Digital Signal Processing

Assignment 3

Deadline: 16th March, 2012.

1. Consider the following difference equation: **[15 marks]**

$$y[n] - 0.25\ y[n-1] - 0.125\ y[n-2] = 3\ x[n].$$

(a) Determine the general form of the homogeneous solution to this difference equation.

(b) Both a causal and an anticausal LTI system are characterized by this difference equation. Find the impulse responses of the two systems.

(c) Show that causal LTI system is stable and the anticausal LTI system is unstable.

(d) Find a particular solution to the difference equation when $x[n] = (1/2)^n u[n]$.

2. (a) Find the frequency response $H(e^{jw})$ of the linear time-invariant system whose input and output satisfy the difference equation **[10 marks]**

$$y[n] - 0.5\ y[n-1] = x[n] + 2x[n-1] + x[n-2].$$

(b) Write a difference equation that characterizes a system whose frequency response is

$$H(e^{jw}) = (1 - 0.5e^{-jw} + e^{-j3w})(1 + 0.5e^{-jw} + 0.75e^{-j2w})^{-1}.$$

3. The input to a causal linear time-invariant system is **[10 marks]**

$$x[n] = u[-n-1] + (0.5)^n u[n].$$

The z-transform of the output of this system is

$$Y(z) = (-0.5z^{-1}).[(1-0.5z^{-1})(1+z^{-1})]^{-1}.$$

(a) Determine H(z), the z-transform of the system impulse response. Sketch the pole-zero plot and shade the region of convergence.

(b) What is the region of convergence for Y(z) ?

(c) Determine y[n].

4. (a) If $H(z) = (1-0.25\ z^{-2})^{-1}$ and $h[n] = A_1\alpha_1^n u[n] + A_2\alpha_2^n u[n]$, determine the values of A_1, A_2, α_1 and α_2.

(b) For the following pair of input and output z-transforms X(z) and Y(z), determine the region of convergence for the system function H(z): **[15 marks]**

$X(z) = (1 - 0.75z^{-1})^{-1} \qquad |z| > 0.75$

$Y(z) = (1 + 2/3\ z^{-1})^{-1} \qquad |z| > 2/3$

(c) Consider a sequence x[n] for which the z-transform is

$$X(z) = \frac{\frac{1}{3}}{1 - \frac{1}{2}z^{-1}} + \frac{\frac{1}{4}}{1 - 2z^{-1}}$$

and for which the region of convergence includes the unit circle. Determine x[0] using the initial value theorem.