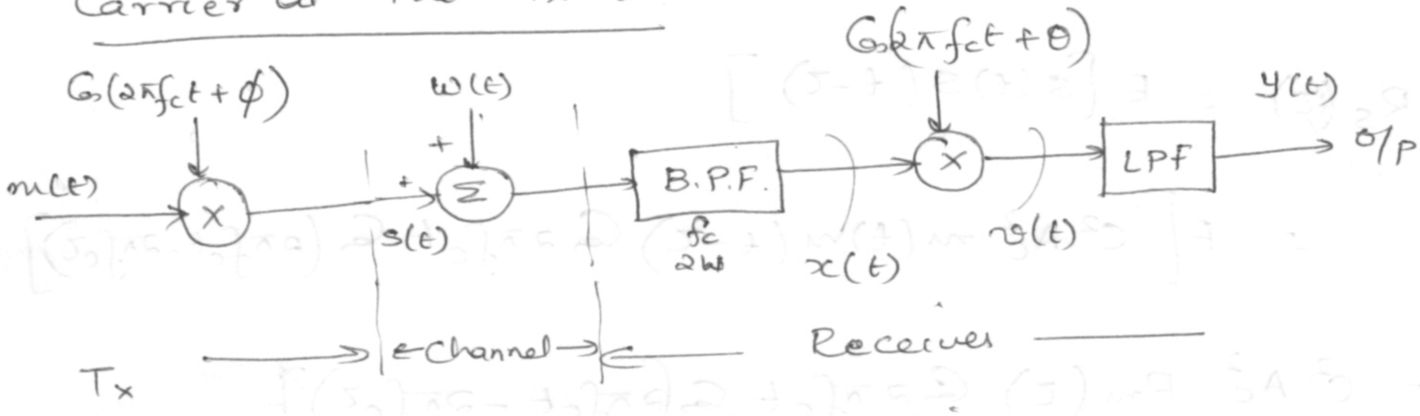
Analysis of DSB-SC ~~using~~ with random phase in the Carrier at the Tx & Rx:



Case-1 ϕ is random in the carrier.

$s(t) = C A_c m(t) \cos(2\pi f_c t + \phi)$

$$R(\tau) = C^2 A_c^2 E\left[s(t)s(t-\tau)\right]$$

$$= C^2 A_c^2 E\left[m(t)m(t-\tau)\cos(2\pi f_c t + \phi)\cos(2\pi f_c t + \phi - 2\pi f_c \tau)\right]$$

$$= C^2 A_c^2 R_m(\tau)\, E\left[\cos(2\pi f_c t + \phi)\cos(2\pi f_c t + \phi - 2\pi f_c \tau)\right]$$

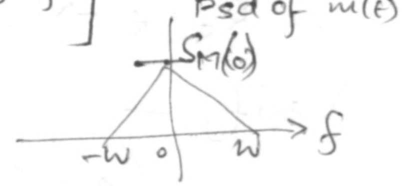
ϕ = Uniformly distributed r.v. between 0 to 2π or $-\pi$ to π

$$= C^2 A_c^2 \frac{R_m(\tau)}{2}\left\{ \underbrace{E\left[\cos(4\pi f_c t + 2\phi - 2\pi f_c \tau)\right]}_{\text{avg. over } \phi \;=\; 0} + E\left[\cos(2\pi f_c \tau)\right] \right.$$

deterministic

$$= C^2 A_c^2 \frac{R_m(\tau)}{2}\cos 2\pi f_c \tau = C^2 A_c^2 \frac{R_m(\tau)}{4}\left[e^{j2\pi f_c \tau} + e^{-j2\pi f \tau}\right]$$

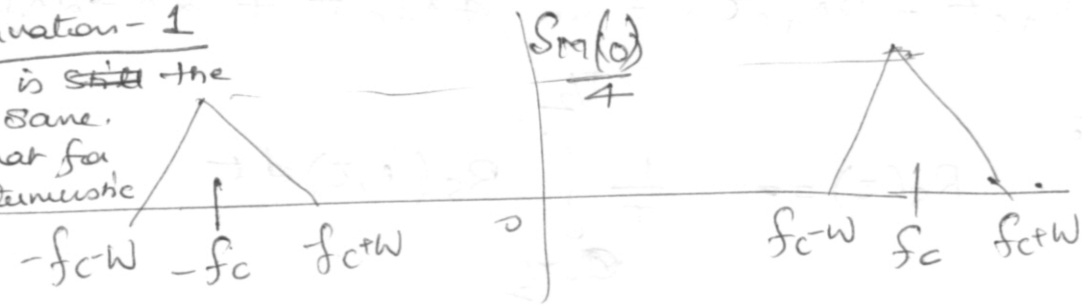
Psd of $m(t)$



~~P. Spectrum of~~ P.S.D. of $s(t)$

Observation-1

PSD is ~~still~~ the same as that for a deterministic signal m(t)



~~Observation~~ $\therefore$ $P_{S_i} = \frac{R_m(0)}{2} \cdot C^2 A_c^2 = \frac{P \cdot C^2 A_c^2}{2}$

Case-2 $\phi = 0$ or a constant

$$s(t) = CA_c m(t) \cos 2\pi f_c t$$

$$R_s(t,\tau) = E[s(t)s(t-\tau)]$$

$$= E[C^2 A_c^2 m(t)m(t-\tau) \cos 2\pi f_c t \cos(2\pi f_c t - 2\pi f_c \tau)]$$

$$= C^2 A_c^2 R_m(\tau) \cos 2\pi f_c t \cos(2\pi f_c t - 2\pi f_c \tau)$$

$$R_s(t,\tau) = \frac{C^2 A_c^2}{2} R_m(\tau) \left[\underbrace{\cos(4\pi f_c t - 2\pi f_c \tau)}_{①} + \underbrace{\cos 2\pi f_c \tau}_{②} \right]$$

Not a ~~Sta~~ WSS process because it is a ~~Comp~~ it is a fn. of both t & τ. (Actually, Cyclostationary random process. In fact, whenever ϕ has any distribution other than uniform, the resulting process is cyclostationary)

~~Find time-avged~~

For cyclostationary —

$$R_s(t,\tau) = R_s(t+T,\tau) \qquad T = \frac{1}{2f_c}$$

Verify —

$$\because \cos(4\pi f_c t - 2\pi f_c \tau) = \cos(4\pi f_c t + 2\pi - 2\pi f_c \tau)$$

$\therefore$ Compute

$$R_s(\tau) = \frac{1}{T}\int_{-T/2}^{T/2} R_s(t,\tau)\,dt$$

$$= \frac{C^2 A_c^2}{2} R_m(\tau) \cos 2\pi f_c \tau \longrightarrow \text{Same as before.}$$

$$x(t) = s(t) + n(t)$$

$$= CA_c \cos(2\pi f_c t + \phi)\, m(t) + n_I(t) \cos 2\pi f_c t$$
$$- n_Q(t) \sin 2\pi f_c t$$

$$v(t) = x(t) \cos(2\pi f_c t + \theta)$$

$$= \underbrace{CA_c \cos(2\pi f_c t + \phi) \cos(2\pi f_c t + \theta)\, m(t)}_{s_1(t)}$$

$$\boxed{+ n_I(t) \cos 2\pi f_c t \cdot \cos(2\pi f_c t + \theta) - n_Q(t) \sin 2\pi f_c t \cos(2\pi f_c t + \theta)} \leftarrow n_1(t)$$

noise

$$s_1(t) = CA_c \cos(2\pi f_c t + \phi) \cos(2\pi f_c t + \theta)\, m(t)$$

$$= \frac{CA_c m(t)}{2} \cos(4\pi f_c t + \phi + \theta)$$

$$+ \frac{CA_c}{2} m(t) \cos(\phi - \theta)$$

$$y(t) = LPF(v(t)) = s_o(t) + n_o(t)$$

$s_o(t)$ = o/p at the filter

$$= \frac{CA_c}{2} m(t) \underbrace{\cos(\phi - \theta)}$$

Use of coherent detector will nullify it

& hence, for the sake of simplicity, we write $\phi = \theta = 0$

Look at noise –

$$n_1(t) = \underbrace{n_I(t)\cos 2\pi f_c t\cos(2\pi f_c t+\theta)}$$

$$-\,n_Q(t)\underbrace{\sin 2\pi f_c t}_{b}\cos\underbrace{(2\pi f_c t+\theta)}_{a}$$

$\sin b\cos a = \frac{1}{2}\left(\sin(a+b) - \sin(a-b)\right)$

$$\cancel{n_I(t)} = \frac{n_I(t)}{2}\left[\cos(4\pi f_c t+\theta) + \cos\theta\right]$$

$$-\frac{n_Q(t)}{2}\left[\sin(4\pi f_c t+\theta) - \sin\theta\right]$$

$$n_0(t) = LPF\left[n_1(t)\right]$$

$$= \frac{n_I(t)}{2}\cos\theta + \frac{n_Q(t)}{2}\sin\theta$$

Power in $n_I'(t) = \frac{n_I(t)}{2}\cos\theta = ?$

$$R_{n_I'}(t, t-\tau) = E\left[\frac{n_I(t)}{2}\cos\theta\,\frac{n_I(t-\tau)}{2}\cos\theta\right]$$

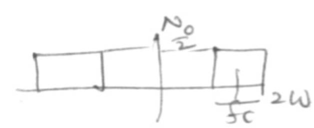
$$= \frac{1}{4} \times R_{n_I}(\tau)E\left[\cos^2\theta\right]$$

$$= \frac{1}{8} R_{n_I}(\tau)E\left[(1+\cos 2\theta)\right]$$

$E[\cos 2\theta] = 0$ θ is uniformly distributed

$$= \frac{1}{8} R_{n_I}(\tau)$$

DSB



$$R_{n_I'}(0) = \frac{1}{8}R_{n_I}(0) = \frac{1}{8}R_n(0) = \frac{1}{8}\times 2\times 2W\times\frac{N_0}{\cancel{2}}$$

$$= \frac{1}{4}WN_0$$

$$R_{n_o}(t,t-\tau) = E\left[\left(\frac{n_I(t)\cos\theta}{2} + \frac{n_Q(t)\sin\theta}{2}\right)\left(\frac{n_I(t-\tau)\cos\theta}{2} + \frac{n_Q(t-\tau)\sin\theta}{2}\right)\right]$$

$$= \cancel{\frac{1}{4}E[n_I} \;\; \frac{1}{4}\Big\{ R_{n_I}(\tau)E[\cos^2\theta] + R_{n_Q}(\tau)E[\sin^2\theta]$$

$$+ E[n_I(t)n_Q(t-\tau)]E[\cos\theta\sin\theta]$$

$$+ E[n_I(t-\tau)n_Q(t)]E[\cos\theta\sin\theta]\Big\}$$

$$= \frac{1}{4}R_{n_I}(\tau)\left\{\frac{1+\cancelto{0}{E(\cos 2\theta)}}{2}\right\}$$

$$+ \frac{1}{4}R_{n_Q}(\tau)\left\{\frac{1-\cancelto{0}{E(\cos 2\theta)}}{2}\right\}$$

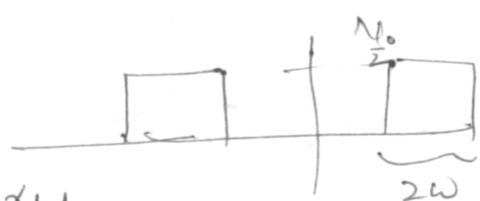
$$+ \frac{1}{8}R_{n_I n_Q}(\tau)\cancelto{0}{E(\sin 2\theta)} + \frac{1}{8}R_{n_I n_Q}(\tau)\cancelto{0}{E(\sin 2\theta)}$$

$$R_{n_o}(\tau) = \frac{1}{8}R_{n_I}(\tau) + \frac{1}{8}R_{n_Q}(\tau)$$

$$P_{n_o} = R_{n_o}(0) = \frac{1}{8}R_{n_I}(0) + \frac{1}{8}R_{n_Q}(0)$$

$$= \cancel{\frac{1}{4}[} \;\; \frac{1}{8}R_n(0) + \frac{1}{8}R_n(0)$$

$$= \frac{1}{4}R_n(0)$$



$$= \frac{1}{\cancel{4}} \times \cancel{2} \times \frac{N_0}{\cancel{2}} \times \cancel{2}W$$

$$= \frac{1}{2}N_0 W.$$

Likewise for SSB —

Again, it is coherent detection —

$\therefore$ Signal power is same as was calculated earlier.

For noise power —

~~$n(t) = n_I$~~

$$v(t) = x(t)\cos(2\pi f_c t + \theta)$$

$$= \left(s(t) + n(t)\right)\cos(2\pi f_c t + \theta)$$

$$= s(t)\cos(2\pi f_c t + \theta) + n(t)\cos(2\pi f_c t + \theta)$$

Noise —

$$\cancel{nt} = n(t)\cos(2\pi f_c t + \theta)$$

$$= \left\{n_I(t)\cos\left[2\pi\left(f_c - \frac{W}{2}\right)t\right] - n_Q(t)\sin\left[2\pi\left(f_c - \frac{W}{2}\right)t\right]\right\} \times \cos(2\pi f_c t + \theta)$$

$$2\pi f_c t = a$$
$$\pi W t = b$$

$$= \left[n_I(t)\cos(a-b) - n_Q(t)\sin(a-b)\right]\cos(a+\theta)$$

$$= \frac{1}{2} n_I(t)\left\{\cos(2a - b + \theta) + \cos(b+\theta)\right\}$$
$$+ \frac{1}{2} n_Q(t)\left\{\sin(2a - b + \theta) + \sin(-b-\theta)\right\}$$

$$\sin a \cos b = \frac{\sin(a+b) + \sin(a-b)}{2}$$

$$= \boxed{\sin a \cos b}$$

$$y(t) = LPF\{v(t)\}$$

$$= S_0(t) + n_0(t)$$

$$\therefore n_0(t) = \frac{1}{2} n_I(t) \cos(b+\theta) - \frac{1}{2} n_Q(t) \sin(b+\theta)$$

$$= \frac{1}{2} n_I(t) \cos(\pi W t + \theta) - \frac{1}{2} n_Q(t) \sin(\pi W t + \theta)$$

$$R_{n_0}(t, t-\tau) = \frac{1}{4} E\Big[\big(n_I(t) \cos(b+\theta) - n_Q(t) \sin(b+\theta)\big)$$

$$\big(n_I(t-\tau) \cos(b-c+\theta) - n_Q(t-\tau) \sin(b-c+\theta)\big)\Big]$$

$$= \frac{1}{4} R_{n_I}(\tau)\, E\left[\cos(b+\theta) \cos(b-c+\theta)\right]$$

$$+ \frac{1}{4} R_{n_Q}(\tau)\, E\left[\sin(b+\theta) \sin(b-c+\theta)\right]$$

$$- \frac{1}{4} \underbrace{E\left[n_I(t)\, n_Q(t-\tau)\right]}_{=0} E\left[\cos(b+\theta) \sin(b-c+\theta)\right]$$

$$- \frac{1}{4} \underbrace{E\left[n_I(t-\tau)\, n_Q(t-\tau)\right]}_{=0} E\left[\sin(b+\theta) \cos(b-c+\theta)\right]$$

<u>Property -7</u> — If a narrowband noise $n(t)$ is Gaussian with zero mean & a PSD $P_N(f)$ that is locally symmetric about the mid-band freq. $\pm f_c$, then the in-phase noise $n_I(t)$ & the quadrature noise $n_Q(t)$ are statistically independent.

$$R_{n_0}(t, t-\tau) = \frac{1}{8} R_{n_I}(\tau)\, E\Big[\cos(\cancel{2b+2\theta}-c) + \cos c\Big]$$

$=0$

$\sin a \sin b = \frac{\cos(a-b) - \cos(a+b)}{2}$

$$+ \frac{1}{8} R_{n_Q}(\tau)\, E\Big[\cos(c) - \cos(\cancel{2b+2\theta}-c)\Big]$$

Avg over $\theta = 0$

$$= \frac{1}{8} R_{n_I}(\tau) \cos c + \frac{1}{8} R_{n_Q}(\tau) \cos c$$

$$R_{n_0}(\tau) = \frac{1}{8} R_{n_I}(\tau) \cos \pi W \tau + \frac{1}{8} R_{n_Q}(\tau) \cos \pi W \tau$$

$$R_{n_0}(0) = \frac{1}{8} R_{n_I}(0) + \frac{1}{8} R_{n_Q}(0)$$

SSB

$$= \frac{1}{4} R_n(0)$$

$$= \frac{1}{4} \times \cancel{2} \cdot \frac{N_0}{\cancel{2}} \times W$$

$$= \frac{N_0 W}{4}$$