ECE 230 : Probability and Random Processes

Assignment 5

Deadline : November 14, 2011

1. Let $\{X_n\}$ be a sequence of i.i.d. normal random variables with mean m and variance σ^2. Find the mean and the variance of the 'sample variance' $V_n = \frac{1}{n}\sum_{j=1}^{n} (X_j - M_n)^2$ where $M_n = \frac{1}{n}\sum_{j=1}^{n} X_j$. Evaluate the asymptotic values of these (as $n \to \infty$) . What do you conclude?

2. Let $\{X_k\}$ be a sequence of i.i.d. random variables with uniform p.d.f. over [0 1]. Let $Y_k = \frac{1}{n}\sum_{j=0}^{n-1} X_{k+j}$ be a moving average. Find and plot $E[Y_k\, Y_{k+j}]$ as a function of j.

3. With $\{X_k\}$ as defined above in Q2 , let $Z_1 = X_1$ and $Z_k = (1-a)Z_{k-1} + a\,X_k$ for $k \geq 2$ and $0 < a < 1$. Z_k is called an autoregressive process. Find and plot the normalized autocorrelation coefficient $\rho(i) = \frac{Cov(Z_k, Z_{k+i})}{\sqrt{Var(Z_k)}\sqrt{Var(Z_{k+i})}}$ for large k and $i = 0, \pm 1, \pm 2, \ldots$

4. Find the first order characteristic function of a) Poisson process b) Wiener – Levy process

5. Let S = {H, T}, P(H) = P(T) = ½ , $\Gamma = \{\, t : t \geq 0 \}$. Let $X(s,t) = \sin(\pi t)$, if s = H, and $X(s,t) = 2t$, if s = T. Find E[X(t)], $F_{X(t)}(x,t)$ for t = 0.25, t = 0.5 and t = 1.

6. A and B are independent normal random variables with zero means and variances σ^2. Let X(t) = A – Bt. Calculate the probability that X(t) crosses the t – axis in the interval (0 T)

(P.T.O)

7. If X(t) is a WSS Gaussian stochastic process $E[X(t)] = m_x$ and $R_x(\tau) = \exp[-\alpha|\tau|]$, find
   a. $f_{X(t)}(x,t)$
   b. $f_{X(t_1)X(t_2)}(x_1,t_1;x_2,t_2)$
   c. $\varphi_{X(t_1)X(t_2)}(u_1,t_1;u_2,t_2)$ (second order joint characteristic function)
   d. n-th order joint characteristic function of X(t) process

8. For each of the correlation functions of a stationary stochastic process X(t)
   a. $R_x(\tau) = (1+\alpha|\tau|)\, e^{-\alpha|\tau|}$
   b. $R_x(\tau) = [\cos\omega|\tau| + \frac{\alpha}{\omega}\sin\omega|\tau|]\, e^{-\alpha|\tau|}$, find whether $Y(t) \triangleq \frac{dX(t)}{dt}$ exists. If yes is your answer, then find
      i. $R_y(\tau)$
      ii. $R_{xy}(\tau)$
      iii. $R_{yx}(\tau)$
      iv. $S_x(f)$
      v. $S_y(f)$
      vi. $S_{xy}(f)$
      vii. $S_{yx}(f)$