Laplace Inverse :-

① $F(s) = \frac{1}{s+a}$, $Re\{s\} > -a$

$f(t) = e^{-at}\, u(t)$

② $F(s) = \frac{10}{(s+6)^3}$

$$\boxed{t^{n} e^{-at}\, u(t) \longleftrightarrow \frac{n!}{(s+a)^{n+1}}}$$

$F(s) = 5 \times \frac{2!}{(s+6)^{2+1}}$

$$\boxed{f(t) = 5 \cdot t^{2} e^{-6t}\, u t}$$

③ $F(s) = \frac{8}{s^2+4s+10} = \frac{8}{\underbrace{s^2+2\cdot s\cdot 2+(2)^2}+6}$

$= \frac{8 \times \sqrt{6}}{\sqrt{6}\,(s+2)^2+(\sqrt{6})^2} = \frac{8}{\sqrt{6}} \frac{\sqrt{6}}{(s+2)^2+(\sqrt{6})^2}$

$f(t) = \frac{8}{\sqrt{6}} e^{-2t} \sin\sqrt{6}t\; u(t)$

eg $X(s) = \frac{2s^2+5s+5}{(s+1)^2(s+2)}$ find causal inverse $x(t)$

let $X(s) = \frac{2s^2+5s+5}{(s+1)^2(s+2)} = \frac{A}{s+1} + \frac{B}{(s+1)^2} + \frac{C}{s+2}$

$$\Rightarrow 2s^2+5s+5 = A(s+1)(s+2) + B(s+2) + C(s+1)^2$$

At $s=-2$ $8-10+5 = 0+0+C$

$$C = 3$$

$$\Rightarrow 2s^2+5s+5 = A(s^2+3s+2) + B(s+2) + C(s^2+2s+1)$$

comparing coeff of s^2 both side.

$$2 = A + C \quad \therefore \quad A = 2 - C = 2-3$$

$$A = -1$$

comparing coeff of s^0 both side.

$$5 = 2A + 2B + C$$

$$5 = -2 + 2B + 3 \Rightarrow 2B = 4 \Rightarrow B = 2$$

$$X(s) = \frac{-1}{s+1} + \frac{2}{(s+1)^2} + \frac{3}{s+2}$$

Taking causal Inverse Laplace.

$$x(t) = -e^{-t}u(t) + 2\cdot t\cdot e^{-t}u(t) + 3e^{-2t}u(t)$$

$$\boxed{x(t) = \left[-e^{-t} + 2te^{-t} + 3e^{-2t}\right]u(t)}$$

eg $$X(s) = \frac{5(s-2)}{s^2(s+4)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+4}$$

$$5(s-2) = As(s+4) + B(s+4) + Cs^2$$

At $s=-4$ $-30 = 16C$ $\quad C = -\frac{30}{16}$

comp. coeff of s^2

$$0 = A + C \qquad A = -C \Rightarrow A = \frac{30}{16}$$

comp coeff of s

$$5 = 4A + B \Rightarrow B = \frac{50}{16}$$

$$X(s) = \frac{30/16}{s} + \frac{50/16}{s^2} + \frac{(-30/16)}{s+4}$$

$$\Rightarrow \quad x(t) = \frac{30}{16}u(t) + \frac{50}{16}t\,u(t) - \frac{30}{16}e^{-4t}u(t)$$

Minimum Phase system:-

→ All poles & zeroes must be located in L.H.S. of s-plane, so that the inverse of the system will also be causal & stable.

$$H(s) = \frac{(s+2)(s+3)}{(s+1)(s+4)(s+5)} \qquad -2, -3 \longrightarrow \text{Minimum plane system}$$

$$H(s) = \frac{(s-2)(s+3)}{(s+1)(s+4)(s+5)} \qquad 2, -3 \longrightarrow \text{Mixed phase system}$$

$$H(s) = \frac{(s-2)(s-3)}{(s+1)(s+4)(s+5)} \qquad 2, 3 \longrightarrow \text{Maximum phase system.}$$

→ Total no. of poles = Total no. of zeros = 3

No. of zeros in finite s-plane $z = 2$

" poles " " " $p = 3$

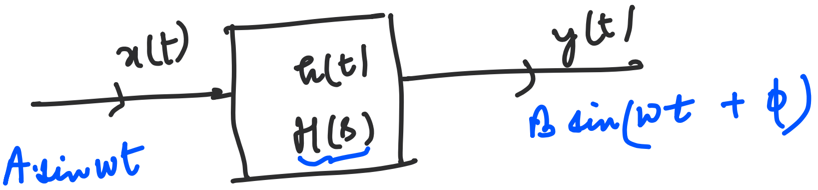
" zeros in infinite s-plane $= p - z = 1$

eg
$$H(s) = \frac{(s+2)(s+3)}{(s+1)(s+4)(s+5)}$$

$$H(j\omega) = \frac{(j\omega+2)(j\omega+3)}{(j\omega+1)(j\omega+4)(j\omega+5)}$$

$$\angle H(j\omega) = \left\{ \tan^{-1}\frac{\omega}{2} + \tan^{-1}\frac{\omega}{3} \right\} - \left\{ \tan^{-1}\omega + \tan\frac{\omega}{4} + \tan^{-1}\frac{\omega}{5} \right\}$$

Steady State Response of an LTI system :-



$H(s) \Rightarrow H(j\omega)$

$H(j\omega) \rightarrow |H(j\omega)| = X$

$H(j\omega) \rightarrow \angle H(j\omega) = \phi$

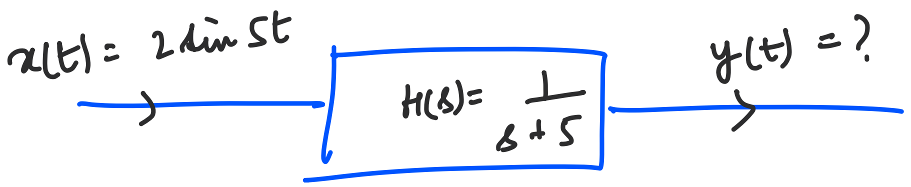
$B = AX$

$\phi = \angle H(j\omega)$

If input to a LTI system is sinusoidal, then its response will also be sinusoidal in nature, having same freq. as input.

Amplitude & phase angle may vary!

eg



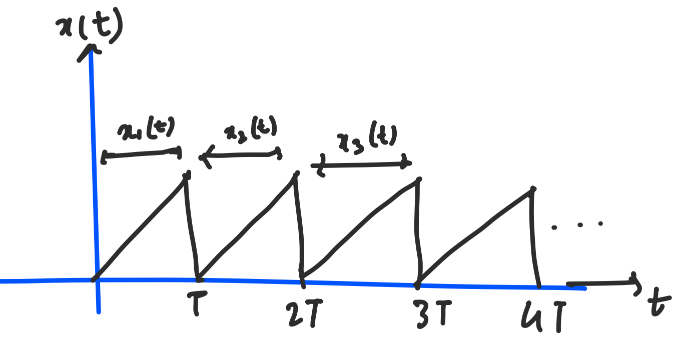
$$H(j\omega) = \frac{1}{j\omega + 5}$$

$$|H(j\omega)| = \frac{1}{\sqrt{\omega^2+25}} = \frac{1}{\sqrt{50}} = X$$

$$\angle H(j\omega) = -\tan^{-1}\frac{\omega}{5} = -45^\circ = \phi$$

$$y(t) = B\sin(5t+\phi) = \frac{2}{\sqrt{50}}\sin(5t-45^\circ)$$

Laplace transform for causal repeating func^n:



$$x(t) = x_1(t) + x_1(t-T) + x_1(t-2T) + x_1(t-3T) + \cdots$$

$$x_1(t) \longleftrightarrow X_1(s)$$

$$x_1(t-T) \longleftrightarrow e^{-sT} X_1(s)$$

$$x_2(t-2T) \longleftrightarrow e^{-2sT} X_1(s)$$

$$\therefore X(s) = X_1(s) + e^{-sT} X_1(s) + e^{-2sT} X_1(s) + e^{-3sT} X_2(s) \cdots$$

$$= X_1(s)\left[1 + e^{-sT} + e^{-2sT} + e^{-3sT} + \cdots\right]$$

$$= X_1(s)\left(\frac{1}{1-e^{-sT}}\right)$$

Partial fraction :-

① $$\frac{1}{x^2-4} = \frac{A}{x+2} + \frac{B}{x-2}$$

② $$\frac{x-4}{x^2+2x-15} = \frac{x-4}{(x+5)(x-3)} = \frac{A}{x+5} + \frac{B}{x-3}$$

③ $$\frac{x}{(x-1)(x-2)^2} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{(x-2)^2}$$

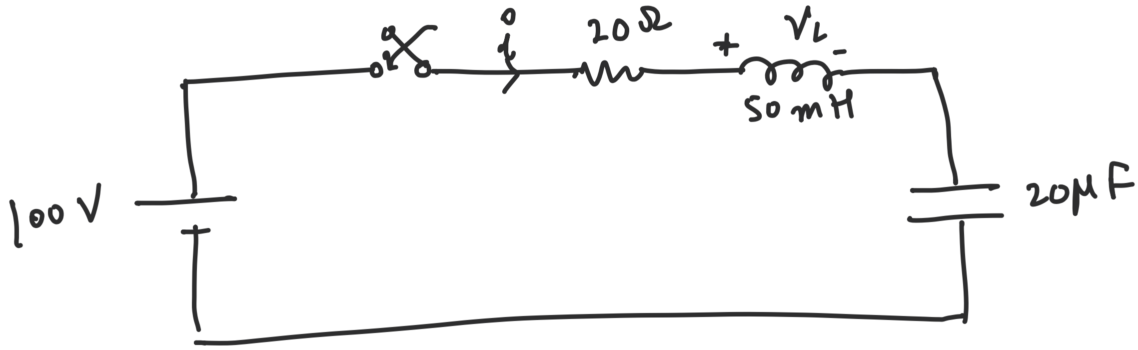
④ $$\frac{1}{(x-3)^3} = \frac{A}{x-3} + \frac{B}{(x-3)^2} + \frac{C}{(x-3)^3}$$

⑤ $$\frac{x^2+9}{(x^2-1)(x^2+4)} = \frac{x^2+9}{(x+1)(x-1)(x^2+4)}$$

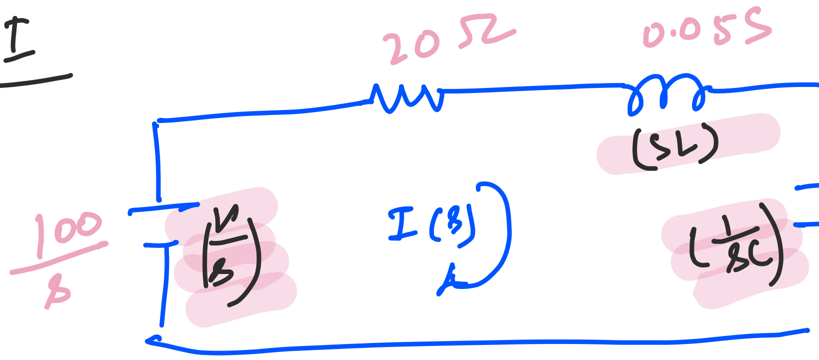
$$= \frac{A}{x+1} + \frac{B}{x-1} + \frac{Cx+D}{x^2+4}$$

Series RLC circuit Analysis :-

Ques :- The switch closes in the circuit of figure at $t=0$. Assuming a relaxed circuit at the time of switching, determine the current (i) for $t>0$. Also find the voltage V_L across the inductance for $t>0$.



Method I



$$\frac{1}{s(20\times10^{-6})} = \frac{5\times10^{4}}{s}$$

By ohm's law

$$\boxed{V = IR}$$

$$\frac{100}{s} = I(s)\left[20 + 0.05s + \frac{5\times10^{4}}{s}\right]$$

$$\Rightarrow \boxed{I(s) = \frac{100/s}{20 + 0.05s + \frac{5\times10^{4}}{s}}}$$

Method 2 Using KVL.

On applying KVL – (Net potential drop in a closed circuit is 0)

$$\boxed{100 - Ri(t) - L\frac{di(t)}{dt} - \frac{1}{C}\int i(t)\,dt = 0}$$

$$100 - 20i(t) - 0.05\frac{di(t)}{dt} - \frac{1}{20\mu}\int i(t)\,dt = 0$$

On taking Laplace Transform

$$\frac{100}{s} - 20I(s) - 0.05\left[sI(s)\right] - \frac{1}{20\times10^{-6}}\left[\frac{I(s)}{s}\right]$$

$$\Rightarrow I(s) = \frac{100/s}{20 + 0.05s + \frac{5\times10^4}{s}}$$

$$\Rightarrow I(s) = \frac{100}{0.05s^2 + 20s + 5\times10^4} = \frac{2000}{s^2 + 400s + 10^6}$$

$$= \frac{2000}{(s+200)^2 + (979.79)^2}$$

$$\left[\because e^{-at}\sin wt = \frac{w}{(s+a)^2 + w^2}\right]$$

$$\left(\frac{2000}{979.79}\right)\frac{979.79}{(s+200)^2 + (979.79)^2}$$

On taking inverse laplace-transform

$$\Rightarrow i(t) = 2.04e^{-200t}\sin(979.79t)$$

→ Now voltage across the inductor -:

$$V_L(s) = I(s)(Ls)$$

$$\frac{2000}{(s+200)^2 + (979.79)^2} \times 0.05s$$

$$V_L(s) = \frac{100s}{(s+200)^2 + (979.79)^2}$$

$$\boxed{\because e^{-at}\cos\omega t = \frac{s+a}{(s+a)^2 + \omega^2}}$$

$$\therefore \frac{100(s+200) - 20000}{(s+200)^2 + (979.79)^2}$$

$$V_L(s) = \frac{100(s+200)}{(s+200)^2 + (979.79)^2} - \frac{20000}{(s+200)^2 + (979.79)^2}$$

$$= \frac{100(s+200)}{(s+200)^2 + (979.79)^2} - \left(\frac{20000}{979.79}\right)\left(\frac{979.79}{(s+200)^2 + (979.79)^2}\right)$$

On taking inverse L.T.

$$V_L(t) = 100\left[e^{-200t}\cos(979.79t) - 20.41e^{-200t}\sin(979.79t)\right]$$

Numericals → Initial value Theorem :-

$$\lim_{t\to 0^+} x(t) = x(0^+) = \lim_{s\to\infty} sX(s) \quad \leftarrow \text{Property}$$

eg $F(s) = \dfrac{4s+5}{s^2+8s+16}$, find initial value of $f(t)$

$$f(0^+) = \lim_{t\to 0^+} f(t) = \lim_{s\to\infty} \frac{s(4s+5)}{s^2+8s+16} = \lim_{s\to\infty} \frac{\cancel{s^2}\left(4+\cancel{\frac{5}{s}}^{\,0}\right)}{\cancel{s^2}\left(1+\cancel{\frac{8}{s}}+\cancel{\frac{16}{s^2}}\right)}$$

$$= 4.$$

eg $F(s) = \dfrac{6s+5}{2s+8}$ → not a proper fraction (degree are same)

$$\begin{array}{r|l|l} 2s+8 & 6s+5 & 3 \\ & \ominus 6s \ominus 24 & \\ \hline & -19 & \end{array}$$

$$\Rightarrow F(s) = 3 - \frac{19}{2s+8}$$

$$\Rightarrow f(0^+) = \lim_{s\to\infty} s\left(\frac{-19}{2s+8}\right) = \lim_{s\to\infty} \frac{\cancel{s}(-19)}{\cancel{s}\left(2+\cancel{\frac{8}{s}}^{\,0}\right)}$$

$$= \frac{-19}{2} = -9.5 \quad \text{Ans}$$

eg $F(s) = \dfrac{s^3+6s^2+8s+10}{s^2+4s+3}$

$$\begin{array}{r|l|l} s^2+4s+3 & s^3+6s^2+8s+10 & s+2 \\ & \ominus s^3 \ominus 4s^2 \ominus 3s & \\ \hline & 2s^2+5s+10 & \\ & \ominus 2s^2 \ominus 8s \ominus 6 & \\ \hline & -3s+4 & \end{array}$$

$$F(s) = (s+2) + \frac{-3s+4}{s^2+4s+3}$$

$$\therefore f(0^+) = \lim_{s \to 0} \frac{s \cdot (-3s+4)}{s^2+4s+3}$$

$$\lim_{s \to \infty} \frac{\cancel{s^2}\left(-3 + \cancel{\frac{4}{s}}\right)}{\cancel{s^2}\left(1 + \cancel{\frac{4}{s}} + \cancel{\frac{3}{s^2}}\right)} = -3$$

$$f(0^+) = -3$$

Numericals on final value theorem -:

$$\lim_{t \to \infty} x(t) = x(\infty) = \lim_{s \to 0} s\,X(s)$$

1. Poles of $sX(s)$ must lie in LHS of s-plane
2. Periodic signal & bounded signal not applicable

eg L.T. of $f(t)$ is $F(s) = \dfrac{5s^2+23s+6}{s(s^2+2s+2)}$

As $t \to \infty$, then $f(t)$ approaches to -:

a) 0

b) $\frac{5}{2}$

c) 17

d) ∞

$$sF(s) = \frac{\cancel{s} \cdot 5s^2+23s+6}{\cancel{s}(s^2+2s+2)}$$

$$sF(s) = \frac{5s^2+23s+6}{s^2+2s+2}$$

$$\therefore \lim_{t \to \infty} f(t) = \lim_{s \to 0} s \cdot F(s) = \lim_{s \to 0} \frac{5s^2+23s+6}{s^2+2s+2}$$

$$= \frac{6}{2} = 3$$

eg $\ddot{x}(t) + 3\dot{x}(t) + 2x(t) = 5$

soln of $x(t)$ approaches, when $t \to \infty$

$$\frac{d^2x(t)}{dt^2} + \frac{3\,dx(t)}{dt} + 2x(t) = 5$$

$$s^2X(s) + 3sX(s) + 2X(s) = \frac{5}{s}$$

$$X(s)\left[s^2 + 3s + 2\right] = \frac{5}{s}$$

$$\therefore \quad X(s) = \frac{5}{s(s^2+3s+2)}$$

$$sX(s) = \frac{5}{s^2+3s+2}$$

final value : $x(\infty) = \lim_{t\to\infty} x(t) = \lim_{s\to 0} \frac{5}{s^2+3s+2} = \frac{5}{2}$

eg $X(s) = \frac{4}{s^2+4}$, then as $t \to \infty$, $x(t) = ?$

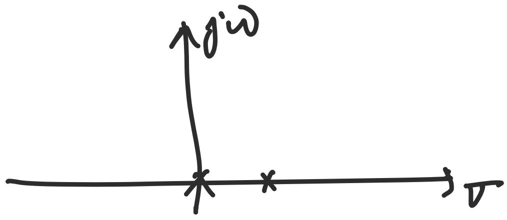
$$\sin \omega_0 t\, u(t) \longleftrightarrow \frac{\omega_0}{s^2+\omega_0^2}$$

$x(t) = \sin 2t\, u(t)$ ← causal periodic signal

← final theorem not applicable.

eg $X(s) = \frac{1}{s(s-2)}$

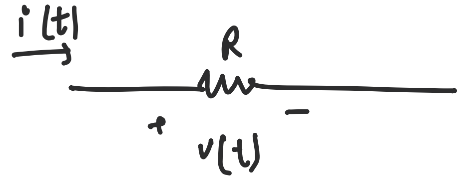
poles of $X(s)$ = $s = 0, 2$

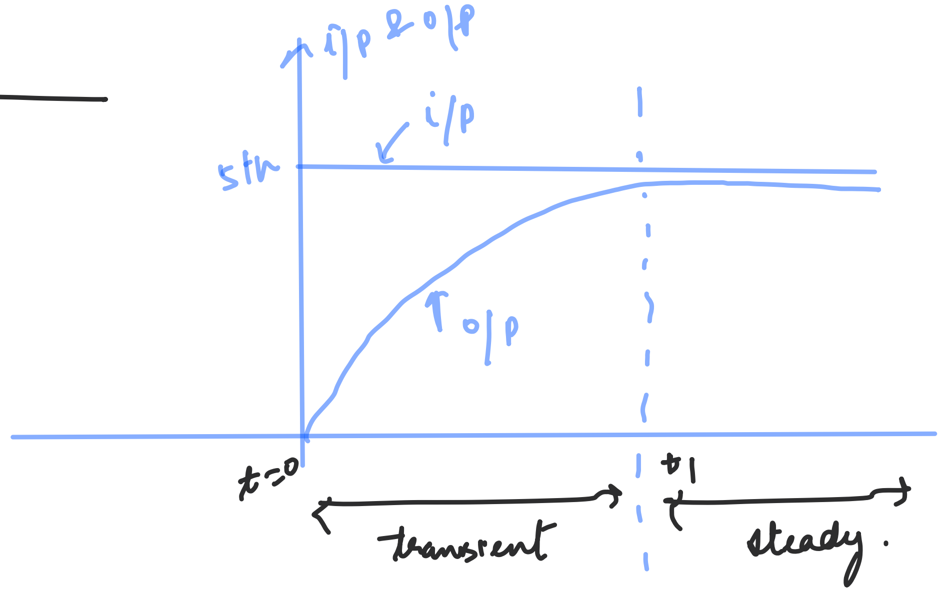


→ unbounded signal

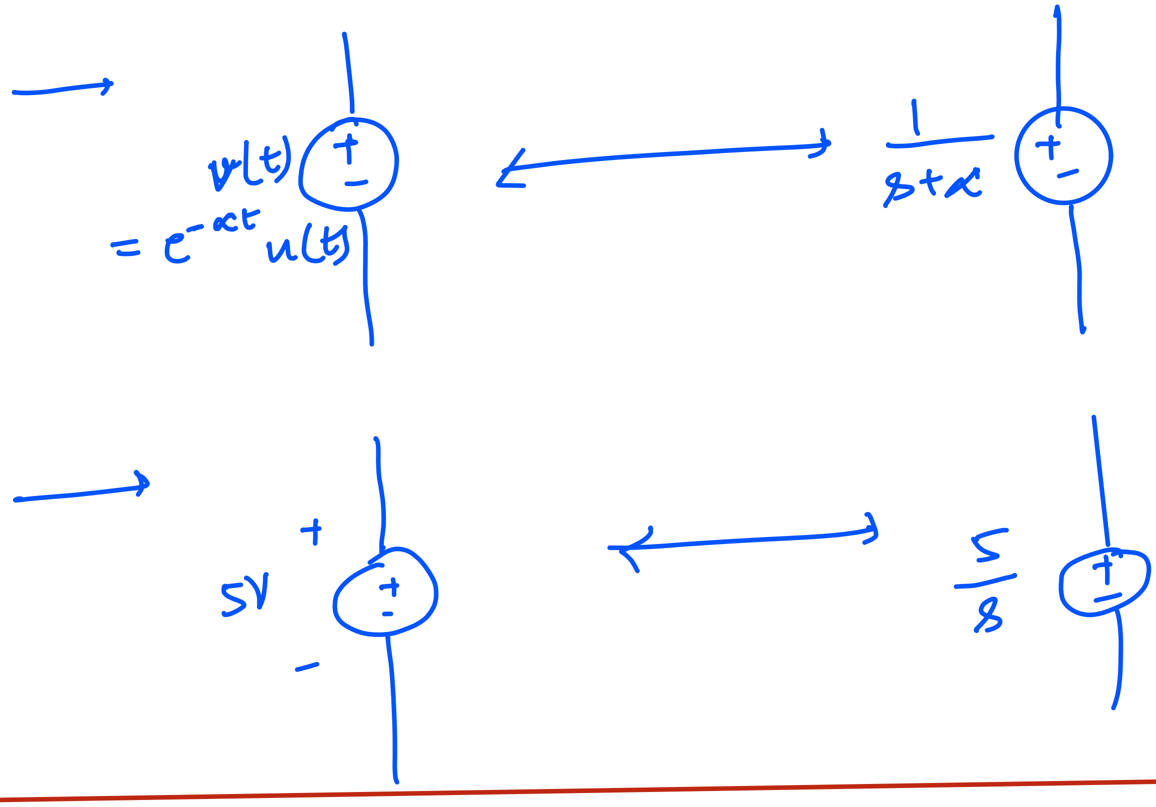
→ cannot be determined!

Application of L.T. in electrical Network -:

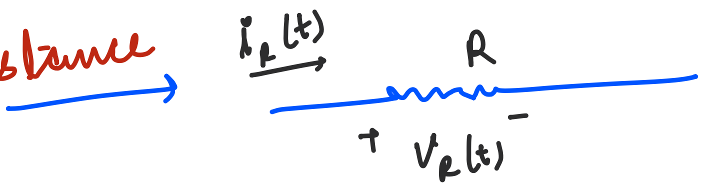




Transform Approach -:



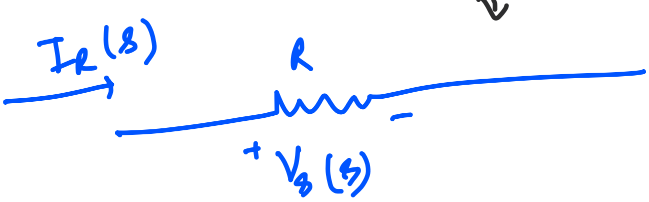
Resistance



$$V_R(t) = i_R(t) \cdot R$$

$$V_R(s) = R \cdot I_R(s)$$

↓ LT



Inductor

$$V_L(t) = L\frac{di_L(t)}{dt}$$

$x(t) \longleftrightarrow X(s)$

$\frac{d\,x(t)}{dt} \longleftrightarrow sX(s) - x(0^+)$

$\frac{d^2 x(t)}{dt} \longleftrightarrow s^2X(s) - s\,x(0^+) - x'(0^+)$

Taking L.T.

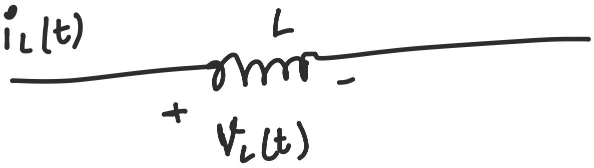
$$V_L(s) = L\left[sI_L(s) - i_L(0^-)\right]$$

$$\boxed{V_L(s) = \underbrace{Ls}_{\text{Impedance}}\,I_L(s) - \underbrace{L\,i_L(0^-)}_{\text{voltage source}}}$$

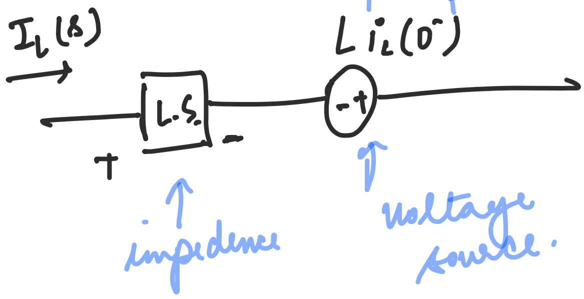
Impedance

voltage source

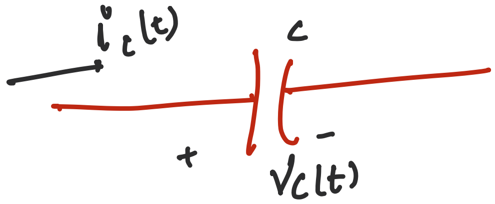
current through the inductor before switching



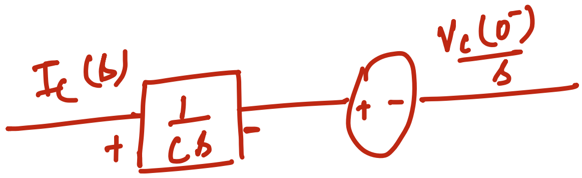
$\xleftrightarrow{LT}$



Capacitor



$\xleftrightarrow{LT}$



$$V_c(t) = \frac{1}{C}\int_{-\infty}^{t} i_c(t)\,dt = \underbrace{\frac{1}{C}\int_{-\infty}^{0} i_c(t)\,dt}_{V_c(0^-)} + \frac{1}{C}\int_{0}^{t} i_c(t)\,dt$$

$V_c(0^-)$ voltage across capacitor before switching

$$V_c(s) = \frac{V_c(0^-)}{s} + \frac{1}{C}\left[\frac{I_c(s)}{s}\right]$$

$$\boxed{V_c(s) = \underbrace{\frac{1}{CS}}_{\text{Impedance}}\,I_c(s) + \underbrace{\frac{V_c(0^-)}{s}}_{\text{Voltage Source}}}$$

Impedance

Voltage Source

	Initial	Steady
Inductor	 open ckt	 short ckt
Capacitor	 short ckt	 open ckt

Inductor

$$V_L(t) = L\left.\frac{di_L}{dt}\right|_{t=0} = 0$$

Capacitor

$$i_c(t) = C\frac{dV_c(t)}{dt} = 0$$

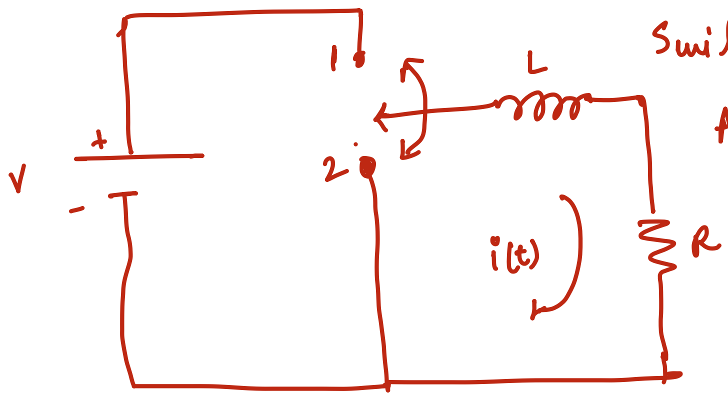
→ In inductor, current before switching = current after switching.

$$i_L(0^-) = i_L(0^+)$$

→ In capacitor, voltage before switching = voltage after switching.

$$V_c(0^-) = V_c(0^+)$$

~~eg~~

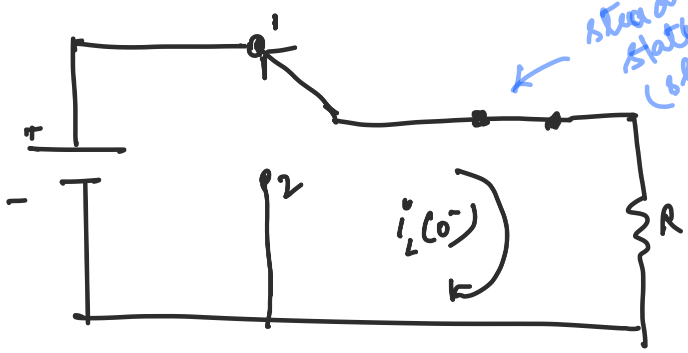


Switch is shifted from $1 \to 2$ at $t = 0$

At $t = 0^- \to$ steady state

Find $i(t)$ for $t \geq 0$

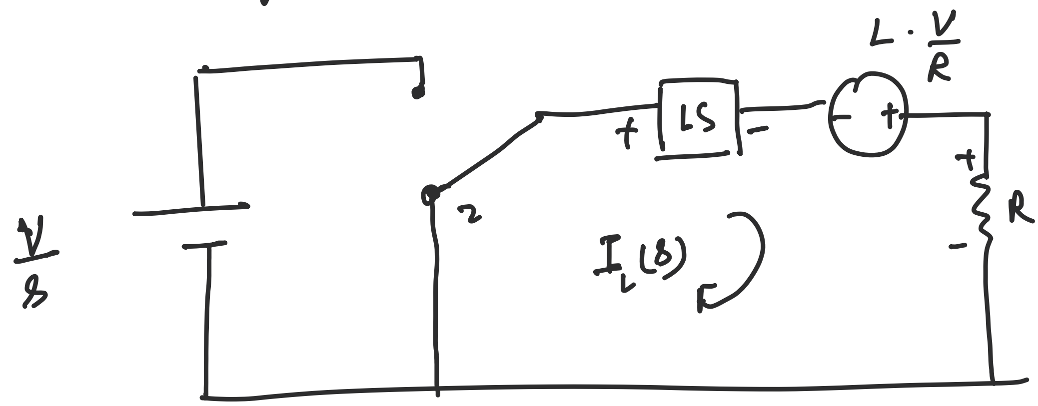
At $t = 0^-$



steady state (short ckt)

$$\Rightarrow i_L(0^-) = \frac{V}{R}$$

Transform Network for $t \geq 0$



$$I_L(s) \cdot LS - \frac{VL}{R} + I_L(s) R = 0$$

$$\Rightarrow \quad I_L(s)\left[LS + R\right] = \frac{VL}{R}$$

$$\Rightarrow \quad I_L(s) = \frac{VL}{R(LS+R)} = \frac{V\cancel{L}}{R\cancel{L}\left(s + \frac{R}{L}\right)}$$

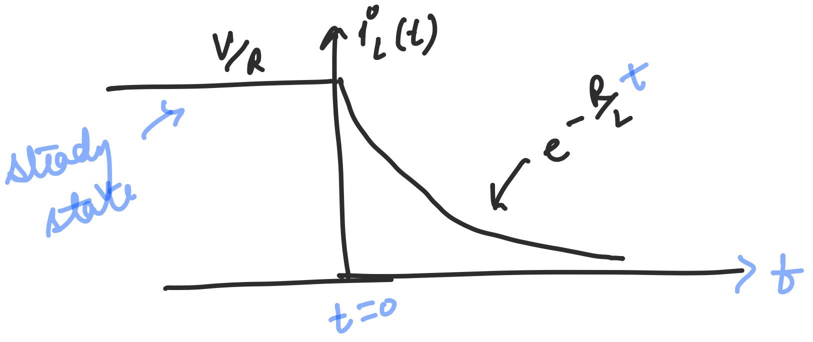
$$\therefore \quad \boxed{i_L(t) = \frac{V}{R} e^{-\frac{R}{L}t}, \quad t \geq 0}$$

To find $V_L(t)$

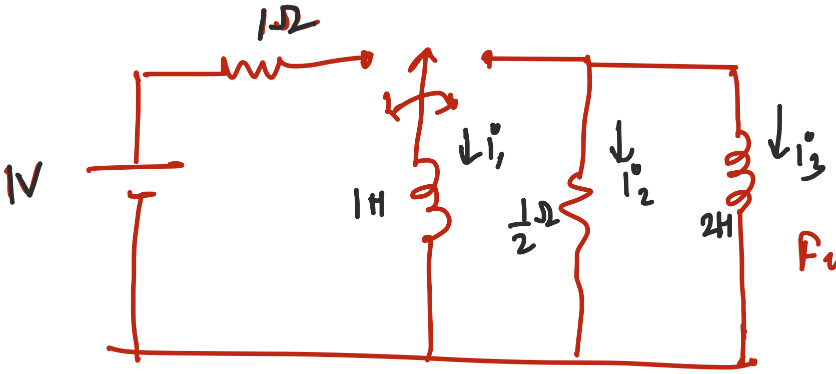
$$V_L(t) = I_L(s) LS - \frac{VL}{R} = \frac{VLs}{R\left(s+\frac{R}{L}\right)} - \frac{VL}{R}$$

OR

$$V_L(t) = L\frac{di}{dt} = L\frac{d}{dt}\left[\frac{V}{R} e^{-\frac{R}{L}}\right]$$



eg ②

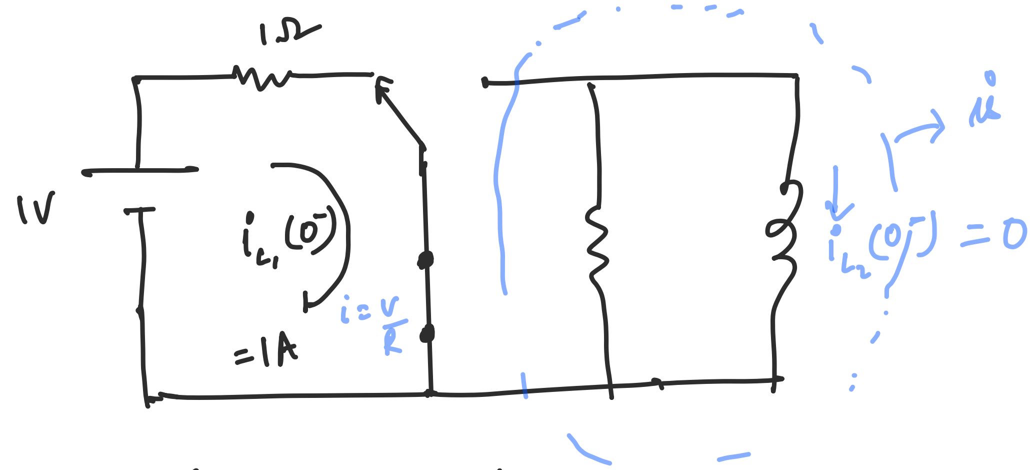


a → b at t = 0

$t = 0^- \Rightarrow$ steady state

Find $v(t)$, $i_1(t)$, $i_2(t)$, $i_3(t)$ for $t \geq 0$

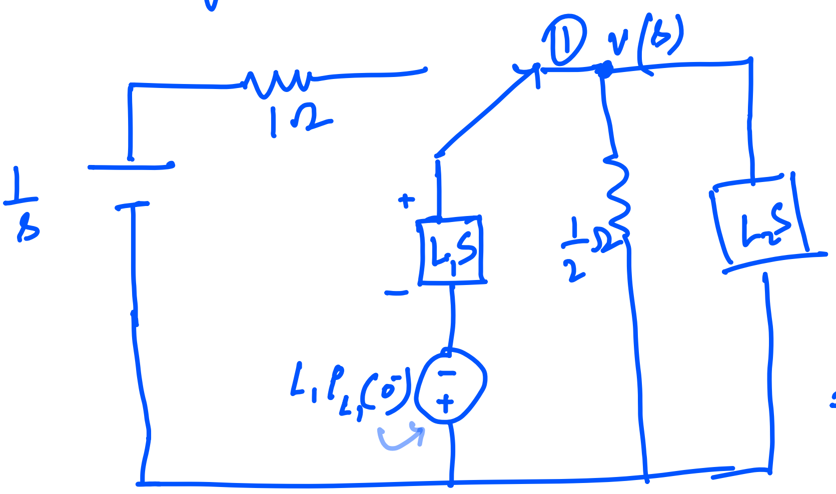
At $t=0^-$ (steady state)



is disconnected

$\therefore\ i_{L_1}(0^-) = 1\ A$

Transform network for $t \geq 0$



KCL at ① :-

$$\frac{V(s) - (-1)}{s} + \frac{V(s)}{1/2} + \frac{V(s)}{2s} = 0$$

$$2V(s) + 2 + 4s\,V(s) + V(s) = 0$$

$$\Rightarrow V(s)\,[4s + 3] = -2$$

$$V(s) = \frac{-2}{4\left(s + \frac{3}{4}\right)}$$

$$\boxed{v(t) = \frac{-1}{2} e^{-3/4\,t}} \quad t \geq 0$$

For $i_1(t)$

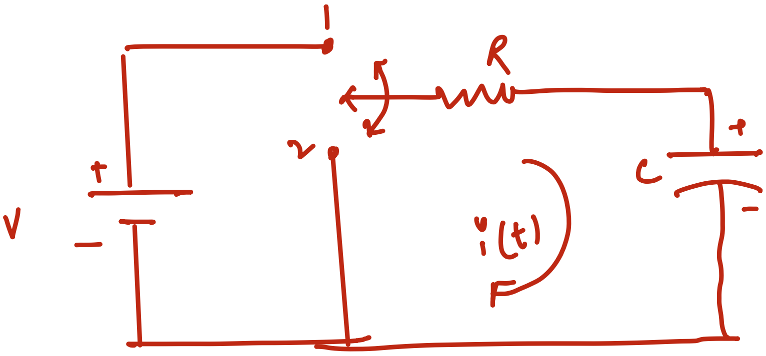
$$i_1(t) = \frac{1}{L}\int_{-\infty}^{t} v(t)\,dt = \frac{1}{L}\int_{-\infty}^{0} v(t)\,dt + \frac{1}{L}\int_{0}^{t} v(t)\,dt$$

$$i_1(t) = 1\,A + \int_0^t \left(-\frac{1}{2}\right) e^{-\frac{3}{4}t}\,dt$$

$$= \frac{1}{3} + \frac{2}{3} e^{-3/4\,t}$$

Ans.

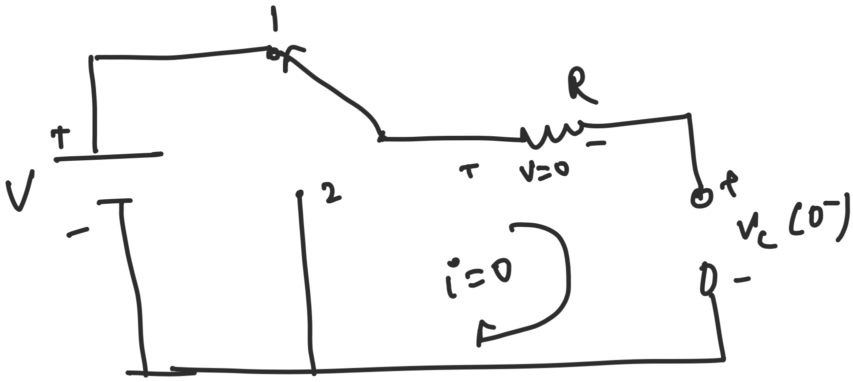
eg



$1 \to 2$ at $t=0$, with $t(0^-)$ = steady state

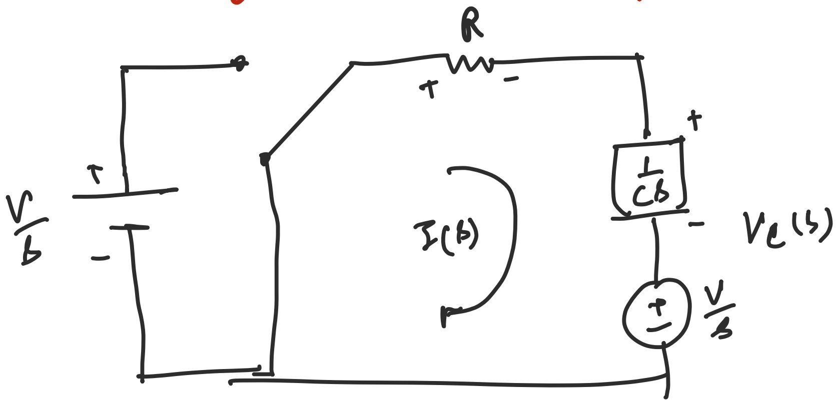
-find $i(t)$ & $v(t)$ for $t \geq 0$

At $t=0^-$



$\therefore V_c(0^-) = V$

Transform network for $t \geq 0$



KVL

$$RI(s) + \frac{1}{Cs} I(s) + \frac{V}{s} = 0$$

$$I(s)\left[R + \frac{1}{Cs}\right] = -\frac{V}{s}$$

$$\Rightarrow I(s) = -\frac{V \cdot Cs}{s(RCs+1)}$$

$$\Rightarrow I(s) = -\frac{VC}{RC\left(s + \frac{1}{RC}\right)}$$

$$\boxed{i(t) = -\frac{V}{R} e^{-\frac{1}{RC}t}}$$

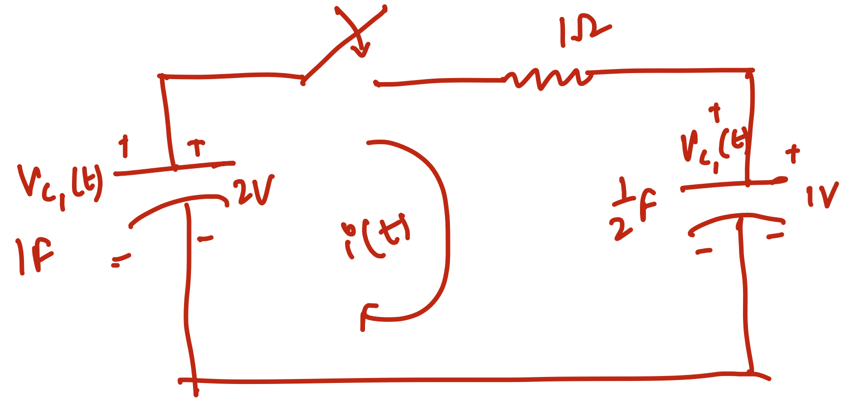
To find $v(t)$:-

$$iR + v(t) = 0$$

$$\therefore v(t) = -iR = \frac{V}{R} e^{-t/RC} \cdot R$$

$$v(t) = V \cdot e^{-t/RC}$$

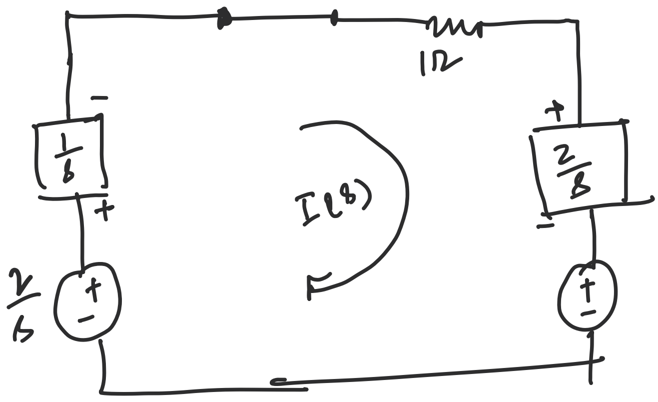
eg



find $i(t)$, $V_{c_1}(t)$, $V_{c_2}(t)$ for $t \geq 0$

$$V_{c_1}(0^-) = 2V \quad , \quad V_{c_2}(0^-) = 1V$$

Transform network for $t>0$



KVL

$$-\frac{2}{s} + I(s)\left(\frac{1}{s}\right) + 1 \cdot I(s) + \frac{2}{s} I(s) + \frac{1}{s} = 0$$

$$\Rightarrow I(s)\left\{\frac{1}{s} + 1 + \frac{2}{s}\right\} = \frac{2}{s} - \frac{1}{s}$$

$$\Rightarrow I(s)\left(\frac{1+s+2}{s}\right) = \frac{1}{s}$$

$$\therefore I(s) = \frac{1}{s+3}$$

$$\boxed{\therefore i(t) = e^{-3t}} \quad t \geq 0$$

To find $V_{c_1}(s)$:-

$$V_{c_1}(s) = -I(s)\left(\frac{1}{s}\right) + \frac{2}{s} = \frac{-1}{s(s+3)} + \frac{2}{s}$$

$$= -1 \, \frac{1}{3}\left\{\frac{1}{s} - \frac{1}{s+3}\right\} + \frac{2}{s} = -\frac{1}{3}\left(\frac{1}{s}\right) + \frac{1}{3}\left(\frac{1}{s+3}\right) + \frac{2}{s}$$

$$V_{c_1}(t) = -\frac{1}{3} + \frac{1}{3}e^{-3t} + 2 = \frac{5}{3} + \frac{1}{3}e^{-3t} \quad , \quad t \geq 0$$

APPLICATION OF LAPLACE X-FORM

eg A system is described by integral differential eqⁿ.

$$\frac{dy(t)}{dt} + 4y(t) + 10\int_{-\infty}^{t} y(\tau)\,d\tau = 5x(t)$$

Find response $y(t)$ for input $x(t) = u(t)$ if intial conditions are $y(0^-) = 1$, $\int_{-\infty}^{0} y(\tau)\,d\tau = 2$

We know,

$$x(t) \longrightarrow X(s)$$

$$\frac{d}{dt}x(t) \longrightarrow sX(s) - x(0^-)$$

$$\int_{-\infty}^{t} x(\tau)\,d\tau \longleftrightarrow \frac{X(s)}{s} + \frac{\int_{-\infty}^{0} x(\tau)\,d\tau}{s}$$

$$sY(s) - y(0^-) + 4X(s) + 10\left[\frac{Y(s)}{s} + \frac{\int_{-\infty}^{0} y(\tau)\,d\tau}{s}\right] = 5X(s)$$

$$\Rightarrow Y(s)\left[s + 4 + \frac{10}{s}\right] = \frac{5}{s} + y(0^-) - \frac{10\int_{-\infty}^{0} y(\tau)\,d\tau}{s}$$

$$\Rightarrow Y(s)\left[\frac{s^2 + 4s + 10}{s}\right] = \frac{5}{s} + 1 - \frac{10(2)}{s}$$

$$\Rightarrow Y(s)\left[\frac{s + 4s + 10}{s}\right] = 1 - \frac{15}{s} = \frac{s-15}{s}$$

$$\Rightarrow Y(s) = \frac{s-15}{s^2 + 4s + 10} = \frac{s-15}{s^2 + 4s + 4 + 6}$$

$$\Rightarrow Y(s) = \frac{s-15}{(s+2)^2 + (\sqrt{6})^2} = \frac{s+2}{(s+2)^2 + (\sqrt{6})^3} - \frac{17 \times \sqrt{6}}{(s+2)^2 + (\sqrt{6})^2} \times \frac{1}{\sqrt{6}}$$

$$\Rightarrow \boxed{y(t) = e^{-2t}\cos\sqrt{6}t\,u(t) - \frac{17}{\sqrt{6}}e^{-2t}\sin\sqrt{6}t\,u(t)}$$

Ans.

→ Total Response :-

→ $c(t) = c(t)_{t.s} + c(t)_{s.s}$

transient state — steady state

→ $y(t)$ = complimentory func (CI) + Particular Integral (PI)

Zero input response

→ input = 0

→ response is due to initial condition

→ responsible for transitent response

→ Natural response

→ C.F.

Zero state response

→ when initial condition = 0

→ due to applied input

→ forced response

→ Steady state

→ P.I.

→ Transfer func^n = $\left.\dfrac{Y(s)}{X(s)}\right|_{IC=0}$

---

eg A system is described by a diff eq^n

$$\frac{d^2y(t)}{dt} + 5\frac{dy(t)}{dt} + 4y(t) = x(t)$$

System is intially relaxed and then excited by an input $x(t) = 10u(t)$. Find response $y(t)$

→ Initial conditions = 0 $\therefore y(0^-) = 0$

→ forced response due to $x(t) = 10u(t)$

$$s^2 Y(s) + 5s Y(s) + 4 Y(s) = \frac{10}{s}$$

$$Y(s)\left[s^2 + 5s + 4\right] = \frac{10}{s}$$

$$\therefore\ Y(s) = \frac{10}{s(s+4)(s+1)} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+4}$$

$$Y(s) = \frac{10/4}{s} - \frac{10/3}{s+1} + \frac{10/12}{s+4}$$

$$y(t) = \Big[\underbrace{\frac{10}{4}}_{\text{steady state}} - \underbrace{\frac{10}{3}e^{-t} + \frac{10}{12}e^{-4t}}_{\text{transient}}\Big]u(t)$$

[depends on 't']

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eg Find the response of the system represented by

$$\frac{d^2y(t)}{dt^2} + 7\frac{dy}{dt} + 10\,y(t) = 3x(t)$$

where $x(t) = u(t)$ & initial conditions $y(0^-) = 1$, $y'(0^-) = 2$

$$\left\{s^2 Y(s) - s\,y(0^-) - y'(0^-)\right\} + 7\left\{sY(s) - y(0^-)\right\} + 10Y(s) = 3X(s)$$

$$\Rightarrow\ Y(s)\left\{s^2 + 7s + 10\right\} - s\underbrace{y(0^-)}_{1} - \underbrace{y'(0^-)}_{2} - 7\underbrace{y(0^-)}_{1} = 3X(s)$$

$$\Rightarrow\ Y(s)\left[s^2 + 7s + 10\right] = 3X(s) + s + 2 + 7$$

$$\Rightarrow\ Y(s)\left(s^2 + 7s + 10\right) = 3X(s) + (s+9)$$

$$Y(s) = \underbrace{\frac{3X(s)}{s^2 + 7s + 10}}_{\text{Forced resp.}} + \underbrace{\frac{s+9}{s^2 + 7s + 10}}_{\text{Natural resp.}}$$

→ due to input $X(s)$

→ due to initial conditions.

$$= \frac{3}{s(s+2)(s+5)} + \frac{s+9}{(s+2)(s+5)}$$

$$Y(s) = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s+5} + \frac{D}{s+2} + \frac{E}{s+5}$$

$$Y(s) = \frac{A}{s} + \frac{B+D}{s+2} + \frac{C+E}{s+5}$$

$$Y(t) = [\underbrace{A}_{\text{steady state}} + \underbrace{(B+D)e^{-2t} + (C+E)e^{-5t}}_{\text{transient response.}}]\,u(t)$$

---

eg $X(s) = \log\left(\frac{s+4}{s+5}\right)$. Find inverse L.T $x(t)$.

We know,

$$x(t) \longleftrightarrow X(s)$$

$$t \cdot x(t) \longleftrightarrow (-1)\frac{dX(s)}{ds}$$

$$X(s) = \log(s+4) - \log(s+5)$$

$$\frac{dX(s)}{ds} = \frac{1}{s+4} - \frac{1}{s+5}$$

$$\therefore \quad -\frac{dX(s)}{ds} = \frac{1}{s+5} - \frac{1}{s+4}$$

Taking I.L.T both sides

$$t \cdot x(t) = e^{-5t}u(t) - e^{-4t}u(t)$$

$$\boxed{x(t) = \frac{1}{t}\left[e^{-5t}u(t) - e^{-4t}u(t)\right]}$$