Exercise3

February 19, 2018

1 Exercise 3

1.1 Problem 3.1

1.1.1 Some Theory

When we want to know about μ in a multi-parameter model with parameter μ and σ^2, we need the average over the nuisance parameter on the joint posterior distribution (in this case σ^2).

$p(\mu|y) = \int p(\mu|\sigma^2, y)p(\sigma^2|y)d\sigma^2$

An easy way of doing this in practice is to factor the joint posterior distribution (as is done above) into the conditional posterior distribution on the nuisance parameter and a marginal posterior distribution on the nuisance parameter. Now we can first draw σ^2 from the marginal posterior distribution, and then use that σ^2 to draw μ from the conditional posterior distribution. This indirectly evaluate the integral with respect to σ^2

how ever while working with the Normal model, and using a noninformative prior distribution such as $(\sigma^2)^{-1}$ the posterior distribution takes on a convenient analytic (closed) form, namely a scaled and translated version of the studen's t form:

$\frac{\bar{y}-\mu}{s/\sqrt{n}}|\mu, \sigma^2 \sim t_{n-1}$

this amounts to a studen's t_{n-1}, with a translation/location of $\bar{y}$ and scale of $\sqrt{s^2/n}$

1.1.2 Code base

```
In [2]: #magic call
        %matplotlib inline
        #imports
        import matplotlib
        import matplotlib.pyplot as plt
        import numpy as np
        from utilities_and_data import plot_tools, sinvchi2 #this is the N-inv Chi^2 distributio
        from scipy import stats
```

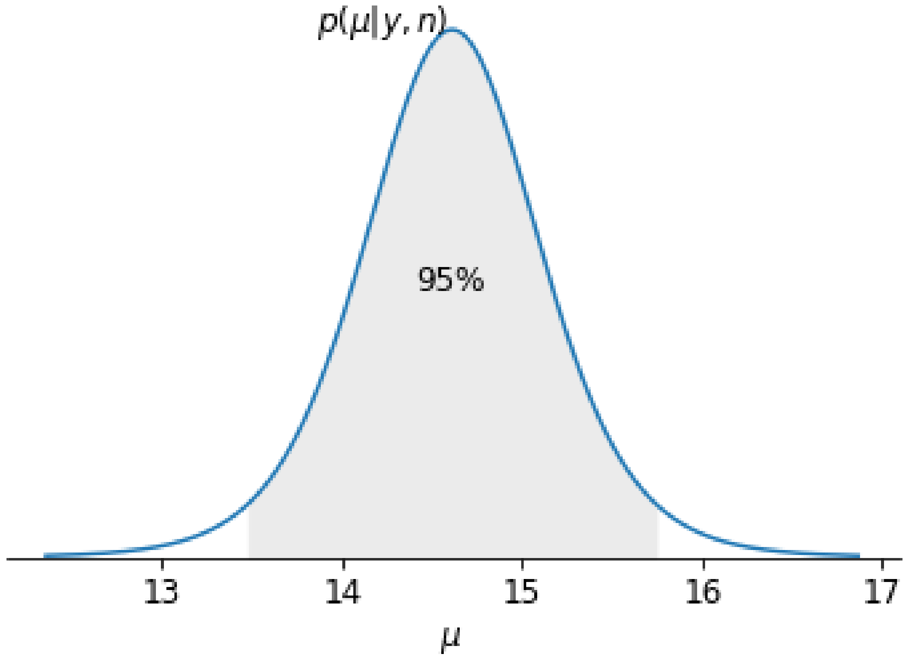
```
In [3]: # edit default plot settings
        plt.rc('font', size=12)
```

```
In [4]: #sample data
        y = np.loadtxt('data/windshieldy1.txt')
        n = len(y)
        y_mean = np.mean(y)
        y_s2 = np.var(y,ddof=1)
```

1.1.3 3.1.a

In [23]:
```
# compute the exact marginal density for mu
# multiplication by 1./sqrt(s2/n) is due to the transformation of variable
# z=(x-mean(y))/sqrt(s2/n), see BDA3 p. 21
pos_dist_mu = stats.t(n-1, loc=y_mean, scale=np.sqrt(y_s2/n))
mean, var = pos_dist_mu.stats('mv')

#create line space (x axis)
x = np.linspace(mean-var*7, mean+var*7, 1000)

pos_pd_mu = pos_dist_mu.pdf(x)
plt.plot(x,pos_pd_mu)

# show only x-axis
plot_tools.modify_axes.only_x()
# annotate the line
plt.annotate(r'$p(\mu|y,n)$',
             (x[len(x)//2]-0.015, pos_pd_mu[len(x)//2]),
             ha='right'
            )

x_95 = pos_dist_mu.interval(0.95)
x_95_idx = ((x>x_95[0])&(x<x_95[1]))

plt.fill_between(x[x_95_idx], pos_pd_mu[x_95_idx], color='0.92')
# add text into the shaded area
plt.text(x[np.argmax(pos_pd_mu)],0.4,"95%", horizontalalignment='center')
# add labels and title
plt.xlabel(r'$\mu$')

mean, var = pos_dist_mu.stats('mv')

print('\n\n\nThe mean of the posterior distribution of μ is ' + str(np.round(mean,decim
      'The variance of the posterior distribution of μ is ' + str(np.round(var,decimals
```

```
The mean of the posterior distribution of μ is 14.61
The variance of the posterior distribution of μ is 0.322
```



We can thus say that μ takes on a value between 13.47 and 15.74 with a 95% certainty

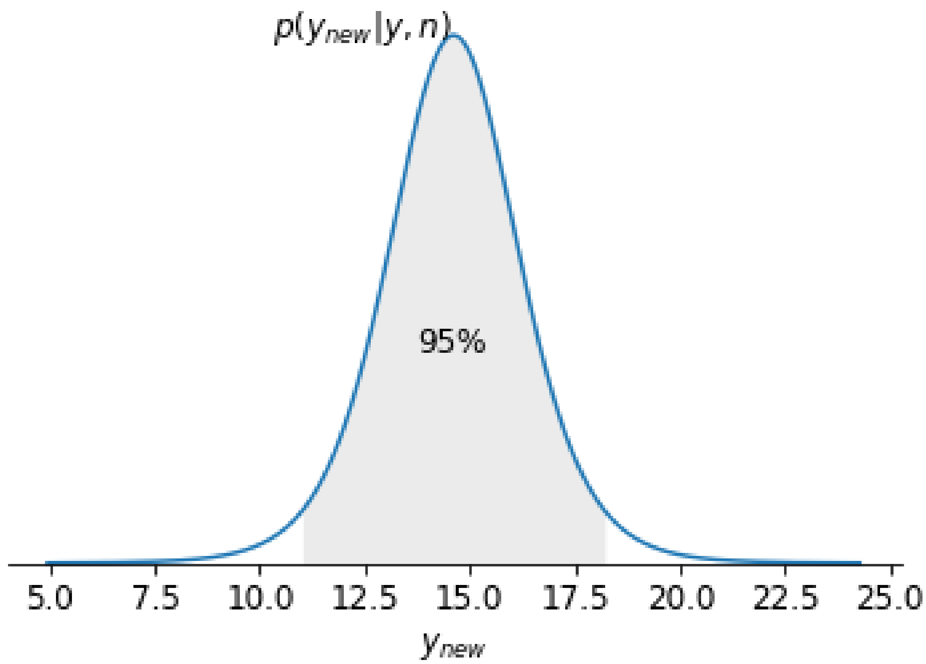
1.1.4 3.1.b

working with the Normal model, and using a noninformative prior distribution such as $(\sigma^2)^{-1}$ the posterior predictive distribution also takes on the form of a studen's t_{n-1}, but now the translation/location is $\bar{y}$ and the scale is $\sqrt{s^2(1+\frac{1}{n})}$

In [37]:
```
#frees the dist.
pos_pred_dist = stats.t(n-1, loc=y_mean, scale=np.sqrt(y_s2*(1+1/n)))
pred_mean, pred_var = pos_pred_dist.stats('mv')

#create sample space/x-axis
x_pred = np.linspace(pred_mean-pred_var*3, pred_mean+pred_var*3, 1000)

pos_pred_pd = pos_pred_dist.pdf(x_pred)
plt.plot(x_pred,pos_pred_pd)

# show only x-axis
plot_tools.modify_axes.only_x()
# annotate the line
plt.annotate(r'$p(y_{new}|y,n)$',
             (x_pred[len(x)//2]-0.015, pos_pred_pd[len(x)//2]),
             ha='right'
            )
```

```
x_pred_95 = pos_pred_dist.interval(0.95)
x_pred_95_idx = ((x_pred>x_pred_95[0])&(x_pred<x_pred_95[1]))

plt.fill_between(x_pred[x_pred_95_idx], pos_pred_pd[x_pred_95_idx], color='0.92')
# add text into the shaded area
plt.text(x_pred[np.argmax(pos_pred_pd)],0.1,"95%", horizontalalignment='center')
# add labels and title
plt.xlabel(r'$y_{new}$')

print('\n\n\nThe mean of the posterior distribution of μ is ' + str(np.round(pred_mean,
      'The variance of the posterior distribution of μ is ' + str(np.round(pred_var,dec
```

```
The mean of the posterior distribution of μ is 14.61
The variance of the posterior distribution of μ is 3.219
```



Thus we can say that the hardness of the next windshield lays with in the interval from 11.03 to 18.19 with a 95 percent certainty.

1.2 problem 3.2

1.2.1 some theory

While working with binomial data, the conjugate prior is a Beta(α, β), and when used the posterior distribution has a closed form as:

$p(p_x | y) ∝ Beta($\alpha + y, \beta + n - y$)$

Lets assign p_0 as the probability of dying in the control group, and p_1 as the probability of dying in the treatment group.

We can then sample from the two posteriors and run the samples through the odds function, and that way do inference on the increase of the odds of surviving under the treatment

additional let us use a weakly informative prior Beta(2,20)

1.2.2 Code base

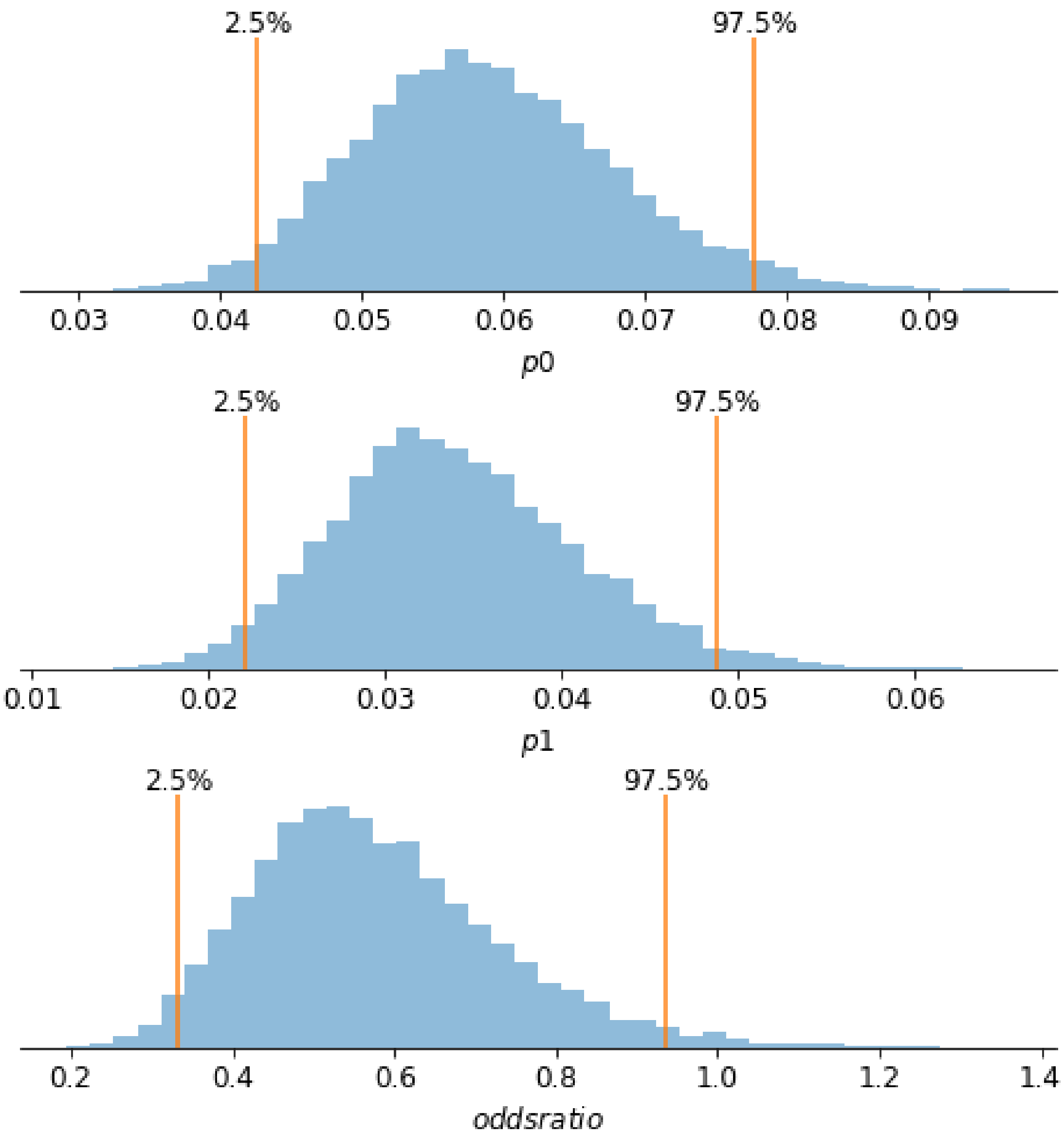
```
In [117]: #freez the distributions
          p0_dist = stats.beta(2+39,20+674-39)
          p1_dist = stats.beta(2+22,20+680-22)

          # create the sample space
          x = np.linspace(0, p0_dist.median()*3, 1000)

          # draw n random samples from the two distributions
          n = 10000
          p0 = p0_dist.rvs(n)  # rvs comes from `random variates`
          p1 = p1_dist.rvs(n)

          # run the odds function
          odd = (p1/(1-p1))/(p0/(1-p0))
```

1.2.3 3.2.a

```
In [124]: # plot 2 subplots
          fig, axes = plt.subplots(nrows=3, ncols=1, figsize=(8, 8))
          # show only x-axis
          plot_tools.modify_axes.only_x(axes)
          # manually adjust spacing
          fig.subplots_adjust(hspace=0.5)

          # plot histogram
          axes[0].hist(p0, bins=40, color=plot_tools.lighten('C0'))
          # compute 2.5% and 97.5% quantile approximation using samples
          p0_25, p0_975 = np.percentile(p0, [2.5, 97.5])
          # draw lines for these
          axes[0].axvline(p0_25, color='C1')
          axes[0].axvline(p0_975, color='C1')
          axes[0].text(
              p0_25,
              axes[0].get_ylim()[1]+15,
```

```
    '2.5%',
    horizontalalignment='center'
)
axes[0].text(
    p0_975,
    axes[0].get_ylim()[1]+15,
    '97.5%',
    horizontalalignment='center'
)
axes[0].set_xlabel(r'$p0$')

# plot histogram for the transformed variable
axes[1].hist(p1, bins=40, color=plot_tools.lighten('C0'))
# compute 2.5% and 97.5% quantile approximation using samples
p1_25, p1_975 = np.percentile(p1, [2.5, 97.5])
# draw lines for these
axes[1].axvline(p1_25, color='C1')
axes[1].axvline(p1_975, color='C1')
axes[1].text(
    p1_25,
    axes[1].get_ylim()[1]+15,
    '2.5%',
     horizontalalignment='center'
)
axes[1].text(
    p1_975,
    axes[1].get_ylim()[1]+15,
    '97.5%',
    horizontalalignment='center'
)
axes[1].set_xlabel(r'$p1$');

# plot histogram for the transformed variable
axes[2].hist(odd, bins=40, color=plot_tools.lighten('C0'))
# compute 2.5% and 97.5% quantile approximation using samples
odd_25, odd_975 = np.percentile(odd, [2.5, 97.5])
# draw lines for these
axes[2].axvline(odd_25, color='C1')
axes[2].axvline(odd_975, color='C1')
axes[2].text(
    odd_25,
    axes[2].get_ylim()[1]+15,
    '2.5%',
     horizontalalignment='center'
)
axes[2].text(
    odd_975,
    axes[2].get_ylim()[1]+15,
```

```
    '97.5%',
    horizontalalignment='center'
)
axes[2].set_xlabel(r'$odds ratio$');
```



It can then be said that the median-ratio_of_deaths in the treatment group is only 56% of that in the control group (almost two times lower), and that it is with in the range of 33.2% and 93.5% with a certainty of 95%

1.2.4 3.2.b

As the prior that i uses is a factor 10 less precise as the derived posterior, and that it is centered around the mean of the data make it have very little influence.

1.3 Problem 3.3

1.3.1 some theory

all is given from problems 3.1 and 3.2

1.3.2 Code base

```
In [5]: #sample data
        y1 = np.loadtxt('data/windshieldy1.txt')
        n1 = len(y)
        y1_mean = np.mean(y)
        y1_s2 = np.var(y,ddof=1)

        y2 = np.loadtxt('data/windshieldy2.txt')
        n2 = len(y)
        y2_mean = np.mean(y)
        y2_s2 = np.var(y,ddof=1)
```

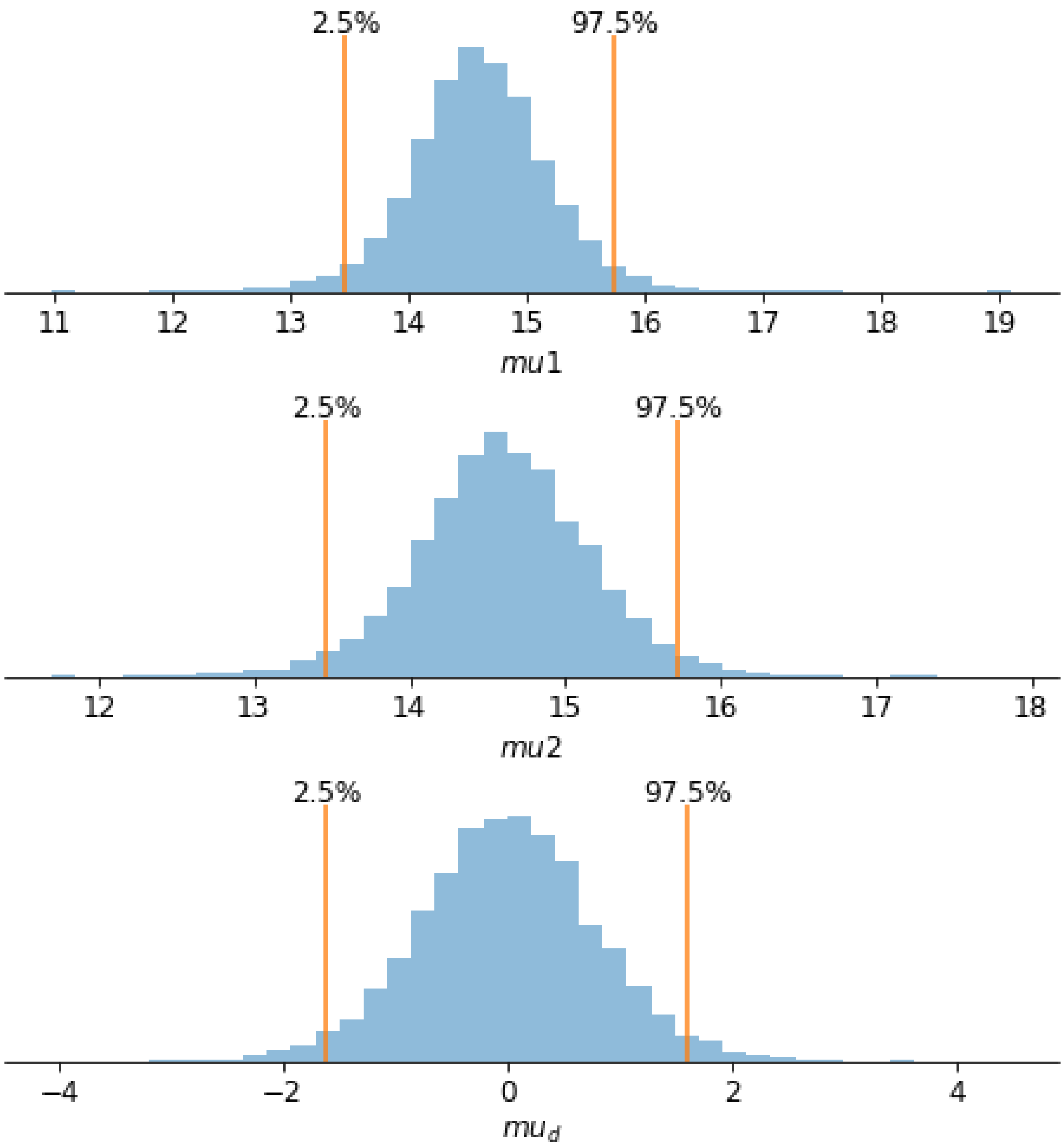
```
In [16]: #freez the distributions
         pos_dist_mu1 = stats.t(n1-1, loc=y1_mean, scale=np.sqrt(y1_s2/n1))
         pos_dist_mu2 = stats.t(n2-1, loc=y2_mean, scale=np.sqrt(y2_s2/n2))

         #create line space (x axis)
         a, b = pos_dist_mu1.stats('mv')
         x = np.linspace(a-b*7, a+b*7, 1000)

         # draw n random samples from the two distributions
         n = 10000
         mu1 = pos_dist_mu1.rvs(n)  # rvs comes from `random variates`
         mu2 = pos_dist_mu2.rvs(n)

         #Do the diff
         mu_d = mu1 - mu2
```

1.3.3 3.3.a

```
In [15]: # plot 2 subplots
         fig, axes = plt.subplots(nrows=3, ncols=1, figsize=(8, 8))
         # show only x-axis
         plot_tools.modify_axes.only_x(axes)
         # manually adjust spacing
         fig.subplots_adjust(hspace=0.5)

         # plot histogram
         axes[0].hist(mu1, bins=40, color=plot_tools.lighten('C0'))
         # compute 2.5% and 97.5% quantile approximation using samples
         mu1_25, mu1_975 = np.percentile(mu1, [2.5, 97.5])
         # draw lines for these
```

```
axes[0].axvline(mu1_25, color='C1')
axes[0].axvline(mu1_975, color='C1')
axes[0].text(
    mu1_25,
    axes[0].get_ylim()[1]+15,
    '2.5%',
    horizontalalignment='center'
)
axes[0].text(
    mu1_975,
    axes[0].get_ylim()[1]+15,
    '97.5%',
    horizontalalignment='center'
)
axes[0].set_xlabel(r'$mu1$')

# plot histogram for the transformed variable
axes[1].hist(mu2, bins=40, color=plot_tools.lighten('C0'))
# compute 2.5% and 97.5% quantile approximation using samples
mu2_25, mu2_975 = np.percentile(mu2, [2.5, 97.5])
# draw lines for these
axes[1].axvline(mu2_25, color='C1')
axes[1].axvline(mu2_975, color='C1')
axes[1].text(
    mu2_25,
    axes[1].get_ylim()[1]+15,
    '2.5%',
     horizontalalignment='center'
)
axes[1].text(
    mu2_975,
    axes[1].get_ylim()[1]+15,
    '97.5%',
    horizontalalignment='center'
)
axes[1].set_xlabel(r'$mu2$');

# plot histogram for the transformed variable
axes[2].hist(mu_d, bins=40, color=plot_tools.lighten('C0'))
# compute 2.5% and 97.5% quantile approximation using samples
mu_d_25, mu_d_975 = np.percentile(mu_d, [2.5, 97.5])
# draw lines for these
axes[2].axvline(mu_d_25, color='C1')
axes[2].axvline(mu_d_975, color='C1')
axes[2].text(
    mu_d_25,
    axes[2].get_ylim()[1]+15,
    '2.5%',
```

```
        horizontalalignment='center'
    )
    axes[2].text(
        mu_d_975,
        axes[2].get_ylim()[1]+15,
        '97.5%',
        horizontalalignment='center'
    )
    axes[2].set_xlabel(r'$mu_d$');
```



The median of mu_d is -0.00014 and the 95% confidence interval is almost symmetric around 0 at [-1.6106, 1.6101]

1.3.4 3.3.b

As 0 is will with in the bounds of the 95% interval we can say that there is no evidence that the two product lines produces different hardness