ECE 230 : Probability and Random Processes

Assignment 6

Deadline: November 24, 2011

1. If a process $X(t)$ is stationary and differentiable with derivative $X'(t)$, show that for a given t, the random variables $X(t)$ and $X'(t)$ are orthogonal and uncorrelated.

2. X(t) is a WSS process with E[X(t)] = 1, $R_x(\tau) = 1 + e^{-2|\tau|}$. Find the mean (ensemble average) and variance for random variable Y, $Y \triangleq \int_0^1 X(t)dt$

3. A Gaussian random process X(t) has mean E[X(t)] = m and auto-covariance $e^{-\alpha|\tau|}$, $\alpha > 0$
   Let $M_T \triangleq \frac{1}{2} \int_{-T}^{T} X(t)dt$
   a. Is X(t) ergodic in the mean.
   b. Find the probability $P[|M_T - m| \leq \varepsilon]$
   c. How large should T be chosen so that $P[|M_T - m| \leq 0.1]$ is not less than 0.95 ?

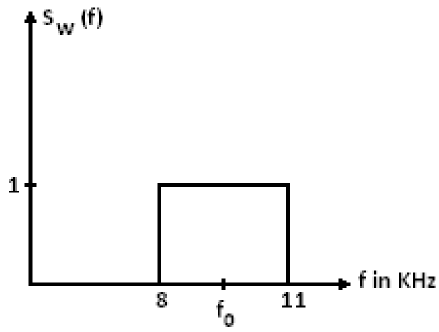
4. If the ACF $R_x(\tau)$ of a stochastic process is given by
   a. $R_x(\tau) = e^{-2\lambda|\tau|}$, $\forall\, \tau$
   b. $R_x(\tau) = \begin{cases} 1 - \frac{|\tau|}{T}, & |\tau| < T \\ 0, & else\ where \end{cases}$

   Find $S_x(f)$ in each case.

5. A zero mean Gaussian process X(t) with ACF $R_x(\tau)$ is applied as the input to a square law detector, $Y(t) = X^2(t)$. Find $f_{Y(t)}(y,t)$, $E\,[Y(t)]$, $R_y(t+\tau, t)$. Is $Y(t)$ WSS? If so, find $S_y(f)$.

(P.T.O)

6. The p.s.d $S_W(f)$ of a zero mean stochastic process $W(t) = X(t)\cos 2\pi f_0 t + Y(t)\sin 2\pi f_0 t$ is shown below (only positive frequency part is shown). Find $S_x(f)$, $S_y(f)$, $S_{yx}(f)$ and sketch them.



$f_0 = 10$ KHz

7. The output X(t) of a bi-stable multi-vibrator switches randomly between ± 1 volts every T seconds and is fed to a delay line with a random delay T_d that is uniformly distributed over [0 T] and is independent of X(t). The output Y(t) of the delay line is applied to a low pass RC filter to yield the output Z(t). Let W(t) = Z(t) – Y(t). Find the power spectrum $S_W(f)$ of W(t) and the variance of W(t)