The Closing of the Gender Gap as a Roy Model Illusion

A Summary of the Model

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Mulligan & Rubinstein (2004) NBER Working Paper

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THE CLOSING OF THE GENDER GAP AS A ROY MODEL ILLUSION

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Figure: The identified paper

Main Focus & Findings

Increased gender equality has been coincident with growing earnings inequality within genders.

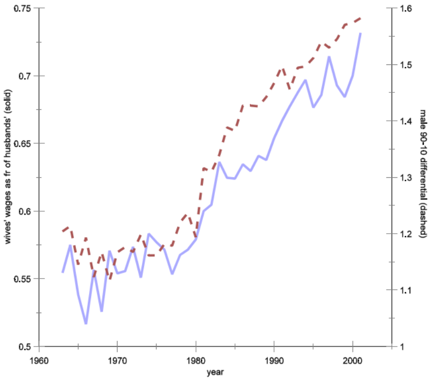

Figure 1 Wage Inequality between and within Genders

Figure: Figure 1 in Mulligan & Rubinstein (2004)

Main Focus & Findings

Hypothesis: Change in the market reward to skills (reflected by male wages) $\Rightarrow$ Change in the nature of female self-selection.

- Self-selection may have become more positive/less negative, or have flipped signs from $-$ to $+$.
- $\Rightarrow$ Reduce gender wage gap, without either a substantial change in female participation, or a real decline in the latent gender wage gap.

Roy Model Setup

$$\text{Wages: } w_{it} = \mu_t^w + \gamma_t + \sigma_t^w \epsilon_{it}^w$$

$$\text{Reservation wages: } r_{it} = \mu_t^r + \sigma_t^r \epsilon_{it}^r$$

Notations:

- γ_t is the "true" (unconditional) gender wage gap.
- σ_t^w is the person-specific market price of skills, σ_t^r is the person-specific non-market price of skills.
- $\epsilon_{it}^w \sim N(0,1)$ can be seen as the person-specific market skills, and $\epsilon_{it}^r \sim N(0,1)$ as the person-specific non-market skills.

Roy Model Setup

Distributions:

$$\begin{bmatrix} \epsilon_{it}^{w} \\ \epsilon_{it}^{r} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix} \right)$$

$$\begin{bmatrix} \sigma_t^{w} \epsilon_{it}^{w} \\ \sigma_t^{r} \epsilon_{it}^{r} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_t^{w\,2}, \rho_t \sigma_t^{w} \sigma_t^{r} \\ \rho_t \sigma_t^{w} \sigma_t^{r}, \sigma_t^{r\,2} \end{bmatrix} \right)$$

where $\rho_t = \dfrac{\sigma_t^{wr}}{\sigma_t^{w} \sigma_t^{r}}$. Assume that $\rho_t = \rho$, constant over-time.
Participate in the labor force $\iff w_{it} > r_{it}$.

$$\mu_t^{w} + \gamma_t + \sigma_t^{w} \epsilon_{it}^{w} > \mu_t^{r} + \sigma_t^{r} \epsilon_{it}^{r} \tag{1a}$$

$$\underbrace{\frac{\sigma_t^{w}}{\sigma_t^{r}} \epsilon_{it}^{w} - \epsilon_{it}^{r}}_{\equiv v_{it}} > \frac{\mu_t^{r} - \mu_t^{w} - \gamma_t}{\sigma_t^{r}} \tag{1b}$$

Selection Bias

The measured gender wage gap:

$$
\begin{aligned}
G_t &= \gamma_t + \sigma_t^w \mathbb{E}\left(\epsilon_t^w \,\middle|\, \frac{\sigma_t^w}{\sigma_t^r}\epsilon_{it}^w - \epsilon_{it}^r > \frac{\mu_t^r - \mu_t^w - \gamma_t}{\sigma_t^r} \right) \\
&= \gamma_t + \sigma_t^w \rho_t^{w\nu} \mathbb{E}\left(\frac{\nu_{it}}{\sigma_t^\nu} \,\middle|\, \frac{\nu_{it}}{\sigma_t^\nu} > \underbrace{\frac{\mu_t^r - \mu_t^w - \gamma_t}{\sigma_t^r \sigma_t^\nu}}_{\equiv z} \right) \\
&= \gamma_t + \sigma_t^w \underbrace{\rho_t^{w\nu} \frac{\phi(z)}{1 - \Phi(z)}}_{\equiv b_t}
\end{aligned}
$$

where b_t is the selection bias on skills evaluated at market prices σ_t^w. Note that

$$
\frac{d}{dz} \frac{\phi(z)}{1 - \Phi(z)} \leq 0
$$

Solve for Selection Bias

The distribution of ν_{it}:

$$\nu_{it} \sim N\left(0, \left[\frac{\sigma_t^w}{\sigma_t^r}\right]^2 + 1 - 2\rho\frac{\sigma_t^w}{\sigma_t^r}\right)$$

by $X - Y \sim N(\mu_x - \mu_y, \sigma_x^2 + \sigma_y^2 - 2\sigma_{xy})$. Then

$$\begin{aligned}
\rho_t^{w\nu} &= \frac{\operatorname{Cov}\left(\nu_{it}, \epsilon_{it}^w\right)}{\sigma_t^\nu \sigma_t^w} \\
&= \frac{\operatorname{Cov}\left(\frac{\sigma_t^w}{\sigma_t^r}\epsilon_{it}^w - \epsilon_{it}^r, \epsilon_{it}^w\right)}{\sigma_t^\nu \sigma_t^w} \\
&= \frac{\frac{\sigma_t^w}{\sigma_t^r} - \rho}{\sigma_t^\nu} \\
&= \frac{\frac{\sigma_t^w}{\sigma_t^r} - \rho}{\left(\left[\frac{\sigma_t^w}{\sigma_t^r}\right]^2 + 1 - 2\rho\frac{\sigma_t^w}{\sigma_t^r}\right)^{0.5}}
\end{aligned}$$

Solve for Selection Bias

Thus, the bias between unconditional and measured gender wage gap is

$$\sigma_t^w \cdot \underbrace{\frac{\frac{\sigma_t^w}{\sigma_t^r} - \rho}{\left(\left[\frac{\sigma_t^w}{\sigma_t^r} \right]^2 + 1 - 2\rho \frac{\sigma_t^w}{\sigma_t^r} \right)^{0.5}} \cdot \frac{\phi(z)}{1 - \Phi(z)}}_{= b_t}$$

Positive bias: $\frac{\sigma_t^w}{\sigma_t^r} - \rho > 0$.

Analysis

Comparative statistics:

$$\frac{\partial b_t}{\partial \frac{\sigma_t^w}{\sigma_t^r}} = \frac{1-\rho^2}{\left(\left[\frac{\sigma_t^w}{\sigma_t^r}\right]^2 + 1 - 2\rho\frac{\sigma_t^w}{\sigma_t^r}\right)^{1.5}} > 0$$

A rise in the relative price of market skill value increases positive self-selection, or decreases negative self-selection $\Rightarrow$ Improves the composition of female workers.

$$\frac{\partial b_t}{\partial \frac{\phi(z)}{1-\Phi(z)}} = \frac{\frac{\sigma_t^w}{\sigma_t^r} - \rho}{\left(\left[\frac{\sigma_t^w}{\sigma_t^r}\right]^2 + 1 - 2\rho\frac{\sigma_t^w}{\sigma_t^r}\right)^{0.5}}$$

If self-selection is negative and more women work (which decreases $\frac{\phi(z)}{1-\Phi(z)}$), this will raise the wage of female workers.