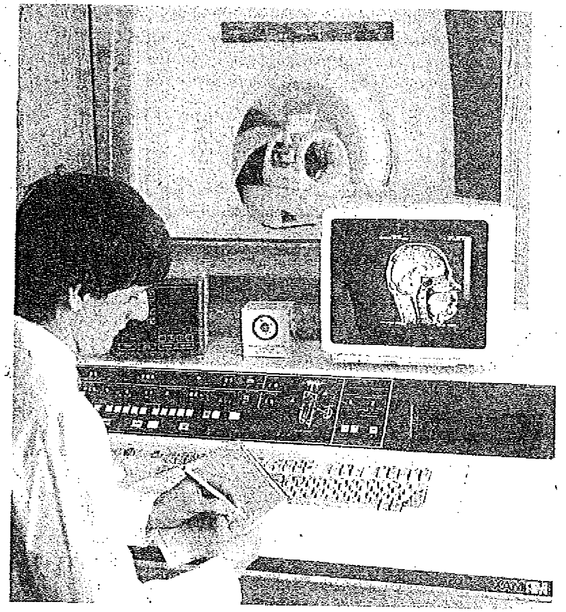
CHAPTER 11

Nuclear Structure



Nuclear magnetic resonance is the basis of a high-resolution method of imaging body tissues. The screen shows a computer-constructed cross section of the head of the person lying inside the powerful magnet at the rear.

Thus far we have been able to regard the nucleus of an atom merely as a tiny, positively charged object whose only roles are to provide the atom with most of its mass and to hold its electrons in thrall. The chief properties (except mass) of atoms, molecules, solids, and liquids can all be traced to the behavior of atomic electrons, not to the behavior of nuclei. Nevertheless, the nucleus turns out to be of paramount importance in the grand scheme of things. To begin with, the very existence of the various elements is due to the ability of nuclei to possess multiple electric charges. Furthermore, the energy involved in almost all natural processes can be traced to nuclear reactions and transformations. And the liberation of nuclear energy in reactors and weapons has affected all our lives in one way or another.

11.1 NUCLEAR COMPOSITION

Atomic nuclei of the same element have the same numbers of protons but can have different numbers of neutrons

The electron structure of the atom was understood before even the composition of its nucleus was known. The reason is that the forces that hold the nucleus together are vastly stronger than the electric forces that hold the electrons to the nucleus, and it is correspondingly harder to break apart a nucleus to find out what is inside. Changes in the electron structure of an atom, such as those that occur when a photon is emitted or absorbed or when a chemical bond is formed or broken, involve energies of only a few electronvolts. Changes in nuclear structure, on the other hand, involve energies in the MeV range, a million times greater.

An ordinary hydrogen atom has as its nucleus a single proton, whose charge is $+e$ and whose mass is 1836 times that of the electron. All other elements have nuclei that contain neutrons as well as protons. As its name suggests, the neutron is uncharged; its mass is slightly greater than that of the proton. Neutrons and protons are jointly called **nucleons.**

The **atomic number** of an element is the number of protons in each of its atomic nuclei, which is the same as the number of electrons in a neutral atom of the element. Thus the atomic number of hydrogen is 1, of helium 2, of lithium 3, and of uranium 92. All nuclei of a given element do not necessarily have equal numbers of neutrons. For instance, although over 99.9 percent of hydrogen nuclei are just single protons, a few also contain a neutron, and a very few two neutrons, along with the proton (Fig. 11.1). The varieties of an element that differ in the numbers of neutrons their nuclei contain are called its **isotopes.**

The hydrogen isotope **deuterium** is stable, but **tritium** is radioactive and eventually changes into an isotope of helium. The flux of cosmic rays from space continually replenishes the earth's tritium by nuclear reactions in the atmosphere. Only about 2 kg of tritium of natural origin is present at any time on the earth, nearly all of it in the oceans. **Heavy water** is water in which deuterium atoms instead of ordinary hydrogen atoms are combined with oxygen atoms.

The conventional symbols for nuclear species, or **nuclides,** follow the pattern $^{A}_{Z}X$,

where X = chemical symbol of the element
Z = atomic number of the element
= number of protons in the nucleus
A = mass number of the nuclide
= number of nucleons in the nucleus

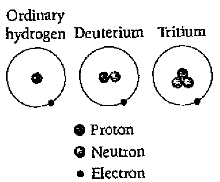


Figure 11.1 The isotopes of hydrogen.

James Chadwick (1891–1974) was educated at the University of Manchester in England and remained there to work on gamma-ray emission under Rutherford. In Germany to investigate beta decay when World War I broke out, Chadwick was interned as an enemy alien. After the war he joined Rutherford at Cambridge, where he used alpha-particle scattering to show that the atomic number of an element equals its nuclear charge. Rutherford and Chadwick suggested an uncharged particle as a nuclear constituent but could not find a way to detect it experimentally.

Then, in 1930, the German physicists W. Bothe and H. Becker found that an uncharged radiation able to penetrate lead is emitted by beryllium bombarded with alpha particles from polonium (Fig. 11.2). Irene Curie and her husband Frederic Joliot, working in France in 1932, discovered that this mysterious radiation could knock protons with energies up to 5.7 MeV out of a paraffin slab. They assumed the radiation consisted of gamma rays (photons more energetic than x-rays) and, on the basis that the protons were knocked out of the hydrogen-rich paraffin in Compton collisions, calculated that the gamma-ray photon energy had to be at least 55 MeV. But this was far too much energy to be produced by the alpha particles interacting with beryllium nuclei.

Chadwick proposed instead that neutral particles with about the same mass as the proton are responsible, in which case their energy need be only 5.7 MeV since a particle colliding head on with another particle of the same mass can transfer all of its KE to the latter. Other experiments confirmed his hypothesis, and he received the Nobel Prize in 1935 for his part in the discovery of the neutron. (Chadwick did not immediately regard the neutron as an elementary particle but instead as "a small dipole, or perhaps better as a proton embedded in an electron." The idea that the neutron is actually an elementary particle was first put forward by the Russian physicist Dmitri Iwanenko.) During World War II Chadwick headed the British group that participated in developing the atomic bomb.

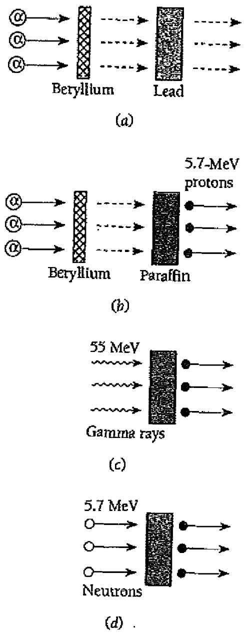
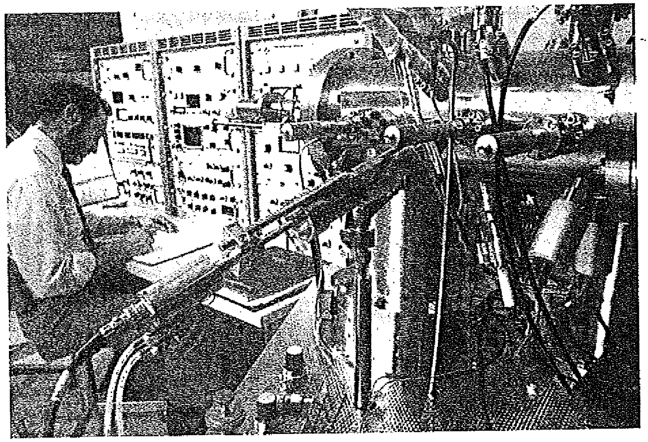


Figure 11.2 (*a*) Alpha particles incident on a beryllium foil cause the emission of a very penetrating radiation. (*b*) Protons of up to 5.7 MeV are ejected when the radiation strikes a paraffin slab. (*c*) If the radiation consists of gamma rays, their energies must be at least 55 MeV. (*d*) If the radiation consists of neutral particles of approximately proton mass, their energies need not exceed 5.7 MeV.

Hence ordinary hydrogen is ^1_1H, deuterium is ^2_1H, and the two isotopes of chlorine ($Z = 17$), whose nuclei contain 18 and 20 neutrons respectively, are $^{35}_{17}Cl$ and $^{37}_{17}Cl$. Because every element has a characteristic atomic number, Z is often omitted from the symbol for a nuclide: ^{35}Cl (read as "chlorine 35") instead of $^{35}_{17}Cl$.

Atomic Masses

Atomic masses refer to the masses of neutral atoms, not of bare nuclei. Thus an atomic mass always includes the masses of its Z electrons. Atomic masses are expressed in



Mass spectrometer being used to study the composition of semiconductor crystals.

mass units (u), which are so defined that the mass of a $^{12}_{6}C$ atom, the most abundant isotope of carbon, is exactly 12 u. The value of a mass unit is

Atomic mass unit $$1\ u = 1.66054 \times 10^{-27}\ kg$$

The energy equivalent of a mass unit is 931.49 MeV. Table 11.1 gives the masses of the proton, neutron, electron, and $^{1}_{1}H$ atom in various units, including the MeV/c^2. The advantage of using this unit is that the energy equivalent of a mass of, say, 10 MeV/c^2 is simply $E = mc^2 = 10$ MeV.

Table 11.2 gives the compositions of the isotopes of hydrogen and chlorine. Chlorine in nature consists of about three-quarters of the ^{35}Cl isotope and one-quarter of the ^{37}Cl isotope, which yields the average atomic mass of 35.46 u that chemists use (see Table 7.2). The chemical properties of an element are determined by the number and arrangement of the electrons in its atoms. Since the isotopes of an element have almost identical electron structures in their atoms, it is not surprising that the two isotopes of chlorine, for instance, have the same yellow color, the same suffocating odor, the same efficiency as poisons and bleaching agents, and the same ability to combine with metals. Because boiling and freezing points depend somewhat on atomic mass, they are slightly different for the two isotopes, as are their densities. Other physical

Table 11.1 Some Masses in Various Units

Particle	Mass (kg)	Mass (u)	Mass (MeV/c^2)
Proton	1.6726×10^{-27}	1.007276	938.28
Neutron	1.6750×10^{-27}	1.008665	939.57
Electron	9.1095×10^{-31}	5.486×10^{-4}	0.511
$^{1}_{1}H$ atom	1.6736×10^{-27}	1.007825	938.79

Table 11.2 The Isotopes of Hydrogen and Chlorine Found in Nature

	Properties of Element		Properties of Isotope					
Element	Atomic Number	Average Atomic Mass, u	Protons in Nucleus	Neutrons in Nucleus	Mass Number	Atomic Mass, u	Relative Abundance, Percent
Hydrogen	1	1.008	1	0	1	1.008	99.985
			1	1	2	2.014	0.015
			1	2	3	3.016	Very small
Chlorine	17	35.46	17	18	35	34.97	75.53
			17	20	37	36.97	24.47

properties of isotopes may very more dramatically with mass number: tritium is radioactive, for instance, whereas ordinary hydrogen and deuterium are not.

Nuclear Electrons

Nuclide masses are always very close to being integral multiples of the mass of the hydrogen atom, as we can see in Table 11.2. Before the discovery of the neutron, it was tempting to regard all nuclei as consisting of protons together with enough electrons to neutralize the positive charge of some of them. This hypothesis is buttressed by the fact that certain radioactive nuclei spontaneously emit electrons, a phenomenon called beta decay. However, there are some strong arguments against the idea of nuclear electrons.

1 Nuclear size. In Example 3.7 we saw that an electron confined to a box of nuclear dimensions must have an energy of more than 20 MeV, whereas electrons emitted during beta decay have energies of only 2 or 3 MeV, an order of magnitude smaller. A similar calculation for protons gives a minimum energy of around 0.2 MeV, which is entirely plausible.

2 Nuclear spin. Protons and electrons are fermions with spins (that is, spin quantum numbers) of $\frac{1}{2}$. Thus nuclei with an even number of protons plus electrons should have 0 or integral spin, those with an odd number of protons plus electrons should have half-integral spins. This prediction is not obeyed. For instance, if a deuterium nucleus, ^2_1H, consisted of two protons and an electron, its nuclear spin should be $\frac{1}{2}$ or $\frac{3}{2}$, but in fact is observed to be 1.

3 Magnetic moment. The proton has a magnetic moment only about 0.15 percent that of the electron. If electrons are part of a nucleus, its magnetic moment ought to be of the order of magnitude of that of the electron. However, observed nuclear magnetic moments are comparable with that of the proton, not with that of the electron.

4 Electron-nuclear interaction. The forces that hold the constituents of a nucleus together lead to typical binding energies of around 8 MeV per particle. If some electrons can bind this strongly to protons in the nucleus of an atom, how can the other electrons in the atom remain outside the nucleus? Furthermore, when fast electrons are scattered by nuclei, they behave as though acted upon solely by electric forces, whereas the scattering of fast protons shows that a different force also acts on them.

Despite these difficulties, the hypothesis of nuclear electrons was not universally abandoned untill the discovery of the neutron in 1932. When he wrote a book on nuclear physics published the year before, George Gamow felt so uneasy about the accepted proton-electron model of the nucleus that he marked each section dealing with nuclear electrons with a skull and crossbones. When the publisher objected, Gamow replied that "It has never been my intention to scare the poor readers more than the text itself will undoubtedly do," and replaced the skull and crossbones with a less dramatic symbol.

11.2 SOME NUCLEAR PROPERTIES

Small in size, a nucleus may have angular momentum and a magnetic moment

The Rutherford scattering experiment provided the first estimates of nuclear sizes. In that experiment, as we saw in Chap. 4, an incident alpha particle is deflected by a target nucleus in a manner consistent with Coulomb's law provided the distance between them exceeds about 10^{-14} m. For smaller separations Coulomb's law is not obeyed because the nucleus no longer appears as a point charge to the alpha particle.

Since Rutherford's time a variety of experiments have been performed to determine nuclear dimensions, with particle scattering still a favored technique. Fast electrons and neutrons are ideal for this purpose, since an electron interacts with a nucleus only through electric forces while a neutron interacts only through specifically nuclear forces. Thus electron scattering provides information on the distribution of charge in a nucleus and neutron scattering provides information on the distribution of nuclear matter. In both cases the de Broglie wavelength of the particle must be smaller than the radius of the nucleus under study. What is found is that the volume of a nucleus is directly proportional to the number of nucleons it contains, which is its mass number A. This suggests that the density of nucleons is very nearly the same in the interiors of all nuclei.

If a nuclear radius is R, the corresponding volume is $\frac{4}{3}\pi R^3$ and so R^3 is proportional to A. This relationship is usually expressed in inverse form as

Nuclear radii $$R = R_0 A^{1/3} \qquad (11.1)$$

The value of R_0 is

$$R_0 \approx 1.2 \times 10^{-15}\ \text{m} \approx 1.2\ \text{fm}$$

It is necessary to be indefinite in expressing R_0 because, as Fig. 11.3 shows, nuclei do not have sharp boundaries. Despite this, the values of R from Eq. (11.1) are representative of effective nuclear sizes. The value of R_0 is slightly smaller when it is deduced from electron scattering, which implies that nuclear matter and nuclear charge are not identically distributed through a nucleus.

Nuclei are so small that the unit of length appropriate in describing them is the **femtometer** (fm), equal to 10^{-15} m. The femtometer is sometimes called the **fermi** in

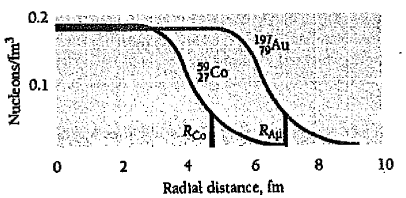


Figure 11.3 The density of nucleons in $^{59}_{27}$Co (cobalt) and $^{197}_{79}$Au (gold) nuclei plotted versus radial distance from the center. The values of the nuclear radius given by $R = 1.2A^{1/3}$ fm are indicated.

honor of Enrico Fermi, a pioneer in nuclear physics. From Eq. (11.1) we find that the radius of the $^{12}_{6}C$ nucleus is

$$R \approx (1.2)(12)^{1/3} \text{ fm} \approx 2.7 \text{ fm}$$

Similarly, the radius of the $^{107}_{47}Ag$ nucleus is 5.7 fm and that of the $^{238}_{92}U$ nucleus is 7.4 fm.

Example 11.1

Find the density of the $^{12}_{6}C$ nucleus.

Solution

The atomic mass of $^{12}_{6}C$ is 12 u. Neglecting the masses and binding energies of the six electrons, we have for the nuclear density

$$\rho = \frac{m}{\frac{4}{3}\pi R^3} = \frac{(12 \text{ u})(1.66 \times 10^{-27} \text{ Kg/u})}{(\frac{4}{3}\pi)(2.7 \times 10^{-15} \text{ m})^3} = 2.4 \times 10^{17} \text{ kg/m}^3$$

This figure—equivalent to 4 billion tons per cubic inch!—is essentially the same for all nuclei. We learned in Sec. 9.11 of the existence of neutron stars, which consist of atoms that have been so compressed that their protons and electrons have interacted to become neutrons. Neutrons in such an assembly, as in a stable nucleus, do not undergo radioactive decay as do free neutrons. The densities of neutron stars are comparable with that of nuclear matter; a neutron star packs the mass of 1.4 to 3 suns into a sphere only about 10 km in radius.

Example 11.2

Find the repulsive electric force on a proton whose center is 2.4 fm from the center of another proton. Assume the protons are uniformly charged spheres of positive charge. (Protons actually have internal structures, as we shall learn in Chapter 13.)

Solution

Everywhere outside a uniformly charged sphere the sphere is electrically equivalent to a point charge located at the center of the sphere. Hence

$$F = \frac{1}{4\pi\epsilon_0}\frac{e^2}{r^2} = \frac{(8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(1.60 \times 10^{-19} \text{ C})^2}{(2.4 \times 10^{-15} \text{ m})^2} = 40 \text{ N}$$

This is equivalent to 9 lb, a familiar enough amount of force—but it acts on a particle whose mass is less than 2×10^{-27} kg! Evidently the attractive forces that bind protons into nuclei despite such repulsions must be very strong indeed.

Spin and Magnetic Moment

Protons and neutrons, like electrons, are fermions with spin quantum numbers of $s = \frac{1}{2}$. This means they have spin angular momenta **S** of magnitude

$$S = \sqrt{s(s+1)}\hbar = \sqrt{\frac{1}{2}\left(\frac{1}{2}+1\right)}\hbar = \frac{\sqrt{3}}{2}\hbar \qquad (11.2)$$

and spin magnetic quantum numbers of $m_s = \pm\frac{1}{2}$ (see Fig. 7.2).

As in the case of electrons, magnetic moments are associated with the spins of protons and neutrons. In nuclear physics, magnetic moments are expressed in **nuclear magnetons** (μ_N), where

Nuclear magneton $$\mu_N = \frac{e\hbar}{2m_p} = 5.051 \times 10^{-27} \text{ J/T} = 3.152 \times 10^{-8} \text{ eV/T} \tag{11.3}$$

Here m_p is the proton mass. The nuclear magneton is smaller than the Bohr magneton of Eq. (6.42) by the ratio of the proton mass to the electron mass, which is 1836. The spin magnetic moments of the proton and neutron have components in any direction of

Proton $$\mu_{pz} = \pm 2.793\ \mu_N$$

Neutron $$\mu_{nz} = \mp 1.913\ \mu_N$$

There are two possibilities for the signs of μ_{pz} and μ_{nz}, depending on whether m_s is $-\frac{1}{2}$ or $+\frac{1}{2}$. The $\pm$ sign is used for μ_{pz} because μ_{pz} is in the same direction as the spin S, whereas $\mp$ is used for μ_{nz} because μ_{nz} is opposite to S (Fig. 11.4).

At first glance it seems odd that the neutron, with no net charge, has a spin magnetic moment. But if we assume that the neutron contains equal amounts of positive and negative charge, a spin magnetic moment could arise even with no net charge. As we shall find in Chap. 13, such a picture has experimental support.

The hydrogen nucleus $^1_1$H consists of a single proton, and its total angular momentum is given by Eq. (11.2). A nucleon in a more complex nucleus may have orbital angular momentum due to motion inside the nucleus as well as spin angular momentum. The total angular momentum of such a nucleus is the vector sum of the spin and orbital angular momenta of its nucleons, as in the analogous case of the electrons of an atom. This subject will be considered further in Sec. 11.6.

When a nucleus whose magnetic moment has the z component μ_z is in a constant magnetic field B, the magnetic potential energy of the nucleus is

Magnetic energy $$U_m = -\mu_z B \tag{11.4}$$

This energy is negative when μ_z is in the same direction as B and positive when μ_z is opposite to B. In a magnetic field, each angular momentum state of the nucleus is therefore split into components, just as in the Zeeman effect in atomic electron states. Figure 11.5 shows the splitting when the angular momentum of the nucleus is due to the spin of a single proton. The energy difference between the sublevels is

$$\Delta E = 2\mu_{pz} B \tag{11.5}$$

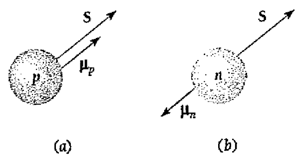


Figure 11.4 (a) The spin magnetic moment μ_p of the proton is in the same direction as its spin angular momentum S. (b) In the case of the neutron, μ_n is opposite to S.

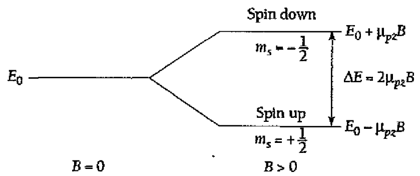


Figure 11.5 The energy levels of a proton in a magnetic field are split into spin-up (S_z parallel to **B**) and spin-down (S_z antiparallel to **B**) sublevels.

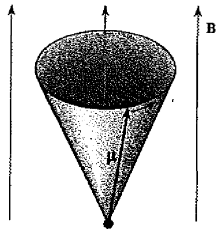


Figure 11.6 A nuclear magnetic moment $\boldsymbol{\mu}$ precesses around an external magnetic field **B** with a frequency called the Larmor frequency that is proportional to B.

A photon with this energy will be emitted when a proton in the upper state flips its spin to fall to the lower state. A proton in the lower state can be raised to the upper one by absorbing a photon of this energy. The photon frequency ν_L that corresponds to ΔE is

Larmor frequency for protons
$$\nu_L = \frac{\Delta E}{h} = \frac{2\mu_{pz}B}{h} \tag{11.6}$$

This is equal to the frequency with which a magnetic dipole precesses around a magnetic field (Fig. 11.6). It is named for Joseph Larmor, who derived ν_L from classical physics for an orbiting electron in a magnetic field; his result can be generalized to any magnetic dipole.

Example 11.3

(*a*) Find the energy difference between the spin-up and spin-down states of a proton in a magnetic field of $B = 1.000$ T (which is quite strong). (*b*) What is the Larmor frequency of a proton in this field?

Solution

(*a*) The energy difference is

$$\Delta E = 2\mu_{pz}B = (2)(2.793)(3.153 \times 10^{-8}\ \text{eV/T})(1.000\ \text{T}) = 1.761 \times 10^{-7}\ \text{eV}$$

If an electron rather than a proton were involved, ΔE would be considerably greater.
(*b*) The Larmor frequency of the proton in this field is

$$\nu_L = \frac{\Delta E}{h} = \frac{1.761 \times 10^{-7}\ \text{eV}}{4.136 \times 10^{-15}\ \text{eV}\cdot\text{s}} = 4.258 \times 10^{7}\ \text{Hz} = 42.58\ \text{MHz}$$

From Fig. 2.2 we see that em radiation of this frequency is in the lower end of the microwave part of the spectrum.

Nuclear Magnetic Resonance

Suppose we put a sample of some substance that contains nuclei with spins of $\frac{1}{2}$ in a magnetic field **B**. The spins of most of these nuclei will become aligned parallel to **B**

Applications of NMR

NMR turns out to be far more useful than just as a way to find nuclear magnetic moments. The electrons around a nucleus partly shield it from an external magnetic field to an extent that depends on the chemical environment of the nucleus. The **relaxation time** needed for the nuclei to drop to the lower state after having been excited also depends on this environment. These properties of NMR enable chemists to use NMR spectroscopy to help unravel details of chemical structures and reactions. For instance, the hydrogen nuclei in the CH_3, CH_2 and OH groups have slightly different resonant frequencies in the same magnetic field. All of these frequencies appear in the NMR spectrum of ethanol with a 3:2:1 ratio of intensities. Ethanol molecules are known to contain two C atoms, six H atoms, and one O atom, so they must consist of the three above groups linked together. The formula CH_3CH_2OH thus better represents methanol than C_2H_6O, which merely lists the atoms in its molecules. The intensity ratio 3:2:1 corroborates this picture since the CH_3 group has three H atoms, CH_2 has two, and OH has one. The NMR spectra of other spin-$\frac{1}{2}$ nuclei, such as ^{13}C and ^{32}P, are also of great help to chemists.

In medicine, NMR is the basis of an imaging method with higher resolution than x-ray tomography. In addition, NMR imaging is safer because rf radiation, unlike x radiation, has too little quantum energy to disrupt chemical bonds and so cannot harm living tissue. What is done is to use a nonuniform magnetic field, which means that the resonance frequency for a particular nucleus depends on the position of the nucleus in the field. Because our bodies are largely water, H_2O, proton NMR is usually employed. By changing the direction of the field gradient, an image that shows the proton density in a thin (3–4 mm) slice of the body can then be constructed by a computer. Relaxation times can also be mapped, which is useful because they are different in diseased tissue. In medicine, NMR imaging is called just magnetic resonance imaging, or MRI, to avoid frightening patients with the word "nuclear."

(spin-up) because this is the lowest energy state; see Fig. 11.5. If we now supply em radiation at the Larmor frequency ν_L to the sample, the nuclei will receive the right amount of energy to flip their spins to the higher state (spin-down). This phenomenon is called **nuclear magnetic resonance** (NMR) and it gives a way to determine nuclear magnetic moments experimentally. In one method, radio frequency (rf) radiation is supplied at a fixed frequency by a coil around the sample, and B is varied until the energy absorbed is a maximum. The resonance frequency is then the Larmor frequency for that value of B, from which μ can be calculated. Another method is to apply a broad-spectrum rf pulse and then measure the frequency (which will be ν_L) of the radiation the sample gives off as its excited nuclei return to the lower energy state.

11.3 STABLE NUCLEI

Why some combinations of neutrons and protons are more stable than others

Not all combinations of neutrons and protons form stable nuclei. In general, light nuclei ($A < 20$) contain approximately equal numbers of neutrons and protons, while in heavier nuclei the proportion of neutrons becomes progressively greater. This is evident from Fig. 11.7, which is a plot of N versus Z for stable nuclides.

The tendency for N to equal Z follows from the existence of nuclear energy levels. Nucleons, which have spins of $\frac{1}{2}$, obey the exclusion principle. As a result, each nuclear energy level can contain two neutrons of opposite spins and two protons of opposite

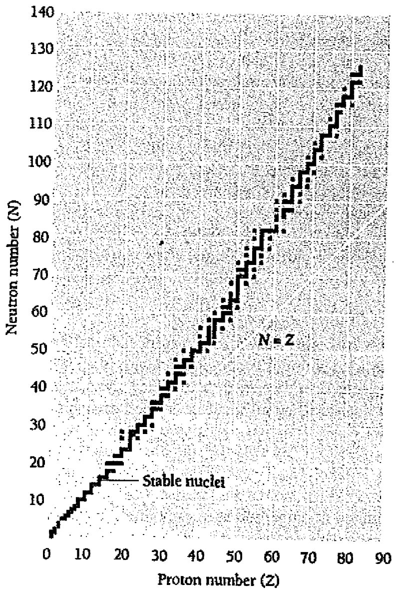


Figure 11.7 Neutron-proton diagram for stable nuclides. There are no stable nuclides with $Z = 43$ or 61, with $N = 19, 35, 39, 45, 61, 89, 115, 126$, or with $A = Z + N = 5$ or 8. All nuclides with $Z > 83$, $N > 126$, and $A > 209$ are unstable.

spins. Energy levels in nuclei are filled in sequence, just as energy levels in atoms are, to achieve configurations of minimum energy and therefore maximum stability. Thus the boron isotope $^{12}_{5}B$ has more energy than the carbon isotope $^{12}_{6}C$ because one of its neutrons is in a higher energy level, and $^{12}_{5}B$ is accordingly unstable (Fig. 11.8). If

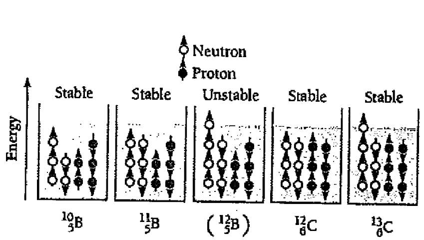


Figure 11.8 Simplified energy-level diagrams of some boron and carbon isotopes. The exclusion principle limits the occupancy of each level to two neutrons of opposite spin and two protons of opposite spin. Stable nuclei have configurations of minimum energy.

created in a nuclear reaction, a $^{12}_{5}B$ nucleus changes by beta decay into a stable $^{12}_{6}C$ nucleus in a fraction of a second.

The preceding argument is only part of the story. Protons are positively charged and repel one another electrically. This repulsion becomes so great in nuclei with more than 10 protons or so that an excess of neutrons, which produce only attractive forces, is required for stability. Thus the curve of Fig. 11.7 departs more and more from the $N = Z$ line as Z increases. Even in light nuclei N may exceed Z, but (except in $^{1}_{1}H$ and $^{3}_{2}He$) is never smaller; $^{11}_{5}B$ is stable, for instance, but not $^{11}_{6}C$.

Sixty percent of stable nuclides have both even Z and even N; these are called "even-even" nuclides. Nearly all the others have either even Z and odd N (even-odd nuclides) or odd Z and even N (odd-even nuclides), with the numbers of both kinds being about equal. Only five stable odd-odd nuclides are known: $^{2}_{1}H$, $^{6}_{3}Li$, $^{10}_{5}Be$, $^{14}_{7}N$, and $^{180}_{73}Ta$. Nuclear abundances follow a similar pattern of favoring even numbers for Z and N. Only about one in eight of the atoms of which the earth is composed has a nucleus with an odd number of protons, for instance.

These observations are consistent with the presence of nuclear energy levels that can each contain two particles of opposite spin. Nuclei with filled levels have less tendency to pick up other nucleons than those with partly filled levels and hence were less likely to participate in the nuclear reactions involved in the formation of the elements.

Nuclear Decay

Nuclear forces are limited in range, and as a result nucleons interact strongly only with their nearest neighbors. This effect is referred to as the **saturation** of nuclear forces. Because the coulomb repulsion of the protons is appreciable throughout the entire nucleus, there is a limit to the ability of neutrons to prevent the disruption of a large nucleus. This limit is represented by the bismuth isotope $^{209}_{83}Bi$, which is the heaviest stable nuclide. All nuclei with $Z > 83$ and $A > 209$ spontaneously transform themselves into lighter ones through the emission of one or more alpha particles, which are $^{4}_{2}He$ nuclei:

Alpha decay

$$^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}He$$

$$\text{Parent nucleus} \rightarrow \text{Daughter nucleus} + \text{Alpha particle}$$

Since an alpha particle consists of two protons and two neutrons, an alpha decay reduces the Z and the N of the original nucleus by two each. If the resulting daughter nucleus has either too small or too large a neutron/proton ratio for stability, it may beta-decay to a more appropriate configuration. In negative beta decay, a neutron is transformed into a proton and an electron is emitted:

Beta decay $\quad n^0 \rightarrow p^+ + e^-$

In positive beta decay, a proton becomes a neutron and a positron is emitted:

Positron emission $\quad p^+ \rightarrow n^0 + e^+$

Thus negative beta decay decreases the proportion of neutrons and positive beta decay increases it. A process that competes with positron emission is the capture by a

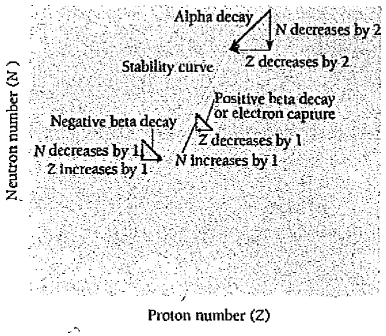


Figure 11.9 Alpha and beta decays permit an unstable nucleus to reach a stable configuration.

nucleus of an electron from its innermost shell. The electron is absorbed by a nuclear proton which is thereby transformed into a neutron:

Electron capture $$p^+ + e^- \rightarrow n^0$$

Figure 11.9 shows how alpha and beta decays enable stability to be achieved. Radioactivity is considered in more detail in Chap. 12, where we will find that another particle, the neutrino, is also involved in beta decay and electron capture.

11.4 BINDING ENERGY

The missing energy that keeps a nucleus together

The hydrogen isotope deuterium, $^2_1$H, has a neutron as well as a proton in its nucleus. Thus we would expect the mass of the deuterium atom to be equal to that of an ordinary $^1_1$H atom plus the mass of a neutron:

Mass of $^1_1$H atom	1.007825 u
+ mass of neutron	+1.008665 u
Expected mass of $^2_1$H atom	2.016490 u

However, the measured mass of the $^2_1$H atom is only 2.014102 u, which is 0.002388 u *less* than the combined masses of a $^1_1$H atom and a neutron (Fig. 11.10).

What comes to mind is that the "missing" mass might correspond to energy given off when a $^2_1$H nucleus is formed from a free proton and neutron. The energy equivalent of the missing mass is

$$\Delta E = (0.002388 \text{ u})(931.49 \text{ MeV/u}) = 2.224 \text{ MeV}$$

To test this interpretation of the missing mass, we can perform experiments to see how much energy is needed to break apart a deuterium nucleus into a separate neutron and

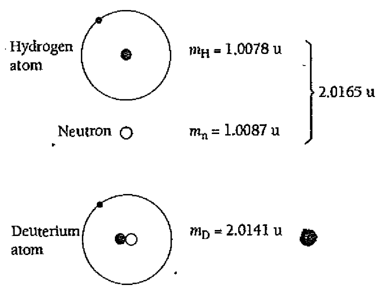


Figure 11.10 The mass of a deuterium atom ($^2_1$H) is less than the sum of the masses of a hydrogen atom ($^1_1$H) and a neutron. The energy equivalent of the missing mass is called the binding energy of the nucleus.

proton. The required energy indeed turns out to be 2.224 MeV (Fig. 11.11). When less energy than 2.224 MeV is given to a $^2_1$H nucleus, the nucleus stays together. When the added energy is more than 2.224 MeV, the extra energy goes into kinetic energy of the neutron and proton as they fly apart.

Deuterium atoms are not the only ones that have less mass than the combined masses of the particles they are composed of—*all* atoms are like that. The energy equivalent of the missing mass of a nucleus is called the **binding energy** of the nucleus. The greater its binding energy, the more the energy that must be supplied to break up the nucleus.

The binding energy E_b in MeV of the nucleus ^A_ZX, which has $N = A - Z$ neutrons, is given by

$$E_b = [Zm(^1_1\text{H}) + Nm(n) - m(^A_ZX)](931.49 \text{ MeV/u}) \tag{11.7}$$

where $m(^1_1\text{H})$ is the atomic mass of $^1_1$H, $m(n)$ is the neutron mass, and $m(^A_ZX)$ is the atomic mass of ^A_ZX, all in mass units. As mentioned before, atomic masses, not nuclear masses, are used in such calculations; the electron masses subtract out.

Nuclear binding energies are strikingly high. The range for stable nuclei is from 2.224 MeV for $^2_1$H (deuterium) to 1640 MeV for $^{209}_{83}$Bi (an isotope of the metal bismuth). To appreciate how high binding energies are, we can compare them with more familiar energies in terms of kilojoules of energy per kilogram of mass. In these units, a typical binding energy is 8×10^{11} kJ/kg—800 billion kJ/kg. By contrast, to boil water

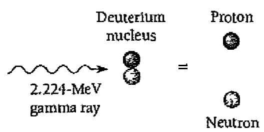


Figure 11.11 The binding energy of the deuterium nucleus is 2.224 MeV. A gamma ray whose energy is 2.224 MeV or more can split a deuterium nucleus into a proton and neutron. A gamma ray whose energy is less than 2.224 MeV cannot do this.

involves a heat of vaporization of a mere 2260 kJ/kg, and even the heat given off by burning gasoline is only 4.7×10^4 kJ/kg, 17 million times smaller.

Example 11.4

The binding energy of the neon isotope $^{20}_{10}\text{Ne}$ is 160.647 MeV. Find its atomic mass.

Solution

Here $Z = 10$ and $N = 10$. From Eq. (11.7),

$$m(^A_Z X) = [Zm(^1_1\text{H}) + Nm(n)] - \frac{E_b}{931.49\ \text{MeV/u}}$$

$$m(^{20}_{10}\text{Ne}) = [10(1.007825\ \text{u}) + 10(1.008665)] - \frac{160.647\ \text{MeV}}{931.49\ \text{MeV/u}} = 19.992\ \text{u}$$

Binding Energy per Nucleon

The **binding energy per nucleon** for a given nucleus is an average found by dividing its total binding energy by the number of nucleons it contains. Thus the binding energy per nucleon for ^2_1H is $(2.2\ \text{MeV})/2 = 1.1$ MeV/nucleon, and for $^{209}_{83}\text{Bi}$ it is $(1640\ \text{MeV})/209 = 7.8$ MeV/nucleon.

Figure 11.12 shows the binding energy per nucleon plotted against the number of nucleons in various atomic nuclei. The greater the binding energy per nucleon, the

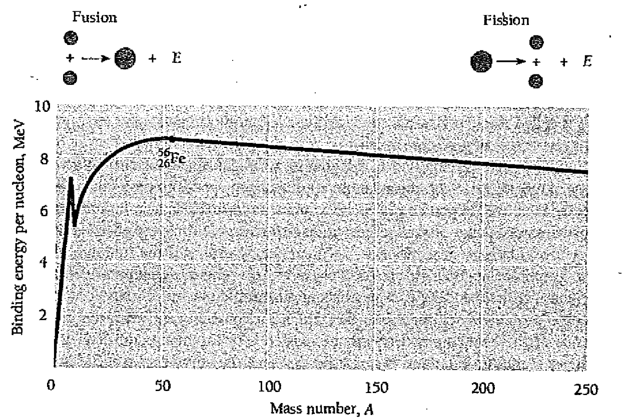


Figure 11.12 Binding energy per nucleon as a function of mass number. The peak at $A = 4$ corresponds to the exceptionally stable ^4_2He nucleus, which is the alpha particle. The binding energy per nucleon is a maximum for nuclei of mass number $A = 56$. Such nuclei are the most stable. When two light nuclei join to form a heavier one, a process called fusion, the greater binding energy of the product nucleus causes energy to be given off. When a heavy nucleus is split into two lighter ones, a process called fission, the greater binding energy of the product nuclei also causes energy to be given off.

The Strong Interaction

The short-range attractive forces between nucleons arise from the **strong interaction.** (There is another fundamental interaction affecting nucleons called the **weak interaction** that will be discussed in Chaps. 12 and 13.) The strong interaction is what holds nucleons together to form nuclei, and it is powerful enough to overcome the electric repulsion of the positively charged protons in nuclei provided neutrons are also present to help. If the strong interaction were a little stronger—perhaps only 1 percent would be enough—two protons could stick together without any neutrons needed. In this case, when the universe came into being in the Big Bang (Sec. 13.8), all its protons would have joined into diprotons almost as soon as they appeared. Then there would be no individual protons to undergo the fusion reactions that power the stars and have created the chemical elements. The universe would be a very different place from what it is today, and we would not exist.

more stable the nucleus is. The graph has its maximum of 8.8 MeV/nucleon when the total number of nucleons is 56. The nucleus that has 56 protons and neutrons is $^{56}_{26}Fe$, an iron isotope. This is the most stable nucleus of them all, since the most energy is needed to pull a nucleon away from it.

Two remarkable conclusions can be drawn from the curve of Fig. 11.12. The first is that if we can somehow split a heavy nucleus into two medium-sized ones, each of the new nuclei will have *more* binding energy per nucleon than the original nucleus did. The extra energy will be given off, and it can be a lot. For instance, if the uranium nucleus $^{235}_{92}U$ is broken into two smaller nuclei, the binding energy difference per nucleon is about 0.8 MeV. The total energy given off is therefore

$$\left(0.8\,\frac{\text{MeV}}{\text{nucleon}}\right)(235 \text{ nucleons}) = 188 \text{ MeV}$$

This is a truly enormous amount of energy to be produced in a single atomic event. As we know, ordinary chemical reactions involve rearrangements of the electrons in atoms and liberate only a few electronvolts per reacting atom. Splitting a heavy nucleus, which is called **nuclear fission,** thus involves 100 million times more energy per atom than, say, the burning of coal or oil.

The other notable conclusion is that joining two light nuclei together to give a single nucleus of medium size also means more binding energy per nucleon in the new nucleus. For instance, if two ^2_1H deuterium nuclei combine to form a ^4_2He helium nucleus, over 23 MeV is released. Such a process, called **nuclear fusion,** is also a very effective way to obtain energy. In fact, nuclear fusion is the main energy source of the sun and other stars.

The graph of Fig. 11.12 has a good claim to being the most significant in all of science. The fact that binding energy exists at all means that nuclei more complex than the single proton of hydrogen can be stable. Such stability in turn accounts for the existence of the elements and so for the existence of the many and diverse forms of matter we see around us (and for us, too). Because the curve peaks in the middle, we have the explanation for the energy that powers, directly or indirectly, the evolution of the universe: it comes from the fusion of light nuclei to form heavier ones.

Example 11.5

(*a*) Find the energy needed to remove a neutron from the nucleus of the calcium isotope $^{42}_{20}Ca$. (*b*) Find the energy needed to remove a proton from this nucleus. (*c*) Why are these energies different?

Solution

(*a*) Removing a neutron from $^{42}_{20}Ca$ leaves $^{41}_{20}Ca$. From the table of atomic masses in the Appendix the mass of $^{41}_{20}Ca$ plus the mass of a free neutron is

$$40.962278 \text{ u} + 1.008665 \text{ u} = 41.970943 \text{ u}$$

The difference between this mass and the mass of $^{42}_{20}Ca$ is 0.012321 u, so the binding energy of the missing neutron is

$$(0.012321 \text{ u})(931.49 \text{ MeV/u}) = 11.48 \text{ MeV}$$

(*b*) Removing a proton from $^{42}_{20}Ca$ leaves the potassium isotope $^{41}_{19}K$. A similar calculation gives a binding energy of 10.27 MeV for the missing proton.
(*c*) The neutron was acted upon only by attractive nuclear forces whereas the proton was also acted upon by repulsive electric forces that decrease its binding energy.

11.5 LIQUID-DROP MODEL

A simple explanation for the binding-energy curve

The short-range force that binds nucleons so securely into nuclei is by far the strongest type of force known. Unfortunately the nuclear force is not as well understood as the electromagnetic force, and the theory of nuclear structure is less complete than the theory of atomic structure. However, even without a full understanding of the nuclear force, much progress has been made in devising nuclear models able to account for prominent aspects of nuclear properties and behavior. We shall examine some of the concepts embodied in these models in this section and the next.

While the attractive forces that nucleons exert upon one another are very strong, their range is short. Up to a separation of about 3 fm, the nuclear attraction between two protons is about 100 times stronger than the electric repulsion between them. The nuclear interactions between protons and protons, between protons and neutrons, and between neutrons and neutrons appear to be identical.

As a first approximation, we can think of each nucleon in a nucleus as interacting solely with its nearest neighbors. This situation is the same as that of atoms in a solid, which ideally vibrate about fixed positions in a crystal lattice, or that of molecules in a liquid, which ideally are free to move about while maintaining a fixed intermolecular distance. The analogy with a solid cannot be pursued because a calculation shows that the vibrations of the nucleons about their average positions would be too great for the nucleus to be stable. The analogy with a liquid, on the other hand, turns out to be extremely useful in understanding certain aspects of nuclear behavior. This analogy was proposed by George Gamow in 1929 and developed in detail by C. F. von Weizsäcker in 1935.

Let us see how the picture of a nucleus as a drop of liquid accounts for the observed variation of binding energy per nucleon with mass number. We start by assuming that the energy associated with each nucleon-nucleon bond has some value U. This energy is actually negative since attractive forces are involved, but is usually written as positive because binding energy is considered a positive quantity for convenience.

Because each bond energy U is shared by two nucleons, each has a binding energy of $\frac{1}{2}U$. When an assembly of spheres of the same size is packed together into the smallest volume, as we suppose is the case of nucleons within a nucleus, each interior sphere

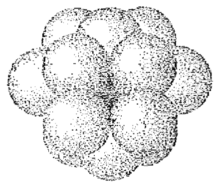


Figure 11.13 In a tightly packed assembly of identical spheres, each interior sphere is in contact with 12 others.

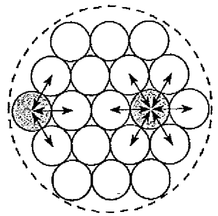


Figure 11.14 A nucleon at the surface of a nucleus interacts with fewer other nucleons than one in the interior of the nucleus and hence its binding energy is less. The larger the nucleus, the smaller the proportion of nucleons at the surface.

has 12 other spheres in contact with it (Fig. 11.13). Hence each interior nucleon in a nucleus has a binding energy of $(12)(\frac{1}{2}U)$ or 6 U. If all A nucleons in a nucleus were in its interior, the total binding energy of the nucleus would be

$$E_v = 6\,AU \tag{11.8}$$

Equation (11.8) is often written simply as

Volume energy
$$E_v = a_1 A \tag{11.9}$$

The energy E_v is called the **volume energy** of a nucleus and is directly proportional to A.

Actually, of course, some nucleons are on the surface of every nucleus and therefore have fewer than 12 neighbors (Fig. 11.14). The number of such nucleons depends on the surface area of the nucleus in question. A nucleus of radius R has an area of $4\pi R^2 = 4\pi R_0^2 A^{2/3}$. Hence the number of nucleons with fewer than the maximum number of bonds is proportional to $A^{2/3}$, reducing the total binding energy by

Surface energy
$$E_s = -a_2 A^{2/3} \tag{11.10}$$

The negative energy E_s is called the **surface energy** of a nucleus. It is most significant for the lighter nuclei since a greater fraction of their nucleons are on the surface. Because natural systems always tend to evolve toward configurations of minimum potential energy, nuclei tend toward configurations of maximum binding energy. Hence a nucleus should exhibit the same surface-tension effects as a liquid drop, and in the absence of other effects it should be spherical, since a sphere has the least surface area for a given volume.

The electric repulsion between each pair of protons in a nucleus also contributes toward decreasing its binding energy. The **coulomb energy** E_c of a nucleus is the work that must be done to bring together Z protons from infinity into a spherical aggregate the size of the nucleus. The potential energy of a pair of protons r apart is equal to

$$V = -\frac{e^2}{4\pi\epsilon_0 r}$$

Since there are $Z(Z-1)/2$ pairs of protons,

$$E_c = \frac{Z(Z-1)}{2}V = -\frac{Z(Z-1)e^2}{8\pi\epsilon_0}\left(\frac{1}{r}\right)_{av} \tag{11.11}$$

where $(1/r)_{av}$ is the value of $1/r$ averaged over all proton pairs. If the protons are uniformly distributed throughout a nucleus of radius R, $(1/r)_{av}$ is proportional to $1/R$ and hence to $1/A^{1/3}$, so that

Coulomb energy
$$E_c = -a_3\frac{Z(Z-1)}{A^{1/3}} \tag{11.12}$$

The coulomb energy is negative because it arises from an effect that opposes nuclear stability.

This is as far as the liquid-drop model itself can go. Let us now see how the result compares with reality.

The total binding energy E_b of a nucleus ought to be the sum of its volume, surface, and coulomb energies:

$$E_b = E_v + E_s + E_c = a_1A - a_2A^{2/3} - a_3\frac{Z(Z-1)}{A^{1/3}} \tag{11.13}$$

The binding energy per nucleon is therefore

$$\frac{E_b}{A} = a_1 - \frac{a_2}{A^{1/3}} - a_3\frac{Z(Z-1)}{A^{4/3}} \tag{11.14}$$

Each of the terms of Eq. (11.14) is plotted in Fig. 11.15 versus A, together with their sum E_b/A. The coefficients were chosen to make the E_b/A curve resemble as closely as possible the empirical binding energy per nucleon curve of Fig. 11.12. The fact that the theoretical curve can be made to agree so well with the empirical one means that the analogy between a nucleus and a liquid drop has at least some validity.

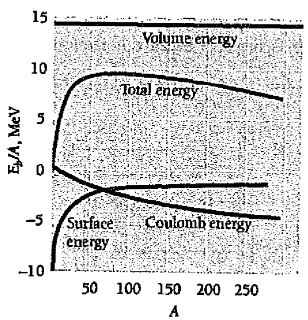


Figure 11.15 The binding energy per nucleon is the sum of the volume, surface, and coulomb energies.

Corrections to the Formula

The binding-energy formula of Eq. (11.13) can be improved by taking into account two effects that do not fit into the simple liquid-drop model but which make sense in terms of a model that provides for nuclear energy levels. (We will see in the next section how these apparently very different approaches can be reconciled.) One of these effects occurs when the neutrons in a nucleus outnumber the protons, which means that higher energy levels have to be occupied than would be the case if N and Z were equal.

Let us suppose that the uppermost neutron and proton energy levels, which the exclusion principle limits to two particles each, have the same spacing ϵ, as in Fig. 11.16. In order to produce a neutron excess of, say, $N - Z = 8$ without changing A, $\frac{1}{2}(N - Z) = 4$ neutrons would have to replace protons in an original nucleus in which $N = Z$. The new neutrons would occupy levels higher in energy by $2\epsilon = 4\epsilon/2$ than those of the protons they replace. In the general case of $\frac{1}{2}(N - Z)$ new neutrons, each must be raised in energy by $\frac{1}{2}(N - Z)\epsilon/2$. The total work needed is

$$\Delta E = (\text{number of new neutrons})\left(\frac{\text{energy increase}}{\text{new neutron}}\right)$$

$$= \left[\frac{1}{2}(N - Z)\right]\left[\frac{1}{2}(N - Z)\frac{\epsilon}{2}\right] = \frac{\epsilon}{8}(N - Z)^2$$

Because $N = A - Z$, $(N - Z)^2 = (A - 2Z)^2$, and

$$\Delta E = \frac{\epsilon}{8}(A - 2Z)^2 \tag{11.15}$$

As it happens, the greater the number of nucleons in a nucleus, the smaller is the energy level spacing ϵ, with ϵ proportional to $1/A$. This means that the **asymmetry energy** E_a due to the difference between N and Z can be expressed as

Asymmetry energy
$$E_a = -\Delta E = -a_4\frac{(A - 2Z)^2}{A} \tag{11.16}$$

The asymmetry energy is negative because it reduces the binding energy of the nucleus.

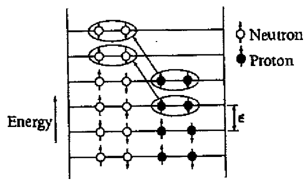


Figure 11.16 In order to replace 4 protons in a nucleus with $N = Z$ by 4 neutrons, the work $(4)(4\epsilon/2)$ must be done. The resulting nucleus has 8 more neutrons than protons.

The last correction term arises from the tendency of proton pairs and neutron pairs to occur (Sec. 11.3). Even-even nuclei are the most stable and hence have higher binding energies than would otherwise be expected. Thus such nuclei as ^4_2He, $^{12}_6C$, and $^{16}_8O$ appear as peaks on the empirical curve of binding energy per nucleon. At the other extreme, odd-odd nuclei have both unpaired protons and neutrons and have relatively low binding energies. The **pairing energy** E_p is positive for even-even nuclei, 0 for odd-even and even-odd nuclei, and negative for odd-odd nuclei, and seems to vary with A as $A^{-3/4}$. Hence

Pairing energy

$$E_p = (\pm, 0)\frac{a_5}{A^{3/4}} \tag{11.17}$$

The final expression for the binding energy of a nucleus of atomic number Z and mass number A, which was first obtained by C. F. von Weizsäcker in 1935, is

Semiempirical binding-energy formula

$$E_b = a_1 A - a_2 A^{2/3} - a_3\frac{Z(Z-1)}{A^{1/3}} - a_4\frac{(A-2Z)^2}{A}(\pm, 0)\frac{a_5}{A^{3/4}} \tag{11.18}$$

A set of coefficients that gives a good fit with the data is as follows:

$a_1 = 14.1$ MeV $\quad a_2 = 13.0$ MeV $\quad a_3 = 0.595$ MeV

$a_4 = 19.0$ MeV $\quad a_5 = 33.5$ MeV

Other sets of coefficients have also been proposed. Equation (11.18) agrees better with observed binding energies than does Eq. (11.13), which suggests that the liquid-drop model, though a good approximation, is not the last word on the subject.

Example 11.6

The atomic mass of the zinc isotope $^{64}_{30}Zn$ is 63.929 u. Compare its binding energy with the prediction of Eq. (11.18).

Solution

The binding energy of $^{64}_{30}Zn$ is, from Eq. (11.7),

$$E_b = [(30)(1.007825\text{ u}) + (34)(1.008665\text{ u}) - 63.929\text{ u}](931.49\text{ MeV/u}) = 559.1\text{ MeV}$$

The semiempirical binding energy formula, using the coefficients in the text, gives

$$E_b = (14.1\text{ MeV})(64) - (13.0\text{ MeV})(64)^{2/3} - \frac{(0.595\text{ MeV})(30)(29)}{(64)^{1/3}} - \frac{(19.0\text{ MeV})(16)}{64} + \frac{33.5\text{ MeV}}{(64)^{3/4}} = 561.7\text{ MeV}$$

The plus sign is used for the last term because $^{64}_{30}Zn$ is an even-even nucleus. The difference between the observed and calculated binding energies is less than 0.5 percent.

Example 11.7

Isobars are nuclides that have the same mass number A. Derive a formula for the atomic number of the most stable isobar of a given A and use it to find the most stable isobar of $A = 25$.

Solution

To find the value of Z for which the binding energy E_b is a maximum, which corresponds to maximum stability, we must solve $dE_b/dZ = 0$ for Z. From Eq. (11.18) we have

$$\frac{dE_b}{dZ} = -\frac{a_3}{A^{1/3}}(2Z-1) + \frac{4a_4}{A}(A-2Z) = 0$$

$$Z = \frac{a_3A^{-1/3} + 4a_4}{2a_3A^{-1/3} + 8a_4A^{-1}} = \frac{0.595A^{-1/3} + 76}{1.19A^{-1/3} + 152A^{-1}}$$

For $A = 25$ this formula gives $Z = 11.7$, from which we conclude that $Z = 12$ should be the atomic number of the most stable isobar of $A = 25$. This nuclide is $^{25}_{12}Mg$, which is in fact the only stable $A = 25$ isobar. The other isobars, $^{25}_{11}Na$ and $^{25}_{13}Al$, are both radioactive.

11.6 SHELL MODEL

Magic numbers in the nucleus

The basic assumption of the liquid-drop model is that each nucleon in a nucleus interacts only with its nearest neighbors, like a molecule in a liquid. At the other extreme, the hypothesis that each nucleon interacts chiefly with a general force field produced by all the other nucleons also has a lot of support. The latter situation is like that of electrons in an atom, where only certain quantum states are permitted and no more than two electrons, which are fermions, can occupy each state. Nucleons are also fermions, and several nuclear properties vary periodically with Z and N in a manner reminiscent of the periodic variation of atomic properties with Z.

The electrons in an atom may be thought of as occupying positions in "shells" designated by the various principal quantum numbers. The degree of occupancy of the outermost shell is what determines certain important aspects of an atom's behavior. For instance, atoms with 2, 10, 18, 36, 54, and 86 electrons have all their electron shells completely filled. Such electron structures have high binding energies and are exceptionally stable, which accounts for the chemical inertness of the rare gases.

The same kind of effect is observed with respect to nuclei. Nuclei that have 2, 8, 20, 28, 50, 82, and 126 neutrons or protons are more abundant than other nuclei of similar mass numbers, suggesting that their structures are more stable. Since complex nuclei arose from reactions among lighter ones, the evolution of heavier and heavier nuclei became retarded when each relatively inert nucleus was formed, which accounts for their abundance.

Other evidence also points up the significance in nuclear structure of the numbers 2, 8, 20, 28, 50, 82, and 126, which have become known as **magic numbers.** An example is the observed pattern of nuclear electric quadrupole moments, which are measures of how much nuclear charge distributions depart from sphericity. A spherical nucleus has no quadrupole moment, while one shaped like a football has a positive

Maria Goeppert-Mayer (1906–1972) was the daughter of the pediatrician of Max Born's children, and she studied at Göttingen under Born. As Born recalled, "She went through all my courses with great industry and conscientiousness, yet remained at the same time a gay and witty member of Göttingen society, fond of parties, of laughter, dancing, and jokes. . . . After she got her doctor's degree with a very good thesis on a problem of quantum mechanics, she married a young American, Joseph Mayer, who worked with me on problems of crystal theory. Both had brilliant careers in the U.S.A., always remaining together." At the University of Chicago in 1948 Goeppert-Mayer reopened the question of periodicities in nuclear stability, which had remained a mystery since their discovery in the early 1930s, and devised a shell model that agreed with the data. J. H. D. Jensen in Germany published a similar theory independently at the same time, and both received the Nobel Prize in 1963 for their work.

moment and one shaped like a pumpkin has a negative moment. Nuclei of magic N and Z are found to have zero quadrupole moments and hence are spherical, while other nuclei are distorted in shape.

The **shell model** of the nucleus is an attempt to account for the existence of magic numbers and certain other nuclear properties in terms of nucleon behavior in a common force field.

Because the precise form of the potential-energy function for a nucleus is not known, unlike the case of an atom, a suitable function $U(r)$ has to be assumed. A reasonable guess on the basis of the nuclear density curves of Fig. 11.3 is a square well with rounded corners. Schrödinger's equation for a particle in a potential well of this kind is then solved, and it is found that stationary states of the system occur that are characterized by quantum numbers n, l, and m_l whose significance is the same as in the analogous case of stationary states of atomic electrons. Neutrons and protons occupy separate sets of states in a nucleus because the latter interact electrically as well as through the specifically nuclear charge. However, the energy levels that come from such a calculation do not agree with the observed sequence of magic numbers. Using other potential-energy functions, for instance that of the harmonic oscillator, gives no better results. Something essential is missing from the picture.

How Magic Numbers Arise

The problem was finally solved independently by Maria Goeppert-Mayer and J. H. D. Jensen in 1949. They realized that it is necessary to incorporate a spin-orbit interaction whose magnitude is such that the consequent splitting of energy levels into sublevels is many times larger than the analogous splitting of atomic energy levels. The exact form of the potential-energy function then turns out not to be critical, provided that it more or less resembles a square well.

The shell theory assumes that LS coupling holds only for the very lightest nuclei, in which the l values are necessarily small in their normal configurations. In this scheme, as we saw in Chap. 7, the intrinsic spin angular momenta $\mathbf{S}_i$ of the particles concerned (the neutrons form one group and the protons another) are coupled together into a total spin momentum $\mathbf{S}$. The orbital angular momenta $\mathbf{L}_i$ are separately coupled together into a total orbital momentum $\mathbf{L}$. Then $\mathbf{S}$ and $\mathbf{L}$ are coupled to form a total angular momentum $\mathbf{J}$ of magnitude $\sqrt{J(J+1)}\hbar$.

After a transition region in which an intermediate coupling scheme holds, the heavier nuclei exhibit ***jj* coupling.** In this case the $\mathbf{S}_i$ and $\mathbf{L}_i$ of each particle are first coupled to form a $\mathbf{J}_i$ for that particle of magnitude $\sqrt{j(j+1)}\hbar$. The various $\mathbf{J}_i$ then couple together to form the total angular momentum $\mathbf{J}$. The *jj* coupling scheme holds for the great majority of nuclei.

When an appropriate strength is assumed for the spin-orbit interaction, the energy levels of either class of nucleon fall into the sequence shown in Fig. 11.17. The levels

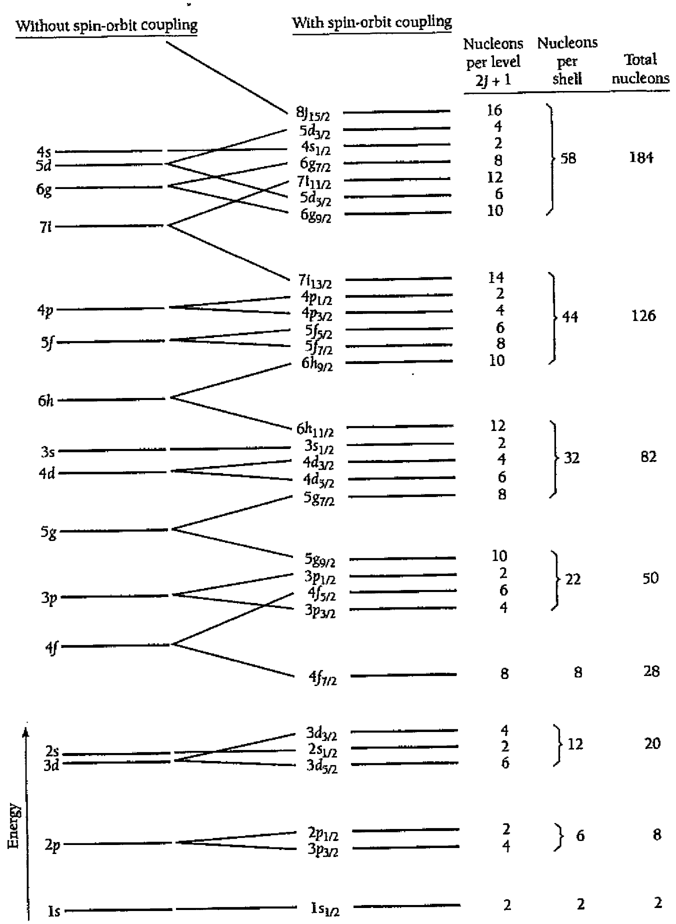


Figure 11.17 Sequence of neutron and proton energy levels according to the shell model (not to scale). The numbers in the right-hand column correspond to the magic numbers expected on the basis of this sequence.

are designated by a prefix equal to the total quantum number n, a letter that indicates l for each particle in that level according to the usual pattern (s, p, d, f, g, . . . corresponding, respectively, to $l = 0, 1, 2, 3, 4, \ldots$), and a subscript equal to j. The spin-orbit interaction splits each state of given j into $2j + 1$ substates, since there are $2j + 1$ allowed orientations of $\mathbf{J}_i$. Large energy gaps appear in the spacing of the levels at intervals that are consistent with the notion of separate shells. The number of available nuclear states in each nuclear shell is, in ascending order of energy, 2, 6, 12, 8, 22, 32, and 44. Hence shells are filled when there are 2, 8, 20, 28, 50, 82, and 126 neutrons or protons in a nucleus.

The shell model accounts for several nuclear phenomena in addition to magic numbers. To begin with, the very existence of energy sublevels that can each be occupied by two particles of opposite spin explains the tendency of nuclear abundances to favor even Z and even N as discussed in Sec. 11.3.

The shell model can also predict nuclear angular momenta. In even-even nuclei, all the protons and neutrons should pair off to cancel out one another's spin and orbital angular momenta. Thus even-even nuclei ought to have zero nuclear angular momenta, as observed. In even-odd and odd-even nuclei, the half-integral spin of the single "extra" nucleon should be combined with the integral angular momentum of the rest of the nucleus for a half-integral total angular momentum. Odd-odd nuclei each have an extra neutron and an extra proton whose half-integral spins should yield integral total angular momenta. Both these predictions are experimentally confirmed.

Reconciling the Models

If the nucleons in a nucleus are so close together and interact so strongly that the nucleus can be considered as analogous to a liquid drop, how can these same nucleons be regarded as moving independently of each other in a common force field as required by the shell model? It would seem that the points of view are mutually exclusive, since a nucleon moving about in a liquid-drop nucleus must surely undergo frequent collisions with other nucleons.

A closer look shows that there is no contradiction. In the ground state of a nucleus, the neutrons and protons fill the energy levels available to them in order of increasing energy in such a way as to obey the exclusion principle (see Fig. 11.8). In a collision, energy is transferred from one nucleon to another, leaving the former in a state of reduced energy and the latter in one of increased energy. But all the available levels of lower energy are already filled, so such an energy transfer can take place only if the exclusion principle is violated. Of course, it is possible for two indistinguishable nucleons of the same kind to merely exchange their respective energies, but such a collision is hardly significant since the system remains in exactly the same state it was in initially. In essence, then, the exclusion principle prevents nucleon-nucleon collisions even in a tightly packed nucleus and thereby justifies the independent-particle approach to nuclear structure.

Both the liquid-drop and shell models of the nucleus are, in their very different ways, able to account for much that is known of nuclear behavior. The **collective model** of Aage Bohr (Niels Bohr's son) and Ben Mottelson combines features of both models in a consistent scheme that has proved quite successful. The collective model takes into account such factors as the nonspherical shape of all but even-even nuclei and the centrifugal distortion experienced by a rotating nucleus. The detailed theory is able to account for the spacing of excited nuclear levels inferred from the gamma-ray spectra of nuclei and in other ways.

Island of Stability

As mentioned in Sec. 11.3, the short range of the strong interaction means that the largest stable nucleus is that of the bismuth isotope $^{209}_{83}Bi$. All nuclei with $Z > 83$ and $A > 209$ undergo radioactive decays until they reach a stable configuration. We can think of the stable nuclei in Fig. 11.7 as representing a peninsula of stability in a sea of instability.

In general, the farther from the peninsula of stability a nucleus is, the faster it decays. For nuclei heavier than $^{209}_{83}Bi$, lifetimes become shorter and shorter with increasing size until they are only milliseconds for $Z = 107$, 108, and 109. (Such superheavy nuclei are created in the laboratory by bombarding targets of heavy atoms with beams of lighter ones.) Since a nucleus with magic numbers of protons or neutrons is exceptionally stable, the question arises whether there might be an island of relative stability among the superheavy nuclei.

In the case of neutrons, Fig. 11.17 shows that the next magic number after $N = 126$ is $N = 184$. For protons the situation is complicated by their electric potential energy, which becomes significant relative to the purely nuclear potential energy (which is independent of charge) when Z is large. The electric potential has a greater effect on proton levels of low l because it is stronger near the nuclear center where the probability densities of such levels are concentrated (see Fig. 6.8). In consequence, the order of proton levels changes from that shown in Fig. 11.17 to make $Z = 114$ a proton magic number instead of $Z = 126$.

A nucleus with $Z = 114$ and $N = 184$ would therefore be doubly magic. This nucleus and nuclei near it in Z and N ought to form an island of stability in the sea of instability that is (so to speak) northeast of the tip of the peninsula of stability in Fig. 11.7.

In 1998 Russian physicists directed a beam of the calcium isotope $^{48}_{20}Ca$ at a target of the plutonium isotope $^{244}_{94}Pu$ to create a nucleus of $Z = 114$ and $N = 175$. Magic in proton number and not far from the middle of the island of stability, this nucleus has a half-life (the time needed for half a sample to decay; see Sec. 12.2) of 30.4 s. As expected, this half-life is much longer than those of nuclei near but outside the island of stability.

When the idea of an island of stability first came up in 1966, it was thought that perhaps the nucleus of $Z = 114$, $N = 184$ might have a half-life in the billions of years. Later calculations gave more modest estimates that range from less than a hundred years to millions of years. When this doubly magic nucleus is eventually produced, we will know. In the meantime, physicists at the Lawrence Berkeley National Laboratory in California have managed to sail past the island of stability to create nuclei of $Z = 116$.

11.7 MESON THEORY OF NUCLEAR FORCES

Particle exchange can produce either attraction or repulsion

In Chap. 8 we saw how a molecule is held together by the exchange of electrons between adjacent atoms. Is it possible that a similar mechanism operates inside a nucleus, with its component nucleons being held together by the exchange of particles of some kind among them?

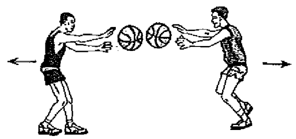
The first approach to this question was made in 1932 by Heisenberg, who suggested that electrons and positrons shift back and forth between nucleons. A neutron, for instance, might emit an electron and become a proton, while a proton absorbing the electron would become a neutron. However, calculations based on beta-decay data showed that the forces resulting from electron and positron exchange by nucleons would be too small by the huge factor of 10^{14} to be significant in nuclear structure.

Hideki Yukawa (1907–1981) grew up in Kyoto, Japan, and attended the university there. After receiving his doctorate at Osaka, he returned to Kyoto where he spent the rest of his career. In the early 1930s Yukawa tackled the problem of what keeps an atomic nucleus together despite the repulsive forces its protons exert on one another. The interaction must be extremely strong but limited in range, and Yukawa found it could be explained on the basis of the exchange between nucleons of particles whose mass is in the neighborhood of 200 electron masses: "Could the neutrons and protons be playing catch?" In 1936, the year after Yukawa published his proposal, a particle of such intermediate mass was found in cosmic rays by C. D. Anderson, who had earlier discovered the positron, and others. But, this particle, today called the muon, did not interact strongly with nuclei, as it should have. The mystery was not cleared up until 1947 when British physicist C. F. Powell discovered the pion, which has the properties Yukawa predicted but decays rapidly into the longer-lived (and hence easier-to-detect) muon. (The pion and muon were originally called the π and μ mesons by Powell because, according to legend, these were the only Greek letters on his typewriter.) Yukawa received the Nobel Prize in 1949, the first Japanese to do so.

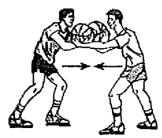
The Japanese physicist Hideki Yukawa was more successful with his 1935 proposal that particles intermediate in mass between electrons and nucleons are responsible for nuclear forces. Today these particles are called **pions**. Pions may be charged (π^+, π^-) or neutral (π^0), and are members of a class of elementary particles collectively called **mesons**. The word pion is a contraction of the original name π meson.

According to Yukawa's theory, every nucleon continually emits and reabsorbs pions. If another nucleon is nearby, an emitted pion may shift across to it instead of returning to its parent nucleon. The associated transfer of momentum is equivalent to the action of a force. Nuclear forces are repulsive at very short range as well as being attractive at greater nucleon-nucleon distances; otherwise the nucleons in a nucleus would mesh together. One of the strengths of the meson theory of such forces is that it can account for both these properties. Although there is no simple way to explain how this comes about, a rough analogy may make it less mysterious.

Let us imagine two boys exchanging basketballs (Fig. 11.18). If they throw the balls at each other, the boys move backward, and when they catch the balls thrown at them,



Repulsive force due to particle exchange



Attractive force due to particle exchange

Figure 11.18 Attractive and repulsive forces can both arise from particle exchange.

their backward momentum increases. Thus this method of exchanging basketballs has the same effect as a repulsive force between the boys. If the boys snatch the basketballs from each other's hands, however, the result will be equivalent to an attractive force acting between them.

A fundamental problem presents itself at this point. If nucleons constantly emit and absorb pions, why are neutrons and protons never found with other than their usual masses? The answer is based upon the uncertainty principle. The laws of physics refer to measurable quantities only, and the uncertainty principle limits the accuracy with which certain combinations of measurements can be made. The emission of a pion by a nucleon which does not change in mass—a clear violation of the law of conservation of energy—can take place provided that the nucleon reabsorbs it or absorbs another pion emitted by a neighboring nucleon so soon afterward that *even in principle* it is impossible to determine whether or not any mass change has actually been involved.

From the uncertainty principle in the form

$$\Delta E \, \Delta t \geq \frac{\hbar}{2} \tag{3.26}$$

an event in which an amount of energy ΔE is not conserved is not prohibited so long as the duration of the event does not exceed $\hbar/2\Delta E$. This condition lets us estimate the pion mass.

Let us assume that a pion travels between nucleons at a speed of $v \sim c$ (actually $v < c$, of course); that the emission of a pion of mass m_π represents a temporary energy discrepancy of $\Delta E \sim m_\pi c^2$ (this neglects the pion's kinetic energy); and that $\Delta E \, \Delta t \sim \hbar$. Nuclear forces have a maximum range r of about 1.7 fm, and the time Δt needed for the pion to travel this far (Fig. 11.19) is

$$\Delta t = \frac{r}{v} \sim \frac{r}{c}$$

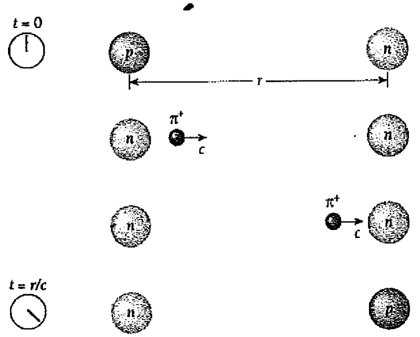


Figure 11.19 The uncertainty principle permits the creation, transfer, and disappearance of a pion to occur without violating conservation of energy provided that the sequence takes place fast enough. Here a positive pion emitted by a proton is absorbed by a neutron; as a result, the proton becomes a neutron and the neutron becomes a proton.

We therefore have

$$\Delta E\,\Delta t \sim \hbar$$

$$(m_\pi c^2)\left(\frac{r}{c}\right) \sim \hbar$$

$$m_\pi \sim \frac{\hbar}{rc} \qquad (11.19)$$

which gives a value for m_π of

$$m_\pi \sim \frac{1.05 \times 10^{-34}\,\mathrm{J\cdot s}}{(1.7 \times 10^{-15}\,\mathrm{m})(3 \times 10^{8}\,\mathrm{m/s})} \sim 2 \times 10^{-28}\,\mathrm{kg}$$

This rough figure is about 220 times the rest mass m_e of the electron.

Discovery of the Pion

A dozen years after Yukawa's proposal, particles with the properties he had predicted were actually discovered. The rest mass of charged pions is 273 m_e and that of neutral pions is 264 m_e, not far from the above estimate.

Two factors contributed to the belated discovery of the free pion. First, enough energy must be supplied to a nucleon so that its emission of a pion conserves energy. Thus at least $m_\pi c^2$ of energy, about 140 MeV, is required. To furnish a stationary nucleon with this much energy in a collision, the incident particle must have considerably more kinetic energy than $m_\pi c^2$ in order that momentum as well as energy be conserved. Particles with kinetic energies of several hundred MeV are therefore required to produce free pions, and such particles are found in nature only in the diffuse stream of cosmic radiation that bombards the earth. Hence the discovery of the pion had to await the development of sufficiently sensitive and precise methods of investigating cosmic-ray

Virtual Photons

Some years before Yukawa's work, particle exchange had been suggested as the mechanism of electromagnetic forces. In this case the particles are photons which, being massless, are not limited in range by Eq.(11.19). However, the greater the distance between two charges, the smaller must be the energies of the photons that pass between them (and hence the less the momenta of the photons and the weaker the resulting force) in order that the uncertainty principle not be violated. For this reason electric forces decrease with distance. Because the photons exchanged in the interactions of electric charges cannot be detected, they are called **virtual photons**. As in the case of pions, they can become actual photons if enough energy is somehow supplied to liberate them from the energy-conservation constraint.

The idea of photons as carriers of electromagnetic forces is attractive on many counts, an obvious one being that it explains why such forces are transmitted with the speed of light and not, say, instantaneously. As subsequently developed, the full theory is called quantum electrodynamics (see Sec. 6.9). Its conclusions have turned out to be in extraordinarily precise agreement with the data on such phenomena as the photoelectric and Compton effects, pair production and annihilation, bremsstrahlung, and photon emission by excited atoms. Unfortunately the details of the theory are too mathematically complex to consider here.

interactions. Later high-energy accelerators were placed in operation which gave the necessary particle energies, and the profusion of pions that were created with their help could be studied readily.

The second reason for the lag between the prediction and experimental discovery of the pion is its instability; the mean lifetime of the charged pion is only 2.6×10^{-8} s and that of the neutral pion is 8.4×10^{-17} s. The lifetime of the π^0 is so short, in fact, that its existence was not established until 1950. The modes of decay of the π^+, π^-, and π^0 are described in Chap. 13. Heavier mesons than the pion have also been discovered, some over a thousand times the electron mass. The contribution of these mesons to nuclear forces is, by Eq. (11.19), limited to shorter distances than those characteristic of pions.

EXERCISES

I hear, and I forget. I see, and I remember. I do, and I understand. —Anon.

11.1 Nuclear Composition

1. State the number of neutrons and protons in each of the following: ${}^{6}_{3}\text{Li}$; ${}^{22}_{10}\text{Ne}$; ${}^{94}_{40}\text{Zr}$; ${}^{180}_{72}\text{Hf}$.
2. Ordinary boron is a mixture of the ${}^{10}_{5}\text{B}$ and ${}^{11}_{5}\text{B}$ isotopes and has a composite atomic mass of 10.82 u. What percentage of each isotope is present in ordinary boron?

11.2 Some Nuclear Properties

3. Electrons of what energy have wavelengths comparable with the radius of a ${}^{197}_{79}\text{Au}$ nucleus? (*Note:* A relativistic calculation is needed.)
4. The greater the atomic number of an atom, the larger its nucleus and the closer its inner electrons are to the nucleus. Compare the radius of the ${}^{238}_{92}\text{U}$ nucleus with the radius of its innermost Bohr orbit.
5. It is believed possible on the basis of the shell model that the nuclide of $Z = 110$ and $A = 294$ may be exceptionally long-lived. Estimate its nuclear radius.
6. Show that the nuclear density of ${}^{1}_{1}\text{H}$ is over 10^{14} times greater than its atomic density. (Assume the atom to have the radius of the first Bohr orbit.)
7. Compare the magnetic potential energies (in eV) of an electron and of a proton in a magnetic field of 0.10 T.
8. One type of magnetometer is based on proton precession. What is the Larmor frequency of a proton in the earth's magnetic field where its magnitude is 3.00×10^{-5} T? In what part of the em spectrum is radiation of this frequency?
9. A system of a million distinguishable protons is in thermal equilibrium at 20°C in a 1.00-T magnetic field. More of the protons are in the lower-energy spin-up state than in the higher-energy spin-down state. (*a*) On the average, how many more? (*b*) Repeat the calculation for a temperature of 20 K. (*c*) What do these results suggest about how strongly such a system will absorb em radiation at the Larmor frequency? (*d*) Could such a system in principle be used as the basis of a laser? If not, why not?

11.3 Stable Nuclei

10. The Appendix at the back of the book lists all known stable nuclides. Are there any for which $Z > N$? Why are such nuclides so rare (or absent)?
11. What limits the size of a stable nucleus?
12. What happens to the atomic number and mass number of a nucleus when it (*a*) emits an alpha particle, (*b*) emits an electron, (*c*) emits a positron, (*d*) captures an electron?
13. Which nucleus would you expect to be more stable, ${}^{7}_{3}\text{Li}$ or ${}^{8}_{3}\text{Li}$; ${}^{13}_{6}\text{C}$ or ${}^{15}_{6}\text{C}$?
14. Both ${}^{14}_{8}\text{O}$ and ${}^{19}_{8}\text{O}$ undergo beta decay. Which would you expect to emit a positron and which an electron? Why?

11.4 Binding Energy

15. Find the binding energy per nucleon in ${}^{20}_{10}\text{Ne}$ and in ${}^{56}_{26}\text{Fe}$.
16. Find the binding energy per nucleon in ${}^{79}_{35}\text{Br}$ and in ${}^{197}_{79}\text{Au}$.
17. Find the energies needed to remove a neutron from ${}^{4}_{2}\text{He}$, then to remove a proton, and finally to separate the remaining neutron and proton. Compare the total with the binding energy of ${}^{4}_{2}\text{He}$.
18. The binding energy of ${}^{24}_{12}\text{Mg}$ is 198.25 MeV. Find its atomic mass.
19. Show that the potential energy of two protons 1.7 fm (the maximum range of nuclear forces) apart is of the correct order of magnitude to account for the difference in binding energy between ${}^{3}_{1}\text{H}$ and ${}^{3}_{2}\text{He}$. How does this result bear upon the question of the dependence of nuclear forces on electric charge?

20. The neutron decays in free space into a proton and an electron. What must be the minimum binding energy contributed by a neutron to a nucleus in order that the neutron not decay inside the nucleus? How does this figure compare with the observed binding energies per nucleon in stable nuclei?

11.5 Liquid-Drop Model

21. Use the semiempirical binding-energy formula to calculate the binding energy of $^{40}_{20}Ca$. What is the percentage discrepancy between this figure and the actual binding energy?

22. Two nuclei with the same mass number for which $Z_1 = N_2$ and $Z_2 = N_1$, so that their atomic numbers differ by 1, are called mirror isobars; for example, $^{15}_{7}N$ and $^{15}_{8}O$. The constant a_3 in the coulomb energy term of Eq. (11.18) can be evaluated from the mass difference between two mirror isobars, one of which is odd-even and the other even-odd (so that their pairing energies are zero). (*a*) Derive a formula for a_3 in terms of the mass difference between two such nuclei, their mass number A, the smaller atomic number Z of the pair, and the masses of the hydrogen atom and the neutron. (*Hint*: First show that $(A - 2Z)^2 = 1$ for both nuclei.) (*b*) Evaluate a_3 for the case of the mirror isobars $^{15}_{7}N$ and $^{15}_{8}O$.

23. The coulomb energy of Z protons uniformly distributed throughout a spherical nucleus of radius R is given by

$$E_C = \frac{3}{5}\frac{Z(Z-1)e^2}{4\pi\epsilon_0 R}$$

(*a*) On the assumption that the mass difference ΔM between a pair of mirror isobars is entirely due to the difference Δm between the $^{1}_{1}H$ and neutron masses and to the difference between their coulomb energies, derive a formula for R in terms of ΔM, Δm, and Z, where Z is the atomic number of the nucleus with the smaller number of protons. (*b*) Use this formula to find the radii of the mirror isobars $^{15}_{7}N$ and $^{15}_{8}O$.

24. Use the formula for E_c of Exercise 23 to calculate a_3 in Eq. (11.12). If this figure is not the same as the value of 0.60 MeV quoted in the text, can you think of any reasons for the difference?

25. (*a*) Find the energy needed to remove a neutron from ^{81}Kr, from ^{82}Kr, and from ^{83}Kr. (*b*) Why is the figure for ^{82}Kr so different from the others?

26. Which isobar of $A = 75$ does the liquid-drop model suggest is the most stable?

27. Use the liquid-drop model to establish which of the mirror isobars $^{127}_{52}Te$ and $^{127}_{53}I$ decays into the other. What kind of decay occurs?

11.6 Shell Model

28. According to the **Fermi gas model** of the nucleus, its protons and neutrons exist in a box of nuclear dimensions and fill the lowest available quantum states to the extent permitted by the exclusion principle. Since both protons and neutrons have spins of $\frac{1}{2}$ they are fermions and obey Fermi-Dirac statistics. (*a*) Find an equation for the Fermi energy in a nucleus under the assumption that $A = 2Z$. Note that the protons and neutrons must be considered separately. (*b*) What is the Fermi energy in such a nucleus for $R_0 = 1.2$ fm? (*c*) In heavier nuclei, $A > 2Z$. What effect will this have on the Fermi energies for each type of particle?

29. A simplified model of the deuteron consists of a neutron and a proton in a square potential well 2 fm in radius and 35 MeV deep. Is this model consistent with the uncertainty principle?

11.7 Meson Theory of Nuclear Forces

30. Van der Waals forces are limited to very short ranges and do not have an inverse-square dependence on distance, yet nobody suggests that the exchange of a special mesonlike particle is responsible for such forces. Why not?