Question 1:

1) **2% - Choose a q value which gives no solution.**
2) **2% - Choose a q value which gives infinitely many solutions.**

$$3x + 6y = 1$$
$$6x + 12y = q$$

Ans1.

The matrix form of the system of equations is as follows (Based on **Ax = b**):

$$\begin{pmatrix} 3 & 6 \\ 6 & 12 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ q \end{pmatrix}$$

Since the determinant of matrix A is zero (3*12 – 6*6 = 0), we can say that it's a singular matrix having no inverse, so either the system of equations will have no solutions or it'll have infinite solutions.

(1) The value of q which has no solutions will result in parallel lines in geometric 2-D space.
Line 1 ➔ 3x + 6y = 1

Line 2 ➔ 2(3x + 6y) = q ➔ 3x + 6y = q/2

To get parallel lines, we can take any real number for q/2 which is not equal to 1

Therefore, a possible value for q/2 = 10 ➔ q = 20

Thus, a value of 4 for the variable q will lead to no solutions for the above system of equations.

(2) The value of q which has infinite solutions will result in same lines in geometric 2-D space.
Line 1 ➔ 3x + 6y = 1

Line 2 ➔ 2(3x + 6y) = q ➔ 3x + 6y = q/2

To get same line, we can equate q/2 = 1

Therefore, the value of q = 2

Thus, a value of 2 for the variable q will lead to infinite solutions for the above system of equations.

Question 2:

1) **5% - Solve the following system of equations using Gaussian Elimination.**

$2x + 3y + z = 12$
$-2x + 3y - 2z = 1$
$x - y + 4z = 16$

2) 2% Validate your answer using Python.

Ans2.

$$\begin{pmatrix} 2 & 3 & 1 \\ -2 & 3 & -2 \\ 1 & -1 & 4 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 12 \\ 1 \\ 16 \end{pmatrix}$$

Row 2 => Row2 + Row1

$$\begin{pmatrix} 2 & 3 & 1 \\ 0 & 6 & -1 \\ 1 & -1 & 4 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 12 \\ 13 \\ 16 \end{pmatrix}$$

Row 1 => Row1/2

$$\begin{pmatrix} 1 & 1.5 & 0.5 \\ 0 & 6 & -1 \\ 1 & -1 & 4 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 6 \\ 13 \\ 16 \end{pmatrix}$$

Row 3 => Row3 – Row1

$$\begin{pmatrix} 1 & 1.5 & 0.5 \\ 0 & 6 & -1 \\ 0 & -2.5 & 3.5 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 6 \\ 13 \\ 10 \end{pmatrix}$$

Row1 => Row2/6

$$\begin{pmatrix} 1 & 1.5 & 0.5 \\ 0 & 1 & (-0.16667) \\ 0 & (-2.5) & 3.5 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 6 \\ 2.16667 \\ 10 \end{pmatrix}$$

Row3 => Row3 + 2.5*Row2

$$\begin{pmatrix} 1 & 1.5 & 0.5 \\ 0 & 1 & (-0.16667) \\ 0 & 0 & 3.0833333 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 6 \\ 2.16667 \\ 15.416675 \end{pmatrix}$$

Row3 => Row3/3.083333

$$\begin{pmatrix} 1 & 1.5 & 0.5 \\ 0 & 1 & (-0.16667) \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 6 \\ 2.16667 \\ 5 \end{pmatrix}$$

⇨ **Upper triangular Matrix**

$$\begin{pmatrix} 1 & 1.5 & 0.5 \\ 0 & 1 & (-0.16667) \\ 0 & 0 & 1 \end{pmatrix}$$

Now using back substitution, we can see that the value of z = 5

For y

y – 5*(-0.16667) = 2.16667 => y = 3

For x

x + 1.5*3 + 0.5*5 = 6 => x = -1

Question 3:

Find the rank of each of the following matrices. Verify your rank calculation using Python.

Ans3.

(a) [[1,3,1,2,0],
[0,0,2,1,3],
[0,0,0,3,2],
[0,0,0,3,-1]]

After converting to upper triangular form, we get 4 pivots, and we see that column 2 is dependent on column1

Thus, the rank of matrix is 4

(b) [[-1, 1, 0, -1],
[-2, 2, 1, -4],
[-1, 1, -2, 3]]

After converting to upper triangular form, we get 2 pivots, and we see that column 2 is dependent on column1 and column4 is dependent on the columns 1 and 3 (col4 = col1 – 2*col3)

Thus, the rank of matrix is 2