Bernoulli Distribution

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Distribution of a single binary random variable.

Suppose $x \in \{0, 1\}$ is the random variable and

$$P(x=1) = p$$
$$P(x=0) = 1-p$$

1) Summation over x

$$\sum_{x \in \{0,1\}} x$$
$$= 0 + 1$$
$$= 1$$

2) Expectation value over the random binary variable

$$E[x] = \sum_{x \in \{0,1\}} x P(x)$$
$$= 1 \cdot P_x(1) + 0 \cdot P_x(0) = p = \mu$$

3) Variance = $E(x-\mu)^2$ ← sum of squared deviations from the mean

$$= E[x^2 - 2\mu x + \mu^2]$$
$$= E[x^2] - E[2\mu x] + E[\mu^2]$$
$$= E[x^2] - 2\mu^2 + \mu^2$$
$$= E[x^2] - \mu^2$$
$$= \mu - \mu^2$$
$$= \mu(1-\mu)$$

4) $H[x] = -\sum P(x) \ln(x)$

$$= -P_y(0) \ln P_y(0) - P_x(1) \ln P_x(1)$$
$$= -(1-p) \ln(1-p) - p \ln p$$