Machine Learning and Extremes for Anomaly Detection

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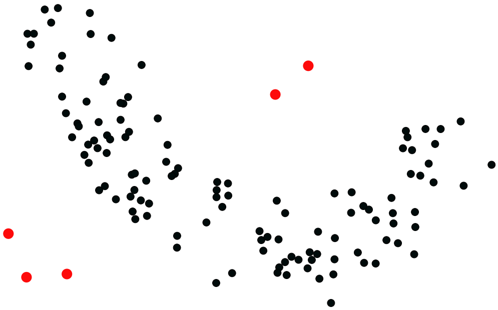
Anomaly Detection (AD)

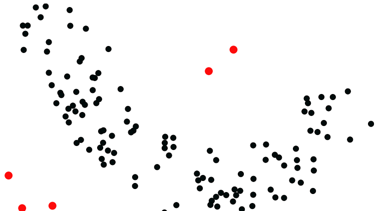
'Finding patterns in the data that do not conform to expected behavior'



Applications: Network intrusions, credit card fraud detection, insurance, finance, military surveillance, predictive maintenance...

Different kinds of Anomaly Detection

- **Supervised** AD (not dealt with)
  Labels available for both normal data and anomalies
  (similar to rare class mining)
- **Novelty Detection** (our theoretical framework)
  Synonym: one-class classification. The algorithm learns on normal data only
- **Outlier Detection** (extended application framework)
  Training set (unlabeled) = normal + abnormal data
  (assumption: anomalies are very rare)



Some litterature in Anomaly Detection

- **Statistical AD techniques**
  [Hawkins 1980, Liu and Weng 1991, Eskin 2000, Agarwal 2006]

- **K-nearest neighbors**
  [Breunig *et al.* 2000, Tang *et al.* 2002, Papadimitriou *et al.* 2002, Hautamaki *et al.* 2004]

- **Support estimation**
  [Einmahl and Mason 92, Polonik 97, Schölkopf *et al.* 2000, Vert and Vert 2006, Scott and Nowak 2006]

- **High-dimensional techniques**
  [Aggarwal and Yu 2001, Shyu *et al.* 2003, Shi and Horvath 2012, Liu *et al.* 2008]

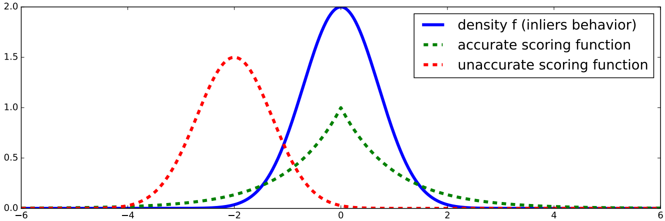
Background: scoring functions

Notation: Normal behavior $\to$ density f.

AD algorithm returns a **scoring function** $s : \mathbb{R}^d \to \mathbb{R}$

- s defines a pre-order on $\mathbb{R}^d$ = 'degree of normality'.
- s level sets are estimates of f level sets.
- s can be interpreted as a continuum of level sets estimates (at different levels).

Remark. Ideal scoring functions: $s = T \circ f$ any increasing transform of f.



Outline

Part I: Performance criterion for scoring functions

Part II: Learning accurate scoring functions on extreme regions

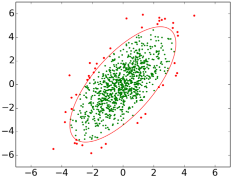
Part III: Heuristic contributions and perspectives

Part I: Performance criterion for scoring functions

Part II: Learning accurate scoring functions on extreme regions

Part III: Heuristic contributions and perspectives

Existing criterion: Mass-Volume curve



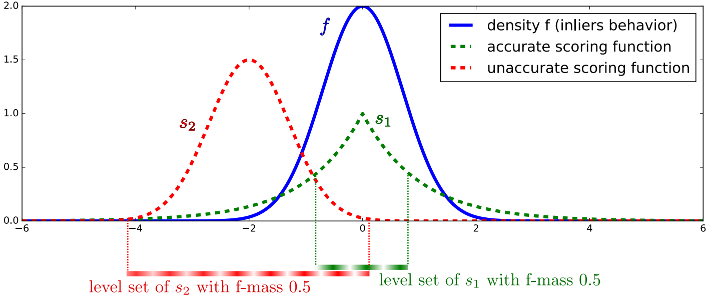
Minimum volume set [Einmahl and Mason 1992, Polonik 1997]

$$\Gamma_\alpha^* = \underset{\Gamma \text{ borelian}}{\arg\min} \quad \mathrm{Leb}(\Gamma) \quad s.t. \quad \mathbb{P}(\mathbf{X} \in \Gamma) \geq \alpha$$

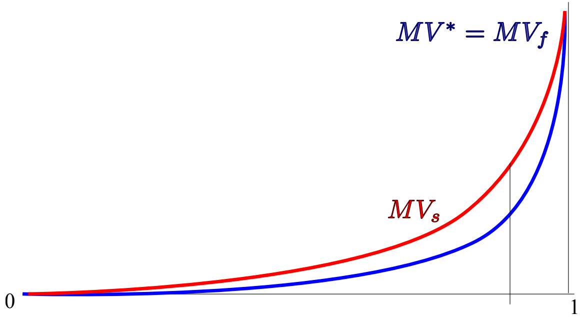
Mass Volume curve of a scoring function s [Clémençon and Jakubowicz, 2013]:

$$MV_s(\alpha) \;:=\; \inf_{\Gamma \text{ level-set of } s} \left\{ \mathrm{Leb}(\Gamma) \quad s.t. \; \mathbb{P}(\mathbf{X} \in \Gamma) \geq \alpha \right\}$$

$$MV^*(\alpha) \;:=\; MV_f(\alpha) \overset{prop}{=} \min_{\Gamma \text{ borelian}} \left\{ \mathrm{Leb}(\Gamma) \quad s.t. \; \mathbb{P}(\mathbf{X} \in \Gamma) \geq \alpha \right\}$$



Existing criterion: Mass-Volume curve



Main drawbacks of MV:

- When optimized *w.r.t.* different levels α over a finite class, produces **not necessarily nested** empirical level sets.
- $\rightarrow$ **low convergence rates** – of order $O(n^{-1/4})$.
- MV **diverges** in 1 in case of unbounded support.

Our solution: Excess-Mass curve

Excess-Mass [Polonik 1995]

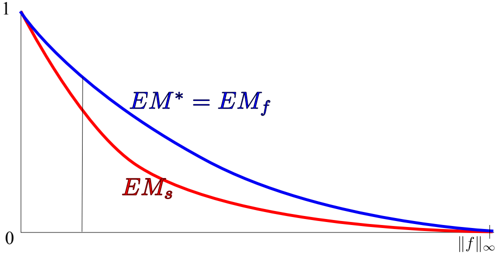
Excess-Mass curve [G., Sabourin, Clémençon 2015]

$$EM_s(t) \quad := \sup_{\Omega \text{ level-set of } s} \left\{ \mathbb{P}(\mathbf{X} \in \Omega) \; - \; t\text{Leb}(\Omega) \right\}$$

$$EM^*(t) \quad := \; EM_f(t) \overset{prop}{=} \max_{\Omega \text{ borelian}} \left\{ \mathbb{P}(\mathbf{X} \in \Omega) - t\text{Leb}(\Omega) \right\}$$

Property: Previous drawbacks are fixed with EM.

- Produces **nested** empirical level sets.
- $\rightarrow$ **convergence rates** of order $O(n^{-1/2})$.
- EM curve **finite** even in case of unbounded support.



Learning a scoring function with M-estimation

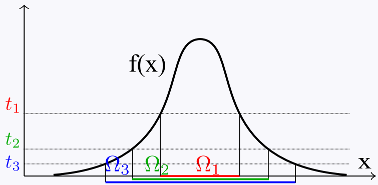
We are looking for nearly optimal scoring functions of the form $s(x) = \sum_{j=1}^{N} a_j \mathbb{1}_{x \in \Omega_j}$, with $a_j \geq 0$, $\Omega_j \in \mathcal{G}$ a VC-class.

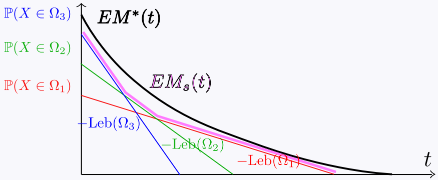
Procedure: Fix $t_0 > 0$
For $k = 1, \dots, N$,

$$t_{k+1} = \frac{t_k}{(1 + \frac{1}{\sqrt{n}})}$$

$$\widehat{\Omega}_{t_{k+1}} = \underset{\Omega \in \mathcal{G},\ \widehat{\Omega}_{t_k} \subset \Omega}{\arg\max} \quad \mathbb{P}_n(X \in \Omega) - t_{k+1} \mathrm{Leb}(\Omega)$$

$s_N(x) := \sum_{j=1}^{N} (t_j - t_{j+1}) \mathbb{1}_{x \in \Omega_{t_j}}$



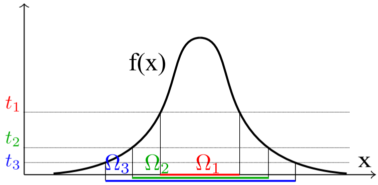


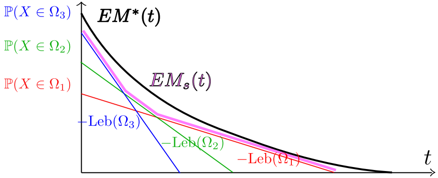
Learning a scoring function with M-estimation

Theorem [G., Sabourin, Clémençon 2015]

Assume the density f bounded, with compact support and without flat parts and $\mathcal{G}$ VC-class. Then if $t_N = \mathcal{O}(n^{-1/2})$, with probability at least $1-\delta$,

$$\sup_{t \in]0, t_1]} |EM^*(t) - EM_{s_N}(t)| \;\leq\; \left[A + \sqrt{2\log(1/\delta)}\right] \frac{1}{\sqrt{n}} + bias(\mathcal{G}).$$





Part I: Performance criterion for scoring functions

Part II: Learning accurate scoring functions on extreme regions

Part III: Heuristic contributions and perspectives

Why dealing with extremes?

General ideas:

- Extreme observations play a special role when dealing with outlying data.
- But no anomaly detection algorithm has **specific treatment for such multivariate extreme observations**. Univariate EVT: [Roberts 99, Lee and Roberts 2008, Clifton *et al.* 2011]
- Our goal:
  - Define a notion of sparsity for extremes observations.
  - Provide a method which can improve performance of standard AD algorithms by combining them with a **multivariate extreme analysis** of the **dependence structure**, using this notion of sparsity.

Purpose

$$\mathbf{X} = (X_1, \ldots, X_d)$$

Goal: find the groups of features which can be large together

ex: $\{X_1, X_2\}$, $\{X_1, X_3, X_4\}$, $\{X_5\}$

Namely: characterize the extreme dependence structure

$\rightarrow$ Anomalies = points which violate this structure

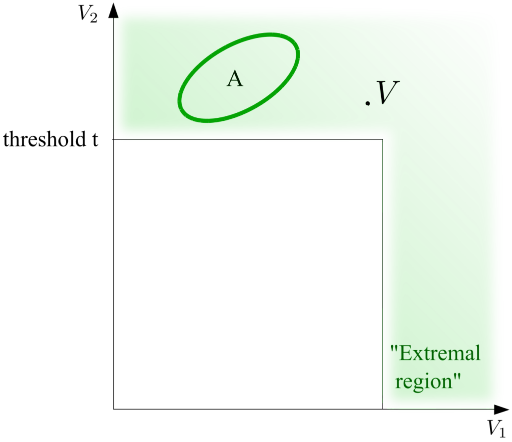
Theoretical framework

- **Context**
  - Random vector $\mathbf{X} = (X_1, \ldots, X_d)$
  - Margins: $X_j \sim F_j$ (F_j continuous)
- **Preliminary step: Standardization of each marginal**
  - Standard Pareto: $V_j = \frac{1}{1-F_j(X_j)}$ $\left(\mathbb{P}(V_j \geq x) = \frac{1}{x},\ \ x \geq 1\right)$

Problematic of Extreme Value Theory

Describe $\mathbf{V}$'s distribution, when $\mathbf{V}$ exceeds some large threshold.

$$\mathbb{P}(\mathbf{V} \in A) = ? \qquad (A \text{ 'far from the origin'}).$$



Fundamental hypothesis and consequences

- Standard assumption: let A extreme region,

$$\mathbb{P}[\mathbf{V} \in t\,A] \simeq t^{-1}\mathbb{P}[\mathbf{V} \in A] \quad \text{(radial homogeneity)}$$

- Formally:

regular variation (after standardization):

$$\text{If} \quad 0 \notin \overline{A}, \qquad t\mathbb{P}[\mathbf{V} \in t\,A] \xrightarrow[t\to\infty]{} \mu(A).$$

μ: exponent measure

Necessarily: $\mu(tA) = t^{-1}\mu(A)$

- $\Rightarrow$ **angular measure** on sphere $\mathbf{S}_{d-1}$: $\Phi(B) = \mu\{tB, t \geq 1\}$

$\mathbb{P}[\mathbf{V} \in A] \quad \simeq \quad \mu(A)$, if A extreme region.

Model for excesses

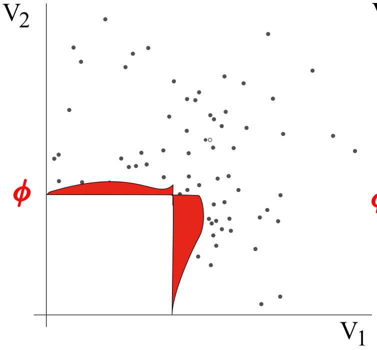
For a large $r > 0$ and a region B on the unit sphere:

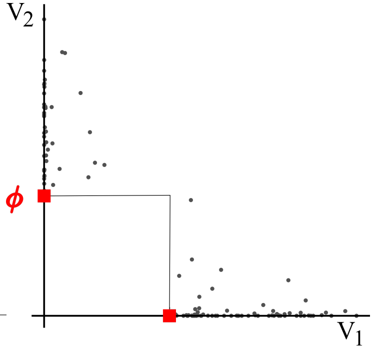
$$\mathbb{P}\left[\|\mathbf{V}\| > r,\ \frac{\mathbf{V}}{\|\mathbf{V}\|} \in B\right] \underset{r\to\infty}{\sim} \frac{1}{r}\,\Phi(B) = \mu(\{tB, t \geq r\})$$

$\Rightarrow \Phi$ (or μ) **rules the joint distribution of extremes** (if margins are known).

Angular distribution

Φ rules the joint distribution of extremes:

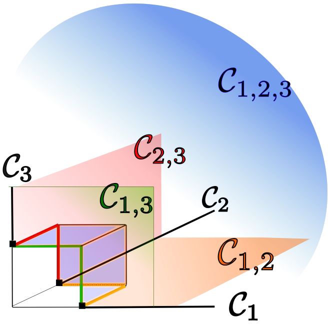




Asymptotic dependence:
(V_1, V_2) may be large together.

Asymptotic independence:
Only V_1 *or* V_2 may be large.

General Case



- Sub-cones: $\mathcal{C}_\alpha = \left\{ \|v\| \geq 1, \quad v_j > 0 \ (j \in \alpha), \quad v_j = 0 \ (j \notin \alpha) \right\}$
- Corresponding sub-spheres: $\left\{ \Omega_\alpha, \alpha \subset \{1, \dots, d\} \right\} \quad (\Omega_\alpha = \mathcal{C}_\alpha \cap \mathbf{S}_{d-1})$

Representation of extreme data

- Natural decomposition of the angular measure :

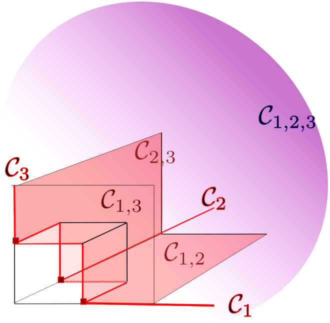
$$\Phi = \sum_{\alpha \subset \{1,\dots,d\}} \Phi_\alpha \qquad \text{with } \; \Phi_\alpha = \Phi_{|\Omega_\alpha} \leftrightarrow \mu_{|\mathcal{C}_\alpha}$$

- $\Rightarrow$ yields a representation

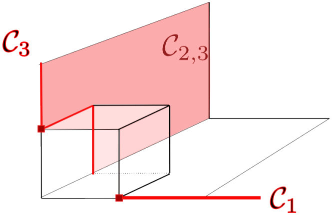
$$\begin{aligned} \mathcal{M} &= \Big\{ \Phi(\Omega_\alpha) : \quad \emptyset \neq \alpha \subset \{1, \dots, d\} \Big\} \\ &= \Big\{ \mu(\mathcal{C}_\alpha) : \quad \emptyset \neq \alpha \subset \{1, \dots, d\} \Big\} \end{aligned}$$

- Assumption: $\frac{d\mu_{|\mathcal{C}_\alpha}}{dv_\alpha} = O(1)$.

Sparse Representation ?



Full pattern :
anything may happen



Sparse pattern
(V_1 not large if V_2 or V_3 large)

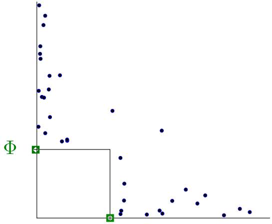
Estimation Problem: $\mathcal{M}$ is an **asymptotic** representation

$$\mathcal{M} \;=\; \left\{\, \Phi(\Omega_\alpha),\ \alpha \,\right\} \;=\; \left\{\, \mu(\mathcal{C}_\alpha),\ \alpha \,\right\}$$

is the restriction of an asymptotic measure

$$\mu(A) = \lim_{t\to\infty} t\mathbb{P}[\mathbf{V} \in t\,A]$$

to a representative class of set $\{\mathcal{C}_\alpha,\ \alpha\}$, but only the central sub-cone has positive Lebesgue measure!

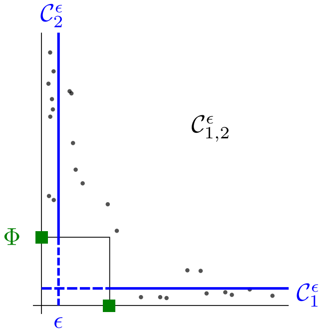


$\Rightarrow$ Cannot just do, for large t:

$$\Phi(\Omega_\alpha) = \mu(\mathcal{C}_\alpha) \simeq t\widehat{\mathbb{P}}(t\mathcal{C}_\alpha)$$

Solution

Fix $\epsilon > 0$. Affect data ϵ-close to an edge, to that edge.



$$\mathcal{C}_\alpha \to \mathcal{C}_\alpha^\epsilon = \{\|v\| \geq 1, \quad v_j > \epsilon \,(j \in \alpha),\, v_j \leq \epsilon \,(j \notin \alpha)\}.$$

New partition of $\mathbf{S}_{d-1}$.

Resulting estimation procedure

$\hat{V}_i^j = \frac{1}{1-\hat{F}_j(X_i^j)}$ with $\hat{F}_j(X_i^j) = \frac{rank(X_i^j)-1}{n}$

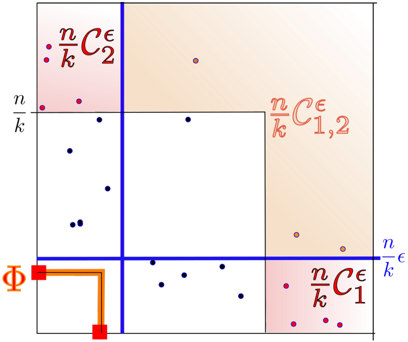
Recall that:

$$\mu(A) = \lim_{t\to\infty} t\mathbb{P}[\mathbf{V} \in t\, A]$$

$\Rightarrow$ get an natural estimate of $\Phi(\Omega_\alpha)$

$$\widehat{\Phi}(\Omega_\alpha) := \frac{n}{k}\mathbb{P}_n(\hat{V} \in \frac{n}{k}\mathcal{C}_\alpha^\epsilon)$$

$(\frac{n}{k}$ large, ϵ small$)$



$\Rightarrow$ we obtain

$$\widehat{\mathcal{M}} := \{\ \widehat{\Phi}(\Omega_\alpha),\ \alpha\ \}$$

Statistical guaranties

Theorem [G., Sabourin, Clémençon 2016]

There is an absolute constant $C > 0$ such that for any $n > 0,\ k > 0,\ 0 < \epsilon < 1,\ \delta > 0$ such that $0 < \delta < e^{-k}$, with probability at least $1 - \delta$,

$$\|\widehat{\mathcal{M}} - \mathcal{M}\|_\infty \ \leq \ Cd\left(\sqrt{\frac{1}{\epsilon k}\log\frac{d}{\delta}} + Md\epsilon\right) + \mathrm{bias}(\epsilon, k, n)$$

Comments:

- $M = \sup(\text{density on sub-cones})$
- Existing litterature (for spectral measure) [Einmahl and Segers 09, Einmahl *et al.* 01]

$$d = 2, \text{ asymptotic behaviour, rates in } 1/\sqrt{k}.$$

- Here: $1/\sqrt{k} \to 1/\sqrt{\epsilon k} + \epsilon$. Price to pay for biasing our estimator with ϵ.

Theorem's proof: key ingredient

Would like to use concentration inequality...

In our case: $\sup_{A \in \mathcal{A}} \frac{n}{k} \left| (\mathcal{P} - \mathcal{P}_n) \left(\frac{k}{n} A \right) \right|$

But usually: $\sup_{A \in \mathcal{A}} \left| (\mathcal{P} - \mathcal{P}_n)(A) \right|$

- scaling $\frac{n}{k}$
- classical VC-inequality: $\frac{k}{n}$ nice but not used !
  $\rightarrow$ high proba bound in

$$\frac{n}{k} \times \sqrt{\frac{1}{n} \log \frac{1}{\delta}} \quad \rightarrow \quad \infty \quad !!$$

$\Rightarrow$ Needs to take into account that the proba of $\frac{k}{n} A$ is small.

Theorem's proof: key ingredient

Key: VC-inequality adapted to rare regions $\rightarrow$ bound in

$$\sqrt{\mathbf{p}}\ \frac{n}{k}\sqrt{\frac{d}{n}\log\frac{1}{\delta}}$$

with p the probability to be in the union class $\cup_{A\in\mathcal{A}} A$.

$$\mathbf{p} \leq d\frac{k}{n}$$

$\Rightarrow$ bound in

$$d\sqrt{\frac{1}{k}\log\frac{1}{\delta}}$$

$k \propto$ number of data considered as extreme (data used for estimation)

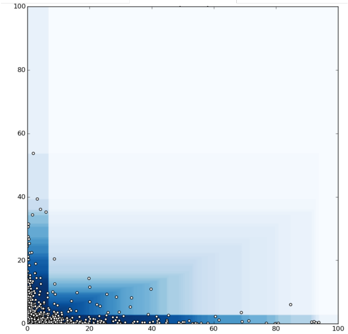
Application to Anomaly Detection

Recall that after standardization of marginals: $\mathbb{P}[R > r, \mathbf{W} \in B] \simeq \frac{1}{r}\,\Phi(B)$

$\rightarrow$ scoring function = $\Phi_n^\epsilon\ \times 1/r$:

$$s_n(\mathbf{x}) := (1/\|\hat{T}(\mathbf{x})\|_\infty) \sum_\alpha \Phi_n^{\alpha,\epsilon} \mathbb{1}_{\hat{T}(\mathbf{x}) \in \mathcal{C}_\alpha^\epsilon}.$$

where $\hat{T} : \mathbf{X} \mapsto \mathbf{V}$ $\qquad (\hat{V}_j = \frac{1}{1-\hat{F}_j(X_j)})$



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EM and MV curves for model selection?

Practical motivations:

Most of the time, data come without any label.

$\rightarrow$ no ROC or PR curves!

Implicit (natural) simplification of the problem:

How good is an anomaly detection algorithm?
$\rightarrow$ How good is it estimating the level sets of the inliers distribution?

Estimation:

$$\widehat{MV}_s(\alpha) = \inf_{u \geq 0} \ \mathrm{Leb}(s \geq u) \quad s.t. \quad \mathbb{P}_n(s \geq u) \geq \alpha \qquad \rightarrow \widehat{\mathcal{C}}^{EM}(s) = \|\widehat{MV}_s\|_{1,J}$$

$$\widehat{EM}_s(t) = \sup_{u \geq 0} \ \mathbb{P}_n(s \geq u) - t\mathrm{Leb}(s \geq u) \qquad \rightarrow \widehat{\mathcal{C}}^{MV}(s) = \|\widehat{EM}_s\|_{1,I}$$

[Thomas et al. 2015, 2016]
Issue in large dimension: The volume $\mathrm{Leb}(s \geq u)$ has to be estimated!

EM and MV curves for model selection?

Heuristic extension for large dimension:

Random projection and averaging

Inputs: AD algorithm $\mathcal{A}$, data set X size $n \times d$, feature sub-sampling size d', number of draws m.

for $k = 1, \dots, m$ **do**
-randomly select a sub-group F_k of d' features
-compute the associated scoring function $s_k = \mathcal{A}\big((x_i^j)_{1 \le i \le n,\ j \in F_k}\big)$
-compute $\widehat{\mathcal{C}}_k^{EM} = \|\widehat{EM}_{s_k}\|_{L^1(I)}$ or $\widehat{\mathcal{C}}_k^{MV} = \|\widehat{MV}_{s_k}\|_{L^1(J)}$
end for

Return performance criteria:

$$\widehat{\mathcal{C}}_{high\_dim}^{EM}(\mathcal{A}) = \frac{1}{m}\sum_{k=1}^{m}\widehat{\mathcal{C}}_k^{EM} \quad \text{or} \quad \widehat{\mathcal{C}}_{high\_dim}^{MV}(\mathcal{A}) = \frac{1}{m}\sum_{k=1}^{m}\widehat{\mathcal{C}}_k^{MV}\ .$$

Seems to work in practice but no statistical guaranties.

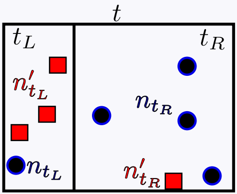
One Class Random Forests

Two-class Random Forests [Breiman, 2001]

Two-Class impurity decrease

$$I_G(t_L, t_R) = \frac{n_{t_L} n'_{t_L}}{n_{t_L} + n'_{t_L}} + \frac{n_{t_R} n'_{t_R}}{n_{t_R} + n'_{t_R}}.$$

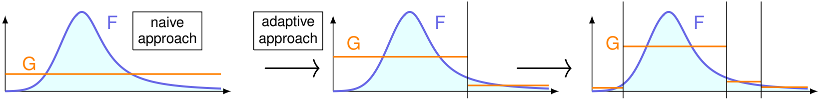
n_t : nb of observations with label 0 in node t.
n'_t: nb of observations with label 1 in node t.



Existing litterature: [Désir et al., 2013, Liu et al., 2008, Shi and Horvath, 2012].

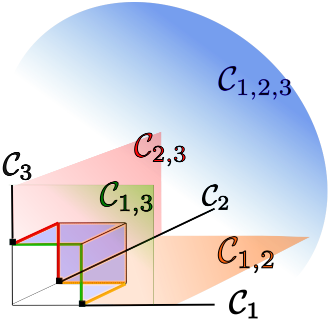
One-Class splitting criterion

$n'_{t_L} \longrightarrow n_t \frac{\text{Leb}(t_L)}{\text{Leb}(t)}$ and $n'_{t_R} \longrightarrow n_t \frac{\text{Leb}(t_R)}{\text{Leb}(t)}$



F is the inliers distribution, G is the assumed outliers distribution.

Does theorical guaranties apply ? [Biau et al. 2008, Biau and Scornet, 2016]



- Add details to the representation?
- An alternative definition of sparsity?

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Corresponding Algorithm

DAMEX in $O(dn \log n)$

Input: parameters $\epsilon > 0, \quad k = k(n)$

for $i = 1, \ldots, n$ **do**

  # Standardize *via* marginal rank-transformation:

  $\hat{V}_i := \big(1/(1 - \hat{F}_j(X_i^j))\big)_{j=1,\ldots,d}$.

  if $\hat{V}_i > \frac{n}{k}$ **then**

    # Assign to each $\hat{V}_i$ the cone $\frac{n}{k}\mathcal{C}_\alpha^\epsilon$ it belongs to:

    $\alpha = \alpha(V_i)$

    c_α ++

  end if

end for

$\Phi_n^{\alpha,\epsilon} := \frac{n}{k} c_\alpha$

Output: (sparse) representation of the dependence structure:
$\Phi_n^{\alpha,\epsilon} = \widehat{\Phi}(\Omega_\alpha) = \frac{n}{k}\mathbb{P}_n(\hat{V} \in \frac{n}{k}\mathcal{C}_\alpha^\epsilon)$, estimate of the α-mass of Φ for every α.

$$\widehat{\mathcal{M}} := (\Phi_n^{\alpha,\epsilon})_{\alpha \subset \{1,\ldots,d\}, \Phi_n^{\alpha,\epsilon} > \Phi_{\min}}$$

Experiments

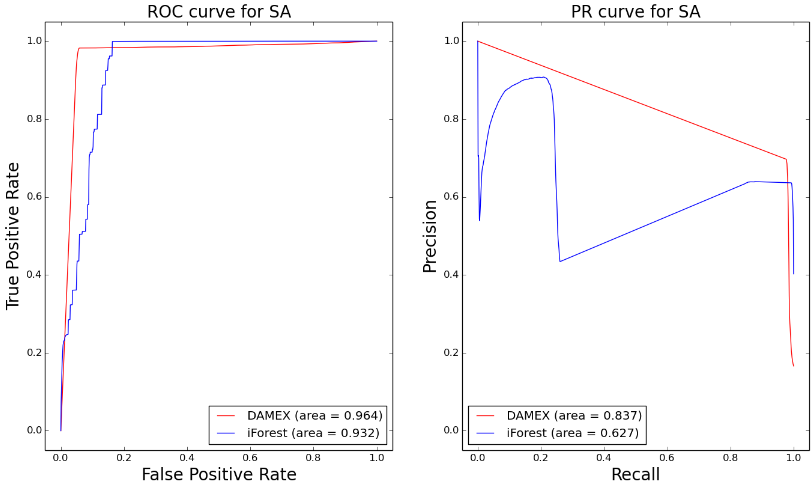

Figure: ROC and PR curve on SA dataset

	number of samples	number of features
shuttle	85849	9
forestcover	286048	54
SA	976158	41
SF	699691	4
http	619052	3

Table: Datasets characteristics

$\epsilon = 0.01,\ k = n^{1/2}$

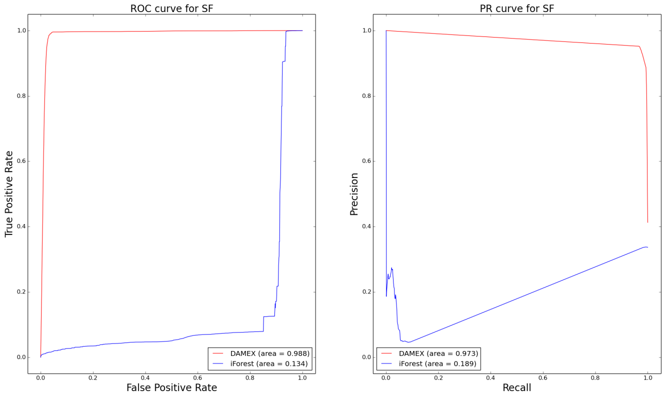

Figure: ROC and PR curve on SF dataset

$\epsilon = 0.01$, $k = n^{1/2}$

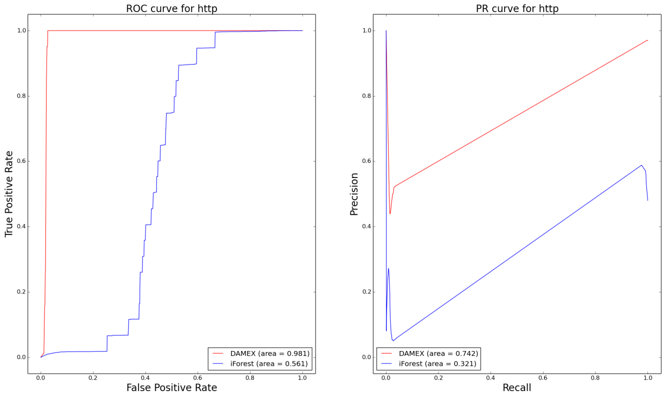

Figure: ROC and PR curve on http dataset

$\epsilon = 0.01$, $k = n^{1/2}$

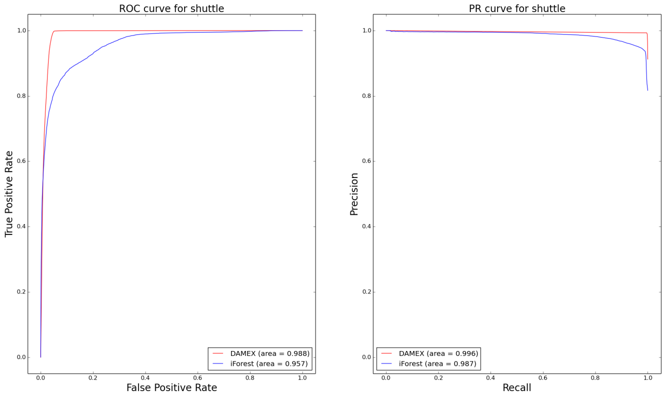

Figure: ROC and PR curve on shuttle dataset

$\epsilon = 0.01,\ k = n^{1/2}$

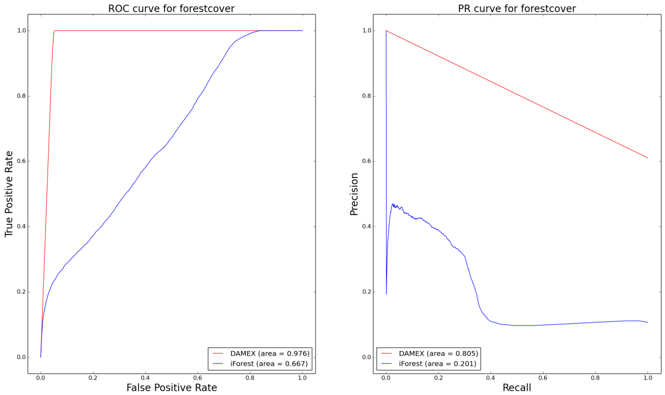

Figure: ROC and PR curve on forestcover dataset

Benchmarks for EM/MV

Does performance in term of EM/MV correspond to performance in term of ROC/PR?

- **Experiments:** 12 datasets, 3 AD algorithms (LOF, OCSVM, iForest) $\rightarrow$ 36 possible pairwise comparisons:

$$\left\{ \Big(A_1 \text{ on } \mathcal{D},\ A_2 \text{ on } \mathcal{D}\Big),\ \ A_1, A_2 \in \{\text{iForest, LOF, OCSVM}\},\ \ \mathcal{D} \in \{\text{adult, http}, \ldots, \text{spambase}\} \right\}.$$

- **Results:** If we only consider the pairs *s.t. ROC and PR agree on which algorithm is the best*, we are able (with EM and MV scores) to recover it in 80% of the cases.

Table: Original Datasets characteristics

	nb of samples	nb of features	anomaly class	
adult	48842	6	class '$> 50K$'	(23.9%)
http	567498	3	attack	(0.39%)
pima	768	8	pos (class 1)	(34.9%)
smtp	95156	3	attack	(0.03%)
wilt	4839	5	class 'w' (diseased trees)	(5.39%)
annthyroid	7200	6	classes $\neq$ 3	(7.42%)
arrhythmia	452	164	classes $\neq$ 1 (features 10-14 removed)	(45.8%)
forestcover	286048	10	class 4 (vs. class 2)	(0.96%)
ionosphere	351	32	bad	(35.9%)
pendigits	10992	16	class 4	(10.4%)
shuttle	85849	9	classes $\neq$ 1 (class 4 removed)	(7.17%)
spambase	4601	57	spam	(39.4%)

Table: Results for the novelty detection setting. One can see that ROC, PR, EM, MV often do agree on which algorithm is the best (in bold), which algorithm is the worse (underlined) on some fixed datasets. When they do not agree, it is often because ROC and PR themselves do not, meaning that the ranking is not clear.

Dataset	iForest				OCSVM				LOF			
	ROC	PR	EM	MV	ROC	PR	EM	MV	ROC	PR	EM	MV
adult	**0.661**	**0.277**	**1.0e-04**	**7.5e01**	0.642	0.206	2.9e-05	4.3e02	0.618	0.187	1.7e-05	9.0e02
http	0.994	0.192	1.3e-03	9.0	**0.999**	**0.970**	**6.0e-03**	**2.6**	0.946	0.035	8.0e-05	3.9e02
pima	0.727	0.182	5.0e-07	**1.2e04**	**0.760**	**0.229**	**5.2e-07**	1.3e04	0.705	0.155	3.2e-07	2.1e04
smtp	0.907	0.005	1.8e-04	9.4e01	0.852	**0.522**	**1.2e-03**	8.2	**0.922**	0.189	1.1e-03	**5.8**
wilt	0.491	0.045	4.7e-05	2.1e03	0.325	0.037	**5.9e-05**	**4.5e02**	**0.698**	**0.088**	2.1e-05	1.6e03
annthyroid	**0.913**	**0.456**	**2.0e-04**	2.6e02	0.699	0.237	6.3e-05	**2.2e02**	0.823	0.432	6.3e-05	1.5e03
arrhythmia	**0.763**	**0.487**	**1.6e-04**	**9.4e01**	0.736	0.449	1.1e-04	1.0e02	0.730	0.413	8.3e-05	1.6e02
forestcov.	0.863	0.046	3.9e-05	2.0e02	0.958	0.110	5.2e-05	1.2e02	**0.990**	**0.792**	**3.5e-04**	**3.9e01**
ionosphere	0.902	0.529	9.6e-05	7.5e01	**0.977**	**0.898**	**1.3e-04**	**5.4e01**	0.971	0.895	1.0e-04	7.0e01
pendigits	0.811	0.197	2.8e-04	2.6e01	0.606	0.112	2.7e-04	2.7e01	**0.983**	**0.829**	**4.6e-04**	**1.7e01**
shuttle	0.996	0.973	1.8e-05	5.7e03	0.992	0.924	**3.2e-05**	**2.0e01**	**0.999**	**0.994**	7.9e-06	2.0e06
spambase	**0.824**	**0.371**	**9.5e-04**	**4.5e01**	0.729	0.230	4.9e-04	1.1e03	0.754	0.173	2.2e-04	4.1e04