SEAT NUMBER:

STUDENT NUMBER:

SURNAME:
(FAMILY NAME)

OTHER NAMES:

UTS
UNIVERSITY OF TECHNOLOGY SYDNEY

This paper and all materials issued must be returned at the end of the examination.
They are not to be removed from the exam centre.

Examination Conditions:

It is your responsibility to fill out and complete your details in the space provided on all the examination material provided to you. Use the time before your examination to do so as you will not be allowed any extra time once the exam has ended.

You are **not** permitted to have on your desk or on your person any **unauthorised material**. This includes but not limited to:

- Mobile phones
- Smart watches and bands
- Electronic devices
- Draft paper (unless provided)
- Textbooks (unless specified)
- Notes (unless specified)

You are **not** permitted to obtain assistance by improper means or ask for help from or give help to any other person.

If you wish to **leave and be re-admitted** (including to use the toilet), you have to wait until **90 mins** has elapsed.

If you wish to **leave the exam room permanently**, you have to wait until **60 mins** has elapsed.

You are **not** permitted to leave your seat (including to use the toilet) during **the final** 15 mins.

During the examination **you must first seek permission** (by raising your hand) from a supervisor before:

- Leaving early
- Using the toilet
- Accessing your bag

Disciplinary action will be taken against you if you infringe university rules.

33130 Mathematical Modelling 1

Time Allowed: 2 hours and 10 mins

Includes 10 minutes of optional reading time.

This reading time is optional. You are permitted to write, calculate or mark your paper in any way during reading time.

This is a Restricted Open Book exam

Please refer to the permitted materials below.

Permitted materials for this exam:

- Calculators (non-programmable only)
- Drawing instruments
  i.e. Rulers, Set Squares and Compasses
- The following material is allowed:
- One (1) revision sheet (A4 double-sided). They must be handwritten only.

Materials provided for this exam:

- This examination paper
- Four (4) answer booklets (5 pages)

Students please note:

- Each question is to be attempted in a separate booklet
- You MUST indicate the question number on the cover of each booklet
- You MUST hand in your sheet of A4 paper

Examiner: Dr D. M. Solina

Assessor: Dr M. Coupland

Do not open your exam paper until instructed.

Rough work space
Do **not** write your answers on this page.

PLEASE BEGIN A NEW BOOKLET

WRITE THE QUESTION NUMBER ON THE COVER

Question 1 : Mastery/Basic Level (3 + 3 + 4 + 4 + 3 + 3 = 20 marks)

(a) For what values of d are the given vectors $\langle 20, d, -19\rangle$ and $\langle d, -d^2, d\rangle$ orthogonal (at 90°)?

(b) Find the equation of the plane through the point (4, 5, 3) and perpendicular to the vector $\langle 9, -4, 3\rangle$.

(c) Each side of a cube is increasing at a rate of 2 cm/s. At what rate is the surface area of the cube increasing when the surface area is 54 cm^2?

(d) Find all the solutions of the equation: $x^2 + 2x + 2 = 0$. Express your answer in exponential polar form with $-\pi < \theta \leq \pi$.

(e) Use logarithmic differentiation to find $\frac{dy}{dx}$ of the function: $y = x^{5x}$

(f) Evaluate the integral: $\int_1^4 \frac{x-12}{\sqrt{x}} dx$. Leave your answer as a fraction.

Remember to start the next question in a separate booklet, and write the question number on the cover.

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PLEASE BEGIN A NEW BOOKLET
WRITE THE QUESTION NUMBER ON THE COVER

Question 2 : Standard Level (4 + 4 + 4 + 4 + 4 = 20 marks)

(a) By writing each vector in terms of its components, e.g. $\underline{u} = \langle u_1, u_2 \rangle$, prove: $\underline{u} + \underline{v} = \underline{v} + \underline{u}$.

(b) Resolve the vector **a** = 5**i** + 7**j** + 4**k** into two vectors, one parallel to **c** = 3**i** – 2**j** + 1**k**, and one perpendicular to **c**.

(c) A hanging cable of constant linear density ρ and horizontal tension T at its lowest point takes the shape of the graph of a solution of the differential equation $\frac{d^2y}{dx^2} = \frac{\rho g}{T}\sqrt{1+\left(\frac{dy}{dx}\right)^2}$ where g is the acceleration due to gravity. Verify that the function $y = \frac{T}{\rho g}\cosh\left(\frac{\rho g x}{T}\right)$ is a solution of the differential equation.

(d) In fluid mechanics, the complex potential function w is given by: $w(r,\theta) = \varphi(r,\theta) + i\psi(r,\theta)$. where $\varphi(r,\theta)$ is the potential function and $\psi(r,\theta)$ the stream function both of which are functions of the two real variables, r and θ. A particular flow is given by $w = z^5 + \frac{c^2}{z^5}$, where c is a non-zero real number and $z = re^{i\theta}$. By considering the real and imaginary parts of w, show that the potential function $\varphi(r,\theta)$ is given by $\varphi(r,\theta) = \left(r^5 + \frac{c^2}{r^5}\right)\cos(5\theta)$.

(e) Show that $\frac{d}{dx}\cos^{-1} x = \frac{-1}{\sqrt{1-x^2}}$ Hint: start by letting $y = \cos^{-1} x$ then complete $x = \ldots$., then differentiate both sides with respect to x using the Chain Rule.

Remember to start the next question in a separate booklet, and write the question number on the cover.

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PLEASE BEGIN A NEW BOOKLET

WRITE THE QUESTION NUMBER ON THE COVER

Question 3 : Standard Level (4 + 4 + 4 + 4 + 4 = 20 marks)

(a) Find the following: $\int x\sqrt{5x+1}\,dx$

(b) Find the length of the curve $y = \cosh x$ for $0 \le x \le 6$. Note: Arc Length $= \int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$

(c) In a particular series RC circuit the voltage $v(t)$ across one of the elements is given by the differential equation $RC\frac{dv}{dt} + v = Ee^{-\frac{t}{RC}}$, with R, C, and E constants. If $v(0) = 2$, find the particular solution.

(d) Find the solution of the following differential equation which satisfies the given initial conditions: $y'' = -10y'$, $y(0) = 1$, $y'(0) = 3$.

(e) Solve the initial value problem: $\frac{dy}{dx} = 4y^2 + 1$, $y(0) = 2$

Remember to start the next question in a separate booklet, and write the question number on the cover.

…/*Over*

PLEASE BEGIN A NEW BOOKLET

WRITE THE QUESTION NUMBER ON THE COVER

Question 4 : Challenge Level (10 + 6 + 4 = 20 marks)

(a) At a mining plant, bauxite ore is stored in a stockpile in a square yard with 100 metre sides. The ore is poured from a conveyor belt 30 metres above the original ground surface of the yard at a rate of 6000 m^3/hour, and forms a conical shaped pile, whose radius is 1.5 times its height.

i. If at a certain time the pile is 20 metres high, how long will it take for the top of the pile to reach the conveyor belt (i.e. 30 metres high)?

ii. How much of the floor area of the yard is uncovered when the pile is 20 metres high? How fast is the floor area of the pile growing at that height?

iii. Suppose that the ore is now being removed from the pile at a rate of 2000 m^3/hour to be used in the next stage of processing when the height of the pile reaches 25 metres. Assuming that the pile continues to maintain its conical shape, how long will it now take for the top of the pile to reach the conveyor belt under these conditions?

(b) Show that $\sin^{-1}(\tanh x) = \tan^{-1}(\sinh x)$.

(c) Consider a container of any shape, full of water, and without a lid. The water evaporates at a rate proportional to the area of the surface of the water. That is, the rate of decrease of the volume is proportional to the surface area. Show that the depth of the water decreases at a constant rate, regardless of the shape of the container.