Bayesian data analysis – exercise 4

This exercise is related to Chapter 3 (additional information on number of required samples will be covered in Chapter 10).

The maximum amount of points from this assignment is 6. In addition to the correctness of the answers, the overall quality and clearness of the report is evaluated.

Report all results to a single, **anonymous** *.pdf -file and return it to peergrade.io. Include also source code to the report (either as an attachment or as a part of the answer). By anonymity it is meant that the report should not contain your name or student number.

Generalized linear model: Bioassay + grid sampling

In the bioassay example (Chapter 3 in the book), replace the uniform prior density by a joint normal prior distribution on (α, β), with $\alpha \sim N(0, 2^2)$, $\beta \sim N(10, 10^2)$, and $\mathrm{corr}(\alpha, \beta) = 0.5$.

a) Repeat all the computations and plots of Section 3.7 with this new prior distribution.
   - Compute the posterior density at a grid of points (α, β)
   - Use the grid to sample 1000 draws from the posterior
   - Draw a posterior contour plot for the parameters α and β (see Figure 3.3a)
   - Draw a scatterplot of the 1000 draws from the posterior (see Figure 3.3b)
   - Draw a histogram of the draws from the posterior distribution of the LD50 conditional on $\beta > 0$ (see Figure 3.4)

b) Check visually that your contour plot and scatter plot look like a compromise between the prior distribution and the likelihood

c) Report an estimate for $p(\beta > 0|x, n, y)$, that is, the probability that the drug is harmful

Hints

- See `demo3_6.R` or `demo3_6.py`
- Check that the range and spacing of grid for α and β are sensible for the alternative prior
- Compute the log-posterior in a grid
- Scale the log-posterior by subtracting its maximum value before exponentiating
- Exponentiate
- Normalize the posterior
- Use 2D grid sampling or the method in the book to sample from the posterior