An algorithm for multiplying two digit numbers

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1 Introduction

This is an algorithm for multiplying two digit numbers mentally and within less time. This algorithm considers two specific cases:

- The first digits are same, and the last ones add up to 10
- The last digits are same, and the first ones add up to 10

Ex:

- Case 1: $\begin{array}{ccc} & 6 & 4 \\ \text{x} & 6 & 6 \\ \hline & 6(6+1) & 4*6 \\ & 42 & 24 \end{array}$

  Ans: 4224

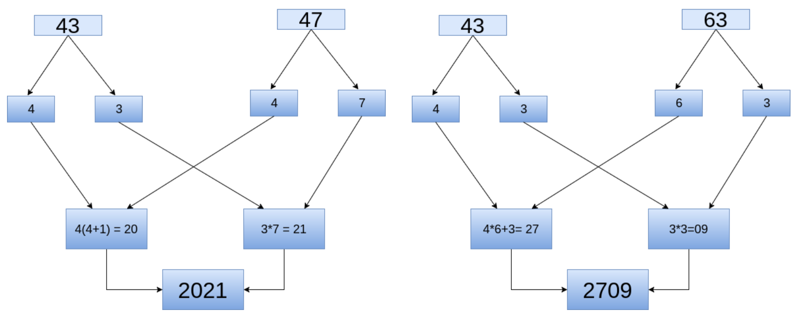
- Case 2: $\begin{array}{ccc} & 3 & 4 \\ \text{x} & 7 & 4 \\ \hline & 3*7+4 & 4*4 \\ & 25 & 16 \end{array}$

  Ans: 2516

2 How to mentally implement it ?

Implementation is really simple for these two cases. Just do the following:

- Case 1:
  - First two digits : just add the first digit to its square
  - Last two digits : multiply the last to digits
- Case 2:
  - First two digits : multiply the first two digits and add the last digit to the product.
  - Last two digits : just square the last digit



3 Mathematical Derivation of the algorithm

Proof. Let the numbers be A and B where $A = 10 * a_1 + a_2$ and $B = 10 * b_1 + b_2$

$$A * B = 100.a_1.a_2 + 10.(a_1.b_2 + a_2.b_1) + b_1.b_2 \tag{1}$$

- Case 1: $a_1 = b_1$ and $a_2 + b_2 = 10$

  Proof.

$$\begin{aligned} a_1.b_2 + a_2.b_1 &= 10.a_1 \\ From\ 1, & \\ A.B &= 100.a_1.a_2 + 10.(a_1.b_2 + a_2.b_1) + b_1.b_2 \\ &= 100.a_1.b_1 + 100.a_1 + a_2.b_2 \\ &= 100(a_1.b_1 + a_1) + a_2.b_2 \\ &= 100.a_1.(a_1 + 1) + a_2.b_2 \end{aligned}$$

  $\square$

- Case 2: $a_2 = b_2$ and $a_1 + b_1 = 10$

  Proof.

$$\begin{aligned} a_1.b_2 + a_2.b_1 &= 10.a_2 \\ From\ 1, & \\ A.B &= 100.a_1.a_2 + 10.(a_1.b_2 + a_2.b_1) + b_1.b_2 \\ &= 100.a_1.b_1 + 100.a_2 + a_2.b_2 \\ &= 100(a_1.b_1 + a_2) + a_2^2 \end{aligned}$$

  $\square$

□

4 Algorithm[1]

Algorithm 1: Multipy 2 two diit numbers A and B

a_1=$A/10$
a_2=$A\%10$
b_1=$B/10$
b_2=$B\%10$
if $a_1 = b_1$ *and* $a_2 + b_2 = 10$ **then**
  $left \leftarrow a_1(a_1 + 1)$
  $right \leftarrow a_2 b_2$
  print $left$ $right$
else if $a_2 = b_2$ *and* $a_1 + b_1 = 10$ **then**
  $left \leftarrow a_1.b_1 + a_2$
  $right \leftarrow a_2^2$
  print $left$ $right$

5 Extension for more numbers

The algorithm that we have studied is valid for two digit numbers only. The next question arises, can we further extend this algo for more than two digits?

The answer is YES. Analogous to the proof for two digit case, we can get the algorithm for a more than two digit case.But there will be certain restrictions.We would be able to divide the numbers into two smaller parts and then if the conditions apply , we could use that algorithm.For further details, see the bibliography.It will be clear with the following example: 125 * 175 = 21875

	1	25
x	1	75
	1.(1+1)	25*75
	2	1875

Thus, the answer is 21875.

All these ideas are not new, but they have their roots in atharvaveda.
This shows that even in acient period, when the vedas were composed, the people of that time were aware of such techniques.[1]

[1]Note that this algorithm is fast only for these two cases.For other cases,refer the bibliography

References

[1] W. V. Kandasamy and F. Smarandache, *Vedic Mathematics, A fuzzy and Neutrosophic Analysis.*