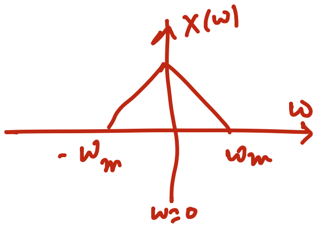
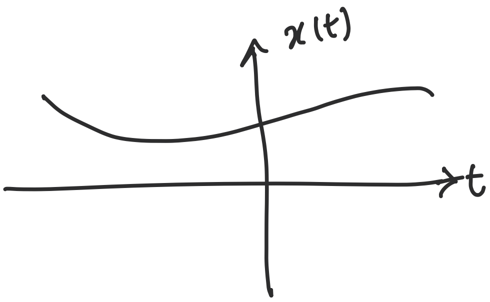
Sampling :-



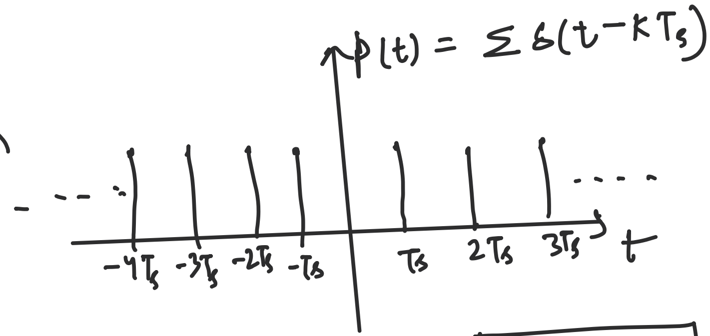
→ To find maximum frequency component.

Sampling Theorem :- (Instantaneous) (For low pass signal)

Any continous time signal band limited to maximum freq. component w_m, can be completely represented by (or recoverable from) knowledge of value of its samples, equally spaced in time, provided the sampling freq. must be greater than or equal to twice of max. freq. component of the signal which is to be sampled. i.e. $\boxed{w_s \geq 2w_m}$ or $\boxed{f_s \geq 2f_m}$



$\times$



Sampling freq.

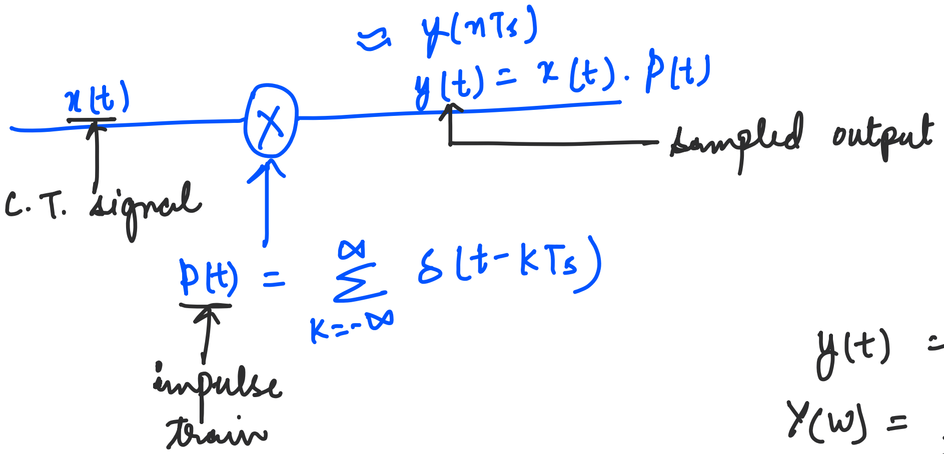
= No. of samples taken per second

$T_0 = T_s$ $\qquad$ $w_0 = \boxed{\frac{2\pi}{T_s} = w_s}$

$\boxed{f_s = \frac{1}{T_s}}$

$\left.\begin{array}{ll} w_m & r/s \\ f_m & Hz \end{array}\right\} \rightarrow$ Maximum freq of $x(t)$ to be sampled.

→ If $w_s = 2w_m$ or $f_s = 2f_m$ then sampling rate is called as Nyquist Sampling rate.



$$y(t) = x(t) \cdot p(t)$$

$$Y(\omega) = \frac{1}{2\pi}\left[X(\omega) * P(\omega)\right]$$

We know,

$$X(\omega) = 2\pi \sum_{k=-\infty}^{\infty} a_k\, \delta(\omega - k\omega_0)$$

For impulse train,

$$a_k = \frac{1}{T_s}$$

$$\therefore\ P(\omega) = \frac{2\pi}{T_s} \sum_{k=-\infty}^{\infty} \delta(\omega - k\underbrace{\omega_s}_{\text{sampling freq.}})$$

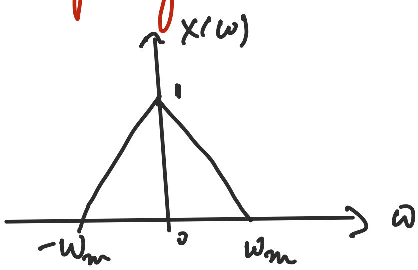
$$Y(\omega) = \frac{1}{2\pi}\left[X(\omega) * \frac{2\pi}{T_s} \sum_{k=-\infty}^{\infty} \delta(\omega - k\omega_s)\right]$$

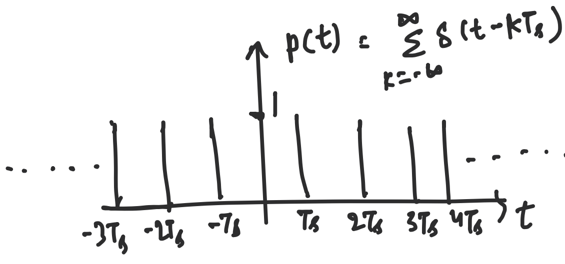
$$= \frac{1}{T_s} \sum_{k=-\infty}^{\infty} X(\omega) * \delta(\omega - k\omega_s)$$

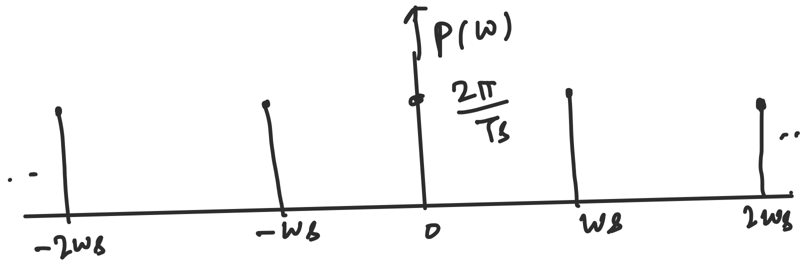
$$\boxed{Y(\omega) = \frac{1}{T_s} \sum_{k=-\infty}^{\infty} X(\omega - k\omega_s)}$$

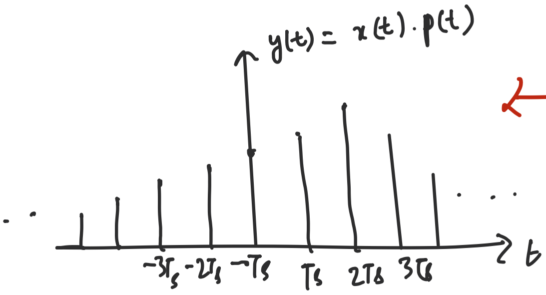
Time domain

Frequency domain







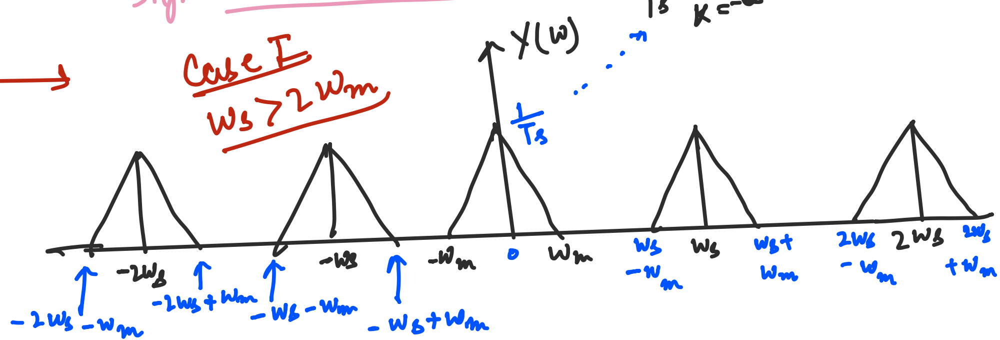


Signal is recoverable

$$= \frac{1}{T_s} \sum_{k=-\infty}^{\infty} X(\omega - k\omega_s)$$

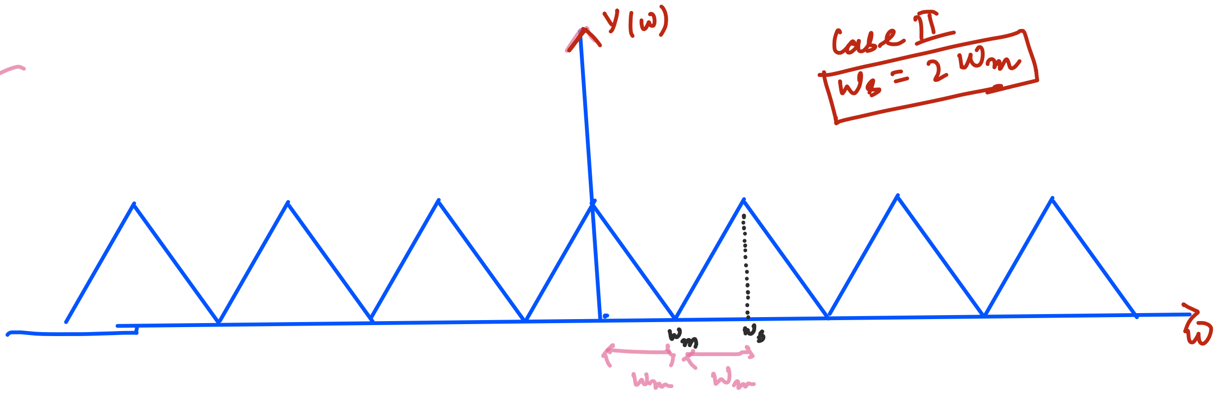
Case I

$\omega_s > 2\omega_m$



This is Theoritical case. (Ideal case)

Case II $\boxed{W_s = 2W_m}$



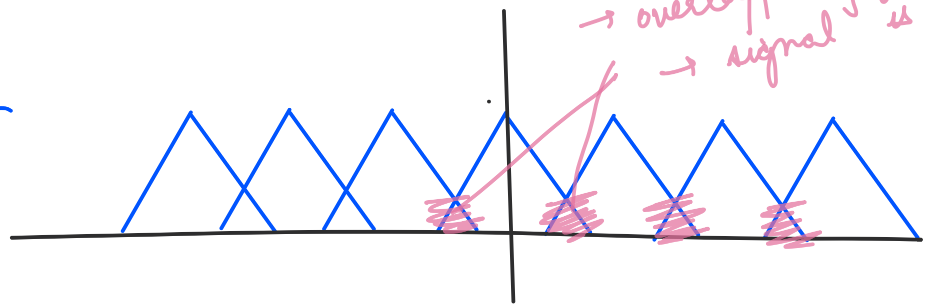
→ overlapping
→ signal is lost

Case III

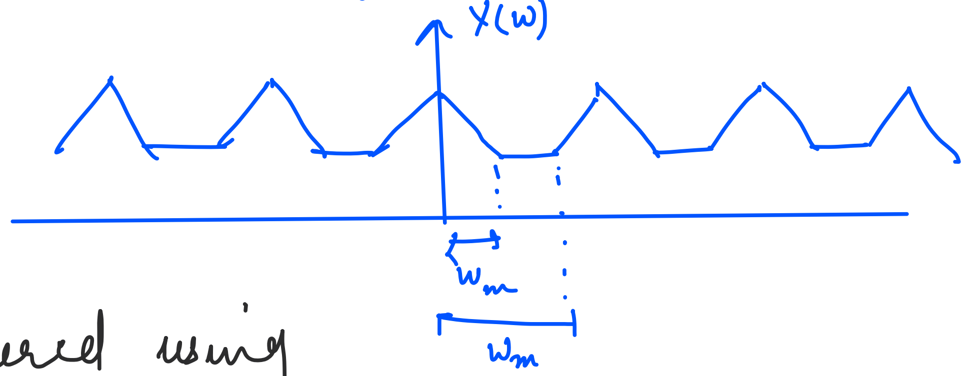
$W_s < 2W_m$

Signal is distorted
∴ cannot be recovered.

Aliasing effect

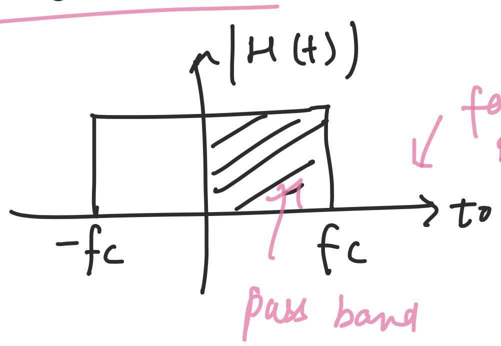


⇓

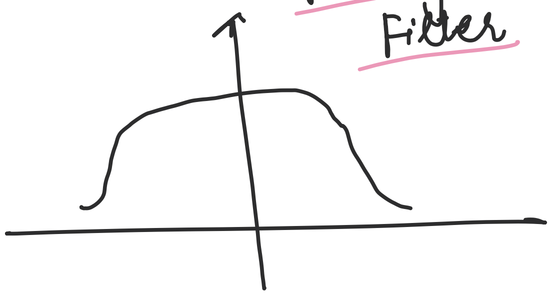


Signal can recovered using Filters :

Ideal Filter



Reality Filter



Notes :- ① Nyquist Sampling Rate $f_s = 2f_m$. $\omega_s = 2\omega_m$

→ Sampling freq, $f_s = 3f_m$, then 3 samples of $x(t)$ are taken in one sampling interval.

eg If $x(t)$ is a sinosuidal signal of 200 Hz & it is sampled at a rate of 480 Hz. Then the total no. of periods of $x(t)$ required to get one sampled period -:

$$\frac{f_s}{f_m} = \frac{480}{200} = 2.4$$

In one period, no. of samples = 2.4

2	"	"	= 4.8
⋮			
5	"	"	= 2.4 × 5 = 12

eg A signal is sampled at 4 times Nyquist rate for 1sec and a total of 800 samples are aquired. Find f_m & f_s.

Given
$f_s = 800$ Hz

Nyquist rate = $2f_m$

$f_s = 4 \times 2f_m = 8f_m$

$f_s = 8f_m = 800$ Hz

$\boxed{f_m = 100 \text{ Hz}}$

eg $x(t) = \cos 2\pi f_0 t$ is sampled at 3 times it's maximum freq for 6 periods. Total no. of samples = ?

$f_s = 3f_m$ ⟶ In 1 period ⟹ 3 samples

6 " ⟹ 3×6 = 18 samples

eg A signal $x(t) = \cos 2\pi f_0 t$ is sampled at a rate of 5 samples / 3 periods. Find f_s.

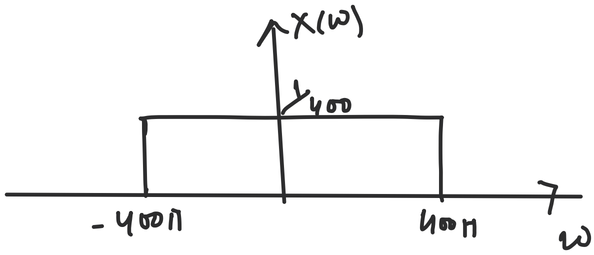
$$3 \text{ periods} \Rightarrow 5$$

$$\therefore \; 1 \text{ period} \Rightarrow \frac{5}{3}$$

$$\boxed{\therefore \; f_s = \frac{5}{3} f_m}$$

eg Find Nyq. sampling rate for sinc (400 t) :-

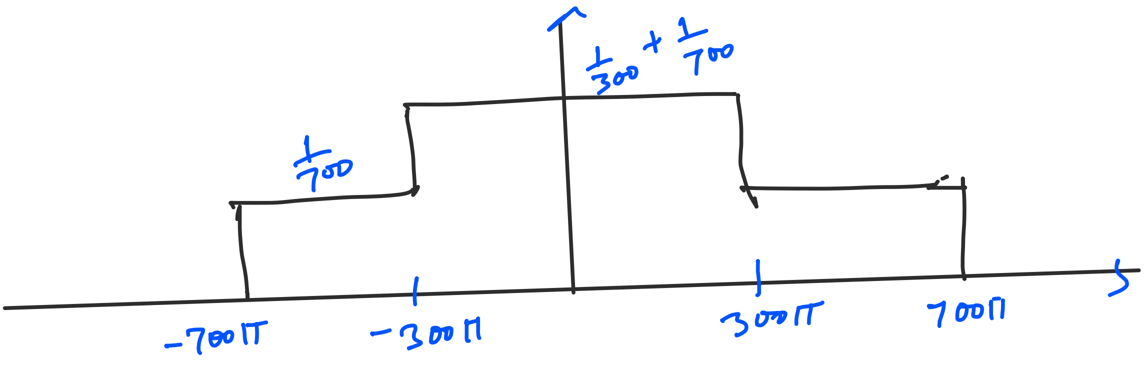
$$\frac{1}{400}\left[\frac{\sin 400\pi t}{\pi t}\right] \longleftrightarrow$$



$$\omega_m = 400\pi \ \frac{rad}{s} \qquad f_s = 200 \ Hz$$

$$\therefore f_s = 2 \cdot f_m = 2 \times 200 \ Hz = 400 \ Hz$$

eg $x(t) = \text{Sinc}(300t) + \text{Sinc}(700t)$:-

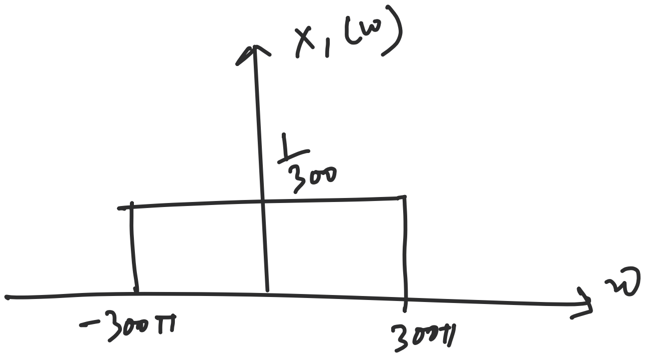


$$\omega_m = 700\pi \qquad f_m = 350 \ Hz \qquad \therefore \; f_s = 700 \ Hz$$

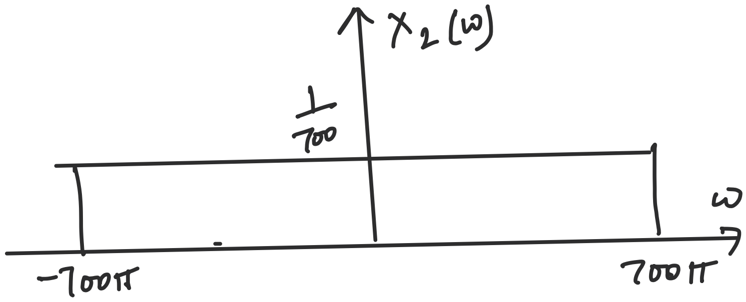
eg $x(t) = \underbrace{\text{Sinc}(300t)}_{x_1(t)} * \underbrace{\text{Sinc}(700t)}_{x_2(t)}$

$$X(\omega) = X_1(\omega) \cdot X_2(\omega)$$

$$\frac{1}{300}\left[\frac{\sin 300\pi t}{\pi t}\right] \longleftrightarrow$$



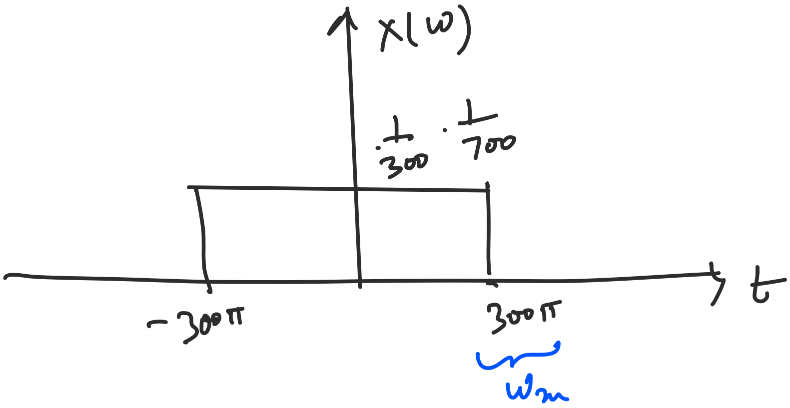
$$\frac{1}{700}\left[\frac{\sin 700\pi t}{\pi t}\right] \longleftrightarrow$$



$\omega_m = 300\pi$

$f_m = 150$ Hz

$f_s = 2 \times 150 = 300$ Hz

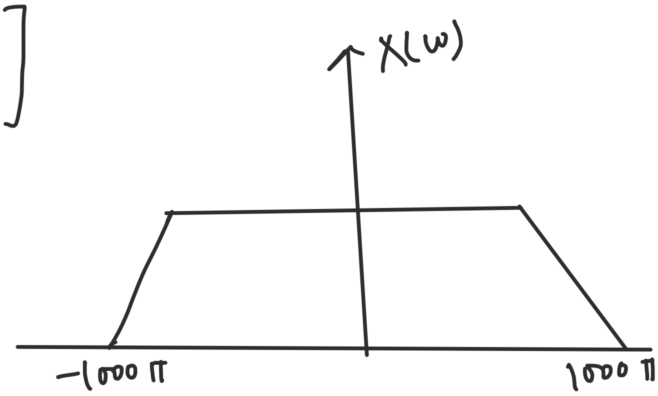


eg. $x(t) = \text{Sinc}(300t) \cdot \text{Sinc}(700t)$

$$X(\omega) = \frac{1}{2\pi}\left[X_1(\omega) * X_2(\omega)\right]$$

$W_m = 1000\pi, \quad f_m = 500$ Hz

$\boxed{\text{Nyq. } f_s = 1000 \text{ Hz}.}$



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3 cases to remember :-

$x_1(t) \longrightarrow \omega_{m_1}$

$x_2(t) \longrightarrow \omega_{m_2}$

Nyquist rate for

1. $x(t) = x_1(t) + x_2(t)$

   $f_s = 2 \times \text{Max}^m$ of $[\omega_{m_1} \ \& \ \omega_{m_2}]$

2. $x(t) = x_1(t) * x_2(t)$

   $f_s = 2 \times \text{Min}^m$ of $[\omega_{m_1} \ \& \ \omega_{m_2}]$

3. $x(t) = x_1(t) \cdot x_2(t)$

   $f_s = 2 \times [\omega_{m_1} + \omega_{m_2}]$

Note :- Frequency components present in sampled output :-

$$Y(\omega) = \frac{1}{T_s} \sum_{K=-\infty}^{\infty} X(\omega - K\omega_s)$$

$$= \frac{1}{T_s} \sum_{K=-\infty}^{\infty} X(f - Kf_s)$$

Frequency components :-

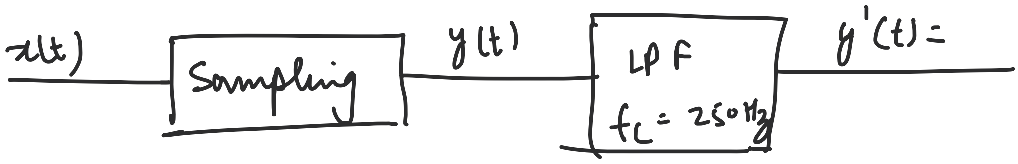
$$\pm f_m , \quad f_s \pm f_m , \quad 2f_s \pm f_m , \quad 3f_s \pm f_m , \cdots$$

For combination of signals :-

$$\pm f_{m_1} , \pm f_{m_2} , \quad f_s \pm f_{m_1} , \quad f_s \pm f_{m_2} , \quad 2f_s \pm f_{m_1} , \quad 2f_s \pm f_{m_2} \cdots$$

eg $x(t) = 5\cos 200\pi t + 10\cos 280\pi t$ is sampled at a rate of 300 samples/sec and the sampled output is passed through a LPF of cut off freq. 250 Hz. Then find freq. components at output of L.P.F.

$$\cos 200\pi t \xleftrightarrow{F.T.} \pi[\delta(\omega - 200\pi) + \delta(\omega + 200\pi)]$$



$f_s = 300$ Hz , $\omega_s = 600\pi$ Hz

$f_c = 250$ Hz , $\omega_c = 500\pi$

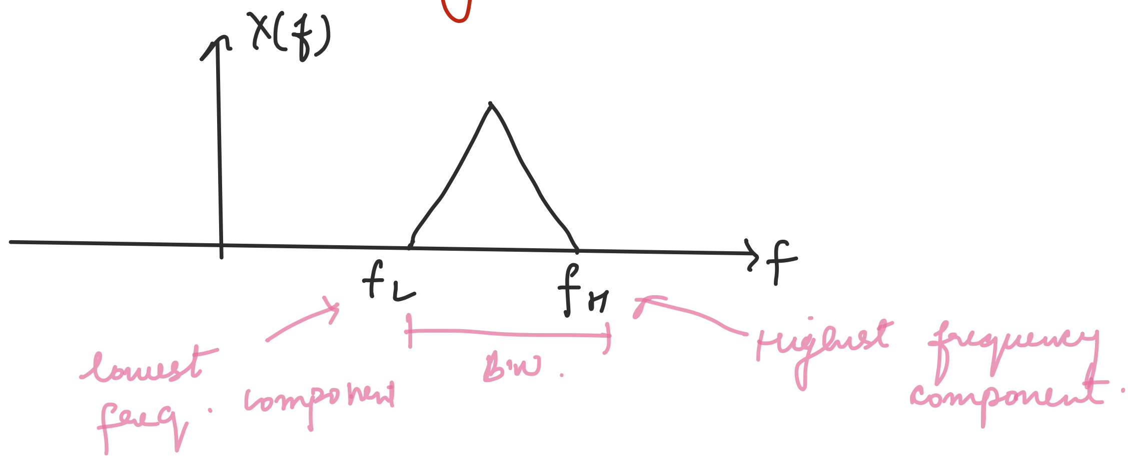
Using this

$$\boxed{\text{For combination of signals :-} \quad \pm f_{m_1} , \pm f_{m_2} , \quad f_s \pm f_{m_1} , \quad f_s \pm f_{m_2} , \quad 2f_s \pm f_{m_1} , \quad 2f_s \pm f_{m_2}}$$

± 100 Hz , ± 140 Hz , 300 ± 100 $= \cancel{200}, 400$, 300 ± 140 $= 440, 160$, 600 ± 100 $700, 500$,

These frequencies will pass through filters because less than 250.

Band Pass Sampling -:



→ used to reduce requirement of minimum sampling frequency.

→ $B.W. = F_H - F_L$

→ Required sampling freq. is given as

$$\boxed{\frac{2f_H}{K} \leq f_s \leq \frac{2f_L}{K-1}}$$

where $K = 1, 2, 3 \ldots N$

& $N = \text{Integer of } \left[\frac{f_H}{B.W.}\right]$

→ 2 conditions -:

① If f_L or f_H is integer multiple of 2 (B.W

i.e. f_L or $f_H = p\, 2(f_H - f_L)$

Then $\boxed{f_s = 2(f_H - f_L)}$