EECS 391 Introduction to Artificial Intelligence
Fall 2019, Written Assignment 3 ("W3")

Due: Fri Oct 11 by midnight

Total Points: 100

Remember: name and case ID, stapled, answers concise, neat, and legible. Submit in class on the due date.

Q1. Basic Probability (10 P.)

Consider the following joint probability table of $p(x, y)$. Each entry in row i and column j is $p(x = i, y = j)$

	y			
	0.06	0.08	0.04	0.02
	0.12	0.16	0.08	0.04
x	0.09	0.12	0.06	0.03
	0.03	0.04	0.02	0.01

a) Show that this is a valid probability distribution.

b) Write the mathematical expression for the distribution p(x) and calculate the distribution from the table.

c) Are x and y independent? Justify your answer.

Q2. R&N Q13.3 (15 P.)

For each of the following statements, either prove it is true or give a counter example.

a) If $P(a|b,c) = P(b|a,c)$, then $P(a|c) = P(b|c)$

b) If $P(a|b,c) = P(a)$, then $P(b|c) = P(b)$

c) If $P(a|b) = P(a)$, then $P(a|b,c) = P(a|c)$

Q3. R&N Q13.8 (20 P.)

Given the full joint distribution shown in Figure 13.3, calculate the following

a) $P(toothache)$

b) $P(Cavity)$

c) $P(Toothache\,|\,cavity)$

d) $P(Cavity\,|\,toothache \vee catch)$

Q4. R&N Q13.13 (20 P.)

Consider two medical tests, A and B, for a virus. Test A is 95% effective at recognizing the virus when it is present, but has a 10% false positive rate (indicating that the virus is present, when it is not). Test B is 90% effective at recognizing the virus, but has a 5% false positive rate. The two tests use independent methods of identifying the virus. The virus is carried by 1% of all people. Say that a person is testsed for the virus using only one of the tests, and that the test comes back positive for carrying the virus. Which test returning positive is more indicative of someone really carrying the virus? Justify your answer mathematically.

Q5. R&N Q13.17 (10 P.) (Note that there is a typo in the R&N textbook on p508 for Q13.17: B should be Y in the second equation) Show that the statement of conditional independence

$$P(X,Y|Z) = P(X|Z)P(Y|Z)$$

is equivalent to each of the statements

$$P(X|Y,Z) = P(X|Z) \quad \text{and} \quad P(Y|X,Z) = P(Y|Z)$$

Q6. R&N Q13.21 (Adapted from Pearl, 1988) (10 P.) Suppose you are a witness to a nighttime hit-and-run accident involving a taxi in Athens. All taxis in Athens are blue or green. You swear, under oath, that the taxi was blue. Extensive testing shows that, under dim lighting conditions, discrimination between blue and green is 75% reliable.

a) Is it possible to calculate the most likely color for the taxi? (Hint: distinguish carefully between the proposition that the taxi *is* blue and the proposition that it *appears* blue.)

b) What if you know that 9 out of 10 Athenian taxis are green?

Q7. Bayesian Inference (15 P.)

You are designing a new system for detecting an explosive device. Your current design has a 1% false negative rate and a 5% false positive rate. The estimated frequency of actual devices through a typical security check point where the system will be employed is 1 in 5,000. Use the variables E to represent the explosive device and D (or D1 and D2) to represent the detector(s).

a) What is the probability that when the detector indicates the presence of an explosive device that the person is actually carrying one? Show your work?

b) On average, how many people will have to be detained and inspected for every device detected?

c) Now suppose you have additional equipment that gives you an independent test for the same types of devices, but it's noisier: the false negative rate is 5% and the false positive rate is 10%. What is the probability of a device when both of them signal a detection? Show your work.