Tuesday, May 01, 2012
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# Geometric transformations: Rotations & Translations

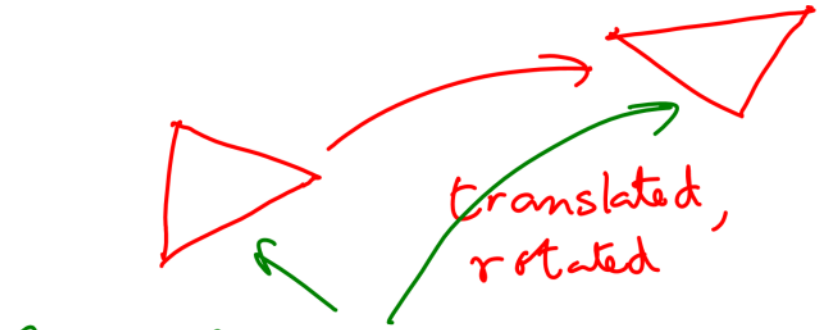
Note Title 4/30/2012

Camera Coordinate system - $\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$ with various points on the object

If object moves, coordinates change!

POSE - relation of object to camera

SHAPE - independent of relationship of the object to the camera




SAME
TRIANGLE

CONGRUENCE

SHAPE of the triangle remains unchanged under the transformation of rotation and translation.

Note The coordinates $\begin{bmatrix} x \\ y \end{bmatrix}$ of various points on the triangle have changed!

POSE changes when we rotate & translate the object

SHAPE does not.

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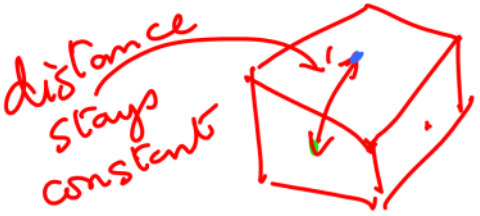
ROTATIONS, TRANSLATIONS

are these special cases of more general transformations?

RIGID BODY TRANSFORMATIONS / ISOMETRY





distances between any pair of points will remain constant!

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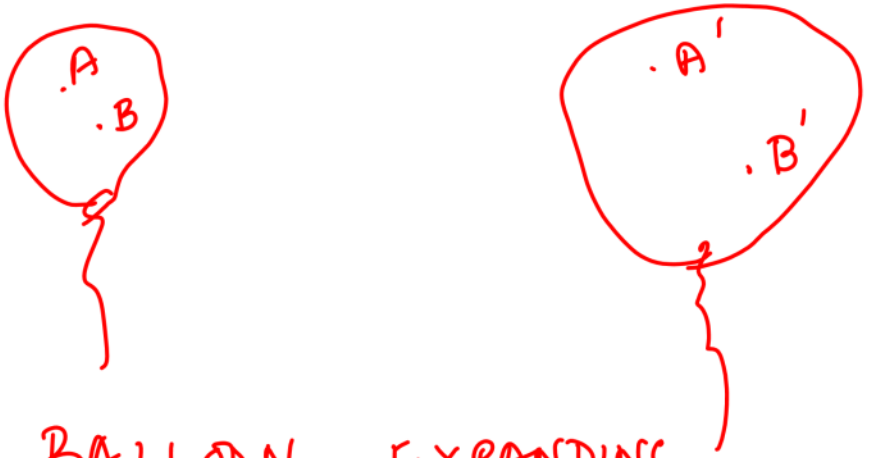


BALLOON EXPANDING

distance between A' and B' $\neq$ distance between A and B

Not a rigid body motion!

**Definition** A rigid body motion is a transformation that preserves distances.

distance $= \| \underset{\sim}{a} - \underset{\sim}{b} \|$

$\underset{\sim}{a} \in \mathbb{R}^3$

$\underset{\sim}{b} \in \mathbb{R}^3$

$\psi$

$\psi(a)$

$\psi(b)$

$\| \psi(a) - \psi(b) \|$

Distance between $\underset{\sim}{a}$ and $\underset{\sim}{b}$ $= \| \underset{\sim}{a} - \underset{\sim}{b} \|$

Suppose we have a vector $\underset{\sim}{v} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$

length

$$\| \underset{\sim}{v} \| = \sqrt{v_1^2 + v_2^2 + v_3^2}$$

Norm of $\underset{\sim}{v}$

If $\psi$ is a rigid body transformation then $\|\underset{\sim}{a} - \underset{\sim}{b}\| = \|\psi(a) - \psi(b)\|$ DEFINITION

**Examples**

1. Is translation a rigid body motion?

$$\psi(\underset{\sim}{a}) = \underset{\sim}{a} + \underset{\sim}{t}$$ definition of translation

**Check** that this is a rigid body motion.

distance $\|\underset{\sim}{a} - \underset{\sim}{b}\|$

$\underset{\sim}{a} \longrightarrow \underset{\sim}{a} + \underset{\sim}{t}$

$\underset{\sim}{b} \longrightarrow \underset{\sim}{b} + \underset{\sim}{t}$

same !!

$\|(\underset{\sim}{a}+\underset{\sim}{t}) - (\underset{\sim}{b}+\underset{\sim}{t})\| = \|\underset{\sim}{a} - \underset{\sim}{b}\|$ distance