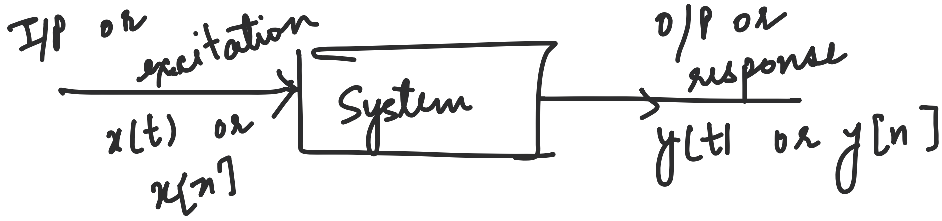
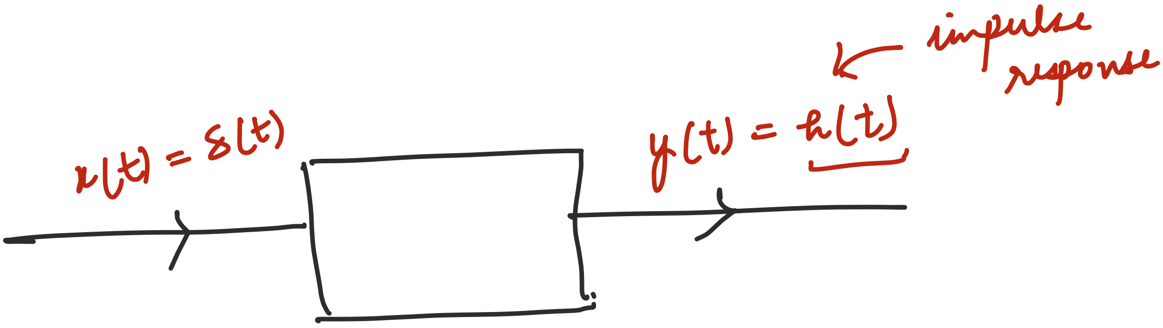
System :-





Impulse Response

Response of a system when impulse input is called impulse response.

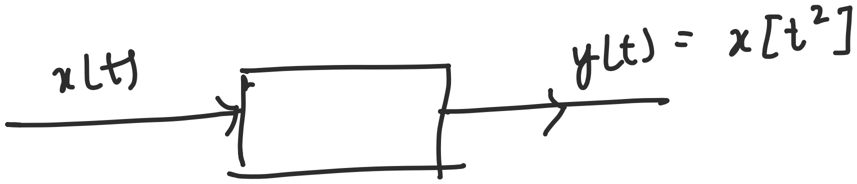
matlab

What o/p System is giving for impulse input !

Properties -:

(1) Static and Dynamic System -:

(Memoryless) (with memory)



A system is said to be static → If present output depends only on present input.

eg $y(t) = x(t^2)$

$y(1) = x(1)$

$y(2) = x(4)$ ← present output depends on future input at $(t = 4.)$

Therefore, system is dynamic!

eg $y(t) = x^2(t) = [x(t)]^2$

$y(0) = [x(0)]^2$

$y(-1) = [x(-1)]^2$

$y(2) = [x(2)]^2$

∴ system depends on present input and present output

∴ System is Static!!

(2) If present output depends

Causal	Non-Causal	Anti causal
on present or present + past inputs	on present + future or present + past + future or past + future inputs	only on future inputs
$h(t) = 0 \quad t<0$ $h[n] = 0 \quad n<0$	$h(t) \neq 0, \ t<0$ $h[n] \neq 0, \ n<0$	$h(t) = 0, \ t>0$ $h[n] = 0, \ n>0$

eg $y(t) = x^2[t]$

We know, this is static system

$\therefore$ present output depends only on present inputs.

$\therefore$ Static systems are always Causal.

(3) Time Variant and Invariant Systems :-

Invariant Systems :-

If time shift in input results in identical time shift in output without changing the nature of output

To check time invariancy :-

(1) Find $y(t-t_0)$ ...... delayed response

: .... replace 't' by '$t-t_0$'

(2) Find $T[x(t-t_0)] = y[(t, t_0)]$ ..... Response of the system for delayed input.

symbol for response

(write the given relation as it is and then provide a shift of t_0 in the input.

③ If $y(t-t_0) = y(t, t_0)$

... System is time invariant.

eg $y(t) = x(t^2)$

① find delayed response -:

$y(t-t_0) = x[(t-t_0)^2]$ ---- ①

② Response of the system for delayed input -:

$y(t, t_0) = x(t^2 - t_0)$ ---- ②

$\therefore$ $y(t-t_0) \neq y(t, t_0)$

$\therefore$ Time Variant!

eg $y(t) = t \cdot x(t)$

① find delayed response -:

$y(t-t_0) = (t-t_0)\, x(t-t_0)$ —— ①

② Find response of system for delayed input -:

$y(t, t_0) = t\; x(t-t_0)$ —— ②

$y(t-t_0) \neq y(t, t_0)$

$\therefore$ Time variant.

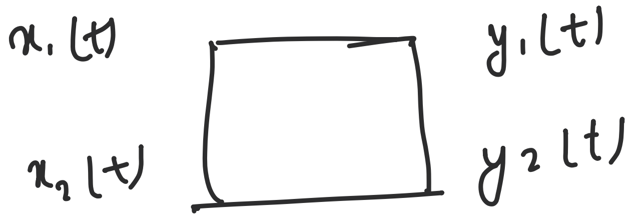
eg $y(t) = x^2(t)$

$y(t-t_0) = x^2(t-t_0)$ —— ①

$y(t, t_0) = x^2(t-t_0)$ —— ②

Since $y(t-t_0) = y(t, t_0)$ $\therefore$ Time invariant

Way to find for linearity :-



$y_3(t) = y_1(t) + y_2(t)$

$x_3'(t) = x_1(t) + x_2(t)$

$y_3'(t) =$ Find it .

$$\boxed{\text{If} \quad y_3'(t) = y_3(t)}$$

System is homogenous !!

~~eg~~ $y(t) = t \cdot x(t)$

$y_1(t) = t \cdot x_1(t)$

$y_2(t) = t \cdot x_2(t)$

$y_3(t) = y_1(t) + y_2(t) = t\, x_1(t) + t\, x_2(t)$

$= t\,(x_1(t) + x_2(t))$ —— 1

$x_3'(t) = x_1(t) + x_2(t)$

$y_3'(t) = t \cdot x_3'(t)$

$y_3'(t) = t\,[x_1(t) + x_2(t)]$ —— 2

$y_3'(t) = y_3(t)$

$\therefore$ Additive

$$\therefore\ y(t, t_0) = \int_{-\infty}^{t-t_0} x(\lambda)\, d\lambda \longrightarrow (3)$$

Comparing (1) & (3)

Since τ & λ are dummy variables

$\therefore$ have no effect on integrals

Since $\boxed{y(t-t_0) = y(t, t_0)}$ $\longrightarrow$ time invariant!

(4) Linear & Non Linear Systems :-

Linear $\longrightarrow$ Superposition.

Homogeneity

$x(t) \longrightarrow y(t)$

$\alpha\, x(t) \longrightarrow \alpha\, y(t)$

This constant can be Real / Imag / Complex

Additivity

$x_1(t) \longrightarrow y_1(t)$

$x_2(t) \longrightarrow y_2(t)$

$x_1(t) + x_2(t) \longrightarrow y_1(t) + y_2(t)$

Combining both,

If $\boxed{\alpha x_1(t) + \beta x_2(t) \longrightarrow \alpha y_1(t) + \beta y_2(t)}$

Linear.

For a system to be linear,

(1) graph b/w O/P & I/P must be throughout a straight line passing through origin without having saturation or dead time.

(2) If system is represented by differential eq^n then the eq^n must be linear.

$$a_n \frac{d^n x(t)}{dt^n} + a_{n-1} \frac{d^{n-1} x(t)}{dt^{n-1}} + \cdots\cdots + a_0 x(t) = y(t)$$

(3) System must follow zero input zero output criteria

$$x(t) \longrightarrow y(t)$$

$$0 = 0 \cdot x(t) \longrightarrow 0 \cdot y(t) = 0$$

~~eg~~ $y(t) = x(t^3)$

Additivity

$$x_1(t) \longrightarrow x_1(t^3) = y_1(t)$$

$$x_2(t) \longrightarrow x_2(t^3) = y_2(t)$$

$$y_3(t) = y_1(t) + y_2(t)$$

$$= x_1(t^3) + x_2(t^3) \text{ ——— } (1)$$

let $x'_3(t) = x_1(t) + x_2(t)$

$$y'_3(t) = x'_3(t^3)$$

$$= x_1(t^3) + x_2(t^3) \text{ ——— } (2)$$

$$y_3'(t) = y_3(t) \longrightarrow \text{Additive.}$$

Homogeneous

$$y(t) = x(t^3)$$

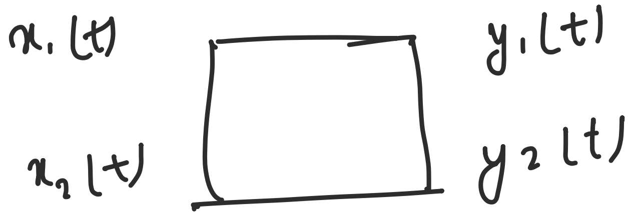
$$\alpha y(t) = \alpha x(t^3) \text{ ——— } (1)$$

If i/p is $\alpha x(t)$

then o/p $y'(t) = \alpha x(t^3)$ ——— (2)

$\therefore$ Homogeneous.

Way to find for linearity :-



$$y_3(t) = y_1(t) + y_2(t)$$

$$x_3'(t) = x_1(t) + x_2(t)$$

$$y_3'(t) = \text{Find it.}$$

$$\boxed{\text{If} \quad y_3'(t) = y_3(t)}$$

System is homogeneous!!

eg $y(t) = t \cdot x(t)$

$$y_1(t) = t \cdot x_1(t)$$

$$y_2(t) = t \cdot x_2(t)$$

$$y_3(t) = y_1(t) + y_2(t) = t\, x_1(t) + t\, x_2(t)$$

$$= t\,(x_1(t) + x_2(t)) \quad \text{—— (1)}$$

$$x_3'(t) = x_1(t) + x_2(t)$$

$$y_3'(t) = t \cdot x_3'(t)$$

$$y_3'(t) = t\,[x_1(t) + x_2(t)] \quad \text{—— (2)}$$

$$y_3'(t) = y_3(t)$$

$\therefore$ Additive

Homogenity

$$\boxed{\begin{array}{c} x(t) \longrightarrow y(t) \\ \alpha\, x(t) \longrightarrow \alpha\, y(t) \end{array}}$$

$y(t) = t\, x(t)$

$\boxed{x(t) \longrightarrow \alpha\, x(t)}$

$y'(t) = t\,[\alpha\, x(t)] = \alpha\, t\, x(t)$

$\alpha \cdot y(t) = \alpha\, t\, x(t)$

$\therefore$ Homogneous.

since system is Homogneous & additive $\therefore$ it is linear !!

eg $y(t) = x(t^3)$

Additive

$x_1(t) \rightarrow y_1(t) = x_1(t^3)$

$x_2(t) \rightarrow y_2(t) = x_2(t^3)$

$y_3(t) = x_1(t^3) + x_2(t^3)$ — (1)

$x_3'(t) = x_1(t^3) + x_2(t^3)$

$y_3'(t) = x_3'(t)$

$y_3'(t) = x_1(t^3) + x_2(t^3)$ — (2)

Since (1) = (2)

$\therefore$ Additive.

Homogenity

$y(t) = x(t^3)$

$\alpha\, y(t) = \alpha\, x(t^3)$ — (2)

If input is $\alpha\, x(t)$ then output $y'(t) = \alpha\, x(t^3)$ — (2)

$\therefore$ Homonenity

eg $y(t) = x[\sin t]$

We know, $\alpha x_1(t) + \beta x_2(t) \longrightarrow \alpha y_1(t) + \beta y_2(t)$

$x_1 \longrightarrow y_1(t) = x_1[\sin t]$

$y_2 \longrightarrow y_2(t) = x_2[\sin t]$

$\therefore \alpha y_1(t) + \beta y_2(t) = \alpha x_1[\sin t] + \beta x_2 \cdot [\sin t]$ — (1)

Let $x_3'(t) = \alpha x_1(t) + \beta x_2(t)$

$y_3'(t) = x_3'[\sin t] = \alpha x_1[\sin t] + \beta x_2[\sin t]$ — (3)

$\therefore y_3'(t) = y_3(t)$ $\therefore$ linear!!

eg $y(t) = x[e^t]$

Add

$x_1(t) \longrightarrow y_1(t) = x_1[e^t]$

$x_2(t) \longrightarrow y_2(t) = x_2[e^t]$

$y_3 = y_1(t) + y_2(t) = x_1[e^t] + x_2[e^t]$ — (1)

$x_3'(t) = x_1(t) + x_2(t)$

$y_3'(t) = x_3'(e^t) = x_1(e^t) + x_2(e^t)$ — (2)

(1) & (2) are equal $\therefore$ Additive!

Homo

$y(t) = x(e^t)$

$y_1(t) = \alpha y(t) = \alpha x[e^t]$ — (1)

For input $\alpha x(t)$, the output will be

$y_1'(t) = \alpha x[e^t]$ — (2)

$\therefore y_1(t) = y_1'(t)$

Homog.

$\therefore$ System is linear!

eg $y(t) = \sin t \cdot x(t)$

$x_1 \longrightarrow y_1(t) = \sin t \ x_1(t)$

$x_2 \longrightarrow y_2(t) = \sin t \ x_2(t)$

$y_3(t) = y_1(t) + y_2(t) = \sin t \ x_1(t) + \sin t \ x_2(t)$

$y_3(t) = \sin t \left[x_1(t) + x_2(t)\right]$ —— ①

$x_3'(t) = x_1(t) + x_2(t)$

$y_3'(t) = \sin t \ x_3'(t) = \sin t \left[x_1(t) + x_2(t)\right]$ —— ②

① = ② $\quad \therefore$ Additive !

It is homogenous too !

eg $y(t) = 4x(t)$

$x_1 \longrightarrow y_1(t) = 4x_1(t)$

$x_2 \longrightarrow y_2(t) = 4x_2(t)$

$y_3(t) = \alpha y_1(t) + \beta y_2(t)$

$= \alpha 4 x_1(t) + \beta 4 x_2(t)$

$y_3(t) = 4\left[\alpha x_1(t) + \beta x_2(t)\right]$ —— ①

$x_3'(t) = \alpha x_1(t) + \beta x_2(t)$

$y_3'(t) = 4x_3'(t)$

$= \alpha 4 x_1(t) + \beta 4 x_2(t)$ —— ②

① = ② $\quad \therefore$ linear

eg $y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$

$x_1 \longrightarrow y_1(t) = \int_{-\infty}^{t} x_1(\tau)\, d\tau$

$x_2 \longrightarrow y_2(t) = \int_{-\infty}^{t} x_2(\tau)\, d\tau$

$y_3(t) = y_1(t) + y_2(t)$

$= \int_{-\infty}^{t} x_1(\tau)\, d\tau + \int_{-\infty}^{t} x_2(\tau)\, d\tau$

$$= \int_{-\infty}^{t} \{ x_1(\tau) + x_2(\tau) \} d\tau \quad \text{—— (1)}$$

$$x_3'(t) = x_1(t) + x_2(t)$$

$$y_3'(t) = \int_{-\infty}^{t} x_3'(\tau) d\tau = \int_{-\infty}^{t} [x_1(\tau) + x_2(\tau)] d\tau \quad \text{—— (2)}$$

$\therefore$ Add

$$y_1(t) = \alpha\, y(t) = \alpha \int_{-\infty}^{t} x(\tau) d\tau \quad \text{—— (1)}$$

$$y_1'(t) = \int_{-\infty}^{t} \alpha\, x(\tau) d\tau = \alpha \int_{-\infty}^{t} x(\tau) d\tau \quad \text{—— (2)}$$

$\therefore$ Hom.

eg $y(t) = \text{Even}[x(t)]$ [for odd $[x(t)]$]

$$y(t) = \frac{x(t) + x(-t)}{2}$$

$$y(t) = \frac{1}{2} x(t) + \frac{1}{2} x(-t)$$

linear from prev. results.

eg $y(t) = x(t-2) + x(t+2)$

$x_1 \rightarrow$ $y_1(t) = x_1(t-2) + x_1(t+2)$

$x_2 \rightarrow$ $y_2(t) = x_2(t-2) + x_2(t+2)$

$$y_3(t) = \alpha\, y_1(t) + \beta\, y_2(t)$$

$$= \alpha [x_1(t-2) + x_1(t+2)] + \beta [x_2(t-2) + x_2(t+2)] \longrightarrow (2)$$

$$x_3'(t) = \alpha x_1(t) + \beta x_2(t)$$

$$y_3'(t) = x_3'(t-2) + x_3'(t+2)$$

$$y_3'(t) = \alpha x_1(t-2) + \beta x_2(t-2) + \alpha x_1(t+2) + \beta x_2(t+2)$$

$$= \alpha\left[x_1(t-2) + x_1(t+2)\right] + \beta\left[x_2(t-2) + x_2(t+2)\right] \quad \text{—— } (2)$$

$(1) = (2)$ $\therefore$ Linear.

eg $y(t) = \begin{cases} x(t-4) & , t>0 \\ x(t+4 & , t>0 \end{cases}$

$$y(t) = \underbrace{u(t)\,x(t-4)}_{\text{linear}} + \underbrace{u(-t)\,x(t-4)}_{\text{linear}}$$

$\therefore$ Overall linear !!

eg $y(t) = \cos[x(t)]$

$x_1 \longrightarrow y_1(t) = \cos[x_1(t)]$

$x_2 \longrightarrow y_2(t) = \cos[x_2(t)]$

$y_3(t) = \alpha y_1(t) + \beta y_2(t) \Rightarrow \alpha \cos[x_1(t)] + \beta \cos[x_2(t)]$ —— (1)

$x_3'(t) = \alpha x_1(t) + \beta x_2(t)$

$y_3'(t) = \cos[x_3'(t)] = \cos[\alpha x_1(t) + \beta x_2(t)]$ —— (2)

$(1) \neq (2)$ $\therefore$ non-linear !

eg $y(t) = x^2(t)$

this means $\longrightarrow$ $x(t) \longrightarrow \boxed{[\ ]^2} \longrightarrow y(t)$

$y_1(t) = [x_1(t)]^2$

$y_2(t) = [x_2(t)]^2$

$y_3(t) = \alpha[x_1(t)]^2 + \beta[x_2(t)]^2$ —— (1)

$x_3'(t) = \alpha x_1(t) + \beta x_2(t)$

$$y_3'(t) = [x_3'(t)]^2 = [\alpha x_1(t) + \beta x_2(t)]^2 \quad \text{——— (2)}$$

$$y_3'(t) \neq y_3(t) \qquad \therefore \text{ non linear}$$

$\longrightarrow$ $y(t) = 4t + x(t)$

$y(t) = 2 + x(t)$

Non linear

$\therefore$ Zero input zero output is not followed.

$\longrightarrow$ $y(t) = x^*(t) = [x(t)]^*$

$\hookrightarrow$ $x(t) \longrightarrow \boxed{[\;\;]^*} \longrightarrow y(t)$

$x_1 \longrightarrow y_1(t) = [x_1(t)]^*$

$x_2 \longrightarrow y_2(t) = [x_2(t)]^*$

$$y_3(t) = \alpha y_1(t) + \beta y_2(t) = \alpha [x_1(t)]^* + \beta [x_2(t)]^* \quad \text{——— (1)}$$

$$x_3'(t) = \alpha x_1(t) + \beta x_2(t)$$

$$y_3'(t) = [x_3'(t)]^* = [\alpha x_1(t) + \beta x_2(t)]^* \quad \text{——— (2)}$$

$$y_3(t) \neq y_3'(t)$$

$\therefore$ Non - linear

eg $y(t) = \text{Real}[x(t)]$

$y(t) = \text{Imag}[x(t)]$

$y(t) = |x(t)|$

$y(t) = \text{Sgn}[x(t)]$

$y(t) = \text{Sinc}[x(t)]$

$y(t) = e^{x(t)}$

$y(t) = \log[x(t)]$

$\longrightarrow$ Non linear systems

Note : For differential Functions :-

$$\frac{d^2y(t)}{dt^2} + t\frac{dy(t)}{dt} + 4y(t) = x(t) \rightarrow \text{Linear}$$

since coeff are constant

$$\frac{d^2y(t)}{dt} + y(t)\frac{dy(t)}{dt} + 4y(t) = x(t) \rightarrow \text{Non Linear}$$

If coeff are func^n of dependent variable. (i.e. $y(t)$)

Following Systems are Linear :-

(1) $y(t) = t\,x(t)$

$y(t) = x[\sin t]$

$y(t) = x[e^t]$

(2) $y(t) = t\,x(t)$

$y(t) = \sin t\; x(t)$

$y(t) = 4x(t)$

$y(t) = x(-t)$

(3) $y(t) = \int_{-\infty}^{t} x(\tau)\,d\tau$

$y(t) = \frac{d\,x(t)}{dt}$

(4) $y(t) = \text{Even}[x(t)]$

$y(t) = \text{Odd}[x(t)]$

(5) $y(t) = a\,x(t+t_0) + b\,x(t-t_0)$

(6) $y(t) = \begin{cases} x(t-4) & t>0 \\ x(t+4) & t\leq 0 \end{cases}$

Invertible and Non Invertible Systems :-

→ If distinct inputs produces distinct outputs

→ If input can be determined by observing the output.

eg $y(t) = 4x(t)$

$x(t)$	$y(t) = 4x(t)$
0	0
-1	-4
$+1$	$+4$

Invertible

$y(t) = x^2(t)$

$x(t)$	$y(t) = x^2(t)$
0	0
1	1
-1	1
2	4
-2	4

Non Invertible

eg $y(t) = |x(t)|$

$x(t)$	$y(t) = \|x(t)\|$
0	0
-1	1
$+1$	1
$+j$	1
$-j$	1

$\therefore$ non - Invertible

We will use test signals :-

$u(t)$	$u(n)$
$-u(t)$	$-u(n)$
$\delta(t)$	$\delta(n)$
$-\delta(t)$	$-\delta(n)$
$2u(t)$	$2u(n)$
$2\delta(t)$	$2\delta(n)$

eg $y(t) = \sin t \, x(t)$

For $x(t) = \delta(t)$

$y(t) = \sin t \, \delta(t)$

$y(t) = \sin(0)\, \delta(t) = 0$

For $x(t) = 4\delta(t)$

$y(t) = \sin t \; 4\delta(t)$

$= 4 \sin 0 \, \delta(t) = 0$

$\therefore$ non- Invertible

eg $y(n) = x(n)\, x(n-1)$

For i/p $u(n)$

$y(n) = u(n)\, u(n-1)$

for i/p $-u(n)$

$y(n) = \{-u(n)\}\{-u(n-1)\}$

$= u(n)\, u(n-1)$

$\therefore$ Non Invertible

eg $y(n) = \begin{cases} x(n+2) & n \geq 0 \\ x(n & n < -1 \end{cases}$

For $x(n) = \delta(n)$

$y(n) = \delta(\underbrace{n+2}_{n=-2})$, for $n \geq 0$

which is not within range

$y(n) = 0$

For $x(n) = 2\delta(n)$

$y(n) = 2 \cdot \delta(n+2)$ for $n \geq 0$

$= 0$

$\therefore$ non invertible

Stable and Unstable System (BIBO)

Bounded input Bounded Output Criterion -:

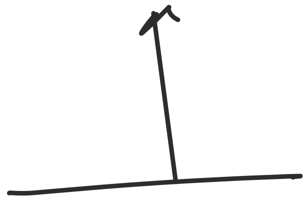
BIBO stability -:

→ The output of the system must be bounded for bounded inputs.

for, $$\boxed{\begin{array}{l} 0 \leq |x(t)| < \infty \\ 0 \leq |y(t)| < \infty \end{array}}$$ then

→ BIBO implies that impulse response must tend to zero, as time 't' tend to ∞.

$$\int_{-\infty}^{\infty} h(\tau)\, d\tau < \infty$$



→ $$c(t) = \underbrace{c(t)_{t.R}}_{\text{Transient response}} + \underbrace{c(t)_{s.s.R}}_{\text{steady state response}}$$

total response

$$\lim_{t \to \infty} c(t)\Big|_{t.R} = 0$$

eg $$c(t) = \underset{\uparrow \atop S.S}{4} + \underbrace{e^{-2t} + e^{-3t}}_{T.R.}$$ ← not stable if there are T.R. in system.

→ Test Signals :-

$x(t) = u(t)$

$\sin(t)$, $\cos(t)$

eg $y(t) = x^2(t)$

for $x(t) = u(t)$

$y(t) = [u(t)]^2 = u(t)$

BIBO stable.

eg $y(t) = \sin t \cdot x(t)$

for input $x(t) = u(t)$

$|y(t)| = \underbrace{|\sin t|}_{<1} \underbrace{|u(t)|}_{1}$

$|y(t)| \leqslant |x(t)|$

Stable.

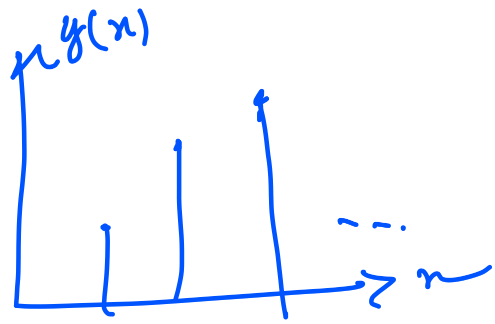
eg $y(n) = n \cdot x(n)$

for $x(n) = u(n)$

$y(n) = n u(n) = r(n)$

It goes till ∞

$\therefore$ unstable!

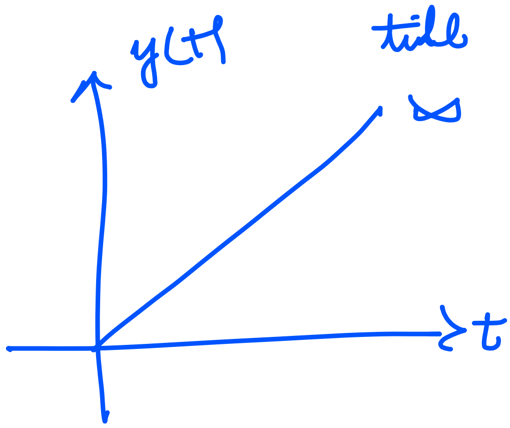


eg $y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$

for $x(t) = u(t)$

$y(t) = \int_{-\infty}^{t} u(\tau)\, d\tau = r(t)$

$\therefore$ unstable.



eg $y(t) = \int_{-\infty}^{\infty} x(t) \sin 2t \, dt$

$x(t) = \sin 2t$

$$y(t) = \int_{-\infty}^{t} \sin^2 2t \, dt = \int_{-\infty}^{t} \frac{1-\cos 4t}{2} \, dt$$

$$= \frac{1}{2}\left[\underbrace{\int_{-\infty}^{t} dt} - \int_{-\infty}^{t} \cos 4t \, dt\right]$$

↳ This value will be infinite

$\therefore$ not BIBO

$\therefore$ Unstable.