Relationships in the extremes and their influence on competition and coexistence

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Abstract

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1 Introduction

2 Theory

For pedgogical clarity, we develop each step of the theory both for a simple classical model, the lottery model, and in general. The model is as follows. Letting $N_i(t)$ denote the population density of species $i = 1, 2$ at time t, and defining $N = N_1(t) + N_2(t)$, model equations are

$$N_i(t+1) = (1-\delta)N_i(t) + \delta N \frac{B_i(t)N_i(t)}{B_1(t)N_1(t) + B_2(t)N_2(t)} \tag{1}$$

for $i = 1, 2$. Here, δ is a mortality rate, and $B_i(t)$ is the fecundiity of species i at time t. The model postulates that individuals die at rate δ at each time step, and are replaced by juveniles in proportion to the reproductive outputs of the two species that year. For each i, we assume the random variables $B_i(t)$ are independent and identically distributed (iid) through time. We let $B_i = \exp(b_i)$, where (b_1, b_2) is some bivariate random variable with the properties that b_i is normally distributed with mean μ_i and variance σ^2. We denote $\operatorname{cov}(b_1, b_2)$ by ρ. However, we do *not* assume that (b_1, b_2) is a bivariate normal distribution. We will consider various distributions of (b_1, b_2) with the above properties, corresponding to symmetric and asymmetric tail association cases. We assume without loss of generality that $\mu_1 \leq \mu_2$, so that species 1 is the weaker competitor. Note that N is constant through time.

Modern coexistence theory (Chesson, 2000) and its recent computational extensions (Ellner *et al.*, 2016, 2019) quantify the contributions of different mechanisms to species coexistence. But we show in Sup Mat section X that, for purely temporal variation, only one mechanism relates to tail associations: storage effects. We here define storage effects abstractly and then for the lottery model, using an approach and notation that follows elements of the presentations in Ellner *et al.* (2016) and Ellner *et al.* (2019) closely (and see those papers for additional details and explanation). We return briefly to other mechanisms of coexistence in the Discussion. It is assumed that the growth rate $r_i(t)$ of species i can be written as an increasing function of an environment-dependent factor $E_i(t)$ and as a decreasing function of a competitive pressure, $C_i(t)$. For the lottery model, we take $E_i(t) = B_i(t)$, henceforth assuming that fecundity depends strictly on the environment. Competition $C_i(t)$ is taken to be

3 Methods

4 Results

5 Discussion

References

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