

Machine Learning and Extremes for Anomaly Detection

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'Finding patterns in the data that do not conform to expected behavior'



Huge number of applications: Network intrusions, credit card fraud detection, insurance, finance, military surveillance,...

Different kinds of Anomaly Detection

- ▶ **Supervised AD** (not dealt with)
Labels available for both normal data and anomalies
(similar to rare class mining)
- ▶ **Novelty Detection** (our theoretical framework)
The algorithm learns on normal data only
- ▶ **Outlier Detection** (extended application framework)
Training set (unlabeled) = normal + abnormal data
(assumption: anomalies are very rare)

- ▶ **Statistical AD techniques**

[Hawkins 1980, Liu and Weng 1991, Eskin 2000, Agarwal 2006]

- ▶ **K-nearest neighbors**

[Breunig *et al.* 2000, Tang *et al.* 2002, Papadimitriou *et al.* 2002, Hautamaki *et al.* 2004]

- ▶ **Support estimation**

[Einmahl and Mason 92, Polonik 97, Schölkopf *et al.* 2000, Vert and Vert 2006, Scott and Nowak 2006]

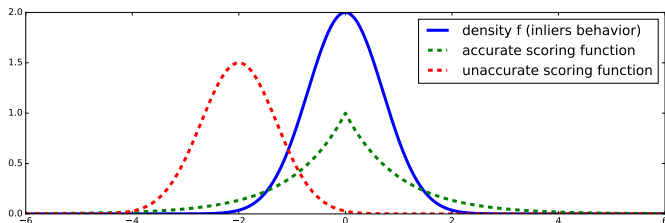
- ▶ **High-dimensional techniques**

[Aggarwal and Yu 2001, Shyu *et al.* 2003, Shi and Horvath 2012, Liu *et al.* 2008]

AD algorithm returns a **scoring function** $s : \mathbb{R}^d \rightarrow \mathbb{R}$

- ▶ s defines a **pre-order** on \mathbb{R}^d = 'degree of normality'.
- ▶ s level sets are estimates of f level sets.
- ▶ s can be interpreted as a **continuum of level sets estimates** (at different levels).

Remark. Ideal scoring functions: $s = T \circ f$ any increasing transform of f .



Part I: Performance criterion for scoring functions

Part II: Learning accurate scoring functions on extreme regions

Part III: Heuristic contributions and perspectives

Part I: Performance criterion for scoring functions

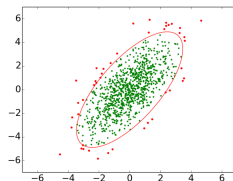
Part II: Learning accurate scoring functions on extreme regions

Part III: Heuristic contributions and perspectives

Existing criterion: Mass-Volume curve

Minimum volume set [Einmahl and Mason 1992, Polonik 1997]

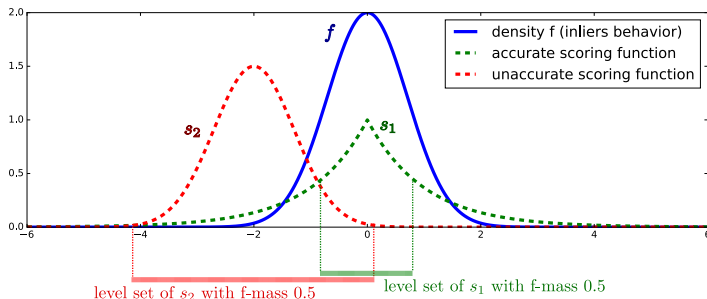
$$\Gamma_{\alpha}^* = \arg \min_{\Gamma \text{ borelian}} \text{Leb}(\Gamma) \quad \text{s.t.} \quad \mathbb{P}(\mathbf{X} \in \Gamma) \geq \alpha$$



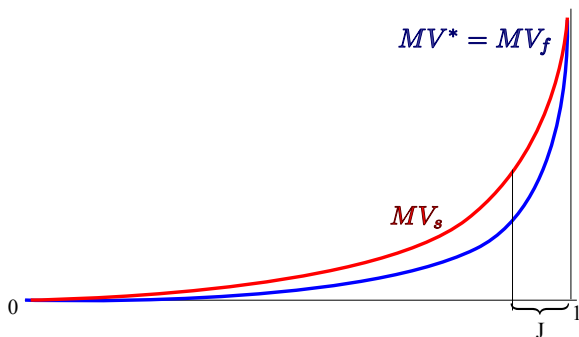
Mass Volume curve of a scoring function s [Cl  men  on and Jakubowicz, 2013]:

$$MV_s(\alpha) := \inf_{\Gamma \text{ level-set of } s} \{ \text{Leb}(\Gamma) \quad \text{s.t.} \quad \mathbb{P}(\mathbf{X} \in \Gamma) \geq \alpha \}$$

$$MV^*(\alpha) := \min_{\Gamma \text{ borelian}} \{ \text{Leb}(\Gamma) \quad \text{s.t.} \quad \mathbb{P}(\mathbf{X} \in \Gamma) \geq \alpha \} \stackrel{\text{prop}}{=} MV_f(\alpha)$$



Existing criterion: Mass-Volume curve



Main drawbacks of MV:

- ▶ When optimized *w.r.t.* different levels α over a finite class, produces **not necessarily nested** empirical level sets.
- ▶ \rightarrow **low convergence rates** – of order $O(n^{-1/4})$.
- ▶ MV **diverges** in 1 in case of unbounded support.

Our solution: Excess-Mass curve

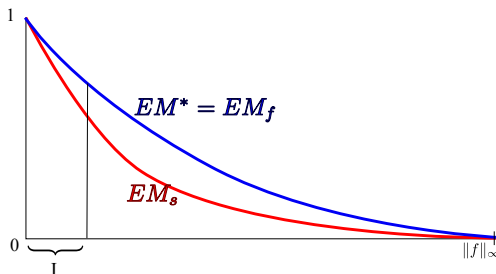
Excess-Mass curve [G., Sabourin, Cléménçon 2015]

$$EM_s(t) := \sup_{\Omega \text{ level-set of } s} \{ \mathbb{P}(\mathbf{X} \in \Omega) - t \text{Leb}(\Omega) \}$$

$$EM^*(t) := \max_{\Omega \text{ borelian}} \{ \mathbb{P}(\mathbf{X} \in \Omega) - t \text{Leb}(\Omega) \} \stackrel{\text{prop}}{=} EM_f(t)$$

Property: Previous drawbacks are fixed with EM.

- ▶ Produces **nested** empirical level sets.
- ▶ \rightarrow **convergence rates** of order $O(n^{-1/2})$.
- ▶ EM curve **finite** even in case of unbounded support.



Learning a scoring function with M-estimation

We are looking for nearly optimal scoring functions of the form

$$s(x) = \sum_{j=1}^N a_j \mathbb{1}_{x \in \Omega_j}, \text{ with } a_j \geq 0, \Omega_j \in \mathcal{G} \text{ a VC-class.}$$

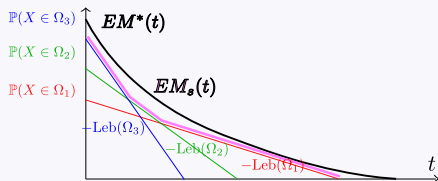
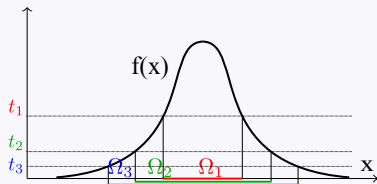
Procedure: Fix $t_0 > 0$

For $k = 1, \dots, N$,

$$t_{k+1} = \frac{t_k}{\left(1 + \frac{1}{\sqrt{n}}\right)}$$

$$\hat{\Omega}_{t_{k+1}} = \arg \max_{\Omega \in \mathcal{G}, \hat{\Omega}_{t_k} \subset \Omega} \mathbb{P}_n(X \in \Omega) - t_{k+1} \text{Leb}(\Omega)$$

$$s_N(x) := \sum_{j=1}^N (t_j - t_{j+1}) \mathbb{1}_{x \in \Omega_{t_j}}$$

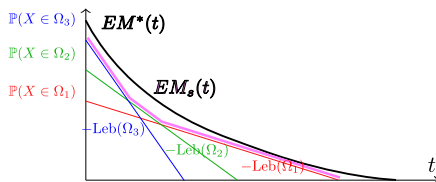
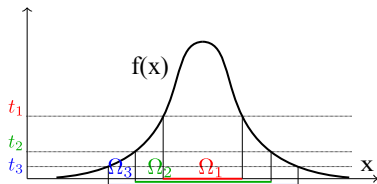


Learning a scoring function with M-estimation

Theorem [G., Sabourin, Cléménçon 2015]

Assume the density f bounded, with compact support and without flat parts and \mathcal{G} VC-class. Then if $t_N = \mathcal{O}(n^{-1/2})$, with probability at least $1 - \delta$,

$$\sup_{t \in]0, t_1]} |EM^*(t) - EM_{s_N}(t)| \leq \left[A + \sqrt{2 \log(1/\delta)} \right] \frac{1}{\sqrt{n}} + \text{bias}(\mathcal{G}).$$



Part I: Performance criterion for scoring functions

Part II: Learning accurate scoring functions on extreme regions

Part III: Heuristic contributions and perspectives

General ideas:

- ▶ Extreme observations play a special role when dealing with outlying data.
- ▶ But no anomaly detection algorithm has **specific treatment for such multivariate extreme observations**. Univariate EVT: [Roberts 99, Lee and Roberts 2008, Clifton *et al.* 2011]
- ▶ Our goal:
 - ▶ Define a notion of sparsity for extremes observations.
 - ▶ Provide a method which can improve performance of standard AD algorithms by combining them with a **multivariate extreme analysis** of the **dependence structure**, using this notion of sparsity.

$$\mathbf{X} = (X_1, \dots, X_d)$$

Goal: find the groups of features which can be large together

ex: $\{X_1, X_2\}$, $\{X_1, X_3, X_4\}$, $\{X_5\}$

Namely: characterize the extreme dependence structure

→ Anomalies = points which violate this structure

- ▶ **Context**

- ▶ Random vector $\mathbf{X} = (X_1, \dots, X_d)$
- ▶ Margins: $X_j \sim F_j$ (F_j continuous)

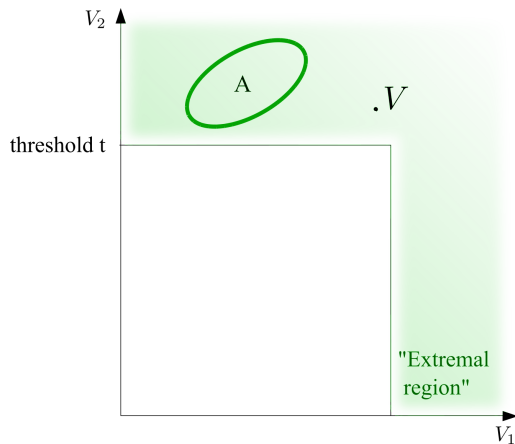
- ▶ **Preliminary step: Standardization of each marginal**

- ▶ Standard Pareto: $V_j = \frac{1}{1-F_j(X_j)}$ ($\mathbb{P}(V_j \geq x) = \frac{1}{x}, x \geq 1$)

Problematic of Extreme Value Theory

Describe \mathbf{V} 's distribution, when \mathbf{V} exceeds some large threshold.

$$\mathbb{P}(\mathbf{V} \in A) = ? \quad (A \text{ 'far from the origin'}).$$



Fundamental hypothesis and consequences

- ▶ Standard assumption: let A extreme region,

$$\mathbb{P}[\mathbf{V} \in tA] \simeq t^{-1} \mathbb{P}[\mathbf{V} \in A] \quad (\text{radial homogeneity})$$

- ▶ Formally:

regular variation (after standardization):

$$\text{If } 0 \notin \bar{A}, \quad t\mathbb{P}[\mathbf{V} \in tA] \xrightarrow[t \rightarrow \infty]{} \mu(A).$$

μ : exponent measure

Necessarily: $\mu(tA) = t^{-1} \mu(A)$

- ▶ \Rightarrow **angular measure** on sphere \mathbf{S}_{d-1} : $\Phi(B) = \mu\{tB, t \geq 1\}$

$\mathbb{P}[\mathbf{V} \in A] \simeq \mu(A)$, if A extreme region.

Model for excesses

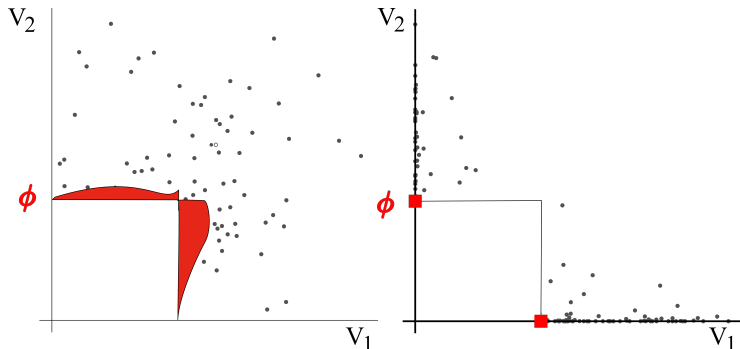
For a large $r > 0$ and a region B on the unit sphere:

$$\mathbb{P} \left[\|\mathbf{V}\| > r, \frac{\mathbf{V}}{\|\mathbf{V}\|} \in B \right] \underset{r \rightarrow \infty}{\sim} \frac{1}{r} \Phi(B) = \mu(\{tB, t \geq r\})$$

$\Rightarrow \Phi$ (or μ) **rules the joint distribution of extremes** (if margins are known).

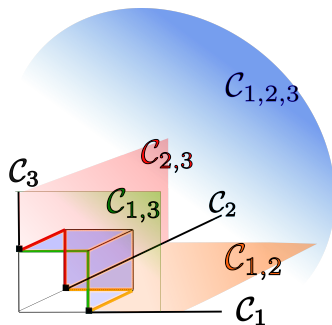
Angular distribution

Φ rules the joint distribution of extremes:



Asymptotic dependence:
(V_1 , V_2) may be large together.

Asymptotic independence:
Only V_1 or V_2 may be large.



- ▶ Sub-cones: $\mathcal{C}_\alpha = \{\|v\| \geq 1, v_j > 0 (j \in \alpha), v_j = 0 (j \notin \alpha)\}$
- ▶ Corresponding sub-spheres: $\{\Omega_\alpha, \alpha \subset \{1, \dots, d\}\}$ ($\Omega_\alpha = \mathcal{C}_\alpha \cap \mathbf{S}_{d-1}$)

- Natural decomposition of the angular measure :

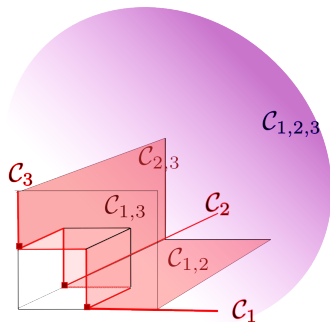
$$\Phi = \sum_{\alpha \subset \{1, \dots, d\}} \Phi_{\alpha} \quad \text{with } \Phi_{\alpha} = \Phi|_{\Omega_{\alpha}} \leftrightarrow \mu|_{\mathcal{C}_{\alpha}}$$

- \Rightarrow yields a representation

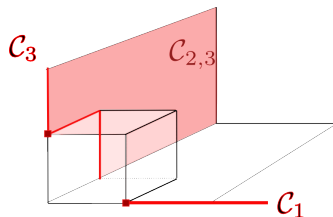
$$\begin{aligned} \mathcal{M} &= \left\{ \Phi(\Omega_{\alpha}) : \emptyset \neq \alpha \subset \{1, \dots, d\} \right\} \\ &= \left\{ \mu(\mathcal{C}_{\alpha}) : \emptyset \neq \alpha \subset \{1, \dots, d\} \right\} \end{aligned}$$

- Assumption: $\frac{d\mu|_{\mathcal{C}_{\alpha}}}{d\nu_{\alpha}} = O(1)$.

Sparse Representation ?



Full pattern :
anything may happen



Sparse pattern
(V_1 not large if V_2 or V_3 large)

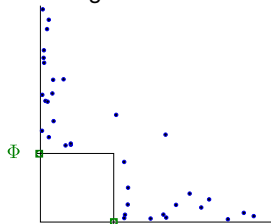
Estimation Problem: \mathcal{M} is an **asymptotic** representation

$$\mathcal{M} = \{ \Phi(\Omega_\alpha), \alpha \} = \{ \mu(\mathcal{C}_\alpha), \alpha \}$$

is the restriction of an asymptotic measure

$$\mu(A) = \lim_{t \rightarrow \infty} t\mathbb{P}[\mathbf{V} \in tA]$$

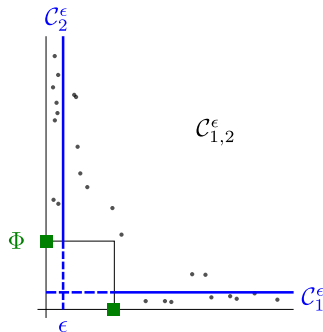
to a representative class of set $\{\mathcal{C}_\alpha, \alpha\}$, but only the central sub-cone has positive Lebesgue measure!



\Rightarrow Cannot just do, for large t :

$$\Phi(\Omega_\alpha) = \mu(\mathcal{C}_\alpha) \simeq \widehat{t\mathbb{P}}(t\mathcal{C}_\alpha)$$

Fix $\epsilon > 0$. Affect data ϵ -close to an edge, to that edge.



$$\mathcal{C}_\alpha \rightarrow \mathcal{C}_\alpha^\epsilon = \{\|v\| \geq 1, \ v_j > \epsilon (j \in \alpha), \ v_j \leq \epsilon (j \notin \alpha)\}.$$

New partition of \mathbf{S}_{d-1} .

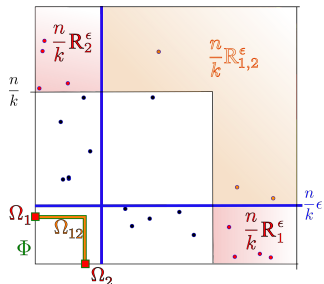
Resulting estimation procedure

$$\hat{V}_i^j = \frac{1}{1 - \hat{F}_j(X_i^j)} \text{ with } \hat{F}_j(X_i^j) = \frac{\text{rank}(X_i^j) - 1}{n}$$

\Rightarrow get an natural estimate of $\Phi(\Omega_\alpha)$

$$\hat{\Phi}(\Omega_\alpha) := \frac{n}{k} \mathbb{P}_n(\hat{V} \in \frac{n}{k} \mathcal{C}_\alpha^\epsilon)$$

($\frac{n}{k}$ large, ϵ small)



\Rightarrow we obtain

$$\widehat{\mathcal{M}} := \{ \hat{\Phi}(\Omega_\alpha), \alpha \}$$

Theorem [G., Sabourin, Cl  men  on 2016]

There is an absolute constant $C > 0$ such that for any $n > 0$, $k > 0$, $0 < \epsilon < 1$, $\delta > 0$ such that $0 < \delta < e^{-k}$, with probability at least $1 - \delta$,

$$\|\widehat{\mathcal{M}} - \mathcal{M}\|_{\infty} \leq Cd \left(\sqrt{\frac{1}{\epsilon k} \log \frac{d}{\delta}} + Md\epsilon \right) + \text{bias}(\epsilon, k, n)$$

Comments:

- ▶ $M = \sup(\text{density on sub-cones})$
- ▶ Existing litterature (for spectral measure) [Einmahl and Segers 09, Einmahl *et al.* 01]

$d = 2$, asymptotic behaviour, rates in $1/\sqrt{k}$.

- ▶ Here: $1/\sqrt{k} \rightarrow 1/\sqrt{\epsilon k} + \epsilon$. Price to pay for biasing our estimator with ϵ .

Theorem's proof: key ingredient

Would like to use concentration inequality...

In our case:
$$\sup_{A \in \mathcal{A}} \frac{n}{k} \left| (\mathcal{P} - \mathcal{P}_n) \left(\frac{k}{n} A \right) \right|$$

But usually:
$$\sup_{A \in \mathcal{A}} |(\mathcal{P} - \mathcal{P}_n)(A)|$$

- ▶ scaling $\frac{n}{k}$
- ▶ classical VC-inequality: $\frac{k}{n}$ nice but not used !
→ high proba bound in

$$\frac{n}{k} \times \sqrt{\frac{1}{n} \log \frac{1}{\delta}} \rightarrow \infty \quad !!$$

⇒ Needs to take into account that the proba of $\frac{k}{n} A$ is small.

Key: VC-inequality adapted to rare regions \rightarrow bound in

$$\sqrt{\mathbf{p}} \frac{n}{k} \sqrt{\frac{d}{n} \log \frac{1}{\delta}}$$

with p the probability to be in the union class $\cup_{A \in \mathcal{A}} A$.

$$\mathbf{p} \leq d \frac{k}{n}$$

\Rightarrow bound in

$$d \sqrt{\frac{1}{k} \log \frac{1}{\delta}}$$

$k \propto$ number of data considered as extreme (data used for estimation)

DAMEX in $O(dn \log n)$

Input: parameters $\epsilon > 0$, $k = k(n)$

for $i = 1, \dots, n$ **do**

 # Standardize via marginal rank-transformation:

$$\hat{V}_i := (1/(1 - \hat{F}_j(X_i^j)))_{j=1, \dots, d}.$$

if $\hat{V}_i > \frac{n}{k}$ **then**

 # Assign to each \hat{V}_i the cone $\frac{n}{k}C_\alpha^\epsilon$ it belongs to:

$$\alpha = \alpha(V_i)$$

$$C_\alpha ++$$

end if

end for

$$\Phi_n^{\alpha, \epsilon} := \frac{n}{k} C_\alpha$$

Output: (sparse) representation of the dependence structure:

$\Phi_n^{\alpha, \epsilon} = \hat{\Phi}(\Omega_\alpha) = \frac{n}{k} \mathbb{P}_n(\hat{V} \in \frac{n}{k} C_\alpha^\epsilon)$, estimate of the α -mass of Φ for every α .

$$\widehat{\mathcal{M}} := (\Phi_n^{\alpha, \epsilon})_{\alpha \in \{1, \dots, d\}, \Phi_n^{\alpha, \epsilon} > \Phi_{\min}}$$

Application to Anomaly Detection

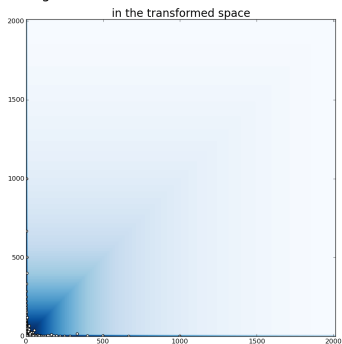
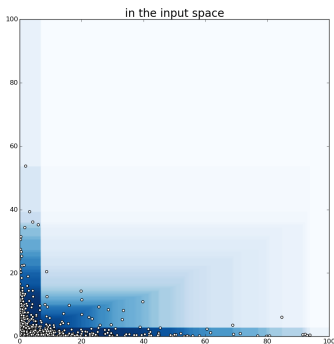
Recall that after standardization of marginals: $\mathbb{P}[R > r, \mathbf{W} \in B] \simeq \frac{1}{r} \Phi(B)$

→ scoring function = $\Phi_n^\epsilon \times 1/r$:

$$s_n(\mathbf{x}) := (1/\|\hat{T}(\mathbf{x})\|_\infty) \sum_{\alpha} \Phi_n^{\alpha, \epsilon} \mathbb{1}_{\hat{T}(\mathbf{x}) \in \mathcal{C}_\alpha^\epsilon}.$$

where $\hat{T} : \mathbf{X} \mapsto \mathbf{V}$ $(\hat{V}_j = \frac{1}{1 - \hat{F}_j(X_j)})$

levels set of DAMEX scoring function



Experiments

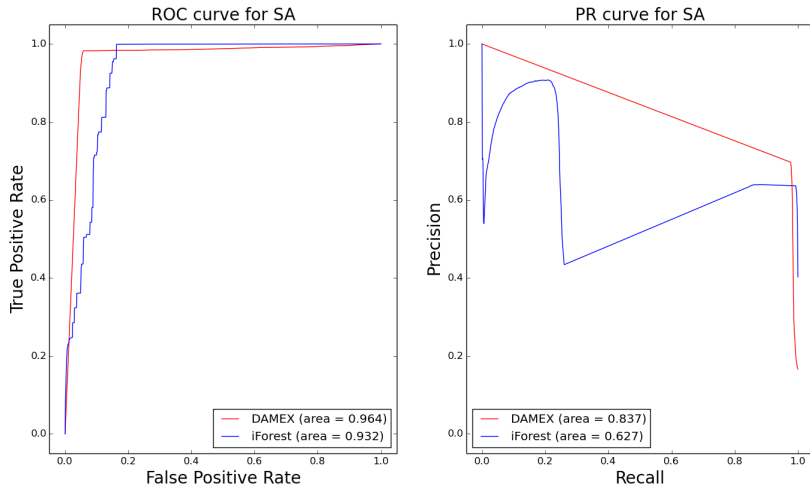


Figure: ROC and PR curve on SA dataset

Part I: Performance criterion for scoring functions

Part II: Learning accurate scoring functions on extreme regions

Part III: Heuristic contributions and perspectives

EM and MV curves for model selection?

In moderate dimension: [Thomas et al. 2015, 2016]

Practical motivations:

Most of the time, data come without any label.

→ no ROC or PR curves!

Implicit (natural) simplification of the problem:

How good is an anomaly detection algorithm?

→ How good is it estimating the level sets of the inliers distribution?

Estimation:

$$\widehat{MV}_s(\alpha) = \inf_{u \geq 0} \text{Leb}(s \geq u) \quad \text{s.t.} \quad \mathbb{P}_n(s \geq u) \geq \alpha \quad \rightarrow \widehat{\mathcal{C}}^{EM}(s) = \|MV_s\|_{1,J}$$

$$\widehat{EM}_s(t) = \sup_{u \geq 0} \mathbb{P}_n(s \geq u) - t \text{Leb}(s \geq u) \quad \rightarrow \widehat{\mathcal{C}}^{MV}(s) = \|EM_s\|_{1,I}$$

Issue in large dimension: The volume $\text{Leb}(s \geq u)$ has to be estimated!

EM and MV curves for model selection?

Heuristic extension for large dimension:

Random projection and averaging

Inputs: AD algorithm \mathcal{A} , data set X size $n \times d$, feature sub-sampling size d' , number of draws m .

for $k = 1, \dots, m$ **do**

-randomly select a sub-group F_k of d' features

-compute the associated scoring function $s_k = \mathcal{A}((x_i^j)_{1 \leq i \leq n, j \in F_k})$

-compute $\widehat{C}_k^{EM} = \|\widehat{EM}_{s_k}\|_{L^1(I)}$ or $\widehat{C}_k^{MV} = \|\widehat{MV}_{s_k}\|_{L^1(J)}$

end for

Return performance criteria:

$$\widehat{C}_{high.dim}^{EM}(\mathcal{A}) = \frac{1}{m} \sum_{k=1}^m \widehat{C}_k^{EM} \quad \text{or} \quad \widehat{C}_{high.dim}^{MV}(\mathcal{A}) = \frac{1}{m} \sum_{k=1}^m \widehat{C}_k^{MV}.$$

Seems to work in practice but no statistical guarantees.

One Class Random Forests

Existing litterature: [Désir et al., 2013, Liu et al., 2008, Shi and Horvath, 2012].

One-Class splitting criterion

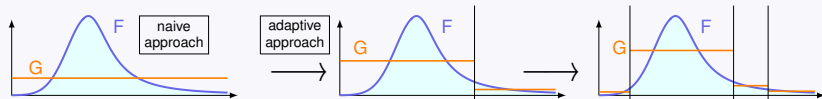
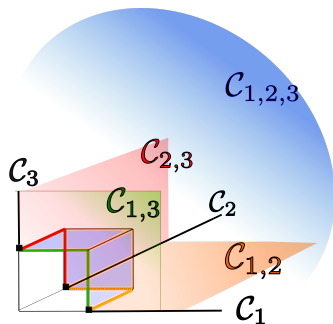


Figure: F is the inliers distribution, G is the assumed outliers distribution.

Does theoretical guaranties on RFs apply ? [Biau et al. 2008, Biau and Scornet, 2016]



A more accurate sparse representation?

Some references:

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- ▶ Thomas, Cléménçon, Feuillard and Gramford 2016, Learning Hyperparameters for Unsupervised Anomaly Detection.
- ▶ Vert and Vert 2006, Consistency and Convergence Rates of One-Class SVMs and Related Algorithms

| | number of samples | number of features |
|-------------|-------------------|--------------------|
| shuttle | 85849 | 9 |
| forestcover | 286048 | 54 |
| SA | 976158 | 41 |
| SF | 699691 | 4 |
| http | 619052 | 3 |

Table: Datasets characteristics

$$\epsilon = 0.01, k = n^{1/2}$$

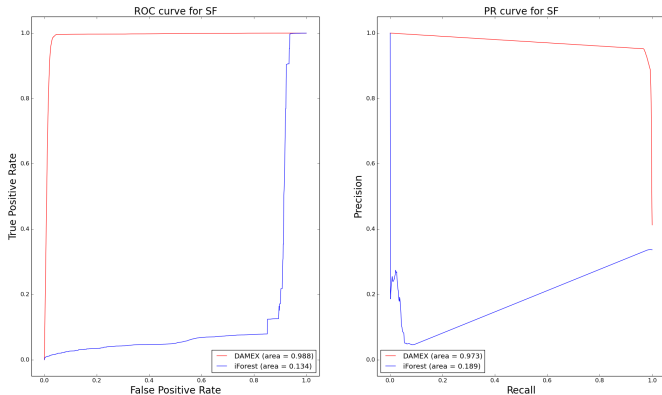


Figure: ROC and PR curve on SF dataset

$$\epsilon = 0.01, k = n^{1/2}$$

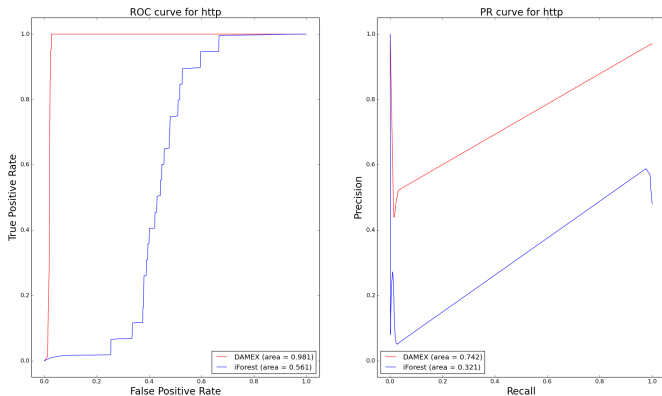


Figure: ROC and PR curve on http dataset

$$\epsilon = 0.01, k = n^{1/2}$$

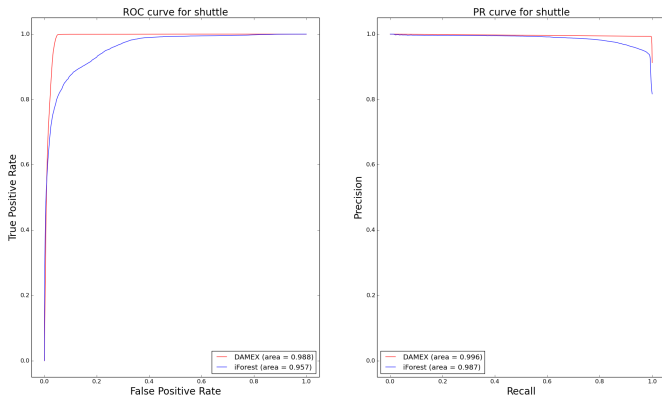


Figure: ROC and PR curve on shuttle dataset

$$\epsilon = 0.01, k = n^{1/2}$$

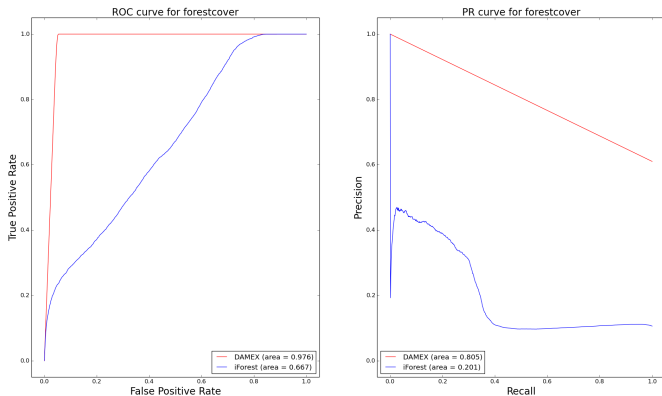


Figure: ROC and PR curve on forestcover dataset

Does performance in term of EM/MV correspond to performance in term of ROC/PR?

- **Experiments:** 12 datasets, 3 AD algorithms (LOF, OCSVM, iForest) \rightarrow 36 possible pairwise comparisons:

$$\left\{ \left(A_1 \text{ on } \mathcal{D}, A_2 \text{ on } \mathcal{D} \right), A_1, A_2 \in \{\text{iForest, LOF, OCSVM}\}, \right. \\ \left. \mathcal{D} \in \{\text{adult, http, } \dots, \text{spambase}\} \right\}.$$

- **Results:** If we only consider the pairs *s.t.* ROC and PR agree on which algorithm is the best, we are able (with EM and MV scores) to recover it in 80% of the cases.

Table: Original Datasets characteristics

| | nb of samples | nb of features | anomaly class | |
|-------------|---------------|----------------|---|---------|
| adult | 48842 | 6 | class '> 50K' | (23.9%) |
| http | 567498 | 3 | attack | (0.39%) |
| pima | 768 | 8 | pos (class 1) | (34.9%) |
| smtpt | 95156 | 3 | attack | (0.03%) |
| wilt | 4839 | 5 | class 'w' (diseased trees) | (5.39%) |
| annthyroid | 7200 | 6 | classes \neq 3 | (7.42%) |
| arrhythmia | 452 | 164 | classes \neq 1 (features 10-14 removed) | (45.8%) |
| forestcover | 286048 | 10 | class 4 (vs. class 2) | (0.96%) |
| ionosphere | 351 | 32 | bad | (35.9%) |
| pendigits | 10992 | 16 | class 4 | (10.4%) |
| shuttle | 85849 | 9 | classes \neq 1 (class 4 removed) | (7.17%) |
| spambase | 4601 | 57 | spam | (39.4%) |

Table: Results for the novelty detection setting. One can see that ROC, PR, EM, MV often do agree on which algorithm is the best (in bold), which algorithm is the worse (underlined) on some fixed datasets. When they do not agree, it is often because ROC and PR themselves do not, meaning that the ranking is not clear.

| Dataset | iForest | | | | OCSVM | | | | LOF | | | |
|------------|--------------|--------------|----------------|---------------|--------------|--------------|----------------|---------------|--------------|--------------|----------------|---------------|
| | ROC | PR | EM | MV | ROC | PR | EM | MV | ROC | PR | EM | MV |
| adult | 0.661 | 0.277 | 1.0e-04 | 7.5e01 | 0.642 | 0.206 | 2.9e-05 | 4.3e02 | <u>0.618</u> | <u>0.187</u> | <u>1.7e-05</u> | <u>9.0e02</u> |
| http | 0.994 | 0.192 | 1.3e-03 | 9.0 | 0.999 | 0.970 | 6.0e-03 | 2.6 | <u>0.946</u> | <u>0.035</u> | <u>8.0e-05</u> | <u>3.9e02</u> |
| pima | 0.727 | 0.182 | 5.0e-07 | 1.2e04 | 0.760 | 0.229 | 5.2e-07 | <u>1.3e04</u> | <u>0.705</u> | <u>0.155</u> | <u>3.2e-07</u> | 2.1e04 |
| smtp | 0.907 | <u>0.005</u> | <u>1.8e-04</u> | <u>9.4e01</u> | <u>0.852</u> | 0.522 | 1.2e-03 | 8.2 | 0.922 | 0.189 | 1.1e-03 | 5.8 |
| wilt | 0.491 | 0.045 | <u>4.7e-05</u> | <u>2.1e03</u> | <u>0.325</u> | <u>0.037</u> | 5.9e-05 | 4.5e02 | 0.698 | 0.088 | <u>2.1e-05</u> | 1.6e03 |
| annthyroid | 0.913 | 0.456 | 2.0e-04 | 2.6e02 | <u>0.699</u> | <u>0.237</u> | <u>6.3e-05</u> | 2.2e02 | 0.823 | 0.432 | 6.3e-05 | <u>1.5e03</u> |
| arrhythmia | 0.763 | 0.487 | 1.6e-04 | 9.4e01 | 0.736 | 0.449 | 1.1e-04 | 1.0e02 | <u>0.730</u> | <u>0.413</u> | <u>8.3e-05</u> | <u>1.6e02</u> |
| forestcov. | <u>0.863</u> | <u>0.046</u> | <u>3.9e-05</u> | <u>2.0e02</u> | 0.958 | 0.110 | 5.2e-05 | 1.2e02 | 0.990 | 0.792 | 3.5e-04 | 3.9e01 |
| ionosphere | <u>0.902</u> | <u>0.529</u> | <u>9.6e-05</u> | <u>7.5e01</u> | 0.977 | 0.898 | 1.3e-04 | 5.4e01 | 0.971 | 0.895 | 1.0e-04 | 7.0e01 |
| pendigits | 0.811 | 0.197 | 2.8e-04 | 2.6e01 | 0.606 | 0.112 | 2.7e-04 | 2.7e01 | 0.983 | 0.829 | 4.6e-04 | 1.7e01 |
| shuttle | 0.996 | 0.973 | 1.8e-05 | 5.7e03 | <u>0.992</u> | <u>0.924</u> | 3.2e-05 | 2.0e01 | 0.999 | 0.994 | <u>7.9e-06</u> | <u>2.0e06</u> |
| spambase | 0.824 | 0.371 | 9.5e-04 | 4.5e01 | <u>0.729</u> | 0.230 | 4.9e-04 | 1.1e03 | 0.754 | <u>0.173</u> | <u>2.2e-04</u> | <u>4.1e04</u> |