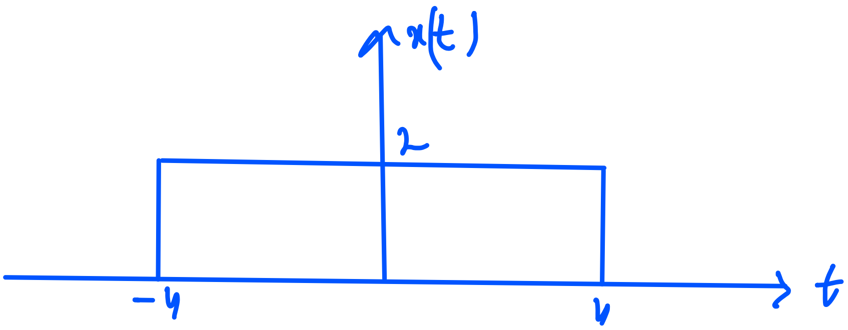
# Classification of signals -:

1) Even and Odd signal -:

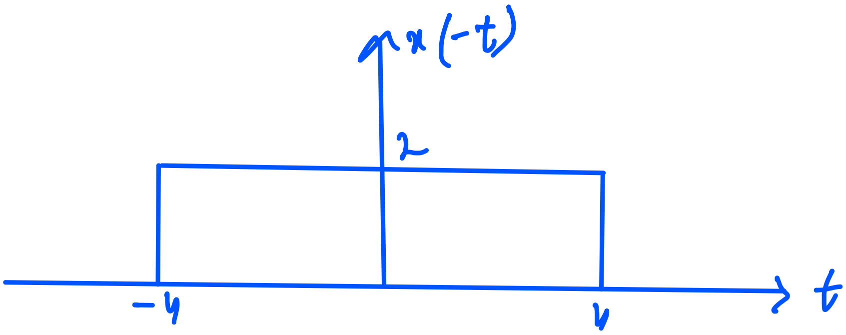
→ If $x(-t) = x(t)$, then signal is Even signal

→ If $x(-t) = -x(t)$, " " " Odd signal

eg

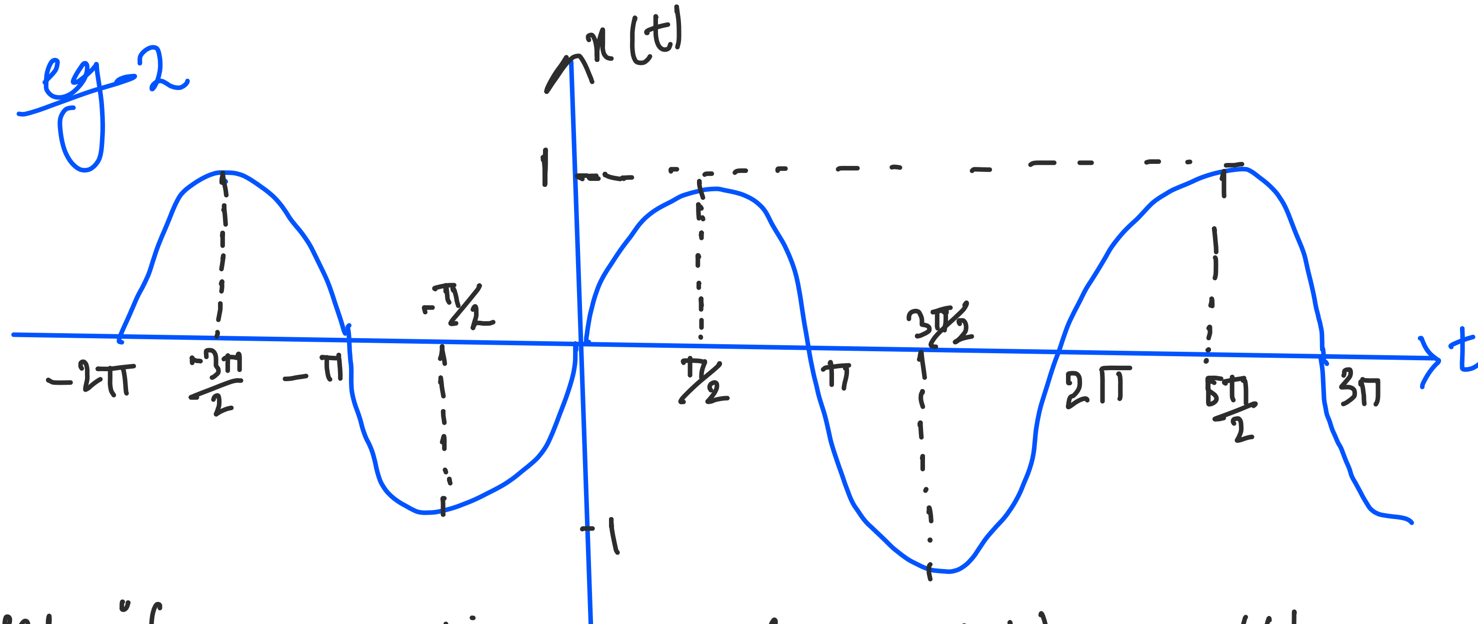



$\Downarrow$




Hence $x(t)$ is an even signal.

eg-2




Here if you do time reversal, $x(-t) = -x(t)$

∴ odd signal!!

Note -1 If a signal is mirrored across y-axis, it is an even signal.

→ A signal can be neither even nor odd.

→ To convert any arbitrary signal, which is neither even nor odd, into it's equivalent even and odd parts :-

$$\boxed{x_e(t) = \frac{x(t) + x(-t)}{2}}$$

$$\boxed{x_o(t) = \frac{x(t) - x(-t)}{2}}$$

eg① Check if following signal is even or odd :-

$x(t) = 2-3t+4t^2$

$x(-t) = 2+3t+4t^2$

This signal is neither even nor odd.

To convert to even signal,

To convert to odd signal,

eg 2 $\quad x(t) = e^{-t}$

Find even and odd parts of $x(t)$

$$x(-t) = e^{t}$$

$$x_e(t) = \frac{x(t) + x(-t)}{2} = \frac{e^{-t} + e^{t}}{2} = \cosh t$$

$$x_o(t) = \frac{x(t) - x(-t)}{2} = \frac{e^{-t} - e^{t}}{2} = -\sinh x$$

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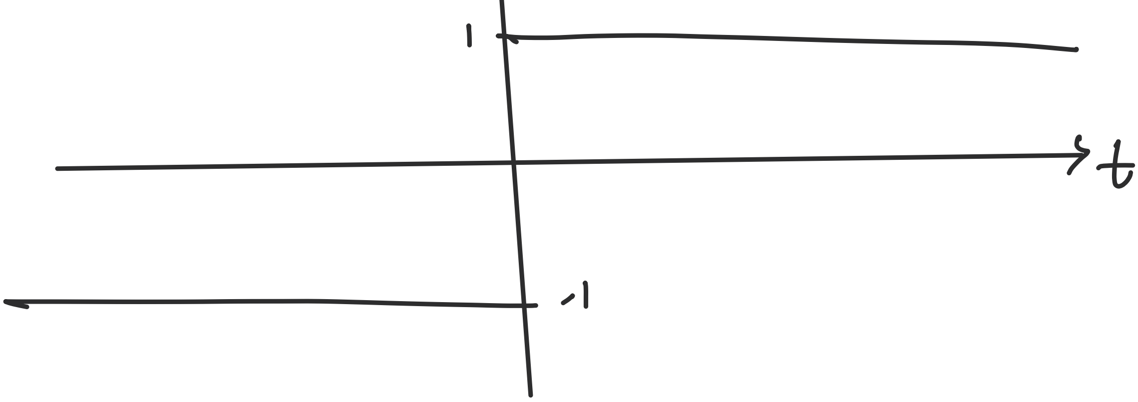
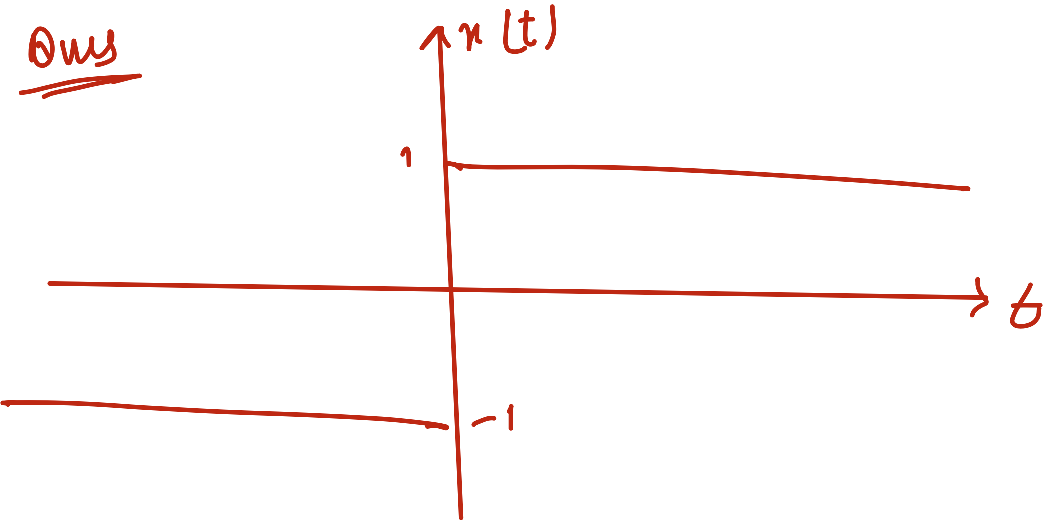
# Signum func$^n$ :-

$$x(t) = u(t) - u(-t) \Rightarrow Sgn(t)$$




**Ques**




Even and odd parts of $u(t)$ can be represented in form of $x(t)$ as

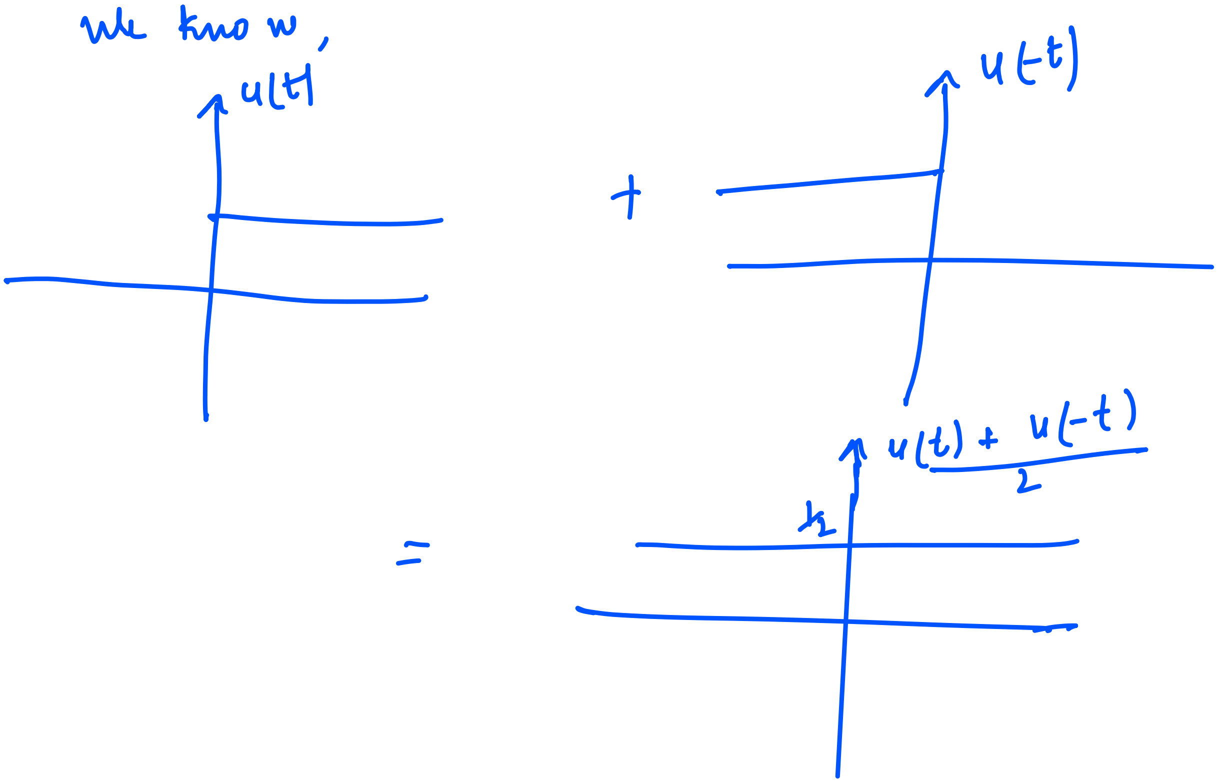
1) $\frac{1}{2}$, $\frac{1}{2}x(t)$
2) $-\frac{1}{2}$, $\frac{1}{2}x(t)$
3) $-\frac{1}{2}$, $-\frac{1}{2}x(t)$
4) $\frac{1}{2}$, $-\frac{1}{2}x(t)$

To calculate even part,

we know, $$x_e(t) = \frac{x(t) + x(-t)}{2}$$

Therefore $$u_e(t) = \frac{u(t) + u(-t)}{2}$$

we know,




divided by 2

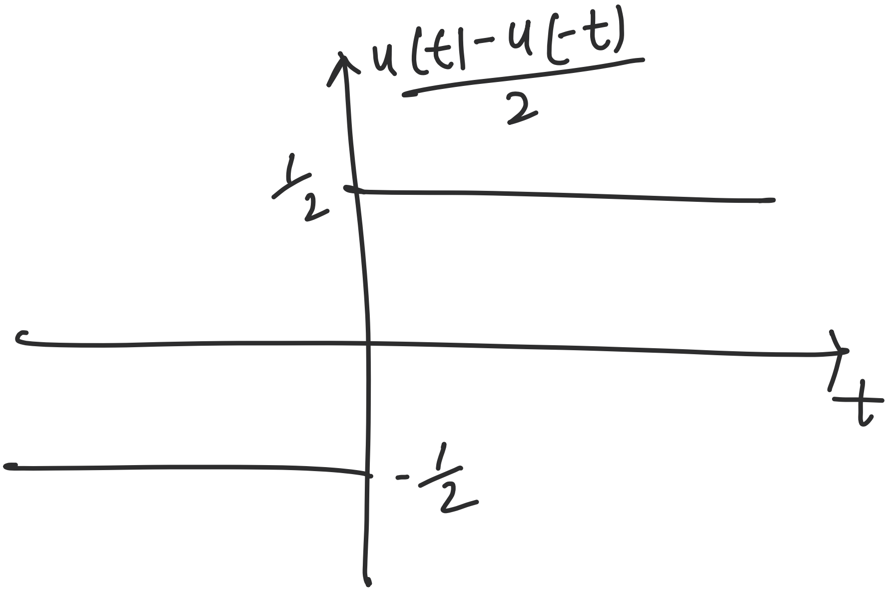
$$\boxed{u_e(t) = \frac{u(t) + u(-t)}{2} = \frac{1}{2}}$$

To calculate odd part,

We know, $$x_o(t) = \frac{x(t) - x(-t)}{2}$$

$$u_o(t) = \frac{u(t) - u(-t)}{2} = \frac{1}{2} x(t)$$

(when compared to original func$^n$)




$\therefore$ (a) part is correct

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# Periodic and Aperiodic Signal -:

A signal is said to be periodic, if

→ It must exist for $-\infty \leq t \leq \infty$

→ It must repeat itself after some constant amount of time '$T$' → ($T$ = Fundamental time period).

fundamental time period → $$\boxed{T = \frac{2\pi}{\omega_0}}$$ ← fundamental frequency (rad/sec)

## 1) Sinusoidal Signals -:

→ General Representation -:

$$x(t) = A \sin(\omega_0 t + \phi)$$

$A$ → Amplitude

$\omega_0 t$ → phase angle

$\phi$ → phase shift

$$\frac{d(\text{phase})}{dt} \doteq \frac{d(\omega_0 t)}{dt} = \omega_0 \quad \underline{\text{frequency}}$$

→ Shifting have no effect on periodicity.

→ Combination of periodic Signals :- will be periodic if ratio of individual time periods is a <u>rational number</u>.

eg $x = \underbrace{4\cos 2\pi t}_{T_1} + \underbrace{3\sin 6t}_{T_2} + \underbrace{5\cos 60\pi t}_{T_3}$

If $\frac{T_2}{T_1} = \frac{T_3}{T_2} = \frac{T_1}{T_3} \Rightarrow$ Rational

Then, period of resultant signal
$T =$ LCM of $[T_1, T_2, T_3 \ldots\ldots]$

or freq $\omega_0 =$ GCD or HCF of $[\omega_{0_1}, \omega_{0_2}, \omega_{0_3} \ldots\ldots]$

**Note** :- In the form of ratio of integers $\frac{p}{q}$ = & if ratio is converted into decimal, then it must be terminating or repeating decimal.

What is meant by rational number? →

eg $\frac{T_1}{T_2} = \frac{10}{3} = 3.333$ → rational

$\frac{T_1}{T_2} = \frac{10}{4} = 2.5$ → Terminating → rational

$$\frac{T_1}{T_2} = \pi = \frac{22}{7} = 3.1428\ldots \quad \neq \text{rational}$$

→ $\pi$, $e$ will be commonly used to make it irrational. because there value is non-terminating.

→ LCM of Rational numbers = $\frac{\text{LCM of Numerators}}{\text{HCF of Denominator}}$.

→ HCF of Rational numbers = $\frac{\text{HCF of Numerator}}{\text{LCM of Denominator}}$

eg LCM of $\left[\frac{1}{2}, \frac{3}{6}, \frac{5}{4}\right] = \frac{15}{2} = 7.5$

Ques Is the given function periodic? If yes what is the fundamental frequency & period?

$$x(t) = \underbrace{4\cos t}_{T_1} + \underbrace{3\sin 2\pi t}_{T_2} + \underbrace{2\sin 3\pi t}_{T_3}$$

$\omega_{0_1} = 1$ $\quad$ $\omega_{0_2} = 2\pi$ $\quad$ $\omega_{0_3} = 3\pi$

$T_1 = \frac{2\pi}{1}$ $\quad$ $T_2 = \frac{2\pi}{2\pi}$ $\quad$ $T_3 = \frac{2\pi}{3\pi}$

$T_1 = 2\pi$ $\quad$ $T_2 = 1$ $\quad$ $T_3 = \frac{2}{3}$

$$\frac{T_1}{T_2} = \frac{2\pi}{1} = 2\pi$$ ← pie is not a rational number.

∴ It not a periodic signal.

**Ques 2**

$$x(t) = \underbrace{4\cos \pi t}_{T_1} + \underbrace{3\sin 2\pi t}_{T_2} + \underbrace{2\sin 3\pi t}_{T_3}$$

$\omega_{0_1} = \pi$ $\omega_{0_2} = 2\pi$ $\omega_{0_3} = 3\pi$

$T_1 = 2$ $T_2 = 1$ $T_3 = \frac{2}{3}$

$$\frac{T_1}{T_2} = \frac{2}{1} = 2 \qquad \frac{T_2}{T_3} = \frac{1}{2/3} = \frac{3}{2}$$

∴ $x(t)$ is periodic!

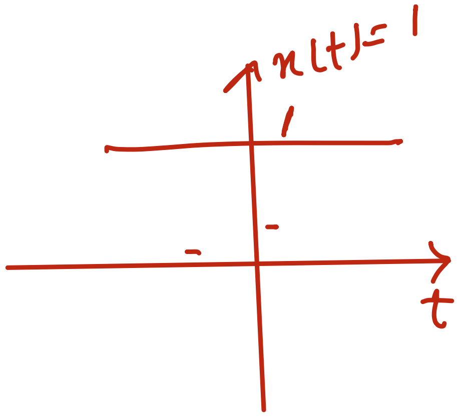
$$T = \text{LCM of } (T_1, T_2, T_3) = \text{LCM of } \left[\frac{2}{1}, \frac{1}{1}, \frac{2}{3}\right]$$

$$\boxed{T = \frac{2}{1} = 2 \text{ sec}}$$

$$\boxed{\omega_0 = \frac{2\pi}{2} = \pi \text{ rad/sec}}$$

**Note-:** constant / DC

$x(t) = 1$




It is a periodic signal, who's period is not defined.

eg $x(t) = 1 + \sin t$

we don't consider it in calculating time period.

$\therefore$ $x(t)$ is periodic.

Ques $x(t) = 4 + \cos^2 4\pi t$

$$x(t) = 4 + \left[\frac{1+\cos 8\pi t}{2}\right]$$

$$= 4 + \frac{1}{2} + \frac{1}{2}\cos 8\pi t$$

$$= \frac{9}{2} + \frac{1}{2}\cos 8\pi t$$

$\omega_0 = 8\pi$ rad/sec

$$\therefore T = \frac{2\pi}{8\pi} = \frac{1}{4} = 0.25 \text{ sec.}$$

Remember the formula

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

$$= \cos^2\theta - 1 + \cos^2\theta$$

$$= 2\cos^2\theta - 1$$

$$\boxed{\cos^2\theta = \frac{1+\cos 2\theta}{2}}$$

Important Point

## Even and odd

If $x(t)$ is real or purely imaginary.

If $x(-t) = x(t) \longrightarrow$ Even | Symmetric

If $x(-t) = -x(t) \longrightarrow$ Odd | Antisymmetric

$\longrightarrow x(t) = jt = 0 + jt$ (Purely Imaginary)

$x(-t) = -jt$

$x(-t) = -x(t) \longrightarrow$ odd / Antisymmetric

## Some properties of Even [E] & Odd [O] :-

$O \pm O = O$

$E \pm E = E$

$O \pm E =$ Neither even nor odd

$E \pm O =$ "

$E \times E = E$

$O \times O = E$

$E \times O = O$

$O \times E = O$

$\frac{E}{E} = E$

$\frac{O}{O} = E$

$\frac{E}{O} = O$

$\frac{O}{E} = O$

$\int E = O$ $\quad \int O = E$

$$\int_{-T/2}^{T/2} \text{Even} = 2 \times \int_{0}^{T/2} (E)$$

$$\int_{-T/2}^{T/2} \text{Odd} = 0$$

$\frac{dE}{dt} = O$

$\frac{dO}{dt} = E$

# If $x(t)$ is complex

$x(t) = a \pm jb$

→ For even or symmetric,

$$x(t) = x^*(-t)$$ ← conjugate eg $3+j4$, $3-j4$

For odd or antisymmetric

$$x(t) = -x^*(-t)$$

→ conjugate symmetric part of $x(t)$

$$x_e(t) = \frac{x(t) + x^*(-t)}{2}$$

conjugate antisymmetric part of $x(t)$

$$x_o(t) = \frac{x(t) - x^*(-t)}{2}$$

eg $x(t) = 5 - jt$

$$x(-t) = 5 + jt$$

$$x^*(-t) = 5 - jt$$

$$\Rightarrow x(t) = x^*(-t)$$

$\therefore$ $x(t)$ is a conjugate symmetric.

$$x_e(t) = \frac{x(t) + x^*(-t)}{2} = \frac{5-jt+5-jt}{2}$$

$$= 5 - jt$$

$$x_o(t) = \frac{x(t) - x^*(-t)}{2} = \frac{\cancel{5-jt} \cancel{-5} \cancel{+jt}}{2}$$

$$= 0$$

eg $x_1(t) =$ Even of $[\sin 4\pi t \; u(t)]$

Is $x_1(t)$ periodic? If yes find period.

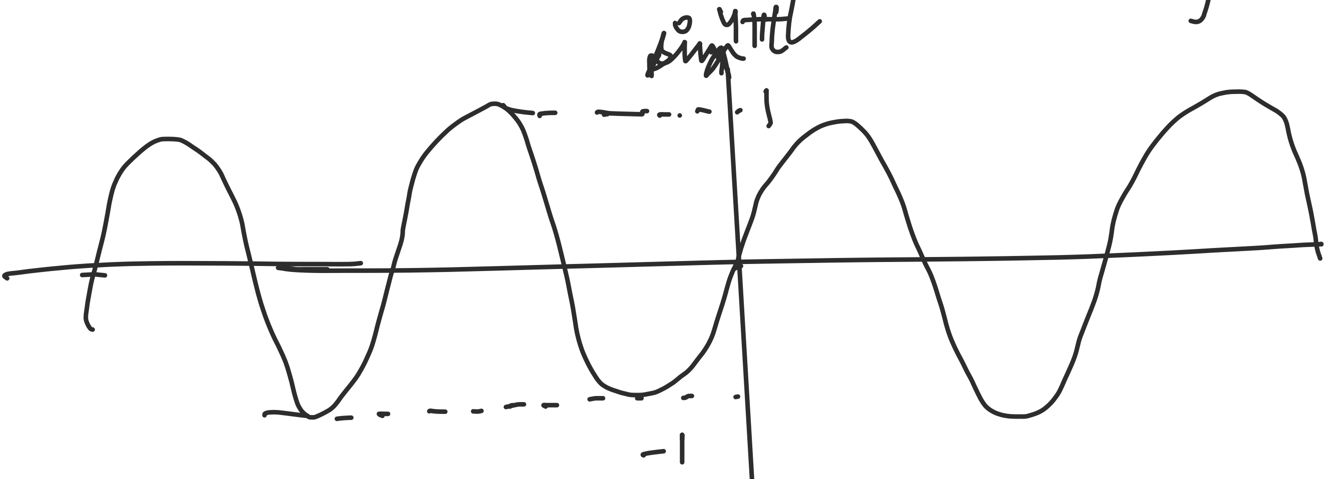
Sol $x(t) = \sin 4\pi t \; u(t)$

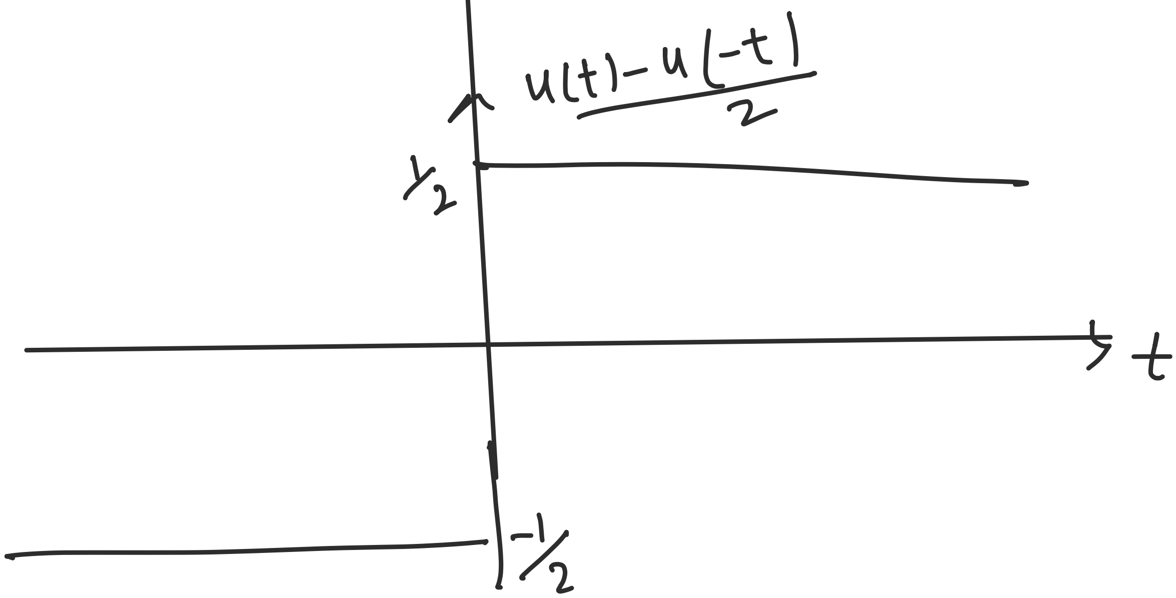
$$x(-t) = \sin 4\pi(-t)\, u(-t)$$
$$= -\sin 4\pi t \; u(-t)$$

$$x_1(t) = x_e(t) = \frac{x(t) + x(-t)}{2}$$

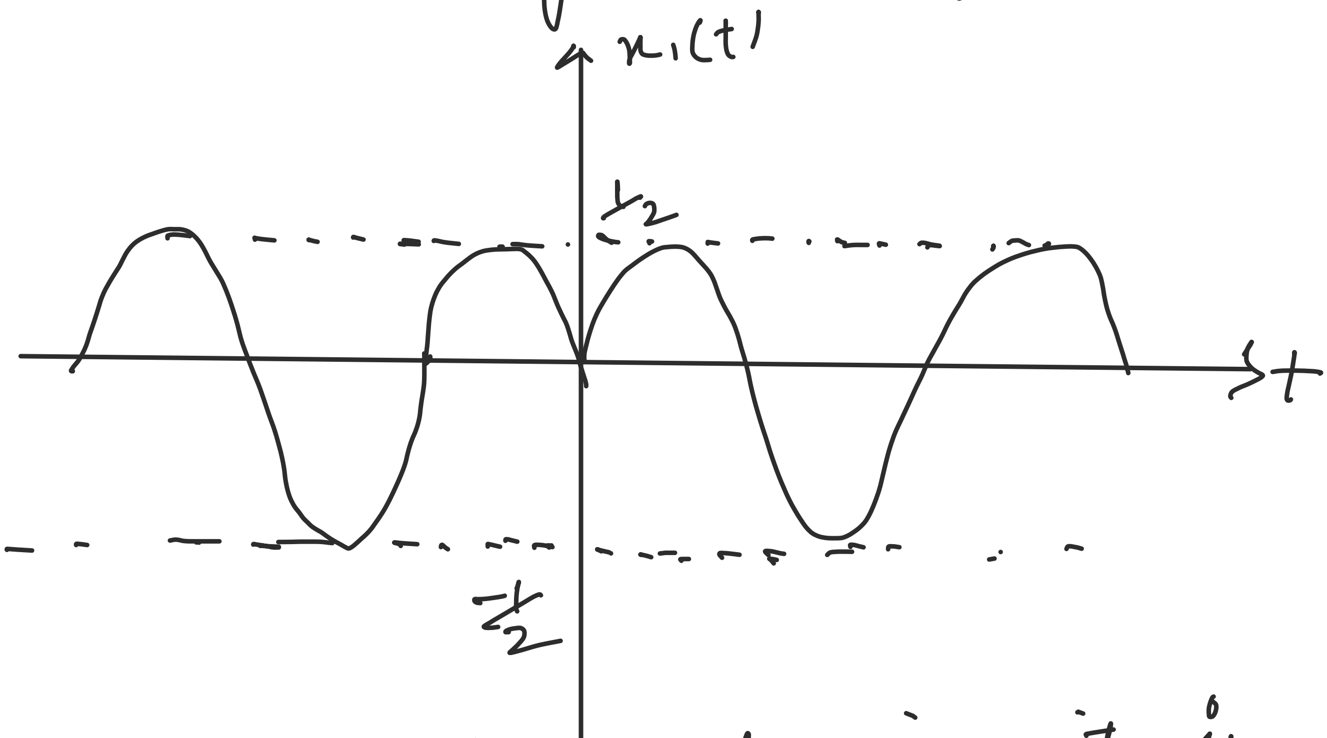
$$= \frac{\sin 4\pi t \; u(t) - \sin 4\pi t \; u(-t)}{2}$$

$$x_1(t) = \sin 4\pi t \left[\frac{u(t) - u(-t)}{2}\right]$$



When we multiply these signals -:




Not a periodic signal, since it is not repeated !!

# Complex Exponential -:

→ Always Periodic !!

→ General Representation →

$$x(t) = A \cdot e^{j\omega_0 t}$$

$\omega_0$ → fundamental freq (r/s)

$j\omega_0 t$ → phase angle

$$\boxed{e^{jx} = \cos x + j \sin x}$$

$$\longrightarrow \quad A e^{j\omega_0 t} = A \left[ \cos \omega_0 t + j \sin \omega_0 t \right]$$

$$\left| A e^{j\omega_0 t} \right| = A \sqrt{\cos^2 \omega_0 t + \sin^2 \omega_0 t}$$

$$= A \cdot 1 = A$$

$\therefore$ A represents magnitude of $e^{j\omega_0 t}$.

$\rightarrow$ Phase Angle $= \angle A e^{j\omega_0 t} = \tan^{-1}\left[\dfrac{\sin \omega_0 t}{\cos \omega_0 t}\right]$

$= \omega_0 t$

Note :- $\rightarrow$ $\cos x = \dfrac{e^{jx} + e^{-jx}}{2}$

$\sin x = \dfrac{e^{jx} - e^{-jx}}{2}$

$\rightarrow$ $x = a + jb$

$|x| = \sqrt{a^2 + b^2}$ $\leftarrow$ magnitude

$\angle x = \tan^{-1}\left(\dfrac{b}{a}\right)$ $\leftarrow$ phase angle.

eg (1) $x(t) = 4e^{j3\pi t}$

Comparing with standard representation $Ae^{j(\omega_0 t + \phi)}$

$\omega_0 = 3\pi$

$\therefore T = \dfrac{2\pi}{\omega_0} = \dfrac{2\pi}{3\pi} = \dfrac{2}{3}$ sec

$\therefore$ periodic

eg (2)

$x(t) = 1 + 5e^{j6\pi t}$

$\omega_0 = 6\pi$

$T = \dfrac{2\pi}{6\pi} = \dfrac{1}{3}$ sec

Note :- Only complex exponential are periodic if their is real part it is not periodic

eg $e^{\pi t}$ 86 is not periodic.

eg ③ $x(t) = 4 + e^{j\frac{2\pi}{5}t} + e^{j\frac{3\pi}{7}t}$

For the signal to be periodic,

$$\frac{T_1}{T_2} = \frac{T_2}{T_2} = \text{rational}$$

$w_{0_1} = \frac{2\pi}{5}$ r/s $\qquad$ $w_{0_2} = \frac{3\pi}{7}$ rad/s

$T_1 = \frac{2\pi}{(2\pi/5)}$ $\qquad$ $T_2 = \frac{2\pi}{(3\pi/7)}$

$T_1 = 5$ sec $\qquad$ $T_2 = \frac{14}{3}$ sec

$\therefore \frac{T_1}{T_2} = \frac{5}{14/3} = \frac{15}{14} = \text{rational}.$

$\therefore$ periodic.

Now fundamental Time $T = \text{LCM}[T_1 \,\&\, T_2]$

$= \text{LCM}\left[\frac{5}{1}, \frac{14}{3}\right]$

$$\boxed{T = \frac{70}{1} = 7 \text{ sec}}$$

$w_0 = \text{GCD}$ or $\text{HCF}\ [w_{0_1}, w_{0_2}]$

$= \text{HCF}\left[\frac{2\pi}{5}, \frac{3\pi}{7}\right] = \frac{\pi}{35}$ rad/sec

# Recalling Complex Numbers

## Rectangular form

→ $x = a + jb$

$r = |x| = \sqrt{a^2+b^2}$

$\theta = \angle x = \tan^{-1}\left(\frac{b}{a}\right)$

→ convenient incase of Addition / subtraction

## Polar form

$x = re^{j\theta}$ or $r\angle\theta$

$= r[\cos\theta + j\sin\theta]$

$= \underbrace{r\cos\theta}_{a} + j\underbrace{r\sin\theta}_{b}$

→ Multiply / Divide / square root

## Few points related to rectangular form :-

① $x = a + jb$

$|x| = \sqrt{a^2+b^2}$

$\angle x = \tan^{-1}\left(\frac{b}{a}\right)$

② $x = a$ or $x = a + j0$

$|x| = a$

$\angle x = 0^\circ$

③ $x = jb$ or $x = 0 + jb$

$|x| = b$

$\angle x = 90^\circ$ $\quad \because \tan^{-1}\left(\frac{b}{0}\right) = \tan^{-1}(\infty) = 90^\circ$

④ $x = -a + jb$

$|x| = \sqrt{a^2+b^2}$

$\angle x = 180^\circ - \tan^{-1}\left(\frac{b}{a}\right)$

eg $x = \dfrac{(4+j5)(3+j6)}{(8+j4)(9+j3)}$

$$= \frac{(x_1 \angle \theta_1)(x_2 \angle \theta_2)}{(x_3 \angle \theta_3)(x_4 \angle \theta_4)} = \frac{x_1 x_2}{x_3 x_4} \angle (\theta_1+\theta_2)-(\theta_3+\theta_4)$$

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# Energy & Power signal :-

→ Total energy transmitted to load

$$E = \int_{-T}^{T} x^2(t)\,dt \quad [\text{finite duration}]$$

$$= \lim_{T\to\infty} \int_{-T}^{T} x^2(t)\,dt \quad [\text{infinite duration}]$$

→ A signal is said to be an power/energy signal if the total energy/power transmitted is finite.

$$0 < E < \infty \quad \& \quad 0 < P < \infty$$

→ Average Power $P = \dfrac{1}{T} \displaystyle\int_{-T/2}^{T/2} x^2(t)\,dt$

(for finite duration periodic)

$$P = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x^2(t)\,dt$$

(for infinite duration)

→ Power of energy signal = 0

Energy of Power signal = $\infty$

→ At a same time signal cannot be both energy or power signal.

# Energy And Power Signal

→ Energy of signal $x(t)$ is given as

$$\boxed{E_x = \int_{-\infty}^{\infty} |x(t)|^2 \, dt}$$

If $0 < E_x < \infty$

Then $x(t)$ is said to be an <u>energy signal</u>

→ Power of $x(t)$ is given as :-

$$P_x = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 \, dt$$

$$P_x = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt$$

$$= \lim_{T \to \infty} \frac{1}{T} \int_{0}^{T} |x(t)|^2 \, dt$$

If $0 < P_x < \infty$, then $x(t)$ is said to a <u>power signal</u>.

→ If Energy of a signal is finite, then power of the signal . will be zero.

→ If Power of a signal is finite Then its energy will be infinite.

→ Energy = Power X Time
= v(t) X i(t)

→ Power = $(RMS)^2$
→ root mean square

→ If signal is periodic,

$$P = \frac{1}{T}\int_{-T/2}^{T/2} |x(t)|^2 \, dt$$

If signal is not periodic,

$$P = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} |x(t)|^2 \, dt$$

→ All periodic signals are power signal.
and there energy is ∞ .

eg $x(t) = e^{-4t} u(t)$

$e^{-\infty} = 0$

$e^{-0} = 1$

$$E_x = \lim_{T \to \infty} \int_{-T/2}^{T/2} |e^{-4t} u(t)|^2 \, dt$$

$$= \int_{-\infty}^{\infty} |e^{-4t} u(t)|^2 \, dt = \int_0^{\infty} |e^{-4t}|^2 \, dt = \int_0^{\infty} e^{-8t} \, dt$$

$$= \left[ \frac{e^{-8t}}{-8} \right]_0^{\infty} = \frac{0-1}{-8} = \frac{1}{8} \text{ Joules.}$$

$\therefore$ $x(t) = e^{-4t} u(t)$ is an energy signal.

& $\therefore$ Power $= 0$.

eg 2 $x(t) = [4e^{-2t} + 3e^{-4t}] u(t)$

$$E_x = \int_0^{\infty} (4e^{-2t} + 3e^{-4t})^2 \, dt$$

$\boxed{0 < E_x < \infty}$ $x(t)$ is a energy signal.

eg $x(t) = u(t)$, Power?

later

eg $x(t) = A \sin \omega t$, Power?

RMS value $= \frac{A}{\sqrt{2}}$ $\Rightarrow$ $P_x = \left(\frac{A}{\sqrt{2}}\right)^2 = \frac{A^2}{2}$

eg $x(t) = A \sin t$ , Power ?

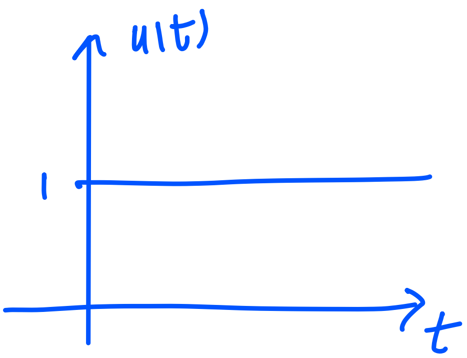
$$P = \frac{1}{T}\int_{-T/2}^{T/2} |A \sin t|^2 \, dt = \frac{1}{2\pi}\int_{-\pi}^{\pi} A^2 \sin^2 t \, dt$$

$$= \frac{A^2}{2\pi}\int_{-\pi}^{\pi}\left(\frac{1-\cos 2t}{2}\right) dt$$

$$= \frac{A^2}{2\pi}\left[\frac{1}{2}\int_{-\pi}^{\pi} dt - \cancelto{0}{\frac{1}{2}\int_{-\pi}^{\pi}\cos 2t \, dt}\right]$$

$$= \frac{A^2}{2\,\cancel{4\pi}}\cancel{2\pi} = \frac{A^2}{2} \quad \text{Ans}$$

eg $x(t) = u(t)$ ; Power ? Energy ?




$$P_x = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} |u(t)|^2 \, dt$$

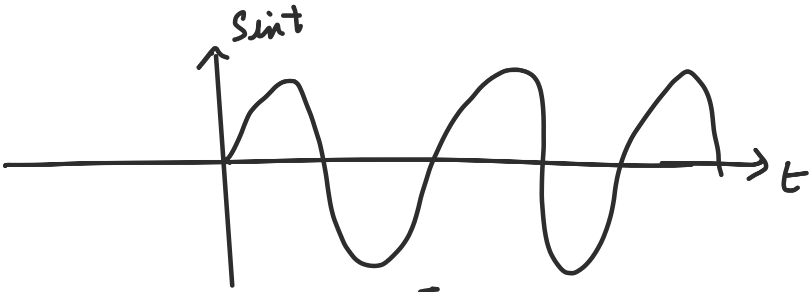
$$= \lim_{T\to\infty}\frac{1}{T}\int_{0}^{T/2} dt$$

$$= \lim_{T\to\infty}\frac{1}{T}[t]_0^{T/2} = \lim_{T\to\infty}\frac{1}{T}\times\frac{T}{2} = \frac{1}{2} \text{ watts}$$

$\therefore$ $u(t)$ is Power signal.

$\therefore$ $E = \infty$

eg $x(t) = \sin t \; u(t)$ , Power ? Energy ?




$$P_x = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} \sin^2 t \; u(t) \, dt = \lim_{T \to \infty} \frac{1}{T} \int_0^{T/2} \left[ \frac{1 - \cos 2t}{2} \right] dt$$

$$= \lim_{T \to \infty} \frac{1}{T} \left[ \frac{1}{2} \int_0^{T/2} dt - \frac{1}{2} \int_0^{T/2} \cos 2t \, dt \right]$$

$$= \lim_{T \to \infty} \frac{1}{T} \left[ \frac{1}{2} \left( \frac{T}{2} \right) - \frac{1}{2} \left\{ \frac{\sin 2t}{2} \right\}_0^{T/2} \right]$$

$$\frac{1}{\infty} = 0$$

$$= \lim_{T \to \infty} \frac{1}{T} \left[ \frac{T}{4} - \frac{1}{4} \{ \sin 2t \} \right]$$

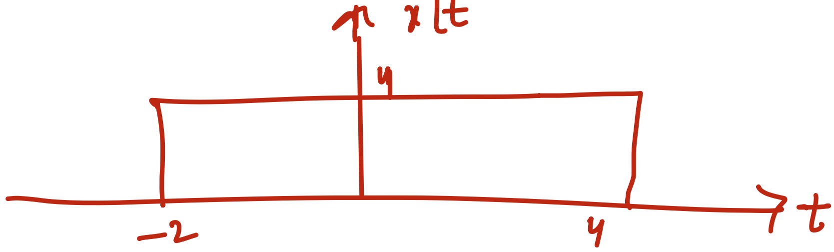
$$= \left[ \lim_{T \to \infty} \frac{1}{T} \left( \frac{T}{4} \right) \right] - \left[ \lim_{T \to \infty} \frac{1}{T} \frac{\sin t}{4} \right]$$

$$= \frac{1}{4} - 0 \implies \boxed{P_x = \frac{1}{4}}$$

$\therefore$ $x(t)$ is a power signal

$\therefore$ Energy $= \infty$

eg




Energy ?
Power ?

$$E_x = \int_{-2}^{4} (16)\, dt = 16\,[t]_{-2}^{4} = 16 \times 6 = 96 \text{ Joules.}$$

$x^2(t) = 4^2$

→ If $x(t)$ is existing for infinite duration & is ↓ing in nature.

i.e. $\lim_{t \to \infty} f(t) = 0$

then $f(t)$ will be an energy signal.

→ If $x(t)$ exists for finite duration & value of $x(t)$ is finite at all points, then $x(t)$ will be an energy signal.

→ All periodic signals are power signals, but converse is not true.

→ If $x(t)$ is not a periodic signal but follows these conditions :-

$$\lim_{t \to \infty} f(t) \neq 0$$

$$\lim_{t \to \infty} f(t) \neq \infty$$

$\longrightarrow$ Signals $\qquad$ Power $[(RMS)^2]$

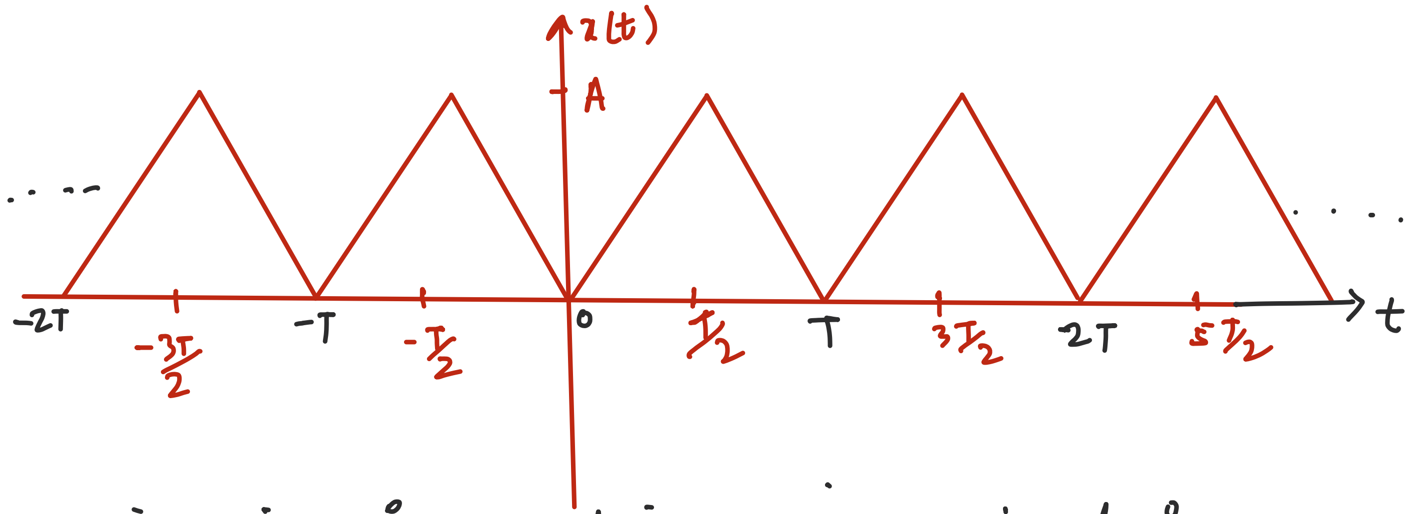
$A \sin \omega t$ or $A \cos \omega t \longrightarrow \frac{A^2}{2}$

$A\,u(t) \longrightarrow \frac{A}{2}$

$A e^{j\omega_0 t} \longrightarrow A^2$

$A \longrightarrow A^2$

eg




Since it is periodic, given signal is <u>power signal</u>.

$$\boxed{P_x = \frac{A^2}{3}}$$ $\leftarrow$ formula to solve this signal.

$\boxed{\text{periodic } \Delta}$

$\longrightarrow$ Time shifting has no effect on power of signal or energy.

→ If Energy of $x(t) = E$

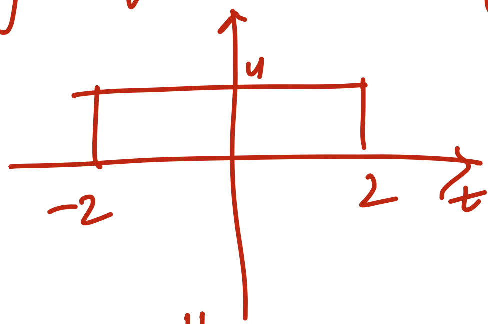
then Energy of $x(\alpha t) = \frac{E}{\alpha}$

→ Time scaling has no effect on power of signal.

→ Amplitude scaling effect on energy signal -:

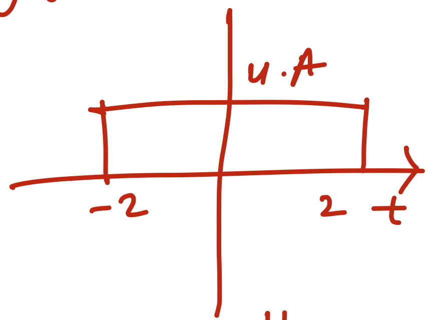
$E_{x(t)} = E$

then $E_{\alpha x(t)} = ?$




$\Downarrow$

$E$




$\Downarrow$

$A^2 . E$

→ $x(t) = x_e(t) + x_o(t)$

$$\int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} |x_e(t)|^2 dt + \int_{-\infty}^{\infty} |x_o(t)|^2 dt$$