STAT 5309 – SP 2022

LAB 5

****CONTENTS: Design with 1, 2 Quantitative factors– Response Surface Model(RSM)**

Due:

A. PRACTICE

1. One Quantitative factor

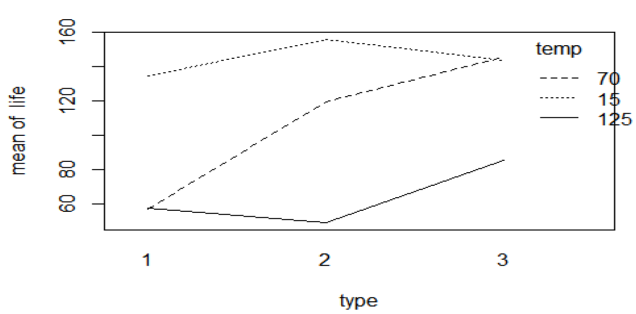
 # Set upt data frame: Battery life

Temp / Type	15	70	125
1	130 155 74 180	34 40 80 75	20 70 82 58
2	150 188 159 126	136 122 106 115	25 70 58 45
3	138 110 168 160	174 120 150 139	96 104 82 60

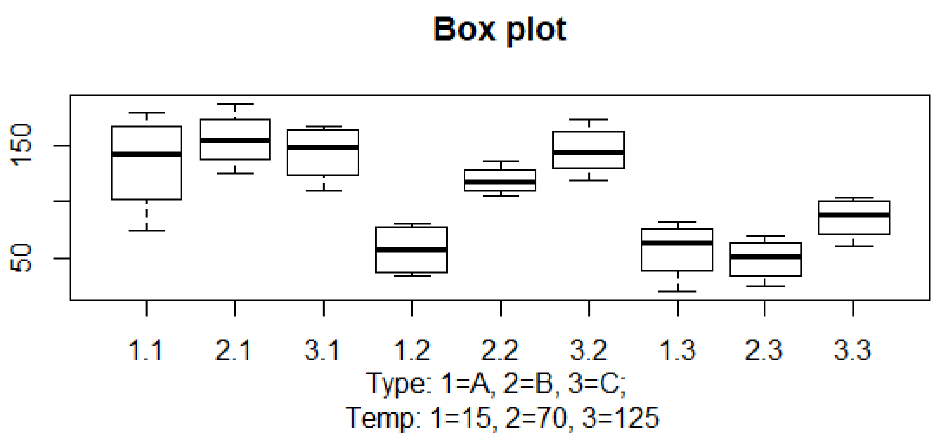
```
type <- rep (c(1,2,3), each=12)
temp <- rep( c(1,2,3),  each=4, times=3)
life <- c(130,155,74,180,34,40, 80, 75, 20, 70, 82, 58, 150, 188, 159, 126, 136, 122, 106, 115, 25, 70,
58, 45, 138, 110, 168, 160, 174, 120, 150, 139, 96, 104, 82, 60)
battery <- data.frame(type,temp,life)
temp <- factor(temp)
type <- factor(type)
attach(type, temp, life)
```

 # Plots

```
interaction.plot(type, temp, life)
```



```
> boxplot(life ~ type+temp , main="Box plot", xlab=c("Type: 1=A, 2=B, 3=C;
\n Temp: 1=15, 2=70, 3=125"))
```



Question : Which combination levels give the longest battery life?

##Regression model

battery.mod <- aov(life ~ type*temp, data=battery)

```
             Df Sum Sq Mean Sq F value   Pr(>F)
type          2  10684    5342   7.911  0.00198 **
temp          2  39119   19559  28.968 1.91e-07 ***
type:temp     4   9614    2403   3.560  0.01861 *
Residuals    27  18231     675
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Note: Main effects (Type, Temp) and Interaction effect (Type:Temp) are significant

Summary of means and effects: model.tables(): list factor level means, effects.

model.tables(battery.mod,type="mean")

```
Tables of means
Grand mean

105.5278
 type
type
     1      2      3
 83.17 108.33 125.08
 temp
temp
     1      2      3
```

```
144.83 107.58  64.17
 type:temp
    temp
type 1      2      3
   1 134.75  57.25  57.50
   2 155.75 119.75  49.50
   3 144.00 145.75  85.50
```

Note: The combination type=2; temp=1 gives highest battery life

2. Two Quantitative factors

Data: CO emmisions.
A data frame with 18 observations . **2 factors**

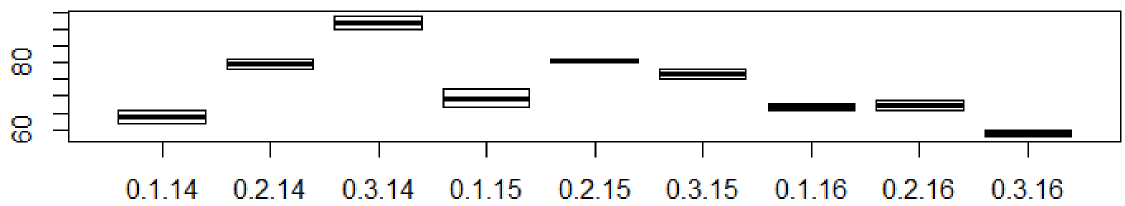
.Eth : a factor with 3 levels 0.1 0.2 0.3.
Ratio: a factor with 3 levels 14 15 16.
CO: response, a numeric vector

library(daewr)
data(COdata)

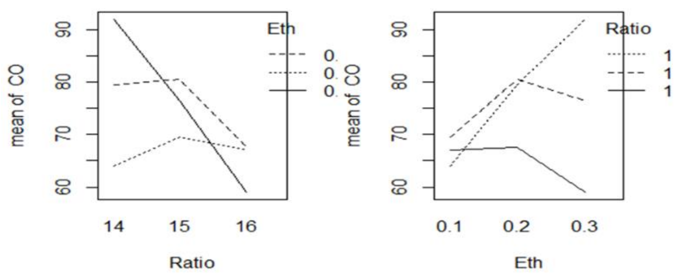
```
   Eth Ratio CO
1  0.1    14 66
2  0.1    15 72
3  0.1    16 68
4  0.2    14 78
5  0.2    15 80
6  0.2    16 66
7  0.3    14 90
8  0.3    15 75
9  0.3    16 60
10 0.1    14 62
11 0.1    15 67
12 0.1    16 66
13 0.2    14 81
14 0.2    15 81
15 0.2    16 69
16 0.3    14 94
17 0.3    15 78
18 0.3    16 58
```

Exploring plots

```
> boxplot(CO ~ Eth*Ratio)
```



interaction.plot(Eth,Ratio, CO)
interaction.plot(Ratio,Eth, CO)



Question: Which combination gives the lowest CO emission?

Regression model
CO.mod <-aov(CO ~Eth *Ratio, data=COdata)   # Consider interaction

summary.aov(CO.mod)

```
            Df Sum Sq Mean Sq F value   Pr(>F)    
Eth          2  324.0   162.0   31.36 8.79e-05 ***
Ratio        2  652.0   326.0   63.10 5.07e-06 ***
Eth:Ratio    4  678.0   169.5   32.81 2.24e-05 ***
Residuals    9   46.5     5.2                     
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

Note: Main and Interaction effects are significant.

Study of means and effects: model.tables(): list level means, effects, grand mean.

model.tables(CO.mod, type=”means”, se=T)

```
Tables of means
Grand mean
72.83333
 Eth
Eth
  0.1   0.2   0.3
66.83 75.83 75.83
 Ratio
Ratio
  14   15   16
78.5 75.5 64.5

 Eth:Ratio
     Ratio
Eth   14   15   16
  0.1 64.0 69.5 67.0
  0.2 79.5 80.5 67.5
  0.3 92.0 76.5 59.0

Standard errors for differences of means
          Eth Ratio Eth:Ratio
        1.312 1.312     2.273
replic.     6     6         2
```

Note: 59.0 is the lowest

3. Response Surface Model (RSM)

RSM is a special Quadratic Regression model, with rich theory, which can help to find the optinal solutions (min, max) of the response variables when dealing with factors which are quantitative. Package rsm, function rsm(). The factors must be in numeric.

Mod.rsm <-rsm(y ~ FO(x1,x2) + TWI(x1,x2) + SO(x1, x2) , data)

There might be more than 2 variables. The regression model is Quadratic.

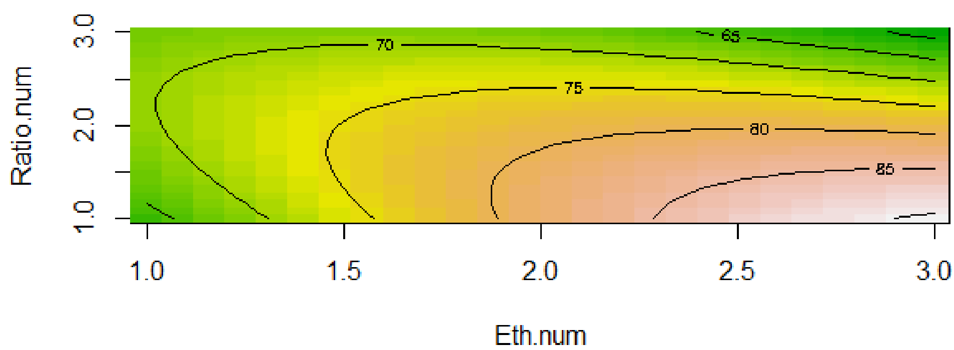
```
>library(rsm)
> Eth.num<- as.numeric(Eth)      #Eth.num is numeric
> Ratio.num <- as.numeric(Ratio)  #Ratio.num is numeric
> COdata.new<-data.frame(Eth.num, Ratio.num, CO)


> CO.rsm<- rsm(CO ~ SO(Eth.num, Ratio.num), data=COdata.new)
> summary(CO.rsm)
```

```
Call:
rsm(formula = CO ~ SO(Eth.num, Ratio.num), data = COdata.new)

                     Estimate Std. Error  t value  Pr(>|t|)
(Intercept)        -1013.5000   284.7158  -3.5597  0.003926 **
Eth.num             1575.0000   143.2928  10.9915 1.278e-07 ***
Ratio.num            131.0000    37.9222   3.4544  0.004766 **
Eth.num:Ratio.num    -90.0000     8.9268 -10.0820 3.279e-07 ***
Eth.num^2           -450.0000   126.2438  -3.5645  0.003891 **
Ratio.num^2           -4.0000     1.2624  -3.1685  0.008093 **
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Multiple R-squared:  0.955,    Adjusted R-squared:  0.9363
F-statistic: 50.95 on 5 and 12 DF,  p-value: 1.146e-07

Analysis of Variance Table
Response: CO
                        Df Sum Sq Mean Sq  F value    Pr(>F)
FO(Eth.num, Ratio.num)   2  831.0  415.50  65.1765 3.588e-07
TWI(Eth.num, Ratio.num)  1  648.0  648.00 101.6471 3.279e-07
PQ(Eth.num, Ratio.num)   2  145.0   72.50  11.3725  0.001697
Residuals               12   76.5    6.37
Lack of fit              3   30.0   10.00   1.9355  0.194443
Pure error               9   46.5    5.17

Stationary point of response surface:
  Eth.num Ratio.num
     -0.9      26.5

Eigenanalysis:
eigen() decomposition
$values
[1]    0.4950549 -454.4950549

$vectors
                 [,1]        [,2]
Eth.num    0.09939545 -0.99504801
Ratio.num -0.99504801 -0.09939545



> canonical(CO.rsm)
$xs
  Eth.num Ratio.num
     -0.9      26.5

$eigen
eigen() decomposition
$values
[1]    0.4950549 -454.4950549

$vectors
                 [,1]        [,2]
Eth.num    0.09939545 -0.99504801
Ratio.num -0.99504801 -0.09939545
```

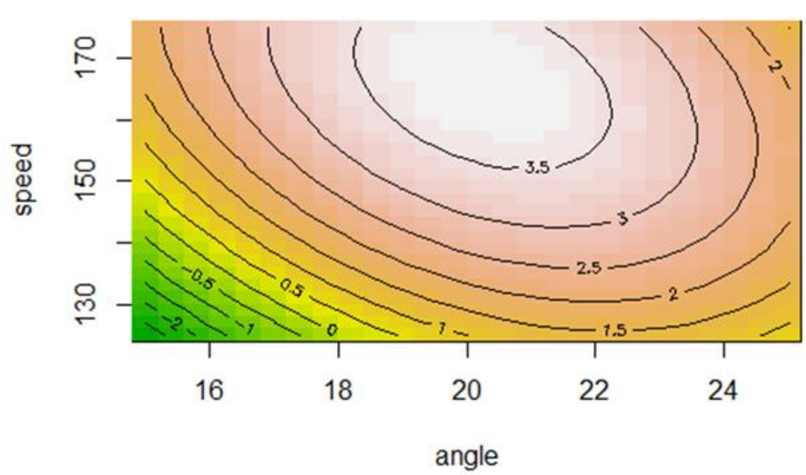
Note: The rsm output contains the location of the stationary point(s). C anonical analysis results : stationary points, eigen values/eigen vectors of the transformed design matrix at the stationary point;

If (a) Both Eigenvaluesa are negative: a maximum
(b) Both eigenvalues are positive: a minimum
(c) Eigenvalues of opposite signs: a saddle point (neither min or max)

```
## Canonical analysis plotting
> xs <- canonical(CO.rsm)$xs
> contour(CO.rsm, ~ Eth.num +Ratio.num, at=xs,image=TRUE  )
```



Note: Eigenvalues of opposite signs, we have saddle point. There are no maximum or minimum.



B. EXERCISE

1. Problem [Dataset 5.9]

5-2. An engineer suspects that the surface finish of a metal part is influenced by the feed rate and the depth of cut. He selects three feed rates and four depths of cut. He then conducts a factorial experiment and obtains the following data:

	Depth of Cut (in)			
Feed Rate (in/min)	0.15	0.18	0.20	0.25
	74	79	82	99
0.20	64	68	88	104
	60	73	92	96
	92	98	99	104
0.25	86	104	108	110
	88	88	95	99
	99	104	108	114
0.30	98	99	110	111
	102	95	99	107

(a) Analyze the data and draw conclusions. Use $\alpha = 0.05$.
(b) Prepare appropriate residual plots and comment on the model's adequacy.
(c) Obtain point estimates of the mean surface finish at each feed rate.
(d) Find the P-values for the tests in part (a).

(c) Build an aov() model: Are the main factors, interaction factor significant? Perform boxplot, interaction plots.

(d) Build a RSM model. Perform Canonical analysis.

2. Problem [Dataset 5.7]

5-7. A mechanical engineer is studying the thrust force developed by a drill press. He suspects that the drilling speed and the feed rate of the material are the most important factors. He selects four feed rates and uses a high and low drill speed chosen to represent the extreme operating conditions. He obtains the following results. Analyze the data and draw conclusions. Use $\alpha = 0.05$.

	Feed Rate			
Drill Speed	0.015	0.030	0.045	0.060
125	2.70	2.45	2.60	2.75
	2.78	2.49	2.72	2.86
200	2.83	2.85	2.86	2.94
	2.86	2.80	2.87	2.88

Set up the data frame , named “drill”, factors “Speed”, “Rate”

(a) Build a linear model, using aov(). Are main and interaction effects significant?

(b) Build a Response Model Surface (RSM).

3. Problem [Dataset 5.14]

5-14. The shear strength of an adhesive is thought to be affected by the application pressure and temperature. A factorial experiment is performed in which both factors are assumed to be fixed. Analyze the data and draw conclusions. Perform a test for nonadditivity.

Pressure (lb/in^2)	Temperature (°F) 250	260	270
120	9.60	11.28	9.00
130	9.69	10.10	9.57
140	8.43	11.01	9.03
150	9.98	10.44	9.80

(a) Build a linear model, using aov(). Are main and interaction effects significant?

(b) Build a Response Model Surface (RSM).