STAT 5309 – SP 2022

LAB 3

****

***CONTENTS : 1-Factor Design : Contrasts- Power of a test; sample size.**

***Due: Thurs, Feb 17**

A. Balanced design ; Linear model

Bacteria under package methods. log(count/cm^2) on meat samples stored in 4 packaging conditions for 9 days.

Packaging Condition log(count/cm^2)	
Commercial plastic wrap	7.66, 6.98, 7.80
Vacuum packaged	5.26, 5.44, 5.80,
1% CO,40% O2, 59% N	7.41, 7.33, 7.04
100% CO2	3.51, 2.91, 3.66

Note: Factor "package" has 4 treatment levels (a =4). Each level has 3 replicates (n=3).

Setup data.

```
package <- rep( c(1,2,3,4) ,each=3)
logcount <- c(7.66,6.98,7.80,5.26,5.44,5.80,7.41,7.33,7.04,3.51,2.91,3.66)
bacteria <- data.frame(package,logcount)
attach(bacteria)                              #attach() before factor()

package <- factor(package)
> bacteria
   package logcount
1        1     7.66
2        1     6.98
3        1     7.80
4        2     5.26
5        2     5.44
6        2     5.80
7        3     7.41
8        3     7.33
9        3     7.04
10       4     3.51
11       4     2.91
12       4     3.66

Note:  by this setup, it is understood that 1-4 are
Commercial plastic wrap, Vacuum packaged,  1% CO, 40% O2, 59% N
100% CO2 respectly.
```

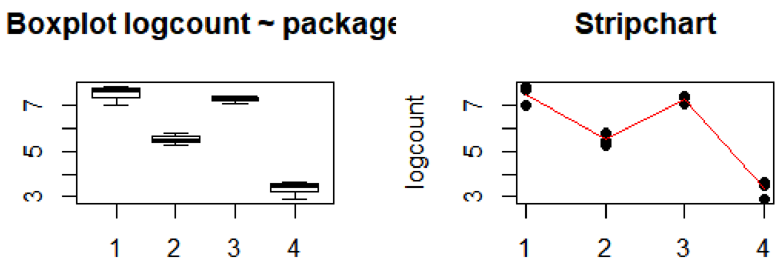
relevel() , 4 as base level:
>package <- relevel(package, ref="4")

Exploring data

```
> tapply(logcount,package,mean)         # treament means, in a vector
   1    2    3    4
7.48 5.50 7.26 3.36
> tapply(logcount,package,sd)             #treatment standard deviations, in a vector
        1         2         3         4
0.4386342 0.2749545 0.1946792 0.3968627
```

Exploring plots: Boxplot, Stripchart

stripchart(logcount~ package, vert=TRUE, pch=16, main="Stripchart")
means<- tapply(logcount,package,mean)
lines(means, col="red")



boxplot(logcount ~ package, main="Boxplot logcount ~ package")

Linear model
>bact.mod <- aov(logcount ~ package)   #linear model

```
> summary.aov(bact.mod)
            Df Sum Sq Mean Sq F value   Pr(>F)
package      3  32.87  10.958   94.58 1.38e-06 ***
Residuals    8   0.93   0.116
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

Notes:

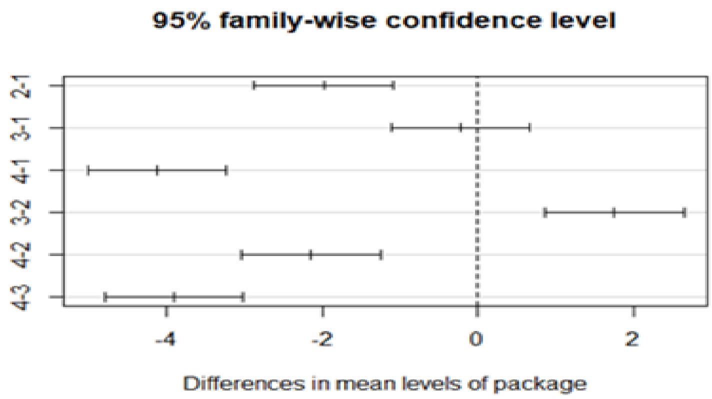
P-value=`1.38e-06 ,` is smaller than alpha=.05. Reject Ho. We say, the factor package is significant

```
> summary.lm(bact.mod)

Call:
aov(formula = logcount ~ package)

Residuals:
   Min     1Q Median     3Q    Max 
-0.500 -0.225  0.110  0.210  0.320 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)   7.4800     0.1965  38.064 2.49e-10 ***
package2     -1.9800     0.2779  -7.125 9.95e-05 ***
package3     -0.2200     0.2779  -0.792    0.451    
package4     -4.1200     0.2779 -14.825 4.22e-07 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.3404 on 8 degrees of freedom
Multiple R-squared:  0.9726,	Adjusted R-squared:  0.9623 
F-statistic: 94.58 on 3 and 8 DF,  p-value: 1.376e-06
```

##--------- **Pairwise comparisons: TukeyHSD()**

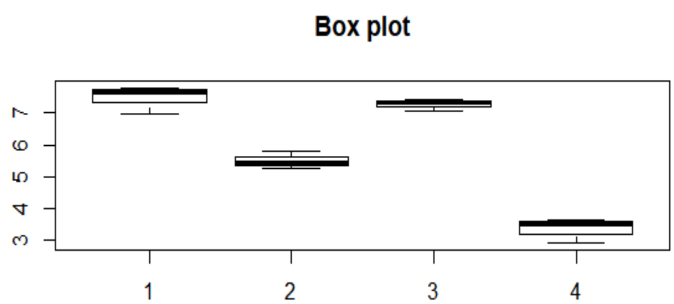
```
> TukeyHSD(bact.mod)
  Tukey multiple comparisons of means
    95% family-wise confidence level

Fit: aov(formula = logcount ~ package)

$package
     diff       lwr       upr     p adj
2-1 -1.98 -2.869962 -1.090038 0.0004549
3-1 -0.22 -1.109962  0.669962 0.8563618
4-1 -4.12 -5.009962 -3.230038 0.0000020
3-2  1.76  0.870038  2.649962 0.0010160
4-2 -2.14 -3.029962 -1.250038 0.0002639
4-3 -3.90 -4.789962 -3.010038 0.0000031
```

Notes: Use P-values or the CI's to check if the difference is significant



B. **CONTRASTS:**

There might be some other relationships among the level means



we might test claims: $\mu_1 = \mu_3$, $\mu_2 = \frac{1}{2}(\mu_3 + \mu_4)$.

A Contrast C is a linear combination of the treatment means,

Testing a contrast is mainly testing it versus 0.

A contrast is a linear combination of the population means

$C = c_1\mu_2 + c_2\mu_2 + \cdots + c_a\mu_a$, $\Sigma c_i = 0$

Simple contrasts: $C = \mu_i - \mu_j$ (compare pair of means)

Testing a contrast:

$$C = c_1\bar{y}_{1.} + c_2\bar{y}_{2.} + \cdots + c_a\bar{y}_{a.}$$

$$var(C) = \frac{\sigma^2(\Sigma\, c_i^2)}{n} = \frac{MSE(\Sigma\, c_i^2)}{n}$$

$$se(C) = \sqrt{\frac{MSE(\Sigma\, c_i^2)}{n}};$$

$$t_o = \frac{\mathrm{C}-0}{se(C)} \sim t_{(N-a)}$$

 # fit.contrast(), package gmodels
A contrast matrix, a matrix of contrasts as rows

```
> C1 <- c(1,0,-1,0)
> C2<- c(0, 1, -1/2, -1/2)
> C <- rbind(C1, C2)                    # rbind()


> rownames(C)<- c("Tr1 = Tr3", "Trt2 =average of Tr3, Tr4")
> C
                          [,1] [,2] [,3] [,4]
Tr1 = Tr3                    1    0 -1.0  0.0
Trt2 =average of Tr3, Tr4    0    1 -0.5 -0.5

> fit.contrast(bact.mod, package, C , conf.int=.95)
                                 Estimate Std. Error   t value  Pr(>|t|)
lower CI
packageTr1 = Tr3                     0.22  0.2779089 0.7916264 0.4514097 -0
.4208590
packageTrt2 =average of Tr3, Tr4     0.19  0.2406761 0.7894426 0.4526137 -0
.3650002
                                  upper CI
packageTr1 = Tr3                 0.8608590
packageTrt2 =average of Tr3, Tr4 0.7450002
attr(,"class")
[1] "fit_contrast"
```

Note:
Use P-values or CI . C1: Support Ho: $\mu_1 - \mu_3 = 0$, $\mu_2 - 0.5\ \mu_3 - 0.5\mu_4 = 0$.

C. Power of a test

β = Type II error of a test = Prob(Accept Ho | Ho is False)

Power of a test= 1-β

Note: Increasing n (sample size) increase the power of a test.

```
> a <- 5
> s <- .707
> n <- 20
> error <- qnorm(0.975)*s/sqrt(n)
> left <- a-error
> right <- a+error
> left
[1] 4.690149
> right
[1] 5.309851
> ##  beta = Prob( Support Ho | HO is FALSE)= Prob(Support Ho| HA is TRUE)
> a1 <- a + 1.5
> Zleft.new <- (left-a1)/(s/sqrt(n))
> Zright.new<-(right-a1)/(s/sqrt(n))
> beta <- pnorm(Zright.new)-pnorm(Zleft.new)
> beta
[1] 2.570223e-14
> power<- 1-beta
> power
[1] 1

> (5.31-6.5)/(.707/sqrt(20))
[1] -7.527358
> (4.69 -6.5)/(.707/sqrt(20))
[1] -11.44917

# the normal area between these values is 0.  So beta=0
As n increases,   beta decrease, so power increases
```

C1. Power of a t-test: power.t.test()

 # Testing under: Ho: mu=16, Ha: mu=18

```
> y1 <- c(16.85,16.40,17.21,16.35,16.52,17.04,16.96,17.15,16.59,16.57)
> mean(y1)
[1] 16.764
> length(y1)
[1] 10
```

 # Under Ho: mu=16; Ha: mu=18

```
> power.t.test(n=10, delta=16-18, sd=sd(y1), sig.level=.05, type="one.sampl
e")

     One-sample t test power calculation 

              n = 10
          delta = 2
             sd = 0.3164455
      sig.level = 0.05
          power = 1
    alternative = two.sided
```

Reversely , given power=.90, then the sample size will be

```
> power.t.test(n=NULL, delta=.05, sd=.01, sig.level=.05, type="one.sample",
power=.90)
```

```
     One-sample t test power calculation

              n = 2.688849
          delta = 0.05
             sd = 0.01
      sig.level = 0.05
          power = 0.9
    alternative = two.sided
```

Note: for a given power, small delta (detection) requires large n.

C2: Power of a F-test: power.anova.test():

trt.means <- tapply(logcount, package, mean)

```
> trt.means
   1    2    3    4
7.48 5.50 7.26 3.36
> bact.mod<- aov(logcount ~ package)
> summary.aov(bact.mod)
            Df Sum Sq Mean Sq F value   Pr(>F)
package      3  32.87  10.958   94.58 1.38e-06 ***
Residuals    8   0.93   0.116
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
> MSE <- 0.116
> power.anova.test(groups=4, n=3, between.var=var(trt.means),
 within.var= MSE, sig.level=.05, power=NULL)

     Balanced one-way analysis of variance power calculation

         groups = 4
              n = 3
    between.var = 3.652533
     within.var = 0.116
      sig.level = 0.05
          power = 1

NOTE: n is number in each group
```

Examples

```
power.anova.test(groups = 4, n = 5, between.var = 1, within.var = 3)
# Power = 0.3535594

power.anova.test(groups = 4, between.var = 1, within.var = 3,
                 power = .80)
# n = 11.92613

## Assume we have prior knowledge of the group means:
groupmeans <- c(120, 130, 140, 150)
power.anova.test(groups = length(groupmeans),
                 between.var = var(groupmeans),
                 within.var = 500, power = .90) # n = 15.18834
```

A. EXERCISE

1. **Problem: (Casting of High Temperature Alloys)** A metal alloy is produced by a high temperature casting process. The tensile strength of the alloy is critical for its future use. The casting process is designed produce bars of alloy with an average tensile strength above minimum requirement. An experiment was planned to isolate the variation in tensile strength due to the effects of different castings. 30 bars or alloy were produced using the 3 casting methods.

	Casting
1	88.0, 88.0, 94.8, 90.8, 93.0, 89.0, 86.0, 92.9, 89.0, 93.0
2	85.9, 88.6, 90.0, 87.1, 85.6, 86.0, 91.0, 89.6, 93.0, 87.5
3	94.2, 91.5, 92.0, 96.5, 95.6, 93.8, 92.5, 93.2, 96.2, 92.5

Set up the data frame. Form a factor vector "methods". Form a vector of response "strength". Form a data frame named "alloy".

(a) Test the equal variance among treatments, using bartlett.test()
(b) Do a boxplot, stripchart.
(c) Build a linear model, using aov(). a summary.aov()
(d) Perform TukeyHSD().
(e) Perform a power analysis.

2. **Problem [dataset 3-26]**

3-10. The response time in milliseconds was determined for three different types of circuits that could be used in an automatic valve shutoff mechanism. The results are shown in the following table:

Circuit Type	Response Time				
1	9	12	10	8	15
2	20	21	23	17	30
3	6	5	8	16	7

(a) Test the hypothesis that the three circuit types have the same response time. Use $\alpha = 0.01$.
(b) Use Tukey's test to compare pairs of treatment means. Use $\alpha = 0.01$.
(c) Use the graphical procedure in Section 3-5.3 to compare the treatment means. What conclusions can you draw? How do they compare with the conclusions from part (b)?
(d) Construct a set of orthogonal contrasts, assuming that at the outset of the experiment you suspected the response time of circuit type 2 to be different from the other two.
(e) If you were the design engineer and you wished to minimize the response time, which circuit type would you select?
(f) Analyze the residuals from this experiment. Are the basic analysis of variance assumptions satisfied?

(c). Skip. Instead, find the number of replicates if the power of the F-test is .90.
(d)Use fit.contrast()

3. Problem [dataset 3-27]

3-15. Four catalysts that may affect the concentration of one component in a three-component liquid mixture are being investigated. The following concentrations are obtained:

Catalyst			
1	2	3	4
58.2	56.3	50.1	52.9
57.2	54.5	54.2	49.9
58.4	57.0	55.4	50.0
55.8	55.3		51.7
54.9			

(a) Do the four catalysts have the same effect on the concentration?
(b) Analyze the residuals from this experiment.
(c) Construct a 99 percent confidence interval estimate of the mean response for catalyst 1.
(d)Calculate from scratch: SST, SSTreatment, SSError. Compare with ANOVA analysis from R.

a 0