# Voter Candidate Range Detection

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# 1 Definition

## 1.1 Voter Candidate Range Property

$\forall(c, v) \in C \times V : c \in ApprovalSet(v) \Leftrightarrow \sqrt{(x_c - x_v)^2} \leq r_c + r_v$

# 2 Integer Linear Programming

## 2.1 Input

The only input is *ApprovalSet* of each voter.
Represented by $2D$ matrix $A_{vc}$

## 2.2 Variables

$x_c$ - candidate $c$ position on $x$ axis
$x_v$ - voter $v$ position on $x$ axis
$r_c$ - candidate $c$ radius
$r_v$ - voter $v$ radius

## 2.3 Transform VCR condition to Linear constraints

### 2.3.1 Positive and Negative Condition and Big M method

Introduce large constant positive parmater
$M$

Introduce new binary variables for each pair of candidate and voter:
$z1_{cv}$
$z2_{cv}$

$$
\begin{aligned}
&\textit{Minimize } 0 \\
&\textit{foreach (c,v) in } C \times V : \\
&\textit{if } c \in \textit{ApprovalSet}(v) \textit{ add constraints} \\
&x_c - x_v \leq r_c + r_v \\
&-(x_c - x_v) \leq r_c + r_v \\
&\textit{if } c \notin \textit{ApprovalSet}(v) \textit{ add constraints} \\
&x_c - x_v + M(1 - z1_{cv}) \geq r_c + r_v + 1 \\
&-(x_c - x_v) + M(1 - z2_{cv}) \geq r_c + r_v + 1 \\
&z1_{cv} + z2_{cv} >= 1
\end{aligned}
$$

### 2.3.2 Mock Distance Variable Condition

Introduce new variable
$D_{cv}$ - distance between candidate $c$ and voter $v$ on $x$ axis

$$
\begin{aligned}
&\textit{Minimize } 0 \\
&\textit{foreach (c,v) in } C \times V : \\
&\textit{add constraints} \\
&D_{cv} = x_c - x_v \\
&D_{cv} = -(x_c - x_v) \\
&\textit{if } c \in \textit{ApprovalSet}(v) \textit{ add constraints} \\
&D_{cv} \leq r_c + r_v \\
&\textit{if } c \notin \textit{ApprovalSet}(v) \textit{ add constraints} \\
&D_{cv} \geq r_c + r_v + 1
\end{aligned}
$$

# References