

Dual Simplex Method Initial Matrices and Vector

$$A = \begin{bmatrix} -2.0 & -1.0 & 1.0 & 0.0 & 0.0 \\ -2.0 & 4.0 & 0.0 & 1.0 & 0.0 \\ -1.0 & 3.0 & 0.0 & 0.0 & 1.0 \end{bmatrix}$$

Initial set of basic and nonbasic indices

$$\beta = \{3, 4, 5\} \quad \text{and} \quad \mathcal{N} = \{1, 2\}$$

Submatrice of A

$$B = \begin{bmatrix} 1.0 & 0.0 & 0.0 \\ 0.0 & 1.0 & 0.0 \\ 0.0 & 0.0 & 1.0 \end{bmatrix} \quad \text{and} \quad N = \begin{bmatrix} -2.0 & -1.0 \\ -2.0 & 4.0 \\ -1.0 & 3.0 \end{bmatrix}$$

Initial values of the basic variables are given by

$$x_B^* = b = \begin{bmatrix} 4.0 \\ -8.0 \\ -7.0 \end{bmatrix}$$

Initial values of the nonbasic dual variables are given by

$$z_N^* = -c_N = \begin{bmatrix} 1.0 \\ 1.0 \end{bmatrix}$$

Since $z_N^ \geq 0$, the initial solution is Dual feasible.*

Iteration No 1

Step 1.

Since x_B^ has some negative components, the current solution is not optimal.*

Step 2.

Since $x_4^ = -8.0$ and this is the most negative dual variables,*

we see that the entering index is $i = 4$

Step 6.

$$\Delta z_N = -(B^{-1}N)^T e_i = - \begin{bmatrix} -2.0 & -2.0 & -1.0 \\ -1.0 & 4.0 & 3.0 \end{bmatrix} \begin{bmatrix} 0.0 \\ 1.0 \\ 0.0 \end{bmatrix} = \begin{bmatrix} 2.0 \\ -4.0 \end{bmatrix}$$

Step 4.

$$t = \left(\max \left\{ \frac{2.0}{1.0}, \frac{-4.0}{1.0} \right\} \right)^{-1} = 0.5$$

Step 5.

In step 4, the ratio corresponds to basic index 1
 $j = 1$

Step 6.

$$\Delta x_B = B^{-1} N e_j = \begin{bmatrix} -2.0 & -1.0 \\ -2.0 & 4.0 \\ -1.0 & 3.0 \end{bmatrix} \begin{bmatrix} 1.0 \\ 0.0 \end{bmatrix} = \begin{bmatrix} -2.0 \\ -2.0 \\ -1.0 \end{bmatrix}$$

Step 7.

$$t = \frac{x_1^*}{\Delta x_1} = \frac{-8.0}{-2.0} = 4.0$$

Step 9.

New set of basic and nonbasic indices
 $\beta = \{3, 1, 5\}$ and $\mathcal{N} = \{4, 2\}$

Corresponding new basis and nonbasis submatrices of A,

$$B = \begin{bmatrix} 1.0 & -2.0 & 0.0 \\ 0.0 & -2.0 & 0.0 \\ 0.0 & -1.0 & 1.0 \end{bmatrix} \quad \text{and} \quad N = \begin{bmatrix} 0.0 & -1.0 \\ 1.0 & 4.0 \\ 0.0 & 3.0 \end{bmatrix}$$

New Basic primal variables and nonbasic dual variables :

$$x_B^* = \begin{bmatrix} x_3^* \\ x_1^* \\ x_5^* \end{bmatrix} = \begin{bmatrix} 12.0 \\ 4.0 \\ -3.0 \end{bmatrix} \quad z_{\mathcal{N}}^* = \begin{bmatrix} z_4^* \\ z_2^* \end{bmatrix} = \begin{bmatrix} 0.5 \\ 3.0 \end{bmatrix}$$

Iteration No 2

Step 1.

Since x_B^ has some negative components, the current solution is not optimal.*

Step 2.

Since $x_5^ = -3.0$ and this is the most negative dual variables,*

we see that the entering index is $i = 5$

Step 6.

$$\Delta z_{\mathcal{N}} = -(B^{-1}N)^T e_i = - \begin{bmatrix} -1.0 & -0.5 & -0.5 \\ -5.0 & -2.0 & 1.0 \end{bmatrix} \begin{bmatrix} 0.0 \\ 0.0 \\ 1.0 \end{bmatrix} = \begin{bmatrix} 0.5 \\ -1.0 \end{bmatrix}$$

Step 4.

$$t = \left(\max \left\{ \frac{0.5}{0.5}, \frac{-1.0}{3.0} \right\} \right)^{-1} = 1.0$$

Step 5.

In step 4, the ratio corresponds to basic index 4

$$j = 4$$

Step 6.

$$\Delta x_{\mathcal{B}} = B^{-1} N e_j = \begin{bmatrix} -1.0 & -5.0 \\ -0.5 & -2.0 \\ -0.5 & 1.0 \end{bmatrix} \begin{bmatrix} 1.0 \\ 0.0 \end{bmatrix} = \begin{bmatrix} -1.0 \\ -0.5 \\ -0.5 \end{bmatrix}$$

Step 7.

$$t = \frac{x_4^*}{\Delta x_4} = \frac{-3.0}{-0.5} = 6.0$$

Step 9.

New set of basic and nonbasic indices

$$\beta = \{3, 1, 4\} \quad \text{and} \quad \mathcal{N} = \{5, 2\}$$

Corresponding new basis and nonbasis submatrices of A,

$$B = \begin{bmatrix} 1.0 & -2.0 & 0.0 \\ 0.0 & -2.0 & 1.0 \\ 0.0 & -1.0 & 0.0 \end{bmatrix} \quad \text{and} \quad N = \begin{bmatrix} 0.0 & -1.0 \\ 0.0 & 4.0 \\ 1.0 & 3.0 \end{bmatrix}$$

New Basic primal variables and nonbasic dual variables :

$$x_{\mathcal{B}}^* = \begin{bmatrix} x_3^* \\ x_1^* \\ x_4^* \end{bmatrix} = \begin{bmatrix} 18.0 \\ 7.0 \\ 6.0 \end{bmatrix} \quad z_{\mathcal{N}}^* = \begin{bmatrix} z_5^* \\ z_2^* \end{bmatrix} = \begin{bmatrix} 1.0 \\ 4.0 \end{bmatrix}$$

Iteration No 3

Step 1.

Since x_B^ has all nonnegative components, the current solution is optimal.*

$$\zeta^* = -1.0x_1^* + -1.0x_2^* = -7.0$$