

MATH-3030 – CRN 10969 – Math Models for CS**Project 3 – Probability and Statistics (but not only...)**

General directions. The project must be completed by individual students or by groups of two or three students; no larger groups are allowed. You may use a computer algebra system (such as *Mathematica*, *Matlab*, etc) or a programming language (such as *C++*, *Java*, etc) or possibly some other appropriate software of your choice.

The problem. Analyze the determinants of random binary, i.e. consisting only of 0s and 1s, matrices, say of size 2×2 . Think of this as a random experiment: you generate randomly such a matrix and calculate its determinant. More specifically:

- (1) Let S be the sample space of this experiment, i.e. the set of all such matrices. We assume that each entry of the matrix can equally likely, i.e. with probability $1/2$, be either 0 or 1. Let X be the random variable that equals the determinant of such a matrix. Answer all of the following questions without using technology, i.e. do them **by hand** on paper. Clearly written solutions to this questions will be the first part of your project. (*4 points for each sub-part here.*)
 - (a) How many matrices are in S ? And therefore, when matrices are randomly generated, what is the probability of each one of them to occur?
 - (b) Make a list of all these matrices and find their determinants. (That will give you a complete list of all possible values of the random variable X).
 - (c) Looking at that list, for each possible value of X , i.e. for each possible value of the determinant, find its probability.
 - (d) Find the mean μ , the variance σ^2 , and the standard deviation σ of the random variable X .
- (2) Create a program (or some kind of software implementation) that automatically answers parts (1b) and (1d) above, i.e. it must output a list of all these matrices along with their determinants, and then it must calculate and output the mean μ , the variance σ^2 , and the standard deviation σ of the random variable X . Print out and attach these outputs to your project. (*8 points for this part.*)
- (3) Create a program (or some kind of software implementation) that using some random number generator does the following: in a cycle, it generates 15 random binary 2×2 matrices and finds the determinant of each — so that now there is a list of 15 instances, i.e. a sample of size 15, of the random variable X ; then it calculates the mean \bar{x} , the variance s^2 , and the standard deviation s of this sample. Print out and attach these outputs to your project. By hand show what formulae for the mean and the variance you used in this part. (*8 points for this part.*)

- (4) You will probably see that the values of the mean, the variance, and the standard deviation found in parts (2) and (3) are not exactly the same. Are they at least close? How can you explain that? Write and attach an explanation to your project. (*2 points for this part.*)
- (5) Submit a paper that includes: a nice cover sheet and all attachments as requested above. (*4 points for the paper submission.*)

Due date. The project is due **Tuesday, April 27**, no later than 7:30 pm. You can submit it to me in person or leave it in my mailbox in the department. Early submissions are welcome. No late submissions will be accepted for any reason whatsoever. You must keep your software implementation files and be able to submit them electronically at any time at my request.