Solutions for Assignment 5:

1. Beiser, Chap. 3, p.118, Prob. 30: Discuss the prohibition of $E = 0$ for a particle trapped in a box L wide in terms of the uncertainty principle. How does the minimum momentum of such a particle compare with the momentum uncertainty required by the uncertainty principle if we take fix $\Delta x = L$?

Ans: The particle is free to move in the coordinate space from $x = 0$ to $x = L$. Thus, maximal uncertainty in the measurement of x, $\triangle x = L$. Corresponding uncertainty in some momentum is then

$$\triangle p \geq \frac{\hbar}{2L}$$

and in kinetic energy ,

$$\triangle E \geq \frac{\hbar^2}{8mL^2}$$

If the particle could have an energy $E = 0$, we cannot measure it with an uncertainty less than $\frac{\hbar^2}{8mL^2}$, which is the energy for $n = 1$. Thus it makes no sense to talk about the probability of the particle having energy $E = 0$.

We know otherwise, that if $E = 0$, the wave function corresponds to n =0 and then it vanishes.

2. Beiser, Chap. 3, p.118, Prob. 31: The atoms in a solid possess a certain minimum zero-point energy even at 0 K, while no such restriction holds for the molecules in an ideal gas. Use the uncertainty principle to explain these statements.

Ans: In the solid, the displacement of an atom from its equilibrium position cannot be larger than a reasonably small value; the atom behaves like a particle in a box. Thus, the system of atoms forming a solid must have a non zero uncertainty in measurement of position and hence a non zero momentum and correspondingly zero point energy. Since the ideal gas consists of particles free to move all over the system,the uncertainty in measurement of momentum and hence kinetic energy and their magnitudes may be zero.Thus no restriction of zero point energy is there for the molecules in ideal gas.

3. Beiser: Chap. 3, p.118, Prob. 34: (a) How much time is needed to measure the kinetic energy of an electron whose speed is 10.0 m/s with an uncertainty of no more than 0.100 percent? How far will the electron have traveled in this period of time? (b) Make the same calculations for a 1.00-g insect whose speed is the same. What do these sets of figures indicate?

Ans: a) $v = 10m/s$

$$\triangle v = 0.1\% \text{ of } 10\text{m/s}$$

$$\implies \triangle v = \frac{0.1}{100} \times 10\text{m/s} = 0.01\text{m/s}$$

$$\therefore \triangle p \leq m_e \times 0.01\text{m/s} = 9.1 \times 10^{-33}\text{kg.m/s}$$

corresponding uncertainty in kinetic energy

$$\triangle E_{\text{kinetic}} = \frac{(\triangle p)^2}{2m} = 4.55 \times 10^{-35}\text{J}$$

Using the energy - time uncertainty in kinetic energy,we get

the uncertainty in measurement of time

$$\triangle t \geq \frac{\hbar}{2\triangle E} \geq \frac{6.6 \times 10^{-34}\text{J.s}}{2\pi \times 2 \times 4.55 \times 10^{-35}\text{J}} \approx 1\text{s}$$

This is the minimum time required to measure the kinetic energy.

The distance moved by the electron in this time$\approx$ 10m.

b) For the insect, the quantities will change due to the difference in mass by the ratio $\frac{m_e}{m_{insect}} = 9.1 \times 10^{-28}$kg

$$\triangle E_{\text{kinetic}} = \frac{10^{-7}}{2}$$

$$\triangle t \geq \frac{\hbar}{2\triangle E} \approx 10^{-27}\text{s}$$

and the distance traveled $\approx 1.05 \times 10^{-27}$m

4. Beiser, Chap. 7, p.264, Prob. 4: In superconductivity, which occurs in certain materials at very low temperatures, electrons are lnked together in Cooper pairs by their interaction with the crystal lattices of the materials. Cooper pairs do not obey the exclusion principle. What aspect of these pairs do you think permits this?

Ans: The Cooper pairs act as individual particles with integral spins. In the process of formation of the pair, the half integral spins of electrons add and the resultant is an integer spin.These pairs do not have to follow the antisymmetry principle, since they are not fermions. Thus, they do not obey the exclusion principle.

5. Beiser, Chap. 8, p.294, Prob.1: The energy needed to detach the electron from a hydrogen atom is 13.6 eV, but the energy needed to detach an electron from a hydrogen molecule is 15.7 eV.. Why do you think the latter energy is greater? ,

Ans: In the formation of H_2molecule, the two H atoms gain a binding energy of 4.2eV.In the ground state,the electronic energy=$-2 \times 13.6 - 4.2 = -31.4$eV. After ionization, we are left with one free H atom,whose electronic energy=-13.6eV. Therefore Ionization energy of H_2molecule $= -31.4 - (-13.6) = 15.7$eV.

6. Beiser, Chap. 8, p.294, Prob.9: The rotational spectrum of HCl contains the following wavelengths:

$$12.03X10^{-5} \text{ m}$$

$$9.60X10^{-5} \text{ m}$$

$$8.04X10^{-5} \text{ m}$$

$$6.89X10^{-5} \text{ m}$$

$$6.04X10^{-5} \text{ m}$$

If the isotopes involved are ^{1}H and ^{35}Cl, find the distance between the hydrogen and chlorine nuclei in an HCI molecule.

Ans:The rotational energy levels are

$$E_j = \frac{j(j+1)\hbar^2}{2I} \quad \text{in energy units}$$

$$= \frac{j\,(j+1)\,\hbar^2}{2I\,(hc)} \quad \text{in inverse length units}$$

$$\text{Let} \quad \text{us write} \quad B = \frac{\hbar^2}{2Ihc} \quad \text{then}$$

$$\triangle E_{j+1\leftarrow j} = B\left[(j+1)\,(j+2) - j\,(j+1)\right]$$

$$= (j+1)\,2B \left(\text{in} \quad m^{-1}\right)$$

Thus the lines in the rotational spectrum will appear at $2B(j = 1 \leftarrow j = 0)$; $4B(j = 2 \leftarrow j = 1)$; $4B(j = 3 \leftarrow j = 2)$ etc., The difference between consecutive lines is always $2B$. Thus, we can take the average of data given and obtain average value as $2061m^{-1}$

$$\implies 2B = 2061m^{-1}$$

$$\frac{\hbar^2}{2I}\frac{1}{hc} = \frac{2061}{2}m^{-1}$$

$$\frac{h}{8\pi^2 Ic} = \frac{h}{8\pi^2 \mu R^2 c} = \frac{2061}{2}$$

$$\implies R^2 = \frac{h}{8 \times 2061 \times \pi^2 \times \mu \times c}$$

$$\mu = \frac{35 \times 1}{36 \times 6.023 \times 10^{23}} \implies R^2 = 1.67 \times 10^{-20} m^2$$

$$R = 1.29 \times 10^{-10}\text{m} = 0.129\text{nm}$$