Assignment 1

May 8, 2012

1. Show that the vanishing points of lines on a plane lie on the vanishing line of the plane.

2. Show that, under typical conditions, the silhouette of a sphere of radius r with center $(X,0,\ Z)$ under planar perspective projection is an ellipse of eccentricity $X/\sqrt{(X^2+Z^2-r^2)}$. Are there circumstances under which the projection could be a parabola or hyperbola? What is the silhouette for spherical perspective?

3. An observer is standing on a ground plane looking straight ahead. We want to calculate the accuracy with which she will be able to estimate the depth Z of points on the ground plane, assuming that she can visually discriminate angles to within $1'$. Derive a formula relating depth error δZ to Z. For simplicity, just consider points straight ahead of the observer($x = 0$). Given a Z value (say 10 m), your formula should be able to predict the δZ.

4. Show that in $\mathcal{R}^2$ reflection about the $\theta = \alpha$ line followed by reflection about the $\theta = \beta$ is equivalent to a rotation of $2(\beta - \alpha)$.

5. Verify Roderigues formula by considering the powers of the skew-symmetric matrix associated with the cross product with a vector.

6. Write a function for computing the orthogonal matrix $\mathbf{R}$ corresponding to rotation ϕ about the axis vector $\mathbf{s}$. Find the eigenvalues and eigenvectors of the orthogonal matrices and study any relationship to the axis vector. Verify the formula $\cos\phi = \frac{1}{2}\{\text{trace}(\mathbf{R}) - 1\}$. Show some points before and after the rotation has been applied.

7. Write a function for the converse of that in the previous problem i.e. given an orthogonal matrix $\mathbf{R}$, compute the axis of rotation $\mathbf{s}$ and ϕ). Hint: Show that $\mathbf{R} - \mathbf{R}^T = (2\sin\phi)\widehat{\mathbf{s}}$

8. Use least squares to find the best estimate of the Euclidean planar transformation (translation + rotation) E that minimizes the error $\sum_{j=1}^{4} \mid E\mathbf{u}_j - \mathbf{v}_j \mid^2$. Here

$$\mathbf{u}_j = [(-3,0),(1,1),(1,0),(1,-1)]$$

and

$$\mathbf{v}_j = [(0,3),(1,0),(0,0),(-1,0)]$$

9. Suppose that the coordinates of a set of points in a plane are measured using an affine basis of 3 non–collinear points. This set of points is then subject to an affine transformation. Show that the transformation leaves the coordinates of this set of points (measured now with respect to the affine–transformed basis) unchanged.