Demuestre que:

3) a)
$$A^{i'}_{k} \tilde{A}^{j}_{i'} = \delta^{j}_{k}$$

teniendo en cuenta que

$$x^{i'} = A^{i'}_{k} x^{j}$$

Considerando que la inversa de $A^{i'}_{k}$ es $\tilde{A}^{k}_{i'}$, vemos que

$$x^{i'} \tilde{A}^{k}_{i'} = x^{j}$$

Por ende

$$x^{j} = \tilde{A}^{k}_{i'} A^{i'}_{k} x^{k}$$

$$x^{j} = x^{k} \delta^{j}_{k}$$

$$\delta^{j}_{k} = A^{i'}_{k} \tilde{A}^{k}_{i'}$$

ya que $j = k$ se puede ver como

$$\delta^{j}_{k} = A^{i'}_{k} \tilde{A}^{j}_{i'}$$

$\theta_{i'1} = \theta_{11'}$

B) ya que $A^{i'}_{j} = \cos(\theta_{i'j})$

$A^{j}_{i'} = \cos(\theta_{ji'})$

Ya que el sistema es ortonormal

Como $A^{i'}_{k} \tilde{A}^{j}_{i'} = \delta^{j}_{k} \rightarrow A^{i'}_{j} \tilde{A}^{j}_{i'} = \cos(\theta_{1'1})\cos(\theta_{11'}) + \cos(\theta_{1'2})\cos(\theta_{21'}) + \cos(\theta_{3'3})\cos(\theta_{33'})$

Entonces $1 = \cos^2(\theta_{1'1}) + \cos^2(\theta_{2'2}) + \cos^2(\theta_{3'3})$ //

4) $r = x^i \hat{I}_i \equiv x\hat{I} + y\hat{J}$ en 2 Dimensiones

a) $(x,y) \to (-y,x)$ $\quad A^{i'}_{j} = \begin{pmatrix} \partial x'/\partial x & \partial x'/\partial y \\ \partial y'/\partial x & \partial y'/\partial y \end{pmatrix}$

$x' = -y$
$y' = x$ $\quad A^{i'}_{j} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

Para que sus componentes sean verdaderos, debe ser ortogonal es decir

$AA^T = I$ donde A es la matriz de transformación

$AA^T = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$

Cumple ✓

b) $(x,y) \to (x,-y)$ $\quad A = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$AA^T = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$ Cumple ✓

c) $(x,y) \to (x-y, x+y)$ $\quad A = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$

$AA^T = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$ $\quad A^T = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$

$AA^T \neq I$ no cumple ✗

d) $(x,y) \to (x+y, x-y)$ $\quad A = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$ $\quad A^T = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$

$AA^T = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \neq I$ no cumple ✗

Taller número 2

a) $(\nabla(\phi\psi))_i = \partial_i(\phi\psi) = \psi\partial_i\phi + \phi\partial_i\psi$

$\nabla(\phi) = \partial_i\phi$ y $\nabla(\psi) = \partial_i\psi \Rightarrow = (\nabla\phi)\psi + (\nabla\psi)\phi$

d)

$$\nabla\cdot(\nabla\times a) = \nabla_i(\nabla\times a)_i = \varepsilon_{ijk}\nabla_i\nabla_j a_k$$

$\varepsilon_{ijk}\partial_i\partial^j a_k = 0 \rightarrow$ debido a que ε_{ijk} es antisimétrico respecto a i,j mientras que $\partial_i\partial^j$ es simétrico respecto a i,j

d.1) $(\nabla\times(\nabla\cdot a))_i = \varepsilon_{ijk}\nabla_j(\nabla\cdot a)_k$

$\varepsilon_{ijk}\nabla_j\nabla_m a_m = \varepsilon_{ijk}\partial_j(\partial_m a_m)$

Esto no tiene sentido, pues incluso, un producto vectorial entre un vector y un escalar no tiene sentido, por ende esta expresión no existe

f) $\nabla\times(\nabla\times a) = \nabla(\nabla\cdot a) - \nabla^2 a$

$(\nabla\times(\nabla\times a))_i = \varepsilon_{ijk}\nabla_j(\nabla\times a)_k = \varepsilon_{ijk}\varepsilon^{kmn}\nabla_j\nabla_m a_n$

$= \delta^i_m\delta^j_n\nabla_j\nabla_m a_n - \delta^i_n\delta^j_m\nabla_j\nabla_m a_n = \nabla_m\nabla_j a_j - \nabla_m\nabla_m a_n$
$\quad j=n \qquad\qquad\qquad j=m$

$\nabla_m(\nabla\cdot a) - \nabla^2 a_n = \nabla_i(\nabla\cdot a) - \nabla^2 a$

Scribe

1.6.7

2) $\left(\cos(\alpha) + i\,\text{sen}(\alpha)\right)^3 = \cos(3\alpha) + i\,\text{sen}(3\alpha)$

Expando el binomio

$\cos^3\alpha + 3i\cos^2\alpha\,\text{sen}\,\alpha - 3\,\text{sen}^2\alpha\cos\alpha - i\,\text{sen}^3\alpha$

Igualo $\mathbb{R}$ con $\mathbb{R}$ e i con i

$\cos^3\alpha - 3\,\text{sen}^2\alpha\cos\alpha = \cos 3\alpha$ a) //

$3i\cos^2\alpha\,\text{sen}\,\alpha - i\,\text{sen}^3\alpha = i\,\text{sen}\,3\alpha$

$3\cos^2\alpha\,\text{sen}\,\alpha - \text{sen}^3\alpha = \text{sen}\,3\alpha$ b) //

5) a) $2e^{i\pi/2}$ Raices $= \sqrt{2}\,e^{i\left(\frac{\pi/2 + 2\pi k}{n}\right)}$

$k = 0, 1$

- $\sqrt{2}\,e^{i\pi/4} = \sqrt{2}(\cos\pi/4 + i\,\text{sen}\,\pi/4)$

$\sqrt{2}\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right)$

$1+i$

- $\sqrt{2}\,e^{i(\pi/4+\pi)} = \sqrt{2}\left(\cos\frac{5\pi}{4} + i\,\text{sen}\,5\pi/4\right)$

$-1-i$

b) $\sqrt{1-\sqrt{3}i}$ $\quad 2e^{i\,\text{Arctan}\sqrt{3}} = 2e^{i\pi/3}$

Raices $\quad \sqrt{2}\,e^{i\pi/6} \Rightarrow \sqrt{2}(\cos\pi/6 + i\,\text{sen}\,\pi/6) = \frac{\sqrt{2}\sqrt{3}}{2} + \frac{\sqrt{2}}{2}i$

$\sqrt{2}\,e^{7\pi/6} \Rightarrow \sqrt{2}(\cos 7\pi/6 + i\,\text{sen}\,7\pi/6) = \frac{-\sqrt{2}\sqrt{3}}{2} + \frac{\sqrt{2}}{2}i$

c) $\sqrt[3]{-1}$ $\quad e^{i\pi}$ raices $e^{i\pi/3} = \cos\pi/3 + i\,\text{sen}\,\pi/3 = \frac{1}{2} + \frac{\sqrt{3}}{2}i$

$e^{i3\pi/3} = \cos\pi + i\,\text{sen}\,\pi = -1$

$e^{5\pi/3} = \cos 5\pi/3 + i\,\text{sen}\,5\pi/3 = \frac{1}{2} - \frac{\sqrt{3}}{2}i$

d) $\sqrt[6]{8} = (8^{1/3})^{1/2} = \sqrt{2} = \sqrt{2}e^{i\cdot 0°}$

Raices: $\sqrt{2}e^{i\cdot 0°} = \sqrt{2}$

- $\sqrt{2}e^{i\pi/3} = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}\sqrt{3}}{2}i$
- $\sqrt{2}e^{i2\pi/3} = \sqrt{2}\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)$
- $\sqrt{2}e^{i\pi} = -\sqrt{2}$
- $\sqrt{2}e^{i4\pi/3} = -\sqrt{2}\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)$
- $\sqrt{2}e^{i5\pi/3} = -\sqrt{2}\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)$

e) $\sqrt[4]{8(-1-\sqrt{3}i)} = 2e^{i\arctan(\sqrt{3}+1)}$

Raices $2e^{i\pi/3} = 1+\sqrt{3}i$

$2e^{i5\pi/6} = -\sqrt{3}+i$

$2e^{i(4\pi/3)} = -1-\sqrt{3}i$

$2e^{i(\pi/3+3\pi/2)} = \sqrt{3}-i$

6) a) ① $Ln(-ie) = Ln(-i) + Ln\,e =$

$w = x+yi \qquad e^{w} = -i$

$e^{x+yi} = -i \qquad e^{x}(\cos y + i\operatorname{sen} y) = -i$

$e^{x}\cos y = 0 \qquad e^{x}\operatorname{sen} y = -1$

$y = -\pi/2 \qquad x = 0$

$e^{(0+(-\pi/2 i))} = -i$ Volviendo a ①

$Ln\,e^{-\pi/2 i} + 1 = -\frac{\pi}{2}i + 1$

b) $Ln(1-i)$ $e^{w} = 1-i = e^{x}e^{y} = e^{x}(\cos y + i \operatorname{sen} y)$

Volvemos a escribir el imaginario

$$1-i = \sqrt{2}\, e^{-i\pi/4}$$

$$e^{x} = \sqrt{2} \qquad e^{y} = e^{-i\pi/4}$$

$$x = \frac{1}{2}Ln 2 \qquad y = -i\pi/4$$

$$Ln\, e^{\frac{1}{2}Ln 2 - i\pi/4} \overset{\text{por leyes de } Ln}{=} \frac{1}{2}Ln 2 - i\pi/4 \quad //$$

c) $Ln(e) = 1 + 2n\pi i$

$$e^{x}(\cos y + i \operatorname{sen} y) = e$$

$$e^{x}\cos\theta = e \qquad i e^{x}\operatorname{sen} y = 0$$

cada $2\pi n$ $\operatorname{sen} y = 0$ ya que $e^{x} \neq 0$

cada $2\pi n$ $\cos\theta = 1$ y por ende

$$e^{x} = e \qquad x = 1$$

$$Ln\, e^{1+2\pi n i} = 1 + 2n\pi i \quad //$$

d) $Ln(i) = (2n + \frac{1}{2})\pi i$

$$e^{x}(\cos y + i \operatorname{sen}\theta) = i$$

$$e^{x}\cos y = 0 \qquad i e^{x}\operatorname{sen}\theta = i \qquad Ln\, e^{(2n+1/2)\pi i}$$

$\cos y = 0$ $x = 0$ $(2n+1/2)\pi i$ //

cada $(2n + 1/2)\pi$ entonces queda como