Page No.:
Date:

Assignment #1

$$A = \begin{bmatrix} 0 & 3 & -1 \\ -1 & 4 & -2 \\ 1 & 3 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & -1 & 2 \\ -1 & 0 & 1 \\ -1 & 2 & 2 \end{bmatrix}$$

$$\vec{v} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix} \qquad \vec{u} = \begin{bmatrix} -2 \\ 1 \\ 5 \end{bmatrix} \qquad \begin{array}{l} a = -2 \\ b = 1 \end{array}$$

Q1 (1) $\|\vec{v}\|_1 \vec{v} + a\vec{u}$

$\|\vec{v}\|_1 =$ L_1 norm of vector $\vec{v}$ or Manhattan distance

$$\|\vec{v}\|_1 = |v_1| + |v_2| + |v_3| + \cdots + |v_n|$$

$\hookrightarrow$ if $\vec{v} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}$

$$\Rightarrow \|V\|_1 = |2| + |-1| + |4| = 7$$

$$\|V\|_1 \vec{V} = 7 \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix} = \begin{bmatrix} 7*2 \\ 7*-1 \\ 7*4 \end{bmatrix} = \begin{bmatrix} 14 \\ -7 \\ 28 \end{bmatrix}$$

$$a\vec{u} = -2 \begin{bmatrix} -2 \\ 1 \\ 5 \end{bmatrix} = \begin{bmatrix} -2*-2 \\ -2*1 \\ -2*5 \end{bmatrix} = \begin{bmatrix} +4 \\ -2 \\ -10 \end{bmatrix}$$

$$\Rightarrow \|\vec{v}\|_1 \vec{v} + a\vec{u} = \begin{bmatrix} 14 + 4 \\ -7 + (-2) \\ 28 + (-10) \end{bmatrix} = \begin{bmatrix} 18 \\ -9 \\ 18 \end{bmatrix}$$

(2) Please refer jupyter notebook attached ~~cell number 2~~

Q2 ① cosine formula

$\rightarrow \quad \cancel{a}\ \vec{u}\cdot\vec{v} = |u||v|\cos\theta$

$\Rightarrow \cos\theta = \dfrac{\vec{u}\cdot\vec{v}}{|u||v|}$

↳ L_2 norms of vectors

$$\Rightarrow \cos\theta = \frac{\begin{bmatrix}-2\\1\\5\end{bmatrix}\begin{bmatrix}2\\-1\\4\end{bmatrix}}{\sqrt{(-2)^2+1^2+5^2}\cdot\sqrt{2^2+(-1)^2+4^2}}$$

$$= \frac{(-2\cdot2)+(-1.1)+(5\cdot4)}{\sqrt{30}\cdot\sqrt{21}}$$

$$= \frac{15}{\sqrt{30\cdot21}} = \frac{15}{\sqrt{630}} = \frac{15}{25.0998} = 0.5976$$

② Please refer jupyter notebook attached

Q3 ① Calculating $a(A\cdot\vec{v})$

$$A\cdot\vec{v} = \begin{bmatrix}0 & 3 & (-1)\\-1 & 4 & -2\\1 & 3 & 1\end{bmatrix}\begin{bmatrix}2\\-1\\4\end{bmatrix}$$

$$= \begin{bmatrix}2\times0 + 3\times-1 + (-1)\times4\\-1\times2 + 4\times-1 + -2\times4\\1\times2 + 3\times-1 + 1\times4\end{bmatrix}$$

$$= \begin{bmatrix}0-3-4\\-2-4-8\\2-3+4\end{bmatrix} = \begin{bmatrix}-7\\-14\\3\end{bmatrix}$$

$$a(A\cdot\vec{v}) = -2\begin{bmatrix}-7\\-14\\3\end{bmatrix} = \begin{bmatrix}-2\times-7\\-2\times-14\\-2\times3\end{bmatrix} = \begin{bmatrix}+14\\28\\-6\end{bmatrix}$$

(2) Please refer jupyter notebook attached

Q4 (1)

Calculate → $A \cdot B^T + \mathrm{tr}(B) * L$

Transpose $B_{ij}^T = B_{ji}$

$$B^T = \begin{bmatrix} 2 & -1 & 2 \\ -1 & 0 & 1 \\ -1 & 2 & 2 \end{bmatrix}^T = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 0 & 2 \\ 2 & 1 & 2 \end{bmatrix}$$

tr(B) = sum of diagonal elements of matrix B
$= 2 + 0 + 2 = 4$

L = lower triangular matrix ⇒ keep diagonal elements and all elements below that & replace elements above diagonal by zero

$$L = \begin{bmatrix} 2 & 0 & 0 \\ -1 & 0 & 0 \\ -1 & 2 & 2 \end{bmatrix}$$

$$A \cdot B^T = \begin{bmatrix} 0 & 3 & -1 \\ -1 & 4 & -2 \\ 1 & 3 & 1 \end{bmatrix} \begin{bmatrix} 2 & -1 & -1 \\ -1 & 0 & 2 \\ 2 & 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 0\times2 + 3\times-1 + -1\times2 & 0*(-1) + 3*0 + (-1)*2 & 0\times2 + 3\times1 + (-1)\times2 \\ -1\times2 + 4\times-1 + -2\times2 & (-1)*(-1) + 4*0 + (-2)*2 & -1\times2 + 4\times1 + (-2)\times2 \\ 1\times2 + 3\times-1 + 1\times2 & 1*(-1) + 3*0 + 1*2 & -1\times2 + 3\times1 + 1\times2 \end{bmatrix}$$

$$= \begin{bmatrix} 0-3-2 & 0+0-2 & 0+3-2 \\ -2-4-4 & 1+0-4 & -2+4-4 \\ 2-3+2 & -1+0+2 & -2+3+2 \end{bmatrix}$$

$$= \begin{bmatrix} -5 & -2 & 1 \\ -10 & -3 & -2 \\ 1 & 1 & 3 \end{bmatrix}$$

$$AB^T = \begin{bmatrix} 0 & 3 & -1 \\ -1 & 4 & -2 \\ 1 & 3 & 1 \end{bmatrix} \begin{bmatrix} 2 & -1 & -1 \\ -1 & 0 & 2 \\ 2 & 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 0\times2 + 3\times-1 - 1\times2 & 0\times-1 + 3\times0 - 1\times1 & 0\times-1 + 3\times2 - 1\times2 \\ -1\times2 + 4\times-1 - 2\times2 & -1\times-1 + 4\times0 - 2\times1 & -1\times-1 + 4\times2 - 2\times2 \\ 1\times2 + 3\times-1 + 1\times2 & -1\times1 + 3\times0 + 1\times1 & 1\times-1 + 3\times2 + 1\times2 \end{bmatrix}$$

$$= \begin{bmatrix} 0-3-2 & 0+0-1 & 0+6-2 \\ -2-4-4 & 1+0-2 & 1+8-4 \\ 2+(-3)+2 & -1+0+1 & -1+6+2 \end{bmatrix}$$

$$= \begin{bmatrix} -5 & -1 & 4 \\ -10 & -1 & 5 \\ 1 & 0 & 7 \end{bmatrix}$$

$$Gr(B)^{*L} = 4 * \begin{bmatrix} 2 & 0 & 0 \\ -1 & 0 & 0 \\ -1 & 2 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 0 & 0 \\ -4 & 0 & 0 \\ -4 & 8 & 8 \end{bmatrix}$$

$$A \cdot B^T + Gr(B) * L = \begin{bmatrix} -5 & -1 & 4 \\ -10 & -1 & 5 \\ 1 & 0 & 7 \end{bmatrix} + \begin{bmatrix} 8 & 0 & 0 \\ -4 & 0 & 0 \\ -4 & 8 & 8 \end{bmatrix}$$

$$= \begin{bmatrix} -5+8 & -1+0 & 4+0 \\ -10-4 & -1+0 & 5+0 \\ 1-4 & 0+8 & 7+8 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 4 \\ -14 & -1 & 5 \\ -3 & 8 & 15 \end{bmatrix}$$

(2) Please refer jupyter notebook attached