Jasleen
102103191
3CO7

DATE

PAGE

Q1

To find max. likelihood estimates of the parameters θ_1 (mean) & θ_2 (variance) for a normal distribution, we will use likelihood fcn. & then maximize it.

Given that $x_1, x_2, ---, x_n$ is a random sample from a normal distⁿ with mean θ_1 & variance θ_2 the likelihood fcn. is:

$$L(\theta_1, \theta_2 \mid x_1, x_2, ---, x_n) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\theta_2}} e^{-\frac{(x-\theta_1)^2}{2\theta_2}}$$

Taking log on both sides:

$$\ln L(\theta_1, \theta_2 \mid x_1, x_2, ---, x_n) = -\frac{n}{2}\ln(2\pi\theta_2) - \frac{1}{2\theta_2}\sum_{i=1}^{n}(x_i - \theta_1)^2$$

To find MLE, we'll differentiate the log-likelihood with respect to θ_1 & θ_2, set derivative = to 0.

i) For θ_1:

$$\frac{d}{d\theta_1} \ln L(\theta_1, \theta_2 \mid x_1, x_2, ---, x_n) = \frac{1}{\theta_2}\sum_{i=1}^{n}(x_i - \theta_1)$$

Setting this = to 0.

$$\frac{1}{\theta_2}\sum_{i=1}^{n}(x_i - \hat{\theta}_1) = 0 \Rightarrow \sum_{i=1}^{n}(x_i - \hat{\theta}_1) = 0$$

DATE
PAGE

$$\therefore \hat{\theta}_1 = \frac{1}{n}\sum_{i=1}^{n} x_i$$

So, The MLE for θ_1 is The sample mean.

ii) For θ_2:

$$\frac{\partial}{\partial \theta_2} \ln L(\theta_1, \theta_2 \mid x_1, x_2, \ldots, x_n) = \frac{-n}{2\theta_2} + \frac{1}{2\theta_2^2}\sum_{i=1}^{n}(x_i - \theta_1)^2$$

Setting this = to 0:

$$\frac{-n}{2\hat{\theta}_2} + \frac{1}{2\hat{\theta}_2^{2}}\sum_{i=1}^{n}(x_i - \hat{\theta}_1)^2 = 0$$

$$\Rightarrow \frac{n}{2\hat{\theta}_2} = \frac{1}{2\hat{\theta}_2^{2}}\sum_{i=1}^{n}(x_i - \hat{\theta}_1)^2$$

$$\hat{\theta}_2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \hat{\theta}_1)^2$$

So, MLE for θ_2 is sample variance.

Q2
To find the MLE of θ for a random sample $x_1, x_2, \ldots, x_n$ from a Bernoulli distribution with parameter $\theta \angle 0$ known m.
The likelihood for this scenario is:

$$L(\theta \mid x_1, x_2, \ldots, x_n) = \prod_{i=1}^{n} P(X_i = x_i \mid \theta)$$

Since, X_i follows a bernoulli distribution,

$P(X_i = x_i | \theta) = \theta^{x_i}(1-\theta)^{m-x_i}$ for each i.

Taking log on both sides:

$$\ln L(\theta | x_1, x_2, ---, x_n) = \sum_{i=1}^{n} \ln \left(\theta^{x_i} (1-\theta)^{m-x_i} \right)$$

$$= \sum_{i=1}^{n} \left(x_i \ln \theta + (m - x_i) \ln (1-\theta) \right)$$

Now, differentiate wrt θ & set to 0.

$$\frac{d}{d\theta} \left(\ln L (\theta | x_1, x_2, ----, x_n) \right) = 0$$

$$\sum_{i=1}^{n} \left(\frac{x_i}{\theta} - \frac{m - x_i}{1-\theta} \right) = 0$$

$$\sum_{i=1}^{n} \frac{x_i}{\theta} = \frac{n.m - \sum_{i=1}^{n} x_i}{1-\theta}$$

$$\therefore \theta = \sum_{i=1}^{n} \frac{x_i}{n.m}$$

So, max likelihood estimate for θ is:

$$\hat{\theta}_{MLE} = \sum_{i=1}^{n} \frac{x_i}{n.m}$$