Fourier Transform :-

Gap between harmonic components : $w_0 = \frac{2\pi}{T}$

As T increases, w_0 i.e. gap b/w harmonic component decreases.

If $T \to \infty$, $w_0 \to 0$

& gap vanishes

& discrete variable 'kw_0' changes into continous variable 'w'

$$\boxed{\lim_{T \to \infty} kw_0 = w}$$

Fourier Transform $\xrightarrow{\text{Yes}}$ Aperiodic (Energy signals)

→ Power signals

→ Impulses.

→ periodic signals

Fourier Transform $\xrightarrow{\text{No}}$

→ neither Energy or Power signals

→ Unstable

→ Unbounded

$$\boxed{\begin{aligned} &\text{F T :} && X(w) = \int_{-\infty}^{\infty} x(t) \cdot e^{-jwt} \, dt \\ &\text{Inverse FT :} && x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(w) \, e^{jwt} \, dt \end{aligned}}$$

We know,

$$a_k = \frac{1}{T}\int_{-\infty}^{\infty} x(t)\, e^{-jk\omega_0 t}\, dt$$

$$x(t) = \sum_{k=-\infty}^{\infty} a_k\, e^{jk\omega_0 t}$$

→ To find F.T from F.S

$$X(\omega) = \lim_{T\to\infty} T.a_k$$

→ To find F.S. from F.T

$$a_k = \frac{1}{T} X(\omega)\Big|_{\omega = k\omega_0}$$

→ $x(t) \xleftrightarrow{F.T.} X(j\omega)$

$$X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt \quad \text{—— (1)}$$

$$x(t) = \frac{1}{2\pi}\int_{-}^{\infty} X(\omega)\, e^{j\omega t}\, d\omega \quad \text{—— (2)}$$

Note :- We calculate Fourier Transform in imaginary-axis.

From eq^n (1) :

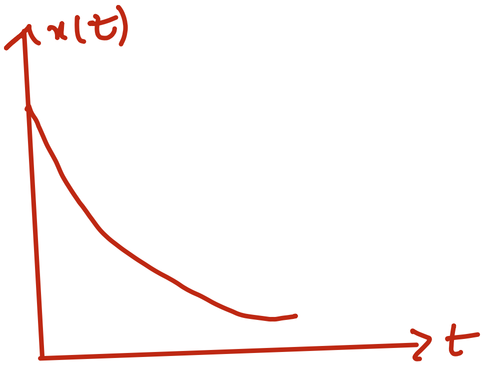
$$X(0) = \int_{-\infty}^{\infty} x(t)\, dt \implies \text{Area under } x(t)$$

From eq^n (2) :

$$x(0) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega)\, d\omega \implies 2\pi\, x(0) = \int_{-\infty}^{\infty} X(\omega)\, d\omega$$

$$\implies \text{Area under } X(\omega)$$

eg $x(t) = e^{-at} u(t)$



$$X(\omega) = \int_{-\infty}^{\infty} e^{-at} u(t)\, e^{-j\omega t}\, dt$$

$$= \int_{0}^{\infty} e^{-at} e^{-j\omega t}\, dt = \int_{0}^{\infty} e^{-(a+j\omega)t}\, dt$$

$e^{0} = 1$

$$= \left[\frac{e^{-(a+j\omega)t}}{-(a+j\omega)} \right]_0^{\infty} = \frac{0-1}{-(a+j\omega)} = \frac{1}{a+j\omega}$$

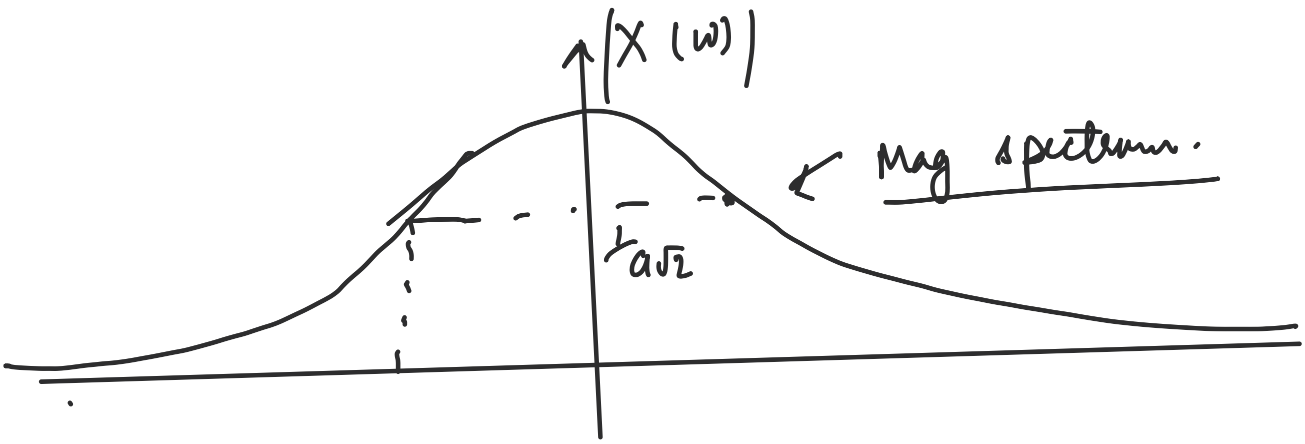
$$e^{-at} u(t) \xleftrightarrow{F.T.} \frac{1}{a+j\omega}$$

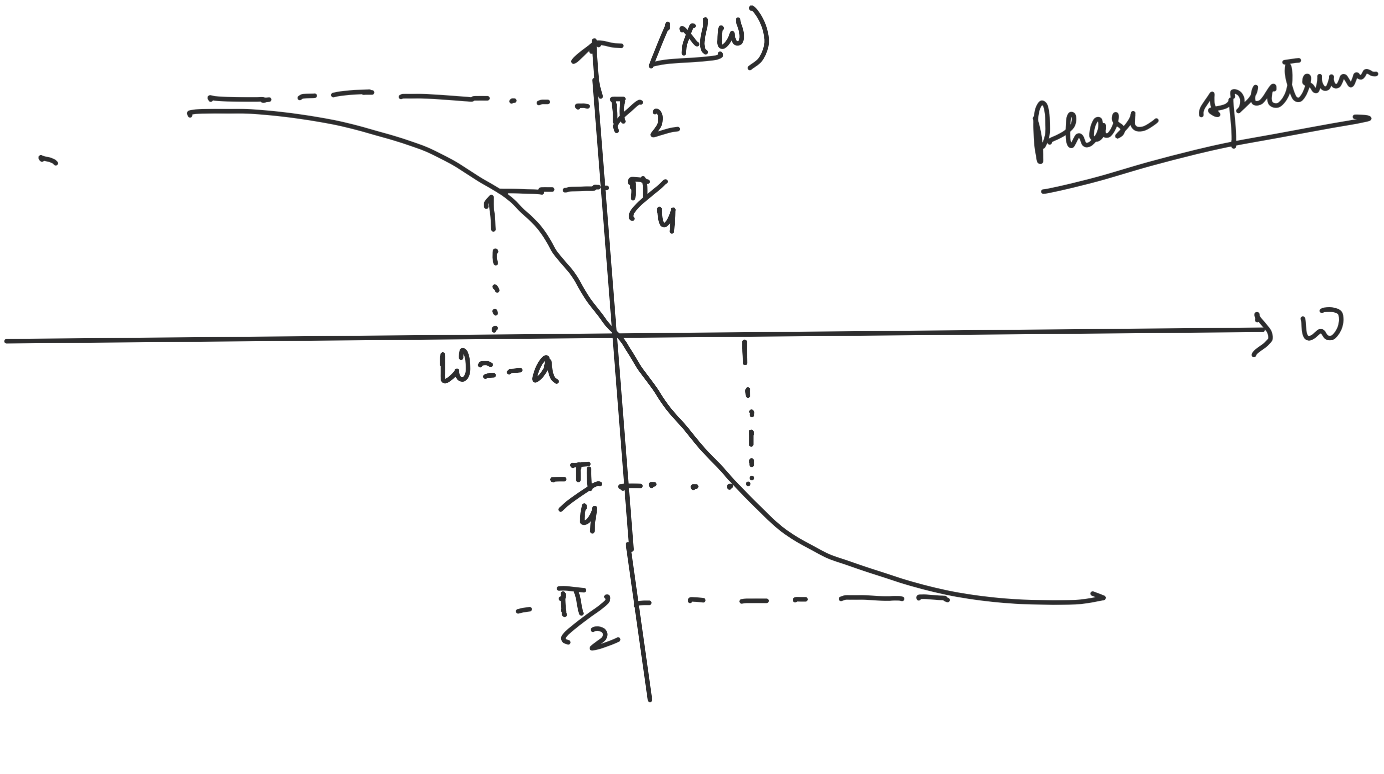
$$X(\omega) = \frac{1}{a+j\omega}$$

Mag $\quad |X(\omega)| = \dfrac{1}{\sqrt{a^2+\omega^2}}$

Phase $\quad \angle X(\omega) = 0 - \tan^{-1}\left(\dfrac{\omega}{a}\right)$

ω	$\lvert X(\omega) \rvert$	$\angle X(\omega) = -\tan^{-1}\left(\frac{\omega}{a}\right)$
0	$\frac{1}{a}$	0°
$-a$	$\frac{1}{a\sqrt{2}}$	$+45^\circ$
$+a$	$\frac{1}{a\sqrt{2}}$	-45°
$\vdots$		
$-\infty$	0	$+90^\circ$
$+\infty$	0	-90°

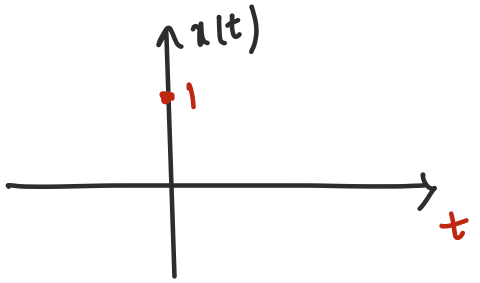


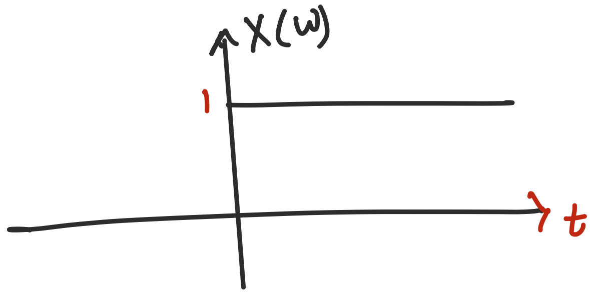


eg (2) $x(t) = \delta(t)$

$$X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt = \int_{-\infty}^{\infty} \delta(t)\, e^{-j\omega t}\, dt$$

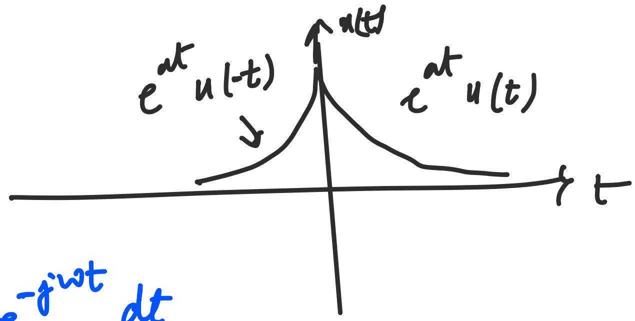
$$X(\omega) = e^{-j\omega t}\Big|_{t=0} = 1 \quad (\text{Constant})$$





Note :- Impulse in one domain will result in constant in other domain.

eg (3) $x(t) = e^{-a|t|} = e^{-at}u(t) + e^{at}u(-t)$



$$X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{0} \underbrace{e^{at} \cdot e^{-j\omega t}}_{e^{(a-j\omega)t}}\, dt + \int_{0}^{\infty} \underbrace{e^{-at} \cdot e^{-j\omega t}\, dt}_{\frac{1}{a+j\omega}}$$

$$= \left[\frac{e^{(a-j\omega)t}}{a-j\omega}\right]_{0}^{-\infty} + \frac{1}{a+j\omega}$$

$$\Rightarrow \quad X(\omega) = \frac{1}{a-j\omega} + \frac{1}{a+j\omega} = \frac{2a}{a^2-(j\omega)^2}$$

$$\boxed{\therefore \quad e^{-a|t|} \xleftrightarrow{F.T.} \frac{2a}{a^2+\omega^2}}$$ Ans

Results from ③ examples :-

$$e^{-4t} u(t) \xleftrightarrow{F.T} \frac{1}{4+j\omega}$$

$$4\,\delta(t) \xleftrightarrow{F.T} 4$$

$$e^{-2|t|} \xleftrightarrow{F.T} \frac{2\times 2}{4+\omega^2}$$

Note :-

$$X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

Applying conjugate on both sides

$$X^*(\omega) = \int_{-\infty}^{\infty} x^*(t)\, e^{j\omega t}\, dt$$

Replacing ω by $-\omega$

$$X^*(-\omega) = \int_{-\infty}^{\infty} x^*(t)\, e^{-j\omega t}\, dt$$

Results :

If $x(t) \longleftrightarrow X(\omega)$

$x^*(t) \longleftrightarrow X^*(-\omega)$

Properties of F.T. -:

① Linearity property

$$x_1(t) \longleftrightarrow X_1(\omega)$$

$$x_2(t) \longleftrightarrow X_2(\omega)$$

$$\alpha\, x_1(t) + \beta x_2(t) \longleftrightarrow \alpha X_1(\omega) + \beta X_2(\omega)$$

② Time Shifting Property -:

$$x(t) \xleftrightarrow{F.T.} X(\omega)$$

$$x(t-t_0) \xleftrightarrow{F.T.} e^{-j\omega t_0}\, X(\omega)$$

$$x(t+t_0) \xleftrightarrow{F.T} e^{j\omega t_0}\, X(\omega)$$

eg

$$\delta(t) \longleftrightarrow 1$$

$$\delta(t-1) \longleftrightarrow e^{-j\omega} \cdot 1$$

$$\delta(t+1) \longleftrightarrow e^{j\omega} \cdot 1$$

eg $x(t) = \frac{1}{2}\left[\delta(t-1) + \delta(t+1)\right]$

Find $X(\omega)$

$$X(\omega) = \frac{1}{2}\left[e^{-j\omega} + e^{j\omega}\right] = \cos\omega$$

③ Time Reversal Property -:

$$x(t) \longleftrightarrow X(\omega)$$

$$x(-t) \longleftrightarrow X(-\omega)$$

eg $x(t) \longleftrightarrow X(\omega)$

$x_1(t) = x(-t-4)$

find $X_1(\omega)$ in terms of $X(\omega)$

$$x(t) \longleftrightarrow X(\omega)$$

$$x(t-4) \longleftrightarrow e^{-j\omega 4} X(\omega)$$

$$x(-t-4) \longleftrightarrow \boxed{e^{j4\omega} X(-\omega) = X_1(\omega)}$$

4 Time Scaling Property :-

$$x(t) \longleftrightarrow X(\omega)$$

$$x(\alpha t) \longleftrightarrow \frac{1}{|\alpha|} X\left(\frac{\omega}{\alpha}\right)$$

eg $x(t) = e^{-at} u(t) \longleftrightarrow \dfrac{1}{a+j\omega}$

$x_1(t) = x(2t) = e^{-2at} u(2t)$

$= e^{-2at} u(t) \longleftrightarrow \dfrac{1}{2a+j\omega}$

$$x(2t) \longleftrightarrow \frac{1}{2}\left(\frac{1}{a+j\left(\frac{\omega}{2}\right)}\right)$$

$$= \frac{1}{2} \times \frac{2}{2a+j\omega} = \frac{1}{2a+j\omega}$$

(5) Duality Property -:

$$x(t) \longleftrightarrow X(\omega)$$

$$X(t) \longleftrightarrow 2\pi . x(-\omega)$$

eg- find F.T of $x(t) = A$

we know, $A\,\delta(t) \longleftrightarrow A$

$$A \longleftrightarrow 2\pi A\,\delta(-\omega)$$

$$\boxed{A \longleftrightarrow 2\pi A\,.\delta(\omega)}$$

eg find F.T. of $x(t) = e^{jt}$

we have, $\delta(t+1) \longleftrightarrow e^{j\omega}$

Applying duality property,

$$e^{jt} \longleftrightarrow 2\pi\ \delta(-\omega+1)$$

$$= 2\pi\ \delta(-(\omega-1))$$

$$\boxed{\therefore\ e^{jt} \longleftrightarrow 2\pi\ \delta(\omega-1)}$$

eg $x(t) = e^{-jt}$

we have, $\delta(t-1) \longleftrightarrow e^{-j\omega}$

$$\therefore\ e^{-jt} \longleftrightarrow 2\pi\ \delta(-\omega-1)$$

$$= 2\pi\ \delta\{-(\omega+1)\}$$

$$\boxed{e^{-jt} \longleftrightarrow 2\pi\ \delta(\omega+1)}$$

eg $$x(t) = \cos t$$
$$= \frac{e^{jt} + e^{-jt}}{2}$$

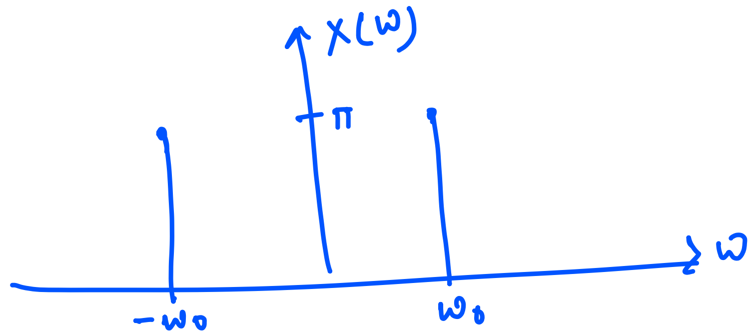
$$\therefore \cos t \longleftrightarrow \frac{1}{2}\left[2\pi\,\delta(\omega-1) + 2\pi\,\delta(\omega+1)\right]$$

Imp Remember!

$$\boxed{\cos t \longleftrightarrow \pi\left[\delta(\omega-1) + \delta(\omega+1)\right]}$$

generalise

$$\boxed{\cos \omega_0 t \longleftrightarrow \pi\left[\delta(\omega-\omega_0) + \delta(\omega+\omega_0)\right]}$$

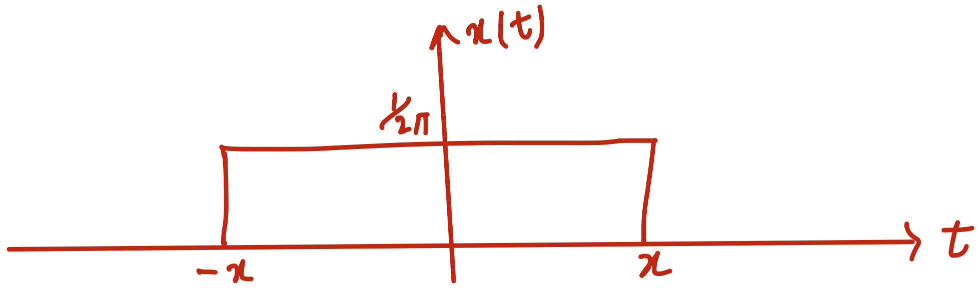


Note -: $$\sin t \longleftrightarrow \frac{\pi}{j}\left[\delta(\omega-1) - \delta(\omega+1)\right]$$

Imp!

$$\boxed{\sin \omega_0 t \longleftrightarrow \frac{\pi}{j}\left[\delta(\omega-\omega_0) - \delta(\omega+\omega_0)\right]}$$

eg Find F.T of following signal -:

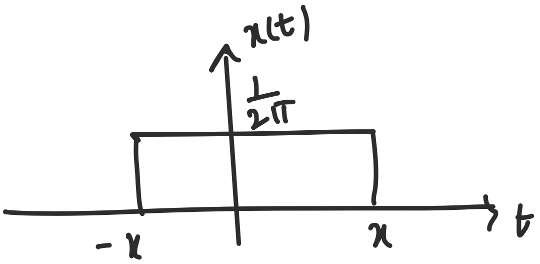


$$X(w) = \frac{1}{2\pi}\int_{-x}^{x} e^{-jwt}\,dt = \frac{1}{2\pi}\left[\frac{e^{-jwt}}{-jw}\right]_{-x}^{x}$$

$$= \frac{1}{2\pi}\left[\frac{e^{-jwx} - e^{jwx}}{-jw}\right] = \frac{1}{\pi w}\cdot\left[\frac{e^{jwx} - e^{-jwx}}{2j}\right]$$

$$= \frac{\sin xw}{\pi w}$$

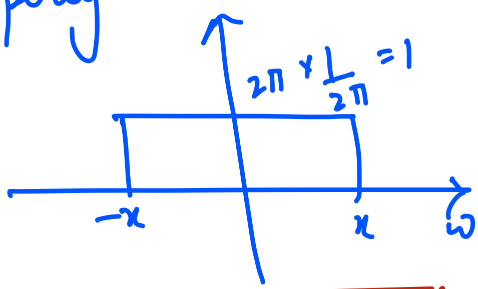
$\therefore$



$$\xleftrightarrow{F.T.} \quad \frac{\sin xw}{\pi w}$$

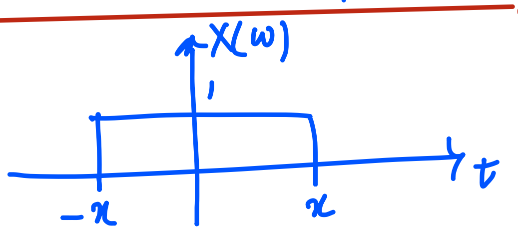
Applying duality property.

$$\frac{\sin xt}{\pi t} \longleftrightarrow$$



Imp Result

$$\frac{\sin xt}{\pi t} \longleftrightarrow$$



eg $\quad x(t) = \underbrace{\text{sinc}(400t)}_{x_1} + \underbrace{\text{sinc}(600t)}_{x_2}$

$$X(w) = X_1(w) + X_2(w)$$

$$x_1(t) = \text{sinc}(400t) = \frac{\sin(400\pi t)}{400\pi t} = \frac{1}{400}\left[\frac{\sin 400\pi t}{\pi t}\right]$$

$$x_2(t) = \text{sinc}(600t) = \frac{\sin 600\pi t}{600\pi t} = \frac{1}{600}\left[\frac{\sin 600\pi t}{\pi t}\right]$$

⑥ Frequency Shifting Property

$$x(t) \longleftrightarrow X(\omega)$$

$$e^{j\omega_0 t}\, x(t) \longleftrightarrow X(\omega-\omega_0)$$

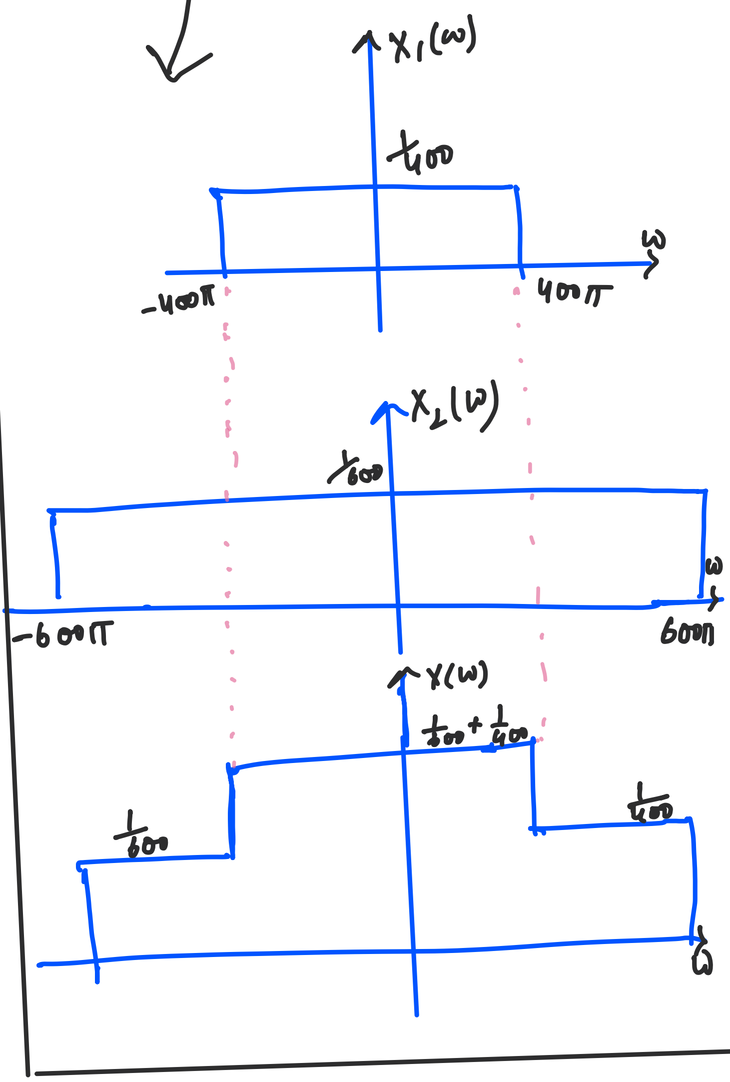
$$e^{-j\omega_0 t}\, x(t) \longleftrightarrow X(\omega+\omega_0)$$

eg $x(t) \longleftrightarrow \dfrac{5}{\omega} \sin \pi \omega$

$x_1(t) = e^{j6t}\, x(t)$

find $X_1(\omega)$

$$e^{j6t} \cdot x(t) \longleftrightarrow \frac{5}{\omega-6} \sin \pi(\omega-6)$$



⑦ Convolution Property :-

$$x(t) \longleftrightarrow X(\omega)$$

$$y(t) \longleftrightarrow Y(\omega)$$

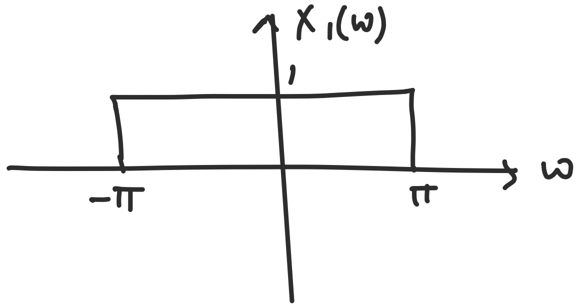
$$\boxed{x(t) * y(t) \longleftrightarrow X(\omega) \cdot Y(\omega)}$$

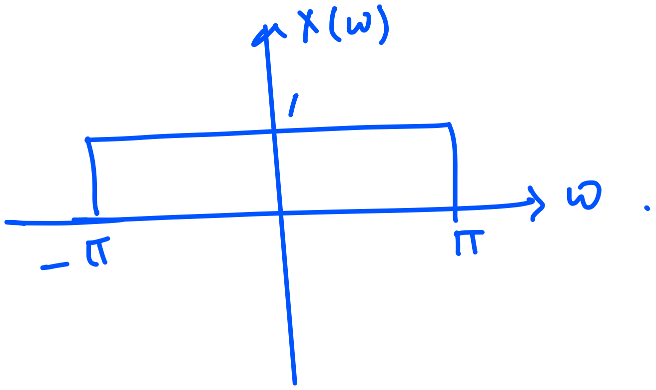
eg $x(t) = e^{-4t} u(t) * e^{-6t} u(t)$, find $X(\omega)$

$$X(\omega) = \frac{1}{4+j\omega} \cdot \left(\frac{1}{6+j\omega}\right)$$

eg $x(t) = \text{Sinc}(t) * \text{Sinc}(t)$, $X(\omega) = ?$

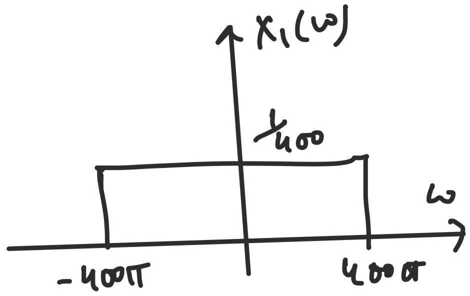
$$\text{Sinc}(t) = \frac{\sin \pi t}{\pi t} \longleftrightarrow$$



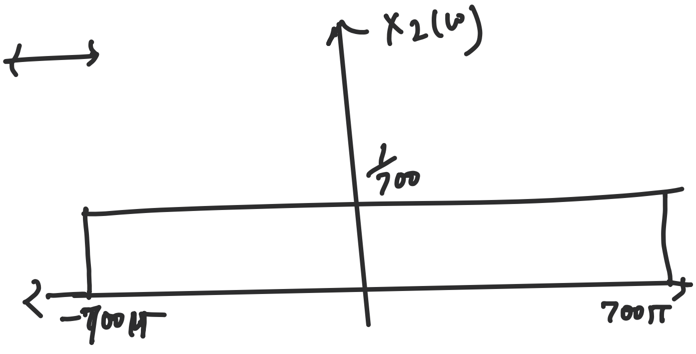


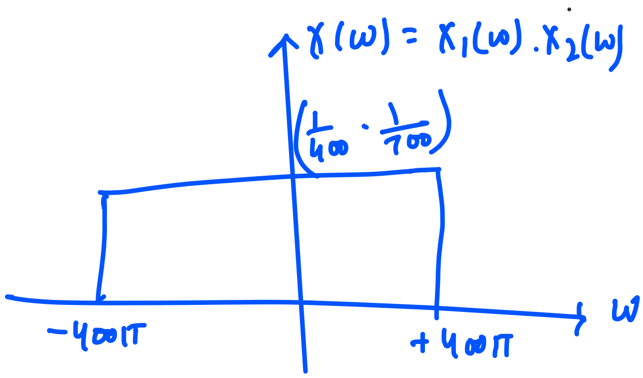
eg $$x(t) = \underbrace{\text{Sinc}(400t)}_{x_1(t)} + \underbrace{\text{Sinc}(700t)}_{x_2(t)}$$

$$x_1(t) = \frac{\sin 400\pi t}{400\pi t} = \frac{1}{400}\left[\frac{\sin 400\pi t}{\pi t}\right] \longleftrightarrow$$



$$x_2(t) = \frac{\sin 700\pi t}{700\pi t} = \frac{1}{700}\left[\frac{\sin 700\pi t}{\pi t}\right] \longleftrightarrow$$





Next sem

SOEN 321

ELEC 321

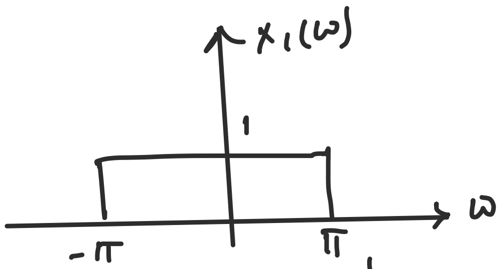
Semi conductor.

8. Multiplication Property :-

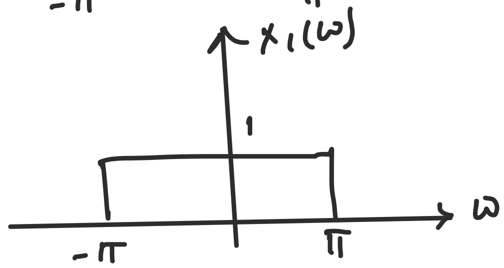
$$x_1(t) \longleftrightarrow X_1(\omega)$$
$$x_2(t) \longleftrightarrow X_2(\omega)$$
$$\boxed{x_1(t) \cdot x_2(t) \longleftrightarrow \frac{1}{2\pi}\left[X_1(\omega) * X_2(\omega)\right]}$$

eg $x(t) = \text{sinc}^2(t) = \underbrace{\text{sinc}(t)}_{x_1(t)} \cdot \underbrace{\text{sinc}(t)}_{x_2(t)}$

$$x_1(t) = \frac{\sin \pi t}{\pi t} \longleftrightarrow$$



$$x_2(t) = \frac{\sin \cdot \pi t}{\pi t} \longleftrightarrow$$



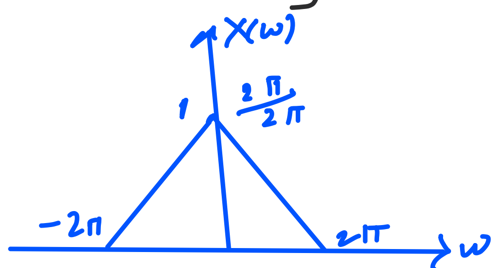
Note :-

$$u(t) * u(t) = t\,u(t) = r(t)$$
$$u(\omega) * u(\omega) = r(\omega)$$

$$X_1(\omega) = u(\omega+\pi) - u(\omega-\pi)$$
$$X_2(\omega) = u(\omega+\pi) - u(\omega-\pi)$$

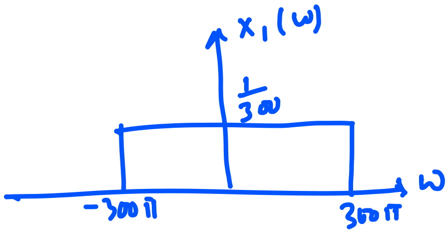
$$X(\omega) = \left[X_1(\omega) * X_2(\omega)\right] \times \frac{1}{2\pi}$$
$$= \left[r(\omega+2\pi) - r(\omega) - r(\omega) + r(\omega-2\pi)\right] \cdot \frac{1}{2\pi}$$

$$X(\omega) = \frac{1}{2\pi}\left[r(\omega+2\pi) - 2r(\omega) + r(\omega-2\pi)\right]$$

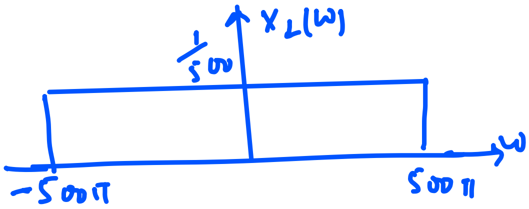


eg $x(t) = \underbrace{\text{Sinc}(300t)}_{x_1(t)} \cdot \text{Sinc}(500t)$

$$x_1(t) = \frac{1}{300}\left[\frac{\sin 300\pi t}{\pi t}\right] \longleftrightarrow$$



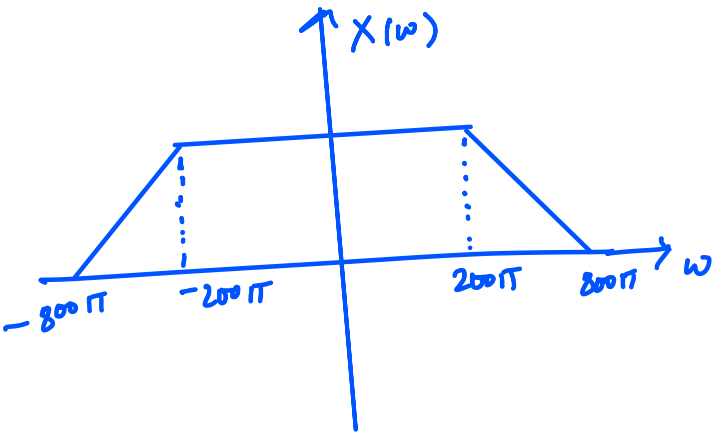
$$x_2(t) = \frac{1}{500}\left[\frac{\sin 500\pi t}{\pi t}\right] \longleftrightarrow$$



$$X_1(\omega) = \frac{1}{300}\left[u(\omega + 300\pi) - u(\omega - 300\pi)\right]$$

$$X_2(\omega) = \frac{1}{500}\left[u(\omega + 500\pi) - u(\omega - 500\pi)\right]$$

$$X(\omega) = \frac{1}{2\pi}\left[X_1(\omega) * X_2(\omega)\right]$$



(9) Differentiation in time domain :-

$$x(t) \longleftrightarrow X(\omega)$$

$$\frac{d}{dt}x(t) \longleftrightarrow j\omega\, X(\omega)$$

$$\frac{d^n}{dt^n}x(t) \longleftrightarrow (j\omega)^n X(\omega)$$

(10) Differentiation in frequency domain :-
(or multiplication by 't' property)

$$x(t) \longleftrightarrow X(\omega)$$

$$t \cdot x(t) \longleftrightarrow j \frac{d}{d\omega}[X(\omega)]$$

$$t^n x(t) \longleftrightarrow (j)^n \frac{d^n}{d\omega^n}[X(\omega)]$$

eg $x(t) = e^{-at} u(t) \longleftrightarrow \dfrac{1}{a+j\omega}$

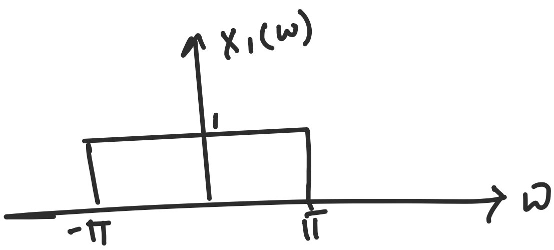
$$x_1(t) = t \cdot x(t) = t\, e^{-at} u(t)$$

$$X_1(\omega) = j \frac{d}{d\omega}\left[\frac{1}{a+j\omega}\right] = j\left[\frac{(a+j\omega)0 - 1(j)}{(a+j\omega)^2}\right]$$

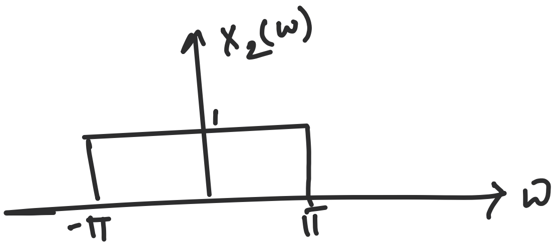
$$= \frac{-j \times j}{(a+j\omega)^2} = \frac{1}{(a+j\omega)^2}$$

eg $x(t) = t \cdot \underbrace{\left[\underbrace{\mathrm{sinc}(t)}_{x_1} * \underbrace{\mathrm{sinc}(t)}_{x_2}\right]}_{x_3}$

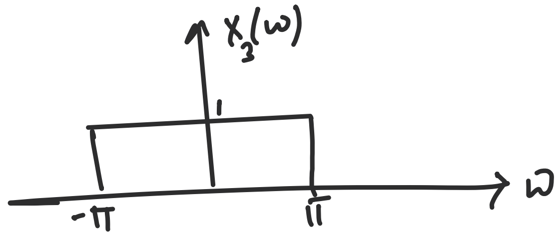
$x_1(t) = \dfrac{\sin \pi t}{\pi t} \longleftrightarrow$



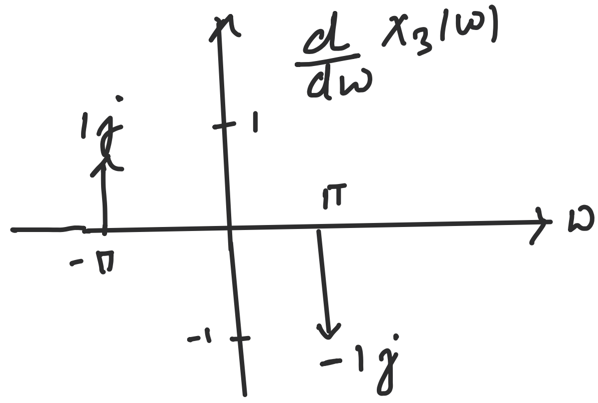
$x_2(t) = \dfrac{\sin \pi t}{\pi t} \longleftrightarrow$



$x_3(t) \longleftrightarrow$



$t \cdot x_3(t) \longleftrightarrow$



$$\Rightarrow j\,\delta(\omega+\pi) - j\,\delta(\omega-\pi)$$

(11) Integration Property :-

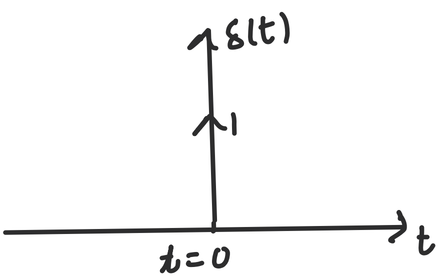
$$x(t) \longleftrightarrow X(\omega)$$

$$\int_{-\infty}^{t} x(\tau)\,d\tau \longleftrightarrow \frac{1}{j\omega}X(\omega) + \pi X(0)\,\delta(\omega)$$

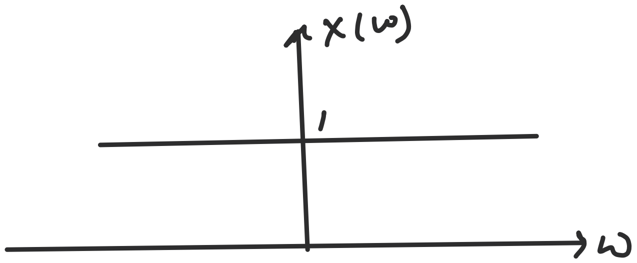
$\hookrightarrow X(\omega)\big|_{\omega=0}$

eg find F.T. of $x(t) = u(t)$:-

$$u(t) = \int_{-\infty}^{t} \delta(\tau)\,d\tau$$



$\longleftrightarrow$



We have

$$\delta(t) \longleftrightarrow 1$$

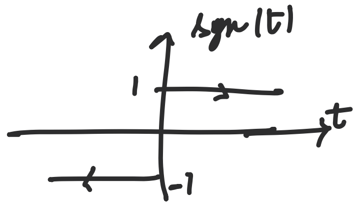
Applying integration property

$$u(t) = \int_{-\infty}^{t} \delta(\tau)\, d\tau \longleftrightarrow \frac{1}{j\omega} \times 1 + \pi \cdot 1 \cdot \delta(\omega)$$

Imp!

$$\therefore \boxed{u(t) \longleftrightarrow \frac{1}{j\omega} + \pi\,\delta(\omega)}$$

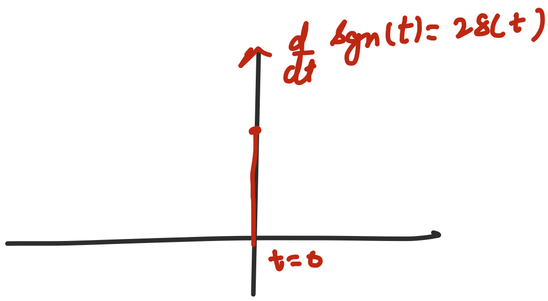
eg Find F.T. of $x(t) = \text{Sgn}(t)$?



Method ① using diff property :-

$$x(t) = \text{Sgn}(t)$$

$$\frac{d}{dt} x(t) = 2\delta(t)$$



Taking F.T. both sides

$$j\omega\, X(\omega) = 2 \cdot 1$$

$$\therefore \boxed{X(\omega) = \frac{2}{j\omega}}$$

Method ② We know

$$x(t) = \text{Sgn}(t) = 2u(t) - 1$$

We have, $u(t) = \frac{1}{j\omega} + \pi\,\delta(\omega)$

$$2u(t) = \frac{2}{j\omega} + 2\pi\,\delta(\omega)$$

$$1 \longleftrightarrow 2\pi\,\delta(\omega)$$

$$\therefore X(\omega) = \left[\frac{2}{j\omega} + 2\pi\,\delta(\omega)\right] - \left[2\pi\,\delta(\omega)\right]$$

$$\therefore \boxed{X(\omega) = \frac{2}{j\omega}}$$

Method ③ $x(t) = \text{Sgn}(t) = u(t) - u(-t)$

$$u(t) \longleftrightarrow \frac{1}{j\omega} + \pi\delta(\omega)$$

Using time reversal prop.

$$u(-t) \longleftrightarrow \frac{1}{j(-\omega)} + \pi\delta(-\omega) = \frac{-1}{j\omega} + \pi\delta(\omega)$$

$$X(\omega) = \left[\frac{1}{j\omega} + \pi\delta(\omega)\right] - \left[-\frac{1}{j\omega} + \pi\delta(\omega)\right]$$

$$\boxed{X(\omega) = \frac{2}{j\omega}}$$

eg $x(t) = \underbrace{\frac{2}{t}}_{x_1(t)} + \underbrace{\pi\delta(t)}_{x_2(t)}$, find $X(\omega)$

We have $\text{Sgn}(t) \longleftrightarrow \dfrac{2}{j\omega}$

$$j\,\text{sgn}(t) \longleftrightarrow \frac{2}{\omega}$$

Applying duality :-

$$\frac{2}{t} \longleftrightarrow 2\pi j\,\text{sgn}(-\omega)$$

$$\frac{2}{t} \longleftrightarrow -2\pi j\,\text{sgn}(\omega)$$

$$\delta(t) \longleftrightarrow 1 \cdot \pi$$

$\therefore\ X(\omega) = X_1(\omega) + X_2(\omega)$

$$\boxed{X(\omega) = -2\pi j\,\text{sgn}(\omega) + \pi}$$ Ans

Note : Hilbert Transform -:

$$\hat{x}(t) = \frac{1}{\pi t} * x(t)$$

we have $\quad \operatorname{sgn}(t) \longleftrightarrow \frac{2}{j\omega}$

$$\frac{j}{2}\operatorname{sgn}(t) \longleftrightarrow \frac{1}{\omega}$$

$$\frac{j}{2\pi}\operatorname{sgn}(t) \longleftrightarrow \frac{1}{\pi\omega}$$

Applying duality -:

$$\frac{1}{\pi t} \longleftrightarrow 2\pi \frac{j}{2\pi} \operatorname{sgn}(-\omega)$$

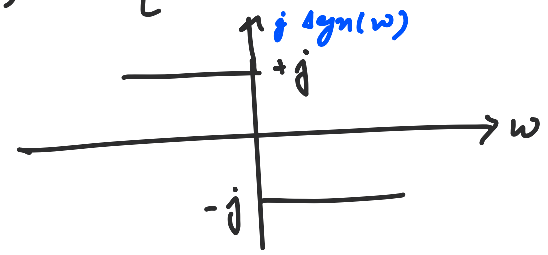
$$\therefore \frac{1}{\pi t} \longleftrightarrow -j \operatorname{sgn}(\omega)$$

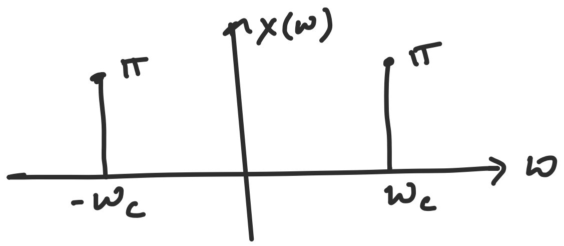
Imp

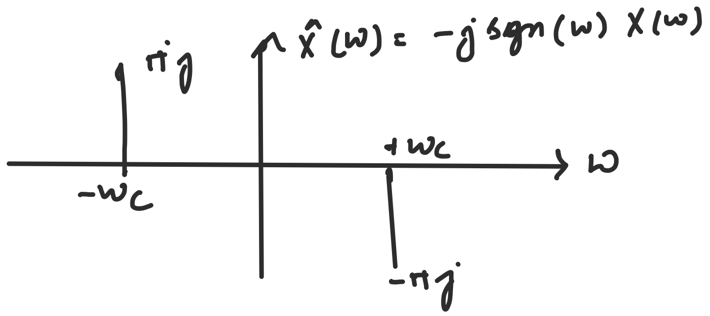
$$\boxed{\hat{X}(\omega) = -j \operatorname{Sgn}(\omega) \cdot X(\omega)}$$

eg find H.T. for $x(t) = \cos \omega_c t$

$$X(\omega) = \pi\left[\delta(\omega-\omega_c) + \delta(\omega+\omega_c)\right]$$







$$\hat{X}(\omega) = \pi j \left[\delta(\omega + \omega_c) - \delta(\omega - \omega_c) \right]$$

$$\hat{X}(\omega) = \frac{\pi}{j} \left[\delta(\omega + \omega_c) - \delta(\omega - \omega_c) \right]$$

$$\boxed{\therefore \; \hat{x}(t) = \sin \omega_c t}$$

12 Fourier transform for periodic signals -:

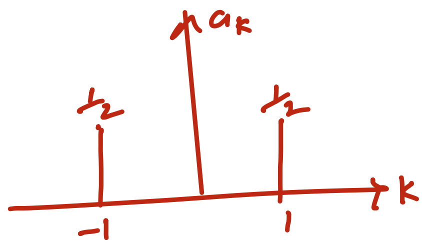
→ will always descrete in nature.

→ $X(\omega) = 2\pi \sum_{k=-\infty}^{\infty} a_k \, \delta(\omega - k\omega_0)$

fundamental freq.

eg $x(t) = \cos \omega_0 t$

$a_{-1} = \frac{1}{2} \qquad a_1 = \frac{1}{2}$



$$\therefore X(\omega) = 2\pi \left[\frac{1}{2} \delta(\omega - \omega_0) + \frac{1}{2} \delta(\omega + \omega_0) \right]$$

$$\boxed{X(\omega) = \pi \left[\delta(\omega - \omega_0) + \delta(\omega + \omega_0) \right]}$$

eg $x(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT_0)$

$a_k = \frac{1}{T_0} \quad \forall \; k$

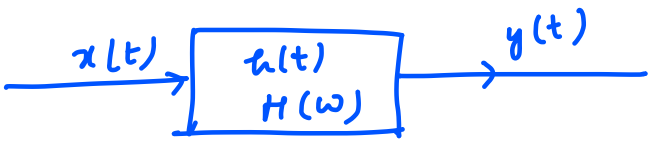
$$X(\omega) = 2\pi \frac{1}{T_0} \sum_{k=-\infty}^{\infty} \delta(\omega - k\omega_0)$$

13 Parsevel's theorem for Energy Signal -:

$$E_x = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(\omega)|^2 \, d\omega$$

$$\Rightarrow \boxed{\int_{-\infty}^{\infty} |x(t)|^2 \, dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(\omega)|^2 \, d\omega}$$

Application of F.T. in LTI systems -:



$$y(t) = x(t) * h(t)$$

$$Y(\omega) = X(\omega) \cdot H(\omega)$$

$$\therefore \boxed{H(\omega) = \frac{Y(\omega)}{X(\omega)}}$$

eg A C.T. LTI system is represented by I/P - O/P relation given as

$$\frac{d^2 y(t)}{dt^2} + 4\frac{dy(t)}{dt} + 5y(t) = 2\frac{dx(t)}{dt} + 3x(t)$$

Find system func^n?

We know,

$$y(t) \longrightarrow Y(\omega)$$

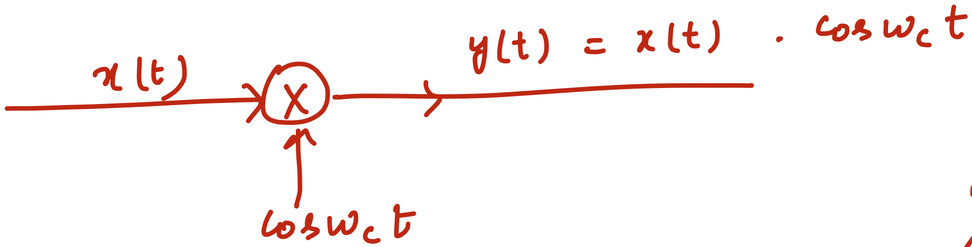
$$\frac{dy(t)}{dt} \longleftrightarrow j\omega Y(\omega)$$

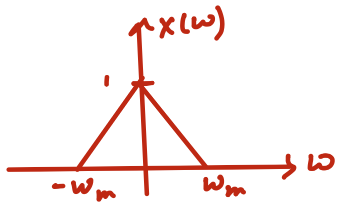
$$(j\omega)^2 Y(\omega) + 4(j\omega) Y(\omega) + 5Y(\omega) = 2j\omega X(\omega) + 3X(\omega)$$

$$\Rightarrow Y(\omega)\left[-\omega^2 + 4j\omega + 5\right] = X(\omega)\left[2j\omega + 3\right]$$

$$\therefore H(\omega) = \frac{Y(\omega)}{X(\omega)} = \frac{2j\omega + 3}{-\omega^2 + 4j\omega + 5}$$

eg





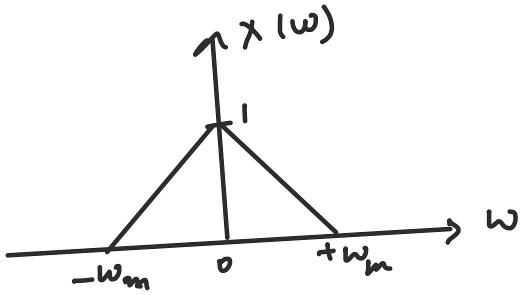
$$\boxed{w_m <<< w_c}$$

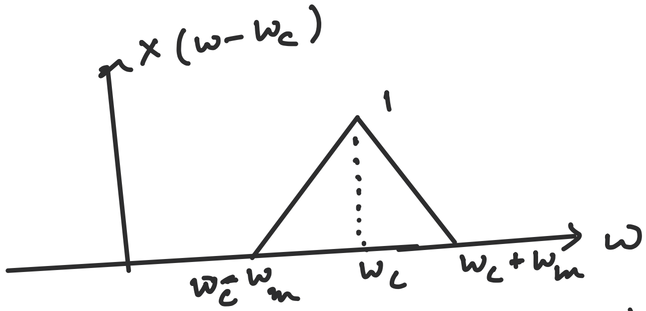
$$y(t) = x(t) \cdot \cos \omega_c t$$

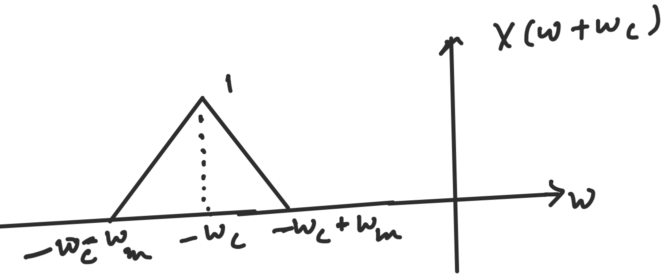
$$Y(\omega) = \frac{1}{2\pi}\left[X(\omega) * \pi\{\delta(\omega-\omega_c) + \delta(\omega+\omega_c)\}\right]$$

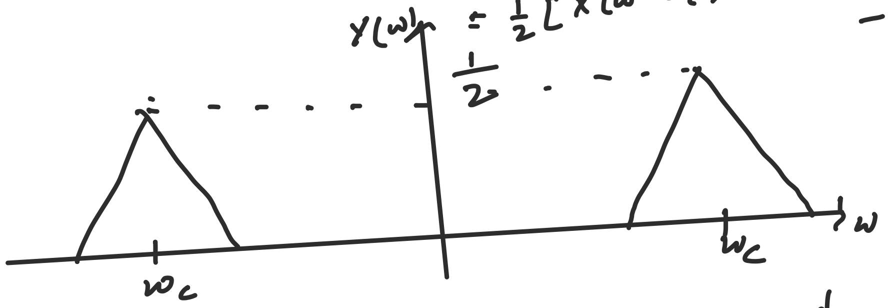
$$= \frac{1}{2}\left[\{X(\omega) * \delta(\omega-\omega_c)\} + \{X(\omega) * \delta(\omega+\omega_c)\}\right]$$

$$\boxed{Y(\omega) = \frac{1}{2}\left[X(\omega-\omega_c) + X(\omega+\omega_c)\right]}$$



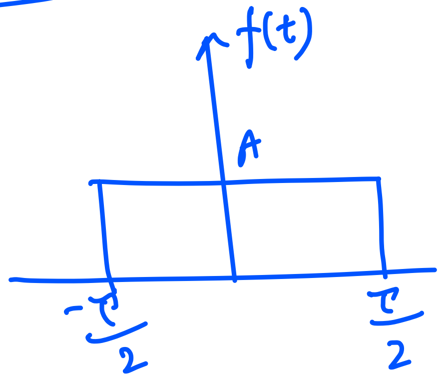






double side width with suppressed carrier.

Examples ①



$\tau = \tau$
$T = \infty$
$A = A$

$$f(t) = \begin{cases} A & -\frac{\tau}{2} < t < \frac{\tau}{2} \\ 0 & |t| > \frac{\tau}{2} \end{cases}$$

$$F(\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-i\omega t}\, dt$$

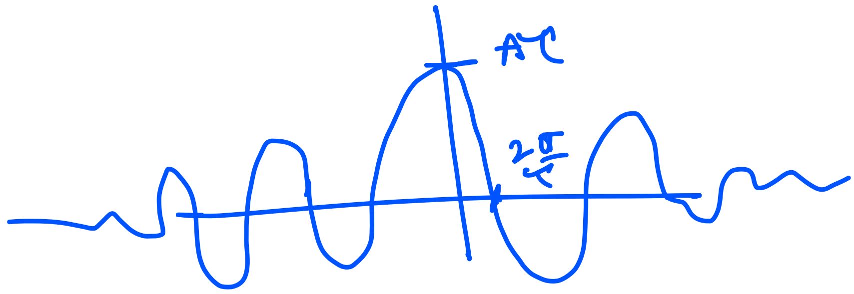
$$= \int_{-\tau/2}^{\tau/2} A\, e^{-i\omega t}\, dt = -\frac{A}{i\omega}\left[e^{-i\omega t}\right]_{-\tau/2}^{\tau/2}$$

$$= -\frac{A}{i\omega}\left[e^{-i\omega\frac{\tau}{2}} - e^{+i\omega\frac{\tau}{2}}\right]$$

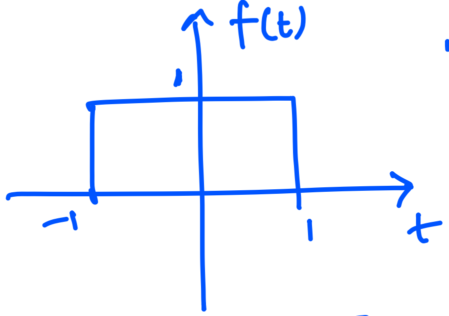
$$= \frac{A}{i\omega}\left[\frac{e^{i\omega\frac{\tau}{2}} - e^{-i\omega\tau/2}}{2i}\right] 2i = \frac{2A}{\omega}\sin\left(\omega\frac{\tau}{2}\right)$$

$$= \frac{\cancel{2}A\left(\frac{\tau}{\cancel{2}}\right)}{\omega\left(\frac{\tau}{2}\right)}\sin\omega\frac{\tau}{2} = A\tau\,\frac{\sin\omega\frac{\tau}{2}}{\omega\frac{\tau}{2}}$$

$$= A\tau\,\mathrm{sinc}\,\omega\frac{\tau}{2}$$



Ex 2



$\tau = 2$

$T = \infty$

$A = 1$

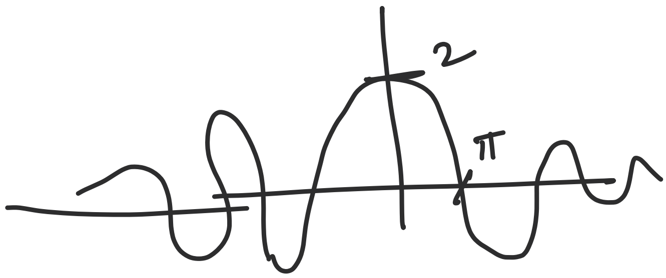
$$f(t) = \begin{cases} 1 & -1 < t < 1 \\ 0 & |t| > 1 \end{cases}$$

$$F(\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-i\omega t}\, dt$$

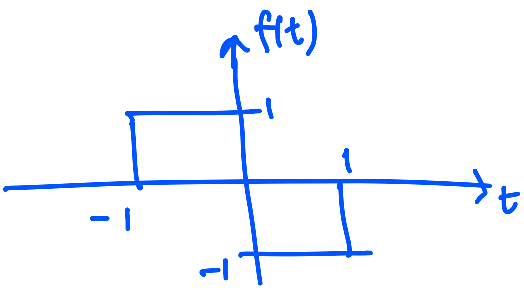
$$= \int_{-1}^{1} 1\, e^{-i\omega t}\, dt = -\left[\frac{e^{-i\omega t}}{i\omega}\right]_{-1}^{1}$$

$$= -\frac{1}{i\omega}\left[e^{-i\omega} - e^{i\omega}\right] = \frac{2}{\omega} \sin\omega = 2\,\frac{\sin\omega}{\omega}$$

$\omega =$



Ex 3



$F(\omega) = ?$

$$f(t) = \begin{cases} 1 & -1 < t < 0 \\ -1 & 0 < t < 1 \\ 0 & |t| > 1 \end{cases}$$

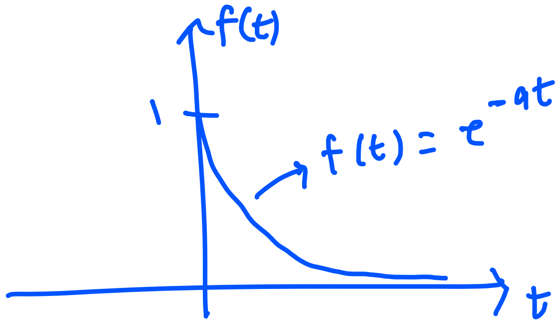
$$F(\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-i\omega t}\, dt = \int_{-1}^{0} (1)\, e^{-i\omega t} + \int_{0}^{1} (-1)\, e^{-i\omega t}\, dt$$

$$= -\frac{1}{i\omega}\left[e^{-i\omega t}\right]_{-1}^{0} + \frac{1}{i\omega}\left[e^{-i\omega t}\right]_{0}^{1}$$

$$= \frac{1}{i\omega}\left[-1 + e^{i\omega}\right] + \frac{1}{i\omega}\left[e^{-i\omega} - 1\right]$$

$$= \frac{2}{i\omega}\left[\cos\omega - 1\right]$$

Ex 4



$$f(t) = \begin{cases} e^{-at} & t>0 \\ 0 & t<0 \end{cases}$$

$$F(\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-i\omega t}\, dt$$

$$= \int_{0}^{\infty} e^{-at}\, e^{-i\omega t}\, dt = \int_{0}^{\infty} e^{-(a+i\omega)t}\, dt$$

$$= \frac{-1}{a+i\omega}\left[e^{-(a+i\omega)}\right]_0^{\infty} = \frac{-1}{a+i\omega}\left[e^{-\infty} - e^{0}\right]$$

$$= \frac{-1}{a+i\omega}\left[0-1\right]$$

$$= \frac{1}{a+i\omega}$$

Ex 5

$F(\omega)$ of $\cos\omega_0 t$?

$$f(t) = \cos\omega_0 t = \frac{e^{i\omega_0 t} + e^{-i\omega_0 t}}{2}$$

$$F(\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-i\omega t}\, dt = \frac{1}{2}\left[\int_{-\infty}^{\infty} e^{i\omega_0 t} e^{-i\omega t}\, dt + \int_{-\infty}^{\infty} e^{-i\omega_0 t} e^{-i\omega t}\, dt\right]$$

$$= \frac{1}{2}\left[\int_{-\infty}^{\infty} e^{-i(\omega-\omega_0)t}\, dt + \int_{-\infty}^{\infty} e^{-i(\omega+\omega_0)t}\, dt\right]$$

We know

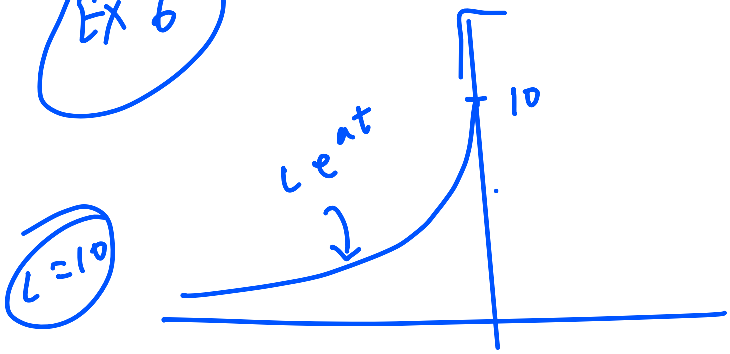
$$f(t) = e^{i\omega_0 t} \xrightarrow{\text{F.T.}} 2\pi\,\delta(\omega-\omega_0)$$

$$= \frac{1}{2}\left[2\pi\,\delta(\omega-\omega_0) + 2\pi\,\delta(\omega+\omega_0)\right]$$

$$\boxed{F(\omega) = \pi\,\delta(\omega-\omega_0) + \pi\,\delta(\omega+\omega_0)}$$

Ex 6

Find F.T. of $f(t) = \begin{cases} c e^{at} & t<0 \\ 0 & t>0 \end{cases}$



$$F(\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-i\omega t}\, dt$$

$$= \int_{-\infty}^{0} 10\, e^{at} e^{-i\omega t}\, dt$$

$$= 10 \int_{-\infty}^{0} e^{(a-i\omega)t}\, dt = \frac{10}{a-i\omega}\left[e^{(a-i\omega)t}\right]_{-\infty}^{0}$$

$$= \frac{10}{a-i\omega}\left[e^{0} - e^{-\infty}\right] = \frac{10}{a-i\omega}\left[1-0\right]$$

$$\boxed{F(\omega) = \frac{10}{a-i\omega}}$$

Ex 7 Ex for linearity property.

$f(t) = \sin w_0 t$. find $F(w)$.

$\mathcal{F}[a_1 f_1(t) + a_2 f_2(t)] = a_1 F_1(w) + a_2 F_2(w)$

$$f(t) = \sin w_0 t = \frac{1}{2i}[e^{iw_0 t} - e^{-iw_0 t}].$$

$$FT\left[\frac{1}{2i} e^{iw_0 t} - \frac{1}{2i} e^{-iw_0 t}\right] = \frac{1}{2j}\left[F(e^{iw_0 t}) - F(e^{-iw_0 t})\right]$$

$$= \frac{1}{2j}[2\pi\, \delta(w - w_0) - 2\pi\, \delta(w + w_0)]$$

$$= \frac{j\,2\pi}{2j\,j}[\delta(w - w_0) - \delta(w + w_0)]$$

$$= j\pi\,[\delta(w + w_0) - \delta(w - w_0)]$$

Ex 8 :- Example for time scaling property :-

Given that $\mathcal{F}[f(t)] = F(w) = \dfrac{10}{(2 + jw)(5 + jw)}$

Then $\mathcal{F}[f(-3t)] = ?$

Property :- $\mathcal{F}[f(at)] = \dfrac{1}{|a|} F\left(\dfrac{w}{a}\right)$

$$\mathcal{F}[f(-3t)] = \frac{1}{|-3|} F\left(\frac{w}{-3}\right) = \frac{1}{3} \frac{10}{2 + j\left(\frac{w}{-3}\right)\left(5 + j\left(\frac{w}{-3}\right)\right)}$$

$$= \frac{1}{3}\left[\frac{10}{\frac{1}{3}(6 - jw)\,\frac{1}{3}(15 - jw)}\right]$$

$$= \frac{30}{(6 - jw)(15 - jw)} \quad \text{Ans}$$

Ex 9 Example on time shifting -:

If $F(w) = \mathcal{F}[f(t)]$ then

$$\mathcal{F}[f(t-t_0)] = e^{-jwt}F(w)$$

Let $F(w) = f(t)$ where $F(w) = \dfrac{5}{(2+jw)(4+jw)}$

$F(w)$?

when $f(t-1)$

$$F(w) = \frac{e^{-jw(1)}\,5}{(2+jw)(4+jw)}$$

when $f(t-2)$

$$F(w) = \frac{e^{-jw2}\,5}{(2+jw)(4+jw)}$$

when $f(2t)$

$$F(w) = \frac{10}{(4+jw)(8+jw)}$$

when $f(2t-1)$

$$F(w) = \frac{e^{-jw\left(\frac{1}{2}\right)}(10)}{(4+jw)(8+jw)}$$

Ex 10

If $F(w) = \mathcal{F}[f(t)]$

then $\mathcal{F}[f(t-t_0)] = e^{-jwt_0}F(w)$

If $\mathcal{F}[e^{-5t}u(t)] = \dfrac{1}{5+jw}$

Then $\mathcal{F}[e^{-5(t-2)}u(t-2)] = ? \Rightarrow \dfrac{e^{-jw(2)}}{5+jw}$

And $\mathcal{F}[e^{-(t-2)}u(t-2)] = ? \Rightarrow \dfrac{e^{-jw(2)}}{1+jw}$