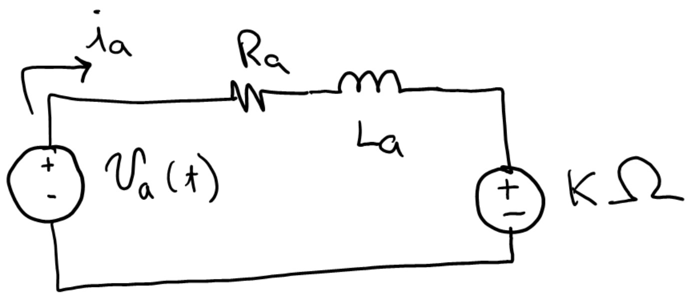
ECGR 2254

Project 2

Due: 12/7/22 at 11:59PM

Problem 1 – A PWM motor drive

In project 1, you were given the model for a DC motor. The circuit model for the DC motor is shown below:



The variable Ω is the speed of the motor in rad/sec. Note that the model includes a speed-dependent voltage source.

The mechanical component of the motor is represented by Newton's Second Law for rotation, which states:

$$(Moment\ of\ Inertia) \times (Angular\ Acceleration) = Net\ Torque$$

Mathematically, this is:

$$J\frac{d\Omega}{dt} = Ki_a - \beta\Omega$$

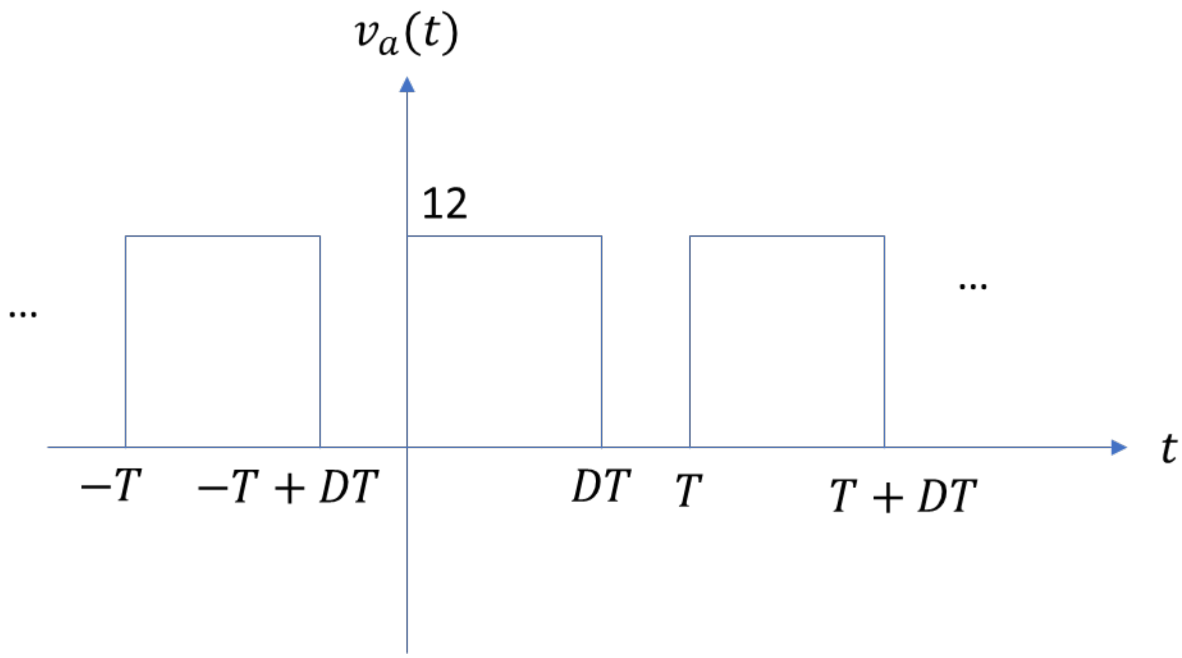
Where:

- β: Damping ratio
- J: Moment of inertia
- K: Motor constant

The motor you are controlling is has the following parameter values:

- $L = 0.01\,H$
- $R = 3.38\,Ohms$
- $K = 0.029\,Vs/rad$
- $J = 2 \times 10^{-4}\,kg\,m^2$
- $\beta = 0.5 \times 10^{-5}\,Nms/rad$

The motor is driven by the pulse width modulated (PWM) signal shown below:



This waveform is periodic with a period T , a peak value of 12, a minimum value of 0, and a duty ratio D. Please note that the duty ratio can vary but is a fraction between 0 and 1.

a) Determine a differential equation for $i_a(t)$ from analyzing the electrical circuit.
b) Using the two differential equations, create a transfer function that has $V_a(\omega)$ as its input and $\Omega(\omega)$ as its output.
c) Plot the magnitude of the transfer function you created in part b. Do so as follows:
   a. Use the logspace command to create 10,000 frequencies between 0.01 rad/sec and 10,000 rad/sec.
   b. Generate a vector containing the elements of the complex function $H(\omega) = \Omega(\omega)/V_a(\omega)$.
   c. Plot $20\log_{10}|H(\omega)|$ versus the log of frequency using the semilogx command.
d) Determine the DC value of $v_a(t)$ in terms of the duty ratio D.
e) We desire the average speed of the motor to be 250 rad/sec (i.e. $|\Omega(0)| = 250$ rad/sec). Use the graph of $|H(\omega)|$ from part c to determine the DC value of $v_a(t)$ needed to achieve this average speed.
f) Determine the value of D needed for the DC value of $v_a(t)$ to meet the requirement given in part e.
g) Determine the complex Fourier series coefficients for a PWM waveform with the value of D you found in part f. What is the amplitude C_1 of the cosine $C_1\cos(\omega_0 t + \phi)$ at the fundamental frequency $\omega_0 = 2\pi/T$?
h) Using the graph from part c, select the frequency of $v_a(t)$ so that the amplitude of the speed variation at the fundamental frequency (i.e. $2|\Omega(\omega_0)|$) will be less than 1% of the average speed (i.e. $2|\Omega(\omega_0)| < 0.01|\Omega(0)|$. Carefully explain your process.

i) Using your work from Project 1, simulate the response of the motor when the voltage is a PWM waveform with the duty ratio D and the fundamental frequency you determined in part h. In this case, you should do the following:

- Use the square command to create $v_a(t)$
- Assume a time step small enough that you can see 10,000 points in one period T.
- Run your simulation for 10 seconds

What is the average value of the speed in steady-state? Does it match your approximation well? It should match very closely (way less than 1% error).

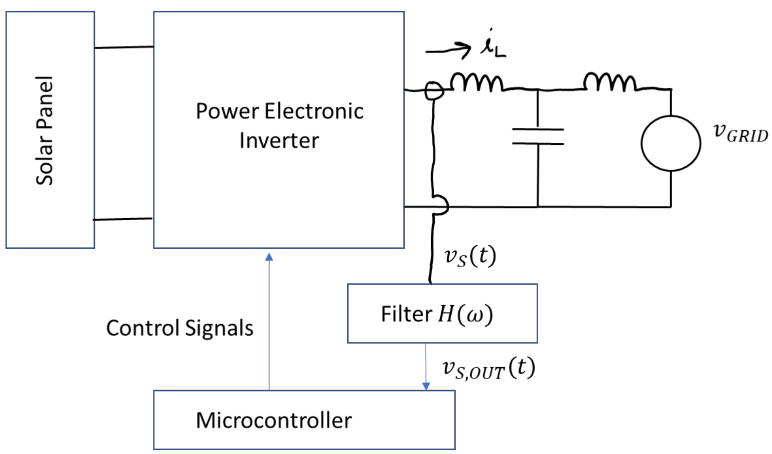
What is the peak-to-peak value of the variation at the switching frequency? This value should be pretty close to your prediction, but the error will be a little bit bigger. Explain why.

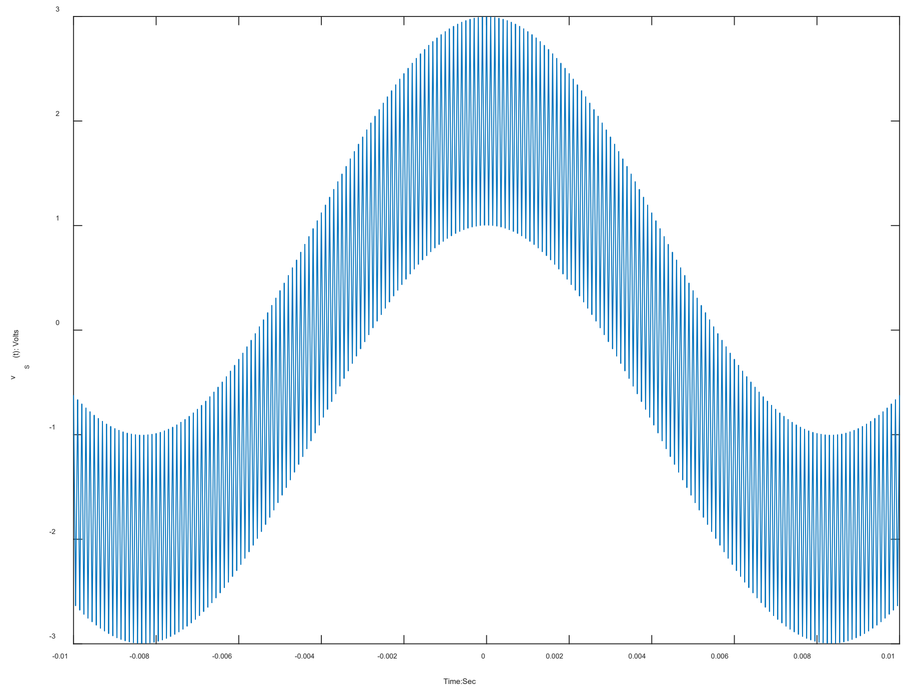
Problem 2 – Filtering a Signal for Control

Solar panels are connected to the grid through a power-electronic inverter that creates a sinusoidal current. The figure below shows an example system. A current sensor measures the current $i_L(t)$ and converts it to an output voltage $v_S(t)$. An example of the voltage $v_S(t)$ is also shown below. This signal has the form

$$v_S(t) = 2\cos(2\pi 60t) + x(t).$$

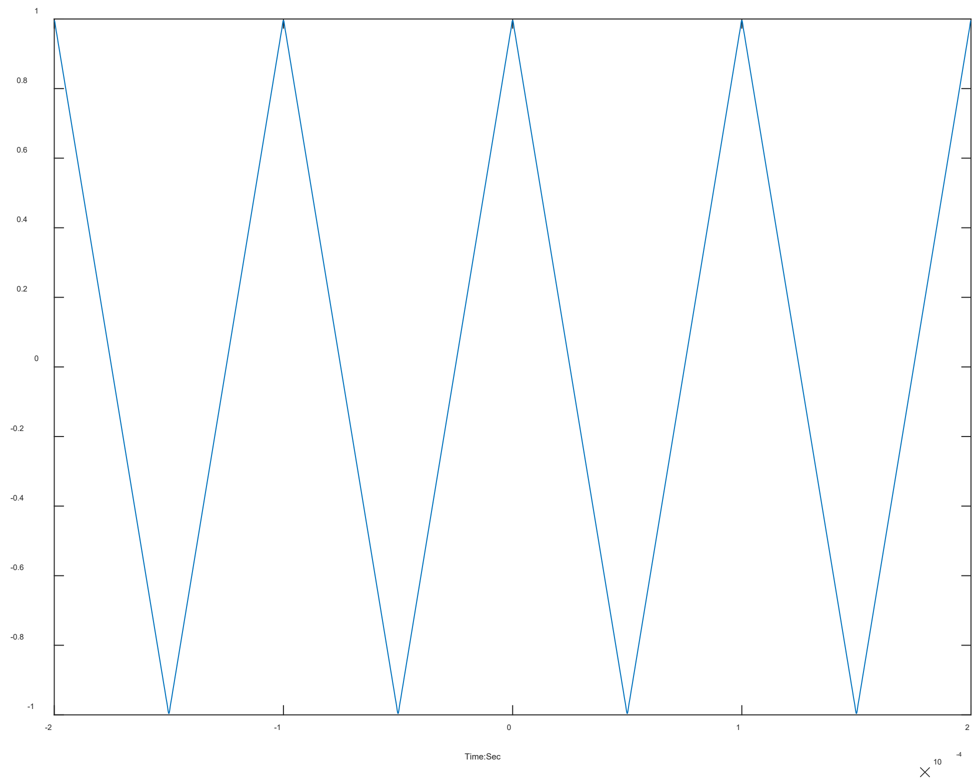
The triangular signal $x(t)$ results from the action of the switches inside the inverter.





In a real control system, the signal $v_S(t)$ must be processed by the microcontroller to decide how to drive the switches. We only want the microcontroller to see the 60Hz signal and not the high-frequency triangle wave. To make sure this happens, we added the filter $H(\omega)$, which is constructed using op-amps, resistors, and capacitors. This is a common problem in embedded controls. Our goal here is to design the filter $H(\omega)$ using typical design specifications.

$x(t)$ is shown below.



a) Determine the Fourier Transform of $x(t)$ for $n = \pm1, \pm2, \pm3$ using numeric integration as described in class on 11/22. To generate the signal $x(t)$:
   a. Determine the period from the figure.
   b. Use the sawtooth command in MATLAB. Check the Mathworks page on the sawtooth command to see an example on how to use the sawtooth command to create a triangle wave that looks like the one in the picture.
   c. Use a time step $\Delta t = T/10000$
   d. Make sure the triangle wave you create has the same phase as the signal shown in the figure above.

   Determine the corresponding signals $C_n \cos(n\omega_o t + \phi_n)$ for $n = 1,2,3$.

   You can always verify your answers by integrating by hand, but it will be messy

b) To design the filter, we need to create the appropriate mathematical constraints. The signal $v_S(t) = 2\cos(2\pi 60t) + C_1\cos(\omega_o t + \phi_1) + C_2\cos(2\omega_o t + \phi_2) + C_3\cos(3\omega_o t + \phi_3) + \cdots$ passes through the filter $H(\omega)$.

   - Constraint 1: Determine a constraint on $|H(\omega_0)|$ if we want the amplitude of the output signal at ω_0 to be about 1% of the amplitude of the input signal at that frequency.
   - Constraint 2: Determine a constraint on $\angle H(2\pi 60)$ if we want the phase shift between the input signal and the output signal to be less than $4°$ at 60Hz
   - Constraint 3: Determine a constraint on $|H(2\pi 60)|$ if we want the amplitude of the input signal and output signal at 60Hz to be approximately the same.

c) You are given the following choices for the filters:

$$H_1(\omega) = \frac{1}{1 + j\frac{\omega}{\omega_c}}$$

$$H_2(\omega) = \frac{\omega_c^2}{(j\omega)^2 + j\frac{2}{\sqrt{2}}\omega_c\omega + \omega_c^2}$$

$$H_4(\omega) = \frac{\omega_c^4}{((j\omega)^2 + j0.7654\omega_c\omega + \omega_c^2)((j\omega)^2 + j1.8478\omega_c\omega + \omega_c^2)}$$

To approach this problem, use the following approach:

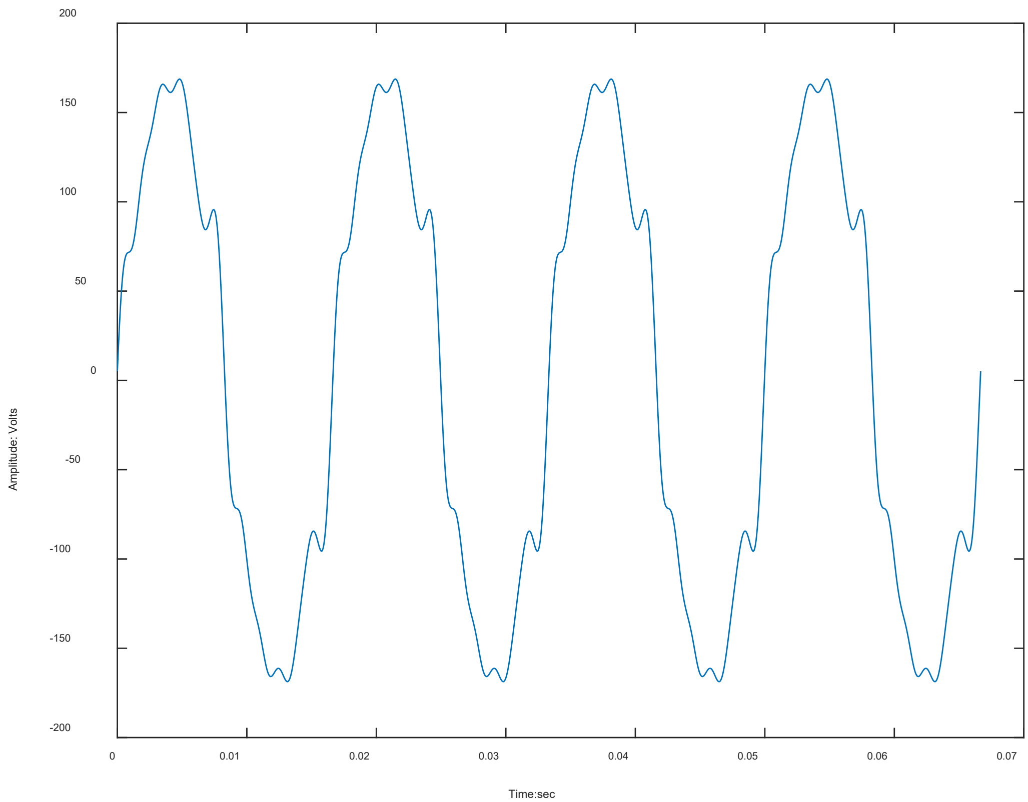
- Consider $H_1(\omega)$. Can you choose a value of ω_c that will meet Constraint 1 and Constraint 3? Can you also then meet Constraint 2? Explain with graphs and words.
- Consider $H_2(\omega)$. Can you choose a value of ω_c that will meet Constraint 1 and Constraint 3? Can you also then meet Constraint 2? Explain with graphs and words.
- Consider $H_3(\omega)$. Can you choose a value of ω_c that will meet Constraint 1 and Constraint 3? Can you also then meet Constraint 2? Explain with graphs and words.

Generate a plot of $20\log_{10}|H(\omega)|$ versus $\log_{10}\omega$ for the filter you choose. You should be able to show that it meets all three constraints. Your answer must be carefully explained using graphs, words, and math.

d) Determine and graph the output signal $v_{s,OUT}(t)$ for your filter choice. You should include the 60Hz term and the first, second, and third harmonics of $x(t)$.

Problem 3 – A simple power quality analyzer

Download the file problem3.mat from the Canvas page. Load this file into MATLAB using the command `load('problem3.mat')`. This loads a variable `x` into your workspace. x contains 4 periods of the grid voltage in a local building. This voltage appears as shown below:



The measured voltage contains several harmonics of the fundamental 60Hz voltage. Please note that the voltage was sampled at a frequency $f_s = 60{,}000$.

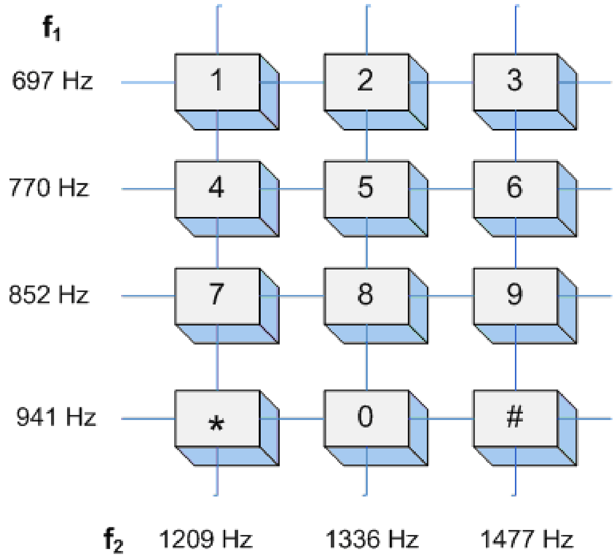
In this problem, you are asked to do the following:

a) Determine the Fourier Transform $X(\omega)$ for the harmonic numbers from $n = -13$ to $n = 13$. This should be determined using numeric integration as described in class on 11/22.
b) Determine a_0 and the amplitudes a_1 through a_{13} and b_1 through b_{13} using your result from part a.
c) Use the values from part c to recreate the input signal. In other words, plot $\frac{a_0}{2} + \sum_{n=1}^{13} a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t)$.

In your answers, carefully explain your work.

Problem 4 – A Touch Tone Recognition System

Touch-tone telephones use signals of different frequencies to indicate which key has been pressed. The sound you hear when you press a key is the sum of two sinusoids. The higher frequency sine wave indicates the column of the key, and the lower frequency sine wave indicates the row. The figure below shows how the frequencies are assigned to each row and column.



For example, the continuous-time signal produced when pressing the "5" is thus

$$d_5(t) = \sin(2\pi 770t) + \sin(2\pi 1336t)$$

In a cellular phone, these signals are stored as digitized waveforms, sampled at 8192Hz. In this assignment, you are going to create each digit. Later you will use the discrete Fourier Transform to decode several numbers in Matlab.

a) In your report, write out $d_0(t)$ through $d_9(t)$. Create the corresponding discrete-time signals $d_0(n)$ through $d_9(n)$. What are the corresponding discrete-time frequencies?

b) In Matlab, create row vectors `d0` through `d9` to represent all 10 digits for the interval $0 \le n \le 999$. Listen to each signal using the command sound. For example, `sound(d2,8192)` should sound like the tone you hear when you push a "2" on the phone.

c) Create a vector called `space` consisting of 100 samples of zeros. Now, create your phone number using a piece of code like the following:

```
>> x = [d7 space d0 space d4 space d6 space d8 space d7 space d8
space d4 space d0 space d2];
```

If you type,

```
>> sound(x,8192)
```

You will hear your phone number.

d) Now you will use MATLAB to compute the discrete Fourier Transform (DFT):

$$X[k] = \sum_{n=0}^{N_p-1} x(n)e^{-j\left(\frac{2\pi}{N_p}\right)kn}$$

Compute 2048 samples of the DFT of `d2` and `d5`. Note that you will need to add zeros to the end of both `d2` and `d5` to compute these samples. In your solution, include your code, and plot the magnitudes of the two DTFTs versus frequency from 0 to 2π. Show that the frequencies correspond to `d2` and `d5`.

e) From the course website, download the file `touch.mat`. Store this file in your current Matlab directory and type `load('touch.mat')` at the Matlab prompt. This loads four "phone number" vectors into Matlab. One of these is called x1. This vector consists of 7 digits of 1000 samples each, separated by 100 samples of silence. Determine the digits of the phone number stored in x1. To do so, compute 2048 samples of the DTFT of each of digit. In your answers, be sure to include the DFT magnitude plots.