Primal Simplex Method Initial Matrices and Vector

$$A = \begin{bmatrix} 1.0 & -1.0 & 1.0 & 0.0 & 0.0 \\ 2.0 & -1.0 & 0.0 & 1.0 & 0.0 \\ 0.0 & 1.0 & 0.0 & 0.0 & 1.0 \end{bmatrix}$$

$$Initial\ set\ of\ basic\ and\ nonbasic\ indices$$

$$\beta = \{3, 4, 5\} \quad and \quad \mathcal{N} = \{1, 2\}$$

$$Submatrice\ of\ A$$

$$B = \begin{bmatrix} 1.0 & 0.0 & 0.0 \\ 0.0 & 1.0 & 0.0 \\ 0.0 & 0.0 & 1.0 \end{bmatrix} \quad and \quad N = \begin{bmatrix} 1.0 & -1.0 \\ 2.0 & -1.0 \\ 0.0 & 1.0 \end{bmatrix}$$

$$Inital\ values\ of\ the\ basic\ variables\ are\ given\ by$$

$$x_B^* = b = \begin{bmatrix} 1.0 \\ 3.0 \\ 5.0 \end{bmatrix}$$

$$Inital\ values\ of\ the\ nonbasic\ dualvariables\ are\ given\ by$$

$$z_N^* = -c_N = \begin{bmatrix} -4.0 \\ -3.0 \end{bmatrix}$$

$$Since\ x_B^*\ \geq\ 0, the\ initial\ solution\ is\ primal\ feasible.$$

Iteration No 1

Step 1.

$$Since\ z_N^*\ has\ some\ negative\ components,\ the\ current\ solution\ is\ not\ optimal.$$

Step 2.

$$Since\ z_1^*\ =\ -4.0\ and\ this\ is\ the\ most\ negative\ dual\ variables,$$

$$we\ see\ that\ the\ entering\ index\ is\ j\ =\ 1$$

Step 3.

$$\Delta x_{\mathcal{B}} = B^{-1}Ne_j = \begin{bmatrix} 1.0 & -1.0 \\ 2.0 & -1.0 \\ 0.0 & 1.0 \end{bmatrix} \begin{bmatrix} 1.0 \\ 0.0 \end{bmatrix} = \begin{bmatrix} 1.0 \\ 2.0 \\ 0.0 \end{bmatrix}$$

Step 4.

$$t = \left(max \left\{ \frac{1.0}{1.0}, \frac{2.0}{3.0}, \frac{0.0}{5.0} \right\} \right)^{-1} \; = \; 1.0$$

Step 5.

$$In\ step\ 4,\ the\ ratio\ corresponds\ to\ basic\ index\ 3$$

$$i \; = \; 3$$

Step 6.

$$\Delta z_{\mathcal{N}} = -(B^{-1}N)^T e_i = -\begin{bmatrix} 1.0 & 2.0 & 0.0 \\ -1.0 & -1.0 & 1.0 \end{bmatrix} \begin{bmatrix} 1.0 \\ 0.0 \\ 0.0 \end{bmatrix} = \begin{bmatrix} -1.0 \\ 1.0 \end{bmatrix}$$

Step 7.

$$s \; = \; \frac{z_1^*}{\Delta z_1} \; = \; \frac{-4.0}{-1.0} \; = \; 4.0$$

Step 8.

$$x_1^* \; = \; 1.0, \quad x_{\mathcal{B}}^* \; = \begin{bmatrix} 1.0 \\ 3.0 \\ 5.0 \end{bmatrix} \; - 1.0 \begin{bmatrix} 1.0 \\ 2.0 \\ 0.0 \end{bmatrix} \; = \begin{bmatrix} 0.0 \\ 1.0 \\ 5.0 \end{bmatrix} \; ,$$

$$z_3^* \; = \; 4.0, \quad z_{\mathcal{N}}^* \; = \begin{bmatrix} -4.0 \\ -3.0 \end{bmatrix} \; - 4.0 \begin{bmatrix} -1.0 \\ 1.0 \end{bmatrix} \; = \begin{bmatrix} 0.0 \\ -7.0 \end{bmatrix} \; ,$$

Step 9.

$$New\ set\ of\ basic\ and\ nonbasic\ indices$$

$$\beta = \{1, 4, 5\} \quad and \quad \mathcal{N} = \{3, 2\}$$

$$Corresponding\ new\ basis\ and\ nonbasis\ submatrices\ of\ A,$$

$$B = \begin{bmatrix} 1.0 & 0.0 & 0.0 \\ 2.0 & 1.0 & 0.0 \\ 0.0 & 0.0 & 1.0 \end{bmatrix} \quad and \quad N = \begin{bmatrix} 1.0 & -1.0 \\ 0.0 & -1.0 \\ 0.0 & 1.0 \end{bmatrix}$$

$$New\ Basic\ primal\ variables\ and\ nonbasic\ dual\ variables:$$

$$x_{\mathcal{B}}^* \; = \begin{bmatrix} x_1^* \\ x_4^* \\ x_5^* \end{bmatrix} \; = \begin{bmatrix} 1.0 \\ 1.0 \\ 5.0 \end{bmatrix} \quad z_{\mathcal{N}}^* \; = \begin{bmatrix} z_3^* \\ z_2^* \end{bmatrix} \; = \begin{bmatrix} 4.0 \\ -7.0 \end{bmatrix}$$

Iteration No 2

Step 1.

$$Since\ z_N^*\ has\ some\ negative\ components,\ the\ current\ solution\ is\ not\ optimal.$$

Step 2.

$$Since\ z_2^* \ = \ -7.0\ and\ this\ is\ the\ most\ negative\ dual\ variables,$$

$$we\ see\ that\ the\ entering\ index\ is\ j \ = \ 2$$

Step 3.

$$\Delta x_{\mathcal{B}} = B^{-1}Ne_j = \begin{bmatrix} 1.0 & -1.0 \\ -2.0 & 1.0 \\ 0.0 & 1.0 \end{bmatrix} \begin{bmatrix} 0.0 \\ 1.0 \end{bmatrix} = \begin{bmatrix} -1.0 \\ 1.0 \\ 1.0 \end{bmatrix}$$

Step 4.

$$t = \left(max \left\{ \frac{-1.0}{1.0}, \frac{1.0}{1.0}, \frac{1.0}{5.0} \right\} \right)^{-1} \ = \ 1.0$$

Step 5.

$$In\ step\ 4,\ the\ ratio\ corresponds\ to\ basic\ index\ 4$$

$$i \ = \ 4$$

Step 6.

$$\Delta z_{\mathcal{N}} = -(B^{-1}N)^T e_i = - \begin{bmatrix} 1.0 & -2.0 & 0.0 \\ -1.0 & 1.0 & 1.0 \end{bmatrix} \begin{bmatrix} 0.0 \\ 1.0 \\ 0.0 \end{bmatrix} = \begin{bmatrix} 2.0 \\ -1.0 \end{bmatrix}$$

Step 7.

$$s \ = \ \frac{z_2^*}{\Delta z_2} \ = \ \frac{-7.0}{-1.0} \ = \ 7.0$$

Step 8.

$$x_2^* \ = \ 1.0, \quad x_{\mathcal{B}}^* \ = \begin{bmatrix} 1.0 \\ 1.0 \\ 5.0 \end{bmatrix} - 1.0 \begin{bmatrix} -1.0 \\ 1.0 \\ 1.0 \end{bmatrix} \ = \begin{bmatrix} 2.0 \\ 0.0 \\ 4.0 \end{bmatrix} \ ,$$

$$z_4^* \ = \ 7.0, \quad z_{\mathcal{N}}^* \ = \begin{bmatrix} 4.0 \\ -7.0 \end{bmatrix} - 7.0 \begin{bmatrix} 2.0 \\ -1.0 \end{bmatrix} \ = \begin{bmatrix} -10.0 \\ 0.0 \end{bmatrix} \ ,$$

Step 9.

$$\textit{New set of basic and nonbasic indices}$$

$$\beta = \{1, 2, 5\} \quad \textit{and} \quad \mathcal{N} = \{3, 4\}$$

$$\textit{Corresponding new basis and nonbasis submatrices of } A,$$

$$B = \begin{bmatrix} 1.0 & -1.0 & 0.0 \\ 2.0 & -1.0 & 0.0 \\ 0.0 & 1.0 & 1.0 \end{bmatrix} \quad \textit{and} \quad N = \begin{bmatrix} 1.0 & 0.0 \\ 0.0 & 1.0 \\ 0.0 & 0.0 \end{bmatrix}$$

$$\textit{New Basic primal variables and nonbasic dual variables}:$$

$$x^*_{\mathcal{B}} \ = \begin{bmatrix} x^*_1 \\ x^*_2 \\ x^*_5 \end{bmatrix} \ = \begin{bmatrix} 2.0 \\ 1.0 \\ 4.0 \end{bmatrix} \quad z^*_{\mathcal{N}} \ = \begin{bmatrix} z^*_3 \\ z^*_4 \end{bmatrix} \ = \begin{bmatrix} -10.0 \\ 7.0 \end{bmatrix}$$

Iteration No 3

Step 1.

$$\textit{Since } z^*_N \textit{ has some negative components, the current solution is not optimal.}$$

Step 2.

$$\textit{Since } z^*_3 \ = \ -10.0 \textit{ and this is the most negative dual variables,}$$

$$\textit{we see that the entering index is } j \ = \ 3$$

Step 3.

$$\Delta x_{\mathcal{B}} = B^{-1} N e_j = \begin{bmatrix} -1.0 & 1.0 \\ -2.0 & 1.0 \\ 2.0 & -1.0 \end{bmatrix} \begin{bmatrix} 1.0 \\ 0.0 \end{bmatrix} = \begin{bmatrix} -1.0 \\ -2.0 \\ 2.0 \end{bmatrix}$$

Step 4.

$$t = \left(max \left\{ \frac{-1.0}{2.0}, \frac{-2.0}{1.0}, \frac{2.0}{4.0} \right\} \right)^{-1} \ = \ 2.0$$

Step 5.

$$\textit{In step 4, the ratio corresponds to basic index } 5$$

$$i \ = \ 5$$

Step 6.

$$\Delta z_{\mathcal{N}} = -(B^{-1}N)^T e_i = -\begin{bmatrix} -1.0 & -2.0 & 2.0 \\ 1.0 & 1.0 & -1.0 \end{bmatrix} \begin{bmatrix} 0.0 \\ 0.0 \\ 1.0 \end{bmatrix} = \begin{bmatrix} -2.0 \\ 1.0 \end{bmatrix}$$

Step 7.

$$s \;=\; \frac{z_3^*}{\Delta z_3} \;=\; \frac{-10.0}{-2.0} \;=\; 5.0$$

Step 8.

$$x_3^* \;=\; 2.0, \quad x_{\mathcal{B}}^* \;=\; \begin{bmatrix} 2.0 \\ 1.0 \\ 4.0 \end{bmatrix} \;-\; 2.0 \begin{bmatrix} -1.0 \\ -2.0 \\ 2.0 \end{bmatrix} \;=\; \begin{bmatrix} 4.0 \\ 5.0 \\ 0.0 \end{bmatrix} \;,$$

$$z_5^* \;=\; 5.0, \quad z_{\mathcal{N}}^* \;=\; \begin{bmatrix} -10.0 \\ 7.0 \end{bmatrix} \;-\; 5.0 \begin{bmatrix} -2.0 \\ 1.0 \end{bmatrix} \;=\; \begin{bmatrix} 0.0 \\ 2.0 \end{bmatrix} \;,$$

Step 9.

$$New\ set\ of\ basic\ and\ nonbasic\ indices$$

$$\beta = \{1, 2, 3\} \quad and \quad \mathcal{N} = \{5, 4\}$$

$$Corresponding\ new\ basis\ and\ nonbasis\ submatrices\ of\ A,$$

$$B = \begin{bmatrix} 1.0 & -1.0 & 1.0 \\ 2.0 & -1.0 & 0.0 \\ 0.0 & 1.0 & 0.0 \end{bmatrix} \quad and \quad N = \begin{bmatrix} 0.0 & 0.0 \\ 0.0 & 1.0 \\ 1.0 & 0.0 \end{bmatrix}$$

$$New\ Basic\ primal\ variables\ and\ nonbasic\ dual\ variables:$$

$$x_{\mathcal{B}}^* \;=\; \begin{bmatrix} x_1^* \\ x_2^* \\ x_3^* \end{bmatrix} \;=\; \begin{bmatrix} 4.0 \\ 5.0 \\ 2.0 \end{bmatrix} \qquad z_{\mathcal{N}}^* \;=\; \begin{bmatrix} z_5^* \\ z_4^* \end{bmatrix} \;=\; \begin{bmatrix} 5.0 \\ 2.0 \end{bmatrix}$$

Iteration No 4

Step 1.

Since z_N^* has all nonnegative components, the current solution is optimal.

$$\zeta^* = 4.0x_1^* \;+ 3.0x_2^* \;=\; 31.0$$