Projections

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Short title

Introduction to Projections

- Projection of a vector onto a subspace is finding the closest point in the subspace to the vector.
- This involves minimizing the error vector, which is orthogonal to the subspace.

$$\mathbf{p} = \mathbf{a}\mathbf{\hat{x}} = \mathbf{a}\frac{\mathbf{a}^T\mathbf{b}}{\mathbf{a}^T\mathbf{a}} = \mathbf{a}(\mathbf{a}^T\mathbf{a})^{-1}\mathbf{a}^T\mathbf{b}.$$

Projection onto a plane

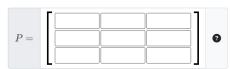
Projection onto a plane 3

Consider the plane x-5y-4z=0. We will compute the projection of the vector $\mathbf{b}=\begin{bmatrix}2\\-2\\0\end{bmatrix}$ onto the plane.

Give $\hat{\mathbf{x}}$ and the projection vector \mathbf{p} .

$$\hat{\mathbf{x}} = \begin{bmatrix} lacksquare \\ lacksquare \end{bmatrix}$$
 $\mathbf{0}$ $\mathbf{p} = \begin{bmatrix} lacksquare \\ lacksquare \end{bmatrix}$

Give the projection matrix P.



Finding a Basis for the Plane

- The plane x 5y 4z = 0 is defined by its normal vector **n**.
- To find a basis, we choose vectors orthogonal to **n**.
- We construct matrix **A** with these basis vectors as columns.

Short title

Computing the Projection Vector

- The projection vector \mathbf{p} is found by solving $\mathbf{A}^T \mathbf{A} \hat{\mathbf{x}} = \mathbf{A}^T \mathbf{b}$.
- We calculate $\hat{\mathbf{x}}$ using the equation $\hat{\mathbf{x}} = (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T \mathbf{b}$.
- Then, $\mathbf{p} = \mathbf{A}\hat{\mathbf{x}}$.

Deriving the Projection Matrix

- The projection matrix P is used to project any vector onto the subspace.
- It is computed as $\mathbf{P} = \mathbf{A}(\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T$.
- ullet This matrix represents a linear transformation mapping \mathbb{R}^3 onto the plane.

Projection with Invertible Matrix

Let A be an invertible matrix and let P be the projection matrix that projects vectors onto the column space of A.

Solution:

- Algebraically, $P = A(A^TA)^{-1}A^T$.
- Since A is invertible, $(A^TA)^{-1} = A^{-1}(A^T)^{-1}$, thus P = I.
- Explanation:
 - A is invertible, so its columns form a basis for \mathbb{R}^n .
 - The column space of A is \mathbb{R}^n .
 - \bullet Projecting any vector onto \mathbb{R}^n returns the vector itself.