

Primal Simplex Method Initial Matrices and Vector

$$A = \begin{bmatrix} 1.0 & -1.0 & 1.0 & 0.0 & 0.0 \\ 2.0 & -1.0 & 0.0 & 1.0 & 0.0 \\ 0.0 & 1.0 & 0.0 & 0.0 & 1.0 \end{bmatrix}$$

Initial set of basic and nonbasic indices

$$\beta = \{3, 4, 5\} \quad \text{and} \quad \mathcal{N} = \{1, 2\}$$

Submatrice of A

$$B = \begin{bmatrix} 1.0 & 0.0 & 0.0 \\ 0.0 & 1.0 & 0.0 \\ 0.0 & 0.0 & 1.0 \end{bmatrix} \quad \text{and} \quad N = \begin{bmatrix} 1.0 & -1.0 \\ 2.0 & -1.0 \\ 0.0 & 1.0 \end{bmatrix}$$

Initial values of the basic variables are given by

$$x_B^* = b = \begin{bmatrix} 1.0 \\ 3.0 \\ 5.0 \end{bmatrix}$$

Initial values of the nonbasic dual variables are given by

$$z_N^* = -c_N = \begin{bmatrix} -4.0 \\ -3.0 \end{bmatrix}$$

Since $x_B^ \geq 0$, the initial solution is Primal feasible.*

Iteration No 1

Step 1.

Since z_N^ has some negative components, the current solution is not optimal.*

Step 2.

Since $z_1^ = -4.0$ and this is the most negative dual variables,*

we see that the entering index is $j = 1$

Step 3.

$$\Delta x_B = B^{-1} N e_j = \begin{bmatrix} 1.0 & -1.0 \\ 2.0 & -1.0 \\ 0.0 & 1.0 \end{bmatrix} \begin{bmatrix} 1.0 \\ 0.0 \end{bmatrix} = \begin{bmatrix} 1.0 \\ 2.0 \\ 0.0 \end{bmatrix}$$

Step 4.

$$t = \left(\max \left\{ \frac{1.0}{1.0}, \frac{2.0}{3.0}, \frac{0.0}{5.0} \right\} \right)^{-1} = 1.0$$

Step 5.

In step 4, the ratio corresponds to basic index 3

$$i = 3$$

Step 6.

$$\Delta z_{\mathcal{N}} = -(B^{-1}N)^T e_i = - \begin{bmatrix} 1.0 & 2.0 & 0.0 \\ -1.0 & -1.0 & 1.0 \end{bmatrix} \begin{bmatrix} 1.0 \\ 0.0 \\ 0.0 \end{bmatrix} = \begin{bmatrix} -1.0 \\ 1.0 \end{bmatrix}$$

Step 7.

$$s = \frac{z_1^*}{\Delta z_1} = \frac{-4.0}{-1.0} = 4.0$$

Step 8.

$$x_1^* = 1.0, \quad x_{\mathcal{B}}^* = \begin{bmatrix} 1.0 \\ 3.0 \\ 5.0 \end{bmatrix} - 1.0 \begin{bmatrix} 1.0 \\ 2.0 \\ 0.0 \end{bmatrix} = \begin{bmatrix} 0.0 \\ 1.0 \\ 5.0 \end{bmatrix},$$

$$z_3^* = 4.0, \quad z_{\mathcal{N}}^* = \begin{bmatrix} -4.0 \\ -3.0 \end{bmatrix} - 4.0 \begin{bmatrix} -1.0 \\ 1.0 \end{bmatrix} = \begin{bmatrix} 0.0 \\ -7.0 \end{bmatrix},$$

Step 9.

New set of basic and nonbasic indices

$$\beta = \{1, 4, 5\} \quad \text{and} \quad \mathcal{N} = \{3, 2\}$$

Corresponding new basis and nonbasis submatrices of A,

$$B = \begin{bmatrix} 1.0 & 0.0 & 0.0 \\ 2.0 & 1.0 & 0.0 \\ 0.0 & 0.0 & 1.0 \end{bmatrix} \quad \text{and} \quad N = \begin{bmatrix} 1.0 & -1.0 \\ 0.0 & -1.0 \\ 0.0 & 1.0 \end{bmatrix}$$

New Basic primal variables and nonbasic dual variables :

$$x_{\mathcal{B}}^* = \begin{bmatrix} x_1^* \\ x_4^* \\ x_5^* \end{bmatrix} = \begin{bmatrix} 1.0 \\ 1.0 \\ 5.0 \end{bmatrix} \quad z_{\mathcal{N}}^* = \begin{bmatrix} z_3^* \\ z_2^* \end{bmatrix} = \begin{bmatrix} 4.0 \\ -7.0 \end{bmatrix}$$

Iteration No 2

Step 1.

Since z_N^ has some negative components, the current solution is not optimal.*

Step 2.

Since $z_2^ = -7.0$ and this is the most negative dual variables,*

we see that the entering index is $j = 2$

Step 3.

$$\Delta x_B = B^{-1}Ne_j = \begin{bmatrix} 1.0 & -1.0 \\ -2.0 & 1.0 \\ 0.0 & 1.0 \end{bmatrix} \begin{bmatrix} 0.0 \\ 1.0 \end{bmatrix} = \begin{bmatrix} -1.0 \\ 1.0 \\ 1.0 \end{bmatrix}$$

Step 4.

$$t = \left(\max \left\{ \frac{-1.0}{1.0}, \frac{1.0}{1.0}, \frac{1.0}{5.0} \right\} \right)^{-1} = 1.0$$

Step 5.

In step 4, the ratio corresponds to basic index 4

$$i = 4$$

Step 6.

$$\Delta z_N = -(B^{-1}N)^T e_i = - \begin{bmatrix} 1.0 & -2.0 & 0.0 \\ -1.0 & 1.0 & 1.0 \end{bmatrix} \begin{bmatrix} 0.0 \\ 1.0 \\ 0.0 \end{bmatrix} = \begin{bmatrix} 2.0 \\ -1.0 \end{bmatrix}$$

Step 7.

$$s = \frac{z_2^*}{\Delta z_2} = \frac{-7.0}{-1.0} = 7.0$$

Step 8.

$$x_2^* = 1.0, \quad x_B^* = \begin{bmatrix} 1.0 \\ 1.0 \\ 5.0 \end{bmatrix} - 1.0 \begin{bmatrix} -1.0 \\ 1.0 \\ 1.0 \end{bmatrix} = \begin{bmatrix} 2.0 \\ 0.0 \\ 4.0 \end{bmatrix},$$

$$z_4^* = 7.0, \quad z_N^* = \begin{bmatrix} 4.0 \\ -7.0 \end{bmatrix} - 7.0 \begin{bmatrix} 2.0 \\ -1.0 \end{bmatrix} = \begin{bmatrix} -10.0 \\ 0.0 \end{bmatrix},$$

Step 9.

New set of basic and nonbasic indices

$$\beta = \{1, 2, 5\} \quad \text{and} \quad \mathcal{N} = \{3, 4\}$$

Corresponding new basis and nonbasis submatrices of A,

$$B = \begin{bmatrix} 1.0 & -1.0 & 0.0 \\ 2.0 & -1.0 & 0.0 \\ 0.0 & 1.0 & 1.0 \end{bmatrix} \quad \text{and} \quad N = \begin{bmatrix} 1.0 & 0.0 \\ 0.0 & 1.0 \\ 0.0 & 0.0 \end{bmatrix}$$

New Basic primal variables and nonbasic dual variables :

$$x_{\mathcal{B}}^* = \begin{bmatrix} x_1^* \\ x_2^* \\ x_5^* \end{bmatrix} = \begin{bmatrix} 2.0 \\ 1.0 \\ 4.0 \end{bmatrix} \quad z_{\mathcal{N}}^* = \begin{bmatrix} z_3^* \\ z_4^* \end{bmatrix} = \begin{bmatrix} -10.0 \\ 7.0 \end{bmatrix}$$

Iteration No 3

Step 1.

Since z_N^ has some negative components, the current solution is not optimal.*

Step 2.

Since $z_3^ = -10.0$ and this is the most negative dual variables,*

we see that the entering index is $j = 3$

Step 3.

$$\Delta x_{\mathcal{B}} = B^{-1} N e_j = \begin{bmatrix} -1.0 & 1.0 \\ -2.0 & 1.0 \\ 2.0 & -1.0 \end{bmatrix} \begin{bmatrix} 1.0 \\ 0.0 \end{bmatrix} = \begin{bmatrix} -1.0 \\ -2.0 \\ 2.0 \end{bmatrix}$$

Step 4.

$$t = \left(\max \left\{ \frac{-1.0}{2.0}, \frac{-2.0}{1.0}, \frac{2.0}{4.0} \right\} \right)^{-1} = 2.0$$

Step 5.

In step 4, the ratio corresponds to basic index 5

$$i = 5$$

Step 6.

$$\Delta z_{\mathcal{N}} = -(B^{-1}N)^T e_i = - \begin{bmatrix} -1.0 & -2.0 & 2.0 \\ 1.0 & 1.0 & -1.0 \end{bmatrix} \begin{bmatrix} 0.0 \\ 0.0 \\ 1.0 \end{bmatrix} = \begin{bmatrix} -2.0 \\ 1.0 \end{bmatrix}$$

Step 7.

$$s = \frac{z_3^*}{\Delta z_3} = \frac{-10.0}{-2.0} = 5.0$$

Step 8.

$$x_3^* = 2.0, \quad x_{\mathcal{B}}^* = \begin{bmatrix} 2.0 \\ 1.0 \\ 4.0 \end{bmatrix} - 2.0 \begin{bmatrix} -1.0 \\ -2.0 \\ 2.0 \end{bmatrix} = \begin{bmatrix} 4.0 \\ 5.0 \\ 0.0 \end{bmatrix},$$

$$z_5^* = 5.0, \quad z_{\mathcal{N}}^* = \begin{bmatrix} -10.0 \\ 7.0 \end{bmatrix} - 5.0 \begin{bmatrix} -2.0 \\ 1.0 \end{bmatrix} = \begin{bmatrix} 0.0 \\ 2.0 \end{bmatrix},$$

Step 9.

New set of basic and nonbasic indices

$$\beta = \{1, 2, 3\} \quad \text{and} \quad \mathcal{N} = \{5, 4\}$$

Corresponding new basis and nonbasis submatrices of A,

$$B = \begin{bmatrix} 1.0 & -1.0 & 1.0 \\ 2.0 & -1.0 & 0.0 \\ 0.0 & 1.0 & 0.0 \end{bmatrix} \quad \text{and} \quad N = \begin{bmatrix} 0.0 & 0.0 \\ 0.0 & 1.0 \\ 1.0 & 0.0 \end{bmatrix}$$

New Basic primal variables and nonbasic dual variables :

$$x_{\mathcal{B}}^* = \begin{bmatrix} x_1^* \\ x_2^* \\ x_3^* \end{bmatrix} = \begin{bmatrix} 4.0 \\ 5.0 \\ 2.0 \end{bmatrix} \quad z_{\mathcal{N}}^* = \begin{bmatrix} z_5^* \\ z_4^* \end{bmatrix} = \begin{bmatrix} 5.0 \\ 2.0 \end{bmatrix}$$

Iteration No 4

Step 1.

Since $z_{\mathcal{N}}^$ has all nonnegative components, the current solution is optimal.*

$$\zeta^* = 4.0x_1^* + 3.0x_2^* = 31.0$$