Dual Simplex Method Initial Matrices and Vector

$$A = \begin{bmatrix} -2.0 & -1.0 & 1.0 & 0.0 & 0.0 \\ -2.0 & 4.0 & 0.0 & 1.0 & 0.0 \\ -1.0 & 3.0 & 0.0 & 0.0 & 1.0 \end{bmatrix}$$

$$Initial\ set\ of\ basic\ and\ nonbasic\ indices$$
$$\beta = \{3, 4, 5\} \quad and \quad \mathcal{N} = \{1, 2\}$$

$$Submatrice\ of\ A$$

$$B = \begin{bmatrix} 1.0 & 0.0 & 0.0 \\ 0.0 & 1.0 & 0.0 \\ 0.0 & 0.0 & 1.0 \end{bmatrix} \quad and \quad N = \begin{bmatrix} -2.0 & -1.0 \\ -2.0 & 4.0 \\ -1.0 & 3.0 \end{bmatrix}$$

$$Inital\ values\ of\ the\ basic\ variables\ are\ given\ by$$
$$x_B^* = b = \begin{bmatrix} 4.0 \\ -8.0 \\ -7.0 \end{bmatrix}$$
$$Inital\ values\ of\ the\ nonbasic\ dualvariables\ are\ given\ by$$
$$z_N^* = -c_N = \begin{bmatrix} 1.0 \\ 1.0 \end{bmatrix}$$

$$Since\ z_N^*\ \geq\ 0, the\ initial\ solution\ is\ Dual\ feasible.$$

Iteration No 1

Step 1.

$$Since\ x_B^*\ has\ some\ negative\ components,\ the\ current\ solution\ is\ not\ optimal.$$

Step 2.

$$Since\ x_4^*\ =\ -8.0\ and\ this\ is\ the\ most\ negative\ dual\ variables,$$

$$we\ see\ that\ the\ entering\ index\ is\ i\ =\ 4$$

Step 3.

$$\Delta z_{\mathcal{N}} = -(B^{-1}N)^T e_i = - \begin{bmatrix} -2.0 & -2.0 & -1.0 \\ -1.0 & 4.0 & 3.0 \end{bmatrix} \begin{bmatrix} 0.0 \\ 1.0 \\ 0.0 \end{bmatrix} = \begin{bmatrix} 2.0 \\ -4.0 \end{bmatrix}$$

Step 4.

$$t = \left(max \left\{ \frac{2.0}{1.0}, \frac{-4.0}{1.0} \right\} \right)^{-1} \; = \; 0.5$$

Step 5.

$$In\ step\ 4,\ the\ ratio\ corresponds\ to\ basic\ index\ 1$$
$$j \; = \; 1$$

Step 6.

$$\Delta x_{\mathcal{B}} = B^{-1} N e_j = \begin{bmatrix} -2.0 & -1.0 \\ -2.0 & 4.0 \\ -1.0 & 3.0 \end{bmatrix} \begin{bmatrix} 1.0 \\ 0.0 \end{bmatrix} = \begin{bmatrix} -2.0 \\ -2.0 \\ -1.0 \end{bmatrix}$$

Step 7.

$$t \; = \; \frac{x_1^*}{\Delta x_1} \; = \; \frac{-8.0}{-2.0} \; = \; 4.0$$

Step 9.

$$New\ set\ of\ basic\ and\ nonbasic\ indices$$
$$\beta = \{3, 1, 5\} \quad and \quad \mathcal{N} = \{4, 2\}$$

$$Corresponding\ new\ basis\ and\ nonbasis\ submatrices\ of\ A,$$
$$B = \begin{bmatrix} 1.0 & -2.0 & 0.0 \\ 0.0 & -2.0 & 0.0 \\ 0.0 & -1.0 & 1.0 \end{bmatrix} \quad and \quad N = \begin{bmatrix} 0.0 & -1.0 \\ 1.0 & 4.0 \\ 0.0 & 3.0 \end{bmatrix}$$

$$New\ Basic\ primal\ variables\ and\ nonbasic\ dual\ variables:$$
$$x_{\mathcal{B}}^* \; = \begin{bmatrix} x_3^* \\ x_1^* \\ x_5^* \end{bmatrix} \; = \begin{bmatrix} 12.0 \\ 4.0 \\ -3.0 \end{bmatrix} \quad z_{\mathcal{N}}^* \; = \begin{bmatrix} z_4^* \\ z_2^* \end{bmatrix} \; = \begin{bmatrix} 0.5 \\ 3.0 \end{bmatrix}$$

Iteration No 2

Step 1.

$$Since\ x_B^*\ has\ some\ negative\ components,\ the\ current\ solution\ is\ not\ optimal.$$

Step 2.

$$Since\ x_5^* \; = \; -3.0\ and\ this\ is\ the\ most\ negative\ dual\ variables,$$

$$we\ see\ that\ the\ entering\ index\ is\ i \; = \; 5$$

Step 3.

$$\Delta z_{\mathcal{N}} = -(B^{-1}N)^T e_i = -\begin{bmatrix} -1.0 & -0.5 & -0.5 \\ -5.0 & -2.0 & 1.0 \end{bmatrix} \begin{bmatrix} 0.0 \\ 0.0 \\ 1.0 \end{bmatrix} = \begin{bmatrix} 0.5 \\ -1.0 \end{bmatrix}$$

Step 4.

$$t = \left(max \left\{ \frac{0.5}{0.5}, \frac{-1.0}{3.0} \right\} \right)^{-1} \; = \; 1.0$$

Step 5.

$$\textit{In step 4, the ratio corresponds to basic index } 4$$

$$j \; = \; 4$$

Step 6.

$$\Delta x_{\mathcal{B}} = B^{-1} N e_j = \begin{bmatrix} -1.0 & -5.0 \\ -0.5 & -2.0 \\ -0.5 & 1.0 \end{bmatrix} \begin{bmatrix} 1.0 \\ 0.0 \end{bmatrix} = \begin{bmatrix} -1.0 \\ -0.5 \\ -0.5 \end{bmatrix}$$

Step 7.

$$t \; = \; \frac{x_4^*}{\Delta x_4} \; = \; \frac{-3.0}{-0.5} \; = \; 6.0$$

Step 9.

New set of basic and nonbasic indices

$$\beta = \{3, 1, 4\} \quad and \quad \mathcal{N} = \{5, 2\}$$

Corresponding new basis and nonbasis submatrices of A,

$$B = \begin{bmatrix} 1.0 & -2.0 & 0.0 \\ 0.0 & -2.0 & 1.0 \\ 0.0 & -1.0 & 0.0 \end{bmatrix} \quad and \quad N = \begin{bmatrix} 0.0 & -1.0 \\ 0.0 & 4.0 \\ 1.0 & 3.0 \end{bmatrix}$$

New Basic primal variables and nonbasic dual variables :

$$x_{\mathcal{B}}^* \; = \begin{bmatrix} x_3^* \\ x_1^* \\ x_4^* \end{bmatrix} \; = \begin{bmatrix} 18.0 \\ 7.0 \\ 6.0 \end{bmatrix} \quad z_{\mathcal{N}}^* \; = \begin{bmatrix} z_5^* \\ z_2^* \end{bmatrix} \; = \begin{bmatrix} 1.0 \\ 4.0 \end{bmatrix}$$

Iteration No 3

Step 1.

Since x_B^* has all nonnegative components, the current solution is optimal.

$$\zeta^* = -1.0x_1^* \; + -1.0x_2^* \; = \; -7.0$$