



Figure 1: Phase portrait of the model given by (1) with $A = 10$, $\mu = 0.005$, $\varepsilon = 0$, $\psi = 0$, $I = 0$, and $s = 50$.

$$\dot{x} = -\pi A \sin\left(\pi \frac{f(x,t)}{s}\right) \cos\left(\pi \frac{y}{s}\right) - \mu x + \eta_1(t), \quad (1a)$$

$$\dot{y} = \pi A \cos\left(\pi \frac{f(x,t)}{s}\right) \sin\left(\pi \frac{y}{s}\right) \frac{df}{dx} - \mu y + \eta_2(t), \quad (1b)$$

$$f(x,t) = \varepsilon \sin(\omega t + \psi) x^2 + (1 - 2\varepsilon \sin(\omega t + \psi)) x. \quad (1c)$$

When $\varepsilon = 0$, the double-gyre flow is time-independent, while for $\varepsilon \neq 0$, the gyres undergo a periodic expansion and contraction in the x direction. In (1), A approximately determines the amplitude of the velocity vectors, $\omega/2\pi$ gives the oscillation frequency, ε determines the amplitude of the left-right motion of the separatrix between the gyres, ψ is the phase, μ determines the dissipation, s scales the dimensions of the workspace, and $\eta_i(t)$ describes a stochastic white noise with mean zero and standard deviation $\sigma = \sqrt{2I}$, for noise intensity I . In this work, $\eta_i(t)$ can be viewed as either measurement or environmental noise. Fig. 1 shows the phase portrait of the time-independent double-gyre model.