Digital Signal Processing

Assignment 2

Deadline: 2nd March, 2012

1. Find the density function of U = XY, where X and Y are independent random variables with density functions as

$$f_x(x) = \begin{cases} 2x & 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases} \quad \text{and} \quad f_y(y) = \begin{cases} 3y^2 & 0 \le y \le 1 \\ 0 & \text{otherwise} \end{cases}$$

[15 marks]

2. Let x[n] and y[n] be stationary, uncorrelated random signals. Show that if w[n]=x[n] + y[n], then $m_w = m_x + m_y$ and $\sigma_w^2 = \sigma_x^2 + \sigma_y^2$. **[10 marks]**

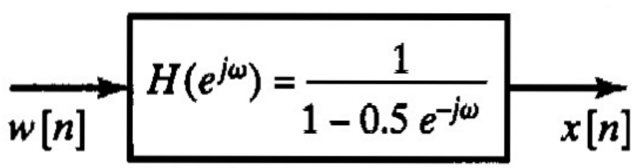
3. Consider a random signal x[n] = s[n] + e[n], where both s[n] and e[n] are independent zero-mean stationary random signals with autocorrelation functions $\phi_{ss}[m]$ and $\phi_{ee}[m]$ respectively. **[10 marks]**

(a) Determine expressions for $\phi_{xx}[m]$ and $\phi_{xx}(e^{jw})$.

(b) Determine expressions for $\phi_{xe}[m]$ and $\phi_{xe}(e^{jw})$.

(c) Determine expressions for $\phi_{xs}[m]$ and $\phi_{xs}(e^{jw})$.

4. Consider a random process x[n] that is the response of the linear time-invariant system shown in the below figure. In the figure, w[n] represents a real zero-mean stationary white noise process with $E\{w^2[n]\} = \sigma_w^2$. **[15 marks]**



(a) Express $E\{x^2[n]\}$ in terms of $\phi_{xx}[n]$ or $\phi_{xx}(e^{jw})$.

(b) Determine $\phi_{xx}(e^{jw})$, the power density spectrum of x[n].

(c) Determine $\phi_{xx}[n]$, the correlation function of x[n].