Introduction to signals -:

Signal -:

Dependent variable or function of one or more independent variable.

ac → signal

dc → not a signal [∵ current is constant]

$$\underbrace{F}_{\text{signal}}(\underbrace{x_1, x_2 \, \ldots \ldots \, x_n}_{\text{independent variable.}})$$

For this course ✓ (i) Single variable signal.

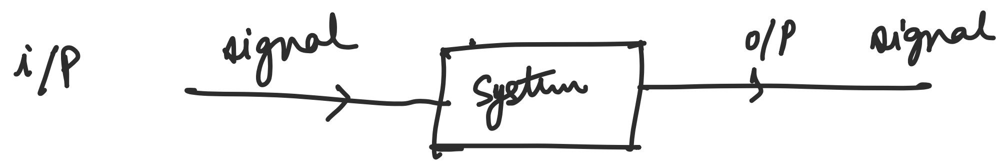
ex-: $F(x)$, $g(t)$

(ii) Multi variable signal.

ex -: $f_1(x_1, x_2)$, $g_1(t_1, t_2, t_3)$

System -:

The meaningful interconnection of physical devices and components is called as system.



System alone can't do anything, but must be linked to a signal!!

i/p signal $\longrightarrow$ $F(x_1, x_2)$

o/p signal $\longrightarrow$ $g(x_1, x_2)$

$\longrightarrow$ 2 types to problem to deal with -:

1) Analysis Problem

i/p signal ✓
system ✓
o/p signal ?

2) Synthesis Problem

i/p signal ✓
system ?
o/p signal ✓

1.1 Classification of signals

(i) continuous-time and discrete-time signals;
(ii) analog and digital signals;
(iii) periodic and aperiodic (or nonperiodic) signals;
(iv) energy and power signals;
(v) deterministic and probabilistic signals;
(vi) even and odd signals.

Continous and Discrete time signals -:

Continous time Signals	Discrete time signals
$\rightarrow$ Specified for every value of time (t)	Specified at discrete time intervals.

operations on signals :-

① Time Shifting

→ $x(t \pm t_0)$

→ no effect to amplitude

→ delay or advancement of signal
(+) (−)

② Time Scaling

→ $x(\alpha t)$

→ α is scaling factor

→ Expansion or compression of signal.
($\alpha > 1$) ($\alpha < 1$)

→ No effect on amplitude

→ Just divide the values on x axiss by α.

→ means you want to move time α X speed.
if twice, 2X speed.

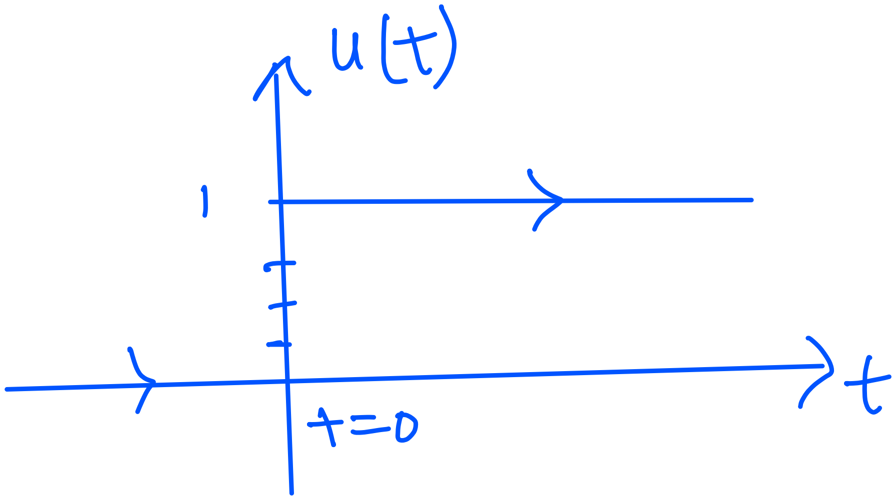
② Time reversal

→ $x(-t)$

→ divide x-axis by -1

Elementary Signal :-

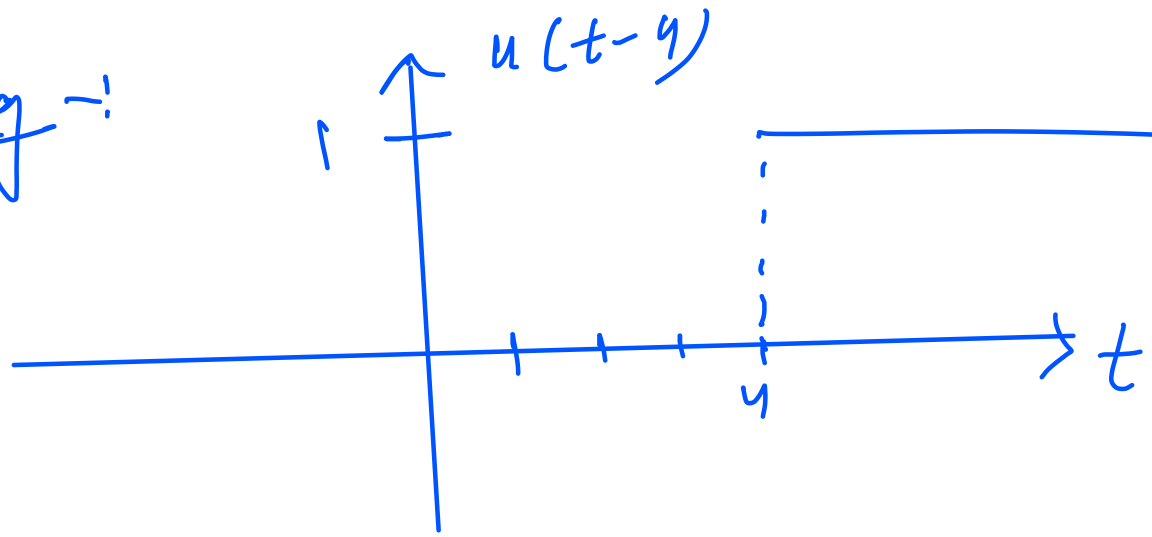
① Unit step Signal (Heaviside step func^n)



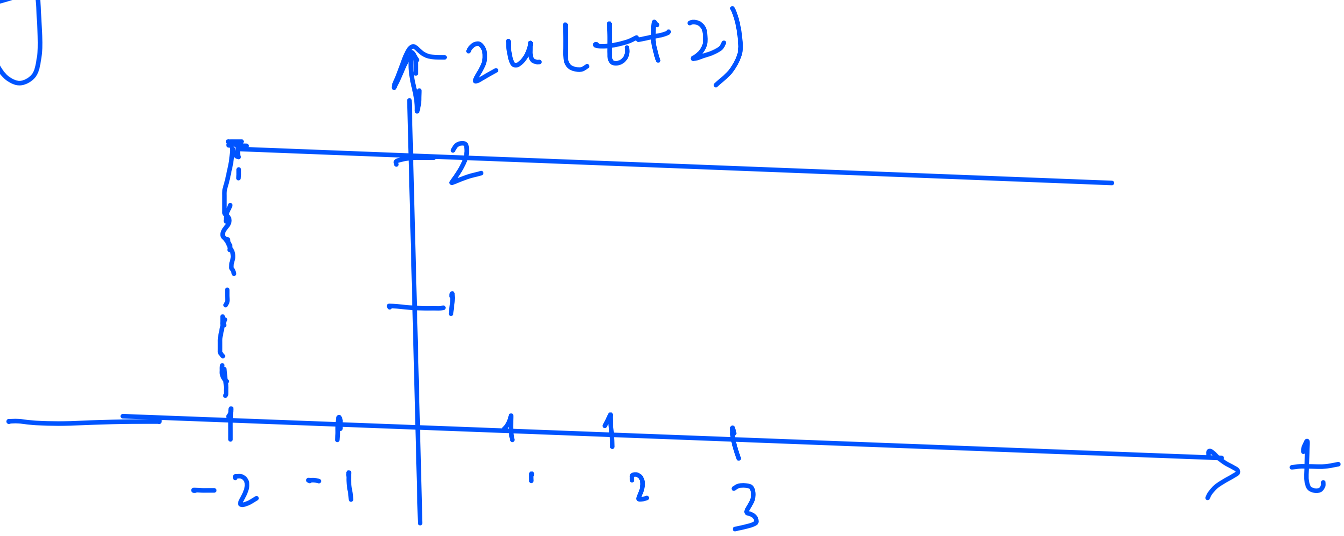
at $t=0$ value is not defined!

$$u(t) = 1 \quad t > 0$$
$$= 0 \quad t < 0$$

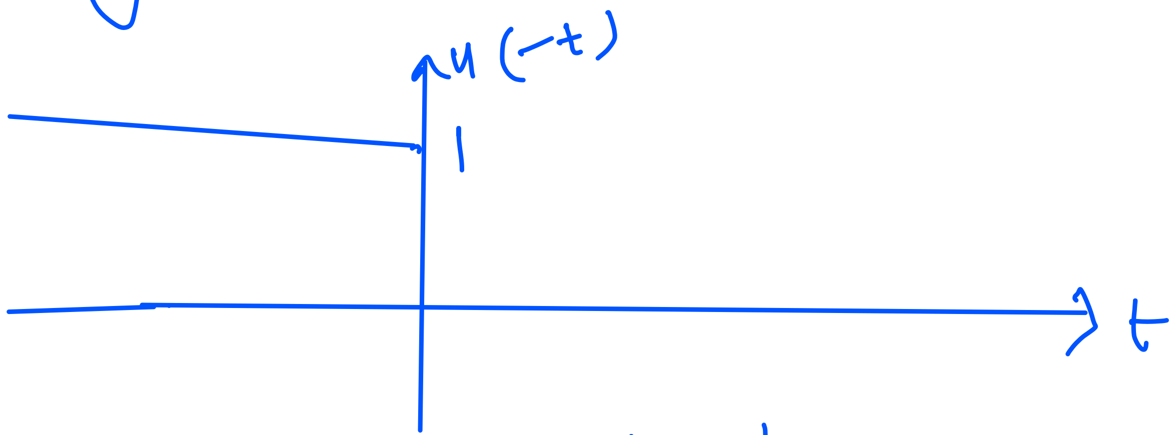
eg :-



eg $2u(t+2)$

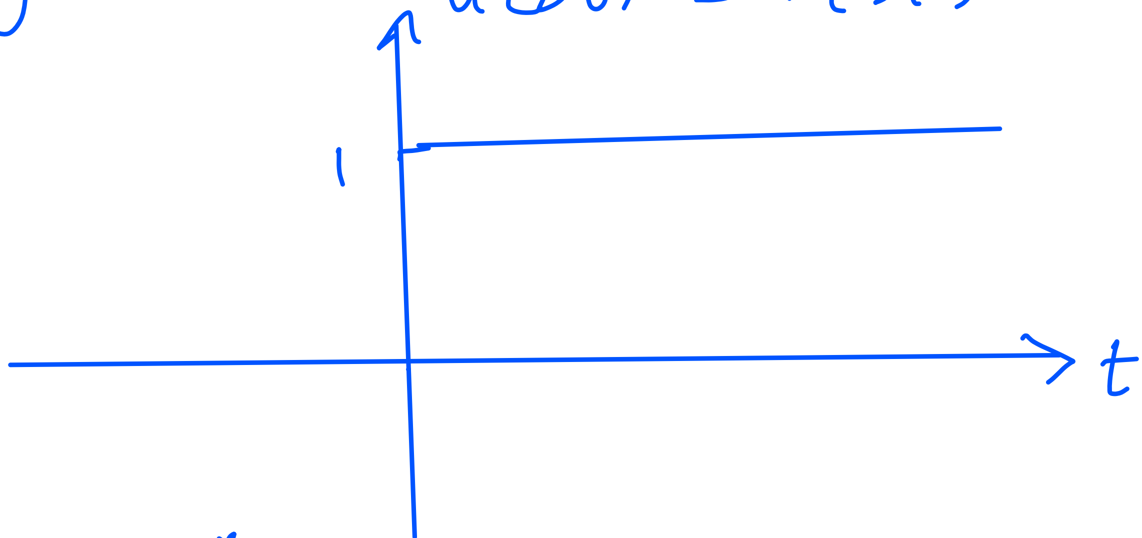


eg draw $u(-t)$



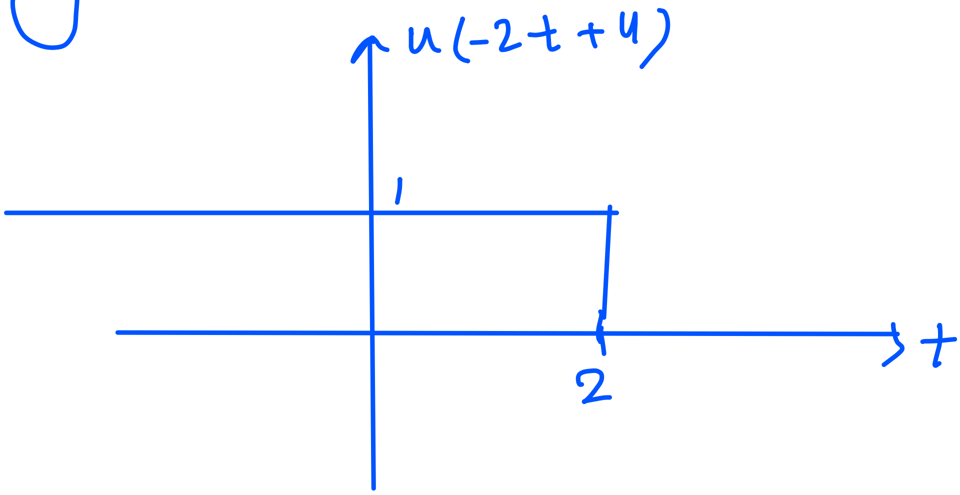
eg draw $u(2t)$

$u(2t) = u(3t) = u(10t)$



Scaling have no effect on step function!!

eg draw $u(-2t+4)$



<u>Note</u> :- $u(-t + \underline{t_0})$

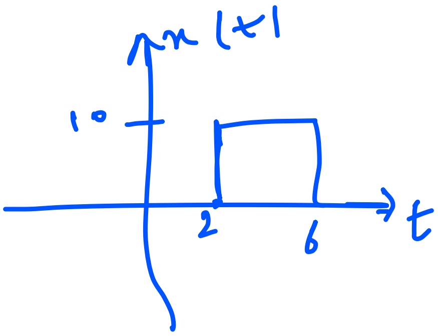
may be +ve or -ve

it always starts from $-\infty$ to the value of t_0 !!

<u>Signals</u>

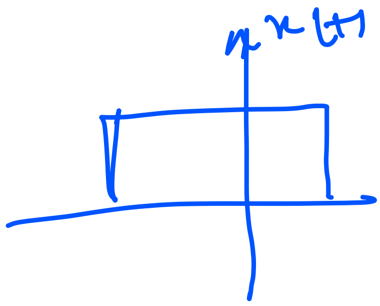
Causal

$x(t) = 0$ at $t < 0$



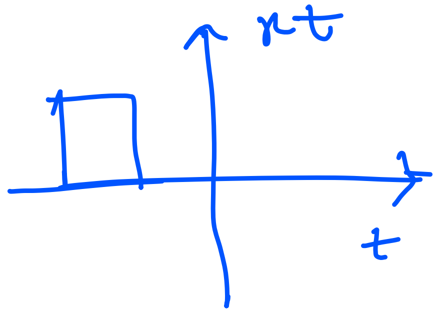
Non-causal

$x(t) \neq 0$ at $t < 0$

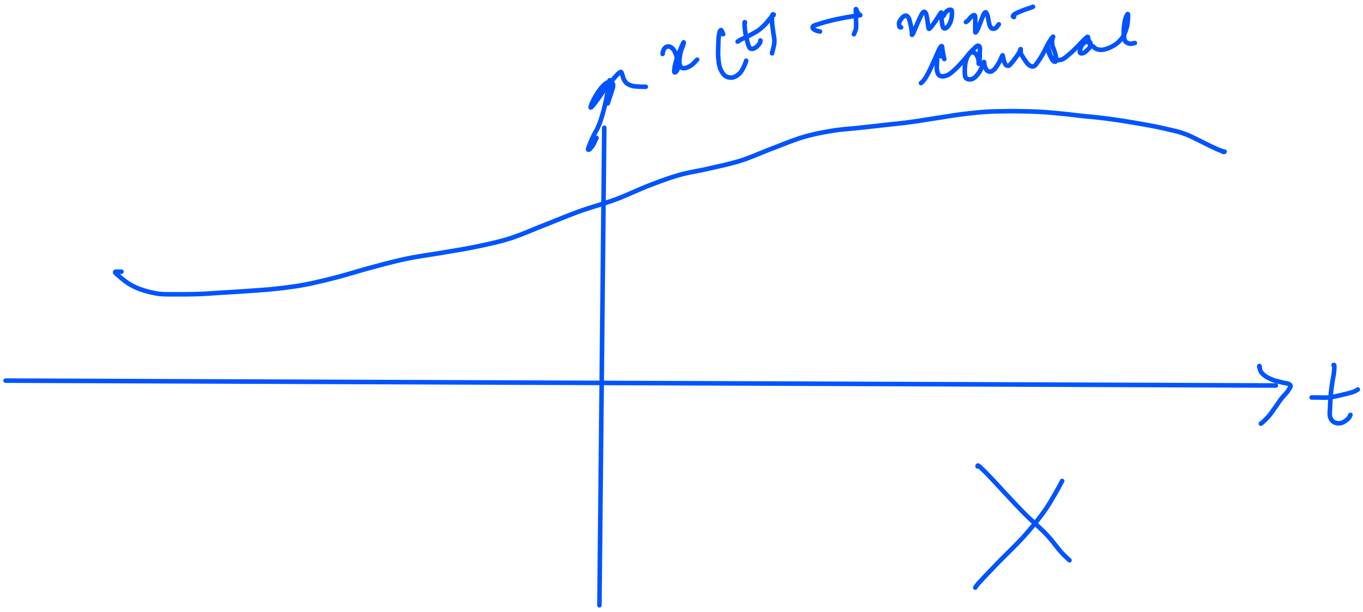


Anti-causal

$x(t) = 0$ at $t > 0$



When we want to design a system where our input remain causal we use unit-step func^n.



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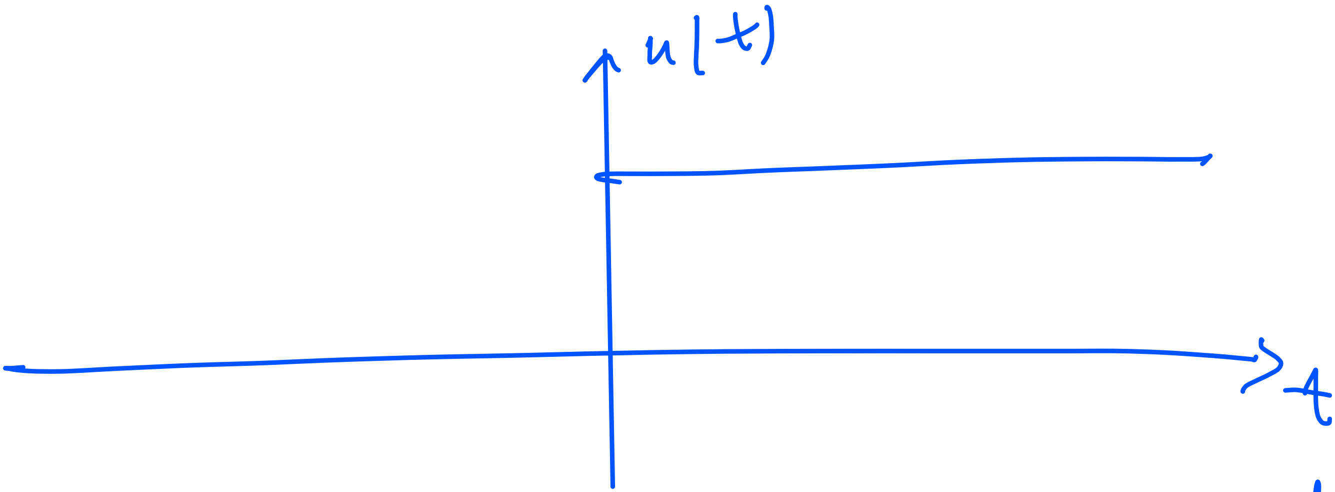
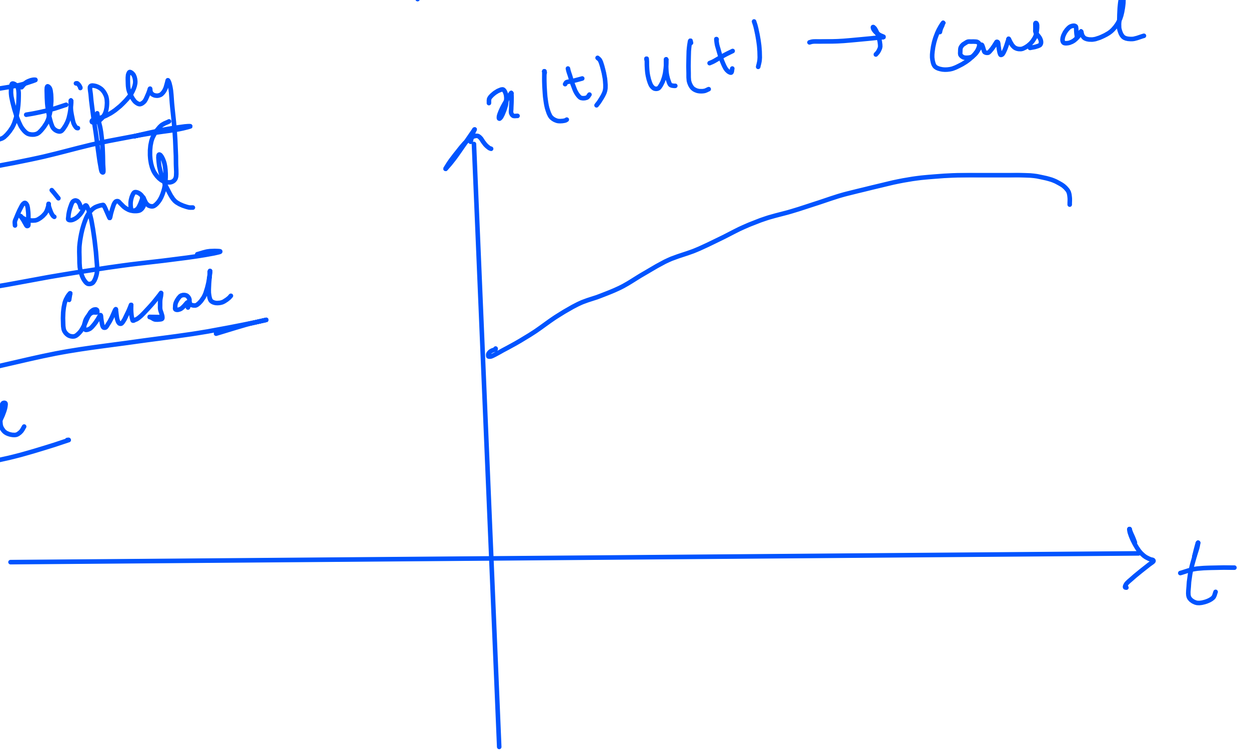


when we multiply both signal we get causal signal



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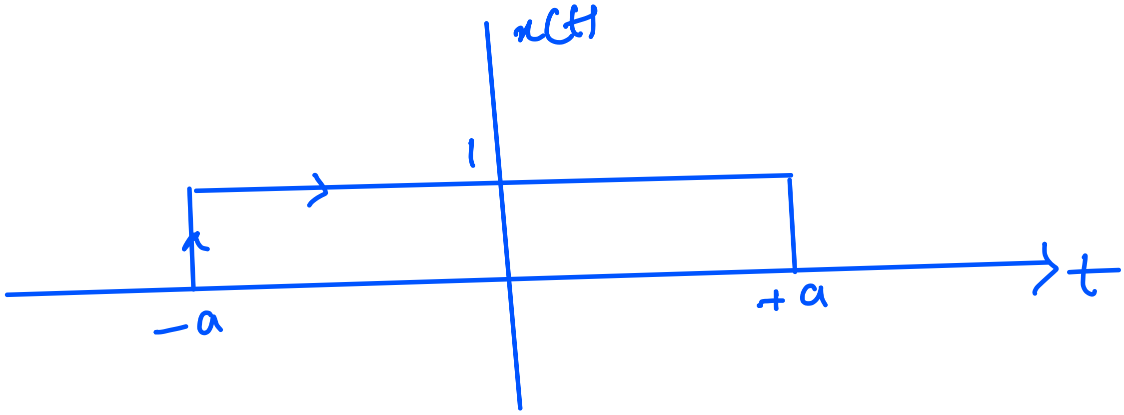
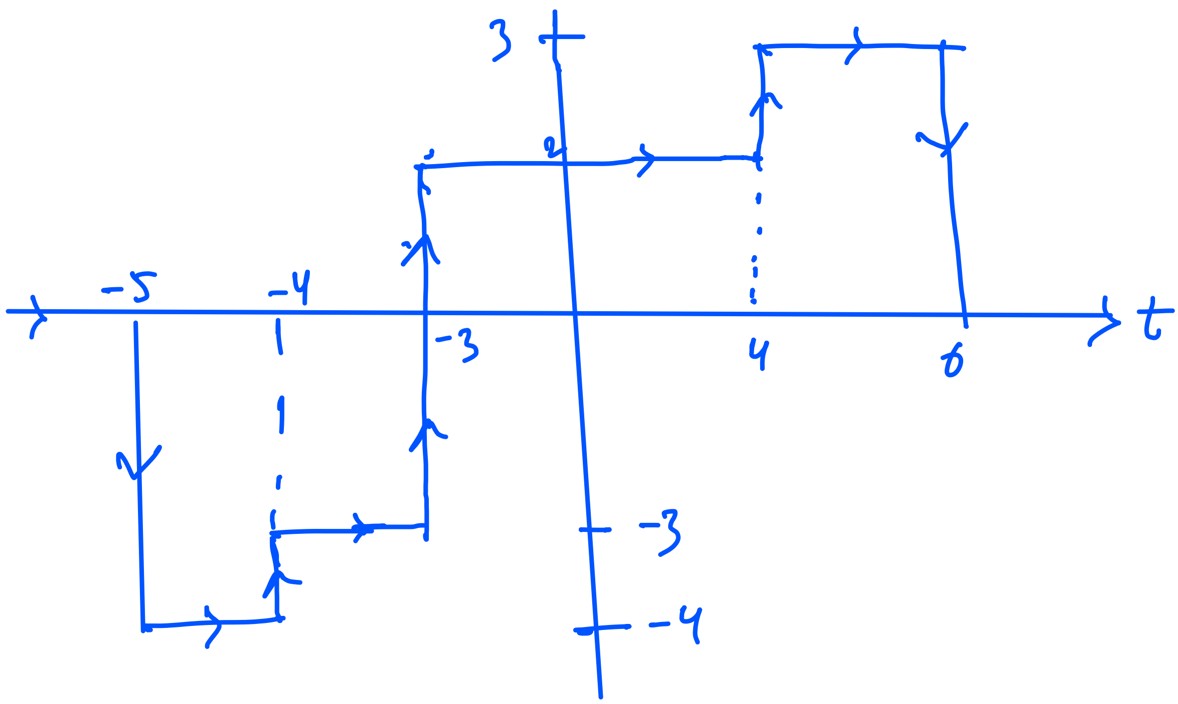


$\longrightarrow x(t) = u(\underbrace{t+a}_{=0}) - u(\underbrace{t-a}_{=0})$

$\Rightarrow t = -a \qquad \Rightarrow t = a$

eg



Write eq^n for $x(t)$!

$$x(t) = -4u(t+5) + u(t+4) + 5u(t+3) + u(t-4) - 3u(t-6)$$

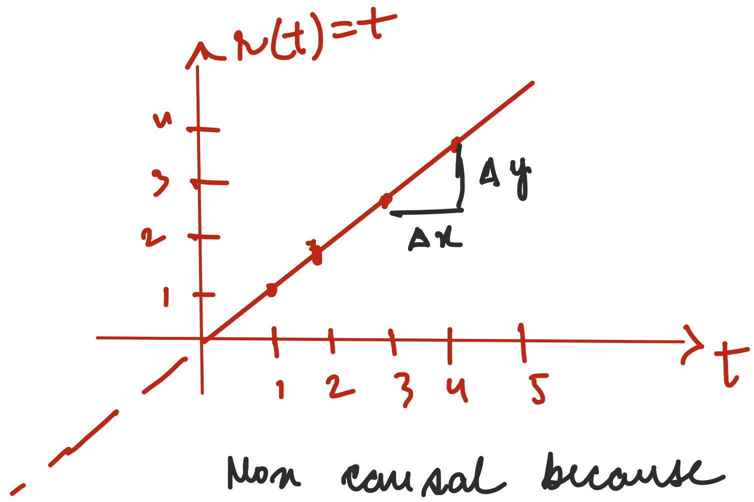
Note -: $u(-t \pm t_0) \longrightarrow$ Left sided signal Extends upto $-\infty$

$u(+t \pm t_0) \longrightarrow$ Right sided signal Extends upto $+\infty$.

② RAMP SIGNAL :-

$$r(t) = t \quad , \quad t \geq 0$$
$$= 0 \quad , \quad \text{otherwise}$$

Note :- Coefficient of step func^n defines its height / amplitude.
& Coeff of ramp func^n defines its SLOPE.

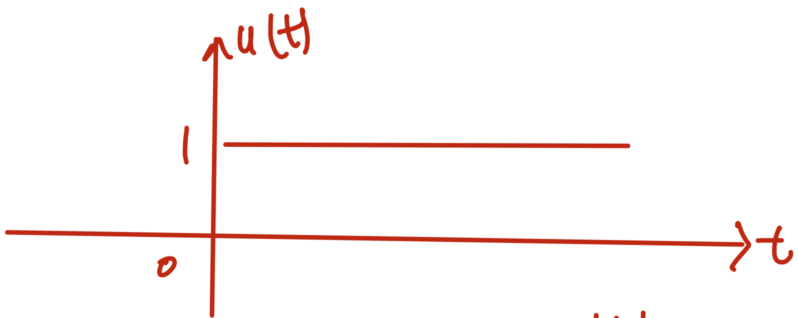


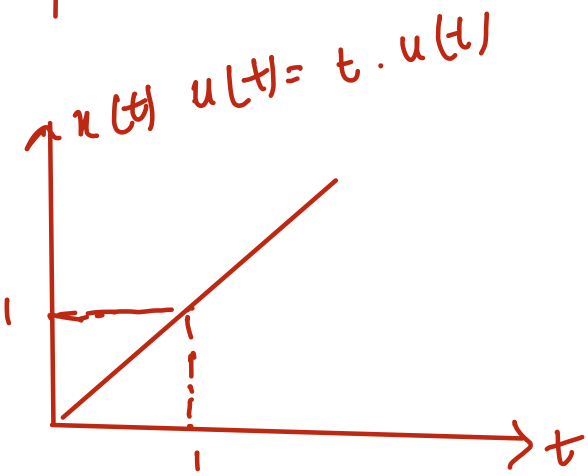
$$\text{slope} = \frac{3-2}{3-2} = 1$$

∴ It's a unit ramp func^n.

Non causal because $t \neq 0$ for $x > 0$

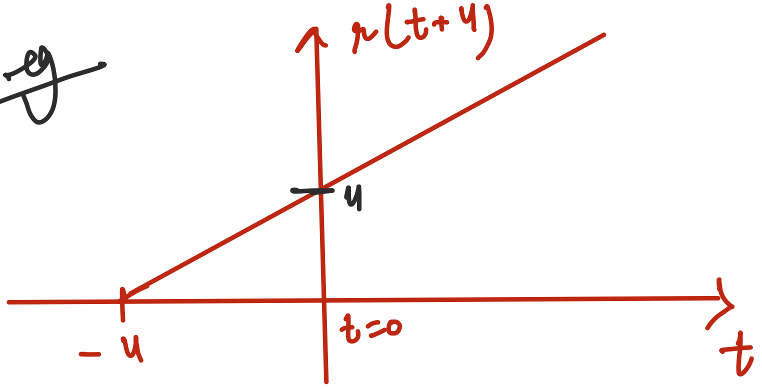
∴ When we multiple this func^n with unit step func^n i.e.





⟹ Therefore we can say $r(t) = t \cdot u(t)$

eg

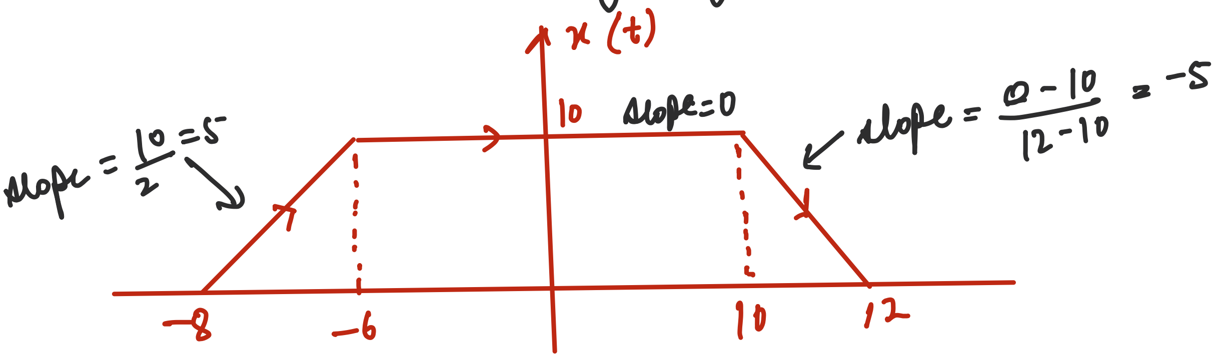


$\Leftarrow r(t+4)$

$$\text{Slope} = \frac{\Delta y}{\Delta x} = 1$$

$$\Delta y = \Delta x$$
$$= 0 - (-4) = 4$$

eg Express the following func^n in ramp func^n form?

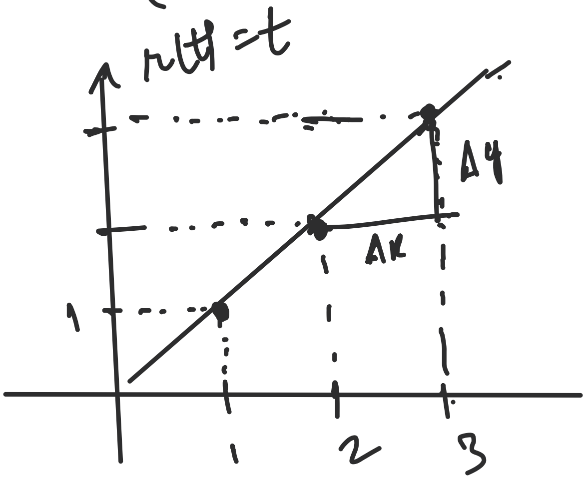


$$x(t) = 5r(t+8) - 5r(t+6) - 5r(t-10) + 5r(t-12)$$

IMPULSE FUNCTIONS -:

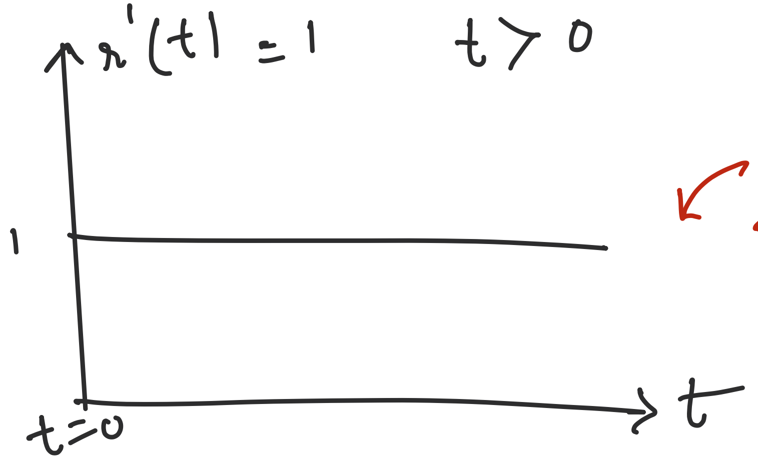
$$r(t) = \begin{cases} t & t \geq 0 \\ 0 & t < 0 \end{cases}$$

$$r(t) = t \cdot u(t)$$



$$\frac{\Delta y}{\Delta x} = 1$$

$$\boxed{\frac{dr(t)}{dt} = \frac{d(t)}{dt} = 1 \quad \text{when } t > 0}$$



we have 'its height '1' because of slope its '1'

∴ Relation b/w unit step function & ramp function is

$$\boxed{\frac{d}{dt} r(t) = u(t)}$$

$$\boxed{r(t) = t \cdot u(t)}$$

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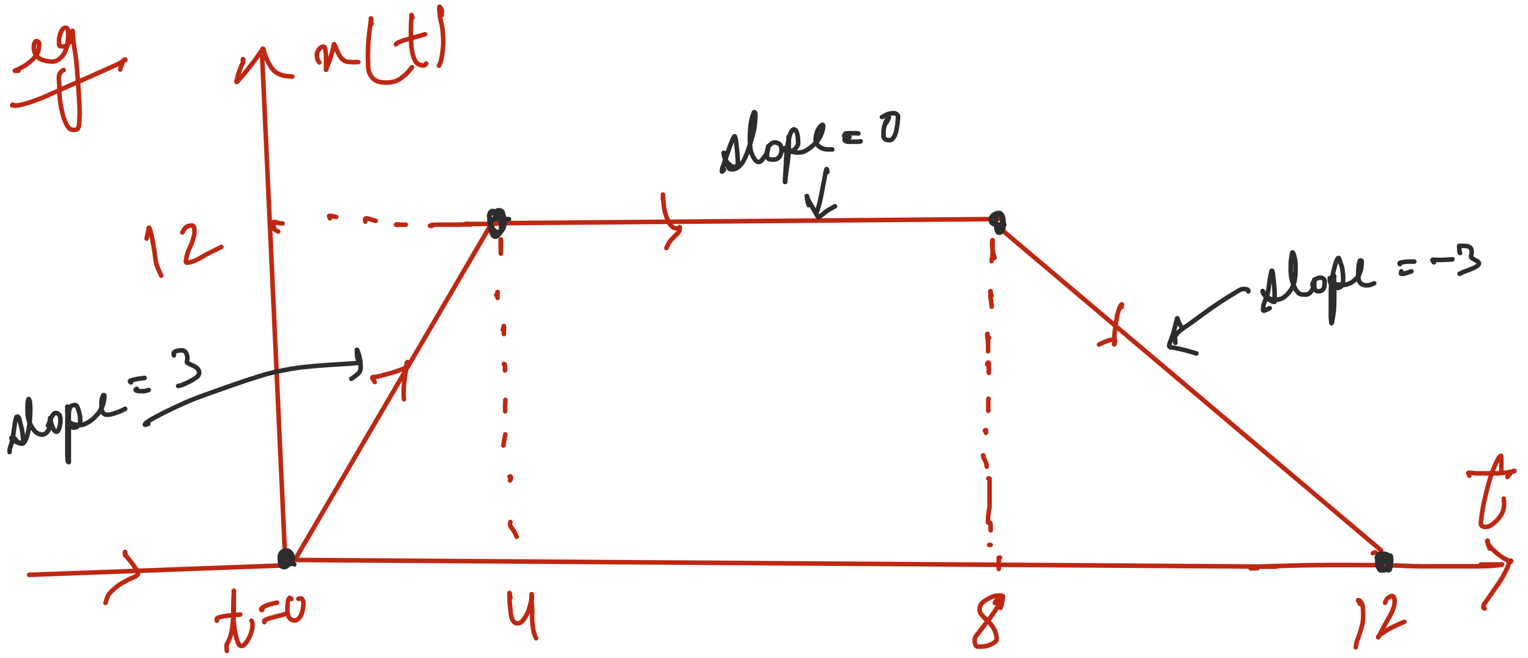
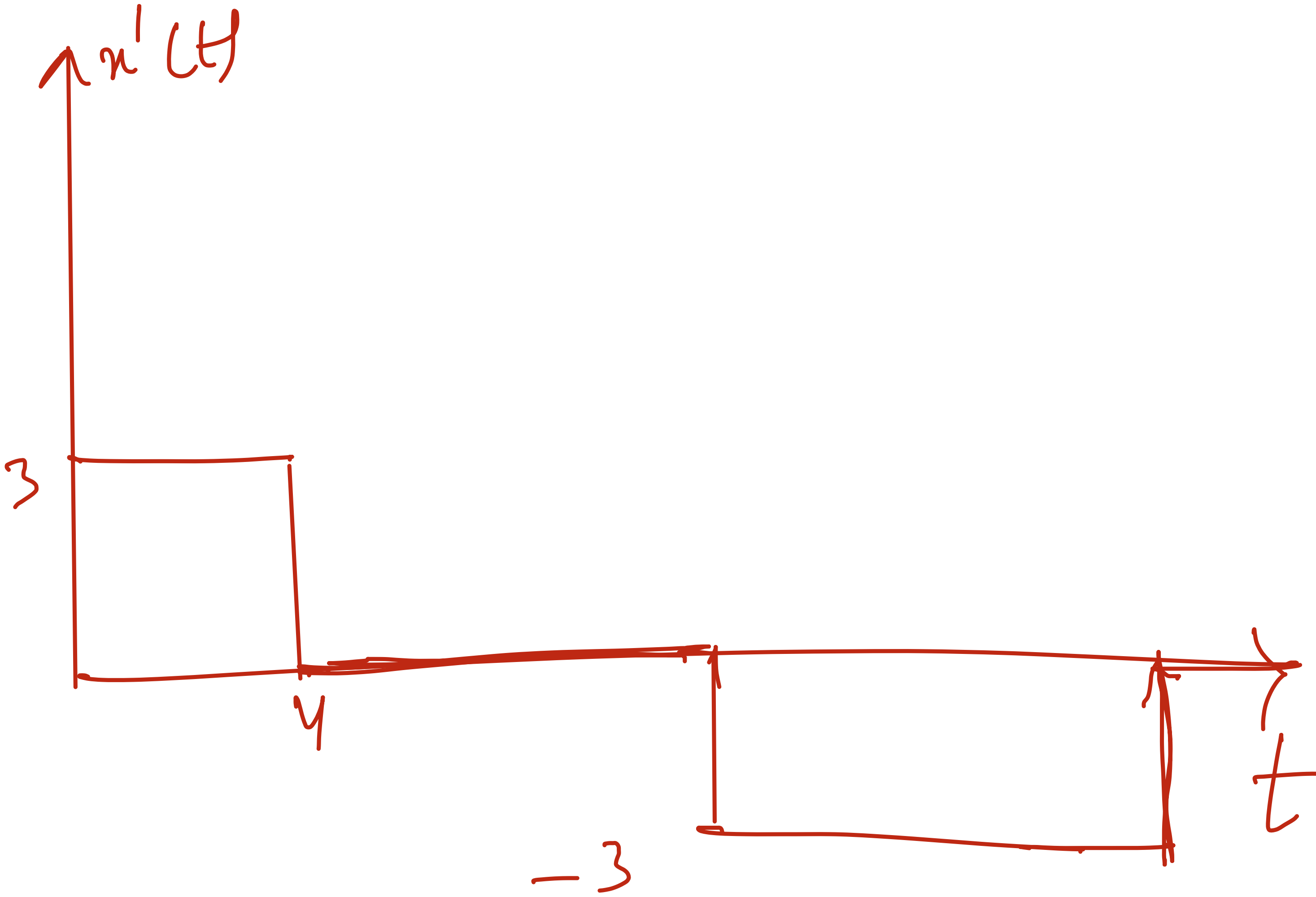
eg

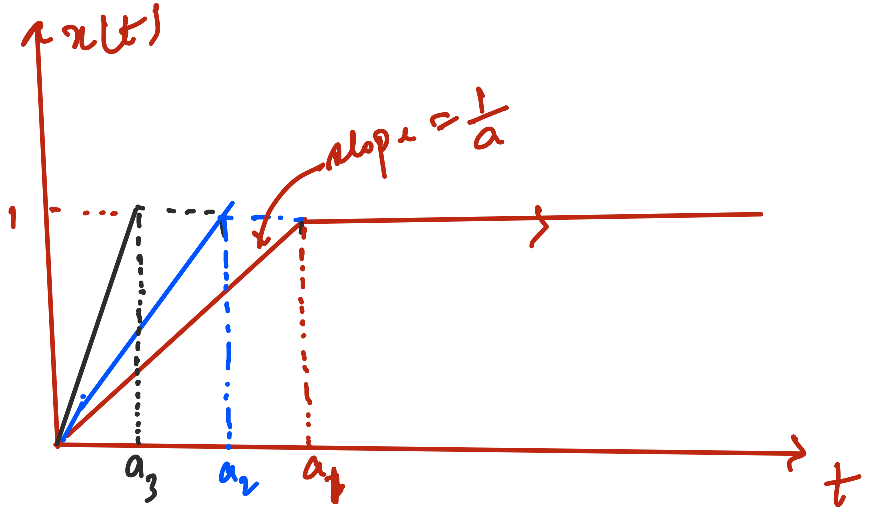


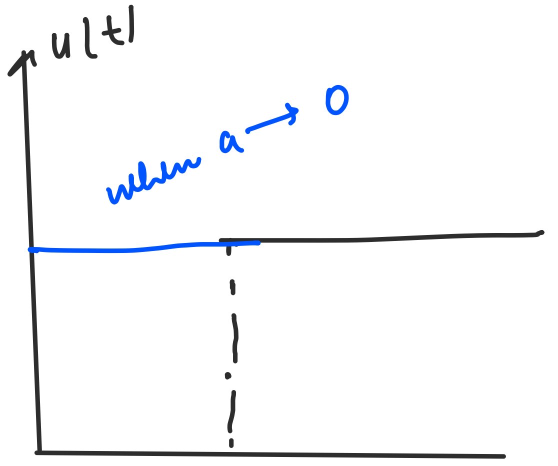
So, basically whenever the slope changes we have to consider that point for writting the funcⁿ x(t).

$$x(t) = 3r(t) - 3r(t-4) - 3r(t-8) + 3r(t-12)$$

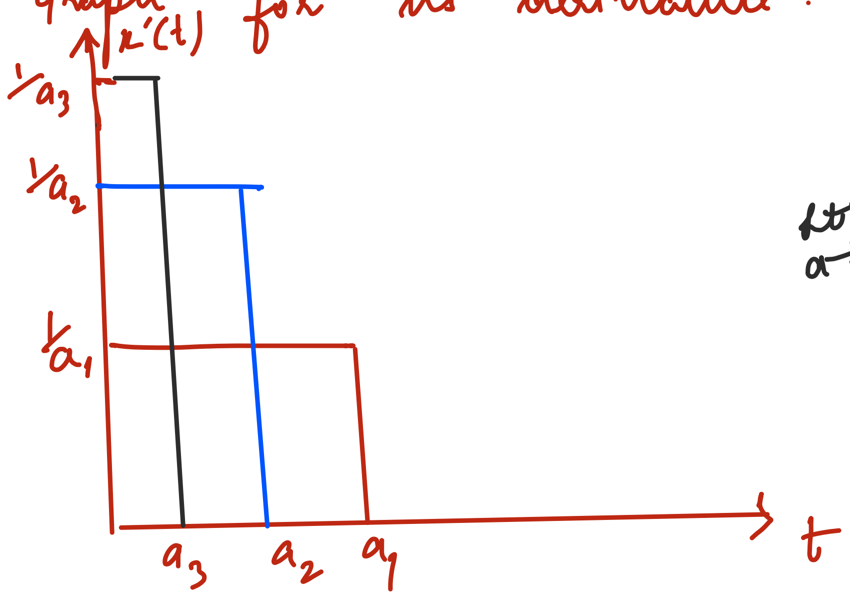
Now we want to differentiate this func^n $x(t)$.

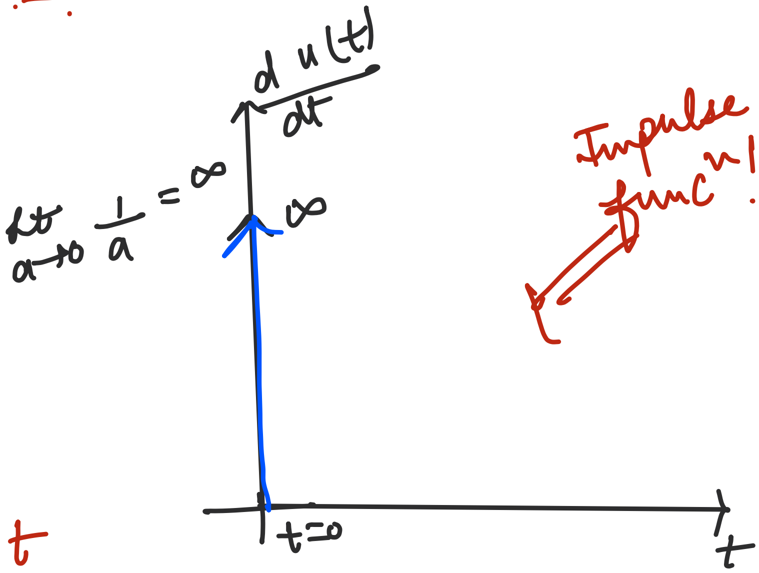






graph for its derivative :-

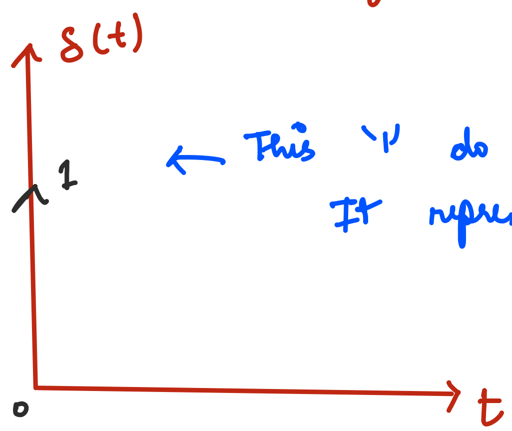




→ Unit impulse function

or

Dirac delta func^n.



← This '1' do not represent height or amplitude. It represents area!

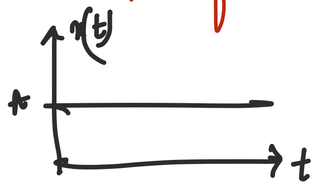
$$\boxed{\begin{aligned} \delta(t) &= \infty \qquad t = 0 \\ &= 0 \qquad t \neq 0 \end{aligned}}$$

$$\boxed{\int_{-\infty}^{\infty} \delta(t)\, dt = 1}$$

Notes

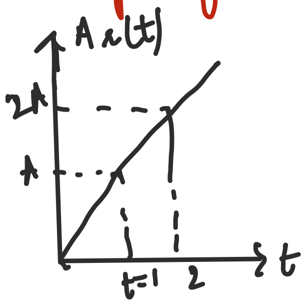
1) Coff. of step func^n represents height / amplitude

$A u(t)$



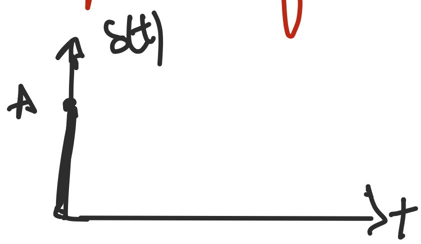
A is height

2) Coff of ramp func^n represents slope.



A is slope

3) Coff of Impulse func^n represents Area.

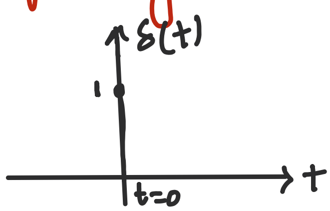


A is Area.

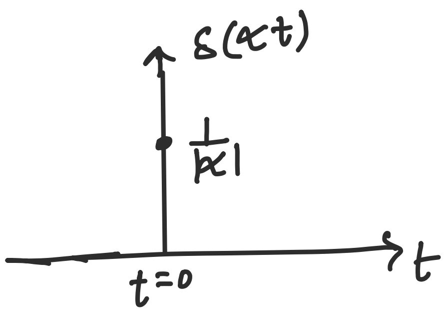
Properties of Impulse Func^n :-

① Time Scaling property -:

$$\delta(\alpha t) = \frac{1}{|\alpha|}\, \delta(t)$$



$\Rightarrow$



$$\delta\{\alpha(t \pm \beta)\} = \frac{1}{|\alpha|}\, \delta(t \pm \beta)$$

$$\boxed{\begin{aligned} \delta(t) &= 1 \quad & t &= 0 \\ &= 0 \quad & t &\neq 0 \end{aligned}}$$

② Product Property :-

Whenever you multiply an impulse function with any other func^n, resultant will be an Impulse func^n !

$$\boxed{x(t) \cdot \underset{=0}{\delta(t)} = x(0)\,\delta(t)}$$

→ This impulse exists at $t=0$

→ Area of resultant impulse would be $x(0)$

$$\boxed{x(t) \cdot \delta(t-t_0) = x(t_0)\,\delta(t-t_0)}$$

③ Shifting Property :-

→ We know $\int_{-\infty}^{\infty} \delta(t)\,dt = 1$

→ $$\boxed{\int_{-\infty}^{\infty} x(t) \cdot \delta(t)\,dt = x(0)}$$

→ $$\boxed{\int_{-\infty}^{\infty} x(t) \cdot \delta(t-t_0)\,dt = x(t_0)}$$

(4) Extension of Scaling property:
(when there is a func^n inside impulse func^n)

$$I = \int_{-\infty}^{\infty} x(t) \cdot \delta\{g(t)\}\, dt$$

$\rightarrow$ $\boxed{\delta\{g(t)\} = 0}$, if g is no where zero.

$\rightarrow$ $\boxed{\delta\{g(t)\} = \dfrac{\delta(t-t_0)}{|g'(t_0)|}}$, if g has a real root at $t = t_0$.

$\rightarrow$ If 'g' has more than one real roots at $t_0, t_1, t_2 \ldots\ldots t_i$

$$\delta\{g(t)\} = \sum_i \frac{\delta(t-t_i)}{|g'(t_i)|} = \frac{\delta(t-t_0)}{|g'(t_0)|} + \frac{\delta(t-t_1)}{|g'(t_1)|} + \cdots$$

ex Evaluate the Integral :-

$$I = \int_{-10}^{10} (t^2+10)\, \delta(t^2-16)\, dt$$

$\delta(g(t)) = \delta(t^2-16) \Rightarrow g(t) = t^2 - 16 = 0$

$t = \pm 4 \qquad t_1 = 4,\ t_2 = -4$

$$\boxed{\delta(t^2-16) = \frac{\delta(t-t_1)}{|g'(t_1)|} + \frac{\delta(t-t_2)}{|g'(t_2)|}}$$

$g'(t) = 2t$

$g'(t_1) = 2(4) = 8$

$g'(t_2) = 2\cdot(-4) = -8$

$$I = \int_{-10}^{10} (t^2+10)\left\{\frac{\delta(t-4)}{8} + \frac{\delta(t+4)}{8}\right\} dt$$

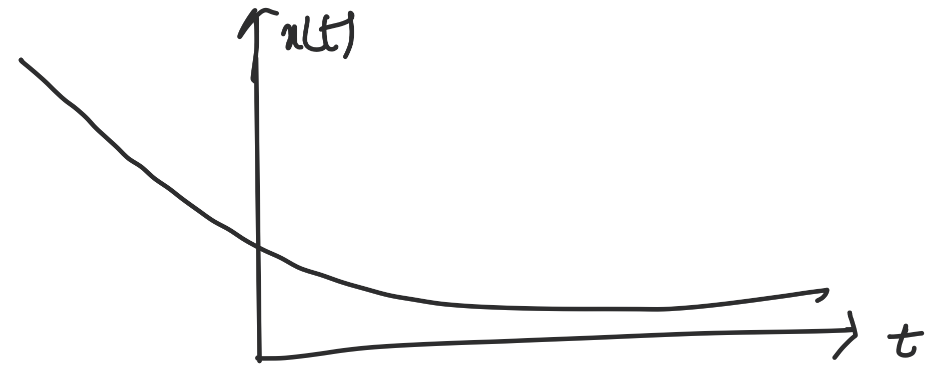
$$I = \frac{1}{8}\left[\int_{-10}^{10}(t^2+10)\,\delta(\underbrace{t-4}_{t=4})\,dt + \int_{-10}^{10}(t^2+10)\,\delta(\underbrace{t+4}_{t=-4})\,dt\right]$$

$$= \frac{1}{8}\left[(t^2+10)\Big|_{t=4} + (t^2+10)\Big|_{t=-4}\right]$$

$$= \frac{1}{8}[26+26] = \frac{52}{8} = \frac{13}{4}$$

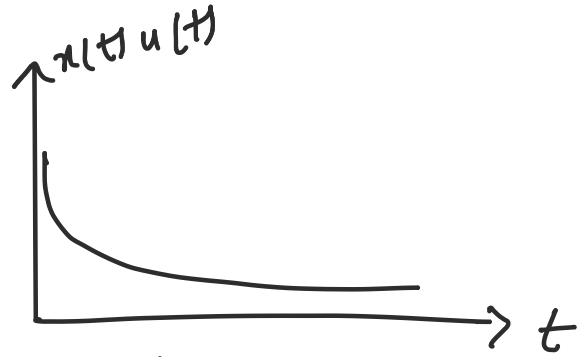
Exponential Functions -:

$\rightarrow$ $x(t) = e^{-at}$, $|a| > 1$



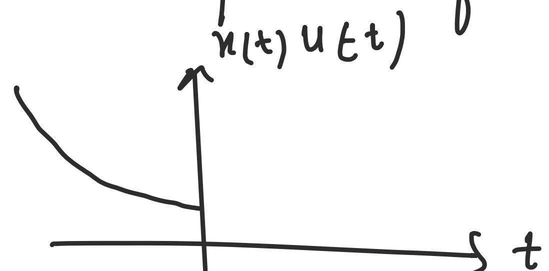
If we multiply this $func^n$ with $u(t)$

It will give all the values of that function from 0 to $+\infty$



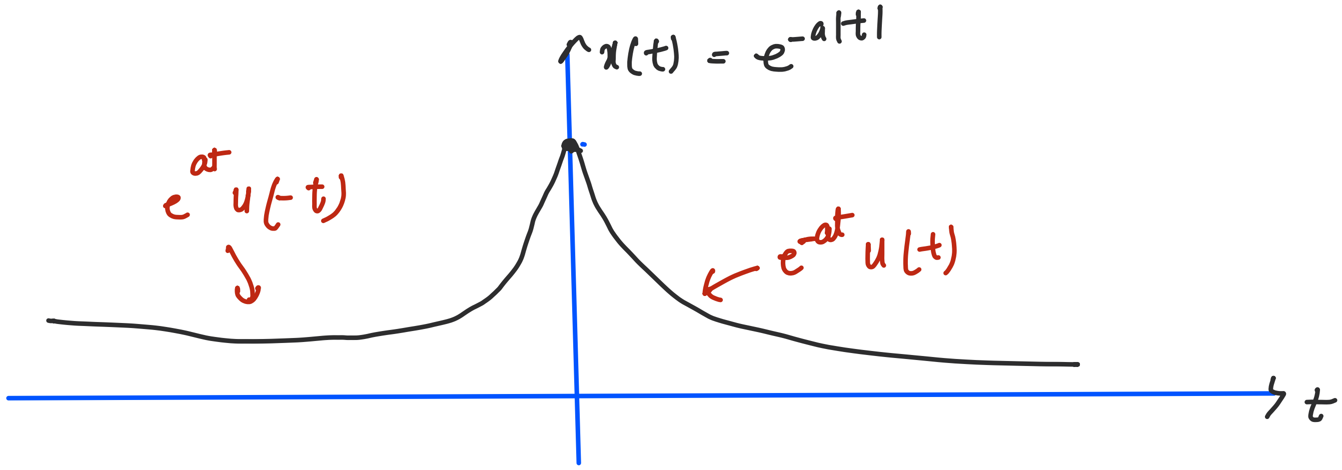
If we multiply this $func^n$ with $u(-t)$

It will give all the values of that $func^n$ from $-\infty$ to 0.



$$\therefore \quad x(t) = e^{-a|t|}$$

$$= \underbrace{e^{-at}\,u(t)}_{\text{when } t \text{ is +ve, } 0 \text{ to } \infty} + \underbrace{e^{at}\,u(-t)}_{\text{when } t \text{ is -ve, } -\infty \text{ to } 0}$$



eg 2 $\quad x(t) = e^{-at^2} \longrightarrow$ Gaussian

t	e^{-at^2}
0	1
1	e^{-a}
-1	e^{-a}
$+2$	e^{-4a}
-2	e^{-4a}
$+\infty$	0
$-\infty$	0