Assignment 1: solutions:

1. Problem 8; p. 89: Under favorable circumstances the human eye can detect $1.0X10^{-18}$ J of electromagnetic energy. How many 600-nm photons does this represent?

Ans. one 600 nm photon has energy $\frac{hc}{\lambda} = \frac{6.63\times10^{-34}Js.3\times10^{8}ms^{-1}}{600\times10^{-9}m} = 3.315 \times 10^{-19}$J.

$\therefore$no. of photons with $1.0X10^{-18}$ J energy $= \frac{10^{-18}}{3.315\times10^{-19}} \approx 3$. What this means is that the eye can detect even three photons of yellow light! Three not Avogadro's number! That's a really small number!

2. Problem 17; p.117: The phase velocity of ocean waves is $\sqrt{\frac{g\lambda}{2\pi}}$, where g is the acceleration of gravity. Find the group velocity of ocean waves.

Ans. The phase velocity $= \frac{\omega}{k} = \sqrt{\frac{g\lambda}{2\pi}} \implies \omega = k \times \sqrt{\frac{g\lambda}{2\pi}} = k \times \sqrt{\frac{g}{k}} = \sqrt{gk} \implies$ group velocity $= \frac{d\omega}{dk} = \sqrt{\frac{g}{2\pi k}} = \frac{1}{2\pi}\sqrt{g\lambda}$

3. Problem 29; p.118: A proton in a one-dimensional box has an energy of 400 keV in its first excited state. How wide is the box?

Ans. The first excited state corresponds to $n = 2$. If the width of the box is L, then

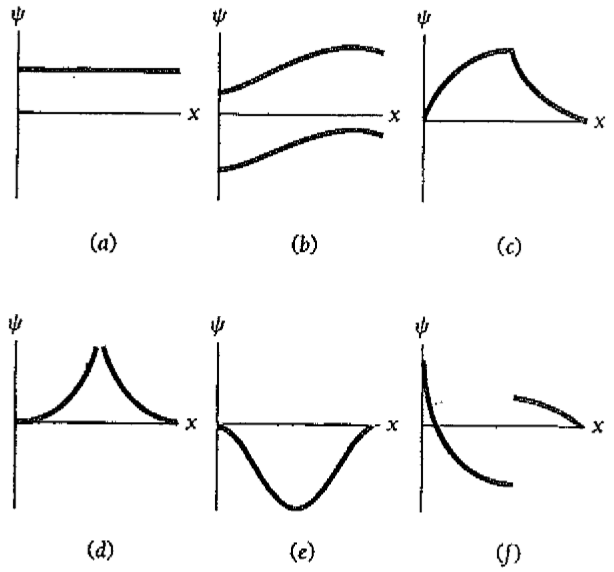
$$\frac{2^2h^2}{8m_pL^2} = 400keV \implies L = \frac{2h}{8.\sqrt{m_p.400eV}}$$

We can do it in any system of units. Let us choose the atomic units: $\hbar = 1$; $m_e = 1$ and unit of energy = 1 Hartree = 13.6eV; unit of length= Bohr radius, $a_0 = 0.0529$nm

Then, using $m_p = 1836$, we get,

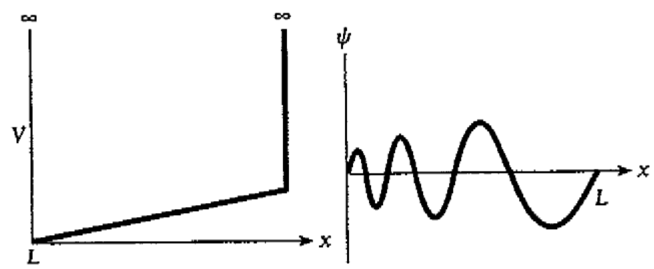
$$L = \frac{2.2\pi}{8.\sqrt{1836.\frac{400}{13.6}}}a_0 = 0.0037a_0 = 0.0037 \times 0.0529 = 1.97 \times 10^{-4}\text{nm}$$

4. Problem 2; p. 197: Which of the wave functions in Fig. cannot have physical significance in the interval shown? Why not?



Ans. All except (e). Reason: (a) does not decay to zero asymptotically; (b) is multi-valued in the domain shown; (c) is not differentiable at the bend (d) The function is singular at one point; and (f) is discontinuous.

5. Prob 13; p. 198: One of the possible wave functions of a particle in the potential well of Fig. below is sketched next to it. Explain why the wavelength and amplitude of ψ vary as they do.



Ans. As the potential energy increases, for a definite total energy, the kinetic energy decreases. The kinetic energy is related to the curvature of the function ($\frac{d^2\psi}{dx^2}$). This has two effects: 1. the system has lower velocities, it spends longer time and hence more likely to be found there (hence higher amplitudes) and second the function is more spread out.