

# Primal Simplex Method Initial Matrices and Vector

$$A = \begin{bmatrix} 1.0 & -1.0 & 1.0 & 0.0 & 0.0 \\ 2.0 & -1.0 & 0.0 & 1.0 & 0.0 \\ 0.0 & 1.0 & 0.0 & 0.0 & 1.0 \end{bmatrix}$$

*Initial set of basic and nonbasic indices*

$$\beta = \{3, 4, 5\} \quad \text{and} \quad \mathcal{N} = \{1, 2\}$$

*Submatrice of A*

$$B = \begin{bmatrix} 1.0 & 0.0 & 0.0 \\ 0.0 & 1.0 & 0.0 \\ 0.0 & 0.0 & 1.0 \end{bmatrix} \quad \text{and} \quad N = \begin{bmatrix} 1.0 & -1.0 \\ 2.0 & -1.0 \\ 0.0 & 1.0 \end{bmatrix}$$

*Initial values of the basic variables are given by*

$$x_B^* = b = \begin{bmatrix} 1.0 \\ 3.0 \\ 5.0 \end{bmatrix}$$

*Initial values of the nonbasic dual variables are given by*

$$z_N^* = -c_N = \begin{bmatrix} -4.0 \\ -3.0 \end{bmatrix}$$

*Since  $x_B^* \geq 0$ , the initial solution is Primal feasible.*

## Iteration No 1

### Step 1.

*Since  $z_N^*$  has some negative components, the current solution is not optimal.*

### Step 2.

*Since  $z_1^* = -4.0$  and this is the most negative dual variables,*

*we see that the entering index is  $j = 1$*

### Step 3.

$$\Delta x_B = B^{-1} N e_j = \begin{bmatrix} 1.0 & -1.0 \\ 2.0 & -1.0 \\ 0.0 & 1.0 \end{bmatrix} \begin{bmatrix} 1.0 \\ 0.0 \end{bmatrix} = \begin{bmatrix} 1.0 \\ 2.0 \\ 0.0 \end{bmatrix}$$

**Step 4.**

$$t = \left( \max \left\{ \frac{1.0}{1.0}, \frac{2.0}{3.0}, \frac{0.0}{5.0} \right\} \right)^{-1} = 1.0$$

**Step 5.**

*In step 4, the ratio corresponds to basic index 3*

$$i = 3$$

**Step 6.**

$$\Delta z_{\mathcal{N}} = -(B^{-1}N)^T e_i = - \begin{bmatrix} 1.0 & 2.0 & 0.0 \\ -1.0 & -1.0 & 1.0 \end{bmatrix} \begin{bmatrix} 1.0 \\ 0.0 \\ 0.0 \end{bmatrix} = \begin{bmatrix} -1.0 \\ 1.0 \end{bmatrix}$$

**Step 7.**

$$s = \frac{z_1^*}{\Delta z_1} = \frac{-4.0}{-1.0} = 4.0$$

**Step 8.**

$$x_1^* = 1.0, \quad x_{\mathcal{B}}^* = \begin{bmatrix} 1.0 \\ 3.0 \\ 5.0 \end{bmatrix} - 1.0 \begin{bmatrix} 1.0 \\ 2.0 \\ 0.0 \end{bmatrix} = \begin{bmatrix} 0.0 \\ 1.0 \\ 5.0 \end{bmatrix},$$

$$z_3^* = 4.0, \quad z_{\mathcal{N}}^* = \begin{bmatrix} -4.0 \\ -3.0 \end{bmatrix} - 4.0 \begin{bmatrix} -1.0 \\ 1.0 \end{bmatrix} = \begin{bmatrix} 0.0 \\ -7.0 \end{bmatrix},$$

**Step 9.**

*New set of basic and nonbasic indices*

$$\beta = \{1, 4, 5\} \quad \text{and} \quad \mathcal{N} = \{3, 2\}$$

*Corresponding new basis and nonbasis submatrices of A,*

$$B = \begin{bmatrix} 1.0 & 0.0 & 0.0 \\ 2.0 & 1.0 & 0.0 \\ 0.0 & 0.0 & 1.0 \end{bmatrix} \quad \text{and} \quad N = \begin{bmatrix} 1.0 & -1.0 \\ 0.0 & -1.0 \\ 0.0 & 1.0 \end{bmatrix}$$

*New Basic primal variables and nonbasic dual variables :*

$$x_{\mathcal{B}}^* = \begin{bmatrix} x_1^* \\ x_4^* \\ x_5^* \end{bmatrix} = \begin{bmatrix} 1.0 \\ 1.0 \\ 5.0 \end{bmatrix} \quad z_{\mathcal{N}}^* = \begin{bmatrix} z_3^* \\ z_2^* \end{bmatrix} = \begin{bmatrix} 4.0 \\ -7.0 \end{bmatrix}$$

## Iteration No 2

### Step 1.

*Since  $z_N^*$  has some negative components, the current solution is not optimal.*

### Step 2.

*Since  $z_2^* = -7.0$  and this is the most negative dual variables,*

*we see that the entering index is  $j = 2$*

### Step 3.

$$\Delta x_B = B^{-1} N e_j = \begin{bmatrix} 1.0 & -1.0 \\ -2.0 & 1.0 \\ 0.0 & 1.0 \end{bmatrix} \begin{bmatrix} 0.0 \\ 1.0 \end{bmatrix} = \begin{bmatrix} -1.0 \\ 1.0 \\ 1.0 \end{bmatrix}$$

### Step 4.

$$t = \left( \max \left\{ \frac{-1.0}{1.0}, \frac{1.0}{1.0}, \frac{1.0}{5.0} \right\} \right)^{-1} = 1.0$$

### Step 5.

*In step 4, the ratio corresponds to basic index 4*

$$i = 4$$

### Step 6.

$$\Delta z_N = -(B^{-1} N)^T e_i = - \begin{bmatrix} 1.0 & -2.0 & 0.0 \\ -1.0 & 1.0 & 1.0 \end{bmatrix} \begin{bmatrix} 0.0 \\ 1.0 \\ 0.0 \end{bmatrix} = \begin{bmatrix} 2.0 \\ -1.0 \end{bmatrix}$$

### Step 7.

$$s = \frac{z_2^*}{\Delta z_2} = \frac{-7.0}{-1.0} = 7.0$$

### Step 8.

$$x_2^* = 1.0, \quad x_B^* = \begin{bmatrix} 1.0 \\ 1.0 \\ 5.0 \end{bmatrix} - 1.0 \begin{bmatrix} -1.0 \\ 1.0 \\ 1.0 \end{bmatrix} = \begin{bmatrix} 2.0 \\ 0.0 \\ 4.0 \end{bmatrix},$$

$$z_4^* = 7.0, \quad z_N^* = \begin{bmatrix} 4.0 \\ -7.0 \end{bmatrix} - 7.0 \begin{bmatrix} 2.0 \\ -1.0 \end{bmatrix} = \begin{bmatrix} -10.0 \\ 0.0 \end{bmatrix},$$

### Step 9.

*New set of basic and nonbasic indices*

$$\beta = \{1, 2, 5\} \quad \text{and} \quad \mathcal{N} = \{3, 4\}$$

*Corresponding new basis and nonbasis submatrices of A,*

$$B = \begin{bmatrix} 1.0 & -1.0 & 0.0 \\ 2.0 & -1.0 & 0.0 \\ 0.0 & 1.0 & 1.0 \end{bmatrix} \quad \text{and} \quad N = \begin{bmatrix} 1.0 & 0.0 \\ 0.0 & 1.0 \\ 0.0 & 0.0 \end{bmatrix}$$

*New Basic primal variables and nonbasic dual variables :*

$$x_{\mathcal{B}}^* = \begin{bmatrix} x_1^* \\ x_2^* \\ x_5^* \end{bmatrix} = \begin{bmatrix} 2.0 \\ 1.0 \\ 4.0 \end{bmatrix} \quad z_{\mathcal{N}}^* = \begin{bmatrix} z_3^* \\ z_4^* \end{bmatrix} = \begin{bmatrix} -10.0 \\ 7.0 \end{bmatrix}$$

## Iteration No 3

### Step 1.

*Since  $z_N^*$  has some negative components, the current solution is not optimal.*

### Step 2.

*Since  $z_3^* = -10.0$  and this is the most negative dual variables,*

*we see that the entering index is  $j = 3$*

### Step 3.

$$\Delta x_{\mathcal{B}} = B^{-1} N e_j = \begin{bmatrix} -1.0 & 1.0 \\ -2.0 & 1.0 \\ 2.0 & -1.0 \end{bmatrix} \begin{bmatrix} 1.0 \\ 0.0 \end{bmatrix} = \begin{bmatrix} -1.0 \\ -2.0 \\ 2.0 \end{bmatrix}$$

### Step 4.

$$t = \left( \max \left\{ \frac{-1.0}{2.0}, \frac{-2.0}{1.0}, \frac{2.0}{4.0} \right\} \right)^{-1} = 2.0$$

### Step 5.

*In step 4, the ratio corresponds to basic index 5*

$$i = 5$$

**Step 6.**

$$\Delta z_{\mathcal{N}} = -(B^{-1}N)^T e_i = - \begin{bmatrix} -1.0 & -2.0 & 2.0 \\ 1.0 & 1.0 & -1.0 \end{bmatrix} \begin{bmatrix} 0.0 \\ 0.0 \\ 1.0 \end{bmatrix} = \begin{bmatrix} -2.0 \\ 1.0 \end{bmatrix}$$

**Step 7.**

$$s = \frac{z_3^*}{\Delta z_3} = \frac{-10.0}{-2.0} = 5.0$$

**Step 8.**

$$x_3^* = 2.0, \quad x_{\mathcal{B}}^* = \begin{bmatrix} 2.0 \\ 1.0 \\ 4.0 \end{bmatrix} - 2.0 \begin{bmatrix} -1.0 \\ -2.0 \\ 2.0 \end{bmatrix} = \begin{bmatrix} 4.0 \\ 5.0 \\ 0.0 \end{bmatrix},$$

$$z_5^* = 5.0, \quad z_{\mathcal{N}}^* = \begin{bmatrix} -10.0 \\ 7.0 \end{bmatrix} - 5.0 \begin{bmatrix} -2.0 \\ 1.0 \end{bmatrix} = \begin{bmatrix} 0.0 \\ 2.0 \end{bmatrix},$$

**Step 9.**

*New set of basic and nonbasic indices*

$$\beta = \{1, 2, 3\} \quad \text{and} \quad \mathcal{N} = \{5, 4\}$$

*Corresponding new basis and nonbasis submatrices of A,*

$$B = \begin{bmatrix} 1.0 & -1.0 & 1.0 \\ 2.0 & -1.0 & 0.0 \\ 0.0 & 1.0 & 0.0 \end{bmatrix} \quad \text{and} \quad N = \begin{bmatrix} 0.0 & 0.0 \\ 0.0 & 1.0 \\ 1.0 & 0.0 \end{bmatrix}$$

*New Basic primal variables and nonbasic dual variables :*

$$x_{\mathcal{B}}^* = \begin{bmatrix} x_1^* \\ x_2^* \\ x_3^* \end{bmatrix} = \begin{bmatrix} 4.0 \\ 5.0 \\ 2.0 \end{bmatrix} \quad z_{\mathcal{N}}^* = \begin{bmatrix} z_5^* \\ z_4^* \end{bmatrix} = \begin{bmatrix} 5.0 \\ 2.0 \end{bmatrix}$$

**Iteration No 4**

**Step 1.**

*Since  $z_{\mathcal{N}}^*$  has all nonnegative components, the current solution is optimal.*

$$\zeta^* = 4.0x_1^* + 3.0x_2^* = 31.0$$