Math Review & Python Intro

CIS 600, Spring 2018

January 18, 2018

Agenda

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- Math - sets, functions, matrices & vectors

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- Python, the language, its features & distributions, session, programs & packages

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- Math - sets, functions, matrices & vectors
- Python, the language, its features & distributions, session, programs & packages
- Vim (with LaTeX)

Math - Notation

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- It is sometimes acceptable to present a set in this way.

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- The *empty set* $\emptyset$ is a special set! More on that later...

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- This is read "A complement".

- We have enough notation now to express *DeMorgan's Laws*. If U and V are sets, then

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- This is quite terse. How can we express this in natural language?

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- With our exmamples, $S \times F$ has such elements as $(\pi, 0)$.
- What are some other elements of $S \times F$?

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- The powerset of a set A is also denoted 2^A.
- Can you give a motivation for this alternative notation?

Math - Special Sets

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- The rational numbers, $\mathbb{Q}$
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- The integers, $\mathbb{Z}$
- The real coordinate space, $\mathbb{R}^n$ (of dimension n)

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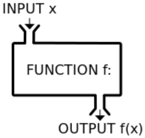
- A function f from *domain* A to *codomain* B is a rule assigning an element of B to any element of A.



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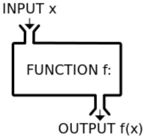

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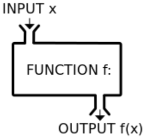

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- We can evaluate f at any element $a \in A$ to get its *value* or *output* $f(a)$. This is read "f of a".

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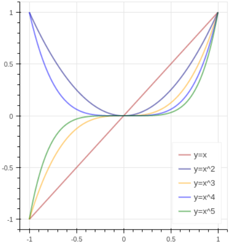

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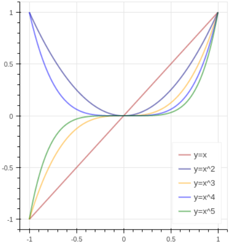
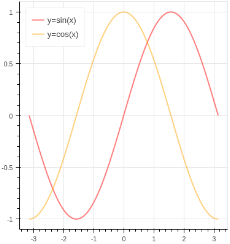


- Trig functions



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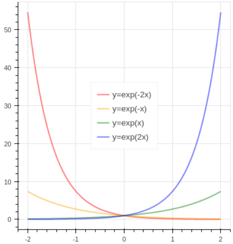

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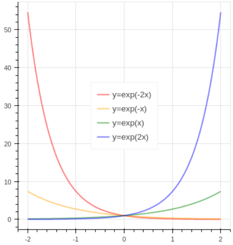

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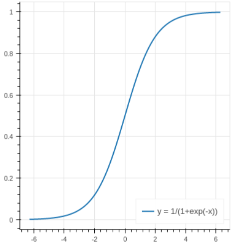

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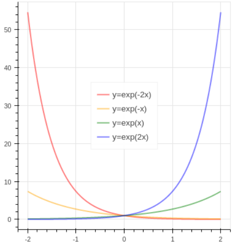

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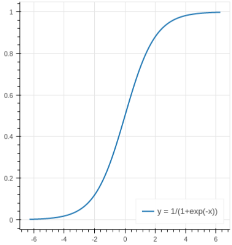

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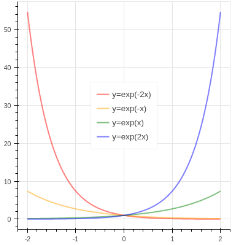

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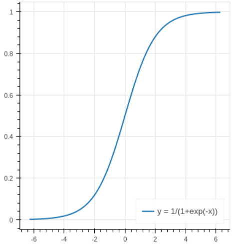

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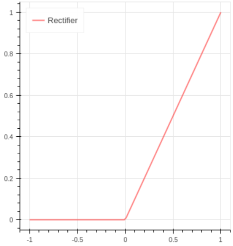

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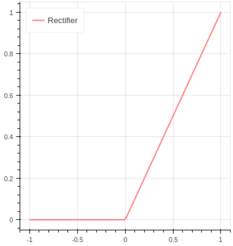
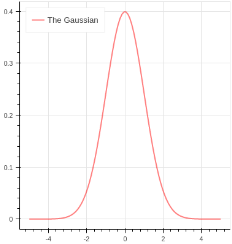


- Gaussians



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- Example

$$\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} \in \mathbb{R}^3$$

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- The entry in the i^{th} row and j^{th} column of the matrix A is denoted $A_{i,j}$. What is $A_{2,2}$?

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- This means that matrix multiplication is a *function*!

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- Let's find some other eigenvectors.

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- A *cumulative distribution function* gives the probability of all univariate outcomes up to and including the given value.

Example