Probabilistic Boolean Logic

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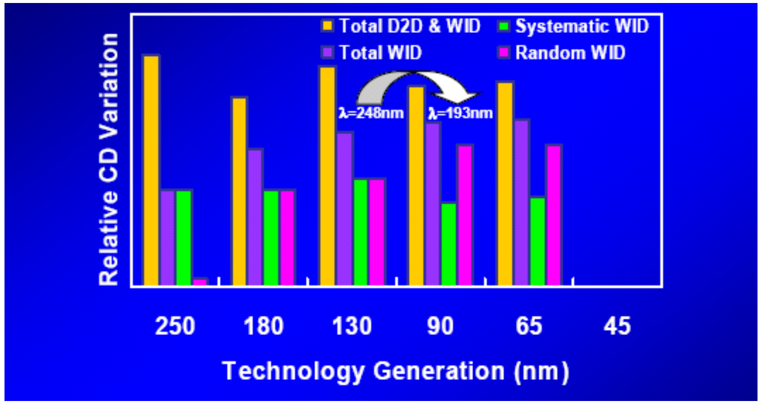
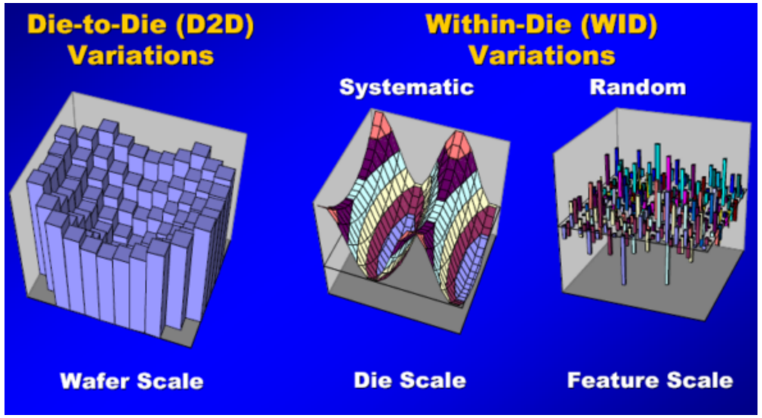
Impact

- A novel Probabilistic Boolean Logic(PBL)
- Validation of PBL through 0.18 μm CHRT(Chartered Semiconductor) technology fabrication.
- Using PBL to implement a ultra-low energy Hyper-Encryption system in CHRT
  - –205 times more efficient through the energy-performance product metric over a conventional design
- Probabilistic arithmetic for ultra low energy signal processing
  - –A FIR used in H-264 realized with half the energy and negligible performance degradation
    - Simulated using HSPICE and software models

The Impediments to Moore's Law



Gate Length Variations*

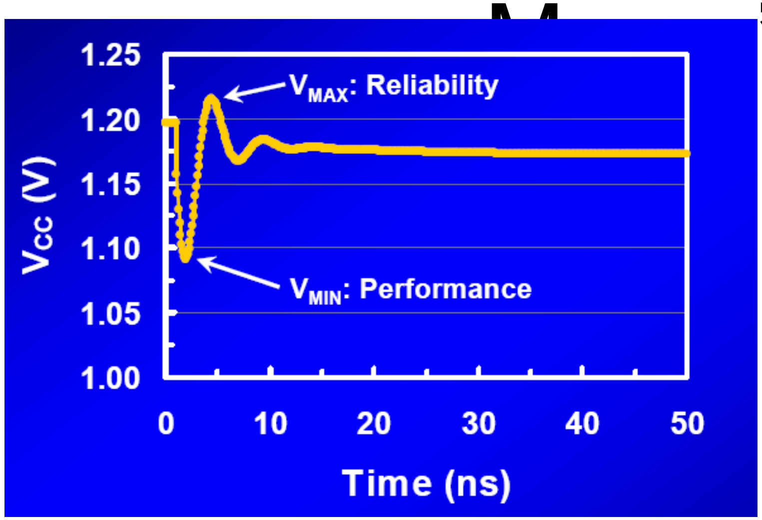


Variations*

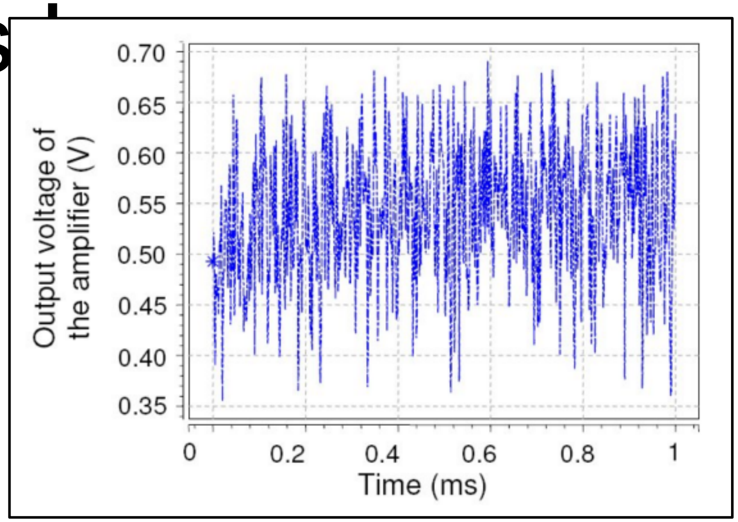
- Decreasing feature size
  - –Shrinking design margins
- Static variations
  - –Channel length, width
  - –Threshold voltage
  - –Interconnect (depth of focus)
  - –Dopant fluctuation

*Courtesy Dr Keith Bowman, Intel Corporation

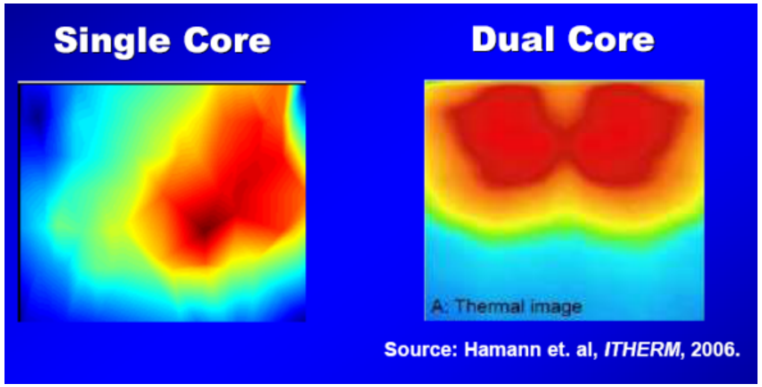
Dynamic Variations as Impediments to



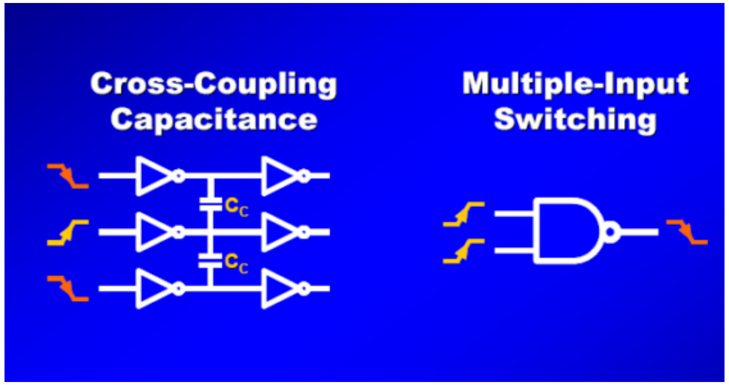
Supply voltage variations*



Thermal noise (amplified)



Temperature variations

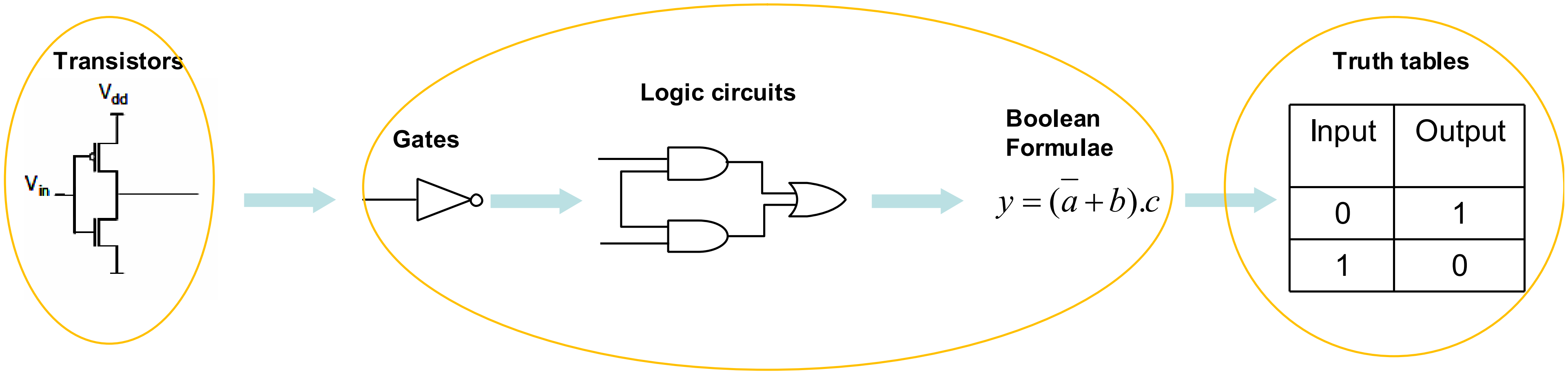


Other dynamic variations*

*Courtesy Dr Keith Bowman, Intel Corporation

Abstraction in Design

- Abstractions have been used to design and automate the design of complex systems

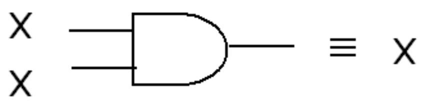


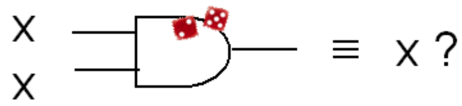
Technology

Structural Abstraction

Behavioral Abstraction

Probabilistic Boolean Logic





- Boolean Logic captures ***"rules of thumb"*** on input and output behaviors (functionality)
- Probability captures ***"uncertainty"***
- Operators are "correct" with a probability p
  - –Operators are subscripted with p
- "Incorrect" with probability (1-p)
- Thermal noise, power supply noise and other dynamic perturbations
  - Does not depend on the source of perturbation
    - » Only on statistics of the source which determines p

Input		Output $x \wedge_p y$	
x	y	0	1
0	0	p	1-p
0	1	p	1-p
1	0	p	1-p
1	1	1-p	p

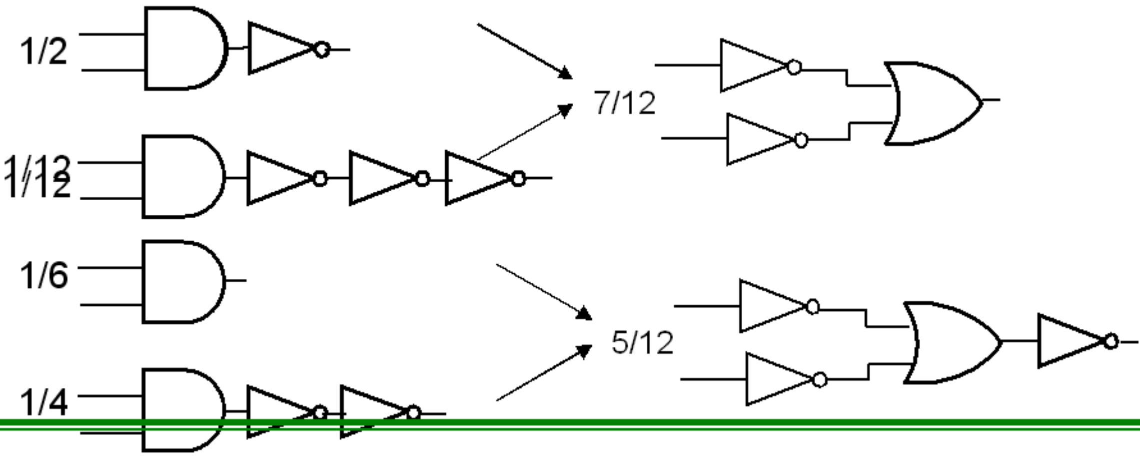
Considering More Complex Formulae

Consider the circuit representation of a probabilistic Boolean formula

Probabilistic De-Morgan's Law

AND	NOT	P(OUT)
Correct	Correct	2/3* 3/4
Wrong	Wrong	1/3*1/4
Correct	Wrong	2/3*1/4
Wrong	Correct	1/3*3/4

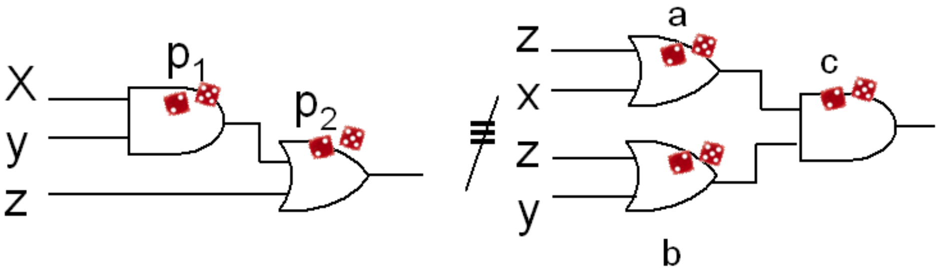
The obvious simplification



OR	P(OUT)
Correct	7/12
Wrong	5/12

Properties of PBL

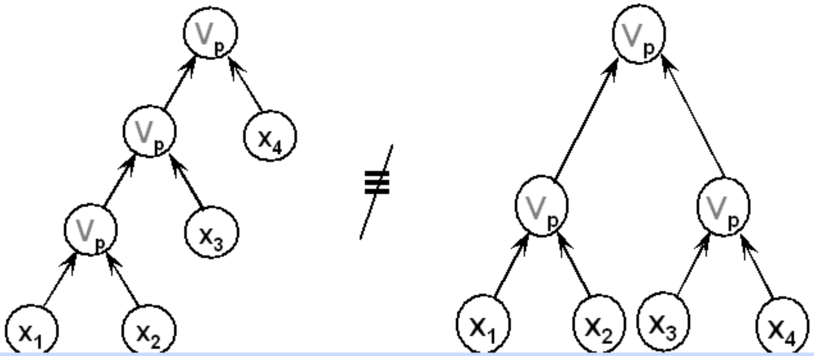
Probabilistic Distributivity



Theorem: *Probabilistic Boolean Logic* is not distributive

Probabilistic Associativity

$(((x_1 \vee_p x_2) \vee_p x_3) \vee_p x_4)$ $\qquad$ $((x_1 \vee_p x_2) \vee_p (x_3 \vee_p x_4))$



Theorem: *Probabilistic Boolean Logic* is not associative

Identities Preserved

1. Commutativity	$(x \vee_p y) \equiv (y \vee_p x)$ $(x \wedge_p y) \equiv (y \wedge_p x)$
2. Double Complementation	$\neg_q(\neg_p x) \equiv \neg_p(\neg_q x)$ $\neg_p 0 \equiv \neg_1(\neg_p 1)$ $\neg_p 1 \equiv \neg_1(\neg_p 0)$
3. Operations with 0 and 1	$(0 \wedge_p x) \equiv (\neg_p 1)$ $(1 \wedge_p x) \equiv \neg_1(\neg_p x)$ $(0 \vee_p x) \equiv \neg_1(\neg_p x)$ $(1 \vee_p x) \equiv (\neg_p 0)$
4. Identity	$(x \vee_p x) \equiv \neg_1(\neg_p x)$ $(x \wedge_p x) \equiv \neg_1(\neg_p x)$
5. Probabilistic Tautology	$(x \vee_p (\neg_1 x)) \equiv \neg_p 0$ $(x \wedge_p (\neg_1 x)) \equiv \neg_p 1$
6. Probabilistic DeMorgan Identity	$\neg_p(x \vee_q y) \equiv (\neg_1 x) \wedge_r (\neg_1 y)$ $\neg_p(x \wedge_q y) \equiv (\neg_1 x) \vee_r (\neg_1 y)$ where $r = pq + (1-p)(1-q)$

Thank You