R Assignment 5

Jasmeet Singh Saini - 0758054

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Question 1 - Local Advertising

Required Data: *advertising.csv*

Upon request, the local newspaper will provide data to help potential advertisers target their ads. The paper provides a random sample of 12 advertisements per day, 4 from each of the three sections of the paper, for the previous week and the number of inquiries the ad generated for the business. Use *advertising.csv* to answer the following questions.

Firstly, let's loading the data set and check the structure of *structure.csv*.

```
# Importing the dataset
advertising_data <- read.csv("advertising.csv")
# Let's view the str()
str(advertising_data)
```

```
## 'data.frame':    60 obs. of  3 variables:
##  $ Day      : chr  "Monday" "Monday" "Monday" "Monday" ...
##  $ Section  : chr  "News" "News" "News" "News" ...
##  $ Inquiries: int  11 8 6 8 9 10 10 12 8 9 ...
```

Here, we can see that "advertising_data" is in long format and the categorical data variables are of "character" data type. Only Inquiries is numeric variable and its data type is integer.

In this part of question, we will assume α, level of significance as **0.01**.

a) Ignoring any potential interaction between Day and Section, are there any differences in the average number of inquiries by day? (Note: this is a hypothesis test and should be structured as such.)

We will use **One-Way ANOVA Test** to check whether there are any differences in the average number of inquiries by day.

Step 1 : Hypothesis & Assumptions

Step 1a: Hypothesis The H_0 is Null hypothesis and H_A is Alternative hypothesis.

H_0 : There is no significant difference in the average number of inquiries by day.

vs.

H_A : There is significant difference in the average number of inquiries by day

Step 1b: Assumptions

We need to check the assumptions of:

- **Independence** : The data was randomly sampled and the sample size is less than 10% of the population size. Thus, the condition is satisfied.
- **Normality** : The residuals should be nearly normal distributed and show normality. It will checked by fitting the linear model and using Q-Q Plot and Shapiro-Wilk's Test of model residuals.
- **Variance** : The variability across the groups should be about equal (equal variance of residuals). It will be checked by using Residual vs Fitted plot of model residuals and Levene's Test

Before we fitting the lm() model and check the the assumptions of Independence, Normality and Variance, we need to convert the categorical variables to factor data type. Here, we will make "Day" variable a ordered factor and "Section" variable a normal factor.

```
advertising_data$Day <- factor(advertising_data$Day
                        , levels = c("Monday","Tuesday","Wednesday","Thursday","Friday")
                        , ordered = TRUE)

advertising_data$Section <- factor(advertising_data$Section)
```

Now let's again check the str() of advertising_data

```
str(advertising_data)
```

```
## 'data.frame':    60 obs. of  3 variables:
##  $ Day      : Ord.factor w/ 5 levels "Monday"<"Tuesday"<..: 1 1 1 1 2 2 2 2 3 3 ...
##  $ Section  : Factor w/ 3 levels "Business","News",..: 2 2 2 2 2 2 2 2 2 2 ...
##  $ Inquiries: int  11 8 6 8 9 10 10 12 8 9 ...
```

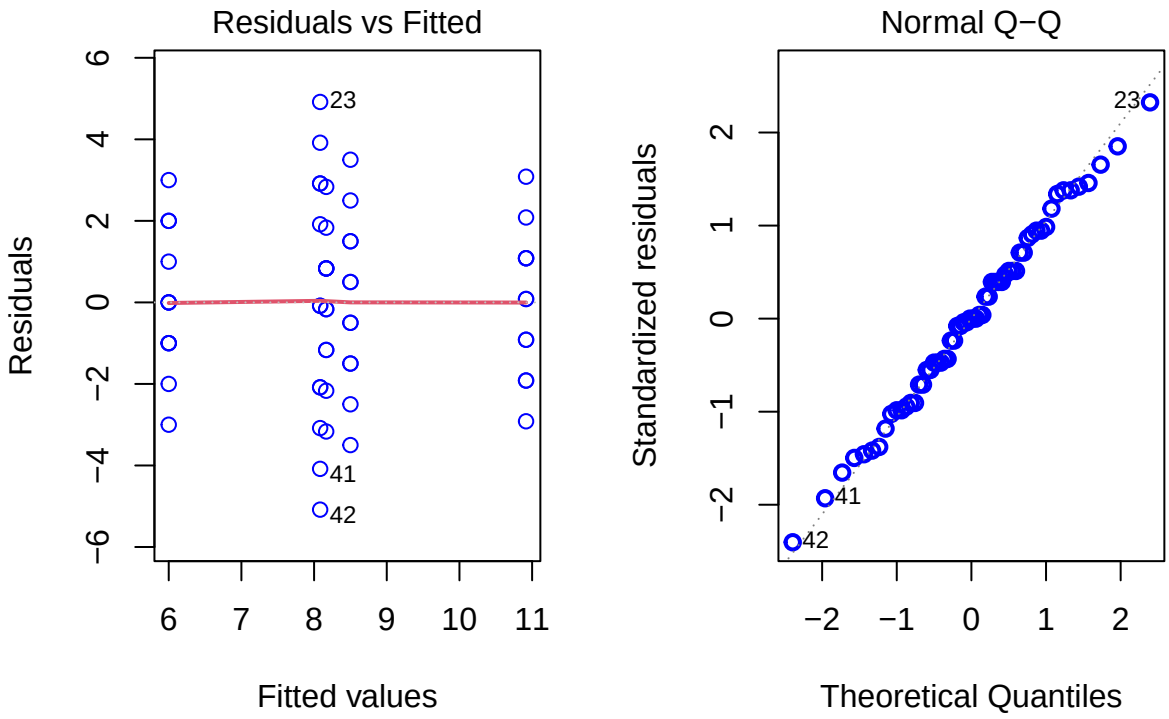
Now, let's fit the linear model:

```
linear_model <- lm(Inquiries ~ Day, data = advertising_data)
```

Checking the **assumptions**:

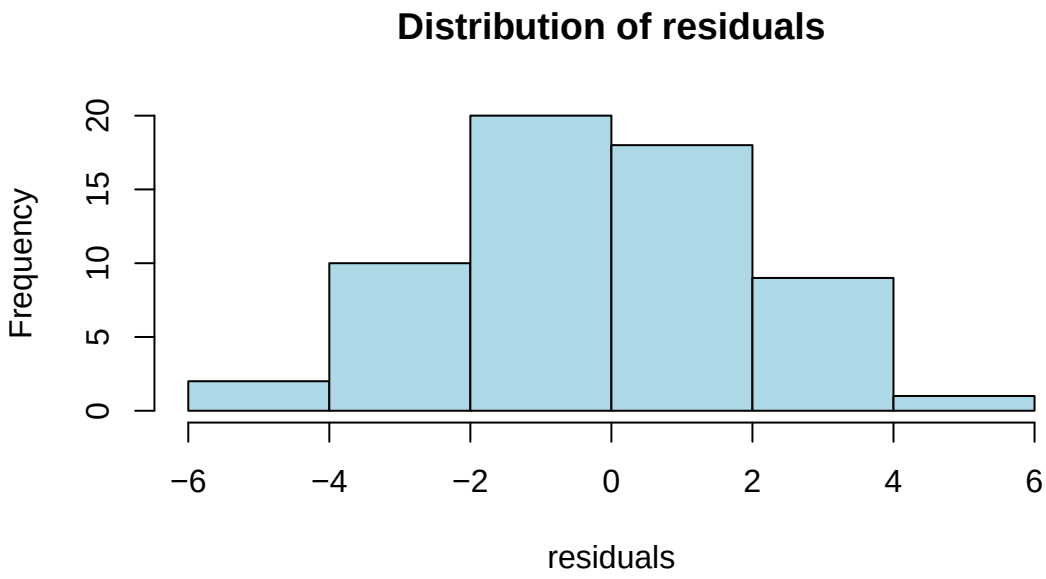
Plot of Residual vs Fitted Plot & QQ Plot:

```
par(mfrow=c(1,2))
plot(linear_model, which = 1:2, col = "blue", lwd = 2, pch = 1)
```



Histogram of residuals:

```
hist(linear_model$residuals, main = "Distribution of residuals"
     , xlab = "residuals", col = "lightblue")
```



Checking normality using Shapiro-Wilk's Test:

```
shapiro.test(linear_model$residuals)
```

```
##
##  Shapiro-Wilk normality test
##
## data:  linear_model$residuals
## W = 0.99351, p-value = 0.9876
```

As it can be observed in the Q-Q Plot that the data points are closely attached to the model fitted line and histogram has a frequency peaks in the center. Also, p-value in Shapiro-Wilk normality test is **0.9876**. Hence, p-value > 0.01 and we can say that the data is normal.

```
 # Levene's Test for Homogeneity of Variance
car::leveneTest(linear_model)
```

```
## Levene's Test for Homogeneity of Variance (center = median)
##       Df F value  Pr(>F)
## group  4  3.0279 0.02503 *
##       55
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

In the Residual vs Fitted plot, all the data points are in the constant distance to the line and p-value in Levene's test is **0.02503**. Hence, $p - value > 0.01$ and we can say that constant variance is reached.

Hence, all of the conditions are met and now we can proceed with the testing using the parametric approach.

Step 2 : Test Statistic and p-value

We need to analyze the result of **One-way ANOVA Test** to make our decision.

Now, let's test the ANOVA and get the summary:

```
aov_advertising <- aov(linear_model)
summary(aov_advertising)
```

```
##             Df Sum Sq Mean Sq F value   Pr(>F)
## Day          4  146.8   36.71   7.519 6.61e-05 ***
## Residuals   55  268.5    4.88
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Hence, the test statistic is **7.519** and p-value is **6.61e-05**.

Step 3 : Statistical Decision

Therefore, we **reject** Null Hypothesis, H_0 because p-value is less than α, level of significance (that is, p-value < 0.01).

Step 4 : Conclusion

There is enough evidence to support the alternative hypothesis, and therefore we conclude that there is significant difference in the average number of inquiries by day, is accurate.

b) If "yes" to a), which day would you prefer to advertise on if maximizing inquiries was your objective?

To check which day would you prefer to advertise on if maximizing inquiries was your objective, we will use Tukey HSD - Statistical Test for Differences.

```
# Tukey for multiple comparisons of means
TukeyHSD(aov_advertising)
```

```
##   Tukey multiple comparisons of means
##     95% family-wise confidence level
##
## Fit: aov(formula = linear_model)
##
## $Day
##                           diff        lwr        upr     p adj
## Tuesday-Monday      0.41666667 -2.1273194 2.96065269 0.9903820
## Wednesday-Monday    0.08333333 -2.4606527 2.62731936 0.9999830
## Thursday-Monday    -2.08333333 -4.6273194 0.46065269 0.1573669
## Friday-Monday       2.83333333  0.2893473 5.37731936 0.0218355
## Wednesday-Tuesday  -0.33333333 -2.8773194 2.21065269 0.9959087
## Thursday-Tuesday   -2.50000000 -5.0439860 0.04398603 0.0563243
## Friday-Tuesday      2.41666667 -0.1273194 4.96065269 0.0702094
## Thursday-Wednesday -2.16666667 -4.7106527 0.37731936 0.1301276
## Friday-Wednesday    2.75000000  0.2060140 5.29398603 0.0279337
## Friday-Thursday     4.91666667  2.3726806 7.46065269 0.0000118
```

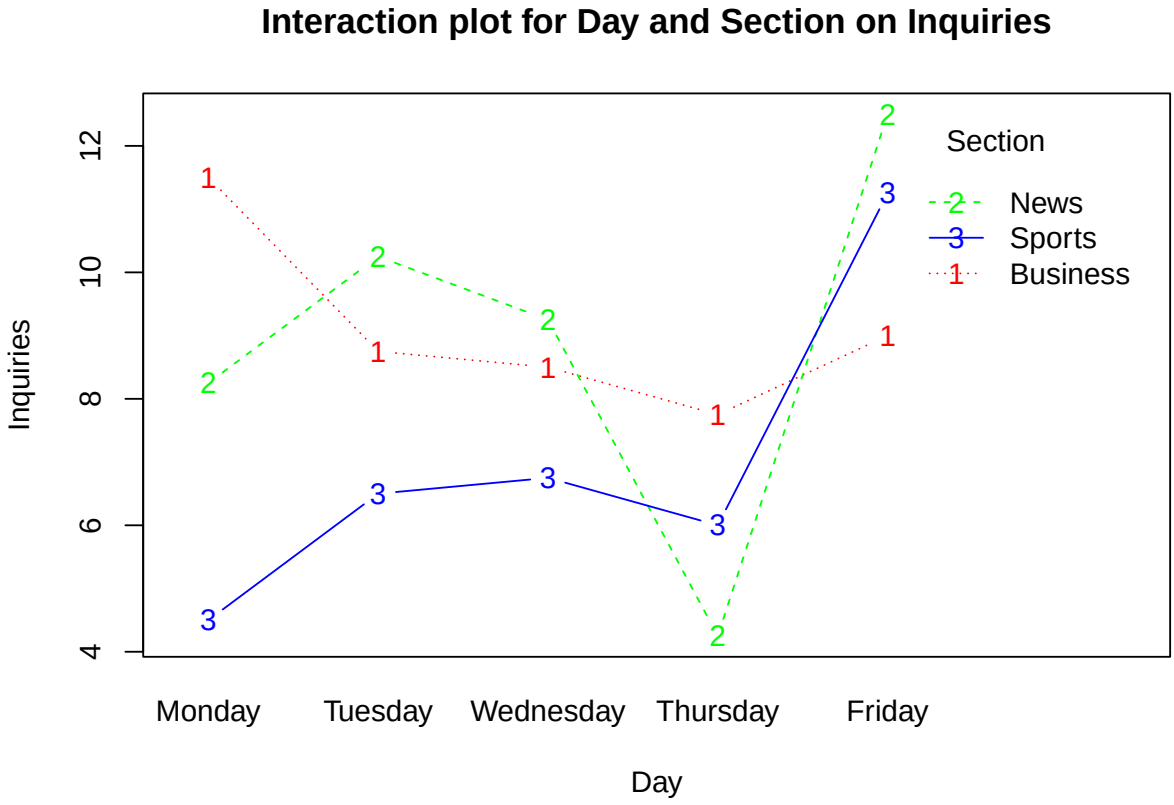
Here, we can observe that there is a significance difference in the mean inquiries of Friday-Thursday group. As the $p - value < 0.01$, the mean inquiries on Friday is greater than mean inquiries on Thursday.

Hence, we can conclude that Friday has a significantly higher mean number of inquiries compared to all other days and we could choosing **Friday** to advertise our ad on.

c) Create an interaction plot for Day and Section on Inquiries. Based on this plot, does there appear to be an interaction; why or why not? (see the examples in the help file for interaction.plot on how to create these plots)

Below is an interaction plot for "Day" and "Section" on "Inquiries".

```
# Interaction Plot
with(advertising_data,
    interaction.plot(x.factor = Day, trace.factor = Section
      , response = Inquiries, type = 'b', ylab = "Inquiries"
      , col = c("red","green","blue")
      , main = "Interaction plot for Day and Section on Inquiries")
    )
```



Yes, Definitely! An interaction between "Day" and "Section" on "Inquiries" can be seen as the lines converge and cross each other and there will be significant increase in inquiries of all sections, particularly "News" section on Fridays.

d) Test for an interaction between Day and Section. Does your result have an effect on your answer in part a) of this question? Bonus for determining which section and day combination sees the most inquiries using emmeans.

We will use **Two-Way ANOVA Test** to check whether there is an interaction between the Day and Section.

Step 1 : Hypothesis & Assumptions

Step 1a: Hypothesis The H_0 is Null hypothesis and H_A is Alternative hypothesis.

H_0 : There is no significant interaction between Day and Section

vs.

H_A : There is significant interaction between Day and Section

Step 1b: Assumptions

We need to check the assumptions of:

- **Independence** : The data was randomly sampled and the sample size is less than 10% of the population size. Thus, the condition is satisfied.
- **Normality** : The residuals should be nearly normal distributed and show normality. It will checked by fitting the linear model and using Q-Q Plot and Shapiro-Wilk's Test of model residuals.
- **Variance** : The variability across the groups should be about equal (equal variance of residuals). It will be checked by using Residual vs Fitted plot of model residuals and Levene's Test

In Part (a), we have checked all the assumptions and are satisfied. Hence, we can proceed with the testing using the parametric approach.

Step 2 : Test Statistic and p-value

We need to analyze the result of **Two-Way ANOVA Test** to make our decision.

Now, let's test the ANOVA and get the summary:

```
# Creating the linear model
linear_model_one <- lm(Inquiries ~ Day * Section, data = advertising_data)
# Performing ANOVA
summary(aov(linear_model_one))
```

```
##             Df Sum Sq Mean Sq F value   Pr(>F)
## Day          4 146.83   36.71  20.910 8.52e-10 ***
## Section      2  53.73   26.87  15.304 8.50e-06 ***
## Day:Section  8 135.77   16.97   9.667 1.12e-07 ***
## Residuals   45  79.00    1.76
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Hence, the test statistic is **9.667** and p-value is **1.12e-07**.

Step 3 : Statistical Decision

Therefore, we **reject** Null Hypothesis, H_0 because p-value is less than α, level of significance (that is, p-value < 0.01).

Step 4 : Conclusion

There is enough evidence to support the alternative hypothesis, and therefore we conclude that there is a significant interaction between Day and Section, is accurate.

Reason:

Yes, Definitely! Our result have an effect on our answer in part (a) of this question. Any chances of interaction between "Day" and "Section" has been ignored in part (a). Also, there is an interaction between "Day" and "Section" which affects the result.

EMMEANS

To check which section and day combination sees the most inquiries, we will use *emmeans()* function:

```
library(emmeans)
```

```
## Warning: package 'emmeans' was built under R version 4.2.3
```

```
ans <- emmeans(linear_model_one, pairwise ~ Day:Section)
ans$emmeans
```

```
##  Day       Section  emmean    SE df lower.CL upper.CL
##  Monday    Business  11.50 0.662 45    10.17    12.83
##  Tuesday   Business   8.75 0.662 45     7.42    10.08
##  Wednesday Business   8.50 0.662 45     7.17     9.83
##  Thursday  Business   7.75 0.662 45     6.42     9.08
##  Friday    Business   9.00 0.662 45     7.67    10.33
##  Monday    News       8.25 0.662 45     6.92     9.58
##  Tuesday   News      10.25 0.662 45     8.92    11.58
##  Wednesday News       9.25 0.662 45     7.92    10.58
##  Thursday  News       4.25 0.662 45     2.92     5.58
##  Friday    News      12.50 0.662 45    11.17    13.83
##  Monday    Sports     4.50 0.662 45     3.17     5.83
##  Tuesday   Sports     6.50 0.662 45     5.17     7.83
##  Wednesday Sports     6.75 0.662 45     5.42     8.08
##  Thursday  Sports     6.00 0.662 45     4.67     7.33
##  Friday    Sports    11.25 0.662 45     9.92    12.58
##
## Confidence level used: 0.95
```

Here, we can see that the pair, that is, Friday and News ("Day" and "Section" respectively) has maximum emmean value of **12.5**. Hence, Friday : News (Day : Section) sees the maximum number of inquiries.

e) The newspaper currently charges the same amount per ad for any day and any section in the newspaper. Assuming you work for the paper, do you have any recommendations for the paper in terms of pricing?

Need to check this

Seeing the Interaction Plot in part-c of this question, it can be seen that maximum inquiries appear on Fridays which suggests that the company shall start charging a premium for posting advertisements on Fridays as advertisers tend to post maximum ads on Fridays. Besides, the inquiries for Sports throughout the week is lower as compared to News and Business sections, thus, the company can lower down the prices for Sports Section so as to attract more advertisers and make profits.

Question 2: A Life of Leisure1 Part I - Model Fitting

It has long been argued that workers with more leisure time are more productive during their work hours. Jay has been trying to convince his employer that he should be allowed to work a four day week for the same annual salary, but so far they have been resistant and require cold hard statistical proof that reduced working hours will help the company's bottom line. After all, Jay is suggesting that he work less while still making the same amount of money. Jay finds data on OECD countries' GDP produced per hour worked which was collected by researchers at a prestigious university (contained in *leisure.csv*) and hires you to analyze the data. This dataset has two columns of interest for us: GDP per hour worked and hours worked.

Firstly, let's loading the data set and check the structure of *leisure.csv*.

```
# Importing the dataset
leisure_data <- read.csv("leisure.csv")
# Let's view the str()
str(leisure_data)
```

```
## 'data.frame':    41 obs. of  3 variables:
##  $ Country: chr  "AUS" "AUT" "BEL" "CAN" ...
##  $ hours  : num  1694 1442 1493 1685 1753 ...
##  $ gdp    : num  48.3 15 35.3 41.3 48.3 ...
```

a) Estimate the linear relationship between GDP per hour and hours worked using a linear model. (This includes checking model assumptions using diagnostic plots.) Comment on how well the model fits the data.

To estimate the linear relationship between GDP per hour and hours worked using a linear model, we will conduct *Linear Regression Model* using the two parameters (β_0 is intercept of the model and β_1 is slope of the model):

$$GDP = \beta_0 + \beta_1 * Hours$$

Fitting the Model

Now let's fit the model:

```
lm <- lm(gdp ~ hours, data = leisure_data)
lm
```

```
##
## Call:
## lm(formula = gdp ~ hours, data = leisure_data)
##
## Coefficients:
## (Intercept)        hours
##    -5.77909      0.02499
```

Here, the intercept, β_0 = -5.779 and slope, β_1 is **0.025**.

$$GDP = (-5.779) + (0.025 * Hours)$$

Assumptions

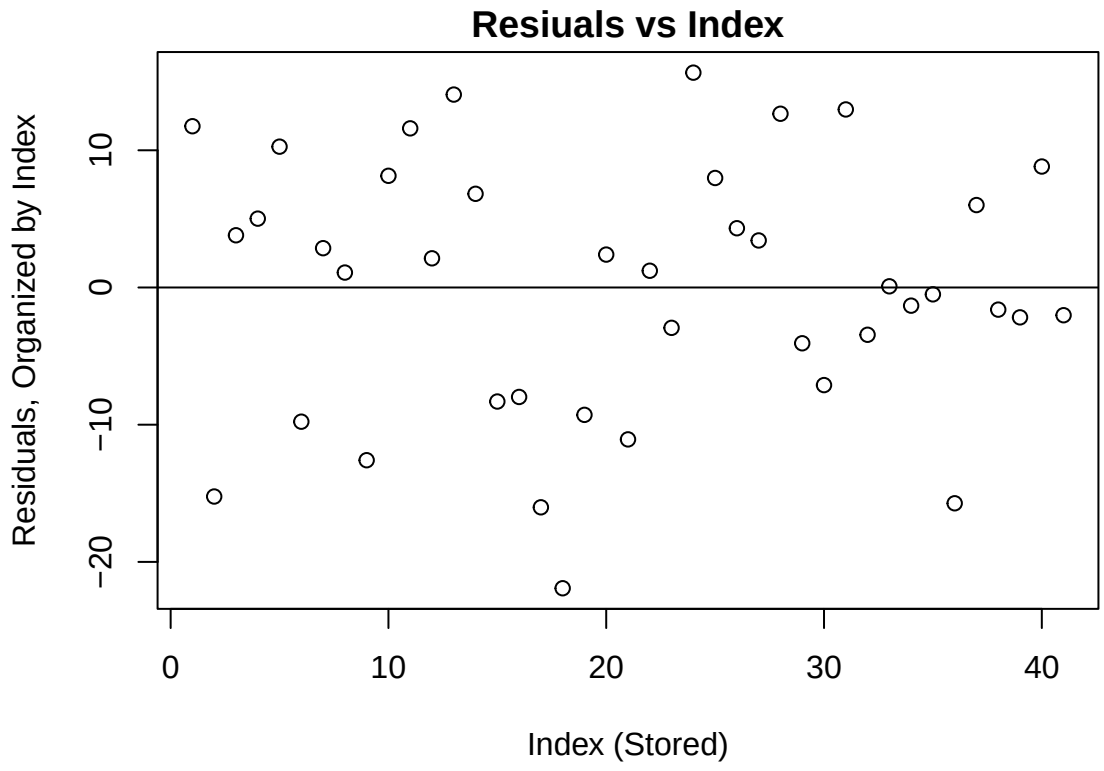
We need to check the assumptions for linear model and tell how well the model fits the data:

- **Independence** : The data was randomly sampled and the sample size is less than 10% of the population size. Thus, the condition is satisfied.
- **Linearity** : There should be a linear relationship between the two variables, that is, "gdp" and "hours"
- **Normality** : The residuals should be nearly normal distributed and show normality.
- **Variance** : The variability across the groups should be about equal (equal variance of residuals).

Now, we will check the assumptions.

Firstly, let's check for *independence*:

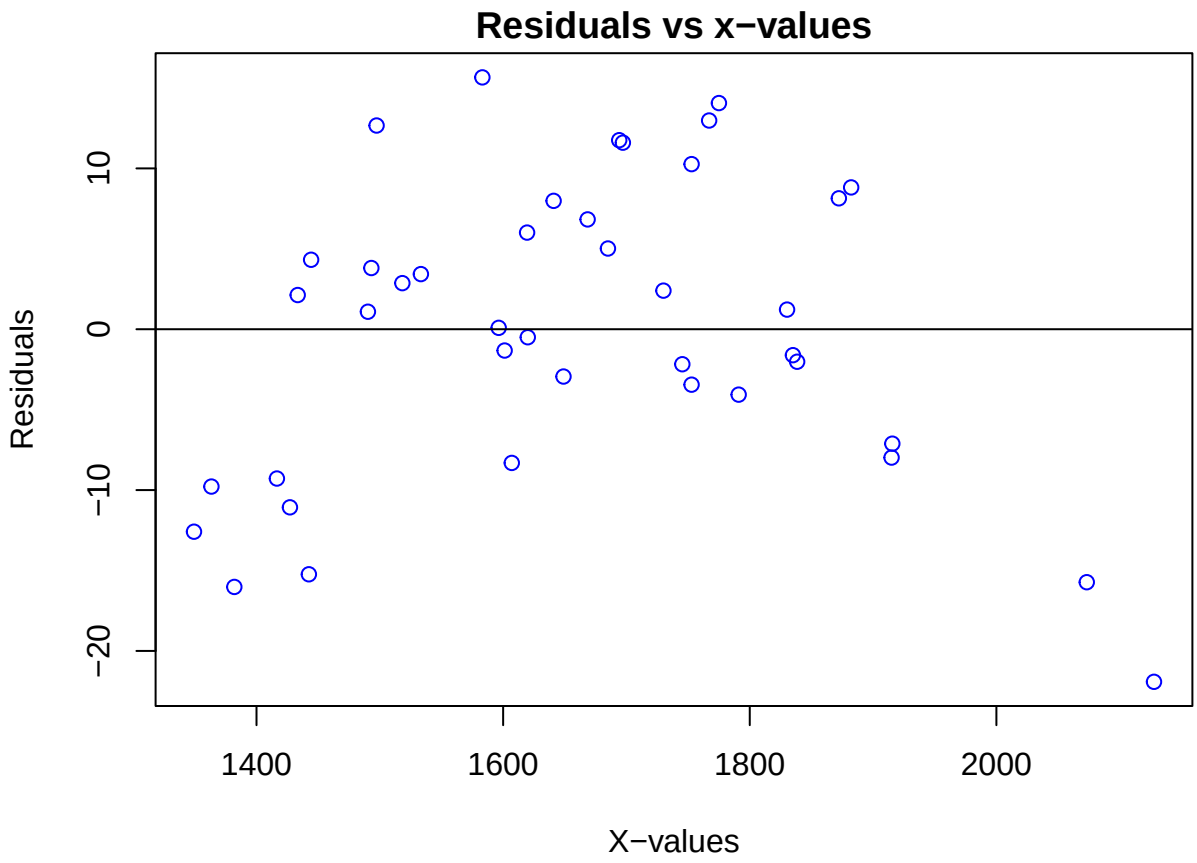
```
par(mar = c(4,4,1.5, 1.5), mgp = c(3, 1, 0))
plot(x = 1:length(lm$residuals), y = lm$residuals
, xlab = "Index (Stored)",ylab = "Residuals, Organized by Index", main = "Resiuals vs Index")
abline(h = 0)
```



It is independent, as no serial correlation can be observed.

Now, we will check the assumptions of *Linearity* among "gdp" and "hours".

```r
par(mar = c(4,4,1.5, 1.5), mgp = c(3, 1, 0))
plot(x = leisure_data$hours, y = lm$residuals
    , xlab = "X-values", ylab = "Residuals", col = "blue"
    , main = "Residuals vs x-values")
abline(h = 0)
```



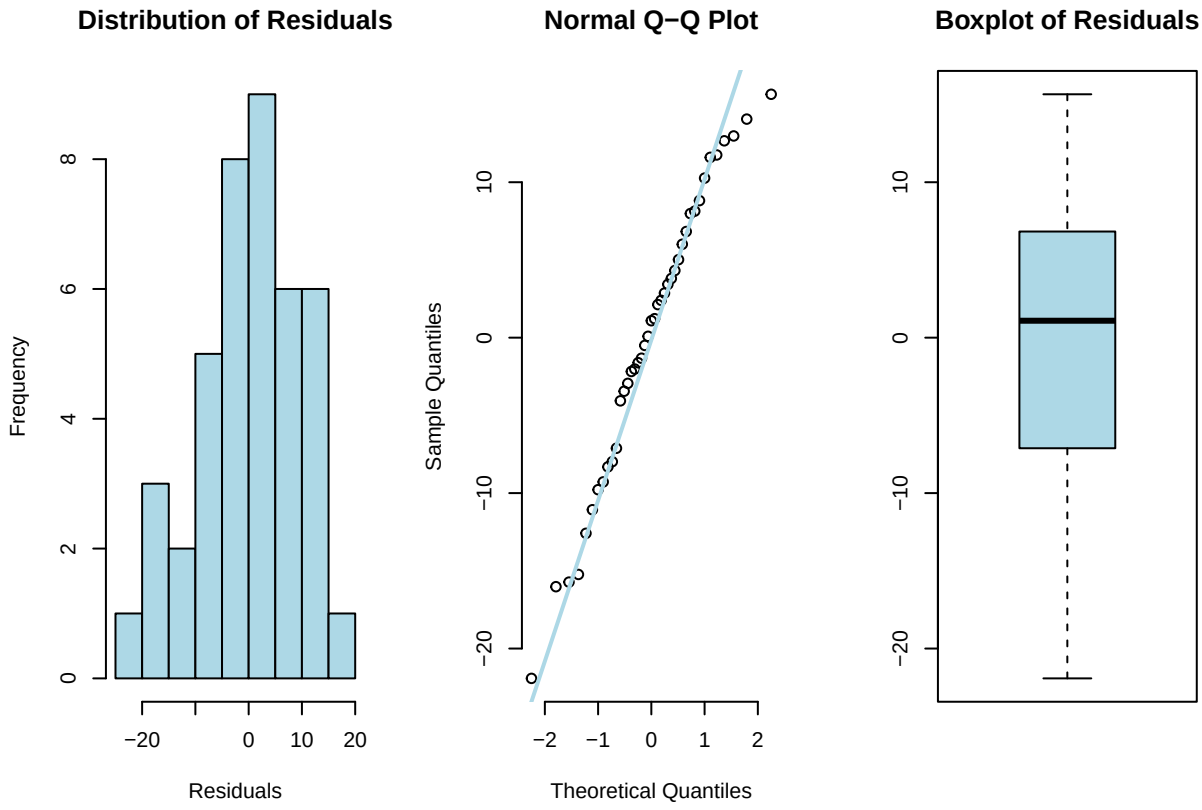
Here, it can be seen that the residuals vs x are nearly linear. Hence, the assumption is met.

Now, we will check the assumptions of *Normality*:

```r
par(mfrow=c(1,3))
# Histogram of Residuals
hist(lm$residuals, main = "Distribution of Residuals"
     , col = "lightblue", xlab = "Residuals")

qqnorm(lm$residuals, pch = 1, frame = FALSE)
qqline(lm$residuals, col = "lightblue", lwd = 2)

# Boxplot of Residuals
boxplot(lm$residuals, col = "lightblue"
        , main = "Boxplot of Residuals")
```



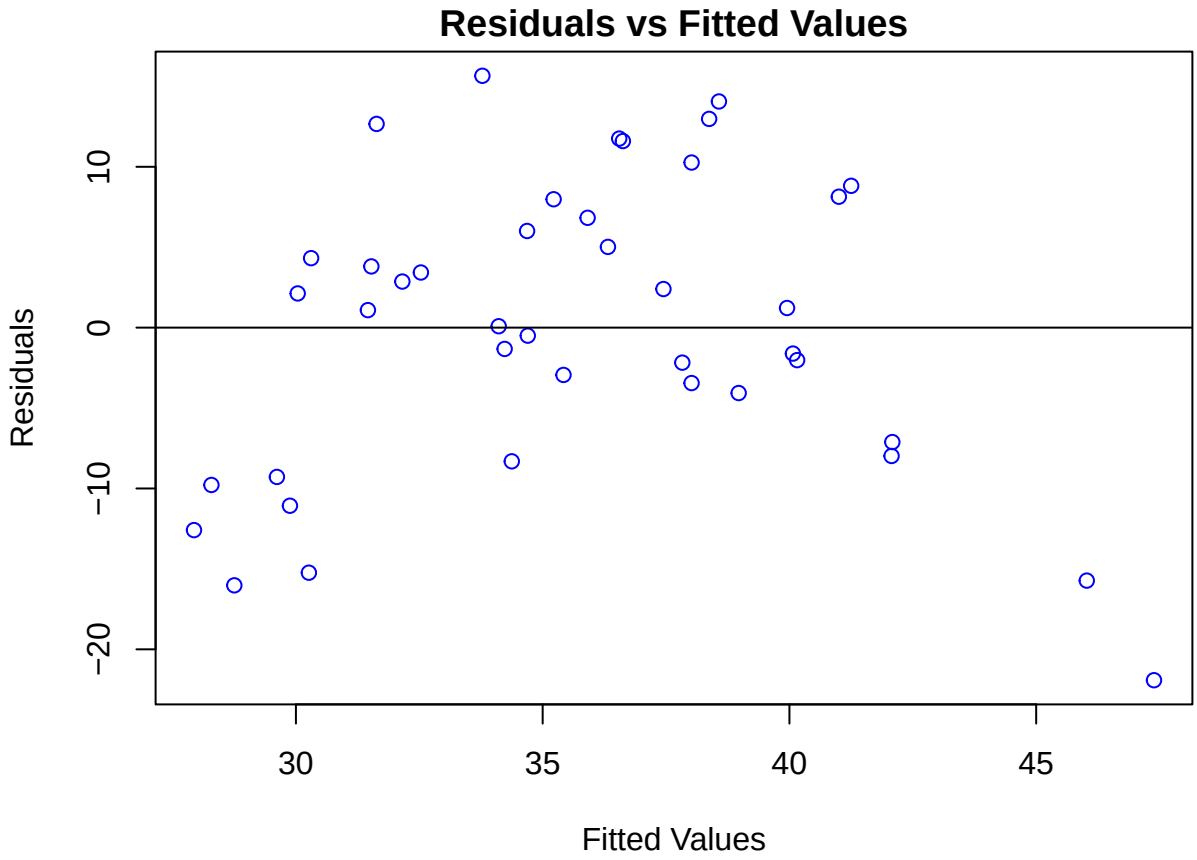

```
# Shapiro-Wilk's normality test
shapiro.test(lm$residuals)
```

```
##
##  Shapiro-Wilk normality test
##
## data:  lm$residuals
## W = 0.97542, p-value = 0.5083
```

Here, it can be seen that the distribution's the frequency peaks at the center and is evenly distributed at the ends, and is confirm by Shapiro-Wilk's test, as p-value > 0.05. Hence, the assumption of *normality* is met.

Finally, we will check the assumptions of *variability*:

```
par(mar = c(4,4,1.5, 1.5), mgp = c(3, 1, 0))
plot(x = lm$fitted.values, y = lm$residuals, col = "blue"
     , xlab = "Fitted Values", ylab = "Residuals"
     , main = "Residuals vs Fitted Values")
abline(h = 0)
```



The data across the line are nearly constant, as the fitted values increase the residuals are constant in "Residuals vs Fitted Values". Hence, the condition of *variability* is met.

Hence, **all of the conditions are met** and we can assume **the model to be a good fit** for the data.

b) Conduct a hypothesis test on the slope parameter of the model in a) to determine if there is a linear relationship between GDP per hour and hours worked.

c) You will have hopefully determined that the relationship here is likely not only linear and therefore the model in a) is not the best model for this data. Use a linear model to fit the quadratic relationship between GDP per hour and hours worked. (Again, use diagnostic plots and comment on how well the model fits the data).

d) Plot the data and your model from a) and c) all on the same plot. Include a legend.

Question 3: A Life of Leisure Part II - Interpretation

For this question "The Model" is your model from Question 2c).

a) Explain why The Model in Q2c) is better than your model in Q2a). There are MANY comparisons to make; to get full marks you must make at least 4.

b) Give The Model equation.

c) Based on The Model equation in b), what is the average number of work hours per person that would maximize GDP per hour?

d) Jay currently works 1992 hours a year. As mentioned, he wants to ask his employer for a four-day work week. Would your analysis convince an employer to move their employees to a four-day work week? Explain your process. If not, is there still a way for Jay to work less than he currently is and still make the same amount of money? What type of compromise could be offerred?