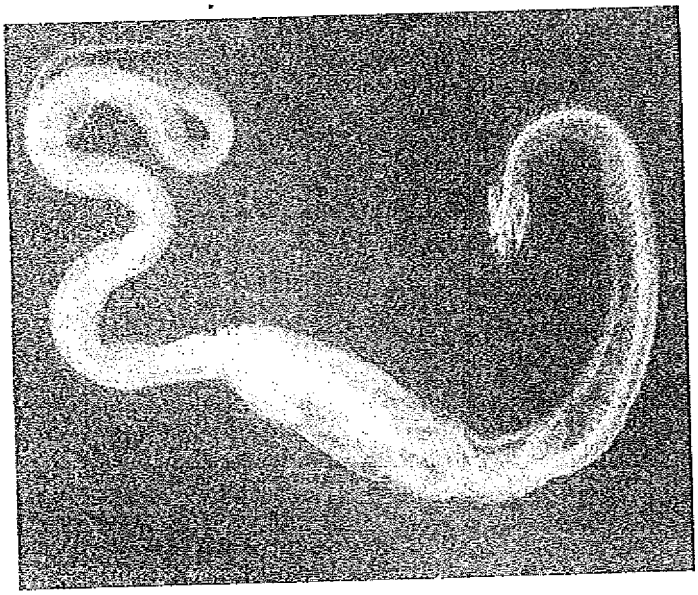
CHAPTER 2

Particle Properties of Waves



The penetrating ability of x-rays enabled them to reveal the frog which this snake had swallowed. The snake's jaws are very loosely joined and so can open widely.

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In our everyday experience there is nothing mysterious or ambiguous about the concepts of **particle** and **wave**. A stone dropped into a lake and the ripples that spread out from its point of impact apparently have in common only the ability to carry energy and momentum from one place to another. Classical physics, which mirrors the "physical reality" of our sense impressions, treats particles and waves as separate components of that reality. The mechanics of particles and the optics of waves are traditionally independent disciplines, each with its own chain of experiments and principles based on their results.

The physical reality we perceive has its roots in the microscopic world of atoms and molecules, electrons and nuclei, but in this world there are neither particles nor waves in our sense of these terms. We regard electrons as particles because they possess charge and mass and behave according to the laws of particle mechanics in such familiar devices as television picture tubes. We shall see, however, that it is just as correct to interpret a moving electron as a wave manifestation as it is to interpret it as a particle manifestation. We regard electromagnetic waves as waves because under suitable circumstances they exhibit diffraction, interference, and polarization. Similarly, we shall see that under other circumstances electromagnetic waves behave as though they consist of streams of particles. Together with special relativity, the wave-particle duality is central to an understanding of modern physics, and in this book there are few arguments that do not draw upon either or both of these fundamental ideas.

2.1 ELECTROMAGNETIC WAVES

Coupled electric and magnetic oscillations that move with the speed of light and exhibit typical wave behavior

In 1864 the British physicist James Clerk Maxwell made the remarkable suggestion that accelerated electric charges generate linked electric and magnetic disturbances that can travel indefinitely through space. If the charges oscillate periodically, the disturbances are waves whose electric and magnetic components are perpendicular to each other and to the direction of propagation, as in Fig. 2.1.

From the earlier work of Faraday, Maxwell knew that a changing magnetic field can induce a current in a wire loop. Thus a changing magnetic field is equivalent in its effects to an electric field. Maxwell proposed the converse: a changing electric field has a magnetic field associated with it. The electric fields produced by electromagnetic induction are easy to demonstrate because metals offer little resistance to the flow of charge. Even a weak field can lead to a measurable current in a metal. Weak magnetic fields are much harder to detect, however, and Maxwell's hypothesis was based on a symmetry argument rather than on experimental findings.

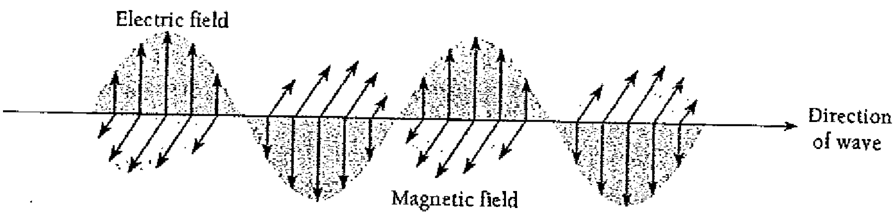


Figure 2.1 The electric and magnetic fields in an electromagnetic wave vary together. The fields are perpendicular to each other and to the direction of propagation of the wave.

James Clerk Maxwell (1831–1879) was born in Scotland shortly before Michael Faraday discovered electromagnetic induction. At nineteen he entered Cambridge University to study physics and mathematics. While still a student, he investigated the physics of color vision and later used his ideas to make the first color photograph. Maxwell became known to the scientific world at twenty-four when he showed that the rings of Saturn could not be solid or liquid but must consist of separate small bodies. At about this time Maxwell became interested in electricity and magnetism and grew convinced that the wealth of phenomena Faraday and others had discovered were not isolated effects but had an underlying unity of some kind. Maxwell's initial step in establishing that unity came in 1856 with the paper "On Faraday's Lines of Force," in which he developed a mathematical description of electric and magnetic fields.

Maxwell left Cambridge in 1856 to teach at a college in Scotland and later at King's College in London. In this period he expanded his ideas on electricity and magnetism to create a single comprehensive theory of electromagnetism. The fundamental equations he arrived at remain the foundations of the subject today. From these equations Maxwell predicted that electromagnetic waves should exist that travel with the speed of light, described the properties the waves should have, and surmised that light consisted of electromagnetic waves. Sadly, he did not live to see his work confirmed in the experiments of the German physicist Heinrich Hertz.

Maxwell's contributions to kinetic theory and statistical mechanics were on the same profound level as his contributions to electromagnetic theory. His calculations showed that the viscosity of a gas ought to be independent of its pressure, a surprising result that Maxwell, with the help of his wife, confirmed in the laboratory. They also found that the viscosity was proportional to the absolute temperature of the gas. Maxwell's explanation for this proportionality gave him a way to estimate the size and mass of molecules, which until then could only be guessed at. Maxwell shares with Boltzmann credit for the equation that gives the distribution of molecular energies in a gas.

In 1865 Maxwell returned to his family's home in Scotland. There he continued his research and also composed a treatise on electromagnetism that was to be the standard text on the subject for many decades. It was still in print a century later. In 1871 Maxwell went back to Cambridge to establish and direct the Cavendish Laboratory, named in honor of the pioneering physicist Henry Cavendish. Maxwell died of cancer at the age of forty-eight in 1879, the year in which Albert Einstein was born. Maxwell had been the greatest theoretical physicist of the nineteenth century; Einstein was to be the greatest theoretical physicist of the twentieth century. (By a similar coincidence, Newton was born in the year of Galileo's death.)

If Maxwell was right, electromagnetic (em) waves must occur in which constantly varying electric and magnetic fields are coupled together by both electromagnetic induction and the converse mechanism he proposed. Maxwell was able to show that the speed c of electromagnetic waves in free space is given by

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}} = 2.998 \times 10^8 \text{ m/s}$$

where ϵ_0 is the electric permittivity of free space and μ_0 is its magnetic permeability. This is the same as the speed of light waves. The correspondence was too great to be accidental, and Maxwell concluded that light consists of electromagnetic waves.

During Maxwell's lifetime the notion of em waves remained without direct experimental support. Finally, in 1888, the German physicist Heinrich Hertz showed that em waves indeed exist and behave exactly as Maxwell had predicted. Hertz generated the waves by applying an alternating current to an air gap between two metal balls. The width of the gap was such that a spark occurred each time the current reached a peak. A wire loop with a small gap was the detector; em waves set up oscillations in the loop that produced sparks in the gap. Hertz determined the wavelength and speed of the waves he generated, showed that they have both electric and magnetic components, and found that they could be reflected, refracted, and diffracted.

Light is not the only example of an em wave. Although all such waves have the same fundamental nature, many features of their interaction with matter depend upon

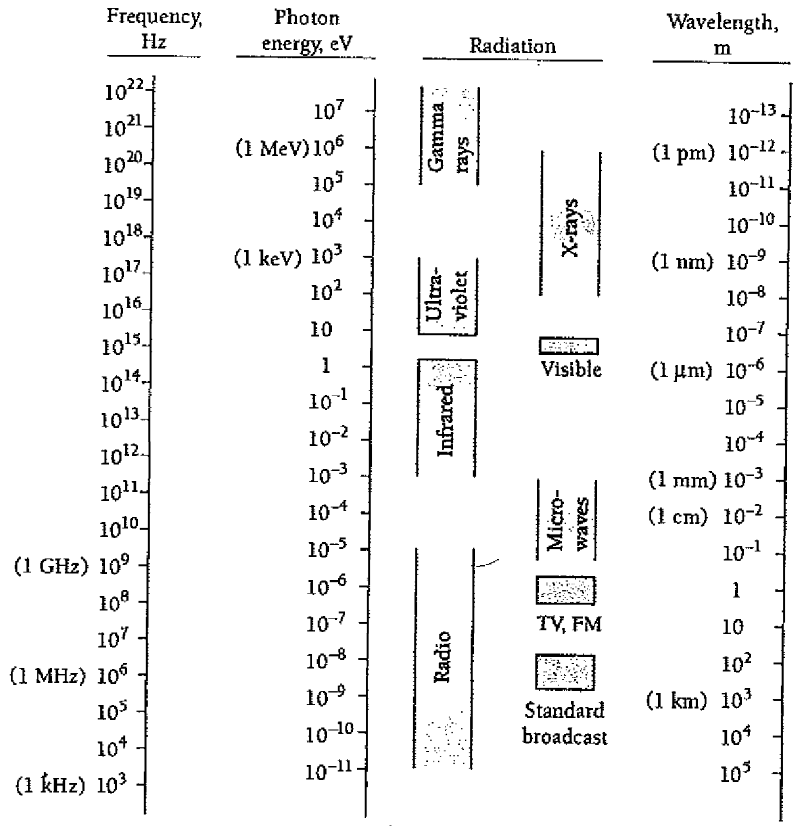


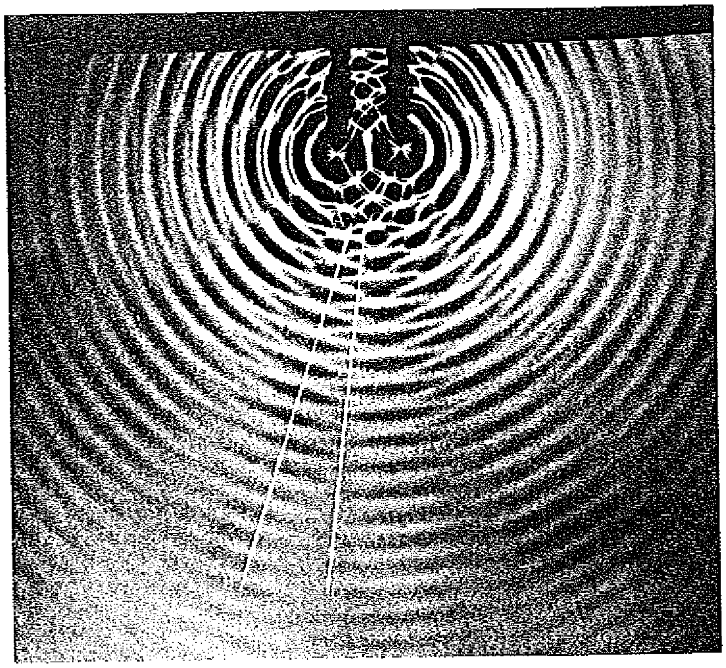
Figure 2.2 The spectrum of electromagnetic radiation.

their frequencies. Light waves, which are em waves the eye responds to, span only a brief frequency interval, from about 4.3×10^{14} Hz for red light to about 7.5×10^{14} Hz for violet light. Figure 2.2 shows the em wave spectrum from the low frequencies used in radio communication to the high frequencies found in x-rays and gamma rays.

A characteristic property of all waves is that they obey the **principle of superposition:**

> When two or more waves of the same nature travel past a point at the same time, the instantaneous amplitude there is the sum of the instantaneous amplitudes of the individual waves.

Instantaneous amplitude refers to the value at a certain place and time of the quantity whose variations constitute the wave. ("Amplitude" without qualification refers to the maximum value of the wave variable.) Thus the instantaneous amplitude of a wave in a stretched string is the displacement of the string from its normal position; that of a water wave is the height of the water surface relative to its normal level; that of a sound wave is the change in pressure relative to the normal pressure. Since the electric and magnetic fields in a light wave are related by $E = cB$, its instantaneous amplitude can be taken as either E or B. Usually E is used, since it is the electric fields of light waves whose interactions with matter give rise to nearly all common optical effects.



The interference of water waves. Constructive interference occurs along the line AB and destructive interference occurs along the line CD.

When two or more trains of light waves meet in a region, they **interfere** to produce a new wave there whose instantaneous amplitude is the sum of those of the original waves. Constructive interference refers to the reinforcement of waves with the same phase to produce a greater amplitude, and destructive interference refers to the partial or complete cancellation of waves whose phases differ (Fig. 2.3). If the original waves have different frequencies, the result will be a mixture of constructive and destructive interference, as in Fig. 3.4.

The interference of light waves was first demonstrated in 1801 by Thomas Young, who used a pair of slits illuminated by monochromatic light from a single source (Fig. 2.4). From each slit secondary waves spread out as though originating at the slit; this is an example of **diffraction**, which, like interference, is a characteristic wave phenomenon. Owing to interference, the screen is not evenly lit but shows a pattern of alternate bright and dark lines. At those places on the screen where the path lengths from the two slits differ by an odd number of half wavelengths ($\lambda/2$, $3\lambda/2$, $5\lambda/2$, . . .), destructive interference occurs and a dark line is the result. At those places where the path lengths are

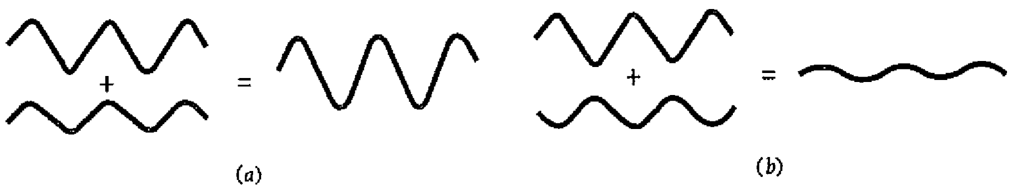


Figure 2.3 (*a*) In constructive interference, superposed waves in phase reinforce each other. (*b*) In destructive interference, waves out of phase partially or completely cancel each other.

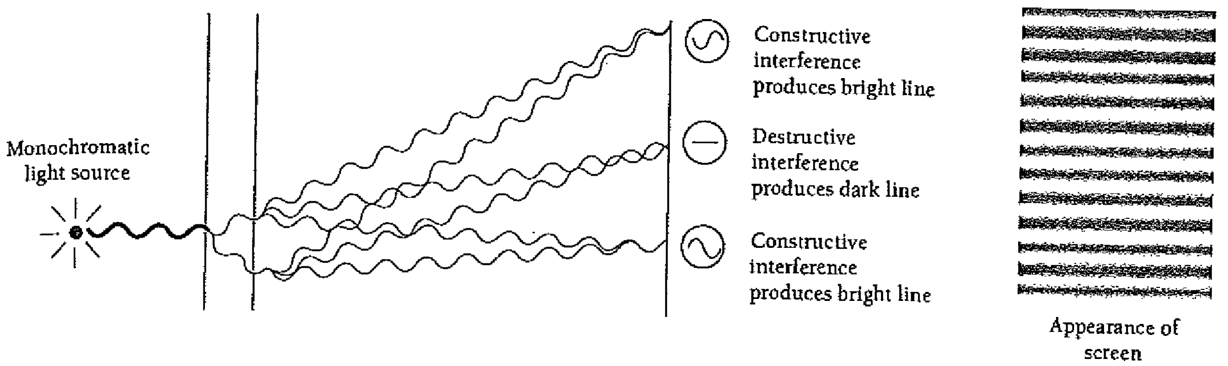


Figure 2.4 Origin of the interference pattern in Young's experiment. Constructive interference occurs where the difference in path lengths from the slits to the screen is $0, \lambda, 2\lambda, \ldots$. Destructive interference occurs where the path difference is $\lambda/2, 3\lambda/2, 5\lambda/2, \ldots$.

equal or differ by a whole number of wavelengths ($\lambda, 2\lambda, 3\lambda, \ldots$), constructive interference occurs and a bright line is the result. At intermediate places the interference is only partial, so the light intensity on the screen varies gradually between the bright and dark lines.

Interference and diffraction are found only in waves—the particles we are familiar with do not behave in those ways. If light consisted of a stream of classical particles, the entire screen would be dark. Thus Young's experiment is proof that light consists of waves. Maxwell's theory further tells us what kind of waves they are: electromagnetic. Until the end of the nineteenth century the nature of light seemed settled forever.

2.2 BLACKBODY RADIATION

Only the quantum theory of light can explain its origin

Following Hertz's experiments, the question of the fundamental nature of light seemed clear: light consisted of em waves that obeyed Maxwell's theory. This certainty lasted only a dozen years. The first sign that something was seriously amiss came from attempts to understand the origin of the radiation emitted by bodies of matter.

We are all familiar with the glow of a hot piece of metal, which gives off visible light whose color varies with the temperature of the metal, going from red to yellow to white as it becomes hotter and hotter. In fact, other frequencies to which our eyes do not respond are present as well. An object need not be so hot that it is luminous for it to be radiating em energy; *all* objects radiate such energy continuously whatever their temperatures, though which frequencies predominate depends on the temperature. At room temperature most of the radiation is in the infrared part of the spectrum and hence is invisible.

The ability of a body to radiate is closely related to its ability to absorb radiation. This is to be expected, since a body at a constant temperature is in thermal equilibrium with its surroundings and must absorb energy from them at the same rate as it emits energy. It is convenient to consider as an ideal body one that absorbs *all* radiation incident upon it, regardless of frequency. Such a body is called a **blackbody**.

The point of introducing the idealized blackbody in a discussion of thermal radiation is that we can now disregard the precise nature of whatever is radiating, since

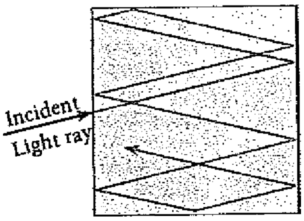


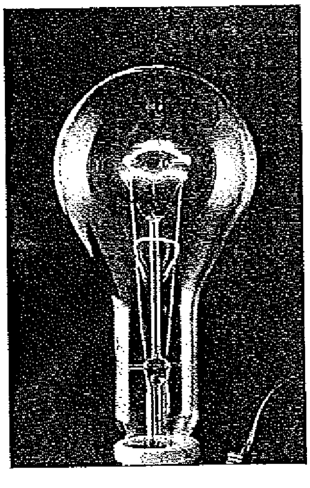
Figure 2.5 A hole in the wall of a hollow object is an excellent approximation of a blackbody.

all blackbodies behave identically. In the laboratory a blackbody can be approximated by a hollow object with a very small hole leading to its interior (Fig. 2.5). Any radiation striking the hole enters the cavity, where it is trapped by reflection back and forth until it is absorbed. The cavity walls are constantly emitting and absorbing radiation, and it is in the properties of this radiation **(blackbody radiation)** that we are interested.

Experimentally we can sample blackbody radiation simply by inspecting what emerges from the hole in the cavity. The results agree with everyday experience. A blackbody radiates more when it is hot than when it is cold, and the spectrum of a hot blackbody has its peak at a higher frequency than the peak in the spectrum of a cooler one. We recall the behavior of an iron bar as it is heated to progressively higher temperatures: at first it glows dull red, then bright orange-red, and eventually it becomes "white hot." The spectrum of blackbody radiation is shown in Fig. 2.6 for two temperatures.

The Ultraviolet Catastrophe

Why does the blackbody spectrum have the shape shown in Fig. 2.6? This problem was examined at the end of the nineteenth century by Lord Rayleigh and James Jeans. The details of their calculation are given in Chap. 9. They started by considering the radiation inside a cavity of absolute temperature T whose walls are perfect reflectors to be a series of standing em waves (Fig. 2.7). This is a three-dimensional generalization of standing waves in a stretched string. The condition



The color and brightness of an object heated until it glows, such as the filament of this light bulb, depends upon its temperature, which here is about 3000 K. An object that glows white is hotter than it is when it glows red, and it gives off more light as well.

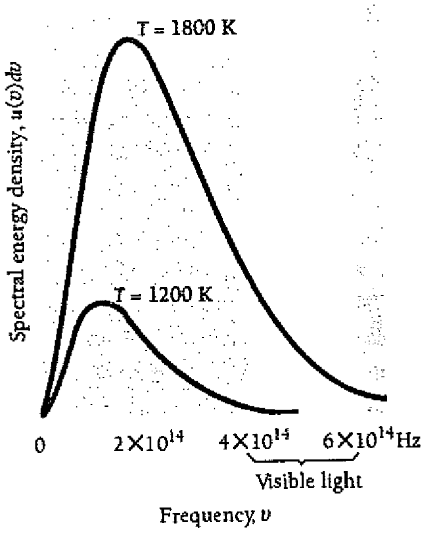


Figure 2.6 Blackbody spectra. The spectral distribution of energy in the radiation depends only on the temperature of the body. The higher the temperature, the greater the amount of radiation and the higher the frequency at which the maximum emission occurs. The dependence of the latter frequency on temperature follows a formula called Wien's displacement law, which is discussed in Sec. 9.6.

for standing waves in such a cavity is that the path length from wall to wall, whatever the direction, must be a whole number of half-wavelengths, so that a node occurs at each reflecting surface. The number of independent standing waves $G(\nu)d\nu$ in the frequency interval between ν and $d\nu$ per unit volume in the cavity turned out to be

Density of standing waves in cavity

$$G(\nu)d\nu = \frac{8\pi\nu^2 d\nu}{c^3} \tag{2.1}$$

This formula is independent of the shape of the cavity. As we would expect, the higher the frequency ν, the shorter the wavelength and the greater the number of possible standing waves.

The next step is to find the average energy per standing wave. According to the **theorem of equipartition of energy**, a mainstay of classical physics, the average energy per degree of freedom of an entity (such as a molecule of an ideal gas) that is a member of a system of such entities in thermal equilibrium at the temperature T is $\frac{1}{2}kT$. Here k is **Boltzmann's constant:**

Boltzmann's constant $\qquad k = 1.381 \times 10^{-23}$ J/K

A degree of freedom is a mode of energy possession. Thus a monatomic ideal gas molecule has three degrees of freedom, corresponding to kinetic energy of motion in three independent directions, for an average total energy of $\frac{3}{2}kT$.

A one-dimensional harmonic oscillator has two degrees of freedom, one that corresponds to its kinetic energy and one that corresponds to its potential energy. Because each standing wave in a cavity originates in an oscillating electric charge in the cavity wall, two degrees of freedom are associated with the wave and it should have an average energy of $2(\frac{1}{2})kT$:

Classical average energy per standing wave

$$\bar{\epsilon} = kT \tag{2.2}$$

The total energy $u(\nu)\,d\nu$ per unit volume in the cavity in the frequency interval from ν to $\nu + d\nu$ is therefore

Rayleigh-Jeans formula

$$u(\nu)\,d\nu = \bar{\epsilon}G(\nu)\,d\nu = \frac{8\pi kT}{c^3}\nu^2\,d\nu \tag{2.3}$$

This radiation rate is proportional to this energy density for frequencies between ν and $\nu + d\nu$. Equation (2.3), the **Rayleigh-Jeans formula,** contains everything that classical physics can say about the spectrum of blackbody radiation.

Even a glance at Eq. (2.3) shows that it cannot possibly be correct. As the frequency ν increases toward the ultraviolet end of the spectrum, this formula predicts that the energy density should increase as ν^2. In the limit of infinitely high frequencies, $u(\nu)\,d\nu$ therefore should also go to infinity. In reality, of course, the energy density (and radiation rate) falls to 0 as $\nu \rightarrow \infty$ (Fig. 2.8). This discrepancy became known as the **ultraviolet catastrophe** of classical physics. Where did Rayleigh and Jeans go wrong?

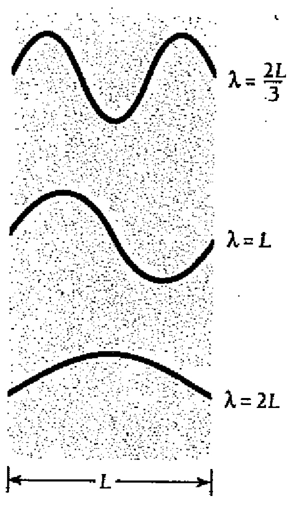


Figure 2.7 Em radiation in a cavity whose walls are perfect reflectors consists of standing waves that have nodes at the walls, which restricts their possible wavelengths. Shown are three possible wavelengths when the distance between opposite walls is L.

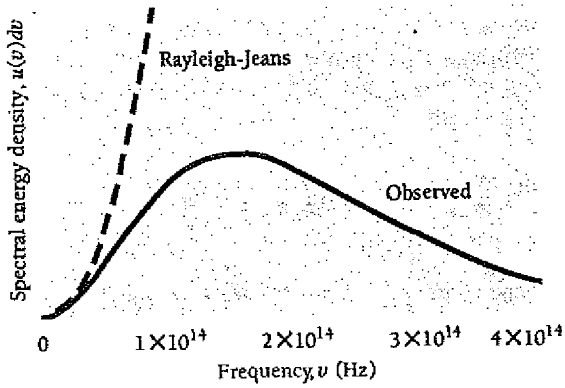


Figure 2.8 Comparison of the Rayleigh-Jeans formula for the spectrum of the radiation from a blackbody at 1500 K with the observed spectrum. The discrepancy is known as the ultraviolet catastrophe because it increases with increasing frequency. This failure of classical physics led Planck to the discovery that radiation is emitted in quanta whose energy is $h\nu$.

Planck Radiation Formula

In 1900 the German physicist Max Planck used "lucky guesswork" (as he later called it) to come up with a formula for the spectral energy density of blackbody radiation:

Planck radiation formula

$$u(\nu)\, d\nu = \frac{8\pi h}{c^3} \frac{\nu^3\, d\nu}{e^{h\nu/kT} - 1} \tag{2.4}$$

Here h is a constant whose value is

Planck's constant

$$h = 6.626 \times 10^{-34}\ \mathrm{J \cdot s}$$

Max Planck (1858–1947) was born in Kiel and educated in Munich and Berlin. At the University of Berlin he studied under Kirchhoff and Helmholtz, as Hertz had done earlier. Planck realized that blackbody radiation was important because it was a fundamental effect independent of atomic structure, which was still a mystery in the late nineteenth century, and worked at understanding it for six years before finding the formula the radiation obeyed. He "strived from the day of its discovery to give it a real physical interpretation." The result was the discovery that radiation is emitted in energy steps of $h\nu$. Although this discovery, for which he received the Nobel Prize in 1918, is now considered to mark the start of modern physics, Planck himself remained skeptical for a long time of the physical reality of quanta. As he later wrote, "My vain attempts to somehow reconcile the elementary quantum with classical theory continued for many years and cost me great effort. . . . Now I know for certain that the quantum of action has a much more fundamental significance than I originally suspected."

Like many physicists, Planck was a competent musician (he sometimes played with Einstein) and in addition enjoyed mountain climbing. Although Planck remained in Germany during the Hitler era, he protested the Nazi treatment of Jewish scientists and lost his presidency of the Kaiser Wilhelm Institute as a result. In 1945 one of his sons was implicated in a plot to kill Hitler and was executed. After World War II the Institute was renamed after Planck and he was again its head until his death.

At high frequencies, $h\nu \gg kT$ and $e^{h\nu/kT} \rightarrow \infty$, which means that $u(\nu)\,d\nu \rightarrow 0$ as observed. No more ultraviolet catastrophe. At low frequencies, where the Rayleigh-Jeans formula is a good approximation to the data (see Fig. 2.8), $h\nu \ll kT$ and $h\nu/kT \ll 1$. In general,

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

If x is small, $e^x \approx 1 + x$, and so for $h\nu/kT \ll 1$ we have

$$\frac{1}{e^{h\nu/kT} - 1} \approx \frac{1}{1 + \frac{h\nu}{kT} - 1} \approx \frac{kT}{h\nu} \qquad h\nu \ll kT$$

Thus at low frequencies Planck's formula becomes

$$u(\nu)\,d\nu \approx \frac{8\pi h}{c^3}\nu^3\left(\frac{kT}{h\nu}\right)d\nu \approx \frac{8\pi kT}{c^3}\nu^2\,d\nu$$

which is the Rayleigh-Jeans formula. Planck's formula is clearly at least on the right track; in fact, it has turned out to be completely correct.

Next Planck had the problem of justifying Eq. (2.4) in terms of physical principles. A new principle seemed needed to explain his formula, but what was it? After several weeks of "the most strenuous work of my life," Planck found the answer: The oscillators in the cavity walls could not have a continuous distribution of possible energies ϵ but must have only the specific energies

Oscillator energies $\qquad \epsilon_n = nh\nu \qquad n = 0, 1, 2, \ldots \qquad$ (2.5)

An oscillator emits radiation of frequency ν when it drops from one energy state to the next lower one, and it jumps to the next higher state when it absorbs radiation of frequency ν. Each discrete bundle of energy $h\nu$ is called a **quantum** (plural **quanta**) from the Latin for "how much."

With oscillator energies limited to $nh\nu$, the average energy per oscillator in the cavity walls—and so per standing wave—turned out to be not $\bar{\epsilon} = kT$ as for a continuous distribution of oscillator energies, but instead

Actual average energy per standing wave $\qquad \bar{\epsilon} = \dfrac{h\nu}{e^{h\nu/kT} - 1} \qquad$ (2.6)

This average energy leads to Eq. (2.4). Blackbody radiation is further discussed in Chap. 9.

Example 2.1

Assume that a certain 660-Hz tuning fork can be considered as a harmonic oscillator whose vibrational energy is 0.04 J. Compare the energy quanta of this tuning fork with those of an atomic oscillator that emits and absorbs orange light whose frequency is 5.00×10^{14} Hz.

Solution

(*a*) For the tuning fork,

$$h\nu_1 = (6.63 \times 10^{-34}\ \text{J} \cdot \text{s})\ (660\ \text{s}^{-1}) = 4.38 \times 10^{-31}\ \text{J}$$

The total energy of the vibrating tines of the fork is therefore about 10^{29} times the quantum energy $h\nu$. The quantization of energy in the tuning fork is obviously far too small to be observed, and we are justified in regarding the fork as obeying classical physics.

(*b*) For the atomic oscillator,

$$h\nu_2 = (6.63 \times 10^{-34}\ \text{J} \cdot \text{s})\ (5.00 \times 10^{14}\ \text{s}^{-1}) = 3.32 \times 10^{-19}\ \text{J}$$

In electronvolts, the usual energy unit in atomic physics,

$$h\nu_2 = \frac{3.32 \times 10^{-19}\ \text{J}}{1.60 \times 10^{-19}\ \text{J/eV}} = 2.08\ \text{eV}$$

This is a significant amount of energy on an atomic scale, and it is not surprising that classical physics fails to account for phenomena on this scale.

The concept that the oscillators in the cavity walls can interchange energy with standing waves in the cavity only in quanta of $h\nu$ is, from the point of view of classical physics, impossible to understand. Planck regarded his quantum hypothesis as an "act of desperation" and, along with other physicists of his time, was unsure of how seriously to regard it as an element of physical reality. For many years he held that, although the energy transfers between electric oscillators and em waves apparently are quantized, em waves themselves behave in an entirely classical way with a continuous range of possible energies.

2.3 PHOTOELECTRIC EFFECT

The energies of electrons liberated by light depend on the frequency of the light

During his experiments on em waves, Hertz noticed that sparks occurred more readily in the air gap of his transmitter when ultraviolet light was directed at one of the metal balls. He did not follow up this observation, but others did. They soon discovered that the cause was electrons emitted when the frequency of the light was sufficiently high. This phenomenon is known as the **photoelectric effect** and the emitted electrons are called **photoelectrons**. It is one of the ironies of history that the same work to demonstrate that light consists of em waves also gave the first hint that this was not the whole story.

Figure 2.9 shows how the photoelectric effect was studied. An evacuated tube contains two electrodes connected to a source of variable voltage, with the metal plate whose surface is irradiated as the anode. Some of the photoelectrons that emerge from this surface have enough energy to reach the cathode despite its negative polarity, and they constitute the measured current. The slower photoelectrons are repelled before they get to the cathode. When the voltage is increased to a certain value V_0, of the order of several volts, no more photoelectrons arrive, as indicated by the current dropping to zero. This extinction voltage corresponds to the maximum photoelectron kinetic energy.

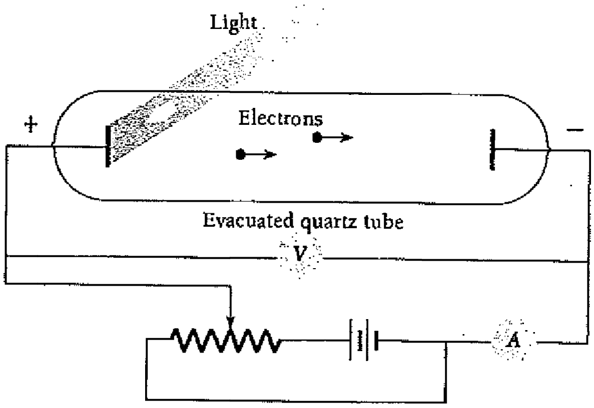


Figure 2.9 Experimental observation of the photoelectric effect.

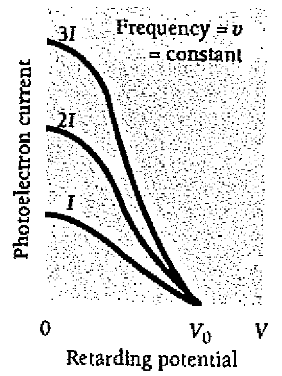


Figure 2.10 Photoelectron current is proportional to light intensity I for all retarding voltages. The stopping potential V_0, which corresponds to the maximum photoelectron energy, is the same for all intensities of light of the same frequency ν.

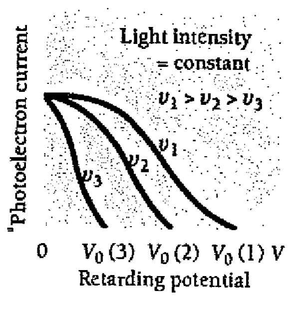


Figure 2.11 The stopping potential V_0, and hence the maximum photoelectron energy, depends on the frequency of the light. When the retarding potential is $V = 0$, the photoelectron current is the same for light of a given intensity regardless of its frequency.

The existence of the photoelectric effect is not surprising. After all, light waves carry energy, and some of the energy absorbed by the metal may somehow concentrate on individual electrons and reappear as their kinetic energy. The situation should be like water waves dislodging pebbles from a beach. But three experimental findings show that no such simple explanation is possible.

1 Within the limits of experimental accuracy (about 10^{-9} s), there is no time interval between the arrival of light at a metal surface and the emission of photoelectrons. However, because the energy in an em wave is supposed to be spread across the wavefronts, a period of time should elapse before an individual electron accumulates enough energy (several eV) to leave the metal. A detectable photoelectron current results when 10^{-6} W/m^2 of em energy is absorbed by a sodium surface. A layer of sodium 1 atom thick and 1 m^2 in area contains about 10^{19} atoms, so if the incident light is absorbed in the uppermost atomic layer, each atom receives energy at an average rate of 10^{-25} W. At this rate over a month would be needed for an atom to accumulate energy of the magnitude that photoelectrons from a sodium surface are observed to have.

2 A bright light yields more photoelectrons than a dim one of the same frequency, but the electron energies remain the same (Fig. 2.10). The em theory of light, on the contrary, predicts that the more intense the light, the greater the energies of the electrons.

3 The higher the frequency of the light, the more energy the photoelectrons have (Fig. 2.11). Blue light results in faster electrons than red light. At frequencies below a certain critical frequency ν_0, which is characteristic of each particular metal, no electrons are emitted. Above ν_0 the photoelectrons range in energy from 0 to a maximum value that increases linearly with increasing frequency (Fig. 2.12). This observation, also, cannot be explained by the em theory of light.

Quantum Theory of Light

When Planck's derivation of his formula appeared, Einstein was one of the first—perhaps the first—to understand just how radical the postulate of energy quantization

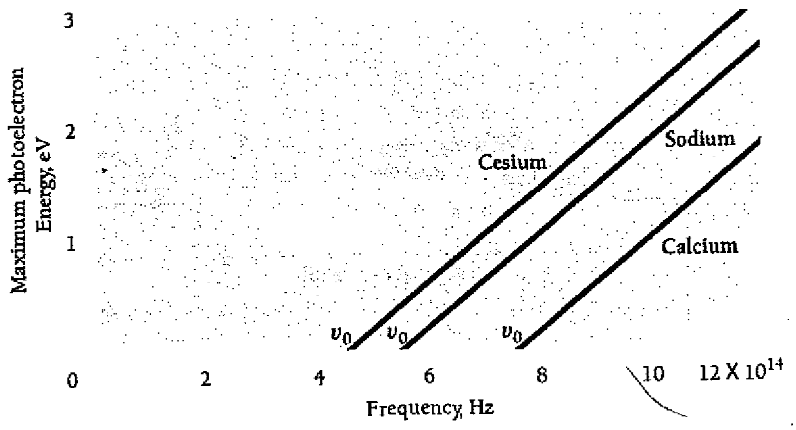


Figure 2.12 Maximum photoelectron kinetic energy KE_{max} versus frequency of incident light for three metal surfaces.

of oscillators was: "It was as if the ground was pulled from under one." A few years later, in 1905, Einstein realized that the photoelectric effect could be understood if the energy in light is not spread out over wavefronts but is concentrated in small packets, or **photons**. (The term photon was coined by the chemist Gilbert Lewis in 1926.) Each photon of light of frequency ν has the energy $h\nu$, the same as Planck's quantum energy. Planck had thought that, although energy from an electric oscillator apparently had to be given to em waves in separate quanta of $h\nu$ each, the waves themselves behaved exactly as in conventional wave theory. Einstein's break with classical physics was more drastic: Energy was not only given to em waves in separate quanta but was also carried by the waves in separate quanta.

The three experimental observations listed above follow directly from Einstein's hypothesis. (1) Because em wave energy is concentrated in photons and not spread out, there should be no delay in the emission of photoelectrons. (2) All photons of frequency ν have the same energy, so changing the intensity of a monochromatic light beam will change the number of photoelectrons but not their energies. (3) The higher the frequency ν, the greater the photon energy $h\nu$ and so the more energy the photoelectrons have.

What is the meaning of the critical frequency ν_0 below which no photoelectrons are emitted? There must be a minimum energy ϕ for an electron to escape from a particular metal surface or else electrons would pour out all the time. This energy is called the **work function** of the metal, and is related to ν_0 by the formula

Work function $$\phi = h\nu_0 \tag{2.7}$$

The greater the work function of a metal, the more energy is needed for an electron to leave its surface, and the higher the critical frequency for photoelectric emission to occur.

Some examples of photoelectric work functions are given in Table 2.1. To pull an electron from a metal surface generally takes about half as much energy as that needed

Table 2.1 Photoelectric Work Functions

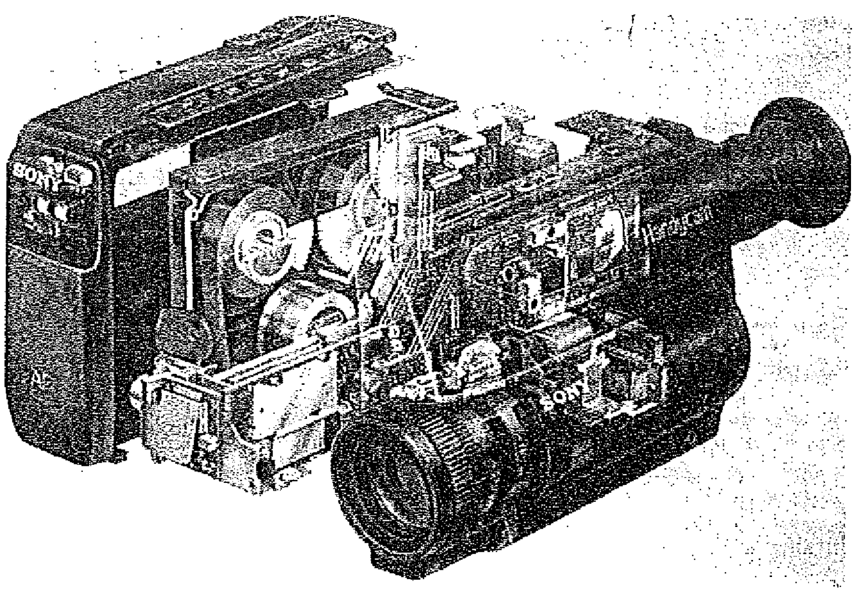
Metal	Symbol	Work Function, eV
Cesium	Cs	1.9
Potassium	K	2.2
Sodium	Na	2.3
Lithium	Li	2.5
Calcium	Ca	3.2
Copper	Cu	4.7
Silver	Ag	4.7
Platinum	Pt	6.4

to pull an electron from a free atom of that metal (see Fig. 7.10); for instance, the ionization energy of cesium is 3.9 eV compared with its work function of 1.9 eV. Since the visible spectrum extends from about 4.3 to about 7.5×10^{14} Hz, which corresponds to quantum energies of 1.7 to 3.3 eV, it is clear from Table 2.1 that the photoelectric effect is a phenomenon of the visible and ultraviolet regions.

According to Einstein, the photoelectric effect in a given metal should obey the equation

Photoelectric effect $$h\nu = \mathrm{KE}_{\max} + \phi \tag{2.8}$$

where $h\nu$ is the photon energy, $\mathrm{KE}_{\max}$ is the maximum photoelectron energy (which is proportional to the stopping potential), and ϕ is the minimum energy needed for an



All light-sensitive detectors, including the eye and the one used in this video camera, are based on the absorption of energy from photons of light by electrons in the atoms the light falls on.

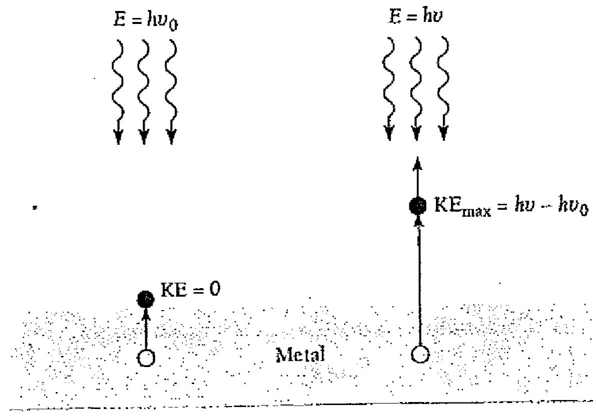


Figure 2.13 If the energy $h\nu_0$ (the work function of the surface) is needed to remove an electron from a metal surface, the maximum electron kinetic energy will be $h\nu - h\nu_0$ when light of frequency ν is directed at the surface.

electron to leave the metal. Because $\phi = h\nu_0$, Eq. (2.8) can be rewritten (Fig. 2.13)

$$h\nu = KE_{max} + h\nu_0$$

$$KE_{max} = h\nu - h\nu_0 = h(\nu - \nu_0) \tag{2.9}$$

This formula accounts for the relationships between KE_{max} and ν plotted in Fig. 2.12 from experimental data. If Einstein was right, the slopes of the lines should all be equal to Planck's constant h, and this is indeed the case.

In terms of electronvolts, the formula $E = h\nu$ for photon energy becomes

Photon energy $$E = \left(\frac{6.626 \times 10^{-34}\ \text{J}\cdot\text{s}}{1.602 \times 10^{-19}\ \text{J/eV}}\right)\nu = (4.136 \times 10^{-15})\nu\ \text{eV}\cdot\text{s} \tag{2.10}$$

If we are given instead the wavelength λ of the light, then since $\nu = c/\lambda$ we have

Photon energy $$E = \frac{(4.136 \times 10^{-15}\ \text{eV}\cdot\text{s})(2.998 \times 10^{8}\ \text{m/s})}{\lambda} = \frac{1.240 \times 10^{-6}\ \text{eV}\cdot\text{m}}{\lambda} \tag{2.11}$$

Example 2.2

Ultraviolet light of wavelength 350 nm and intensity 1.00 W/m^2 is directed at a potassium surface. (*a*) Find the maximum KE of the photoelectrons. (*b*) If 0.50 percent of the incident photons produce photoelectrons, how many are emitted per second if the potassium surface has an area of 1.00 cm^2?

Solution

(*a*) From Eq. (2.11) the energy of the photons is, since 1 nm = 1 nanometer = 10^{-9} m,

$$E_p = \frac{1.24 \times 10^{-6}\ \text{eV}\cdot\text{m}}{(350\ \text{nm})(10^{-9}\ \text{m/nm})} = 3.5\ \text{eV}$$

Table 2.1 gives the work function of potassium as 2.2 eV, so

$$KE_{max} = h\nu - \phi = 3.5 \text{ eV} - 2.2 \text{ eV} = 1.3 \text{ eV}$$

(*b*) The photon energy in joules is 5.68×10^{-19} J. Hence the number of photons that reach the surface per second is

$$n_p = \frac{E/t}{E_p} = \frac{(P/A)(A)}{E_p} = \frac{(1.00 \text{ W/m}^2)(1.00 \times 10^{-4} \text{ m}^2)}{5.68 \times 10^{-19} \text{ J/photon}} = 1.76 \times 10^{14} \text{ photons/s}$$

The rate at which photoelectrons are emitted is therefore

$$n_e = (0.0050)n_p = 8.8 \times 10^{11} \text{ photoelectrons/s}$$

Thermionic Emission

Einstein's interpretation of the photoelectric effect is supported by studies of thermionic emission. Long ago it was discovered that the presence of a very hot object increases the electric conductivity of the surrounding air. Eventually the reason for this effect was found to be the emission of electrons from such an object. Thermionic emission makes possible the operation of such devices as television picture tubes, in which metal filaments or specially coated cathodes at high temperature supply dense streams of electrons.

The emitted electrons evidently obtain their energy from the thermal agitation of the particles of the metal, and we would expect the electrons to need a certain minimum energy to escape. This minimum energy can be determined for many surfaces, and it is always close to the photoelectric work function for the same surfaces. In photoelectric emission, photons of light provide the energy required by an electron to escape, while in thermionic emission heat does so.

2.4 WHAT IS LIGHT?

Both wave and particle

The concept that light travels as a series of little packets is directly opposed to the wave theory of light (Fig. 2.14). Both views have strong experimental support, as we have seen. According to the wave theory, light waves leave a source with their energy spread out continuously through the wave pattern. According to the quantum theory, light consists of individual photons, each small enough to be absorbed by a single electron. Yet, despite the particle picture of light it presents, the quantum theory needs the frequency of the light to describe the photon energy.

Which theory are we to believe? A great many scientific ideas have had to be revised or discarded when they were found to disagree with new data. Here, for the first time, two different theories are needed to explain a single phenomenon. This situation is not the same as it is, say, in the case of relativistic versus newtonian mechanics, where one turns out to be an approximation of the other. The connection between the wave and quantum theories of light is something else entirely.

To appreciate this connection, let us consider the formation of a double-slit interference pattern on a screen. In the wave model, the light intensity at a place on the screen depends on $\overline{E^2}$, the average over a complete cycle of the square of the instantaneous magnitude E of the em wave's electric field. In the particle model, this

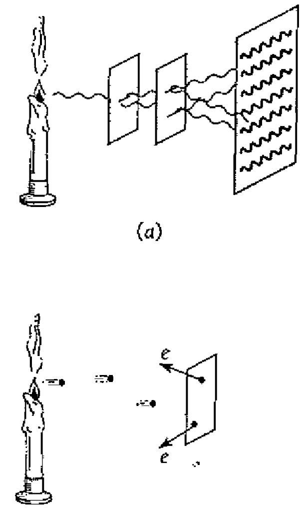


Figure 2.14 (*a*) The wave theory of light explains diffraction and interference, which the quantum theory cannot account for. (*b*) The quantum theory explains the photoelectric effect, which the wave theory cannot account for.

intensity depends instead on $Nh\nu$, where N is the number of photons per second per unit area that reach the same place on the screen. Both descriptions must give the same value for the intensity, so N is proportional to $\bar{E}^2$. If N is large enough, somebody looking at the screen would see the usual double-slit interference pattern and would have no reason to doubt the wave model. If N is small—perhaps so small that only one photon at a time reaches the screen—the observer would find a series of apparently random flashes and would assume that he or she is watching quantum behavior.

If the observer keeps track of the flashes for long enough, though, the pattern they form will be the same as when N is large. Thus the observer is entitled to conclude that the *probability* of finding a photon at a certain place and time depends on the value of $\bar{E}^2$ there. If we regard each photon as somehow having a wave associated with it, the intensity of this wave at a given place on the screen determines the likelihood that a photon will arrive there. When it passes through the slits, light is behaving as a wave does. When it strikes the screen, light is behaving as a particle does. Apparently light travels as a wave but absorbs and gives off energy as a series of particles.

We can think of light as having a dual character. **The wave theory and the quantum theory complement each other.** Either theory by itself is only part of the story and can explain only certain effects. A reader who finds it hard to understand how light can be both a wave and a stream of particles is in good company: shortly before his death, Einstein remarked that "All these fifty years of conscious brooding have brought me no nearer to the answer to the question, 'What are light quanta?'" The "true nature" of light includes both wave and particle characters, even though there is nothing in everyday life to help us visualize that.

2.5 X-RAYS

They consist of high-energy photons

The photoelectric effect provides convincing evidence that photons of light can transfer energy to electrons. Is the inverse process also possible? That is, can part or all of the kinetic energy of a moving electron be converted into a photon? As it happens, the inverse photoelectric effect not only does occur but had been discovered (though not understood) before the work of Planck and Einstein.

In 1895 Wilhelm Roentgen found that a highly penetrating radiation of unknown nature is produced when fast electrons impinge on matter. These **x-rays** were soon found to travel in straight lines, to be unaffected by electric and magnetic fields, to pass readily through opaque materials, to cause phosphorescent substances to glow, and to expose photographic plates. The faster the original electrons, the more penetrating the resulting x-rays, and the greater the number of electrons, the greater the intensity of the x-ray beam.

Not long after this discovery it became clear that x-rays are em waves. Electromagnetic theory predicts that an accelerated electric charge will radiate em waves, and a rapidly moving electron suddenly brought to rest is certainly accelerated. Radiation produced under these circumstances is given the German name **bremsstrahlung** ("braking radiation"). Energy loss due to bremsstrahlung is more important for electrons than for heavier particles because electrons are more violently accelerated when passing near nuclei in their paths. The greater the energy of an electron and the greater the atomic number of the nuclei it encounters, the more energetic the bremsstrahlung.

Wilhelm Konrad Roentgen (1845–1923) was born in Lennep, Germany, and studied in Holland and Switzerland. After periods at several German universities, Roentgen became professor of physics at Würzburg where, on November 8, 1895, he noticed that a sheet of paper coated with barium platinocyanide glowed when he switched on a nearby cathode-ray tube that was entirely covered with black cardboard. In a cathode-ray tube electrons are accelerated in a vacuum by an electric field, and it was the impact of these electrons on the glass end of the tube that produced the penetrating "x" (since their nature was then unknown) rays that caused the salt to glow. Roentgen said of his discovery that, when people heard of it, they would say, "Roentgen has probably gone crazy." In fact, x-rays were an immediate sensation, and only two months later were being used in medicine. They also stimulated research in new directions; Becquerel's discovery of radioactivity followed within a year. Roentgen received the first Nobel Prize in physics in 1902. He refused to benefit financially from his work and died in poverty in the German inflation that followed the end of World War I.

In 1912 a method was devised for measuring the wavelengths of x-rays. A diffraction experiment had been recognized as ideal, but as we recall from physical optics, the spacing between adjacent lines on a diffraction grating must be of the same order of magnitude as the wavelength of the light for satisfactory results, and gratings cannot be ruled with the minute spacing required by x-rays. Max von Laue realized that the wavelengths suggested for x-rays were comparable to the spacing between adjacent atoms in crystals. He therefore proposed that crystals be used to diffract x-rays, with their regular lattices acting as a kind of three-dimensional grating. In experiments carried out the following year, wavelengths from 0.013 to 0.048 nm were found, 10^{-4} of those in visible light and hence having quanta 10^4 times as energetic.

Electromagnetic radiation with wavelengths from about 0.01 to about 10 nm falls into the category of x-rays. The boundaries of this category are not sharp: the shorter-wavelength end overlaps gamma rays and the longer-wavelength end overlaps ultraviolet light (see Fig. 2.2).

Figure 2.15 is a diagram of an x-ray tube. A cathode, heated by a filament through which an electric current is passed, supplies electrons by thermionic emission. The high potential difference V maintained between the cathode and a metallic target accelerates the electrons toward the latter. The face of the target is at an angle relative to the electron beam, and the x-rays that leave the target pass through the

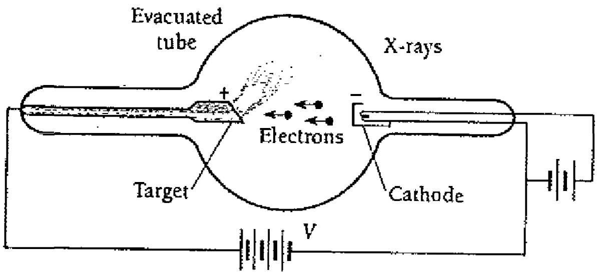
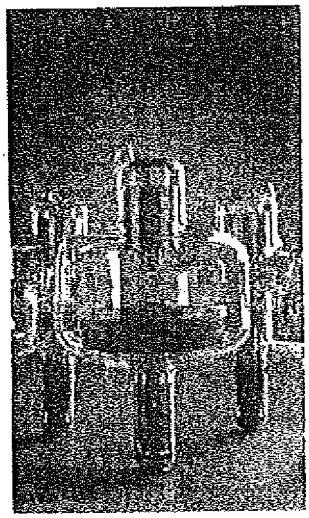


Figure 2.15 An x-ray tube. The higher the accelerating voltage V, the faster the electrons and the shorter the wavelengths of the x-rays.



In modern x-ray tubes like these, circulating oil carries heat away from the target and releases it to the outside air through a heat exchanger. The use of x-rays as a diagnostic tool in medicine is based upon the different extents to which different tissues absorb them. Because of its calcium content, bone is much more opaque to x-rays than muscle, which in turn is more opaque than fat. To enhance contrast, "meals" that contain barium are given to patients to better display their digestive systems, and other compounds may be injected into the bloodstream to enable the condition of blood vessels to be studied.

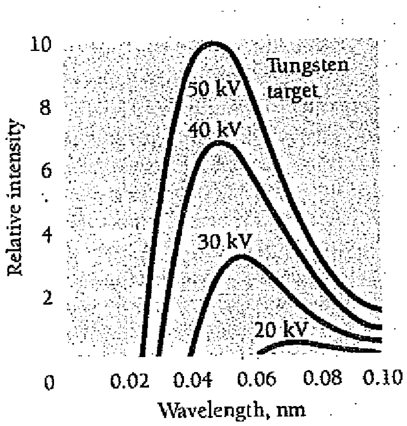


Figure 2.16 X-ray spectra of tungsten at various accelerating potentials.

side of the tube. The tube is evacuated to permit the electrons to get to the target unimpeded.

As mentioned earlier, classical electromagnetic theory predicts bremsstrahlung when electrons are accelerated, which accounts in general for the x-rays produced by an x-ray tube. However, the agreement between theory and experiment is not satisfactory in certain important respects. Figures 2.16 and 2.17 show the x-ray spectra that result when tungsten and molybdenum targets are bombarded by electrons at several different accelerating potentials. The curves exhibit two features electromagnetic theory cannot explain:

1 In the case of molybdenum, intensity peaks occur that indicate the enhanced production of x-rays at certain wavelengths. These peaks occur at specific wavelengths for each target material and originate in rearrangements of the electron structures of the

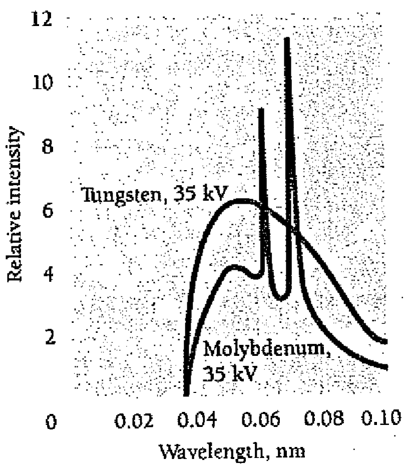
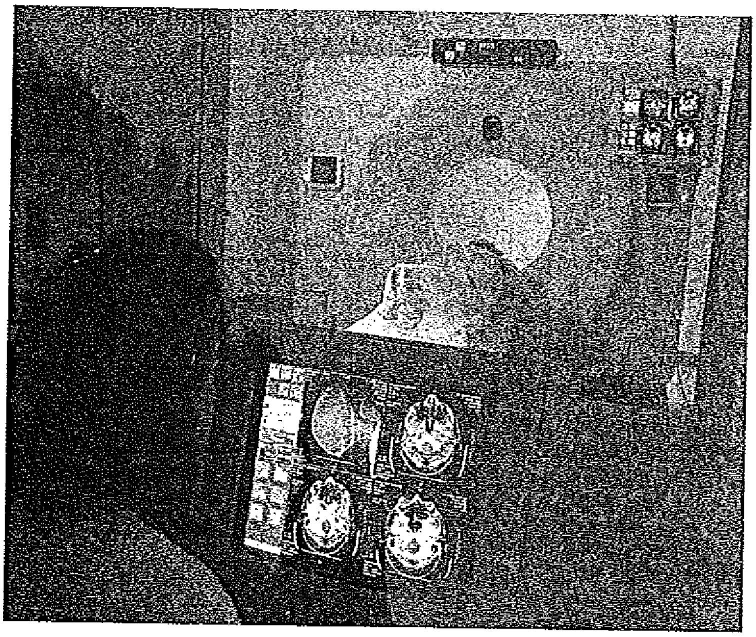


Figure 2.17 X-ray spectra of tungsten and molybdenum at 35 kV accelerating potential.



In a CT (computerized tomography) scanner, a series of x-ray exposures of a patient taken from different directions are combined by a computer to give cross-sectional images of the parts of the body being examined. In effect, the tissue is sliced up by the computer on the basis of the x-ray exposures, and any desired slice can be displayed. This technique enables an abnormality to be detected and its exact location established, which might be impossible to do from an ordinary x-ray picture. (The word tomography comes from tomos, Greek for "cut.")

target atoms after having been disturbed by the bombarding electrons. This phenomenon will be discussed in Sec. 7.9; the important thing to note at this point is the presence of x-rays of specific wavelengths, a decidedly nonclassical effect, in addition to a continuous x-ray spectrum.

2 The x-rays produced at a given accelerating potential V vary in wavelength, but none has a wavelength shorter than a certain value λ_{min}. Increasing V decreases λ_{min}. At a particular V, λ_{min} is the *same* for both the tungsten and molybdenum targets. Duane and Hunt found experimentally that λ_{min} is inversely proportional to V; their precise relationship is

X-ray production

$$\lambda_{min} = \frac{1.24 \times 10^{-6}}{V} \text{ V} \cdot \text{m} \tag{2.12}$$

The second observation fits in with the quantum theory of radiation. Most of the electrons that strike the target undergo numerous glancing collisions, with their energy going simply into heat. (This is why the targets in x-ray tubes are made from high-melting-point metals such as tungsten, and a means of cooling the target is usually employed.) A few electrons, though, lose most or all of their energy in single collisions with target atoms. This is the energy that becomes x-rays.

X-rays production, then, except for the peaks mentioned in observation 1 above, represents an inverse photoelectric effect. Instead of photon energy being transformed into electron KE, electron KE is being transformed into photon energy. A short wavelength means a high frequency, and a high frequency means a high photon energy $h\nu$.

Since work functions are only a few electronvolts whereas the accelerating potentials in x-ray tubes are typically tens or hundreds of thousands of volts, we can ignore the work function and interpret the short wavelength limit of Eq. (2.12) as corresponding to the case where the entire kinetic energy KE $= Ve$ of a bombarding electron is given up to a single photon of energy $h\nu_{max}$. Hence

$$Ve = h\nu_{max} = \frac{hc}{\lambda_{min}}$$

$$\lambda_{min} = \frac{hc}{Ve} = \frac{1.240 \times 10^{-6}}{V} \text{ V} \cdot \text{m}$$

which is the Duane-Hunt formula of Eq. (2.12)—and, indeed, the same as Eq. (2.11) except for different units. It is therefore appropriate to regard x-ray production as the inverse of the photoelectric effect.

Example 2.3

Find the shortest wavelength present in the radiation from an x-ray machine whose accelerating potential is 50,000 V.

Solution

From Eq. (2.12) we have

$$\lambda_{min} = \frac{1.24 \times 10^{-6} \text{ V} \cdot \text{m}}{5.00 \times 10^{4} \text{ V}} = 2.48 \times 10^{-11} \text{ m} = 0.0248 \text{ nm}$$

This wavelength corresponds to the frequency

$$\nu_{max} = \frac{c}{\lambda_{min}} = \frac{3.00 \times 10^{8} \text{ m/s}}{2.48 \times 10^{-11} \text{ m}} = 1.21 \times 10^{19} \text{ Hz}$$

2.6 X-RAY DIFFRACTION

How x-ray wavelengths can be determined

A crystal consists of a regular array of atoms, each of which can scatter em waves. The mechanism of scattering is straightforward. An atom in a constant electric field becomes polarized since its negatively charged electrons and positively charged nucleus experience forces in opposite directions. These forces are small compared with the forces holding the atom together, and so the result is a distorted charge distribution equivalent to an electric dipole. In the presence of the alternating electric field of an em wave of frequency ν, the polarization changes back and forth with the same frequency ν. An oscillating electric dipole is thus created at the expense of some of the energy of the incoming wave. The oscillating dipole in turn radiates em waves of frequency ν, and these secondary waves go out in all directions except along the dipole axis. (In an assembly of atoms exposed to unpolarized radiation, the latter restriction does not apply since the contributions of the individual atoms are random.)

In wave terminology, the secondary waves have spherical wave fronts in place of the plane wave fronts of the incoming waves (Fig. 2.18). The scattering process, then,

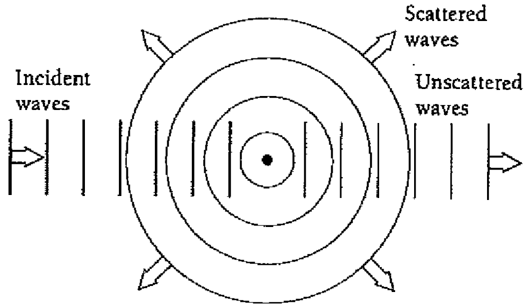


Figure 2.18 The scattering of electromagnetic radiation by a group of atoms. Incident plane waves are reemitted as spherical waves.

involves atoms that absorb incident plane waves and reemit spherical waves of the same frequency.

A monochromatic beam of x-rays that falls upon a crystal will be scattered in all directions inside it. However, owing to the regular arrangement of the atoms, in certain directions the scattered waves will constructively interfere with one another while in others they will destructively interfere. The atoms in a crystal may be thought of as defining families of parallel planes, as in Fig. 2.19, with each family having a characteristic separation between its component planes. This analysis was suggested in 1913 by W. L. Bragg, in honor of whom the above planes are called **Bragg planes.**

The conditions that must be fulfilled for radiation scattered by crystal atoms to undergo constructive interference may be obtained from a diagram like that in Fig. 2.20. A beam containing x-rays of wavelength λ is incident upon a crystal at an angle θ with a family of Bragg planes whose spacing is d. The beam goes past atom A in the first plane and atom B in the next, and each of them scatters part of the beam in random directions. Constructive interference takes place only between those scattered rays that are parallel and whose paths differ by exactly λ, 2λ, 3λ, and so on. That is, the path difference must be $n\lambda$, where n is an integer. The only rays scattered by A and B for which this is true are those labeled I and II in Fig. 2.20.

The first condition on I and II is that their common scattering angle be equal to the angle of incidence θ of the original beam. (This condition, which is independent

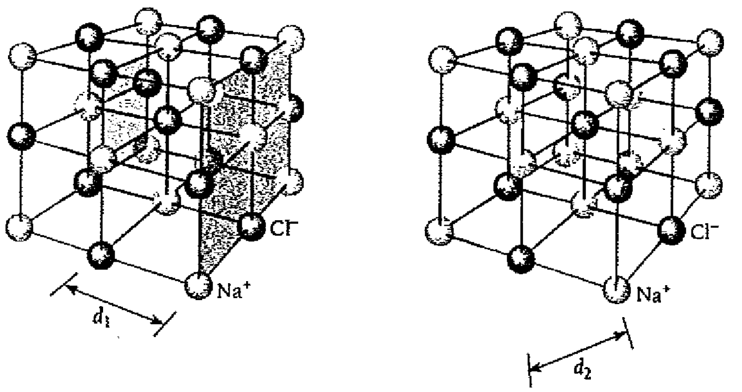
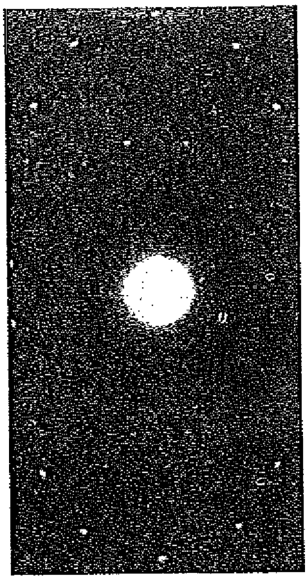


Figure 2.19 Two sets of Bragg planes in a NaCl crystal.



The interference pattern produced by the scattering of x-rays from ions in a crystal of NaCl. The bright spots correspond to the directions where x-rays scattered from various layers in the crystal interfere constructively. The cubic pattern of the NaCl lattice is suggested by he fourfold symmetry of the pattern. The large central spot is due to the unscattered x-ray beam.

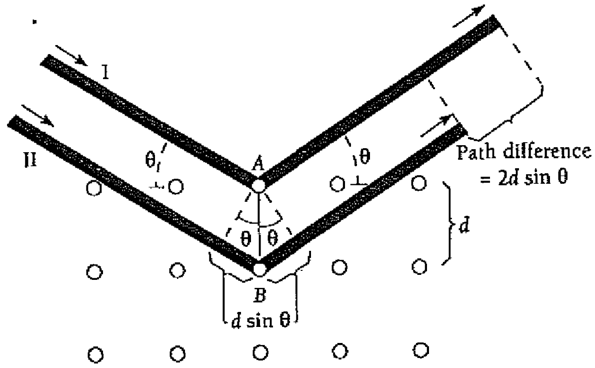


Figure 2.20 X-ray scattering from a cubic crystal.

of wavelength, is the same as that for ordinary specular reflection in optics: angle of incidence = angle of reflection.) The second condition is that

$$2d \sin \theta = n\lambda \qquad n = 1, 2, 3, \ldots \tag{2.13}$$

since ray II must travel the distance $2d \sin \theta$ farther than ray I. The integer n is the order of the scattered beam.

The schematic design of an x-ray spectrometer based upon Bragg's analysis is shown in Fig. 2.21. A narrow beam of x-rays falls upon a crystal at an angle θ, and a detector is placed so that it records those rays whose scattering angle is also θ. Any x-rays reaching the detector therefore obey the first Bragg condition. As θ is varied, the detector

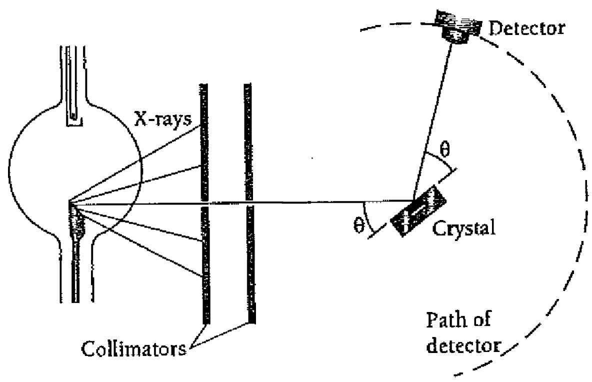


Figure 2.21 X-ray spectrometer.

will record intensity peaks corresponding to the orders predicted by Eq. (2.13). If the spacing d between adjacent Bragg planes in the crystal is known, the x-ray wavelength λ may be calculated.

2.7 COMPTON EFFECT

Further confirmation of the photon model

According to the quantum theory of light, photons behave like particles except for their lack of rest mass. How far can this analogy be carried? For instance, can we consider a collision between a photon and an electron as if both were billiard balls?

Figure 2.22 shows such a collision: an x-ray photon strikes an electron (assumed to be initially at rest in the laboratory coordinate system) and is scattered away from its original direction of motion while the electron receives an impulse and begins to move. We can think of the photon as losing an amount of energy in the collision that is the same as the kinetic energy KE gained by the electron, although actually separate photons are involved. If the initial photon has the frequency ν associated with it, the scattered photon has the lower frequency ν', where

$$\text{Loss in photon energy} = \text{gain in electron energy}$$

$$h\nu - h\nu' = \text{KE} \tag{2.14}$$

From Chap. 1 we recall that the momentum of a massless particle is related to its energy by the formula

$$E = pc \tag{1.25}$$

Since the energy of a photon is $h\nu$, its momentum is

Photon momentum

$$p = \frac{E}{c} = \frac{h\nu}{c} \tag{2.15}$$

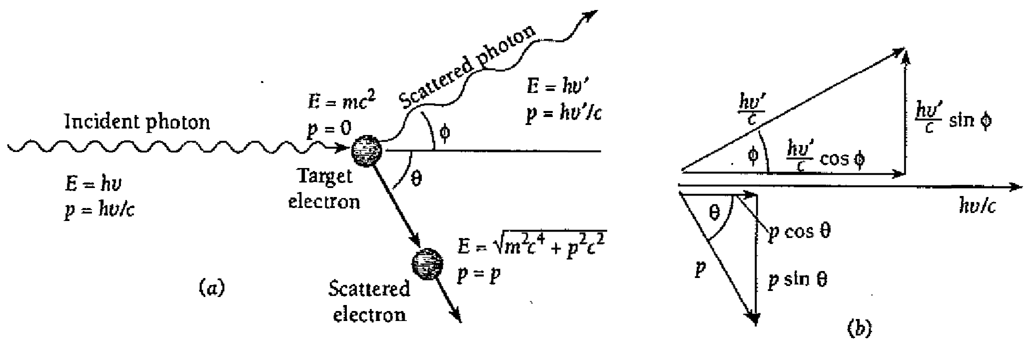


Figure 2.22 (*a*) The scattering of a photon by an electron is called the Compton effect. Energy and momentum are conserved in such an event, and as a result the scattered photon has less energy (longer wavelength) than the incident photon. (*b*) Vector diagram of the momenta and their components of the incident and scattered photons and the scattered electron.

Momentum, unlike energy, is a vector quantity that incorporates direction as well as magnitude, and in the collision momentum must be conserved in each of two mutually perpendicular directions. (When more than two bodies participate in a collision, momentum must be conserved in each of three mutually perpendicular directions.) The directions we choose here are that of the original photon and one perpendicular to it in the plane containing the electron and the scattered photon (Fig. 2.22).

The initial photon momentum is $h\nu/c$, the scattered photon momentum is $h\nu'/c$, and the initial and final electron momenta are respectively 0 and p. In the original photon direction

Initial momentum = final momentum

$$\frac{h\nu}{c} + 0 = \frac{h\nu'}{c} \cos\phi + p\cos\theta \tag{2.16}$$

and perpendicular to this direction

Initial momentum = final momentum

$$0 = \frac{h\nu'}{c} \sin\phi - p\sin\theta \tag{2.17}$$

The angle ϕ is that between the directions of the initial and scattered photons, and θ is that between the directions of the initial photon and the recoil electron. From Eqs. (2.14), (2.16), and (2.17) we can find a formula that relates the wavelength difference between initial and scattered photons with the angle ϕ between their directions, both of which are readily measurable quantities (unlike the energy and momentum of the recoil electron).

The first step is to multiply Eqs. (2.16) and (2.17) by c and rewrite them as

$$pc\cos\theta = h\nu - h\nu'\cos\phi$$

$$pc\sin\theta = h\nu'\sin\phi$$

By squaring each of these equations and adding the new ones together, the angle θ is eliminated, leaving

$$p^2c^2 = (h\nu)^2 - 2(h\nu)(h\nu')\cos\phi + (h\nu')^2 \tag{2.18}$$

Next we equate the two expressions for the total energy of a particle

$$E = \text{KE} + mc^2 \tag{1.20}$$

$$E = \sqrt{m^2c^4 + p^2c^2} \tag{1.24}$$

from Chap. 1 to give

$$(\text{KE} + mc^2)^2 = m^2c^4 + p^2c^2$$

$$p^2c^2 = \text{KE}^2 + 2mc^2\,\text{KE}$$

Since

$$\text{KE} = h\nu - h\nu'$$

we have

$$p^2c^2 = (h\nu)^2 - 2(h\nu)(h\nu') + (h\nu')^2 + 2mc^2(h\nu - h\nu') \qquad (2.19)$$

Substituting this value of p^2c^2 in Eq. (2.18), we finally obtain

$$2mc^2(h\nu - h\nu') = 2(h\nu)(h\nu')(1 - \cos\phi) \qquad (2.20)$$

This relationship is simpler when expressed in terms of wavelength λ. Dividing Eq. (2.20) by $2h^2c^2$,

$$\frac{mc}{h}\left(\frac{\nu}{c} - \frac{\nu'}{c}\right) = \frac{\nu}{c}\frac{\nu'}{c}(1 - \cos\phi)$$

and so, since $\nu/c = 1/\lambda$ and $\nu'/c = 1/\lambda'$,

$$\frac{mc}{h}\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right) = \frac{1 - \cos\phi}{\lambda\lambda'}$$

Compton effect

$$\lambda' - \lambda = \frac{h}{mc}(1 - \cos\phi) \qquad (2.21)$$

Equation (2.21) was derived by Arthur H. Compton in the early 1920s, and the phenomenon it describes, which he was the first to observe, is known as the **Compton effect**. It constitutes very strong evidence in support of the quantum theory of radiation.

Equation (2.21) gives the change in wavelength expected for a photon that is scattered through the angle ϕ by a particle of rest mass m. This change is independent of the wavelength λ of the incident photon. The quantity

Compton wavelength

$$\lambda_C = \frac{h}{mc} \qquad (2.22)$$

is called the **Compton wavelength** of the scattering particle. For an electron $\lambda_C = 2.426 \times 10^{-12}$ m, which is 2.426 pm (1 pm = 1 picometer = 10^{-12} m). In terms of λ_C, Eq. (2.21) becomes

Compton effect

$$\lambda' - \lambda = \lambda_C(1 - \cos\phi) \qquad (2.23)$$

The Compton wavelength gives the scale of the wavelength change of the incident photon. From Eq. (2.23) we note that the greatest wavelength change possible corresponds to $\phi = 180°$, when the wavelength change will be twice the Compton wavelength λ_C. Because $\lambda_C = 2.426$ pm for an electron, and even less for other particles owing to their larger rest masses, the maximum wavelength change in the Compton effect is 4.852 pm. Changes of this magnitude or less are readily observable only in x-rays: the shift in wavelength for visible light is less than 0.01 percent of the initial wavelength, whereas for x-rays of $\lambda = 0.1$ nm it is several percent. The Compton effect is the chief means by which x-rays lose energy when they pass through matter.

Arthur Holly Compton (1892–1962), a native of Ohio, was educated at College of Wooster and Princeton. While at Washington University in St. Louis he found that x-rays increase in wavelength when scattered, which he explained in 1923 on the basis of the quantum theory of light. This work convinced remaining doubters of the reality of photons.

After receiving the Nobel Prize in 1927, Compton, now at the University of Chicago, studied cosmic rays and helped establish that they are fast charged particles (today known to be atomic nuclei, largely protons) that circulate in space and are not high-energy gamma rays as many had thought. He did this by showing that cosmic-ray intensity varies with latitude, which makes sense only if they are ions whose paths are influenced by the earth's magnetic field. During World War II Compton was one of the leaders in the development of the atomic bomb.

Example 2.4

X-rays of wavelength 10.0 pm are scattered from a target. (*a*) Find the wavelength of the x-rays scattered through 45°. (*b*) Find the maximum wavelength present in the scattered x-rays. (*c*) Find the maximum kinetic energy of the recoil electrons.

Solution

(*a*) From Eq. (2.23), $\lambda' - \lambda = \lambda_C(1 - \cos\phi)$, and so

$$\begin{aligned}\lambda' &= \lambda + \lambda_C(1 - \cos 45^\circ)\\ &= 10.0\ \text{pm} + 0.293\lambda_C\\ &= 10.7\ \text{pm}\end{aligned}$$

(*b*) $\lambda' - \lambda$ is a maximum when $(1 - \cos\phi) = 2$, in which case

$$\lambda' = \lambda + 2\lambda_C = 10.0\ \text{pm} + 4.9\ \text{pm} = 14.9\ \text{pm}$$

(*c*) The maximum recoil kinetic energy is equal to the difference between the energies of the incident and scattered photons, so

$$\text{KE}_{\text{max}} = h(\nu - \nu') = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right)$$

where λ' is given in (*b*). Hence

$$\begin{aligned}\text{KE}_{\text{max}} &= \frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{10^{-12}\ \text{m/pm}}\left(\frac{1}{10.0\ \text{pm}} - \frac{1}{14.9\ \text{pm}}\right)\\ &= 6.54\times10^{-15}\ \text{J}\end{aligned}$$

which is equal to 40.8 keV.

The experimental demonstration of the Compton effect is straightforward. As in Fig. 2.23, a beam of x-rays of a single, known wavelength is directed at a target, and the wavelengths of the scattered x-rays are determined at various angles ϕ. The results, shown in Fig. 2.24, exhibit the wavelength shift predicted by Eq. (2.21), but at each angle the scattered x-rays also include many that have the initial wavelength. This is not hard to understand. In deriving Eq. (2.21) it was assumed that the scattering particle is able to move freely, which is reasonable since many of the electrons in matter

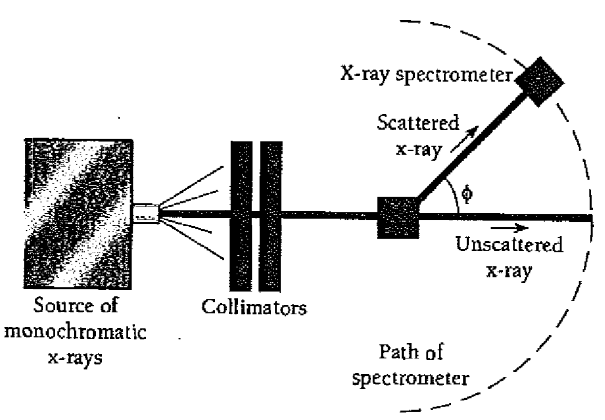


Figure 2.23 Experimental demonstration of the Compton effect.

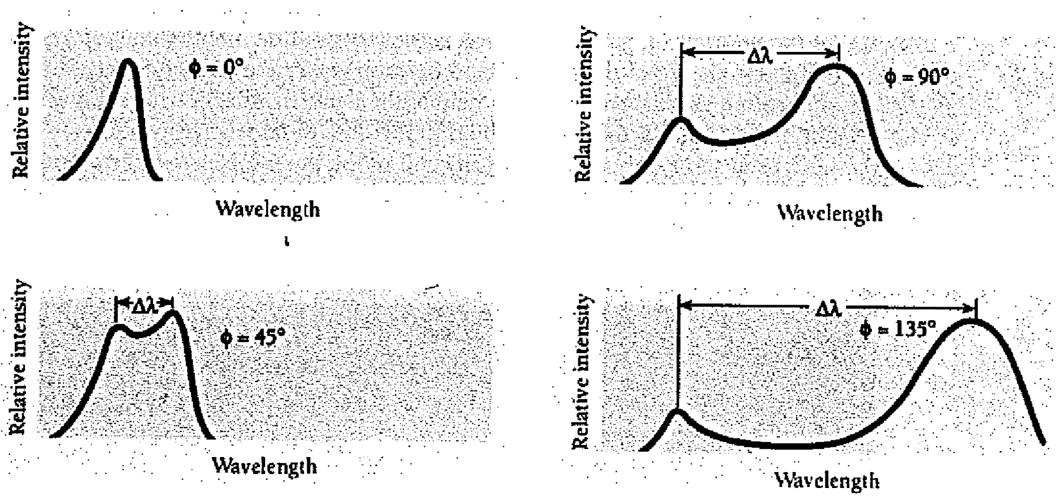


Figure 2.24 Experimental confirmation of Compton scattering. The greater the scattering angle, the greater the wavelength change, in accord with Eq. (2.21).

are only loosely bound to their parent atoms. Other electrons, however, are very tightly bound and when struck by a photon, the entire atom recoils instead of the single electron. In this event the value of m to use in Eq. (2.21) is that of the entire atom, which is tens of thousands of times greater than that of an electron, and the resulting Compton shift is accordingly so small as to be undetectable.

2.8 PAIR PRODUCTION

Energy into matter

As we have seen, in a collision a photon can give an electron all of its energy (the photoelectric effect) or only part (the Compton effect). It is also possible for a photon to materialize into an electron and a positron, which is a positively charged electron. In this process, called **pair production**, electromagnetic energy is converted into matter.

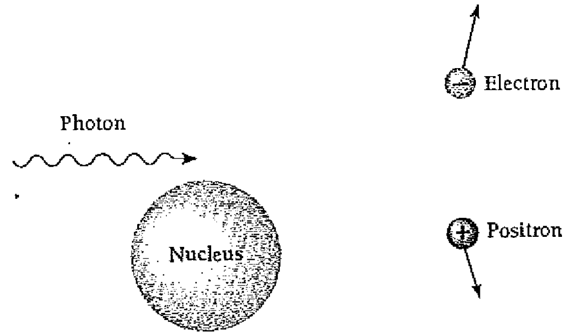
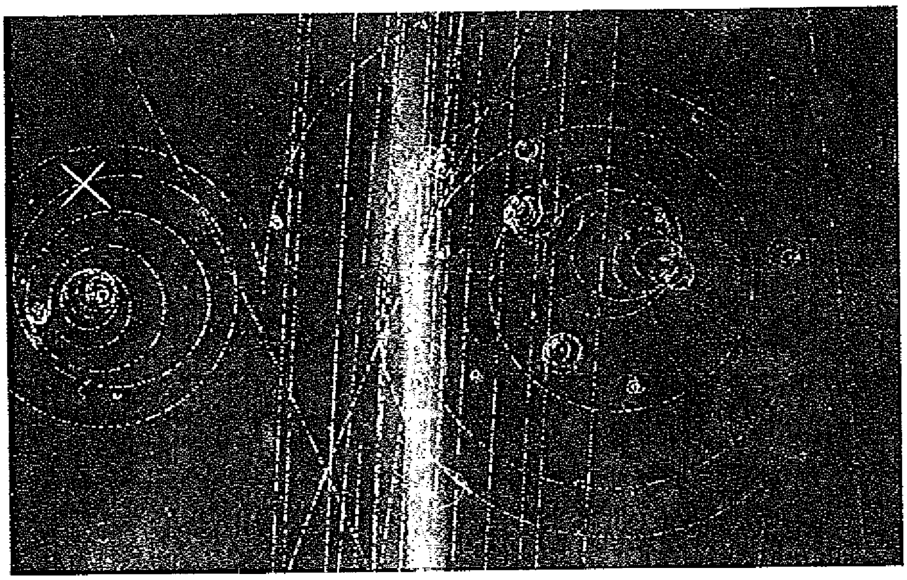


Figure 2.25 In the process of pair production, a photon of sufficient energy materializes into an electron and a positron.

No conservation principles are violated when an electron-positron pair is created near an atomic nucleus (Fig. 2.25). The sum of the charges of the electron ($q = -e$) and of the positron ($q = +e$) is zero, as is the charge of the photon; the total energy, including rest energy, of the electron and positron equals the photon energy; and linear momentum is conserved with the help of the nucleus, which carries away enough photon momentum for the process to occur. Because of its relatively enormous mass, the nucleus absorbs only a negligible fraction of the photon energy. (Energy and linear momentum could not both be conserved if pair production were to occur in empty space, so it does not occur there.)



Bubble-chamber photograph of electron-positron pair formation. A magnetic field perpendicular to the page caused the electron and positron to move in opposite curved paths, which are spirals because the particles lost energy as they moved through the chamber. In a bubble chamber, a liquid (here, hydrogen) is heated above its normal boiling point under a pressure great enough to keep it liquid. The pressure is then released, and bubbles form around any ions present in the resulting unstable superheated liquid. A charged particle moving through the liquid at this time leaves a track of bubbles that can be photographed.

The rest energy mc^2 of an electron or positron is 0.51 MeV, hence pair production requires a photon energy of at least 1.02 MeV. Any additional photon energy becomes kinetic energy of the electron and positron. The corresponding maximum photon wavelength is 1.2 pm. Electromagnetic waves with such wavelengths are called **gamma rays**, symbol γ, and are found in nature as one of the emissions from radioactive nuclei and in cosmic rays.

The inverse of pair production occurs when a positron is near an electron and the two come together under the influence of their opposite electric charges. Both particles vanish simultaneously, with the lost mass becoming energy in the form of two gamma-ray photons:

$$e^+ + e^- \rightarrow \gamma + \gamma$$

The total mass of the positron and electron is equivalent to 1.02 MeV, and each photon has an energy $h\nu$ of 0.51 MeV plus half the kinetic energy of the particles relative to their center of mass. The directions of the photons are such as to conserve both energy and linear momentum, and no nucleus or other particle is needed for this **pair annihilation** to take place.

Example 2.5

Show that pair production cannot occur in empty space.

Solution

From conservation of energy,

$$h\nu = 2\gamma mc^2$$

where $h\nu$ is the photon energy and γmc^2 is the total energy of each member of the electron-position pair. Figure 2.26 is a vector diagram of the linear momenta of the photon, electron, and positron. The angles θ are equal in order that momentum be conserved in the transverse direction. In the direction of motion of the photon, for momentum to be conserved it must be true that

$$\frac{h\nu}{c} = 2p\cos\theta$$

$$h\nu = 2pc\cos\theta$$

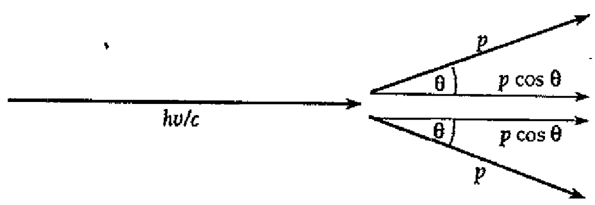


Figure 2.26 Vector diagram of the momenta involved if a photon were to materialize into an electron-positron pair in empty space. Because such an event cannot conserve both energy and momentum, it does not occur. Pair production always involves an atomic nucleus that carries away part of the initial photon momentum.

Since $p = \gamma mv$ for the electron and positron,

$$h\nu = 2\gamma mc^2\left(\frac{v}{c}\right)\cos\theta$$

Because $v/c < 1$ and $\cos\theta \leq 1$,

$$h\nu < 2\gamma mc^2$$

But conservation of energy requires that $h\nu = 2\gamma mc^2$. Hence it is impossible for pair production to conserve both energy and momentum unless some other object is involved in the process to carry away part of the initial photon momentum.

Example 2.6

An electron and a positron are moving side by side in the $+x$ direction at $0.500c$ when they annihilate each other. Two photons are produced that move along the x axis. (*a*) Do both photons move in the $+x$ direction? (*b*) What is the energy of each photon?

Solution

(*a*) In the center-of-mass (CM) system (which is the system moving with the original particles), the photons move off in opposite directions to conserve momentum. They must also do so in the lab system because the speed of the CM system is less than the speed c of the photons.

(*b*) Let p_1 be the momentum of the photon moving in the $+x$ direction and p_2 be the momentum of the photon moving in the $-x$ direction. Then conservation of momentum (in the lab system) gives

$$p_1 - p_2 = 2\gamma mv = \frac{2(mc^2)(v/c^2)}{\sqrt{1 - v/c^2}}$$

$$= \frac{2(0.511\ \text{MeV}/c^2)(c^2)(0.500c)/c^2}{\sqrt{1-(0.500)^2}} = 0.590\ \text{MeV}/c$$

Conservation of energy gives

$$p_1c + p_2c = 2\gamma mc^2 = \frac{2mc^2}{\sqrt{1 - v^2/c^2}} = \frac{2(0.511\ \text{MeV})}{\sqrt{1-(0.500)^2}} = 1.180\ \text{MeV}$$

and so

$$p_1 + p_2 = 1.180\ \text{MeV}/c$$

Now we add the two results and solve for p_1 and p_2:

$$(p_1 - p_2) + (p_1 + p_2) = 2p_1 = (0.590 + 1.180)\ \text{MeV}/c$$

$$p_1 = 0.885\ \text{MeV}/c$$

$$p_2 = (p_1 + p_2) - p_1 = 0.295\ \text{MeV}/c$$

The photon energies are accordingly

$$E_1 = p_1c = 0.885\ \text{MeV} \qquad E_2 = p_2c = 0.295\ \text{MeV}$$

Photon Absorption

The three chief ways in which photons of light, x-rays, and gamma rays interact with matter are summarized in Fig. 2.27. In all cases photon energy is transferred to electrons which in turn lose energy to atoms in the absorbing material.

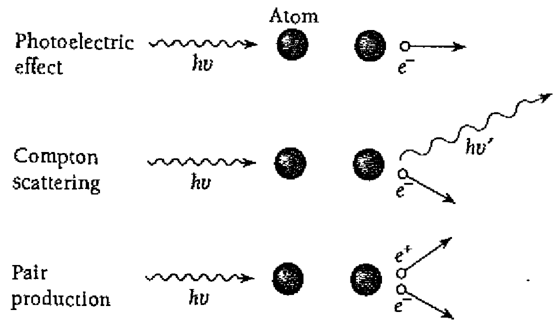


Figure 2.27 X- and gamma rays interact with matter chiefly through the photoelectric effect, Compton scattering, and pair production. Pair production requires a photon energy of at least 1.02 MeV.

At low photon energies the photoelectric effect is the chief mechanism of energy loss. The importance of the photoelectric effect decreases with increasing energy, to be succeeded by Compton scattering. The greater the atomic number of the absorber, the higher the energy at which the photoelectric effect remains significant. In the lighter elements, Compton scattering becomes dominant at photon energies of a few tens of keV, whereas in the heavier ones this does not happen until photon energies of nearly 1 MeV are reached (Fig. 2.28).

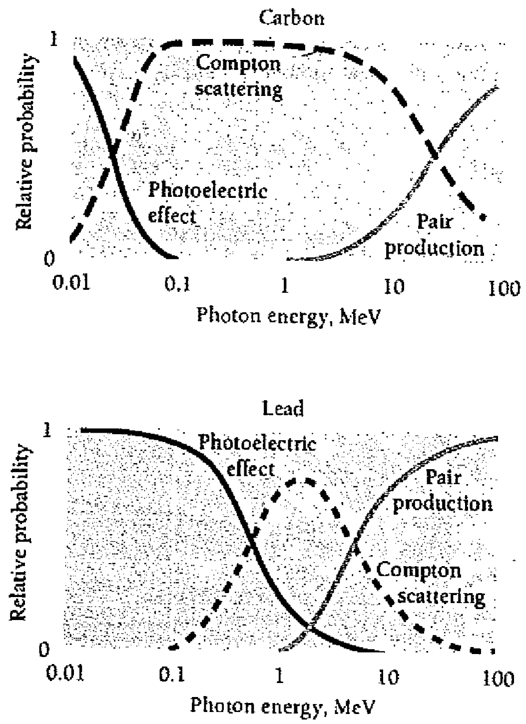


Figure 2.28 The relative probabilities of the photoelectric effect, Compton scattering, and pair production as functions of energy in carbon (a light element) and lead (a heavy element).

Pair production becomes increasingly likely the more the photon energy exceeds the threshold of 1.02 MeV. The greater the atomic number of the absorber, the lower the energy at which pair production takes over as the principal mechanism of energy loss by gamma rays. In the heaviest elements, the crossover energy is about 4 MeV, but it is over 10 MeV for the lighter ones. Thus gamma rays in the energy range typical of radioactive decay interact with matter largely through Compton scattering.

The intensity I of an x- or gamma-ray beam is equal to the rate at which it transports energy per unit cross-sectional area of the beam. The fractional energy $-dI/I$ lost by the beam in passing through a thickness dx of a certain absorber is found to be proportional to dx:

$$-\frac{dI}{I} = \mu\, dx \qquad (2.24)$$

The proportionality constant μ is called the **linear attenuation coefficient** and its value depends on the energy of the photons and on the nature of the absorbing material. Integrating Eq. (2.24) gives

Radiation intensity
$$I = I_0 e^{-\mu x} \qquad (2.25)$$

The intensity of the radiation decreases exponentially with absorber thickness x. Figure 2.29 is a graph of the linear attenuation coefficient for photons in lead as a function of photon energy. The contribution to μ of the photoelectric effect, Compton scattering, and pair production are shown.

We can use Eq. (2.25) to relate the thickness x of absorber needed to reduce the intensity of an x- or gamma-ray beam by a given amount to the attenuation coefficient μ. If the ratio of the final and initial intensities is I/I_0,

$$\frac{I}{I_0} = e^{-\mu x} \qquad \frac{I_0}{I} = e^{\mu x} \qquad \ln\frac{I_0}{I} = \mu x$$

Absorber thickness
$$x = \frac{\ln (I_0/I)}{\mu} \qquad (2.26)$$

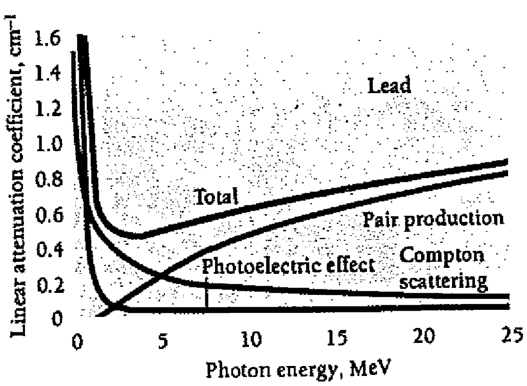


Figure 2.29 Linear attentuation coefficients for photons in lead.

Example 2.7

The linear attenuation coefficient for 2.0-MeV gamma rays in water is 4.9 m^{-1}. (*a*) Find the relative intensity of a beam of 2.0-MeV gamma rays after it has passed through 10 cm of water. (*b*) How far must such a beam travel in water before its intensity is reduced to 1 percent of its original value?

Solution

(*a*) Here $\mu x = (4.9\ m^{-1})(0.10\ m) = 0.49$ and so, from Eq. (2.25)

$$\frac{I}{I_0} = e^{-\mu x} = e^{-0.49} = 0.61$$

The intensity of the beam is reduced to 61 percent of its original value after passing through 10 cm of water.

(*b*) Since $I_0/I = 100$, Eq. (2.26) yields

$$x = \frac{\ln(I_0/I)}{\mu} = \frac{\ln 100}{4.9\ m^{-1}} = 0.94\ m$$

2.9 PHOTONS AND GRAVITY

Although they lack rest mass, photons behave as though they have gravitational mass

In Sec. 1.10 we learned that light is affected by gravity by virtue of the curvature of spacetime around a mass. Another way to approach the gravitational behavior of light follows from the observation that, although a photon has no rest mass, it nevertheless interacts with electrons as though it has the inertial mass

Photon "mass"
$$m = \frac{p}{v} = \frac{h\nu}{c^2} \tag{2.27}$$

(We recall that, for a photon, $p = h\nu/c$ and $v = c$.) According to the principle of equivalence, gravitational mass is always equal to inertial mass, so a photon of frequency ν ought to act gravitationally like a particle of mass $h\nu/c^2$.

The gravitational behavior of light can be demonstrated in the laboratory. When we drop a stone of mass m from a height H near the earth's surface, the gravitational pull of the earth accelerates it as it falls and the stone gains the energy mgH on the way to the ground. The stone's final kinetic energy $\frac{1}{2}mv^2$ is equal to mgH, so its final speed is $\sqrt{2gH}$.

All photons travel with the speed of light and so cannot go any faster. However, a photon that falls through a height H can manifest the increase of mgH in its energy by an increase in frequency from ν to ν' (Fig. 2.30). Because the frequency change is extremely small in a laboratory-scale experiment, we can neglect the corresponding change in the photon's "mass" $h\nu/c^2$.

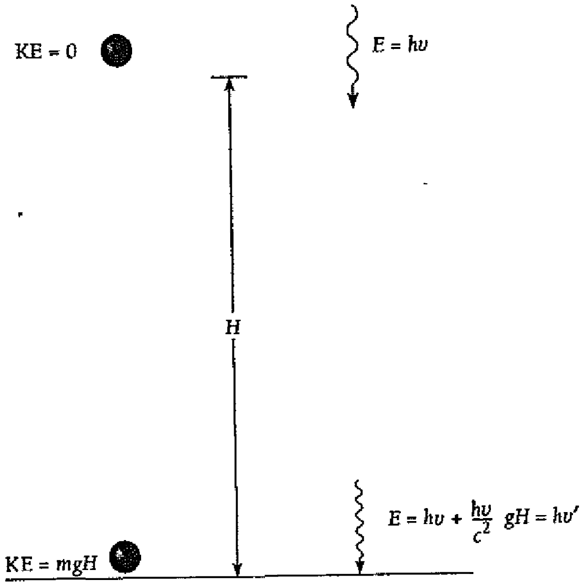


Figure 2.30 A photon that falls in a gravitational field gains energy, just as a stone does. This gain in energy is manifested as an increase in frequency from ν to ν'.

Hence,

$$\text{final photon energy} = \text{initial photon energy} + \text{increase in energy}$$

$$h\nu' = h\nu + mgH$$

and so

$$h\nu' = h\nu + \left(\frac{h\nu}{c^2}\right)gH$$

Photon energy after falling through height H

$$h\nu' = h\nu\left(1 + \frac{gH}{c^2}\right) \tag{2.28}$$

Example 2.8

The increase in energy of a fallen photon was first observed in 1960 by Pound and Rebka at Harvard. In their work H was 22.5 m. Find the change in frequency of a photon of red light whose original frequency is 7.3×10^{14} Hz when it falls through 22.5 m.

Solution

From Eq. (2.28) the change in frequency is

$$\nu' - \nu = \left(\frac{gH}{c^2}\right)\nu$$

$$= \frac{(9.8\ \text{m/s}^2)(22.5\ \text{m})(7.3 \times 10^{14}\ \text{Hz})}{(3.0 \times 10^8\ \text{m/s})^2} = 1.8\ \text{Hz}$$

Pound and Rebka actually used gamma rays of much higher frequency, as described in Exercise 53.

Gravitational Red Shift

An interesting astronomical effect is suggested by the gravitational behavior of light. If the frequency associated with a photon moving toward the earth increases, then the frequency of a photon moving away from it should decrease.

The earth's gravitational field is not particularly strong, but the fields of many stars are. Suppose a photon of initial frequency ν is emitted by a star of mass M and radius R, as in Fig. 2.31. The potential energy of a mass m on the star's surface is

$$\text{PE} = -\frac{GMm}{R}$$

where the minus sign is required because the force between M and m is attractive. The potential energy of a photon of "mass" $h\nu/c^2$ on the star's surface is therefore

$$\text{PE} = -\frac{GMh\nu}{c^2R}$$

and its total energy E, the sum of PE and its quantum energy $h\nu$, is

$$E = h\nu - \frac{GMh\nu}{c^2R} = h\nu\left(1 - \frac{GM}{c^2R}\right)$$

At a larger distance from the star, for instance at the earth, the photon is beyond the star's gravitational field but its total energy remains the same. The photon's energy is now entirely electromagnetic, and

$$E = h\nu'$$

where ν' is the frequency of the arriving photon. (The potential energy of the photon in the earth's gravitational field is negligible compared with that in the star's field.) Hence

$$h\nu' = h\nu\left(1 - \frac{GM}{c^2R}\right)$$

$$\frac{\nu'}{\nu} = 1 - \frac{GM}{c^2R}$$

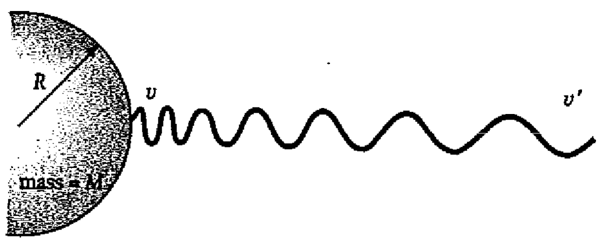


Figure 2.31 The frequency of a photon emitted from the surface of a star decreases as it moves away from the star.

and the relative frequency change is

$$\text{Gravitational red shift} \qquad \frac{\Delta\nu}{\nu} = \frac{\nu - \nu'}{\nu} = 1 - \frac{\nu'}{\nu} = \frac{GM}{c^2R} \tag{2.29}$$

The photon has a *lower* frequency at the earth, corresponding to its loss in energy as it leaves the field of the star.

A photon in the visible region of the spectrum is thus shifted toward the red end, and this phenomenon is accordingly known as the **gravitational red shift**. It is different from the doppler red shift observed in the spectra of distant galaxies due to their apparent recession from the earth, a recession that seems to be due to a general expansion of the universe.

As we shall learn in Chap. 4, when suitably excited the atoms of every element emit photons of certain specific frequencies only. The validity of Eq. (2.29) can therefore be checked by comparing the frequencies found in stellar spectra with those in spectra obtained in the laboratory. For most stars, including the sun, the ratio M/R is too small for a gravitational red shift to be apparent. However, for a class of stars known as **white dwarfs**, it is just on the limit of measurement—and has been observed. A white dwarf is an old star whose interior consists of atoms whose electron structures have collapsed and so it is very small: a typical white dwarf is about the size of the earth but has the mass of the sun.

Black Holes

An interesting question is, what happens if a star is so dense that $GM/c^2R \geq 1$? If this is the case, then from Eq. (2.29) we see that no photon can ever leave the star, since to do so requires more energy than its initial energy $h\nu$. The red shift would, in effect, have then stretched the photon wavelength to infinity. A star of this kind cannot radiate and so would be invisible—a **black hole** in space.

In a situation in which gravitational energy is comparable with total energy, as for a photon in a black hole, general relativity must be applied in detail. The correct criterion for a star to be a black hole turns out to be $GM/c^2R \geq \frac{1}{2}$. The **Schwarzschild radius** R_S of a body of mass M is defined as

Quasars and Galaxies

In even the most powerful telescope, a **quasar** appears as a sharp point of light, just as a star does. Unlike stars, quasars are powerful sources of radio waves; hence their name, a contraction of *quasi-stellar radio sources*. Hundreds of quasars have been discovered, and there seem to be many more. Though a typical quasar is smaller than the solar system, its energy output may be thousands of times the output of our entire Milky Way galaxy.

Most astronomers believe that at the heart of every quasar is a black hole whose mass is at least that of 100 million suns. As nearby stars are pulled toward the black hole, their matter is squeezed and heated to produce the observed radiation. While being swallowed, a star may liberate 10 times as much energy as it would have given off had it lived out a normal life. A diet of a few stars a year seems enough to keep a quasar going at the observed rates. It is possible that quasars are the cores of newly formed galaxies. Did all galaxies once undergo a quasar phase? Nobody can say as yet, but there is evidence that all galaxies, including the Milky Way, contain massive black holes at their centers.

Schwarzschild radius

$$R_S = \frac{2GM}{c^2} \tag{2.30}$$

The body is a black hole if all its mass is inside a sphere with this radius. The boundary of a black hole is called its **event horizon.** The escape speed from a black hole is equal to the speed of the light c at the Schwarzschild radius, hence nothing at all can ever leave a black hole. For a star with the sun's mass, R_S is 3 km, a quarter of a million times smaller than the sun's present radius. Anything passing near a black hole will be sucked into it, never to return to the outside world.

Since it is invisible, how can a black hole be detected? A black hole that is a member of a double-star system (double stars are quite common) will reveal its presence by its gravitational pull on the other star; the two stars circle each other. In addition, the intense gravitational field of the black hole will attract matter from the other star, which will be compressed and heated to such high temperatures that x-rays will be emitted profusely. One of a number of invisible objects that astronomers believe on this basis to be black holes is known as Cygnus X-1. Its mass is perhaps 8 times that of the sun, and its radius may be only about 10 km. The region around a black hole that emits x-rays should extend outward for several hundred kilometers.

Only very heavy stars end up as black holes. Lighter stars evolve into white dwarfs and neutron stars, which as their name suggests consist largely of neutrons (see Sec. 9.11). But as time goes on, the strong gravitational fields of both white dwarfs and neutron stars attract more and more cosmic dust and gas. When they have gathered up enough mass, they too will become black holes. If the universe lasts long enough, then everything in it may be in the form of black holes.

Black holes are also believed to be at the cores of galaxies. Again, the clues come from the motions of nearby bodies and from the amount and type of radiation emitted. Stars close to a galactic center are observed to move so rapidly that only the gravitational pull of an immense mass could keep them in their orbits instead of flying off. How immense? As much as a billion times the sun's mass. And, as in the case of black holes that were once stars, radiation pours out of galactic centers so copiously that only black holes could be responsible.

EXERCISES

"Why," said the Dodo, "the best way to explain it is to do it." —Lewis Carroll, *Alice's Adventures in Wonderland*

2.2 Blackbody Radiation

1. If Planck's constant were smaller than it is, would quantum phenomena be more or less conspicuous than they are now?
2. Express the Planck radiation formula in terms of wavelength.

2.3 Photoelectric Effect

3. Is it correct to say that the maximum photoelectron energy KE_{max} is proportional to the frequency ν of the incident light? If not, what would a correct statement of the relationship between KE_{max} and ν be?
4. Compare the properties of particles with those of waves. Why do you think the wave aspect of light was discovered earlier than its particle aspect?
5. Find the energy of a 700-nm photon.
6. Find the wavelength and frequency of a 100-MeV photon.
7. A 1.00-kW radio transmitter operates at a frequency of 880 kHz. How many photons per second does it emit?
8. Under favorable circumstances the human eye can detect 1.0×10^{-18} J of electromagnetic energy. How many 600-nm photons does this represent?

9. Light from the sun arrives at the earth, an average of 1.5 $\times 10^{11}$ m away, at the rate of 1.4 $\times 10^3$ W/m^2 of area perpendicular to the direction of the light. Assume that sunlight is monochromatic with a frequency of 5.0 $\times 10^{14}$ Hz. (*a*) How many photons fall per second on each square meter of the earth's surface directly facing the sun? (*b*) What is the power output of the sun, and how many photons per second does it emit? (*c*) How many photons per cubic meter are there near the earth?

10. A detached retina is being "welded" back in place using 20-ms pulses from a 0.50-W laser operating at a wavelength of 632 nm. How many photons are in each pulse?

11. The maximum wavelength for photoelectric emission in tungsten is 230 nm. What wavelength of light must be used in order for electrons with a maximum energy of 1.5 eV to be ejected?

12. The minimum frequency for photoelectric emission in copper is 1.1 $\times 10^{15}$ Hz. Find the maximum energy of the photoelectrons (in electronvolts) when light of frequency 1.5 $\times 10^{15}$ Hz is directed on a copper surface.

13. What is the maximum wavelength of light that will cause photoelectrons to be emitted from sodium? What will the maximum kinetic energy of the photoelectrons be if 200-nm light falls on a sodium surface?

14. A silver ball is suspended by a string in a vacuum chamber and ultraviolet light of wavelength 200 nm is directed at it. What electrical potential will the ball acquire as a result?

15. 1.5 mW of 400-nm light is directed at a photoelectric cell. If 0.10 percent of the incident photons produce photoelectrons, find the current in the cell.

16. Light of wavelength 400 nm is shone on a metal surface in an apparatus like that of Fig. 2.9. The work function of the metal is 2.50 eV. (*a*) Find the extinction voltage, that is, the retarding voltage at which the photoelectron current disappears. (*b*) Find the speed of the fastest photoelectrons.

17. A metal surface illuminated by 8.5 $\times 10^{14}$ Hz light emits electrons whose maximum energy is 0.52 eV. The same surface illuminated by 12.0 $\times 10^{14}$ Hz hight emits electrons whose maximum energy is 1.97 eV. From these data find Planck's constant and the work function of the surface.

18. The work function of a tungsten surface is 5.4 eV. When the surface is illuminated by light of wavelength 175 nm, the maximum photoelectron energy is 1.7 eV. Find Planck's constant from these data.

19. Show that it is impossible for a photon to give up all its energy and momentum to a free electron. This is the reason why the photoelectric effect can take place only when photons strike bound electrons.

2.5 X-Rays

20. What voltage must be applied to an x-ray tube for it to emit x-rays with a minimum wavelength of 30 pm?

21. Electrons are accelerated in television tubes through potential differences of about 10 kV. Find the highest frequency of the electromagnetic waves emitted when these electrons strike the screen of the tube. What kind of waves are these?

2.6 X-Ray Diffraction

22. The smallest angle of Bragg scattering in potassium chloride (KCl) is 28.4° for 0.30-nm x-rays. Find the distance between atomic planes in potassium chloride.

23. The distance between adjacent atomic planes in calcite ($CaCO_3$) is 0.300 nm. Find the smallest angle of Bragg scattering for 0.030-nm x-rays.

24. Find the atomic spacing in a crystal of rock salt (NaCl), whose structure is shown in Fig. 2.19. The density of rock salt is 2.16 $\times 10^3$ kg/m^3 and the average masses of the Na and Cl atoms are respectively 3.82 $\times 10^{-26}$ kg and 5.89 $\times 10^{-26}$ kg.

2.7 Compton Effect

25. What is the frequency of an x-ray photon whose momentum is 1.1 $\times 10^{-23}$ kg · m/s?

26. How much energy must a photon have if it is to have the momentum of a 10-MeV proton?

27. In Sec. 2.7 the x-rays scattered by a crystal were assumed to undergo no change in wavelength. Show that this assumption is reasonable by calculating the Compton wavelength of a Na atom and comparing it with the typical x-ray wavelength of 0.1 nm.

28. A monochromatic x-ray beam whose wavelength is 55.8 pm is scattered through 46°. Find the wavelength of the scattered beam.

29. A beam of x-rays is scattered by a target. At 45° from the beam direction the scattered x-rays have a wavelength of 2.2 pm. What is the wavelength of the x-rays in the direct beam?

30. An x-ray photon whose initial frequency was 1.5 $\times 10^{19}$ Hz emerges from a collision with an electron with a frequency of 1.2 $\times 10^{19}$ Hz. How much kinetic energy was imparted to the electron?

31. An x-ray photon of initial frequency 3.0 $\times 10^{19}$ Hz collides with an electron and is scattered through 90°. Find its new frequency.

32. Find the energy of an x-ray photon which can impart a maximum energy of 50 keV to an electron.

33. At what scattering angle will incident 100-keV x-rays leave a target with an energy of 90 keV?

34. (*a*) Find the change in wavelength of 80-pm x-rays that are scattered 120° by a target. (*b*) Find the angle between the directions of the recoil electron and the incident photon. (*c*) Find the energy of the recoil electron.

35. A photon of frequency ν is scattered by an electron initially at rest. Verify that the maximum kinetic energy of the recoil electron is $KE_{max} = (2h^2\nu^2/mc^2)/(1 + 2h\nu/mc^2)$.

36. In a Compton-effect experiment in which the incident x-rays have a wavelength of 10.0 pm, the scattered x-rays at a certain angle have a wavelength of 10.5 pm. Find the momentum (magnitude and direction) of the corresponding recoil electrons.

37. A photon whose energy equals the rest energy of the electron undergoes a Compton collision with an electron. If the electron moves off at an angle of 40° with the original photon direction, what is the energy of the scattered photon?

38. A photon of energy E is scattered by a particle of rest energy E_0. Find the maximum kinetic energy of the recoiling particle in terms of E and E_0.

2.8 Pair Production

39. A positron collides head on with an electron and both are annihilated. Each particle had a kinetic energy of 1.00 MeV. Find the wavelength of the resulting photons.

40. A positron with a kinetic energy of 2.000 MeV collides with an electron at rest and the two particles are annihilated. Two photons are produced; one moves in the same direction as the incoming positron and the other moves in the opposite direction. Find the energies of the photons.

41. Show that, regardless of its initial energy, a photon cannot undergo Compton scattering through an angle of more than 60° and still be able to produce an electron-positron pair. (*Hint:* Start by expressing the Compton wavelength of the electron in terms of the maximum photon wavelength needed for pair production.)

42. (*a*) Verify that the minimum energy a photon must have to create an electron-positron pair in the presence of a stationary nucleus of mass M is $2mc^2(1 + m/M)$, where m is the electron rest mass. (*b*) Find the minimum energy needed for pair production in the presence of a proton.

43. (*a*) Show that the thickness $x_{1/2}$ of an absorber required to reduce the intensity of a beam of radiation by a factor of 2 is given by $x_{1/2} = 0.693/\mu$. (*b*) Find the absorber thickness needed to produce an intensity reduction of a factor of 10.

44. (*a*) Show that the intensity of the radiation absorbed in a thickness x of an absorber is given by $I_0\mu x$ when $\mu x \ll 1$. (*b*) If $\mu x = 0.100$, what is the percentage error in using this formula instead of Eq. (2.25)?

45. The linear absorption coefficient for 1-MeV gamma rays in lead is 78 m^{-1}. Find the thickness of lead required to reduce by half the intensity of a beam of such gamma rays.

46. The linear absorption coefficient for 50-keV x-rays in sea-level air is 5.0×10^{-3} m^{-1}. By how much is the intensity of a beam of such x-rays reduced when it passes through 0.50 m of air? Through 5.0 m of air?

47. The linear absorption coefficients for 2.0-MeV gamma rays are 4.9 m^{-1} in water and 52 m^{-1} in lead. What thickness of water would give the same shielding for such gamma rays as 10 mm of lead?

48. The linear absorption coefficient of copper for 80-keV x-rays is 4.7×10^4 m^{-1}. Find the relative intensity of a beam of 80-keV x-rays after it has passed through a 0.10-mm copper foil.

49. What thickness of copper is needed to reduce the intensity of the beam in Exercise 48 by half?

50. The linear absorption coefficients for 0.05-nm x-rays in lead and in iron are, respectively, 5.8×10^4 m^{-1} and 1.1×10^4 m^{-1}. How thick should an iron shield be in order to provide the same protection from these x-rays as 10 mm of lead?

2.9 Photons and Gravity

51. The sun's mass is 2.0×10^{30} kg and its radius is 7.0×10^8 m. Find the approximate gravitational red shift in light of wavelength 500 nm emitted by the sun.

52. Find the approximate gravitational red shift in 500-nm light emitted by a white dwarf star whose mass is that of the sun but whose radius is that of the earth, 6.4×10^6 m.

53. As discussed in Chap. 12, certain atomic nuclei emit photons in undergoing transitions from "excited" energy states to their "ground" or normal states. These photons constitute gamma rays. When a nucleus emits a photon, it recoils in the opposite direction. (*a*) The $^{57}_{27}Co$ nucleus decays by K capture to $^{57}_{26}Fe$, which then emits a photon in losing 14.4 keV to reach its ground state. The mass of a $^{57}_{26}Fe$ atom is 9.5×10^{-26} kg. By how much is the photon energy reduced from the full 14.4 keV available as a result of having to share energy and momentum with the recoiling atom? (*b*) In certain crystals the atoms are so tightly bound that the entire crystal recoils when a gamma-ray photon is emitted, instead of the individual atom. This phenomenon is known as the **Mössbauer effect.** By how much is the photon energy reduced in this situation if the excited $^{57}_{26}Fe$ nucleus is part of a 1.0-g crystal? (*c*) The essentially recoil-free emission of gamma rays in situations like that of *b* means that it is possible to construct a source of virtually monoenergetic and hence monochromatic photons. Such a source was used in the experiment described in Sec. 2.9. What is the original frequency and the change in frequency of a 14.4-keV gamma-ray photon after it has fallen 20 m near the earth's surface?

54. Find the Schwarzschild radius of the earth, whose mass is 5.98×10^{24} kg.

55. The gravitational potential energy U relative to infinity of a body of mass m at a distance R from the center of a body of mass M is $U = -GmM/R$. (*a*) If R is the radius of the body of mass M, find the escape speed v_e of the body, which is the minimum speed needed to leave it permanently. (*b*) Obtain a formula for the Schwarzschild radius of the body by setting $v_e = c$, the speed of light, and solving for R. (Of course, a relativistic calculation is correct here, but it is interesting to see what a classical calculation produces.)