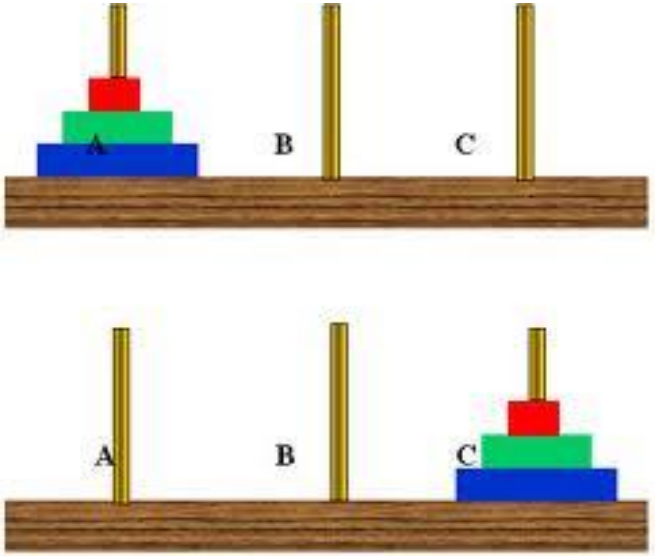
Linear Recurrence Relations

[AnilKishore]

IIIT, Hyderabad, India

Towers of Hanoi (TOH)



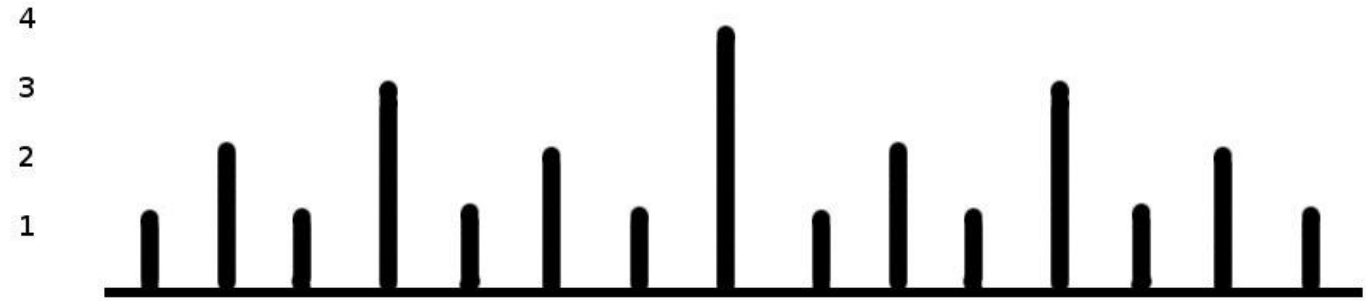
$T(n) = 2\ T(n – 1) + 1$

What is the size of the disk moved in Kth step ?

n = 4

Disk moved in Kth step : 1 2 1 3 1 2 1 4 1 2 1 3 1 2 1

Ruler Markings



Height: 1 2 1 3 1 2 1 4 1 2 1 3 1 2 1

Height of Marking at Kth point

= Size of Disk moved in Kth step in TOH :)

Lowest Set Bit

3 bit numbers :	3 2 1	
	0 0 1	1
	0 1 0	2
	0 1 1	1
	1 0 0	3
	1 0 1	1
	1 1 0	2
	1 1 1	1

: Position of the Lowest Set Bit

Position (1-based) of lowest set bit in K
= Height of Marking at Kth point on the Ruler :)
= Size of Disk moved in Kth step in TOH :) :)

Find the Linear Recurrences

- Simpler Josephus Problem: Every 2^{nd} person is killed
  Idea: Consider odd & even cases separately.

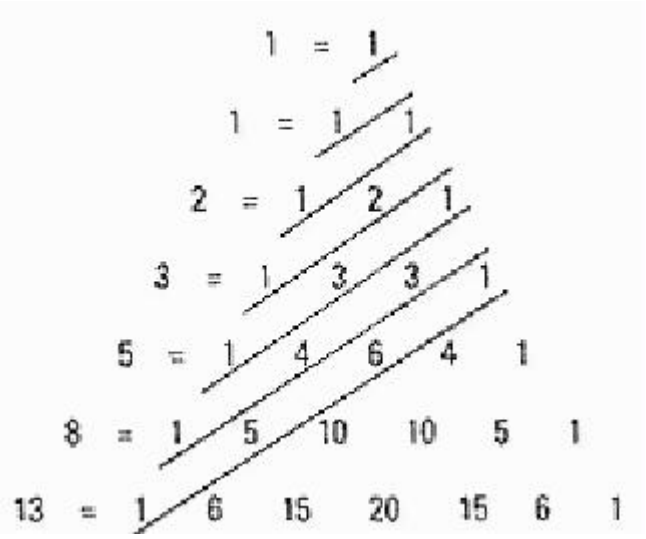
- Given points on each side of an N-gon , how many K-gons can be formed using those points.
  Idea: Consider the sides in order and try taking 0,1,2 points From each side.

- How many ways to partition an integer N in to exactly K parts
  Idea: (1 + 2 + 4) = 7 is same as (1 + 4 + 2) = 7. So consider x1 + x2 + . . .+ xK = N , Xi's in sorted order, and try the two cases, Xi is zero and Xi is non-zero

Fibonacci Sequence

F(0) = 0
F(1) = 1
F(n) = F(n-1) + F(n-2) : n > 1

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55 . . . . . .



Pascals Triangle

- Given M (< 2*10^9) & a large N (< 10^18) , Find F(N) mod M

- Given M (< 2*10^9) & a large N (< 10^18) ,
  Find (F(1) + F(2) + F(3) … F(N)) mod M

Try these simple Problems on SPOJ.
All you need is matrix power code, and can solve them all using it easily, once you construct the matrix A

http://www.spoj.pl/problems/REC/

http://www.spoj.pl/problems/RABBIT1/

http://www.spoj.pl/problems/SEQ/

http://www.spoj.pl/problems/SPP/

TC SRM 397- Div1 Lev 2 ： SumOfPowers

You are given ints **n** and **k**. Return the value of the sum $1^{\mathbf{k}} + 2^{\mathbf{k}} + 3^{\mathbf{k}} + \ldots + \mathbf{n}^{\mathbf{k}}$ modulo 1000000007.

Constraints

- **n** will be between 1 and 10^9, inclusive.
- **k** will be between 1 and 50, inclusive.

Uhh!! Last Slide :)

Simpler version of TC SRM 428 – Div1 Lev2

- How many strings on length N can be made with at most K distinct letters chosen from 'a' to 'z'

- 1 < = N < = 10^9 & 1 < = K < = 26

References:

http://www.topcoder.com/tc?module=Static&d1=features&d2=010408

http://forums.topcoder.com/?module=Thread&threadID=643404&start=0

Thats it !