The Financial Crisis: Securitization, Mathematics, and Systemic Risk

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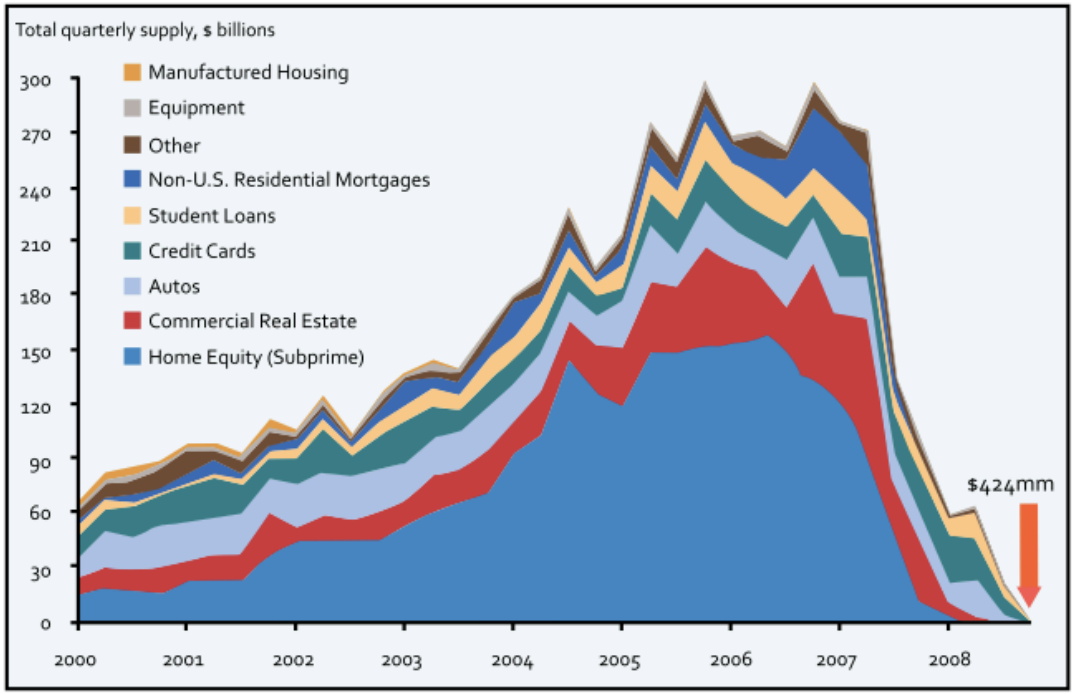
The Financial Crisis (2007-present)

- Who was Responsible for the Financial Crisis?
  - Homeowners
  - Investment banks and hedge funds
  - Credit rating agencies (Standard and Poor, Moody, Fitch)
  - Government (SEC, Congress, President)
  - Quants

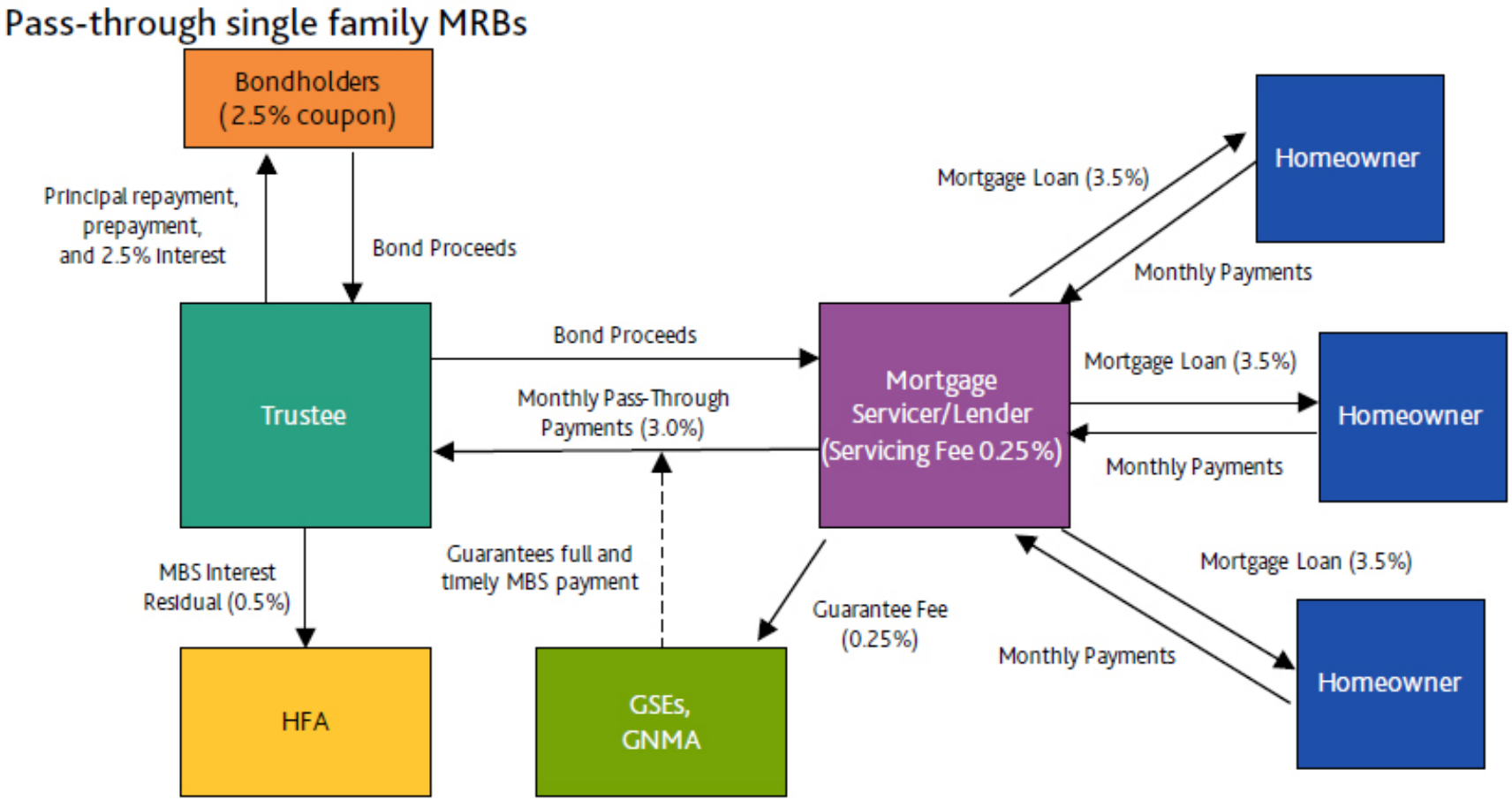
- In the leadup to the financial crisis, a high number of loans were made to borrowers of questionable credit. This was fueled by low interest rates from the Fed and aggressive (perhaps questionable) lending practices by banking and other mortgage institutions.
- The subprime crisis was sparked by the default of subprime mortgage borrowers in unprecedented numbers beginning in 2007.
- When the housing bubble collapsed, why did the crisis spread to the broader banking system?
  - Securitization
  - Shadow Banking System
  - Systemic Risk

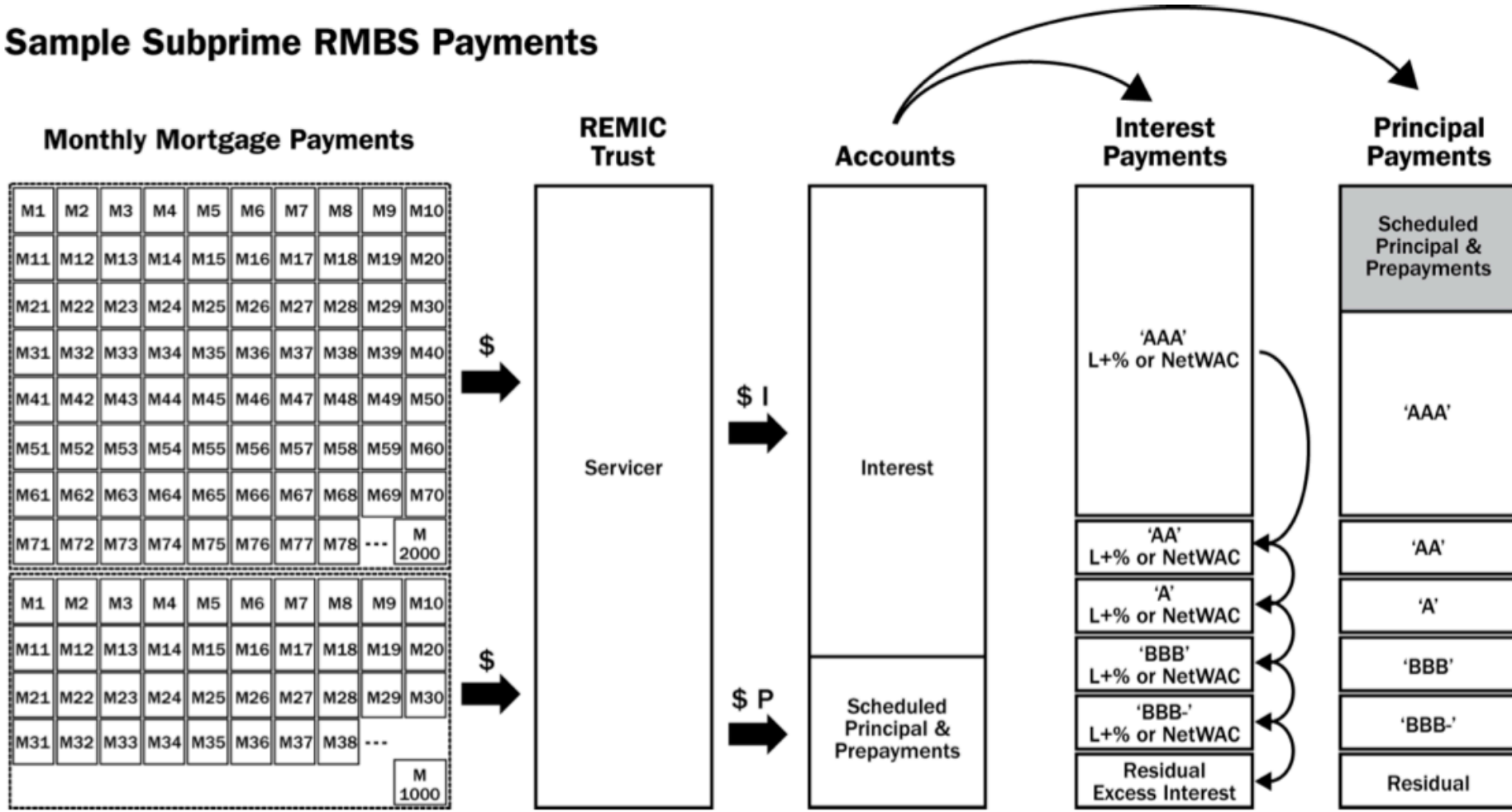
- Securitization is the pooling of debt into a security. Asset-backed securities (ABS) include almost any type of loan:
  - Residential mortgage backed securities (RMBS)
  - Commercial mortgage backed securities (CMBS)
  - Auto Loans
  - Credit cards, air plane leases, student loans, and other equipment leases or loans.
- A bank or lending institution initially originates the loans.
- Tranches of the ABS pool are sold off to investors
  - More senior tranches have prioritized claims on cashflows
  - Defaults in the underlying pool will hit the junior tranches first



Source: J. P. Morgan Securities Inc. Data as of 10/31/2008.

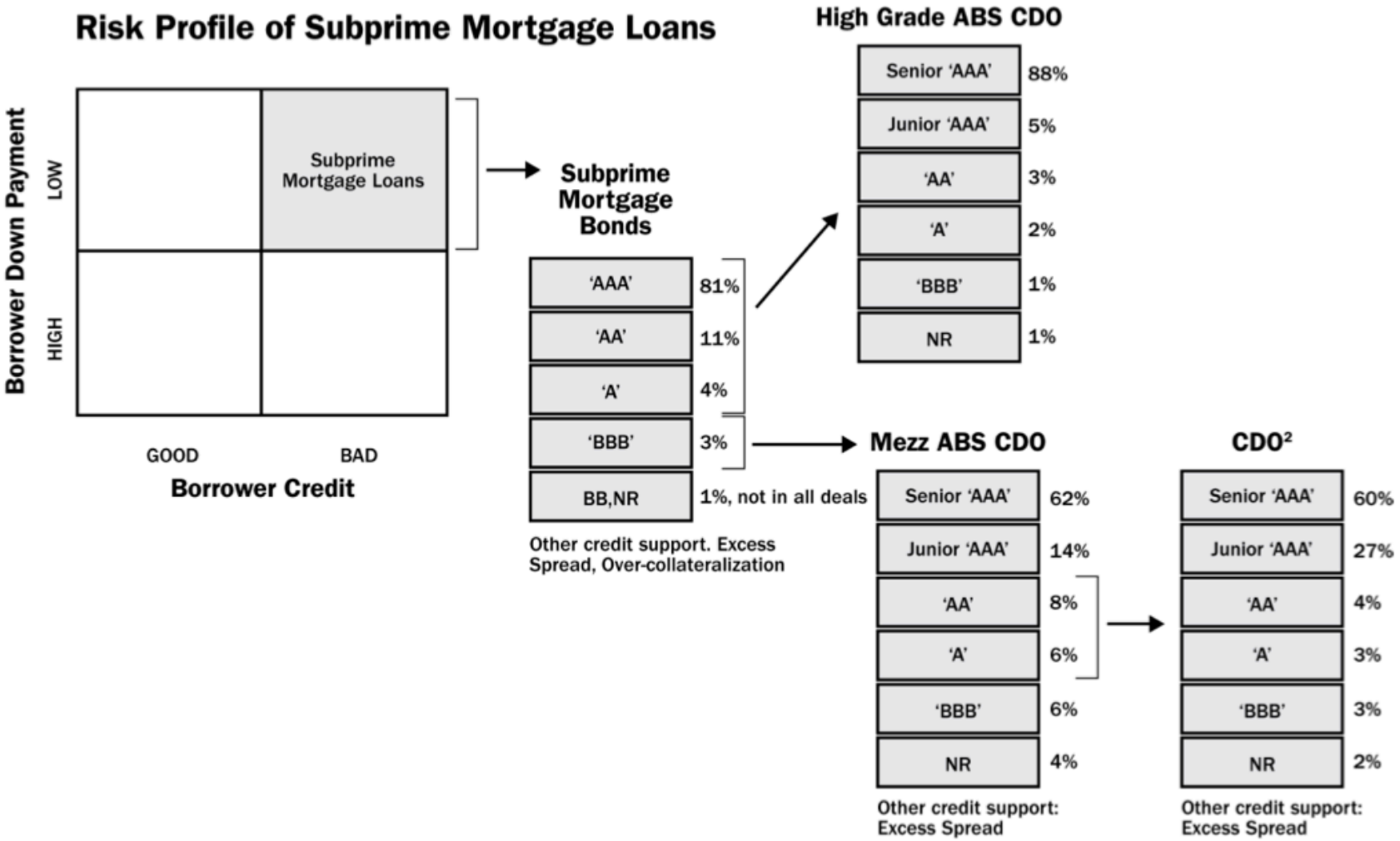
Passthrough structure (Source: NCSHA)





Source: Kevin Kendra, Fitch, "Tranche ABX and Basis Risk in Subprime RMBS Structured Portfolios,"

- Things get a lot more complicated..
  - MBS can be non-tranched (pass-through)
  - Triggers can divert cashflows from senior tranches to more junior tranches
- MBS can then be packaged into collateralized debt obligations (CDOs)
- What's the difference between an MBS and a CDO?
  - CDOs are always tranched
  - CDOs can contain many different types of ABS.
- CDO^2 are CDOs composed of other CDOs



Source: UBS, "Market Commentary," December 13, 2007.

Understanding Correlation

- Why is understanding correlation essential to pricing MBS and CDOs?
- Let's take life insurance policies as an example.
  - Life insurance on an individual
  - Life insurance on a pool of individuals
  - Does it help if the pool is very large?
  - What if the individuals in the pool are somehow correlated?
- How does this relate to an MBS?

Systemic Risk

- What is systemic risk?
  - "The risk of collapse of an entire financial system or entire market"
  - This is the opposite of idiosyncratic risk, which is risk specific to a particular entity.
- One of the main causes of the financial crisis was the mispricing of ABS and CDOs due to underestimation and misunderstanding of the effects of systemic risk.
- Assets can become highly correlated in times of crisis due to systemic risk.

Systemic Risk in the Housing Market

- Subprime mortgages had teaser rates that after a couple of years jumped to much higher rates.
- For this system to work, mortgage holders had to refinance. This depended upon housing prices increasing.
- When housing prices began to fall, mortgage holders began to default. This further depressed housing prices, which caused a negative feedback loop.
- Housing defaults became highly correlated!

An Example

- Senior tranches of MBS, considered AAA, were particularly susceptible to systemic risk. Market participants widely mispriced these securities, failing to take into account systemic risk.
- Consider an MBS with two tranches, a junior and a senior tranche.
- There are two mortgages underlying the MBS. If both default, no one gets any money. If one defaults, the senior tranche gets a payoff of 1. If neither default, both the junior and senior tranches get a payoff of 1.

- Suppose that each mortgage defaults with probability $p = \frac{1}{2}$. Take the discount rate to be $r = 0$.
- If the defaults are uncorrelated, the price of the junior tranche is $\frac{1}{4}$ and the price of the senior tranche is also $\frac{3}{4}$.
- If the defaults are perfectly correlated, the price of the junior tranche is $\frac{1}{2}$ and the price of the senior tranche is $\frac{1}{2}$.
- So, senior tranches (considered extremely safe!) are very sensitive to systemic risk!

The Failure of Diversification

- The pooling of risky loans failed to diversify away risk due to those loans becoming highly correlated in the time of crisis.
- Furthermore, the tranching structure–which was supposed to protect the senior tranches–failed in its purpose due to highly correlated defaults.
- As a result, AAA-rated CDOs became worthless over night.

The Role of Mathematics in Finance

"The stupefying losses in mortgage-related securities came in large part because of flawed, history-based models used by salesmen, rating agencies and investors. These parties looked at loss experience over periods when home prices rose only moderately and speculation in houses was negligible [...] Investors should be skeptical of history-based models. Constructed by a nerdy-sounding priesthood using esoteric terms such as beta, gamma, sigma and the like, these models tend to look impressive. Too often, though, investors forget to examine the assumptions behind the symbols. **Our advice: Beware of geeks bearing formulas.** "

–Warren Buffett

- Pre-crisis, CDOs were almost universally priced using the Gaussian Copula model
- Major assumption: constant correlation (over time) between securities
  - Do you see any potential problem with this?
- Originally invented by David Li at JP Morgan, who himself said "The most dangerous part [of a model] is when people believe everything coming out of it."

"The Formula that Killed Wall Street"

"A year ago, it was hardly unthinkable that a math wizard like David X. Li might someday earn a Nobel Prize [...] Today, though, as dazed bankers, politicians, regulators, and investors survey the wreckage of the biggest financial meltdown since the Great Depression, Li is probably thankful he still has a job in finance at all [...] Li's Gaussian copula formula will go down in history as instrumental in causing the unfathomable losses that brought the world financial system to its knees."

–Felix Salmon, WIRED Magazine

- Which comes first? Mathematics or these complex securities?
  - Black Scholes $\rightarrow$ Options trading
  - Gaussian Copula model $\rightarrow$ CDO trading
- Do these complex securities serve any economic purpose or are they just tools to make large bets?
- Many of these securities do, in fact, serve important economic purposes:
  - Options help investors manage and reduce financial risk
  - Mortgage-backed securities have funded the home purchases for millions of Americans. MBS match investors seeking a return with people who need loans to buy houses!

A Look at the Gaussian Copula Model

- Consider a portfolio of N names, where the n-th name defaults of the random variable X_n falls below the level $B(t)$.
- Let $Z, \epsilon_1, \ldots, \epsilon_N$ be independent standard Gaussian random variables.

$$X_n = \sqrt{\rho} Z + \sqrt{1-\rho}\epsilon_n. \tag{1}$$

- The n-th firm defaults by time t with probability:

$$p(t) = \Phi\left(\frac{B(t) - \sqrt{\rho}X}{\sqrt{1-\rho}}\right), \tag{2}$$

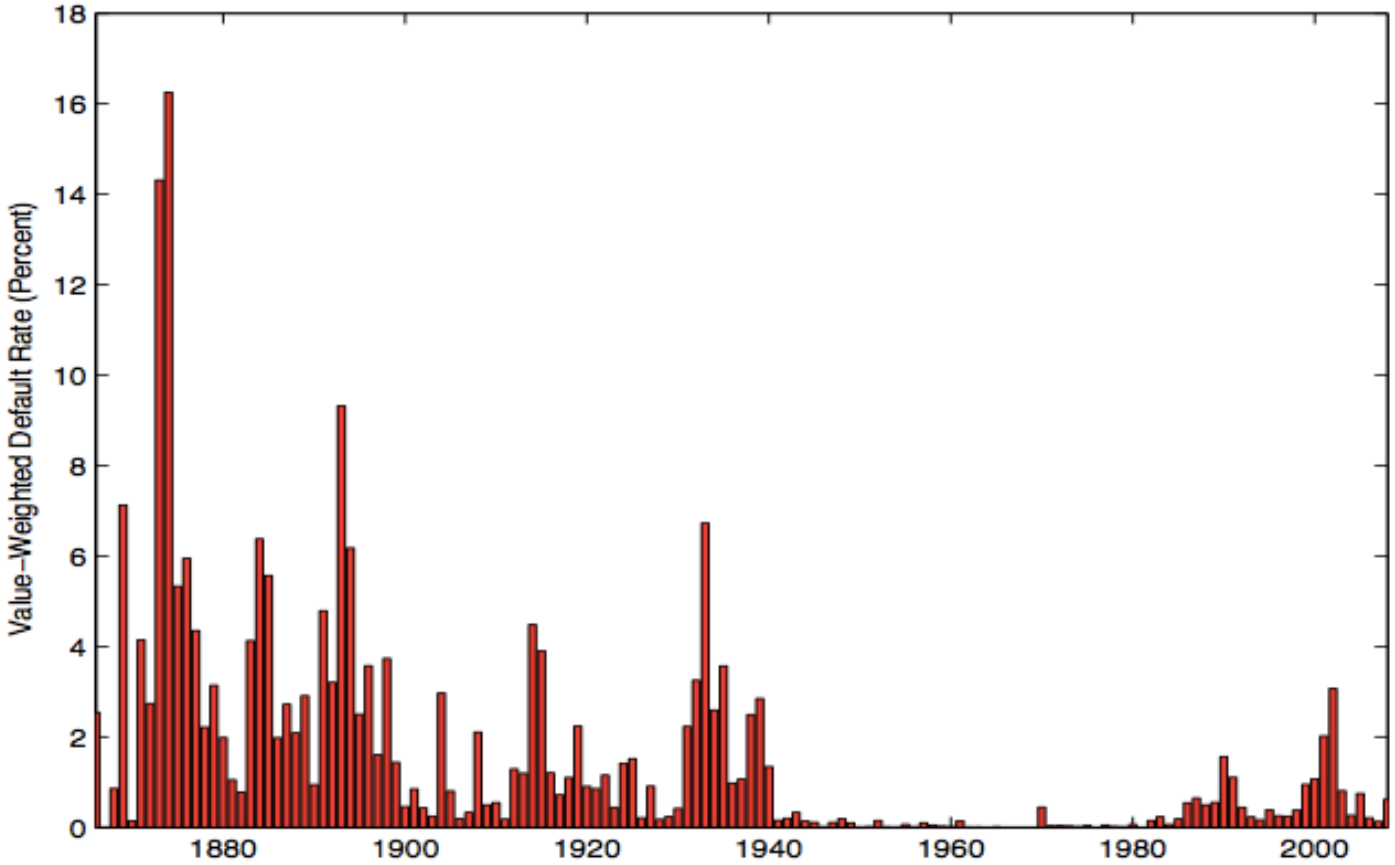
where Φ is the cumulative distribution function of a standard Gaussian random variable.

Contagion

- Contagion is the spread of risk from one financial institution or entity to others through complex legal, financial, and economic ties.
- One default can spark a cascade of defaults
  - Lehman Brothers
- This is currently a big focus of mathematical modeling.
  - Feedback loops

Corporate defaults cluster

Value-weighted default rate 1865–2008, US nonfinancial (Source: Giesecke, Longstaff, Strebulaev & Schaefer (2011))



Shadow Banking System

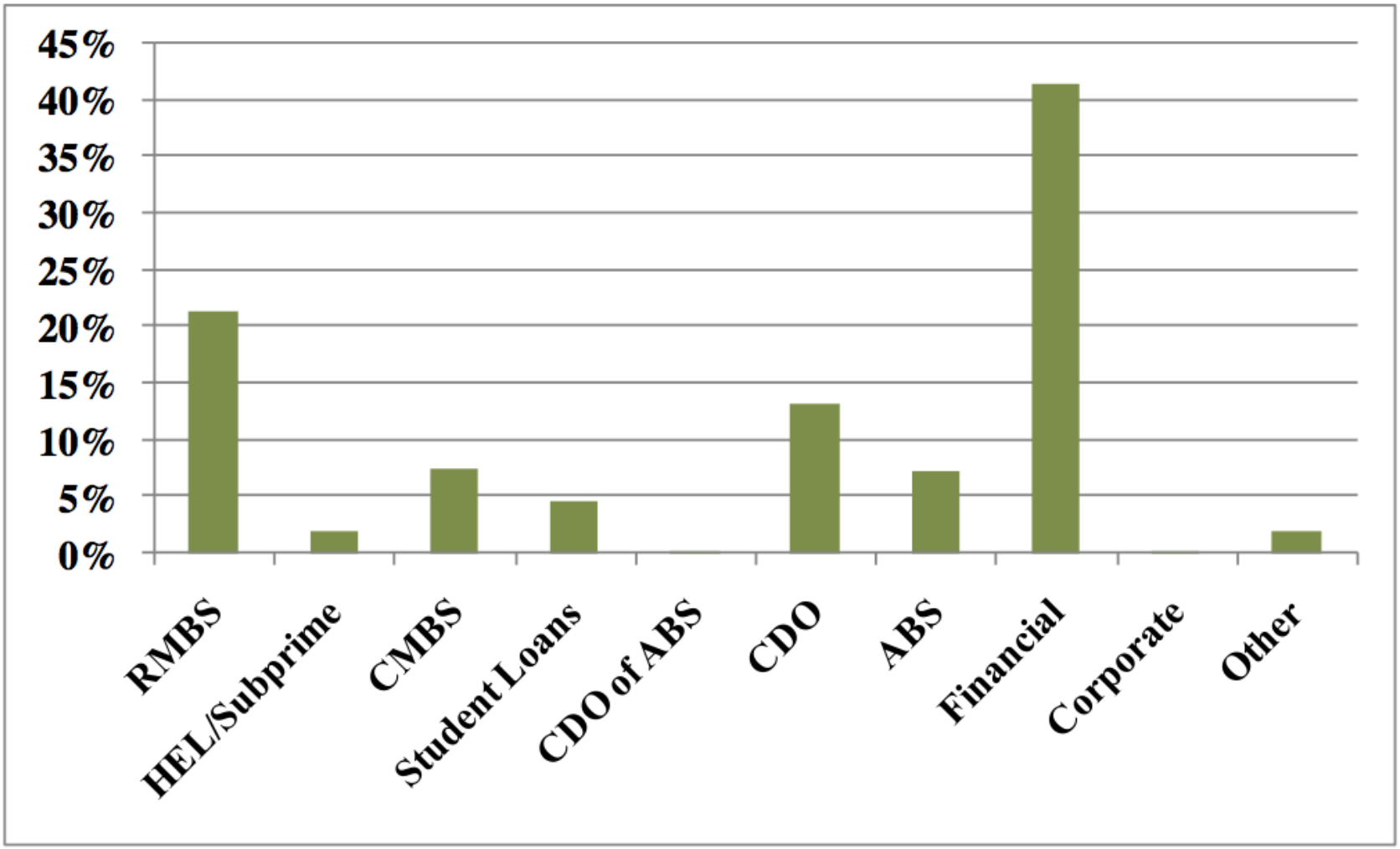
- The shadow banking system includes non-bank financial entities such as hedge funds, structured investment vehicles (SIVs), special purpose vehicle (SPVs), and money market funds.

- These financial entities often act like banks
  - Provide credit like banks, but do not take deposits
  - Often rely on short-term funding
  - Lightly regulated and do not have access to central bank funding
  - Highly leveraged
  - Often used by banks to avoid regulations

Structured Investment Vehicles

- Structured investment vehicles (SIVs) were off-balance sheet investment companies that bought ABS (often MBS)
- Funded ABS with short-term lending
- MBS is longer maturity: "arbitrage" between low lending rate and higher long-term rates
- Since SIVs are off-balance sheet, banks did not have to hold capital against the MBS.
- When MBS' prices began to decline, many of the SIVs' assets were absorbed back into the banks' balance sheets.
  - Banks had actually backed many of the SIVs with guarantees
  - If the banks had not assumed the SIVs' debt back onto their books, the SIVs would have been forced to sell that debt, further depressing MBS prices.

Composition of SIV Portfolios



Source: S&P (August 2007)

Collapse in ABS prices

- Loss of information in the complex chain of securitization and derivatives
  - Difficult to tell what a particular CDO holds or how sensitive it is to the housing collapse
  - Whether than take the risk, why not just sell?
  - Example: even though only a small percentage of SIVs' assets were MBS, the SIVs almost universally endured runs.
- Failure of ratings agencies
  - Incentives to give securities high ratings
- Market illiquidity

"The Lemon Problem"

- Theory: Lending institutions had no incentive to make loans to credit-worthy individuals since they securitized the loans and passed them on to investors, pocketing a fee in the process.
- Banks and originators warehoused mortgages in preparation for securitization.
- In fact, many banks held onto portions of MBS deals. In particular, these portions were often the riskiest tranches. The originators did bear some of the risk.
- Nearly all of the major subprime originators are bankrupt today (except for the investment banks that received bailouts).

Decline in Lending Standards

Underwriting Standards for Subprime Mortgages

	ARM Share	Interest Only Share	Low/No Doc Share	Debt-to-Income Ratio	Average Loan-to-Value Ratio
2001	73.%	0.0%	28.5%	39.7	84.0
2002	80.0%	2.3%	38.6%	40.1	84.4
2003	80.1%	8.6%	42.8%	40.5	86.1
2004	89.4%	27.2%	45.2%	41.2	84.7
2005	93.3%	37.8%	50.7%	41.8	83.2
2006	91.3%	22.8%	50.8%	42.4	83.4

Source: Freddie Mac; see Joint Economic Committee (October 2007).

Risk-taking by the Banks

"When the music stops in terms of liquidity, things will get complicated. But as long as the music is playing, you've got to get up and dance. We're still dancing."

– Chuck Prince, CEO of Citigroup

Risk-taking by the Banks

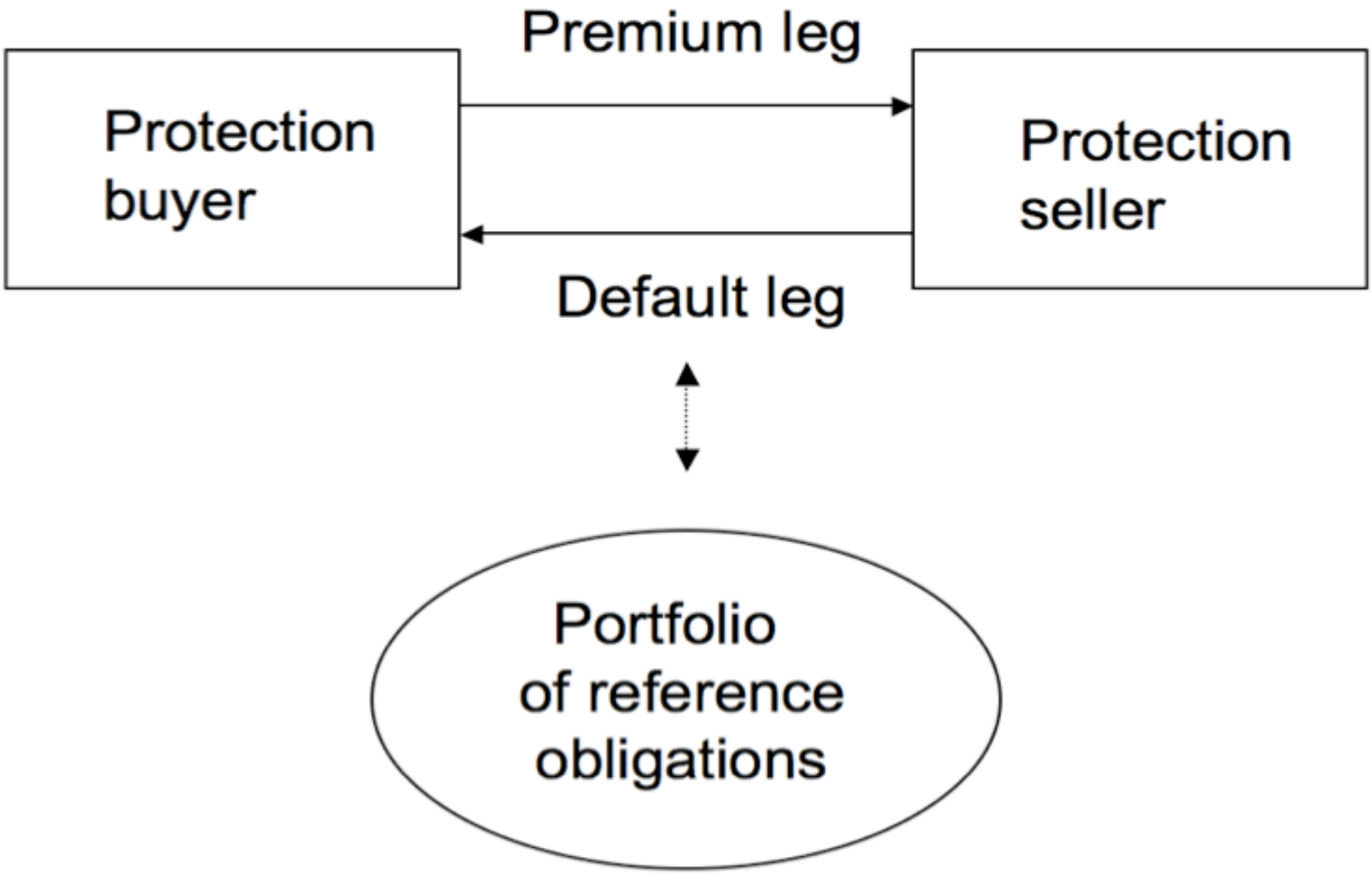
- Financial incentives for short-term performance instead of long-term performance
  - AIG and credit default swaps (CDS)
  - In a sense, traders have a "call option" on the revenue they generate.
- Moral hazard
- Banks felt that if they cut back on risk, their best traders would leave for competitors.

Rating Agencies and the Government

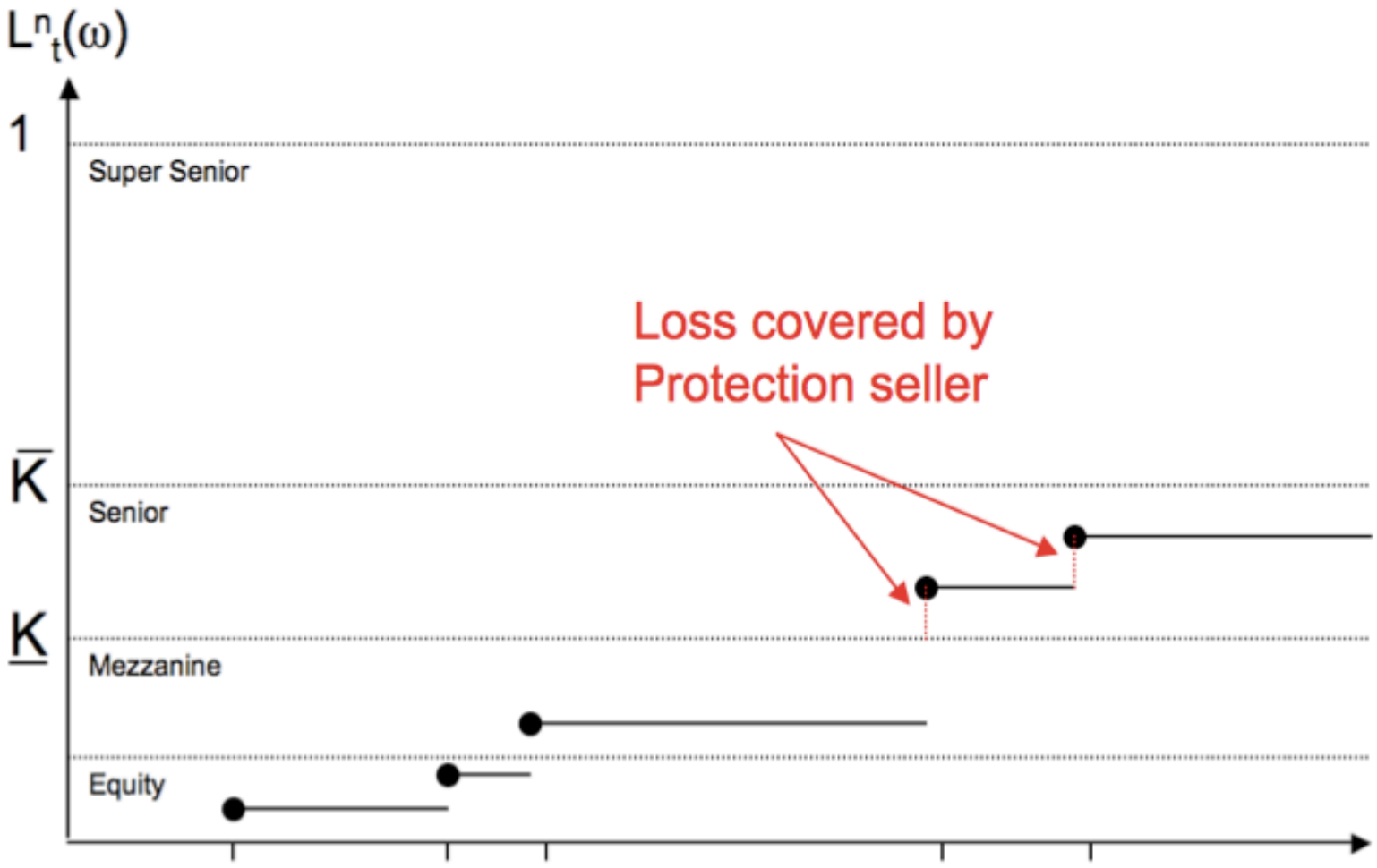
- What role did rating agencies (Standard and Poor, Moody, Fitch) play?
- What role did the Federal Reserve play?
- Did government policies pushing home ownership lead to the Financial Crisis?

Synthetic CDOs

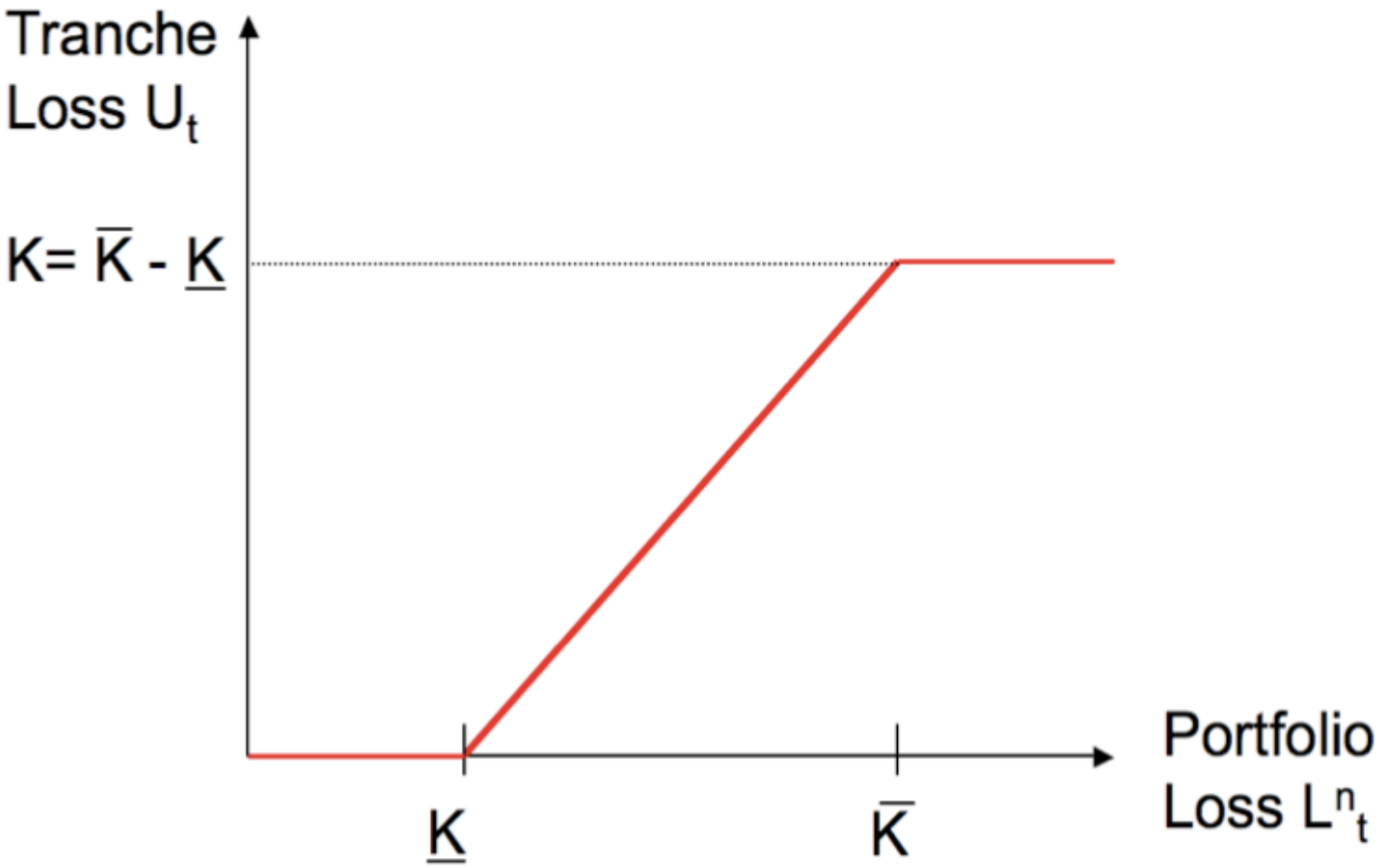
- Up until now, we have discussed cash CDOs. This means they are backed by actual assets.
- Synthetic CDOs are derivatives referenced on a particular set of assets but not actually backed by them.
- Banks, hedge funds, and other financial institutions were able to increase their exposure to the mortgage market through such synthetic CDOs.
- During the Financial Crisis, synthetic CDOs were used to make speculative bets on the continued collapse of the housing market, arguably exacerbating the crisis.


Premium leg
Protection buyer
Protection seller
Default leg
Portfolio of reference obligations

Tranche swap default leg



Tranche loss = call spread on portfolio loss



Abacus CDO

Goldman Sachs

Reference Portfolio

Security	Type	Notional Amount	CUSIP	Fitch	Moody's	S&P	Base WAL (yrs)	Dated Date	Legal Final	Servicer
ABFC 2006-OPT1 M8	Subprime	22,222,222	00075QAM4	BBB	Baa2	BBB	3.9	8/10/2006	9/25/2036	OOMC
ABFC 2006-OPT2 M8	Subprime	22,222,222	00075XAP2	BBB	Baa2	BBB	4.1	10/12/2006	10/25/2036	OOMC
ABSHE 2006-HE3 M7	Subprime	22,222,222	04541GXK3	BBB	Baa2	BBB	3.8	4/17/2006	3/25/2036	OOMC
ABSHE 2006-HE4 M7	Subprime	22,222,222	04544GAP4	BBB	Baa2	BBB	3.8	4/28/2006	5/25/2036	SPS
ACE 2006-FM2 M8	Midprime	22,222,222	00442CAN9		Baa2	BBB	4.5	10/30/2006	8/25/2036	WFB
ACE 2006-OP2 M9	Subprime	22,222,222	00441YAP7		Baa2	BBB-	4.3	10/30/2006	8/25/2036	WFB
ARSI 2006-W1 M8	Subprime	22,222,222	040104RQ6	BBB+	Baa2	BBB+	3.8	2/7/2006	3/25/2036	AQMC
CARR 2006-FRE1 M9	Subprime	22,222,222	144538AN5	BBB+	Baa2	A	3.8	6/28/2006	7/25/2036	FREM
CARR 2006-FRE2 M8	Subprime	22,222,222	14454AAN9		Baa2	BBB+	4.2	10/18/2006	10/25/2036	FREM
CARR 2006-NC1 M8	Midprime	22,222,222	144531FF2	BBB	Baa2	BBB+	3.6	2/8/2006	1/25/2036	NCMC
CARR 2006-NC2 M8	Subprime	22,222,222	14453FAM1	BBB	Baa2	BBB	3.8	6/21/2006	6/25/2036	CARR
CARR 2006-NC3 M9	Subprime	22,222,222	144528AN6	BBB-	Baa2	BBB-	4.0	8/10/2006	8/25/2036	NCMC
CARR 2006-OPT1 M8	Subprime	22,222,222	144531FV7	BBB+	Baa2	A-	3.6	3/14/2006	2/25/2036	OOMC
CMLTI 2006-AMC1 M8	Subprime	22,222,222	17309PAL0		Baa2	BBB	4.1	9/28/2006	9/25/2036	AQMC
CMLTI 2006-NC1 M8	Subprime	22,222,222	172983AN8		Baa2	BBB	3.8	6/29/2006	8/25/2036	WFB
CMLTI 2006-WFH2 M9	Subprime	22,222,222	17309MAN3		Baa2	BBB-	4.0	8/30/2006	8/25/2036	WFB
CMLTI 2006-WMC1 M8	Midprime	22,222,222	17307G2F4	A-	Baa2	BBB+	3.7	1/31/2006	12/25/2035	WFB
CMLTI 2007-WFH1 M9	Subprime	22,222,222	17311CAM3		Baa2	BBB-	4.5	2/9/2007	1/25/2037	WFB
CWL 2006-24 M8	Subprime	22,222,222	23243HAN1		Baa2	BBB	4.9	12/29/2006	5/25/2037	CHLS
FFML 2006-FF11 M8	Midprime	22,222,222	32028PAP0	BBB	Baa2	BBB	3.9	9/6/2006	8/25/2036	WFB
FFML 2006-FF12 M8	Midprime	22,222,222	32027GAN6	BBB	Baa2	BBB	4.2	8/25/2006	9/25/2036	ALS
FFML 2006-FF14 M8	Midprime	22,222,222	32027LAP0	BBB	Baa2	BBB	4.2	9/25/2006	10/25/2036	AURA
FFML 2006-FF15 M8	Midprime	22,222,222	32028GAP0	BBB	Baa2	BBB	4.3	10/25/2006	11/25/2036	AURA
FFML 2006-FF16 M8	Midprime	22,222,222	320275AN0		Baa2	BBB+	4.3	11/30/2006	12/25/2036	NCHL
FFML 2006-FF17 M8	Midprime	22,222,222	32028KAP1	BBB	Baa2	BBB	4.4	11/25/2006	12/25/2036	ALS
FFML 2006-FF7 M8	Midprime	22,222,222	320277AP1	BBB	Baa2	BBB	3.6	5/31/2006	5/25/2036	WFB
FFML 2006-FF9 M8	Midprime	22,222,222	320276AP3	BBB+	Baa2	BBB+	3.7	7/7/2006	6/25/2036	WFB
FHLT 2006-A M7	Subprime	22,222,222	35729RAN6	BBB+	Baa2	BBB	3.9	5/10/2006	5/25/2036	WFB
FHLT 2006-B M8	Midprime	22,222,222	35729QAN8	BBB+	Baa2	BBB	4.4	8/3/2006	8/25/2036	WFB
FMIC 2006-2 M8	Midprime	22,222,222	31659EAM0		Baa2	BBB+	4.1	7/6/2006	7/25/2036	WFB
FMIC 2006-3 M8	Midprime	22,222,222	316599AN9		Baa2	BBB	4.4	10/27/2006	11/25/2036	WFB
GSAMP 2006-FM2 M8	Midprime	22,222,222	36245DAN0		Baa2	BBB+	4.0	9/29/2006	9/25/2036	WFB
HEAT 2006-3 M8	Midprime	22,222,222	437084UZ7	BBB+	Baa2	BBB+	3.5	3/30/2006	7/25/2036	SPS
HEAT 2006-5 M8	Midprime	22,222,222	437096AQ3	BBB+	Baa2	BBB+	3.8	6/25/2006	10/25/2036	WFB
HEAT 2006-6 M8	Midprime	22,222,222	437097AP3	A-	Baa2	A-	4.0	8/1/2006	11/25/2036	SPS
HEAT 2006-7 M8	Midprime	22,222,222	43709NAP8	BBB+	Baa2	BBB+	4.2	10/3/2006	1/25/2037	SPS
HEAT 2006-8 M8	Midprime	22,222,222	43709QAP1	BBB	Baa2	BBB+	4.4	12/1/2006	3/25/2037	SPS
IXIS 2006-HE3 B2	Midprime	22,222,222	46602UAM0	BBB	Baa2	BBB	4.8	9/29/2006	1/25/2037	WFB

Controversy over Abacus CDO

"The whole building is about to collapse anytime now [...] Only potential survivor, the [F]abulous Fab [...] standing in the middle of all these complex, highly leveraged, exotic trades he created without necessarily understanding all of the implications of those monstruosities!!!"

–Fabrice Tourre, CDO Structurer at Goldman Sachs

Why was this crisis much worse than the Tech Bubble in 2000?

"Any theory of the crisis that ties its magnitude to the size of the housing bust must also explain why the fall of dot-com stock prices just a few years earlier, which destroyed as much or more paper wealth – more than 8 trillion dollars– resulted in a relatively short and mild recession and no major financial instability."

–Ben Bernanke

Why was this crisis much worse than the Tech Bubble in 2000?

- The tech bubble mainly hit mutual funds, pension funds, and investors (low leverage agents) while the financial crisis hit banks and hedge funds (highly leveraged agents)
- Securitized products such as CDOs were not transferred to investors, but instead many banks kept them on their own books.
- Banks relied upon short-term funding which dried up when the first financial instability hit.
  - Important to note that banks rarely fail due to actual bankruptcy, but instead fail from lack of liquidity.

Types of Mathematics used to Analyze Systemic Risk

- Network analysis
- Stochastic modeling (SDEs, PDEs, Monte Carlo simulation)
- Statistics
- Game Theory
- Asymptotic analysis of large banking systems and large pools of securities

Policy

- What are the mathematical challenges with understanding systemic risk?
- So what do we do about systemic risk?

Potential Policies

- Break up financial institutions which are "too big to fail"
  - What's a potential problem with this?
- Greater transparency
  - Exchanges for credit default swaps (CDS)
  - Counterparty exposures
- Caps on leverage at banks and other financial institutions
- Systemic risk tax
  - What are potential problems with this?
- Change the incentive structure on Wall Street