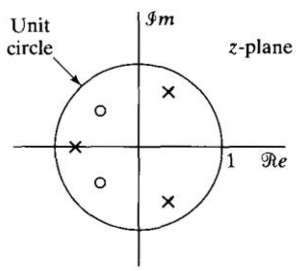
Digital Signal Processing

Assignment – 4

Deadline : 13th April, 2012.

1. If the system function H(z) of a LTI system has a pole-zero diagram as shown below and the system is causal, can the inverse system $H_i(z)$, where $H(z)H_i(z) = 1$, be causal and stable ? Justify your answer.



[5 marks]

2. A discrete time causal LTI system has the system function **[10 marks]**

$$H(z) = \frac{(1 + 0.2z^{-1})(1 - 9z^{-2})}{(1 + 0.81z^{-2})}.$$

(a) Is the system stable ?

(b) Find expressions for a minimum-phase system $H_1(z)$ and an all pass system $H_{ap}(z)$ such that $H(z) = H_1(z)H_{ap}(z)$.

3.(a) Determine the group delay for $0 < \omega < \pi$ for **[10 marks]**

$$x_1[n] = \begin{cases} n - 1, & 1 \le n \le 5, \\ 9 - n, & 5 < n \le 9, \\ 0, & \text{otherwise.} \end{cases}$$

(b) For the following system function H(z), specify a minimum-phase system function $H_{min}(z)$ such that the frequency-response magnitudes of the two systems are equal.

$$H\ (z) = \frac{(1 - 3z^{-1})\left(1 - \frac{1}{4}z^{-1}\right)}{\left(1 - \frac{3}{4}z^{-1}\right)\left(1 - \frac{4}{3}z^{-1}\right)}$$

4. (a) Suppose we design a discrete-time filter using the impulse invariance technique with an ideal continuous-time low pass filter as a prototype. The prototype filter has a cutoff frequency of Ω_c = $2\pi(1000)$ rad/s, and the impulse invariance transformation uses T = 0.2 ms. What is the cutoff frequency ω_c for the resulting discrete-time filter?

(b) The bilinear transformation is used to design an ideal discrete-time lowpass filter with cutoff frequency $\omega_c = 3\pi/5$ from an ideal continuous-time lowpass filter with cutoff frequency $\Omega_c = 2\pi(300)$ rad/s. Given a choice for the parameter T that is consistent with this information. Is this choice unique ? If not, give another choice which is consistent with the information. **[10 marks]**

5. We wish to use the Kaiser window method to design a discrete-time filter with generalized linear phase that meets specifications of the following form : **[15 marks]**

$$\begin{aligned} |H(e^{j\omega})| &\le 0.01, & 0 &\le |\omega| \le 0.25\pi, \\ 0.95 \le |H(e^{j\omega})| &\le 1.05, & 0.35\pi &\le |\omega| \le 0.6\pi, \\ |H(e^{j\omega})| &\le 0.01, & 0.65\pi &\le |\omega| \le \pi. \end{aligned}$$

(a) Determine the minimum length (M+1) of the impulse response and the value of the Kaiser window parameter β for a filter that meets the preceding specifications.

(b) What is the delay of the filter?

(c) Determine the ideal impulse response $h_d[n]$ to which the Kaiser window should be applied.