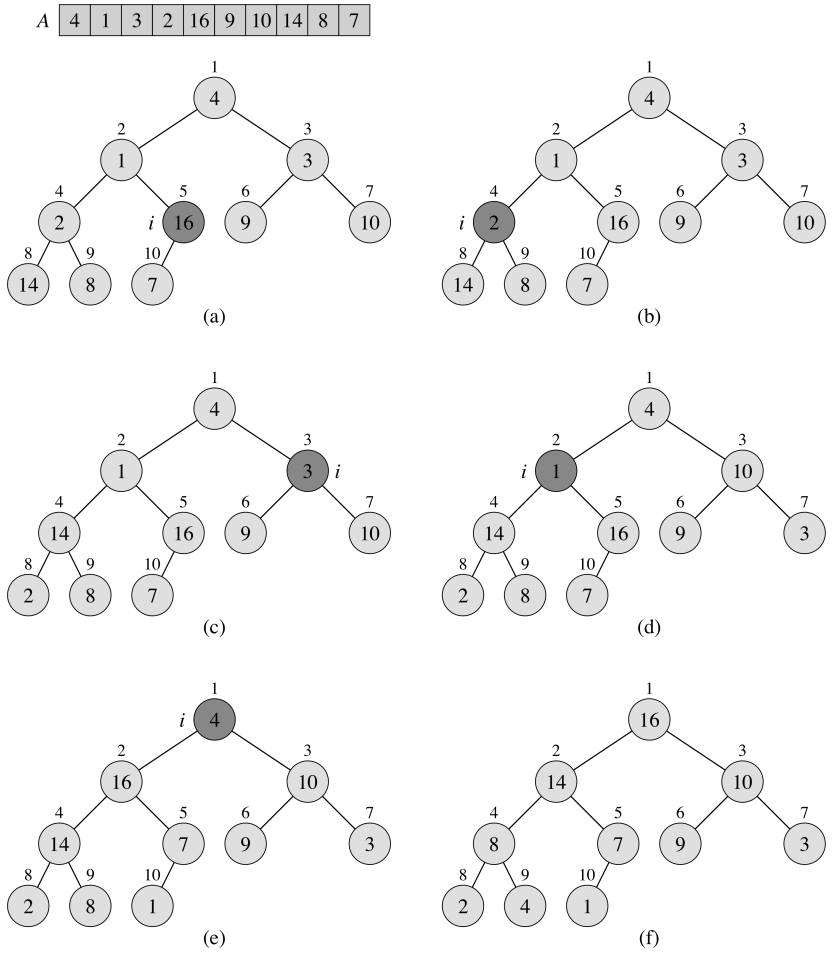

Figure : BuildHEAP

Algorithm BUILD-MAX-HEAP

BUILD-MAX-HEAP(A, n)

for $i = \lfloor n/2 \rfloor$ **downto** 1

MAX-HEAPIFY(A, i, n)

Figure : The BUILD-MAX-HEAP procedure

Heap Sort: Ideas

We can make use `BUILD-MAX-HEAP` and `MAX-HEAPIFY` for sorting an array A in ascending order by "thinking inductively".

Some thoughts ...

1. In a heap, the largest element is always stored at $A[1]$
2. `BUILD-MAX-HEAP` is supported by running `MAX-HEAPIFY`. Does it run fast enough ? (Note: We want to acheive $\Theta(n \, lg \, n)$ time.)

Discussions ....

The Heap Sort Algorithm

HEAPSORT(A, n)

  BUILD-MAX-HEAP(A, n)
  for $i = n$ **downto** 2
    exchange $A[1]$ with $A[i]$
    MAX-HEAPIFY$(A, 1, i-1)$

Figure : The Heapsort Algorithm

Analyzing Heap Sort

- It calls `BUILD-MAX-HEAP` once and `MAX-HEAPIFY` $(n-1)$ times
- It is an $\Theta(n \lg n)$ algorithm since
  1. `MAX-HEAPIFY` runs in $\Theta(\lg n)$ time.
     (Why? See page 155-156)
  2. `BUILD-MAX-HEAP` runs in $\Theta(n)$ time.
     (Why? See page 157-159)

Implementing a Priority Queue via a Heap

Reading: Priority Queues CLRS Section 6.5

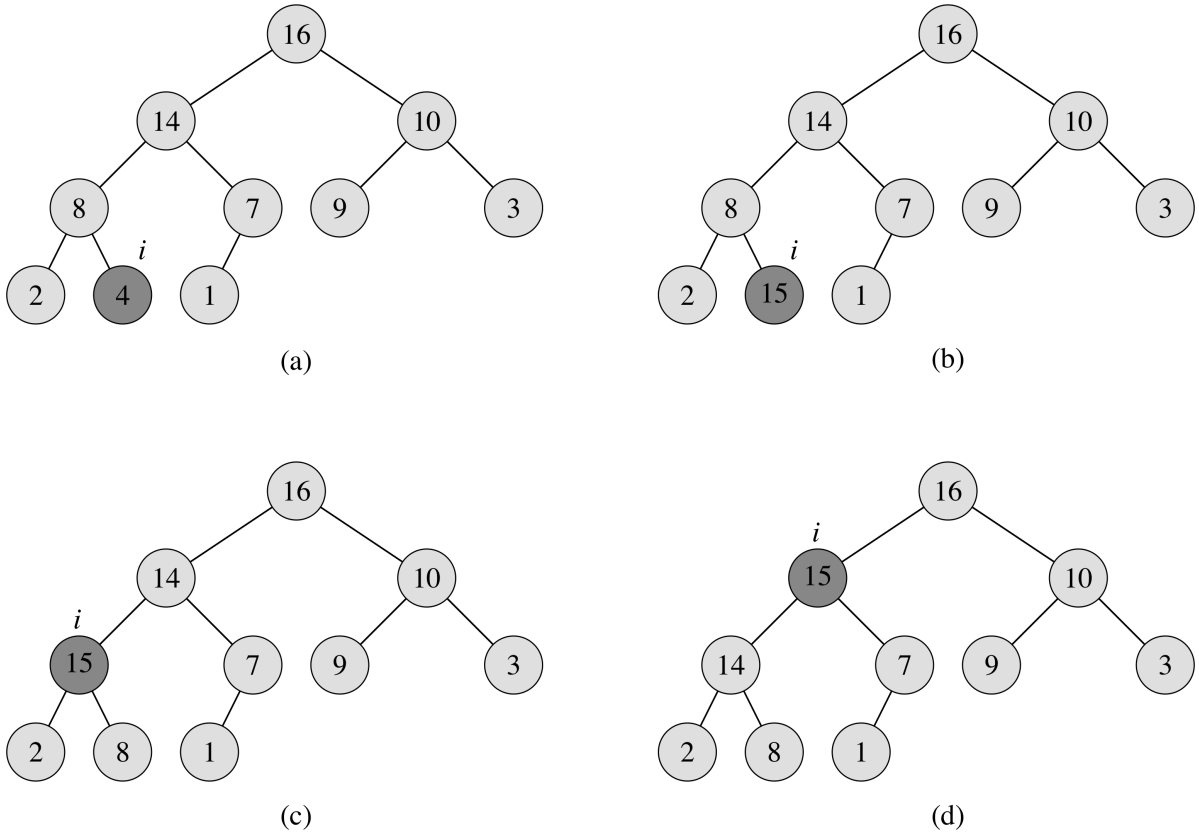

Figure : Example: A Maintenance Step for a Priority Queue