Engineering Mathematics
Volume 2

DIEGO INOCENCIO TAPANG GILLESANIA
Civil Engineer

BSCE, LIT – Magna Cum Laude
5th Place, PICE National Students' Quiz, 1989
Awardee, Most Outstanding Student, 1989
3rd Place, CE Board November 1989
Review Director & Reviewer in all Subjects
Gillesania Engineering Review Center
Reviewer in Mathematics and General Engineering Sciences
MERIT Philippines Review, Manila
Author of Various Engineering Books

to my mother iluminada,

my wife imelda,

and my children kim deunice,

ken dainiel, and

karla denise

Table of Contents

Part 1: Differential Calculus 1

Limits 1
- L' Hospital's Rule 1
- Gillesania's Principle on Limits 1

Differentiation Formulas 3
- Algebraic Function 3
- Logarithmic and Exponential Functions 3
- Trigonometric Functions 4
- Inverse Trigonometric Functions 4
- Hyperbolic Functions 4
- Inverse Hyperbolic Functions 5

Slope of the Curve 6

Rate of Change 6

Curvature and Radius of Curvature 6
- Circle of Curvature 6

Graph of a Function 7
- Points of Inflection 8

Applications of Maxima and Minima 8

Variable Relationships for Maximum or Minimum Values 9

Time Rates 13
- Basic Time Rates 14

PROBLEMS
- Set 1 – Limits, Differentiation, Rate of Change 15
- Set 2 – Maxima & Minima, Time Rates 31

Part 2: Integral Calculus 57

Integration Formulas 57
- Algebraic, Exponential, and Logarithmic Functions 57
- Trigonometric Functions 57
- Inverse Trigonometric Functions 58
- Hyperbolic Functions 59
- Other Functions 59
- Integration by Parts 59
- Trigonometric Substitution 59
- Wallis' Formula 61

Plane Areas 64
Using Vertical Strip 64
Using Horizontal Strip 65
By Polar Coordinates 65
Are of Some Polar Curves 65
Length of Arc 66
Centroid pf Plane Areas 66
Using Horizontal Strip 67
Using Vertical Strip 67
Centroid of Parabolic Segment and Spandrel 67
Moment of Inertia of Plane Areas (First Moment of Area) 67
Polar Moment of Inertia 68
Product of Inertia 68
Mass Moment of Inertia 68
Properties of Common Shapes 69
Solids of Revolution 71
Volume Using Circular Disk 71
Volume Using Hallow Cylindrical Shell 72
Surface Area 73
Volume of Other Solids with Known Cross Sections 73
Centroid of Volume 73
Centroid of Common Solids 74
Work 75
Constant Force 75
Work Under Variable Force 75
Work to Stretch a Spring 76
Work in Winding Up a Load 77

PROBLEMS
Set 3 – Integration 78
Set 4 – Plane Areas, Volumes, Surfaces, Centroid, Etc. 94

Part 3: Differential Equation 119

Definitions 119
Variable Separable 119
Differential Equation of type $dy/dx = f(x)$ 119
Differential Equation of type $dy/dx = f(y)$ 120
Differential Equation of type $dy/dx = f(x)\ g(y)$ 120
Differential Equation of type $dQ/dt = kQ$ 120
Homogeneous First Order Differential Equation 121
Linear First Order Differential Equation 122
Linear Second Order Differential Equation 122

Exact Differential Equation 123
Bernoulli's Equation 124
Finding the Differential Equation from a General Solution 124
Some Applications of Differential Equation 129
Population Growth 129
Exponential Growth and Decay 127
Cooling and Heating Problems 127
Flow Problems 127
Continuous Compound Interest 127
Motion Problems 127

PROBLEMS
Set 5 – Differential Equation and Application 129

Part 4: Engineering Mechanics 147

Definitions 147
Outline of Engineering Mechanics 147
Statics 148
Resultant of Force System 148
Resultant of Two Concurrent Coplanar Forces 148
Resultant of Two or More Concurrent Coplanar Forces 148
Resultant of Concurrent Forces in Space 148
Resultant of Parallel Forces 150
Equilibrium of Forces 150
General Conditions of Equilibrium 150
Equilibrium of Two Forces 150
Equilibrium of Two or More Concurrent Forces 150
Equilibrium of Three Coplanar Forces 151
Parabolic Cables 151
Cables under Concentrated Loads 153
Friction 155
Properties of Section 156
Centroid of Plane Area 156
Center of Gravity of Flat Plates 156
Centroid of Composite Figures 156
Moment of Inertia 157
Polar Moment of Inertia 157
Radius of Gyration 157
Transfer Formula for Moment of Inertia 157
Moment of Inertia with Respect to Inclined Axis 157
Mohr's Circle for Moment of Inertia 158

Dynamics 158
Branches of Dynamics 159
Kinematics 159
Translation 159
Rectilinear Translation 159
Uniform Motion 159
Variable Acceleration 160
Free-Falling Body 160
Curvilinear Translation 166
Motion Curves 161
Rotation 162
Uniform Motion 162
Uniform Acceleration 162
Relationship between Translation and Rotation 163
Kinetics 163
Newton's Laws of Motion 163
D' Alemberts Principle 163
Centrifugal Force 164
Conical Pendulum 165
Banking of Curves 165
Ideal Angle of Banking 166
Horizontal Rotating Platform 166
Work and Energy 167
Impulse and Momentum 167
Law of Conservation of Momentum 168
Coefficient of Restitution 168

PROBLEMS
Set 6 – Translation and Rotation 171
Set 7 – Kinetics, Work, Energy, Momentum, Etc. 192

Part 5: Strength of Materials 217
Simple Stress 217
Normal Stress 217
Shearing Stress 217
Bearing Stress 218
Thin-Walled Pressure Vessels 218
Thin-Walled Cylindrical Vessel 218
Spherical Shell 218
Thick-Walled Cylinder 219
Simple Strain 219
Stress-Strain Diagram 219

Axial Deformation 221
Shearing Deformation 223
Poisson's Ratio 223
Biaxial Deformation 224
Triaxial Deformation 224
Thermal Stress 225
Torsion 226
Torsional Shearing Stress 226
Angle of Twist 227
Power Transmitted by the Shaft 227
Torsion on thin-Walled Tubes 227
Helical Springs 228
A. M. Wahl Formula 228
Springs in Series 229
Springs in Parallel 229
Shear and Moment in Beams 229
Statically Determinate Beams 229
Statically Indeterminate Beams 230
Shear and Moment Diagrams 230
Relationship between Load, Shear, and Moment 231
Properties of Shear and Moment Diagrams 231
Stresses in Beams 233
Radius of Curvature 234
Flexure Formula 234
Shearing Stress Formula 235
Superimposed Beams 235
Spacing of Rivets or Bolt in Built-Up Beams 236
Economic Sections 236
Combined Stresses 237
Combined Axial and Flexure 237
Kern of a Section 238
Combined Axial and Shearing Stress 238
Stresses on an Inclined Plane 240
Mohr's Circle 241
Combined Torsional and Flexural Stress 243

PROBLEMS
Set 8 – Stress, Strain, Torsion, Shear & Moment, Etc. 244

Part 6: Fluid Mechanics 257

Properties of Fluids 257
Unit Weight 257
Mass Density 257
Density of Gases 257
Specific Volume 257
Specific Gravity 258
Viscosity 258
Surface Tension 258
Pressure Inside a Droplet of Liquid 258
Capillarity 258
Bulk Modulus of Elasticity 259
Compression of Gases 260
Boyle's Law 260
Charle's Law 260
Pressure Disturbances 260
Unit Pressure 261
Variations in Pressure 261
Pressure below Layers of Different Liquids 261
Total hydrostatic Pressure 261
Total Hydrostatic Pressure on Plane Surfaces 261
Total Hydrostatic Pressure on Curved Surfaces 262
Buoyancy 262
Relative Equilibrium of Liquids 263
Horizontal Motion 263
Inclined Motion 263
Vertical Motion 263
Rotation 264
Fluid Flow and Pipes 264
Flow Rate 264
Continuity Equation 264
Reynold's Number 265
Energy Equation 265
Bernoulli's Energy Theorem 266
Head Lost in Pipe Flow 266
Darcy-Weisbach Formula 266
Manning's Formula 266
Hazen William's Formula 267
Minor Head Lost 267
Pipes in Series 268
Pipes in Parallel 268
Equivalent Pipes 268

Orifice and Tubes 268
Unsteady Flow 269
Weir 270
Hydrodynamics 272
Drag Force 274

PROBLEMS
Set 9 – Fluid Mechanics 275

Part 7: Engineering Economics 285

Interest 285
Cash Flow Diagrams 285
Simple Interest 286
Ordinary and Exact Simple Interests 286
Compound Interest 287
Continuous Compounding 289
Nominal and Effective Rates of Interest 289
Equivalent Nominal Rates 290
Annuity 290
Types of Annuity 291
Ordinary Annuity 291
Deferred Annuity 392
Annuity Due 392
Perpetuity 393
Uniform Gradient 393
Arithmetic Gradient 393
Geometric Gradient 394
Capitalized Cost and Annual Cost 394
Cost Comparison of Different Alternatives 396
Depreciation 396
Methods of Computing Depreciation 397
Straight line Method 397
Sinking Fund Method 397
Sum of the Years' Digit Method 397
Declining Balance Method 398
Double Declining Balance Method 398
Capital Recovery (Depletion) 398
Bond 399
Break-Even Analysis 300

PROBLEMS
Set 10 - Simple and Compound Interest 302
Set 11 - Compound Interest, Annuity 318
Set 12 - Depreciation, Capitalized Cost. Bonds, Etc. 343

Part 8: Elements and Miscellaneous Problems 363
PROBLEMS
Set 13 - Algebra, Trigonometry, Geometry 363
Set 14 - Calculus, Physics, Mechanics, Economics 373
Set 15 - Miscellaneous Problems 390
Set 16 - Miscellaneous Problems 427

Part 1 DIFFERENTIAL CALCULUS

LIMITS

Theorems on Limits

1. If $f(x) = c$, a constant, then $\lim_{x \to a} f(x) = c$.

Let $\lim_{x \to a} f(x) = A$ and $\lim_{x \to a} g(x) = B$, then;

2. $\lim_{x \to a} k f(x) = kA$, k being constant.
3. $\lim_{x \to a} [f(x) \pm g(x)] = \lim_{x \to a} f(x) \pm \lim_{x \to a} g(x) = A \pm B$.
4. $\lim_{x \to a} [f(x)\, g(x)] = \lim_{x \to a} f(x) \lim_{x \to a} g(x) = A\, B$.
5. $\lim_{x \to a} \dfrac{f(x)}{g(x)} = \dfrac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)} = \dfrac{A}{B}$, provided $B \neq 0$.
6. $\lim_{x \to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \to a} f(x)} = \sqrt[n]{A}$, provided $\sqrt[n]{A}$ is a real number.

L' Hospital's Rule (Indeterminate type 0/0)

If a is a number, if $f(x)$ and $g(x)$ are differentiable and $g(x) \neq 0$ for all x on some interval $0 < |x - a| < \delta$, and if $\lim_{x \to a} f(x) = 0$ and $\lim_{x \to a} g(x) = 0$, then, when $\lim_{x \to a} \dfrac{f'(x)}{g'(x)}$ exists or is infinite,

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

Gillesania's Principle for Limits

To evaluate the $\lim_{x \to a} f(x)$, substitute for x a value that is very close to a then use a calculator.

That is;

For $x \to 2$, substitute $x = 1.9999$ or $x = 2.0001$
For $x \to -5$, substitute $x = -4.9999$ or -5.0001
For $x \to 0$, substitute $x = 0.00001$
For $x \to \infty$, substitute $x = 99999$

Consider the following examples:

Ex. 1: $\lim_{x \to 3} \dfrac{x^4 - 81}{x - 3}$

Substitute $x = 2.9999$

$$\frac{(2.9999)^4 - 81}{(2.9999) - 3} = 107.99 \approx 108$$

Ex. 2: $\lim_{x \to 0} \left(\dfrac{1}{x} - \dfrac{1}{e^x - 1} \right)$

Substitute $x = 0.0001$

$$\left(\frac{1}{0.0001} - \frac{1}{e^{0.0001} - 1} \right) = 0.49999 \approx 0.5$$

Ex. 3: $\lim_{x \to 0} \dfrac{\cot x}{\cot 2x}$

Substitute $x = 0.0001°$

$$\frac{\cot(0.0001)}{\cot[2(0.0001)]} = 2$$

Ex. 4: $\lim_{x \to 0} \dfrac{e^x + e^{-x} - x^2 - 2}{\sin^2 x - x^2}$

Substitute $x = 0.0001$ radian

Note: set your calculator to radian mode

$$\frac{e^{0.0001} + e^{-0.0001} - (0.0001)^2 - 2}{\sin^2(0.0001) - (0.0001)^2} = -0.25$$

DIFFERENTIATION FORMULAS

In the following formulas u, v, and w are differentiable functions of x and a and n are constants.

Algebraic Functions

1. $\dfrac{d}{dx}(c) = 0$
2. $\dfrac{d}{dx}(cu) = c\dfrac{d(u)}{dx}$
3. $\dfrac{d}{dx}(u + v) = \dfrac{d(u)}{dx} + \dfrac{d(v)}{dx}$
4. $\dfrac{d}{dx}(uv) = u\dfrac{d(v)}{dx} + v\dfrac{d(u)}{dx}$
5. $\dfrac{d}{dx}(u^n) = nu^{n-1}\dfrac{d(u)}{dx}$
6. $\dfrac{d}{dx}(\sqrt{u}) = \dfrac{\frac{d(u)}{dx}}{2\sqrt{u}}; u \neq 0$
7. $d\left(\dfrac{u}{v}\right) = \dfrac{v\frac{d(u)}{dx} - u\frac{d(v)}{dx}}{v^2}; v \neq 0$
8. $d\left(\dfrac{c}{v}\right) = \dfrac{-c\frac{d(v)}{dx}}{v^2}$

Logarithmic & Exponential Functions

9. $\dfrac{d}{dx}(\log_a u) = \dfrac{\frac{d(u)}{dx}}{u \ln a}$
10. $\dfrac{d}{dx}(\log_{10} u) = \log_{10} e \dfrac{\frac{d(u)}{dx}}{u} = 0.43429 \dfrac{\frac{d(u)}{dx}}{u}$
11. $\dfrac{d}{dx}(\ln u) = \dfrac{\frac{d(u)}{dx}}{u}; u \neq 0$
12. $\dfrac{d}{dx}(a^u) = a^u \ln a \dfrac{d(u)}{dx}$
13. $\dfrac{d}{dx}(e^u) = e^u \dfrac{d(u)}{dx}$
14. $\dfrac{d}{dx}(u^v) = vu^{v-1}\dfrac{d(u)}{dx} + u^v \ln u \dfrac{d(v)}{dx}$

Trigonometric Functions

15. $\frac{d}{dx}(\sin u) = \cos u \frac{d(u)}{dx}$
16. $\frac{d}{dx}(\cos u) = -\sin u \frac{d(u)}{dx}$
17. $\frac{d}{dx}(\tan u) = \sec^2 u \frac{d(u)}{dx}$
18. $\frac{d}{dx}(\cot u) = -\csc^2 u \frac{d(u)}{dx}$
19. $\frac{d}{dx}(\sec u) = \sec u \tan u \frac{d(u)}{dx}$
20. $\frac{d}{dx}(\csc u) = -\csc u \cot u \frac{d(u)}{dx}$

Inverse Trigonometric Functions

21. $\frac{d}{dx}(\arcsin u) = \frac{d(u)/dx}{\sqrt{1-u^2}}$
22. $\frac{d}{dx}(\arccos u) = -\frac{d(u)/dx}{\sqrt{1-u^2}}$
23. $\frac{d}{dx}(\arctan u) = \frac{d(u)/dx}{1+u^2}$
24. $\frac{d}{dx}(\text{arccot } u) = -\frac{d(u)/dx}{1+u^2}$
25. $\frac{d}{dx}(\text{arcsec } u) = \frac{d(u)/dx}{u\sqrt{u^2-1}}$
26. $\frac{d}{dx}(\text{arccsc } u) = -\frac{d(u)/dx}{u\sqrt{u^2-1}}$

Hyperbolic Functions

27. $\frac{d}{dx}(\sinh u) = \cosh u \frac{d(u)}{dx}$
28. $\frac{d}{dx}(\coth u) = -\text{csch}^2 u \frac{d(u)}{dx}$
29. $\frac{d}{dx}(\cosh u) = \sinh u \frac{d(u)}{dx}$
30. $\frac{d}{dx}(\text{sech } u) = -\text{sech } u \tanh u \frac{d(u)}{dx}$
31. $\frac{d}{dx}(\tanh u) = \text{sech}^2 u \frac{d(u)}{dx}$
32. $\frac{d}{dx}(\text{csch } u) = -\text{csch } u \coth u \frac{d(u)}{dx}$

where:

$$\sinh x = \frac{e^x - e^{-x}}{2} \qquad \cosh x = \frac{e^x + e^{-x}}{2}$$

$$\tanh x = \frac{\sinh x}{\cosh x} \qquad \coth x = \frac{1}{\tanh x}$$

$$\text{sech } x = \frac{1}{\cosh x} \qquad \text{csch } x = \frac{1}{\sinh x}$$

Inverse Hyperbolic Functions

33. $\frac{d}{dx}(\text{arcsinh } u) = \frac{d(u)/dx}{\sqrt{u^2+1}}$
34. $\frac{d}{dx}(\text{arctanh } u) = \frac{d(u)/dx}{1-u^2}$
35. $\frac{d}{dx}(\text{arccosh } u) = \frac{d(u)/dx}{\sqrt{u^2-1}}$
36. $\frac{d}{dx}(\text{arcsech } u) = \frac{-d(u)/dx}{u\sqrt{1-u^2}}$
37. $\frac{d}{dx}(\text{arccot } u) = \frac{-d(u)/dx}{u^2-1} = \frac{d(u)/dx}{1-u^2}$
38. $\frac{d}{dx}(\text{arccsch } u) = \frac{-d(u)/dx}{u\sqrt{1+u^2}}$

where:

$$\text{arcsinh } x = \ln\left(x + \sqrt{x^2+1}\right)$$

$$\text{arccosh } x = \ln\left(x + \sqrt{x^2-1}\right)$$

$$\text{arctanh } x = \frac{1}{2}\ln\frac{1+x}{1-x}$$

$$\text{arccoth } x = \frac{1}{2}\ln\frac{x+1}{x-1}$$

$$\text{arcsech } x = \ln\frac{1+\sqrt{1-x^2}}{x}$$

$$\text{arccsch } x = \ln\frac{1+\sqrt{1+x^2}}{x} \quad x > 0$$

SLOPE OF THE CURVE

The slope of the curve $y = f(x)$ at any point is identical to the derivative of the function $\frac{dy}{dx}$ or y'.

$$\text{Slope at any point} = y' = \frac{dy}{dx}$$

RATE OF CHANGE

The derivative of a function is identical to its rate of change. Thus, the rate of change of the volume (V) of a sphere with respect to its radius (r) is dV/dr.

CURVATURE AND RADIUS OF CURVATURE

CURVATURE

Curvature refers to the rate of change of the direction of the curve. Thus, a circle with a smaller radius has great curvature, or is sharply curved.

The curvature, k, of the curve $y = f(x)$ is:

$$k = \frac{y''}{[1+(y')^2]^{3/2}}$$

where y'' is the second derivative of the function, and $|y''|$ is the absolute or positive value of y''.

RADIUS OF CURVATURE

The radius of curvature, ρ, is the reciprocal of the curvature, k, or:

$$\rho = \frac{1}{k} = \frac{[1+(y')^2]^{3/2}}{|y''|}$$

Circle of Curvature

At any point on a curve $y = f(x)$, where y' and y'' exist and $y'' \neq 0$, there is associated with the curve a circle, which is called the circle of curvature with the following equation:

$$(x-h)^2 + (y-k)^2 = \rho^2$$

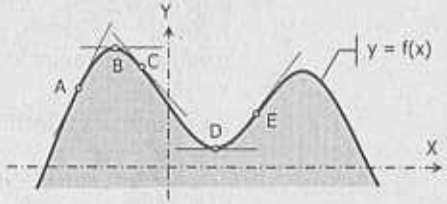
The center (h, k) and radius ρ of the curve is:

$$h = x - \frac{y'[1+(y')^2]}{y''}; \quad k = y + \frac{1+(y')^2}{y''}$$

$$\rho = \frac{[1+(y')^2]^{3/2}}{|y''|}$$

GRAPH OF A FUNCTION $y = f(x)$

The graph of a function $y = f(x)$ may be plotted using calculus. Consider the graph shown below.



As x increases, **the curve rises if the slope is positive**, as of arc AB; **it falls if the slope is negative**, as of arc BC.

Relative Maximum and Minimum Points

At a point such as B, where the function is algebraically greater than that at any neighboring point, the point is said to have a maximum value, and the point is called a maximum point (relative to adjacent points). Similarly, at D the function has a minimum value (relative to adjacent points). At maximum or minimum points, the tangent is horizontal or the slope is zero.

$$\frac{dy}{dx} = y' = 0$$

This does not necessarily mean that at these points the function is maximum or minimum. It does only mean that the tangent is parallel to the x-axis, or the curve is either *concave up* or *concave down*. The points at which $dy/dx = 0$ are called critical points, and the corresponding values of x are critical values.

The *second derivative* of a function is the *rate of change of the first derivative or slope*. It follows that as x increases and y'' is positive, y' is increasing and the tangent turns in a counterclockwise direction and the curve is concave upward. When y'' is negative, y' decreases and the tangent turns in a clockwise direction and the curve is concave downward.

If $y' = 0$ and y'' is negative (i.e. $y'' < 0$), the point is a maximum point (concave downward).

If $y' = 0$ and y'' is positive ($y'' > 0$), the point is a minimum point (concave upward).

Points of Inflection

A point of inflection is a point at which the curve changes from concave upward to concave downward, or vice verse (see point E from the figure). At these points, the tangent changes its rotation from clockwise to counterclockwise or vice versa.

At points of inflection, the *second derivative of y is zero* ($y'' = 0$)

APPLICATIONS OF MAXIMA AND MINIMA

As an example, the area of a rectangular lot, expressed in terms of its length and width, may also be expressed in terms of the cost of fencing. Thus, the area can be expressed as $A = f(x)$. The common task here is to find the value of x that will give a maximum value of A. To find this value, we set $dA/dx = 0$.

Steps in Solving Maxima Minima Problems

1. Identify the variable to be maximized or minimized, say area A.
2. Express this variable in terms of the other relevant variable(s), say $A = f(x,y)$.
3. If the function shall consist of more that one variable, expressed it in terms of one variable (if possible) using the conditions in the problem, say $A = f(x)$.
4. Differentiate and equation to zero, $dA/dx = 0$.

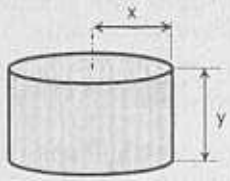
Example

A food manufacturer packages his product in 1-liter cans that are right circular cylinders. If the material for the top costs five times as much per square centimeter as that used for the bottom and side, find the dimension of the can if the cost of the material is to be minimum.

Solution

The variable to be minimized is the cost.

Let: C = total cost of materials
k = cost of materials for the sides and bottom
$5k$ = cost of materials at the top
x = radius of the cylinder
y = height of the cylinder



Cost, C = cost of top + cost of sides and bottom

$C = \pi x^2 \times 5k + [\pi x^2 + 2\pi xy] \times k$

$C = 6\pi kx^2 + 2\pi kxy$

Express y in terms of x:

Volume $= \pi x^2 y = 1$ liter $= 1000$ cm^3

$y = 1000/(\pi x^2)$

Then,

$$C = 6\pi kx^2 + 2\pi kx\frac{1000}{\pi x^2}$$

$$C = 6\pi kx^2 + \frac{2000k}{x}$$

Differentiate and equate to zero:

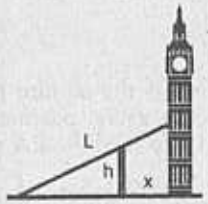
$$\frac{dC}{dx} = 12\pi kx + \frac{-2000k}{x^2} = 0$$

$$12\pi kx = \frac{2000k}{x^2}$$

$$x^3 = 53.0516$$

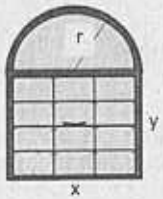
x = **3.76 cm** (radius) y = **22.55 cm** (height)

VARIABLE RELATIONSHIPS FOR MAXIMUM OR MINIMUM VALUES



Minimum length of ladder leaning against a building with one end on the ground outside the wall

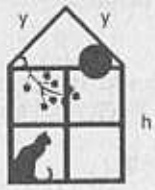
$$L^{2/3} = h^{2/3} + x^{2/3}$$



For maximum area (to admit the most light) of a Norman window of given perimeter

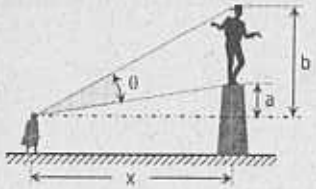
$$y = \frac{x}{2} \qquad r = \frac{x}{2}$$

$$h = x$$



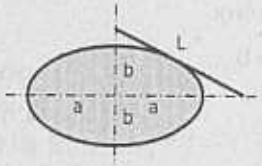
For maximum area of a rectangular window surmounted by a right isosceles triangle of known perimeter.

$$h = y$$



For maximum viewing angle θ of an object

$$x = \sqrt{ab}$$

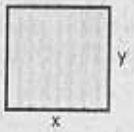


Minimum length L of a line tangent to an ellipse

$$L = a + b$$

For minimum perimeter of a rectangular lot of known area to be fenced on three sides only

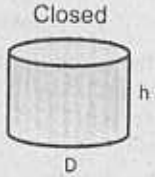
$$x = 2y$$



For minimum perimeter of a rectangle of known area or for maximum area for known perimeter

$$x = y \text{ (a square)}$$

Note: For a given area of rectangle, the square has the LEAST PERIMETER, OR for a given perimeter of rectangle, the square has the LARGEST AREA.

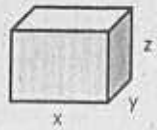


Maximum volume of a CLOSED (both ends) cylindrical tank of given surface area, or minimum surface area for given volume

Diameter = height
$D = h$

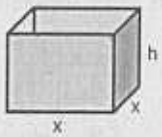
Maximum volume of a OPEN (one end) cylindrical tank of given surface area, or minimum surface area for given volume

Diameter = 2 × height
$D = 2h$



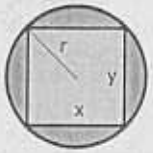
For maximum volume of a closed rectangular box with given sum of all edges or given total area

$$x = y = z \text{ (a cube)}$$



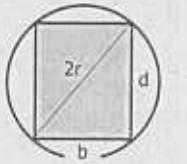
Maximum volume of open (one end) rectangular box of square base and given surface area, or minimum surface area for given volume.

$$x = 2y$$



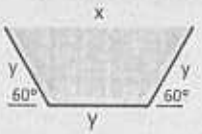
For a rectangle of maximum area or perimeter that can be cut from a circle of radius r

$$x = y \text{ (a square)}$$



For the strongest (stiffest) rectangular beam that can be cut from log of radius r

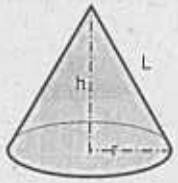
$$d = b\sqrt{3}$$



For maximum capacity of a trapezoidal gutter or canal of known perimeter, or minimum perimeter of known capacity

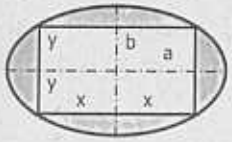
$$x = 2y$$

A half-regular hexagon



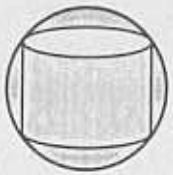
For maximum volume of a right circular cone with known lateral area, or minimum lateral area for known volume

$$h = r\sqrt{2}$$



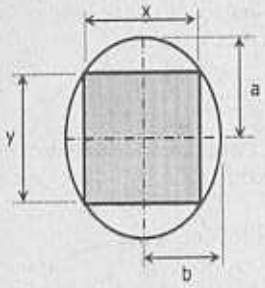
For the largest rectangle that can be inscribed in an ellipse

$$x = a/\sqrt{2}$$
$$y = b/\sqrt{2}$$



Weight of heaviest cylinder that can be cut from a sphere of weight W, or largest cylinder that can be cut from a sphere of volume V

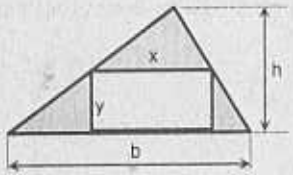
$$W_{cyl} = W/\sqrt{3}$$
$$V_{cyl} = V/\sqrt{3}$$



Strongest beam that can be cut from an elliptical section

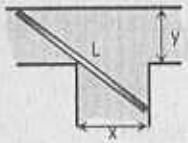
$$x = \frac{2b}{\sqrt{3}}$$

$$y = \frac{2\sqrt{2}a}{\sqrt{3}}$$



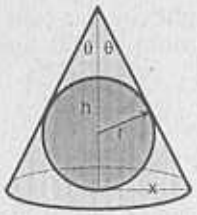
Largest rectangle that can be cut from a given triangle

$$x = b/2$$
$$y = h/2$$



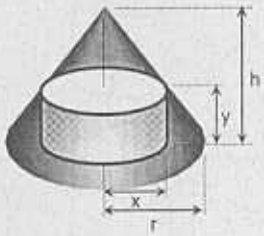
Longest beam that can pass through a corridor

$$L^{2/3} = x^{2/3} + y^{2/3}$$



Smallest cone that can inscribe a sphere of radius r

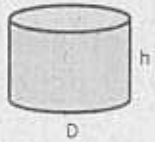
$$\sin\theta = 1/3$$



Largest cylinder that can be inscribed in a given cone

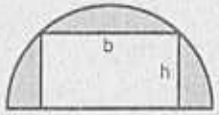
$$y = \frac{h}{3}$$

Closed



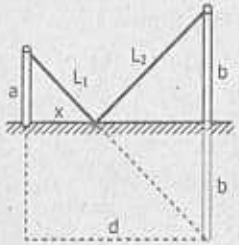
For minimum cost of closed cylindrical tank of known volume or maximum volume for a given cost.

$$\text{Height} = \frac{\text{Cost of ends}}{\text{Cost of sides}} \times \text{Base Diameter}$$



Largest rectangle that can be inscribed in a semi-circle

$$b = 2h$$



For minimum length of wire running from the top of one post to the stake on the ground and to the top of another post

$$\frac{x}{a} = \frac{d}{a+b}$$

For maximum product of n numbers whose sum is A, each number is $\frac{A}{n}$ and the maximum product is $(A/n)^n$

For maximum product of $x^n \times y^m$ where $x + y = K$:

$$x = \frac{K}{m+n}n$$

$$y = \frac{K}{m+n}m$$

TIME RATES

If a quantity x is a function of time t, the *time rate of change* of x is given by dx/dt.

When two or more quantities, all functions of t, are related by an equation, the relation between their rates of change may be obtained by differentiating both sides of the equation with respect to t.

Basic Time Rates

Velocity, $v = \dfrac{dS}{dt}$

where S is the distance

Acceleration, $a = \dfrac{dv}{dt}$

Discharge, $Q = \dfrac{dV}{dt}$

where V is the volume at any time

Angular Speed, $\omega = \dfrac{d\theta}{dt}$

where θ is the angle at any time

Problems - Set 1
Limits, Differentiation, Rate of Change, Slope

PROBLEM 1 - 1

Evaluate $\lim\limits_{\theta \to 0}(1-\sin^2\theta)^{1/2}$.

A. 0
B. 1
C. 2
D. 3

PROBLEM 1 - 2
ME April 1996

Simplify the expression: $\lim\limits_{x \to 4}\dfrac{x^2-16}{x-4}$

A. 1
B. 8
C. 0
D. 16

PROBLEM 1 - 3
CE Nov. 1997

Evaluate the following limit, $\lim\limits_{x \to 1}\dfrac{x^2-1}{x^2+3x-4}$

A. 2/5
B. infinity
C. 0
D. 5/2

PROBLEM 1 - 4
ECE April 1998

Evaluate the limit $(x - 4)/(x^2 - x - 12)$ as x approaches 4.

A. 0
B. undefined
C. 1/7
D. infinity

PROBLEM 1 - 5
ECE Nov. 1997

Evaluate the limit $(\ln x)/x$ as x approaches positive infinity.

A. 1
B. 0
C. e
D. infinity

PROBLEM 1 - 6
ME Oct. 1997

Evaluate the following limit: $\lim\limits_{x \to \infty}\dfrac{x+4}{x-4}$.

A. 1
B. indefinite
C. 0
D. 2

PROBLEM 1 - 7
EE April 1995

Evaluate: $\lim\limits_{x \to 0}\dfrac{1-\cos x}{x^2}$

A. 0
B. ½
C. 2
D. -½

PROBLEM 1 - 8
ECE Nov. 1994

Evaluate the following: $\lim\limits_{x \to 1}(2-x)^{\tan\frac{\pi x}{2}}$.

A. infinity
B. e^{π}
C. 0
D. $e^{2/\pi}$

PROBLEM 1 - 9

Find dy/dx if $y = 5^{2x+1}$.

A. 5^{2x+1} ln 5
B. 5^{2x+1} ln 25
C. 5^{2x+1} ln 10
D. 5^{2x+1} ln 2

PROBLEM 1 - 10

Find dy/dx if $y = e^{\sqrt{x}}$.

A. $e^{\sqrt{x}}/2\sqrt{x}$
B. $e^{\sqrt{x}}/\sqrt{x}$
C. $e^{x}/\sqrt{x}$
D. $e^{\sqrt{x}-2\sqrt{x}}$

PROBLEM 1 - 11

Find dy/dt if $y = x^2 + 3x + 1$ and $x = t^2 + 2$.

A. $4t^3 + 14t^2$
B. $t^3 + 4t$
C. $4t^3 + 14t$
D. $4t^3 + t$

PROBLEM 1 - 12

Evaluate the first derivative of the implicit function: $4x^2 + 2xy + y^2 = 0$

A. $\dfrac{4x+y}{x+y}$
B. $-\dfrac{4x+y}{x+y}$
C. $\dfrac{4x-y}{x+y}$
D. $-\dfrac{4x+y}{x-y}$

PROBLEM 1 - 13
ME April 1996

Find the derivative of $(x + 5)/(x^2 - 1)$ with respect to x.

A. $DF(x) = (-x^2 - 10x - 1)/(x^2 - 1)^2$
B. $DF(x) = (-x^2 + 10x - 1)/(x^2 - 1)^2$
C. $DF(x) = (x^2 - 10x - 1)/(x^2 - 1)^2$
D. $DF(x) = (-x^2 - 10x + 1)/(x^2 + 1)^2$

PROBLEM 1 - 14
ME April 1996

If a is a simple constant, what is the derivative of $y = x^a$?

A. $a\,x^{a-1}$
B. $(a - 1)x$
C. x^{a-1}
D. ax

PROBLEM 1 - 15
ME April 1996

Find the derivative of the function $2x^2 + 8x + 9$ with respect to x.

A. $Df(x) = 4x - 8$
B. $Df(x) = 2x + 9$
C. $Df(x) = 2x + 8$
D. $Df(x) = 4x + 8$

PROBLEM 1 - 16
ME April 1997

What is the first derivative dy/dx of the expression $(xy)^x = e$?

A. $-y(1 + \ln xy) / x$
B. 0
C. $-y(1 - \ln xy) / x^2$
D. y/x

PROBLEM 1 - 17
CE Nov. 1996

Find the derivative of $(x + 1)^3/x$

A. $\dfrac{3(x+1)^2}{x} + \dfrac{(x+1)^3}{x^2}$
B. $\dfrac{3(x+1)^2}{x} - \dfrac{(x+1)^3}{x^2}$
C. $\dfrac{2(x+1)^2}{x} - \dfrac{(x+1)^3}{x^2}$
D. $\dfrac{3(x+1)^2}{x} + \dfrac{2(x+1)^3}{x^2}$

PROBLEM 1 - 18
ECE Nov. 1996

Given the equation: $y = (e^{\ln x})^2$, find y'.

A. $\ln x$
B. $2 (\ln x) / x$
C. $2x$
D. $2\,e^{\ln x}$

PROBLEM 1 - 19
ME April 1998

Find the derivatives with respect to x of the function $\sqrt{2-3x^2}$.

A. $-2x^2 / \sqrt{2-3x^2}$
B. $-3x / \sqrt{2-3x^2}$
C. $-2x^2 / \sqrt{2+3x^2}$
D. $-3x / \sqrt{2-3x^2}$

PROBLEM 1 - 20
ME April 1998

Differentiate $ax^2 + b$ to the ½ power.

A. $-2ax$
B. $2ax$
C. $2ax + b$
D. $ax + 2b$

PROBLEM 1 - 21

Find dy/dx if $y = \ln\sqrt{x}$

A. $\sqrt{x} / \ln x$
B. $x / \ln x$
C. $1 / 2x$
D. $2 / x$

PROBLEM 1 - 22

Evaluate the differential of tan θ.

A. $\ln \sec\theta\, d\theta$
B. $\ln \cos\theta\, d\theta$
C. $\sec\theta \tan\theta\, d\theta$
D. $\sec^2\theta\, d\theta$

PROBLEM 1 - 23
ME April 1997

If $y = \cos x$, what is dy/dx?

A. $\sec x$
B. $-\sec x$
C. $\sin x$
D. $-\sin x$

PROBLEM 1 - 24

Find dy/dx: $y = \sin(\ln x^2)$.

A. $2 \cos(\ln x^2)$
B. $2 \cos(\ln x^2) / x$
C. $2x \cos(\ln x^2)$
D. $2 \cos(\ln x^2) / x^2$

PROBLEM 1 - 25
ECE March 1996

The derivative of ln (cos x) is:

A. $\sec x$
B. $-\sec x$
C. $-\tan x$
D. $\tan x$

PROBLEM 1 - 26
CE May 1997

Find the derivative of arccos $4x$ with respect to x.

A. -4 / [1 - (4x)^2]^2
B. -4 / [1 - (4x)]^0.5
C. 4 / [1 - (4x)^2]^0.5
C. -4 / [(4x)^2 - 1]^0.5

PROBLEM 1 - 27
CE Nov. 1997

What is the first derivative of $y = \arcsin 3x$.

A. $-\dfrac{3}{1+9x^2}$
B. $\dfrac{3}{1+9x^2}$
C. $-\dfrac{3}{\sqrt{1-9x^2}}$
D. $\dfrac{3}{\sqrt{1-9x^2}}$

PROBLEM 1 - 28
ECE Nov. 1997

If $y = x\,(\ln x)$, find d^2y/dx^2.

A. $1 / x^2$
B. $-1 / x$
C. $1 / x$
D. $-1 / x^2$

PROBLEM 1 - 29
CE May 1999

Find the second derivative of $y = x^{-2}$ at $x = 2$.
A. 96 C. -0.25
B. 0.375 D. -0.875

PROBLEM 1 - 30
ME Oct. 1997

Given the function $f(x) = x^3 - 5x + 2$, find the value of the first derivative at $x = 2$, $f'(2)$
A. 7 C. 2
B. $3x^2 - 5$ D. 8

PROBLEM 1 - 31
ME April 1998

Given the function $f(x) = x$ to the 3rd power - $6x + 2$, find the value of the first derivative at $x = 2$, $f'(2)$.
A. 6 C. 7
B. $3x^2 - 5$ D. 8

PROBLEM 1 - 32
ME April 1998

Find the partial derivatives with respect to x of the function: $xy^2 - 5y + 6$.
A. $y^2 - 5$ C. y^2
B. $xy - 5y$ D. $2xy$

PROBLEM 1 - 33
ECE April 1998

Find the point in the parabola $y^2 = 4x$ at which the rate of change of the ordinate and abscissa are equal.
A. (1, 2) C. (4, 4)
B. (2, 1) D. (-1, 4)

PROBLEM 1 - 34
ME April 1996

Find the slope of the line tangent to the curve $y = x^3 - 2x + 1$ at $x = 1$.
A. 1 C. 1/3
B. ½ D. 1/4

PROBLEM 1 - 35
CE Nov. 1998

Determine the slope of the curve $x^2 + y^2 - 6x - 4y - 21 = 0$ at (0,7).
A. 3/5 C. -3/5
B. -2/5 D. 2/5

PROBLEM 1 - 36
ME April 1996

Find the slope of the tangent to a parabola $y = x^2$ at a point on the curve where $x = 1/2$.
A. 0 C. 1/4
B. 1 D. -1/2

PROBLEM 1 - 37
CE May 1996

Find the slope of the ellipse $x^2 + 4y^2 - 10x + 16y + 5 = 0$ at the point where $y = -2 + 8^{0.5}$ and $x = 7$.
A. -0.1654 C. -0.1768
B. -0.1538 D. -0.1463

PROBLEM 1 - 38

Find the slope of the tangent to the curve $y = x^4 - 2x^2 + 8$ through point (2, 16).
A. 20 C. 24
B. 1/24 D. 1/20

PROBLEM 1 - 39

Find the slope of the tangent to the curve $y^2 = 3x^2 + 4$ through point (-2, 4)
A. -3/2 C. 2/3
B. 3/2 D. -2/3

PROBLEM 1 - 40
CE May 1998

Find the slope of the line whose parametric equations are $x = 4t + 6$ and $y = t - 1$.
A. -4 C. 4
B. ¼ D. -¼

PROBLEM 1 - 41
CE May 1998

What is the slope of the curve $x^2 + y^2 - 6x + 10y + 5 = 0$ at (1, 0).
A. 2/5 C. -2/5
B. 5/2 D. -5/2

PROBLEM 1 - 42
CE Nov. 1999

Find the slope of the curve $y = 6(4 + x)^{1/2}$ at (0, 12).
A. 0.67 C. 1.33
B. 1.5 D. 0.75

PROBLEM 1 - 43

Find the acute angle that the curve $y = 1 - 3x^3$ cut the x-axis.
A. 77° C. 79°
B. 75° D. 120°

PROBLEM 1 - 44
CE May 1999

Find the angle that the line $2y - 9x - 18 = 0$ makes with the x-axis.
A. 74.77° C. 47.77°
B. 4.5° D. 77.47°

PROBLEM 1 - 45

Find the equation of the tangent to the curve $y = x + 2x^{1/3}$ through point (8,12)
A. $7x - 6y + 14 = 0$ C. $5x - 6y - 15 = 0$
B. $8x + 5y + 21 = 0$ D. $3x - 2y - 1 = 0$

PROBLEM 1 - 46
CE Nov. 1998

What is the radius of curvature at point (1, 2) of the curve $4x - y^2 = 0$?
A. 6.21 C. 5.66
B. 5.21 D. 6.66

PROBLEM 1 - 47
ECE Nov. 1996

Find the radius of curvature at any point of the curve $y + \ln(\cos x) = 0$.
A. $\cos x$ C. $\sec x$
B. 1.5707 D. 1

PROBLEM 1 - 48
CE Nov. 1997

Determine the radius of curvature at (4, 4) of the curve $y^2 - 4x = 0$.
A. 24.4 C. 23.4
B. 25.4 D. 22.4

PROBLEM 1 - 49
CE Nov. 1999

Find the radius of curvature of the curve $x = y^3$ at (1, 1)

A. 4.72 C. 4.67
B. 3.28 D. 5.27

PROBLEM 1 - 50
CE Nov. 1998

The chords of the ellipse $64x^2 + 25y^2 = 1600$ having equal slopes of 1/5 are bisected by its diameter. Determine the equation of the diameter of the ellipse.

A. $5x - 64y = 0$ C. $5x + 64y = 0$
B. $64x - 5y = 0$ D. $64x + 5y = 0$

ANSWER SHEET

1.	A	B	C	D	E	21.	A	B	C	D	E	41.	A	B	C	D	E
2.	A	B	C	D	E	22.	A	B	C	D	E	42.	A	B	C	D	E
3.	A	B	C	D	E	23.	A	B	C	D	E	43.	A	B	C	D	E
4.	A	B	C	D	E	24.	A	B	C	D	E	44.	A	B	C	D	E
5.	A	B	C	D	E	25.	A	B	C	D	E	45.	A	B	C	D	E
6.	A	B	C	D	E	26.	A	B	C	D	E	46.	A	B	C	D	E
7.	A	B	C	D	E	27.	A	B	C	D	E	47.	A	B	C	D	E
8.	A	B	C	D	E	28.	A	B	C	D	E	48.	A	B	C	D	E
9.	A	B	C	D	E	29.	A	B	C	D	E	49.	A	B	C	D	E
10.	A	B	C	D	E	30.	A	B	C	D	E	50.	A	B	C	D	E
11.	A	B	C	D	E	31.	A	B	C	D	E	51.	A	B	C	D	E
12.	A	B	C	D	E	32.	A	B	C	D	E	52.	A	B	C	D	E
13.	A	B	C	D	E	33.	A	B	C	D	E	53.	A	B	C	D	E
14.	A	B	C	D	E	34.	A	B	C	D	E	54.	A	B	C	D	E
15.	A	B	C	D	E	35.	A	B	C	D	E	55.	A	B	C	D	E
16.	A	B	C	D	E	36.	A	B	C	D	E	56.	A	B	C	D	E
17.	A	B	C	D	E	37.	A	B	C	D	E	57.	A	B	C	D	E
18.	A	B	C	D	E	38.	A	B	C	D	E	58.	A	B	C	D	E
19.	A	B	C	D	E	39.	A	B	C	D	E	59.	A	B	C	D	E
20.	A	B	C	D	E	40.	A	B	C	D	E	60.	A	B	C	D	E

Solutions to Set 1 Limits, Differentiation, Rate of Change, Slope

SOLUTION 1 - 1
Ans: B

$$\lim_{\theta \to 0} (1 - \sin^2\theta)^{1/2} = (1 - \sin^2 0)^{1/2}$$

$$\lim_{\theta \to 0} (1 - \sin^2\theta)^{1/2} = 1$$

SOLUTION 1 - 2
Ans: B

$$\lim_{x \to 4} \frac{x^2 - 16}{x - 4} = \lim_{x \to 4} \frac{(x+4)(x-4)}{(x-4)}$$

$$\lim_{x \to 4} \frac{x^2 - 16}{x - 4} = \lim_{x \to 4} (x+4) = 4 + 4 = 8$$

Another Solution:

$$\lim_{x \to 4} \frac{x^2 - 16}{x - 4} = \frac{0}{0} \text{ (Apply L'Hospital's Rule)}$$

$$\lim_{x \to 4} \frac{x^2 - 16}{x - 4} = \lim_{x \to 4} \frac{2x}{1} = \frac{2(4)}{1} = 8$$

Another Solution (Gillesania's Principle®)

Set $x = 3.9999$

$$\frac{3.9999^2 - 16}{3.9999 - 4} = 7.9999 \approx 8$$

Note: Gillesania's Principle can be applied to any type of problem. This is based from the fact that "as x **approaches** 4 ($x \to 4$)" or very near 4, we substitute a number that is very close to 4. You may try 3.9999 or 4.00001. The nearer is your assumed value to 4 is, the more accurate is the result.

Here are some values to assume:

$x \to 0$	Set $x = 0.00001$
$x \to 1$	Set $x = 0.99999$ or 1.00001
$x \to 5$	Set $x = 4.99999$ or 5.00001
$x \to -3$	Set $x = -2.99999$ or -3.00001
$x \to \infty$	Set $x = 999999$

SOLUTION 1 - 3
Ans: A

$$\lim_{x\to 1}\frac{x^2-1}{x^2+3x-4} = \lim_{x\to 1}\frac{(x+1)(x-1)}{(x-1)(x+4)}$$
$$= \lim_{x\to 1}\frac{(x+1)}{(x+4)} = \frac{1+1}{1+4} = \frac{2}{5}$$

Another Solution

$$\lim_{x\to 1}\frac{x^2-1}{x^2+3x-4} = \frac{0}{0} \text{ (Apply L'Hospital's Rule)}$$
$$\lim_{x\to 1}\frac{x^2-1}{x^2+3x-4} = \lim_{x\to 1}\frac{2x}{2x+3} = \frac{2(1)}{2(1)+3} = \frac{2}{5}$$

Another Solution: (Gillesania's Principle®)
Set $x = 0.9999$

$$\frac{0.9999^2-1}{0.9999^2+3(0.9999)-4} = 0.39999 \approx 0.4 = \mathbf{2/5}$$

SOLUTION 1 - 4
Ans: C

$$\lim_{x\to 4}\frac{x-4}{x^2-x-12} = \lim_{x\to 4}\frac{x-4}{(x-4)(x+3)}$$
$$= \lim_{x\to 4}\frac{1}{x+3} = \frac{1}{4+3} = \mathbf{1/7}$$

Another Solution

$$\lim_{x\to 4}\frac{x-4}{x^2-x-12} = \frac{0}{0} \text{ (Apply L'Hospital's Rule)}$$
$$\lim_{x\to 4}\frac{x-4}{x^2-x-12} = \lim_{x\to 4}\frac{1}{2x-1} = \frac{1}{2(4)-1} = \frac{1}{7}$$

Another Solution: (Gillesania's Principle®)
Set $x = 3.9999$ (approaches 4)

$$\frac{3.9999-4}{3.9999^2-3.9999-12} = 0.14286 = 1/7$$

SOLUTION 1 - 5
Ans: B

$$\lim_{x\to\infty}\frac{\ln x}{x}$$

Set $x = 999{,}999$ (Gillesania's Principle®)

$$\frac{\ln 999{,}999}{999{,}999} \approx 0$$

SOLUTION 1 - 6
Ans: A

$$\lim_{x\to\infty}\frac{x+4}{x-4} = \lim_{x\to\infty}\frac{x+4}{x-4}\times\frac{1/x}{1/x}$$
$$= \lim_{x\to\infty}\frac{1+4/x}{1-4/x} = \frac{1+0}{1+0} = \mathbf{1}$$

Another Solution: (Gillesania's Principle®)
Set $x = 999{,}999$

$$\frac{999999+4}{999999-4} = 1$$

SOLUTION 1 - 7
Ans: B

$$\lim_{x\to 0}\frac{1-\cos x}{x^2} = \frac{0}{0} \text{ (Apply L'Hospital's Rule)}$$

$$\lim_{x\to 0}\frac{1-\cos x}{x^2} = \lim_{x\to 0}\frac{\sin x}{2x} = \frac{0}{0} \text{ (Apply L'Hospital's Rule again)}$$

$$\lim_{x\to 0}\frac{\sin x}{2x} = \frac{\cos x}{2} = \frac{\cos 0^\circ}{2} = \mathbf{1/2}$$

Another Solution: (Gillesania's Principle®)
Set $x = 0.0001$

$$\frac{1-\cos x}{x^2} = \frac{1-\cos(0.0001\times\frac{180^\circ}{\pi})}{(0.0001)^2} = 0.5$$

SOLUTION 1 - 8
Ans: D

$$\lim_{x\to 1}(2-x)^{\tan\frac{\pi x}{2}} = 1^\infty \text{ (indeterminate)}$$

$$\text{limit} = (2-x)^{\tan\frac{\pi x}{2}}$$

Take the natural logarithm (ln) of both sides

$$\ln \text{limit} = \tan\frac{\pi x}{2}\ \ln(2-x) = \frac{\ln(2-x)}{\cot\frac{\pi x}{2}}$$

As x approaches 1, $\dfrac{\ln(2-x)}{\cot\frac{\pi x}{2}} = \dfrac{0}{0}$ (Apply L'Hospital's Rule)

$$\ln \text{limit} = \frac{\frac{-1}{2-x}}{-\csc^2\frac{\pi x}{2}\times\frac{\pi}{2}} = \frac{\frac{-1}{2-1}}{-\csc^2\frac{\pi(1)}{2}\times\frac{\pi}{2}} = \frac{-1}{(-1)\frac{\pi}{2}}$$

$$\ln \text{limit} = \frac{2}{\pi}$$

Taking the exponential of both sides:

$$e^{\ln \text{limit}} = e^{2/\pi}$$

$$\text{limit} = e^{2/\pi}$$

Another Solution: (Gillesania's Principle®)

Set $x = 0.9999$

$$\lim_{x\to 1} (2-x)^{\tan\frac{\pi x}{2}} = (2-0.9999)^{\tan[(\pi(0.9999)/2)\times 180/\pi]}$$

$$\lim_{x\to 1} (2-x)^{\tan\frac{\pi x}{2}} = 1.89 = e^{2/\pi}$$

SOLUTION 1 - 9
Ans: B

From $d(a^u) = a^u \ln a\, du$

$$y = 5^{2x+1}$$

$$dy/dx = 5^{2x+1} \ln 5\,(2) = 5^{2x+1} \ln 25$$

SOLUTION 1 - 10
Ans: A

$y = e^{\sqrt{x}}$; From $d(e^u) = e^u\, du$

$$dy/dx = e^{\sqrt{x}}\, d(\sqrt{x}) = e^{\sqrt{x}} \frac{1}{2\sqrt{x}} = \frac{e^{\sqrt{x}}}{2\sqrt{x}}$$

SOLUTION 1 - 11
Ans: C

$y = x^2 + 3x;$ $\quad dy/dx = 2x + 3$
$x = t^2 + 2;$ $\quad dx/dt = 2t$

$$(dy/dx)(dx/dt) = (2x+3)(2t)$$

but $x = t^2 + 2$

$$dy/dt = [2(t^2+2)+3](2t) = (2t^2+7)(2t)$$
$$dy/dt = \mathbf{4t^3 + 14t}$$

SOLUTION 1 - 12
Ans: B

$$4x^2 + 2xy + y^2 = 0$$

First derivative:

$$8x + 2xy' + 2y + 2yy' = 0$$
$$4x + xy' + + y + yy' = 0$$
$$xy' + yy' = -4x - y$$
$$y'(x+y) = -(4x+y)$$
$$\frac{dy}{dx} = y' = -\frac{4x+y}{x+y}$$

SOLUTION 1 - 13
Ans: A

Let $y = \dfrac{x+5}{x^2-1}$

$$\frac{dy}{dx} = \frac{(x^2-1)(1)-(x+5)(2x)}{(x^2-1)^2}$$

$$\frac{dy}{dx} = \frac{x^2-1-2x^2-10x}{(x^2-1)^2} = \frac{-x^2-10x-1}{(x^2-1)^2}$$

SOLUTION 1 - 14
Ans: A

$y = x^a;\ \dfrac{dy}{dx} = a\,x^{a-1}$

SOLUTION 1 - 15
Ans: D

$f(x) = 2x^2 + 8x + 0$
$Df(x) = \mathbf{4x + 8}$

SOLUTION 1 - 16
Ans: A

$(xy)^x = e$

Taking the natural logarithm of both side:

$$\ln e = \ln (xy)^x = x \ln (xy)$$
$$1 = x(\ln x + \ln y)$$
$$1 = x \ln x + x \ln y$$

Differentiate with respect to x:

$$0 = x\frac{1}{x} + \ln x + x\frac{dy/dx}{y} + \ln y$$

$$\frac{x}{y}\frac{dy}{dx} = -1 - \ln x - \ln y = -1 - \ln xy$$

$$\frac{dy}{dx} = \mathbf{-y(1 + \ln xy)/x}$$

SOLUTION 1 - 17
Ans: B

From $d\left(\dfrac{u}{v}\right) = \dfrac{v\,du - u\,dv}{v^2}$

$$\frac{d\left[\dfrac{(x+1)^3}{x}\right]}{dx} = \frac{x[3(x+1)^2(1)]-(x+1)^3(1)}{x^2}$$

$$= \frac{3(x+1)^2}{x} - \frac{(x+1)^3}{x^2}$$

SOLUTION 1 - 18
Ans: C

$y = (e^{\ln x})^2$

By definition, $e^{\ln u} = u$

$$y = (x)^2 = x^2$$
$$y' = \mathbf{2x}$$

SOLUTION 1 - 19
Ans: B

$f(x) = \sqrt{2-3x^2}$

$$\frac{d\,f(x)}{dx} = \frac{\dfrac{d(2-3x^2)}{dx}}{2\sqrt{2-3x^2}} = \frac{-6x}{2\sqrt{2-3x^2}} = \frac{-3x}{\sqrt{2-3x^2}}$$

SOLUTION 1 - 20
Ans: B

Let $y = ax^2 + b^{1/2}$
$dy/dx = a(2x) + 0 = 2ax$

SOLUTION 1 - 21
Ans: C

$$y = \ln\sqrt{x}\,;\ dy/dx = \frac{d(\sqrt{x})/dx}{\sqrt{x}} = \frac{\dfrac{1}{2\sqrt{x}}}{\sqrt{x}} = \frac{1}{2x}$$

SOLUTION 1 - 22
Ans: D

$d(\tan\theta) = \sec^2\theta\, d\theta$

SOLUTION 1 - 23
Ans: D

$y = \cos x\,;\ \dfrac{dy}{dx} = -\sin x$

SOLUTION 1 - 24
Ans: B

$y = \sin(\ln x^2)$
$dy/dx = \cos(\ln x^2) \times d(\ln x^2) = \cos(\ln x^2) \times (2x/x^2)$
$dy/dx = 2\cos(\ln x^2)/x$

SOLUTION 1 - 25
Ans: C

From the formula:

$$\frac{d(\ln u)}{dx} = \frac{du/dx}{u}$$

$u = \cos x$

$$\frac{d[\ln(\cos x)]}{dx} = \frac{d(\cos x)/dx}{\cos x} = \frac{-\sin x}{\cos x} = -\tan x$$

SOLUTION 1 - 26
Ans: B

From $d(\arccos u) = -\dfrac{du}{\sqrt{1-u^2}}$

$$\frac{d(\arccos 4x)}{dx} = -\frac{4}{\sqrt{1-(4x)^2}}$$

SOLUTION 1 - 27
Ans: D

$y = \arcsin 3x$

From $d(\arcsin u) = \dfrac{du}{\sqrt{1-u^2}}$

$$\frac{dy}{dx} = \frac{3}{\sqrt{1-9x^2}}$$

SOLUTION 1 - 28
Ans: C

$y = x\ln x$
$y' = x(1/x) + \ln x(1) = 1 + \ln x$
$y'' = 0 + 1/x = 1/x$

SOLUTION 1 - 29
Ans: B

$y = x^{-2}$
$y' = -2x^{-3}\,;\ y'' = 6x^{-4}\,;$
At $x = 2$: $y'' = 6(2)^{-4} = 0.375$

SOLUTION 1 - 30
Ans: A

$f(x) = x^3 - 5x + 2$
$f'(x) = 3x^2 - 5;\ f'(2) = 3(2)^2 - 5 = 7$

SOLUTION 1 - 31
Ans: A

$f(x) = x^3 - 6x + 2$
$f'(x) = 3x^2 - 6$
At $x = 2$: $f'(2) = 3(2)^2 - 6 = 12 - 6 = 6$

SOLUTION 1 - 32
Ans: A

$f(xy) = xy^2 - 5y + 6$

To get the partial derivative with respect to x, we differentiate the function treating the other variable y as constant.

$$\frac{\partial f(xy)}{\partial x} = y^2 - 5$$

SOLUTION 1 - 33
Ans: A

The rate of change of the ordinate and abscissa is dy/dx.
$y^2 = 4x$
$2y\,dy/dx = 4$; where $dy/dx = 1$
$2y(1) = 4$
$y = 2$ and $x = 1$; The point is **(1, 2)**

SOLUTION 1 - 34
Ans: A

$y = x^3 - 2x + 1$

Slope, $dy/dx = 3x^2 - 2$

At $x = 1$: Slope, $3(1)^2 - 2 = 1$

SOLUTION 1 - 35
Ans: A

$x^2 + y^2 - 6x - 4y - 21 = 0$
$2x + 2y\,y' - 6 - 4\,y' = 0$

At (0, 7) $2(0) + 2(7)\,y' - 6 - 4\,y' = 0$
$y' = 6/10 = 3/5$

SOLUTION 1 - 36
Ans: B

$y = x^2$
Slope of the curve at any point = $y' = 2x$
At $x = 1/2$
$y' = 2(1/2) = 1$

SOLUTION 1 - 37
Ans: C

Slope = dy/dx
$x^2 + 4y^2 - 10x + 16y + 5 = 0$
$2x + 8y\,(dy/dx) - 10 + 16\,(dy/dx) = 0$

At $(7, 2 + 8^{0.5})$
$2(7) + 8(-2 + 8^{0.5})\,dy/dx - 10 + 16\,(dy/dx) = 0$
$dy/dx = -0.1768$

SOLUTION 1 - 38
Ans: C

The slope of the curve $y = f(x)$ at any point is dy/dx.

$y = x^4 - 2x^2 + 8; \quad y' = 4x^3 - 4x$

Since (8, 12) lies on the given curve, the:
Slope = $y' = 4(8)3 - 4(8) = 24$

SOLUTION 1 - 39
Ans: A

The slope of the curve $y = f(x)$ at any point is dy/dx.
$y^2 = 3x^2 + 4$
$2y\,y' = 6x; \quad y' = 3x / y$

At (-2, 4): Slope = $y' = 3(-2) / 4 = -3/2$

SOLUTION 1 - 40
Ans: B

The slope of the line m is equal to dy/dx
$y = t - 1;\ dy = dt$
$x = 4t + 6,\ dx = 4dt$
Slope, $m = dt / (4dt) = ¼$

SOLUTION 1 - 41
Ans: A

$x^2 + y^2 - 6x + 10y + 5 = 0$
$2x + 2y\,y' - 6 + 10y' + 0 = 0$

@(1, 0)
$2(1) + 2(0)\,y' - 6 + 10y' = 0$
$y' = 4/10 = 2/5$

SOLUTION 1 - 42
Ans: B

$y = 6(4 + x)^{1/2}; \quad y' = 3(4 + x)^{-1/2}$

At (0, 12): $y' = 3(4 + 0)^{-1/2} = 1.5$

SOLUTION 1 - 43
Ans: A

$y = 1 - 3x^3$

$\dfrac{dy}{dx} = -9x^2$ = slope of the curve at any point = tan θ

Where θ is the angle that the curve makes with the horizontal.

The curve will cut the x-axis at $(x,0)$
$0 = 1 - 3x^3;\ x = 0.693$
$\tan\theta = -9(0.693)^2;\ \theta = 76.99°$

SOLUTION 1 - 44
Ans: D

The tangent of the angle that the line curve makes with the x-axis is the slope of the curve.

$\tan\theta$ = slope = y'
$2y - 9x - 18 = 0$
$2y' - 9 = 0;\ y' = 4.5$

$\tan\theta = 4.5;\ \theta = 77.47°$

SOLUTION 1 - 45
Ans: A

The equation of the tangent is $y - y_1 = m(x - x_1)$

Solving for the slope m (y') at (8, 12)
$y = x + 2x^{1/3}$
slope = $y' = 1 + (2/3)x^{-2/3}$
$m = 1 + (2/3)(8)^{-2/3} = 7/6$

Equation of line:
$y - 12 = (7/6)(x - 8)$
$6y - 72 = 7x - 58$
$7x - 6y + 14 = 0$

SOLUTION 1 - 46
Ans: C

$4x - y^2 = 0$

$4 - 2y\,y' = 0;\ y' = \dfrac{2}{y};\ y'' = -\dfrac{2y'}{y^2}$

At (1, 2): $y' = 2/2 = 1$
$y'' = -2(1)/2^2 = -1/2$

$$R = \frac{(1 + y'^2)^{3/2}}{|y''|} = \frac{\left[1 + (1)^2\right]^{3/2}}{|-1/2|} = 5.66$$

SOLUTION 1 - 47
Ans: C

For the curve $y = f(x)$, the radius of curvature at any point is:

$$R = \frac{(1 + y'^2)^{3/2}}{|y''|}$$

$y + \ln\cos x = 0;\ \ y = -\ln\cos x = \ln\sec x$

$$y' = \frac{\sec x \tan x}{\sec x} = \tan x$$

$y'' = \sec^2 x$

$$R = \frac{(1 + \tan^2 x)^{3/2}}{\sec^2 x} = \frac{(\sec^2 x)^{3/2}}{\sec^2 x}$$

$$R = \frac{\sec^3 x}{\sec^2 x} = \sec x$$

SOLUTION 1 - 48
Ans: D

Radius of curvature, $R = \dfrac{[1+(y')^2]^{3/2}}{|y''|}$

$y^2 - 4x = 0$
$2y\,y' - 4 = 0$

$y' = 2/y$ and $y'' = \dfrac{-2\,y'}{y^2}$

@ (4, 4)
$y' = 2/4 = ½$
$y'' = -2(1/2) / 4^2 = -1/16$

$R = \dfrac{[1+(1/2)^2]^{3/2}}{|-1/16|} = \mathbf{22.36\ units}$

SOLUTION 1 - 49
Ans: D

$R = \dfrac{[1+(y')^2]^{3/2}}{|y''|}$

$x = y^3;\ \ y = x^{1/3}$
$y' = \dfrac{1}{3}x^{-2/3};\ \ y'' = -\dfrac{2}{9}x^{-5/3}$

At (1, 1)
$y' = 1/3$ and $y'' = -2/9$

$R = \dfrac{[1+(1/3)^2]^{3/2}}{|-2/9|} = \mathbf{5.27}$

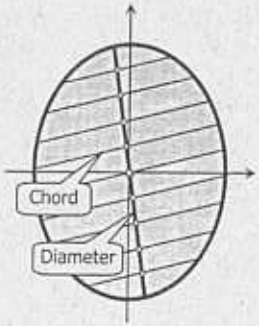
SOLUTION 1 - 50
Ans: D

$64x^2 + 25y^2 = 1600$

Differentiate with respect to x:
$128\,x + 50\,y\,y' = 0$

Where y' = slope of the chords
$y' = 1/5$

$128\,x + 50\,y\,(1/5) = 0$
$64\,x + 5\,y = 0$



Problems - Set 2
Maxima & Minima, Time Rates

PROBLEM 2 - 1
ME Oct. 1997

A function is given below, what x value maximizes y?

$$y^2 + y + x^2 - 2x = 5$$

A. 2.23 C. 5
B. -1 D. 1

PROBLEM 2 - 2
CE May 1999

The number of newspaper copies distributed is given by $C = 50\,t^2 - 200\,t + 10000$, where t is in years. Find the minimum number of copies distributed from 1995 to 2002.

A. 9850 C. 10200
B. 9800 D. 7500

PROBLEM 2 - 3
ME Oct. 1996

Given the following profit-versus-production function for a certain commodity:

$$P = 200000 - x - \left(\frac{1.1}{1+x}\right)^8$$

Where P is the profit and x is unit of production. Determine the maximum profit.

A. 190000 C. 250000
B. 200000 D. 550000

PROBLEM 2 - 4
ME April 1996

The cost C of a product is a function of the quantity x of the product is given by the relation: $C(x) = x^2 - 4000x + 50$. Find the quantity for which the cost is a minimum.

A. 3000 C. 1000
B. 2000 D. 1500

PROBLEM 2 - 5
ME April 1998

If $y = x$ to the 3rd power - $3x$. Find the maximum value of y.

A. 0 C. 1
B. -1 D. 2

PROBLEM 2 - 6

Divide 120 into two parts so that the product of one and the square of the other is maximum. Find the numbers.

A. 60 & 60 C. 70 & 50
B. 100 & 20 D. 80 & 40

PROBLEM 2 - 7

If the sum of two numbers is C, find the minimum value of the sum of their squares.

A. $C^2/2$ C. $C^2/6$
B. $C^2/4$ D. $C^2/8$

PROBLEM 2 - 8 A certain travel agency offered a tour that will cost each person P 1500.00 if not more than 150 persons will join, however the cost per person will be reduced by P 5.00 per person in excess of 150. How many persons will make the profit a maximum?

A. 75 C. 225
B. 150 D. 250

PROBLEM 2 - 9 Two cities A and B are 8 km and 12 km, respectively, north of a river which runs due east. City B being 15 km east of A. A pumping station is to be constructed (along the river) to supply water for the two cities. Where should the station be located so that the amount of pipe is a minimum?

A. 3 km east of A C. 9 km east of A
B. 4 km east of A D. 6 km east of A

PROBLEM 2 - 10 A boatman is at A, which is 4.5 km from the nearest point B on a straight shore BM. He wishes to reach, in minimum time, a point C situated on the shore 9 km from B. How far from C should he land if he can row at the rate of 6 kph and walk at the rate of 7.5 kph?

A. 1 km C. 5 km
B. 3 km D. 8 km

PROBLEM 2 - 11 The shortest distance from the point (5,10) to the curve $x^2 = 12y$ is:

A. 4.331 C. 5.127
B. 3.474 D. 6.445

PROBLEM 2 - 12
ECE April 1998
A statue 3 m high is standing on a base of 4 m high. If an observer's eye is 1.5 m above the ground, how far should he stand from the base in order that the angle subtended by the statue is a maximum?

A. 3.41 m C. 3.71 m
B. 3.51 m D. 4.41 m

PROBLEM 2 - 13 An iron bar 20 m long is bent to form a closed plane area. What is the largest area possible?

A. 21.56 square meter C. 28.56 square meter
B. 25.68 square meter D. 31.83 square meter

PROBLEM 2 - 14
CE Nov. 1996
A Norman window is in the shape of a rectangle surmounted by a semi-circle. What is the ratio of the width of the rectangle to the total height so that it will yield a window admitting the most light for a given perimeter?

A. 1 C. 1/3
B. 2/3 D. ½

PROBLEM 2 - 15
ECE Nov. 1996
A rectangular field is to be fenced into four equal parts. What is the size of the largest field that can be fenced this way with a fencing length of 1500 feet if the division is to be parallel to one side?

A. 65,200 C. 64,500
B. 62,500 D. 63,500

PROBLEM 2 - 16
ECE Nov. 1997
Three sides of a trapezoid are each 8 cm long. How long is the 4th side, when the area of the trapezoid has the greatest value?

A. 16 cm C. 12 cm
B. 15 cm D. 10 cm

PROBLEM 2 - 17 An open top rectangular tank with square bases is to have a volume of 10 cubic meters. The material for its bottom cost P 150.00 per square meter, and that for the sides is P 60.00 per square meter. The most economical height is:

A. 2 meters C. 3 meters
B. 2.5 meters D. 3.5 meters

PROBLEM 2 - 18
CE May 1999
A rectangular box having a square base and open at the top is to have a capacity of 16823 cc. Find the height of the box to use the least amount of material.

A. 16.14 cm C. 18.41 cm
B. 32.28 cm D. 28.74 cm

PROBLEM 2 - 19
ECE March 1996, Nov. 1998
The altitude of a cylinder of maximum volume that can be inscribed in a right circular cone of radius r and height h is:

A. $h/3$ C. $3h/2$
B. $2h/3$ D. $h/4$

PROBLEM 2 - 20
CE May 1997
What is the least amount of tin sheet, in sq. inches, that can be made into a closed cylindrical can having a volume of 108 cu. inches?

A. 125 square meter C. 150 square meter
B. 137 square meter D. 120 square meter

PROBLEM 2 - 21
CE May 1998, Nov. 1999
The volume of the closed cylindrical tank is 11.3 cubic meter. If the total surface area is a minimum, what is its base radius, in m?

A. 1.44 C. 1.22
B. 1.88 D. 1.66

PROBLEM 2 - 22 A cylindrical steam boiler is to be constructed having a capacity of 1000 cu. m. The material for the sides cost P2000.00 per square meter and for the ends P3000.00 per square meter. Find the radius so that the cost is least.

A. 3.52 m C. 4.73 m
B. 4.12 m D. 5.25 m

PROBLEM 2 - 23
ME April 1998

A box is to be constructed from a piece of zinc 20 inches square by cutting equal squares from each corner and turning up the zinc to form the side. What is the volume of the largest box that can be so constructed?
A. 599.95 cubic inches C. 592.59 cubic inches
B. 579.50 cubic inches D. 622.49 cubic inches

PROBLEM 2 - 24

A load of 40 kN is to be raised by means of a lever weighing 250 N/m, which is supported at one end. If the load is placed 1 m from the support, how long should the lever be so that the force required be a minimum?
A. 13.43 m C. 18.56 m
B. 20.19 m D. 17.89 m

PROBLEM 2 - 25
ME April 1998

As x increases uniformly at the rate of 0.002 feet per second, at what rate is the expression $(1 + x)$ to the 3rd power increasing when x becomes 8 feet?
A. 430 cfs C. 0.486 cfs
B. 0.300 cfs D. 0.346 cfs

PROBLEM 2 - 26
ME April 1998

The distance a body travels is a function of time and is given by $x(t) = 16t + 8t^2$. Find its velocity at $t = 3$.
A. 64 C. 54
B. 56 D. 44

PROBLEM 2 - 27

The distance traveled by a train is given by the equation $x = 3t^2 + 2t + 4$ where x is the distance in kilometers and t is the time in hours. Determine the velocity of the train after traveling 10 kilometers.
A. 8.72 kph C. 23.5 kph
B. 18.72 kph D. 75.3 kph

PROBLEM 2 - 28
CE May 1998

An object moves along a straight line such that, after t minutes, its distance from its starting point is $D = 20t + 5 / (t + 1)$ meters. At what speed, in m/minute will it be moving at the end of 4 minutes?
A. 39.8 C. 29.8
B. 49.8 D. 19.8

PROBLEM 2 - 29
CE May 1996

The speed of the traffic flowing past a certain downtown exit between the hours of 1:00 P.M. and 6:00 P.M. is approximately $V = t^3 - 10.5t^2 + 30t + 20$ miles per hour, where t = number of hours past noon. What is the fastest speed of the traffic between 1:00 P.M. and 6:00 P.M. in mph?

A. 50 C. 32.5
B. 46 D. 52

PROBLEM 2 - 30

A van is 5 km due north of a bus at 2:00 p.m. If the van is traveling northward at the rate of 60 kph and the bus is traveling westward at the rate of 75 kph, how fast will the two be separating at 5:00 p.m.?
A. 85.36 kph C. 98.65 kph
B. 96.04 kph D. 102.54 kph

PROBLEM 2 - 31
CE May 1997

A car drives east from point A at 30 kph. Another car starting from B at the same time, drives S 30° W toward A at 60 kph. B is 30 km away from A. How fast in kph is the distance between the two cars changing after one hour?
A. 76.94 kph C. 75.94 kph
B. 78.94 kph D. 77.94 kph

PROBLEM 2 - 32
CE Nov. 1996

A car starting at 12:00 noon travels west at a speed of 30 kph. Another car starting from the same point at 2:00 pm travels north at 45 kph. Find how fast the two are separating at 4:00 pm?
A. 55 C. 51
B. 57 D. 53

PROBLEM 2 - 33
CE May 1996

Two railroad tracts are perpendicular with each other. At 12:00 P.M. there is a train on each track approaching the crossing at 50 kph, one being 100 km., the other 150 km. away from the crossing. How fast in kph is the distance between the two trains changing at 4:00 P.M.?
A. 68.08 C. 69.08
B. 67.08 D. 70.08

PROBLEM 2 - 34
ECE April 1998

A balloon is rising vertically over a point A on the ground at the rate of 15 ft/sec. A point B on the ground is level with and 30 ft. from A. When the balloon is 40 ft. from A, at what rate is its distance from B changing?
A. 10 ft/s C. 13 ft/s
B. 12 ft/s D. 15 ft/s

PROBLEM 2 - 35

A hemispherical dome has a diameter of 100m. A searchlight was placed at a point A located at the circumference at the base. At the middle of the dome at B, a balloon was released vertically at a velocity of 4 m/s. How fast is the shadow of the balloon move along the roof when the balloon is 25 m high?
A. 6.4 m/s C. 7.2 m/s
B. 4.6 m/s D. 4 m/s

PROBLEM 2 - 36
If the distance y from the origin at time t is given by $y = -16t^2 + 3000t + 50000$, find the initial velocity when $t = 0$.
A. 0 C. 5300
B. 3000 D. 50000

PROBLEM 2 - 37
An airplane is flying horizontally at an altitude of 9000 m. An observer on the ground noticed that when the angle of elevation of the plane is 60°, the angle decreases at the rate of 0.15 radians/second. What is the velocity of the airplane?
A. 600 m/s C. 1400 m/s
B. 900 m/s D. 1800 m/s

PROBLEM 2 - 38
The surface area of the sphere (initially zero) increases uniformly at the rate of 26 sq. cm. per second. Find the rate at which the radius is increasing after two seconds.
A. 0.59 cm/sec C. 0.509 cm/sec
B. 0.62 cm/sec D. 0.52 cm/sec

PROBLEM 2 - 39
ECE Nov. 1995
The height of a right circular cylinder is 50 inches and decreases at the rate of 4 inches per second, while the radius of the base is 20 inches and increases at the rate of one inch per second. At what rate is the volume changing?
A. 11310 cu. in./sec C. 11130 cu. in./sec
B. 1275 cu. in./sec D. 1257 cu. in./sec

PROBLEM 2 - 40
CE Nov. 1998
There is a constant inflow of a liquid into a conical vessel 15 feet deep and 7.5 feet in diameter at the top. Water is rising at the rate of 2 feet per minute when the water is 4 feet deep. What is the rate if inflow in cu. ft. per minute?
A. 8.14 C. 9.33
B. 7.46 D. 6.28

PROBLEM 2 - 41
CE May 2003
What is the curvature of the curve $y^2 = 16x$ at the point (4, 8)?
A. -0.044 C. -0.066
B. -0.088 D. -0.033

PROBLEM 2 - 42
CE Nov. 2002
Suppose that x years after founding in 1975, a certain employee association had a membership of $f(x) = 100(2x^3 - 45x^2 + 264x)$, at what time between 1975 and 1989 was the membership smallest?
A. 1983 C. 1984
B. 1985 D. 1986

PROBLEM 2 - 43
CE Nov. 2002
A 3-meter long steel pipe has its upper end leaning against a vertical wall and lower end on a level ground. The lower end moves away at a constant rate of 2 cm/s.

How fast is the upper end moving down, in cm/s, when the lower end is 2 m from the wall?
A. 1.81 C. 1.79
B. 1.66 D. 1.98

PROBLEM 2 - 44
CE May 2002
A particle moves according to the parametric equations:
$$y = 2t^2$$
$$x = t^3$$
where x and y are displacements (in meters) in x and y direction, respectively and t is time in seconds. Determine the acceleration of the body after $t = 3$ seconds.
A. 12.85 m/s² C. 21.47 m/s²
B. 18.44 m/s² D. 5.21 m/s²

PROBLEM 2 - 45
CE May 2002
Determine the shortest distance from point (4, 2) to the parabola $y^2 = 8x$.
A. 2.83 C. 2.41
B. 3.54 D. 6.32

PROBLEM 2 - 46
CE Nov. 2001
Water flows into a tank having the form of a frustum of a right circular cone. The tank is 4 m tall with upper radius of 1.5 m and the lower radius of 1 m. When the water in the tank is 1.2 m deep, the surface rises at the rate of 0.012 m/s. Calculate the discharge of water flowing into the tank in m³/s.
A. 0.02 C. 0.08
B. 0.05 D. 0.12

PROBLEM 2 - 47
CE Nov. 2003
The motion of a particle is defined by the parametric equation $x = t^3$ and $y = 2t^2$. Determine the velocity when $t = 2$.
A. 14.42 C. 12.74
B. 16.25 D. 18.63

PROBLEM 2 - 48
CE Nov. 2003
The sum of two numbers is K. The product of one by the cube of the other is to be a maximum. Determine one of the numbers.
A. 3K/4 C. 3K/2
B. 3K/8 D. 3K/7

PROBLEM 2 - 49
Find the height of the largest right pyramid with a square base that can be inscribed in a sphere of radius 10 cm.
A. 12.2 cm C. 13.3 cm
B. 14.4 cm D. 15.5 cm

PROBLEM 2 - 50 Sand is pouring from a spout at the rate of 25 cc/sec. It forms a cone whose height is always 1/3 the radius of its base. At what rate in cm/sec is the height increasing when the cone is 50 cm high?

A. 0.000785 C. 0.000447
B. 0.000214 D. 0.000354

ANSWER SHEET

1.	A	B	C	D	E	21.	A	B	C	D	E	41.	A	B	C	D	E
2.	A	B	C	D	E	22.	A	B	C	D	E	42.	A	B	C	D	E
3.	A	B	C	D	E	23.	A	B	C	D	E	43.	A	B	C	D	E
4.	A	B	C	D	E	24.	A	B	C	D	E	44.	A	B	C	D	E
5.	A	B	C	D	E	25.	A	B	C	D	E	45.	A	B	C	D	E
6.	A	B	C	D	E	26.	A	B	C	D	E	46.	A	B	C	D	E
7.	A	B	C	D	E	27.	A	B	C	D	E	47.	A	B	C	D	E
8.	A	B	C	D	E	28.	A	B	C	D	E	48.	A	B	C	D	E
9.	A	B	C	D	E	29.	A	B	C	D	E	49.	A	B	C	D	E
10.	A	B	C	D	E	30.	A	B	C	D	E	50.	A	B	C	D	E
11.	A	B	C	D	E	31.	A	B	C	D	E	51.	A	B	C	D	E
12.	A	B	C	D	E	32.	A	B	C	D	E	52.	A	B	C	D	E
13.	A	B	C	D	E	33.	A	B	C	D	E	53.	A	B	C	D	E
14.	A	B	C	D	E	34.	A	B	C	D	E	54.	A	B	C	D	E
15.	A	B	C	D	E	35.	A	B	C	D	E	55.	A	B	C	D	E
16.	A	B	C	D	E	36.	A	B	C	D	E	56.	A	B	C	D	E
17.	A	B	C	D	E	37.	A	B	C	D	E	57.	A	B	C	D	E
18.	A	B	C	D	E	38.	A	B	C	D	E	58.	A	B	C	D	E
19.	A	B	C	D	E	39.	A	B	C	D	E	59.	A	B	C	D	E
20.	A	B	C	D	E	40.	A	B	C	D	E	60.	A	B	C	D	E

Solutions to Set 2
Maxima & Minima, Time Rates

SOLUTION 2 - 1
Ans: D

The value of y is maximum when $dy/dx = 0$ ($y' = 0$)

$$y^2 + y + x^2 - 2x = 5$$
$$2yy' + y' + 2x - 2 = 0$$

when $y' = 0$: $0 + 0 + 2x - 2 = 0$; $x = 1$

SOLUTION 2 - 2
Ans: B

$$C = 50\,t^2 - 200\,t + 10000$$
$$\frac{dC}{dt} = 100\,t - 200 = 0;\ t = 2$$
$$C = 50(2)^2 - 200(2) + 10000 = 9800$$

SOLUTION 2 - 3
Ans: B

$$P = 200000 - x - \left(\frac{1.1}{1+x}\right)^8$$
$$\frac{dP}{dx} = -1 - 8\left(\frac{1.1}{1+x}\right)^7 \frac{-1.1}{(1+x)^2} = 0$$
$$\frac{8(1.1)^8}{(1+x)^9} = 1;\ 8(1.1)^8 = (1+x)^9;\ x = 0.371$$
$$P = 200000 - 0.371 - \left(\frac{1.1}{1+0.371}\right)^8 = 199{,}999.46$$

SOLUTION 2 - 4
Ans: B

$$C = x^2 - 4000x + 50$$
$$\frac{dC}{dx} = 2x - 4000 = 0;\quad x = 2000$$

SOLUTION 2 - 5
Ans: D

$$y = x^3 - 3x$$

At maximum y, $dy/dx = 0$
$dy/dx = 3x^2 - 3 = 0$, $x = \pm 1$

To determine the maximum and minimum points, y is maximum if the second derivative y'' is negative and is minimum when y'' is positive.

$$y' = 3x^2 - 3;\ y'' = 6x$$

At $x = +1$: $y'' = 6(1) = +6$ (minimum)
At $x = -1$: $y'' = 6(-1) = -1$ (maximum)
$y = (-1)^3 - 3(-1) = -1 + 3$; $y = 2$

SOLUTION 2 - 6
Ans: D

Let x and y be the numbers, then:
$x + y = 120$, or $y = 120 - x$

Product, $P = y\,x^2 = (120 - x)\,x^2$
$P = 120x^2 - x^3$
$dP/dx = 240x - 3x^2 = 0$
$x = 80$ and $y = 40$

SOLUTION 2 - 7
Ans: A

Let x and y be the number, then $x + y = C$ or $y = C - x$

$S = x^2 + y^2 = x^2 + (C - x)^2$
$S = x^2 + C^2 - 2Cx + x^2 = 2x^2 - 2Cx + C^2$
$\frac{dS}{dx} = 4x - 2C = 0;\ x = C/2$

$S = 2(C/2)^2 - 2C(C/2) + C^2$
$S = 2(C^2/4) - C^2 + C^2 = C^2/2$

SOLUTION 2 - 8
Ans: C

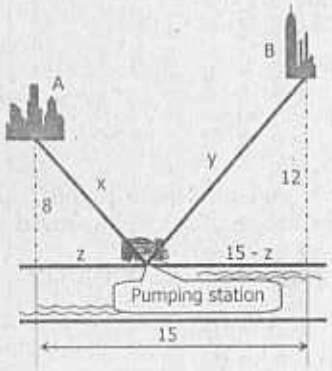
Let x be the number of persons above 150.
Total number of persons = $150 + x$
Cost per person = $1500 - 5x$

Profit, $P = (150 + x)(1500 - 5x) = 225000 + 750x - 5x^2$
$dP/dx = 750 - 10x = 0;\ x = 75$

Number of persons = 150 + 75 = **225**

SOLUTION 2 - 9
Ans: D

$L = x + y$
$x = \sqrt{8^2 + z^2}\ ;\ \ y = \sqrt{12^2 + (15 - z)^2}$
$L = \sqrt{8^2 + z^2} + \sqrt{12^2 + (15 - z)^2}$



$$\frac{dL}{dz} = \frac{2z}{2\sqrt{64 + z^2}} + \frac{2(15 - z)(-1)}{2\sqrt{144 + (15 - z)^2}} = 0$$

$$\frac{z}{\sqrt{64 + z^2}} = \frac{(15 - z)}{\sqrt{144 + (15 - z)^2}} \quad \rightarrow (1)$$

Square both sides of Eq. (1)

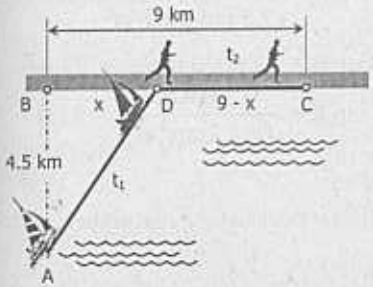
$$\frac{z^2}{64 + z^2} = \frac{225 - 30z + z^2}{144 + 225 - 30z + z^2}$$

$z^2(z^2 - 30z + 369) = z^4 - 30z^3 + 289z^2 - 1920z + 14400$
$z^4 - 30z^3 + 369z^2 = z^4 - 30z^3 + 289z^2 - 1920z + 14400$
$80z^2 + 1920z - 14400 = 0$

$$z = \frac{-1920 \pm \sqrt{(1920)^2 - 4(80)(-14400)}}{2(80)}$$

z = **6 km**

SOLUTION 2 - 10
Ans: B



From the figure shown:
Total time of travel, $T = t_1 + t_2$

$$t_1 = \frac{S_{AD}}{v_{AD}}$$

$$t_1 = \frac{\sqrt{x^2 + 4.5^2}}{6} = \frac{\sqrt{x^2 + 20.25}}{6}$$

$$t_2 = \frac{S_{DC}}{v_{DC}} = \frac{9 - x}{7.5}$$

$$T = \frac{\sqrt{x^2 + 20.25}}{6} + \frac{9 - x}{7.5}$$

$$\frac{dT}{dx} = (1/6)\frac{2x}{2\sqrt{x^2 + 20.25}} + \frac{1}{7.5}(-1) = 0$$

$$\frac{x}{6\sqrt{x^2 + 20.25}} = \frac{1}{7.5}$$

$7.5x = 6\sqrt{x^2 + 20.25}$ square both sides
$56.25\,x^2 = 36(x^2 + 20.25)$
$20.25\,x^2 = 729;\ x = 6$ km; $9 - x = 3$ km

SOLUTION 2 - 11
Ans: C

The distance, d between two points is given by the formula:
$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$

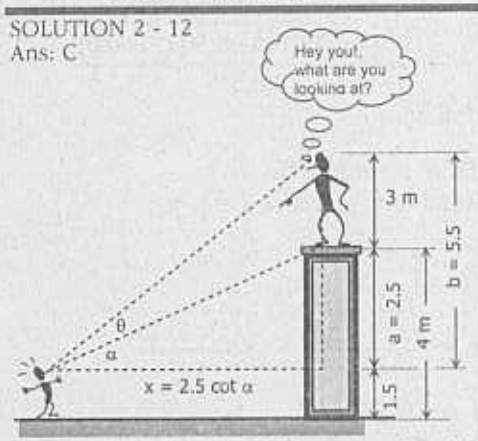
where: $x_2 = 5,\ y_2 = 10\ ;\ x_1 = x,\ y_1 = x^2/12$

$d^2 = (x - 5)^2 + (x^2/12 - 10)^2 \quad \rightarrow (1)$
$2d\frac{dd}{dx} = 2(x - 5)(1) + 2(x^2/12 - 10)(2x/12) = 0$
$x - 5 + x^3/72 - 5x/3 = 0$
$x^3 - 48x - 360 = 0$

By trial and error, $x = 9.31$

In Eq. (1):
$d^2 = (9.31 - 5)^2 + [(9.31)^2/12 - 10]^2 = 26.288$
d = **5.127 units**

SOLUTION 2 - 12
Ans: C



Maximize θ

$$\tan(\theta + \alpha) = \frac{5.5}{2.5\cot\alpha} = 2.2\tan\alpha$$

$$\frac{\tan\theta + \tan\alpha}{1 - \tan\theta\tan\alpha} = 2.2\tan\alpha$$

$$\tan\theta + \tan\alpha = 2.2\tan\alpha - 2.2\tan\theta\tan^2\alpha$$

$$\tan\theta + 2.2\tan\theta\tan^2\alpha = 1.2\tan\alpha$$

$$\tan\theta = \frac{1.2\tan\alpha}{1 + 2.2\tan^2\alpha} \rightarrow (1)$$

Differentiate Eq. (1) with respect to α and equate to zero:
$\sec^2\theta\,(d\theta/d\alpha) =$

$$\frac{1.2\tan\alpha(4.4\tan\alpha\sec^2\alpha) - (1 + 2.2\tan^2\alpha)1.2\sec^2\alpha}{(1 + 2.2\tan^2\alpha)^2} = 0$$

$0 = 5.28\tan^2\alpha\ \sec^2\alpha - 1.2\sec^2\alpha - 2.64\tan^2\alpha\ \sec^2\alpha$
$2.64\tan^2\alpha\ \sec^2\alpha = 1.2\sec^2\alpha$
$\tan^2\alpha = 1.2/2.64$
$\alpha = 33.9878°$
$x = 2.5\cot 33.9878° =$ **3.708 m**

This can also be solved using the relationship that for θ to be maximum, the value of $x = \sqrt{ab} = \sqrt{2.5(5.5)} =$ **3.708 m**

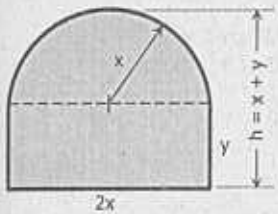
SOLUTION 2 - 13
Ans: D

For a given perimeter (20 m), the figure that will give the largest area
is the circle.
Then;
$P = 2\pi r = 20;\ r = 3.183$ m
Area $= \pi r^2 = \pi(3.183)^2 =$ **31.83 sq. m.**

SOLUTION 2 - 14
Ans: A

For the window to admit the most light, maximize its area: (See figure)
$A = (2x)(y) + ½\,\pi\,x^2$
$A = 2xy + ½\,\pi\,x^2$

$P = 2x + 2y + \pi x$ = constant
$y = ½(P - 2x - \pi x)$

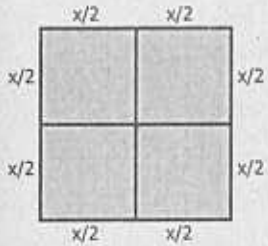


$A = 2x[\,½(P - 2x - \pi x)] + ½\pi x^2$
$A = Px - 2x^2 - \pi x^2 + ½\pi x^2 = Px - 2x^2 - ½\pi x^2$
$\dfrac{dA}{dx} = P - 4x - \pi x = 0;\ P = 4x + \pi x$

$P = 2x + 2y + \pi x = 4x + \pi x$
$2y = 2x$ or $y = x$
Then $h = x + x = 2x$

Thus, the base of the rectangle equals the height of the window, therefore the ratio of the width to the height is 1.

SOLUTION 2 - 15
Ans: B



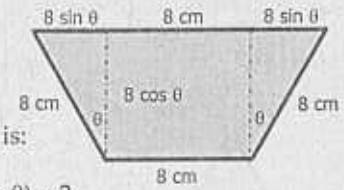
For a given perimeter, the square has the largest area. Hence the rectangular field be a square and divided as shown:

Length of fencing = 12 (x/2) = 1500
$x = 250$

Area $= x^2 = 250^2$
Area = **62500 sq. m.**

SOLUTION 2 - 16
Ans: A

From the figure shown, the fourth side is
8 + 16 sin θ

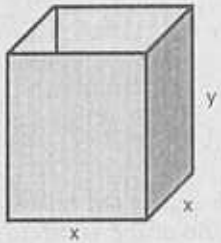


The area of the trapezoid is:
$A = 8 \times 8\cos\theta$
$\quad + ½(8\sin\theta)(8\cos\theta) \times 2$
$A = 64\cos\theta + 64\sin\theta\cos\theta$

Differentiate A with respect to θ and equate to zero:
$dA/d\theta = 64(-\sin\theta) + 64\,[\,\sin\theta\,(-\sin\theta) + \cos\theta\,(\cos\theta)\,] = 0$
$-\sin\theta - \sin^2\theta + \cos^2\theta = 0$
$-\sin\theta - \sin^2\theta + 1 - \sin^2\theta = 0$
$2\sin^2\theta + \sin\theta - 1 = 0$
$(2\sin\theta - 1)(\sin\theta + 1) = 0$
$2\sin\theta - 1 = 0$
$\sin\theta = ½;\ \theta = 30°$

Top width = 8 + 16 sin 30° = **16 cm**

SOLUTION 2 - 17
Ans: B



Let x be the dimension of the square base and y be the height.

Cost = base area × 150 + lateral area × 60

Cost = $x^2(150) + 4xy(60) = 150x^2 + 240xy$

But volume = $x^2 y = 10$ m³; $y = 10/x^2$, then

Cost, $C = 150x^2 + 240x(10/x^2)$

$C = 150x^2 + 2400/x$

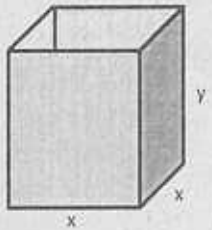
$$\frac{dC}{dx} = 300x - 2400/x^2 = 0$$

$300x = 2400/x^2$

$x^3 = 8;\ x = 2$

$y = 10/2^2 = \mathbf{2.5\ m}$

SOLUTION 2 - 18
Ans: B



Minimize the total surface area:

$A = x^2 + 4xy$

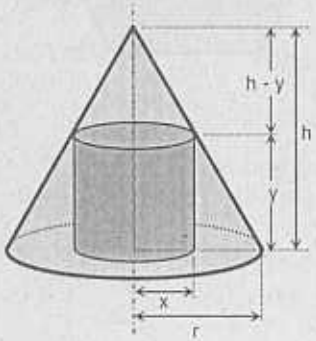
$$V = x^2 y = 16823;\quad y = \frac{16823}{x^2}$$

$$A = x^2 + 4x\frac{16823}{x^2} = x^2 + \frac{67292}{x}$$

$$\frac{dA}{dx} = 2x - \frac{67292}{x^2} = 0;\ x^3 = 33646$$

$$x = 32.28;\quad y = \frac{16823}{(32.28)^2} = \mathbf{16.14\ cm}$$

SOLUTION 2 - 19
Ans: A

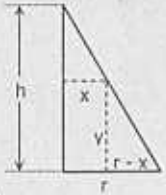


$V = \pi x^2 y$

Expressing y in terms of x:

$$\frac{y}{r-x} = \frac{h}{r}$$

$$y = \frac{h}{r}(r - x)$$



$$V = \pi x^2\left[\frac{h}{r}(r-x)\right] = \pi\frac{h}{r}(rx^2 - x^3)$$

$$\frac{dV}{dx} = \pi\frac{h}{r}(2rx - 3x^2) = 0$$

$x(2r - 3x) = 0$

$x = 0$

$x = \frac{2}{3}r$ (radius of cylinder)

$y = \frac{h}{r}(r - 2r/3);\ y = h/3$ (altitude of cylinder)

SOLUTION 2 - 20
Ans: A

For the closed cylinder of base diameter D and height H:

$$A = 2 \times \frac{\pi}{4}D^2 + \pi DH = \frac{\pi}{2}D^2 + \pi DH$$

$$V = \frac{\pi}{4}D^2 H = 108;\ H = \frac{432}{\pi D^2}$$

$$A = \frac{\pi}{2}D^2 + \pi D\frac{432}{\pi D^2} = \frac{\pi}{2}D^2 + \frac{432}{D}$$

$$\frac{dA}{dD} = \pi D - \frac{432}{D^2} = 0$$

$$D^3 = \frac{432}{\pi};\quad D = 5.16$$

$$H = \frac{432}{\pi(5.16)^2} = 5.16$$

Least surface area = $2 \times \frac{\pi}{4}(5.16)^2 + \pi(5.16)(5.16)$

Least surface area = **125.54 sq. m.**

Note: For a closed cylinder of known volume, the surface area is least when its diameter (D) is equal to its height (H). Also, for a close cylinder of known surface area, the volume is maximum when $D = H$.

SOLUTION 2 - 21
Ans: C

This problem is similar to **Problem 19-20.** For a closed cylindrical tank of a given volume, the total surface are is minimum if the diameter D equals the altitude h.

$V = A_b \times h$; but $h = D$

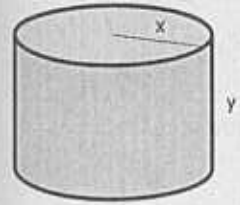
$V = \frac{\pi}{4}D^2 \times D = \frac{\pi}{4}D^3$

$11.3 = \frac{\pi}{4}D^3$

$D = 2.432$ m $= 2r$

$r = \mathbf{1.216\ m}$

SOLUTION 2 - 22
Ans: C



Cost, $C = (\pi x^2 \times 2)(3000) + (2\pi x y)(2000)$

$C = 6000\pi x^2 + 4000\pi xy$

$V = \pi x^2 y = 1000;\ y = 1000/\pi x^2$

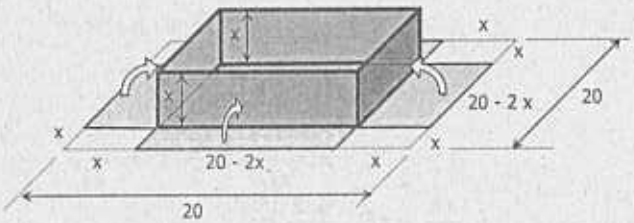
$C = 6000\pi x^2 + 4000\pi x(1000/\pi x^2)$

$$C = 6000\pi x^2 + \frac{4000000}{x}$$

$$\frac{dC}{dx} = 12000\pi x - \frac{4000000}{x^2} = 0$$

$x = \mathbf{4.734\ m}$

SOLUTION 2 - 23
Ans: C



From the figure shown,

Volume, $V = (20 - 2x)(20 - 2x)\,x$

$V = (400 - 80x + 4x^2)x = 400x - 80x^2 + 4x^3$

$$\frac{dV}{dx} = 400 - 160x + 12x^2 = 0$$

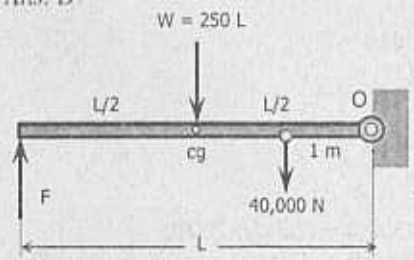
$$x = \frac{-(-160) \pm \sqrt{(-160)^2 - 4(12)(400)}}{2(12)}$$

$$x = \frac{160 \pm 80}{24} = 10 \text{ (absurd) and } x = \underline{3.333\ (OK)}$$

Maximum volume = $400(3.333) - 80(3.333)^2 + 4(3.333)^3$

Maximum volume = **592.59 cubic inches**

SOLUTION 2 - 24
Ans: D



$\Sigma M_O = 0$

$F\,(L) = 40000(1) + 250L(L/2)$

$F = 40000/L + 125L$

To minimize the value of F, differentiate F with respect to L and equate to zero.

$$\frac{dF}{dL} = -40000/L^2 + 125 = 0$$

$125L^2 = 40000;\ L = 17.89$ m

SOLUTION 2 - 25
Ans: C

Let $y = (1 + x)^3$

$dy/dt = 3(1 + x)^2\, dx/dt$

Given that $dx/dt = 0.002$ feet per second and $x = 8$ feet,

$dy/dt = 3(1 + 8)^2(0.002)$

$dy/dt =$ **0.486 cfs**

SOLUTION 2 - 26
Ans: A

Let $S = x(t) = 16t + 8t^2$

Velocity = dS/dt

Velocity = $dS/dt = 16 + 16t$

At $t = 3$:

Velocity = $16 + 16(3) = 64$

SOLUTION 2 - 27
Ans: A

Distance, $x = 3t^2 + 2t + 4$

Velocity: $dx/dt = 6t + 2$

when $x = 10$

$10 = 3t^2 + 2t + 4;\ 3t^2 + 2t + -6 = 0$

$t = 1.12$ hr.

Velocity = $dx/dt = 6(1.12) + 2$

Velocity = **8.72 kph**

SOLUTION 2 - 28
Ans: D

The speed of the object at any time is dD/dt,

$D = 20t + 5\ /\ (t + 1)$

$$\frac{dD}{dt} = 20 - \frac{5}{(t+1)^2}$$

When $t = 4$:

$$\frac{dD}{dt} = 20 - \frac{5}{(4+1)^2} = \mathbf{19.8\ m/min}$$

SOLUTION 2 - 29
Ans: B

$V = t^3 - 10.5t^2 + 30t + 20$

$$\frac{dV}{dt} = 3t^2 - 21t + 30 = 0$$

$0 = (3t - 6)(t - 5)$

$t = 2$ and $t = 5$

When $t = 2$

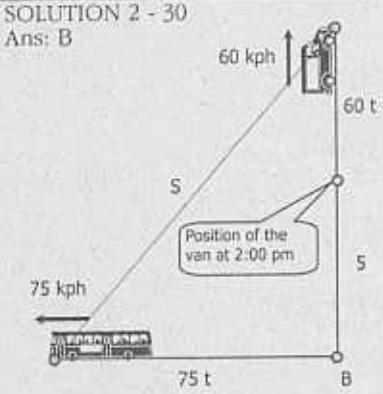
$V = (2)^3 - 10.5(2)^2 + 30(2) + 20 = 46$ kph

When $t = 5$

$V = (5)^3 - 10.5(5)^2 + 30(5) + 20 = 32.5$ kph

Therefore; the fastest speed of the traffic is **46 kph**

SOLUTION 2 - 30
Ans: B



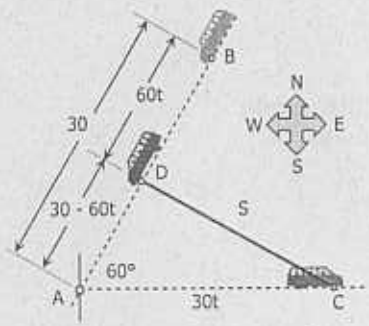
From the figure shown:

$S^2 = (75\,t)^2 + (5 + 60\,t)^2$
$S^2 = 5625t^2 + 25 + 600t + 3600t^2$
$S^2 = 9225t^2 + 600t + 25 \quad \rightarrow (1)$
$2S\,dS/dt = 18450t + 600 \quad \rightarrow (2)$

At 5 pm ($t = 3$), from Eq. (1)
$S^2 = 9225(3)^2 + 600(3) + 25$
$S = 291.29$ km

In Eq. (2)
$2(291.29)\,dS/dt = 18450(3) + 600$
$dS/dt =$ **96.03 kph**

SOLUTION 2 - 31
Ans: D



From the figure shown:

$S^2 = (30t)^2 + (30 - 60t)^2$
$\quad - 2(30t)(30 - 60t)\cos 60°$
$S^2 = 900\,t^2 + 900 - 3600\,t + 3600\,t^2 - 900\,t$
$\quad + 1800\,t^2$
$S^2 = 6300\,t^2 - 4500\,t + 900 \quad \rightarrow (1)$

$2S\dfrac{dS}{dt} = 12600\,t - 4500 \quad \rightarrow (2)$

After one hour ($t = 1$)

In Eq. (1):
$S^2 = 6300\,(1)^2 - 4500\,(1) + 900$
$S = 51.961$ km

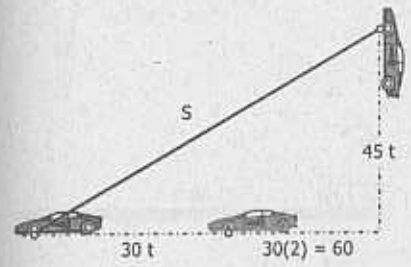
In Eq. (2):
$2(51.961)\dfrac{dS}{dt} = 12600(1) - 4500$

$\dfrac{dS}{dt} =$ **77.943 kph**

SOLUTION 2 - 32
Ans: C

Let t = any time after 2:00 pm
S = distance between the cars at any time t

From the figure shown:
$S^2 = (45t)^2 + (30t + 60)^2$
$S^2 = 2025\,t^2 + 900\,t^2 + 3600\,t + 3600$
$S^2 = 2925\,t^2 + 3600\,t + 3600 \quad \rightarrow (1)$



$2S\dfrac{dS}{dt} = 5850\,t + 3600 \quad \rightarrow (2)$

At 4:00 pm, $t = 2$

Solve for S in Eq. (1):
$S^2 = 2925(2)^2 + 3600(2) + 3600;\ S =$
150 km

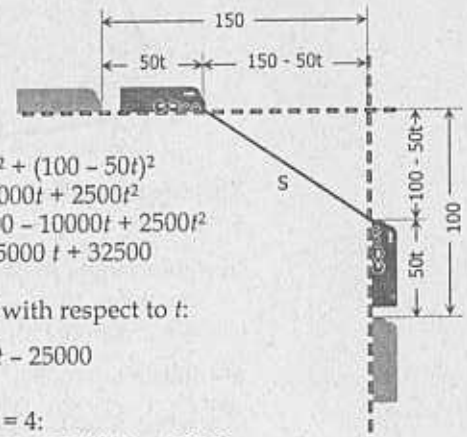
In Eq. (2)
$2(150)\dfrac{dS}{dt} = 5850(2) + 3600$

$\dfrac{dS}{dt} =$ **51 kph**

SOLUTION 2 - 33
Ans: B

Let t be any time after 12 p.m.

The distance S between the trains at any time is:



$S^2 = (150 - 50t)^2 + (100 - 50t)^2$
$S^2 = 22500 - 15000t + 2500t^2$
$\quad + 10000 - 10000t + 2500t^2$
$S^2 = 5000\,t^2 - 25000\,t + 32500$

Differentiate S with respect to t:
$2S\dfrac{dS}{dt} = 10000t - 25000$

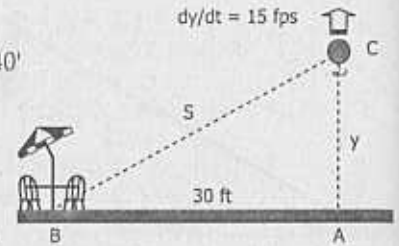
At 4:00 p.m., $t = 4$:
$S^2 = 5000(4)^2 - 25000(4) + 32500$
$S = 111.803$ km

then;
$2(111.803)\dfrac{dS}{dt} = 10000(4) - 25000$

$\dfrac{dS}{dt} =$ **67.08 kph**

SOLUTION 2 - 34
Ans: B

From the triangle shown:
$S^2 = y^2 + 30^2$



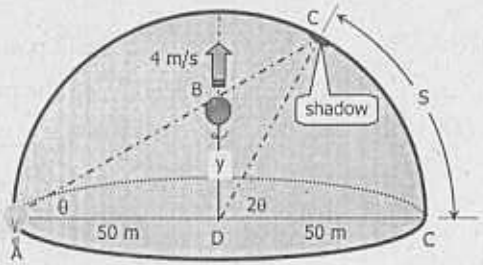
Find dS/dt when $y = 40'$

$S^2 = y^2 + 30^2$
$2S\, dS/dt = 2y\, dy/dt$

When $y = 40'$, $S = 50'$
then;
$50\, dS/dt = 40(15)$
$dS/dt =$ **12 fps**

SOLUTION 2 - 35
Ans: A

Required: dS/dt when $y = 25$ m



The rate the balloon rises is $dy/dt = 4$ m/s
$S = 50 \times 2\theta = 100\,\theta$

In right triangle ADB:
$\tan\theta = y/50$
$\theta = \arctan(y/50)$

$S = 100 \arctan(y/50)$

$$\frac{dS}{dt} = 100\,\frac{(1/50)(dy/dt)}{1+(y/50)^2}$$

$$\frac{dS}{dt} = \frac{4}{1+(25/50)^2} = \mathbf{6.4\ m/s}$$

SOLUTION 2 - 36
Ans: B

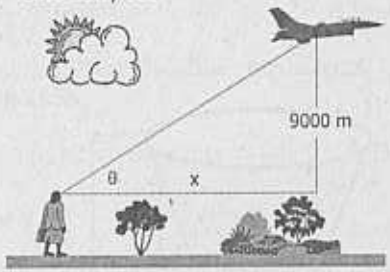
Distance, $y = -16t^2 + 3000t + 50000$
Velocity, $v = dy/dt = -32t + 3000$

When $t = 0$
$v = -32(0) + 3000 =$ **3000**

SOLUTION 2 - 37
Ans: D

When $\theta = 60°$, $d\theta/dt = -0.15$ rad/sec

Velocity of
the airplane $= dx/dt$



$x = 9000 \cot\theta$
$dx/dt = 9000\,(-\csc^2\theta\ \ d\theta/dt)$
$dx/dt = -9000\,(\csc^2 60°)(-0.15)$
$dx/dt = 1800$ m/s

SOLUTION 2 - 38
Ans: C

Surface are of sphere, $A_s = 4\,\pi r^2$
$dA_s/dt = 8\pi r\, dr/dt = 26$ cm²/sec → (1)

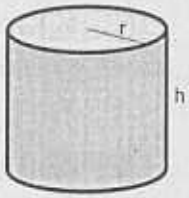
After 2 seconds:
$A_s = 26(2) = 52$ cm²
$4\,\pi r^2 = 52$; $r = 2.034$

Substitute r to Eq. (1):
$8\pi(2.034)\, dr/dt = 26$
$dr/dt =$ **0.509 cm/s**

SOLUTION 2 - 39
Ans: D

$V = \pi(r^2)h$

$$\frac{dV}{dt} = \pi\left[(r^2)\frac{dh}{dt} + h\left(2r\frac{dr}{dt}\right)\right]$$



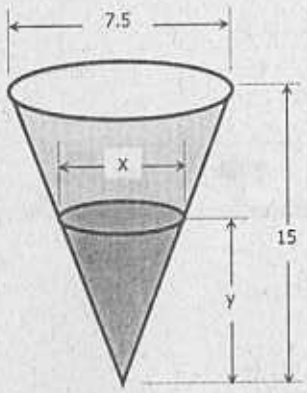
$h = 50$, $dh/dt = -4$

$r = 20$, $dr/dt = 1$

$$\frac{dV}{dt} = \pi[(20)^2(-4) + 50(2)(20)(1)]$$

$$\frac{dV}{dt} = \mathbf{1257\ cu.\ in./sec.}$$

SOLUTION 2 - 40
Ans: D



The volume of water at any time is:

$$V = \frac{1}{3}\frac{\pi}{4}x^2 y = \frac{\pi}{12}x^2 y$$

By ratio and proportion:

$$\frac{x}{y} = \frac{7.5}{15};\ x = y/2$$

$$V = \frac{\pi}{12}(y/2)^2 y = \frac{\pi}{48}y^3$$

$$\frac{dV}{dt} = \frac{\pi}{48}3y^2\frac{dy}{dt}$$

When $y = 4$ ft., $dy/dt = 2$ ft/min.

$$\frac{dV}{dt} = \frac{\pi}{48}(3)(4)^2(2) = \mathbf{6.28\ ft^3/min}$$

SOLUTION 2 - 41
Ans: A

The curvature of the function $y = f(x)$ at any point is given by:

$$k = \frac{y''}{[1+(y')^2]^{3/2}}$$

$y^2 = 16x;\ 2yy' = 16$

$y' = 8/y$

@ (4, 8); $y' = 8/8 = 1$

$$y'' = \frac{8y'}{y^2}$$

$$@\ (4, 8);\ y'' = -\frac{8(1)}{8^2} = -0.125$$

$$\text{Curvature, } k = \frac{-0.125}{[1+(1)^2]^{3/2}} = \mathbf{-0.0442}$$

SOLUTION 2 - 42
Ans: D

$f(x) = 100(2x^3 - 45x^2 + 264x)$

Differentiate the given function and equate to zero:

$f'(x) = 100(6x^2 - 90x + 264) = 0$

$6x^2 - 90x + 264 = 0$

$x^2 - 15x + 44 = 0$

$(x - 11)(x - 4) = 0$

$x = 11$ and $x = 4$

When $x = 11$

$f(11) = 100[2(11)^3 - 45(11)^2 + 264(11)] = 12{,}100$

When $x = 4$

$f(4) = 100[2(4)^3 - 45(4)^2 + 264(4)] = 46{,}400$

Thus, the membership is smallest (relative minimum) when $x = 11$ and that is in 1986.

The membership is largest (relative maximum) when $x = 4$ and that is in 1979.

SOLUTION 2 - 43
Ans: C

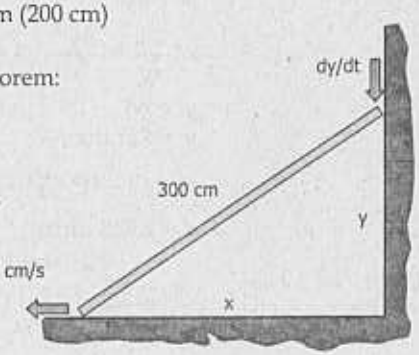
From the figure shown,

$\frac{dy}{dt} = ?$ when $x = 2$ m (200 cm)

By Pythagorean Theorem:

$$y = \sqrt{300^2 - x^2}$$

$$\frac{dy}{dt} = \frac{-2x\frac{dx}{dt}}{2\sqrt{300^2 - x^2}}$$



$$\frac{dy}{dt} = \frac{-200(2)}{\sqrt{300^2 - 200^2}}$$

$$\frac{dy}{dt} = -1.79\ \text{cm/s}$$

Therefore, the upper end moves down at the rate of 1.79 cm/s

SOLUTION 2 - 44
Ans: B

$$a = \sqrt{a_x^2 + a_y^2}$$

$$v_x = \frac{dx}{dt} = 3t^2;\quad a_x = \frac{dv_x}{dt} = 6t$$

$$v_y = \frac{dy}{dt} = 4t;\quad a_y = \frac{dv_y}{dt} = 4$$

$$a = \sqrt{(6t)^2 + (4)^2} = \sqrt{36t^2 + 16}$$

When $t = 3$ sec:

$$a = \sqrt{36(3)^2 + 16} = \mathbf{18.44\ m/s^2}$$

SOLUTION 2 - 45
Ans: A

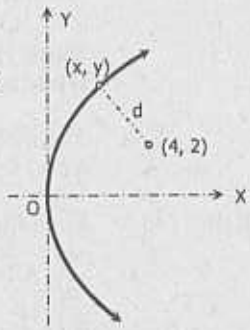
$d^2 = (x-4)^2 + (y-2)^2$

$x = y^2/8$

$d^2 = (y^2/8 - 4)^2 + (y-2)^2$

$d^2 = \frac{y^4}{64} - y^2 + 16 + y^2 - 4y + 4$

$d^2 = \frac{y^4}{64} - 4y + 20$



Differentiate & set dd/dy to zero:

$2d\frac{dd}{dy} = \frac{4y^3}{64} - 4 = 0$

$y^3 = 64$

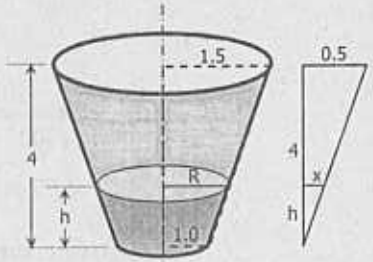
$y = 4$ and $x = 2$

$d^2 = (2-4)^2 + (4-2)^2$

d = **2.828 units**

SOLUTION 2 - 46
Ans: B

$\frac{x}{h} = \frac{0.5}{4}$; $x = 0.125h$



$R = 1 + x = 1 + 0.125h$

$V = \frac{\pi h}{3}(R^2 + Rr + r^2)$

$V = \frac{\pi h}{3}[(1 + 0.125h)^2 + (1 + 0.125h)(1) + 1^2]$

$V = \frac{\pi h}{3}[0.015625h^2 + 0.375h + 3]$

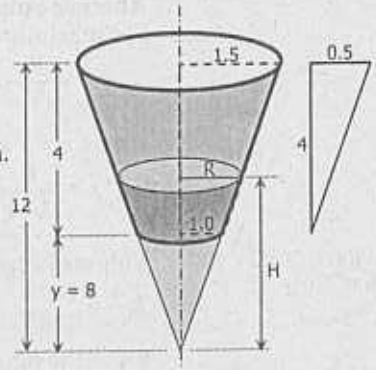
$V = \frac{\pi}{3}[0.015625h^3 + 0.375h^2 + 3h]$

$\frac{dV}{dt} = \frac{\pi}{3}[0.046875\,h^2 + 0.75\,h + 3]\frac{dh}{dt}$

$\frac{dV}{dt} = \frac{\pi}{3}[0.046875\,(1.2)^2 + 0.75\,(1.2) + 3](0.012)$

$\frac{dV}{dt}$ = **0.05 m³/s**

Another Solution:
To avoid complicated variable relationship, the tank may be considered as a cone with dimensions shown.



$\frac{y}{1} = \frac{4}{0.5}$; $y = 8$ m

$V = \frac{1}{3}\pi R^2 H$

$\frac{R}{H} = \frac{0.5}{4}$

$R = 0.125H$

$V = \frac{1}{3}\pi(0.125H)^2 H$

$V = 0.016362\,H^3$

$\frac{dV}{dt} = 0.049087\,H^2\,\frac{dH}{dt}$

when $H = 8 + 1.2 = 9.2$

$\frac{dV}{dt} = 0.049087(9.2)^2(0.012)$ = **0.05 m³/s**

SOLUTION 2 - 47
Ans: A

$x = t^3$; $y = 2t^2$

$v_x = \frac{dx}{dt} = 3t^2 = 3(2)^2 = 12$

$v_y = \frac{dy}{dt} = 4t = 4(2) = 8$

$v = \sqrt{(v_x)^2 + (v_y)^2} = \sqrt{12^2 + 8^2}$ = **14.42**

SOLUTION 2 - 48
Ans: A

Let x and y be the numbers.

$x + y = K$ or $y = K - x$

Product, $P = x^3y$

$P = x^3(K - x) = Kx^3 - x^4$

$\frac{dP}{dx} = 3Kx^2 - 4x^3 = 0$

$x^2(3K - 4x) = 0$

$x^2 = 0$; $x = 0$ (absurd)

$3K - 4x = 0$

$x = 3K/4$

Another Solution:
For maximum product of $x^n \times y^m$ where $x + y = K$:

$$x = \frac{K}{m+n}n; \quad y = \frac{K}{m+n}m$$

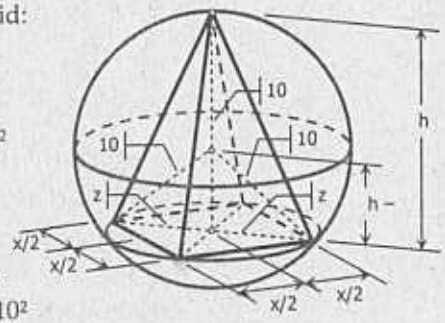
where $n = 1$ and $m = 3$

$$x = \frac{K}{3+1}3 = 3K/4$$

SOLUTION 2 - 49
Ans: C

Volume of pyramid:
$V = \frac{1}{3}x^2h$

From the figure:
$z^2 = (x/2)^2 + (x/2)^2$
$z^2 = x^2/2$



$z^2 + (h - 10)^2 = 10^2$
$x^2/2 + (h - 10)^2 = 10^2$
$x^2 = 200 - 2(h - 10)^2$
$x^2 = 200 - 2h^2 + 40h - 200 = 40h - 2h^2$
$V = \frac{1}{3}(40h - 2h^2)h = \frac{1}{3}(40h^2 - 2h^3)$

$$\frac{dV}{dh} = \frac{1}{3}(80h - 6h^2) = 0$$

$6h^2 = 80h$; $h = \mathbf{13.33\ cm}$

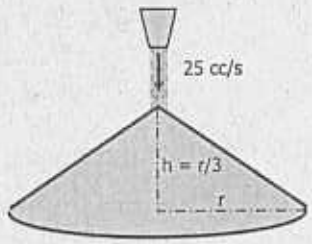
SOLUTION 2 - 50
Ans: D

$V = \frac{1}{3}\pi r^2 h$
$r = 3h$
$V = \frac{\pi}{3}(3h)^2 h = 3\pi h^3$

$$\frac{dV}{dt} = 9\pi h^2 \frac{dh}{dt}$$

$$25 = 9\pi(50)^2 \frac{dh}{dt}$$

$$\frac{dh}{dt} = \mathbf{0.000354\ cm/sec}$$



Part 2
INTEGRAL CALCULUS

INTEGRATION FORMULAS

Algebraic, Exponential, and Logarithmic Functions

1. $\int a\,du = a\int du = au + C$
2. $\int a^u du = \frac{a^u}{\ln a} + C,\ a > 1,\ a \neq 1$
3. $\int u^n du = \frac{1}{n+1}u^{n+1} + C$ for $n \neq -1$
4. $\int e^u du = e^u + C$
5. $\int u^{-1} du = \int \frac{du}{u} = \ln|u| + C$
6. $\int \ln u\,du = u\ln|u| - u + C$

Trigonometric Functions

7. $\int \sin u\,du = -\cos u + C$
8. $\int \cos u\,du = \sin u + C$
9. $\int \tan u\,du = \ln|\sec u| + C$

Another Solution:
For maximum product of $x^n \times y^m$ where $x + y = K$:

$$x = \frac{K}{m+n}n; \quad y = \frac{K}{m+n}m$$

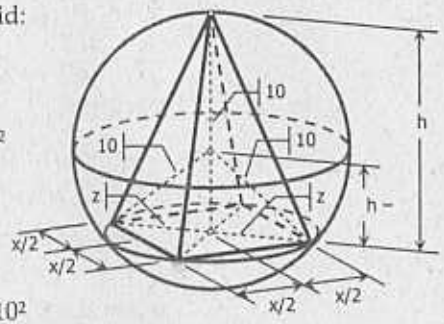
where $n = 1$ and $m = 3$

$$x = \frac{K}{3+1}3 = 3K/4$$

SOLUTION 2 - 49
Ans: C

Volume of pyramid:
$V = \frac{1}{3}x^2h$

From the figure:
$z^2 = (x/2)^2 + (x/2)^2$
$z^2 = x^2/2$



$z^2 + (h - 10)^2 = 10^2$
$x^2/2 + (h - 10)^2 = 10^2$
$x^2 = 200 - 2(h - 10)^2$
$x^2 = 200 - 2h^2 + 40h - 200 = 40h - 2h^2$
$V = \frac{1}{3}(40h - 2h^2)h = \frac{1}{3}(40h^2 - 2h^3)$

$$\frac{dV}{dh} = \frac{1}{3}(80h - 6h^2) = 0$$

$6h^2 = 80h$; $h = 13.33$ cm

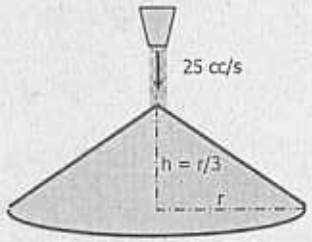
SOLUTION 2 - 50
Ans: D

$V = \frac{1}{3}\pi r^2 h$
$r = 3h$
$V = \frac{\pi}{3}(3h)^2 h = 3\pi h^3$

$$\frac{dV}{dt} = 9\pi h^2 \frac{dh}{dt}$$

$$25 = 9\pi(50)^2 \frac{dh}{dt}$$

$$\frac{dh}{dt} = 0.000354 \text{ cm/sec}$$



Part 2
INTEGRAL CALCULUS

INTEGRATION FORMULAS

Algebraic, Exponential, and Logarithmic Functions

1. $\int a\,du = a\int du = au + C$
2. $\int a^u du = \frac{a^u}{\ln a} + C,\ a > 1, a \neq 1$
3. $\int u^n du = \frac{1}{n+1}u^{n+1} + C$ for $n \neq -1$
4. $\int e^u du = e^u + C$
5. $\int u^{-1}du = \int \frac{du}{u} = \ln|u| + C$
6. $\int \ln u\,du = u\ln|u| - u + C$

Trigonometric Functions

7. $\int \sin u\,du = -\cos u + C$
8. $\int \cos u\,du = \sin u + C$
9. $\int \tan u\,du = \ln|\sec u| + C$

10. $\int \cot u\,du = \ln|\sin u| + C$

11. $\int \sec u\,du = \ln|\sec u + \tan u| + C$

12. $\int \csc u\,du = \ln|\csc u - \cot u| + C$

$= -\ln|\csc u + \cot u| + C$

13. $\int \sec u \tan u\,du = \sec u + C$

14. $\int \csc u \cot u\,du = -\csc u + C$

15. $\int \sec^2 u\,du = \tan u + C$

16. $\int \csc^2 u\,du = -\cot u + C$

Inverse Trigonometric Functions

17. $\int \frac{du}{\sqrt{a^2-u^2}} = \arcsin \frac{u}{a} + C$

18. $\int \frac{du}{a^2+u^2} = \frac{1}{a}\arctan\frac{u}{a} + C$

19. $\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\operatorname{arcsec}\frac{u}{a} + C$

20. $\int \arcsin u\,du = u\arcsin u + \sqrt{1-u^2} + C$

21. $\int \arctan u\,du = u\arctan u - \ln\sqrt{1+u^2}$

Hyperbolic Functions

22. $\int \sinh u\,du = \cosh u + C$

23. $\int \cosh u\,du = \sinh u + C$

24. $\int \tanh u\,du = \ln\cosh u + C$

25. $\int \coth u\,du = \ln|\sinh u| + C$

26. $\int \operatorname{sech}^2 u\,du = \tanh u + C$

27. $\int \operatorname{csch}^2 u\,du = -\coth u + C$

28. $\int \operatorname{sech} u \tanh u\,du = -\operatorname{sech} u + C$

29. $\int \operatorname{csch} u \coth u\,du = -\operatorname{csch} u + C$

30. $\int \frac{du}{\sqrt{u^2+a^2}} = \sinh^{-1}\frac{u}{z} + C$

31. $\int \frac{du}{\sqrt{u^2-a^2}} = \cosh^{-1}\frac{u}{a} + C, u > a > 0$

32. $\int \frac{du}{a^2-u^2} = \frac{1}{a}\tanh^{-1}\frac{u}{a} + C, u^2 < a^2$

33. $\int \frac{du}{a^2-u^2} = -\frac{1}{a}\coth^{-1}\frac{u}{a} + C, x^2 > a^2$

Other Functions

34. $\int \frac{du}{\sqrt{u^2 \pm a^2}} = \ln|u + \sqrt{u^2 \pm a^2}| + C$

35. $\int \frac{du}{a^2-u^2} = \frac{1}{2a}\ln\left|\frac{a+u}{a-u}\right| + C,\ u^2 < a^2$

36. $\displaystyle\int \frac{du}{u^2-a^2} = \frac{1}{2a}\ln\left|\frac{u-a}{u+a}\right| + C,\ u^2 > a^2$

37. $\displaystyle\int \sqrt{a^2-u^2}\,du = \frac{u}{2}\sqrt{a^2-u^2} + \frac{a^2}{2}\arcsin\frac{u}{a} + C$

38. $\displaystyle\int \sqrt{u^2 \pm a^2}\,du = \frac{u}{2}\sqrt{u^2 \pm a^2} + \frac{a^2}{2}\ln\left|u+\sqrt{u^2 \pm a^2}\right| + C$

Integration by Parts

39. $\displaystyle\int u\,dv = uv - \int v\,du$

Trigonometric Substitution

Some integrations may be simplified with the following substitutions:

1. If an integrand contain $\sqrt{a-x^2}$, substitute $x^2 = a\sin^2\theta$
2. If an integrand contain $\sqrt{a+x^2}$, substitute $x^2 = a\tan^2\theta$
3. If an integrand contain $\sqrt{x^2-a}$, substitute $x^2 = a\sec^2\theta$

More generally, an integrand that contains one of the forms $\sqrt{a-bx^2}$, $\sqrt{a+bx^2}$, or $\sqrt{bx^2-a}$ but no other irrational factor may be transformed into another involving trigonometric functions of a new variable as follows:

For	Use	To obtain
$\sqrt{a-bx^2}$	$x^2 = \frac{a}{b}\sin^2\theta$	$\sqrt{a}\,\sqrt{1-\sin^2\theta} = \sqrt{a}\cos\theta$
$\sqrt{a+bx^2}$	$x^2 = \frac{a}{b}\tan^2\theta$	$\sqrt{a}\,\sqrt{1+\tan^2\theta} = \sqrt{a}\sec\theta$
$\sqrt{bx^2-a}$	$x^2 = \frac{a}{b}\sec^2\theta$	$\sqrt{a}\,\sqrt{\sec^2\theta-1} = \sqrt{a}\tan\theta$

Wallis Formula

$$\int_0^{\pi/2} \sin^m\theta\cos^n\theta\,d\theta = \frac{[(m-1)(m-3)\ldots(2\text{ or }1)][(n-1)(n-3)\ldots(2\text{ or }1)]}{(m+n)(m+n-2)\ldots(2\text{ or }1)}\,\alpha$$

where:

$\alpha = \pi/2$ when both m and n are even
$\alpha = 1$ if otherwise
m & n = positive integer, not equal to 1

EXAMPLES

Wallis Formula

Ex. 1: $\displaystyle\int_0^{\frac{\pi}{2}} \sin^5 x\,dx = \frac{(4)(2)}{(5)(3)(1)}\times 1 = \frac{8}{15}$

Ex. 2: $\displaystyle\int_0^{\frac{\pi}{2}} \cos^8 x\,dx = \frac{(7)(5)(3)(1)}{(8)(6)(4)(2)}\times\frac{\pi}{2} = \frac{105\pi}{768}$

Ex. 3: $\displaystyle\int_0^{\frac{\pi}{2}} \sin^4 x\cos^5 x\,dx = \frac{[(3)(1)][(4)(2)]}{(9)(7)(5)(3)(1)}\times 1 = \frac{8}{315}$

Double Integration

Ex. 4: $\displaystyle\int_0^1\int_0^{3x} y^3\,dy\,dx = \int_0^1 \left.\frac{y^4}{4}\right]_0^{3x} dx$

$$= \int_0^1 \frac{(3x)^4}{4}dx = \int_0^1 \frac{81}{4}x^4 dx$$

$$= \left.\frac{81}{4}\frac{x^5}{5}\right]_0^1 = \frac{81}{20}$$

Ex. 5: $\displaystyle\int_0^1\int_2^{2+y} x^2\,dx\,dy = \int_0^1 \left.\frac{x^3}{3}\right]_2^{2+y} dy$

$$= \frac{1}{3}\int_0^1 [(2+y)^3 - 2^3]\,dy$$

$$= \frac{1}{3}\int_0^1 (y^3 + 6y^2 + 12y)\,dy$$

$$= \frac{1}{3}\left[\frac{y^4}{4} + 2y^3 + 6y^2\right]_0^1 = \frac{33}{12}$$

Triple Integration

Ex. 6: $\int_0^3\int_0^2\int_0^y dx\,dy\,dz = \int_0^3\int_0^2 \left[x\right]_0^y dy\,dx$

$$= \int_0^3\int_0^2 y\,dy\,dx = \int_0^3 \left[\frac{y^2}{2}\right]_0^2 dx$$

$$= \int_0^3 2\,dx = 2x\Big]_0^3 = 6$$

Integration by Parts

Ex. 7: $\int x\sin x\,dx$

Let $u = x$, $du = dx$
$dv = \sin x\,dx$, $v = -\cos x$

$$\int u\,dv = uv - \int v\,du$$

$$\int x\sin x\,dx = x(-\cos x) - \int -\cos x\,dx$$

$$= -x\cos x + \sin x + C$$

Ex. 8: $\int x\cos 2x\,dx$

Let $u = x$, $du = dx$
$dv = \cos 2x\,dx$, $v = (1/2)\sin 2x$

$$\int x\cos 2x\,dx = x(1/2)\sin 2x - \int (1/2)\sin 2x\,dx$$

$$= \tfrac{1}{2}x\sin 2x + \tfrac{1}{4}\cos 2x + C$$

Algebraic Substitution

Ex. 9: $\int_0^1 \frac{x\,dx}{(x+1)^8}$

Let $z = x + 1$ $\quad x = z - 1$
$dx = dz$

Change the limits:
when $x = 0$; $z = 1$
when $x = 1$; $z = 2$

$$\int_1^2 \frac{(z-1)\,dz}{z^8} = \int_1^2 (z^{-7} - z^{-8})\,dz$$

$$= \left[\frac{1}{-6}z^{-6} - \frac{1}{-7}z^{-7}\right]_1^2 = 0.0223$$

Trigonometric Substitution

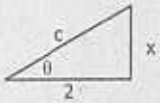
Ex. 10: Find $\int \frac{dx}{x^2\sqrt{4+x^2}}$

Let $x^2 = 4\tan^2\theta$
$x = 2\tan\theta$
$dx = 2\sec^2\theta\,d\theta$

$$\int \frac{dx}{x^2\sqrt{4+x^2}} = \int \frac{2\sec^2\theta\,d\theta}{(4\tan^2\theta)\sqrt{4+4\tan^2\theta}}$$

$$= \frac{1}{4}\int \frac{\sec\theta}{\tan^2\theta}\,d\theta$$

$$= \frac{1}{4}\int \sin^{-2}\theta\cos\theta\,d\theta = -\frac{1}{4\sin\theta} + C$$



$\tan\theta = x/2$

From the right triangle:

$c = \sqrt{2^2 + x^2}$

$\sin\theta = \frac{x}{c} = \frac{x}{\sqrt{4+x^2}}$

$$\int \frac{dx}{x^2\sqrt{4+x^2}} = -\frac{1}{4\sin\theta} + C = -\frac{1}{4\frac{x}{\sqrt{4+x^2}}}$$

$$\int \frac{dx}{x^2\sqrt{4+x^2}} = -\frac{\sqrt{4+x^2}}{4x} + C$$

Integration by Partial Fraction

Ex. 11: Find $\int \frac{dx}{x^2-4}$

Resolve $\frac{1}{x^2-4}$ into partial fraction:

$$\frac{1}{x^2-4} = \frac{1}{(x-2)(x+2)} = \frac{A}{x-2} + \frac{B}{x+2}$$

$$1 = A(x+2) + B(x-2)$$

Set $x = 2$; $A = \frac{1}{4}$

Set $x = -2$; $B = -\frac{1}{4}$

$$\frac{1}{x^2-4} = \frac{1}{4(x-2)} - \frac{1}{4(x+2)}$$

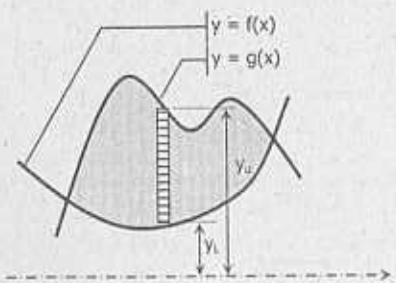
$$\int \frac{dx}{x^2-4} = \frac{1}{4}\int \frac{dx}{x-2} - \frac{1}{4}\int \frac{dx}{x+2}$$

$$= \frac{1}{4}\ln|x-2| - \frac{1}{4}\ln|x+2| + C$$

$$\int \frac{dx}{x^2-4} = \frac{1}{4}\ln\left|\frac{x-2}{x+2}\right| + C$$

PLANE AREAS

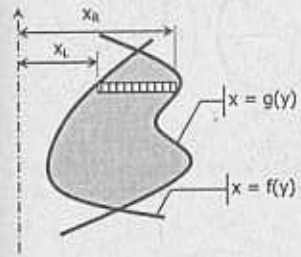
Using Vertical Strip



$$A = \int_{x_1}^{x_2} (y_U - y_L)\,dx$$

$y_U = f(x)$; $y_L = g(x)$

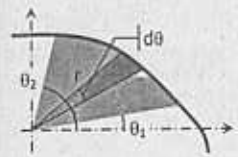
Using Horizontal Strip



$$A = \int_{y_1}^{y_2} (x_R - x_L)\,dy)$$

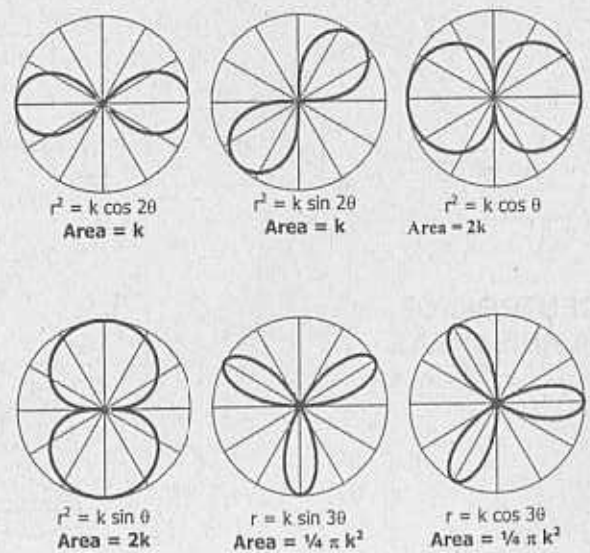
$x_R = g(y)$; $x_L = f(y)$

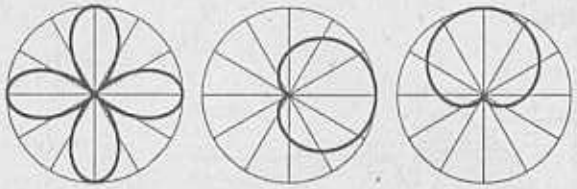
By Polar Coordinates



$$A = \tfrac{1}{2}\int_{\theta_1}^{\theta_2} r^2\,d\theta$$

Area of Some Polar Curves

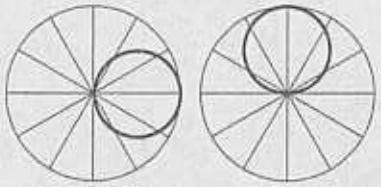




$r = k \cos 2\theta$
Area = ½ π k²

$r = k(1 + \cos \theta)$
Area = 1.5 π k²
Perimeter = 2πa

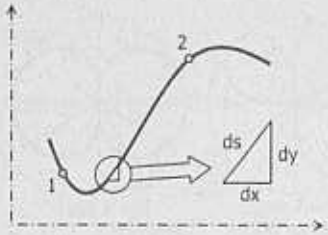
$r = k(1 + \sin \theta)$
Area = 1.5 π k²
Perimeter = 2πa



$r = 2k \cos \theta$
Length = 2πk
Area = πk²

$r = 2k \sin \theta$
Length = 2πk
Area = πk²

LENGTH OF ARC

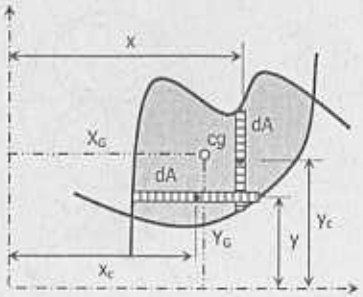


$$(ds)^2 = (dx)^2 + (dy)^2$$

$$s = \int_{x_1}^{x_2} \sqrt{1 + (dy/dx)^2}\, dx$$

$$\text{or} \quad s = \int_{y_1}^{y_2} \sqrt{1 + (dx/dy)^2}\, dy$$

CENTROID OF PLANE AREAS



Using Horizontal Strip

$$A\,X_G = \int_{x_1}^{x_2} x_c\, dA$$

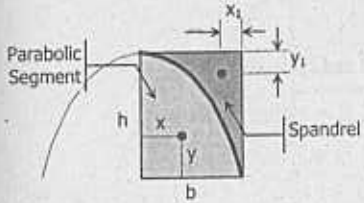
$$A\,Y_G = \int_{y_1}^{y_2} y\, dA$$

Using Vertical Strip

$$A\,X_G = \int_{x_1}^{x_2} x\, dA$$

$$A\,Y_G = \int_{y_1}^{y_2} y_c\, dA$$

Centroid of Parabolic Segment & Spandrel

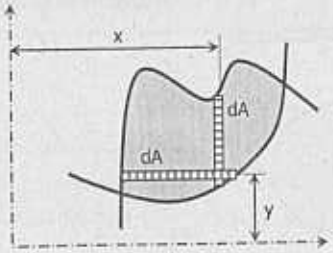


$$x = \frac{3}{8} b \qquad y = \frac{2}{5} h$$

$$x_1 = \frac{1}{4} b \qquad y_1 = \frac{3}{10} h$$

$$A_{parabola} = \frac{2}{3} bh \qquad A_{spandrel} = \frac{1}{3} bh$$

MOMENT OF INERTIA OF PLANE AREAS (FIRST MOMENT OF AREA)



$$I_x = \int_{y_1}^{y_2} y^2\, dA \qquad I_y = \int_{x_1}^{x_2} x^2\, dA$$

Polar moment of Inertia

$$J = I_x + I_y$$

Product of inertia

$$I_{xy} = \int xy\, dA$$

Mass Moment of Inertia

Problems concerned with the rotation of solid bodies involve the mathematical expression $\int \rho^2 \frac{dW}{g}$, which is known as the moment of inertia of a body, also called the *mass moment of inertia* because the ration W/g is widely known as the mass of a body.

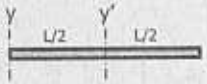
Solid Right Circular Cylinder

$$I = \frac{1}{2} M r^2; \quad M = \text{mass}; r = \text{radius}$$

Hallow Right Circular Cylinder

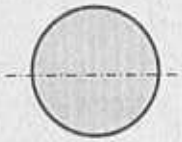
$$I = \frac{1}{2} M (R^2 - r^2); \quad R = \text{outer radius}$$

r = inner radius



Uniform Slender Rod

$$I_y = \frac{1}{3} M L^2; \quad I_{y'} = \frac{1}{12} M L^2$$

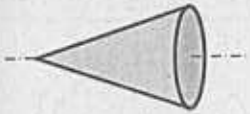


Solid Sphere

$$I = \frac{2}{5} M r^2$$

Spherical Shell

$$I = \frac{2}{3} M r^2; \quad r = \text{mean radius}$$



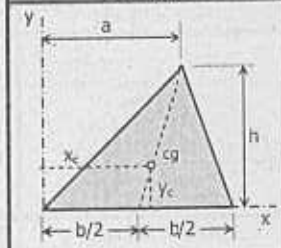
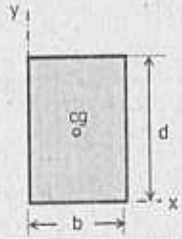
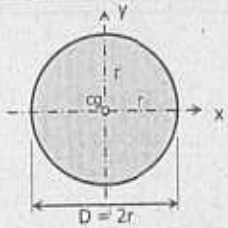
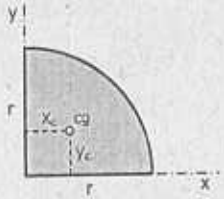
Right Circular Cone

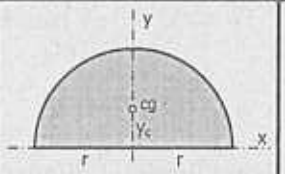
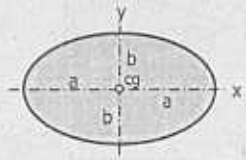
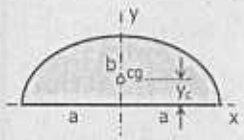
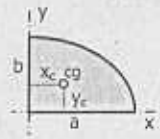
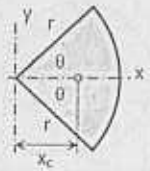
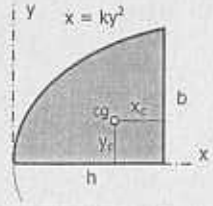
$$I = \frac{3}{10} M r^2; \quad r = \text{base radius}$$

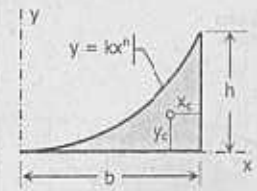
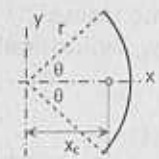
Right Elliptical Cylinder

$$I = \frac{1}{4} M(a^2 + b^2)$$

PROPERTIES OF COMMON SHAPES

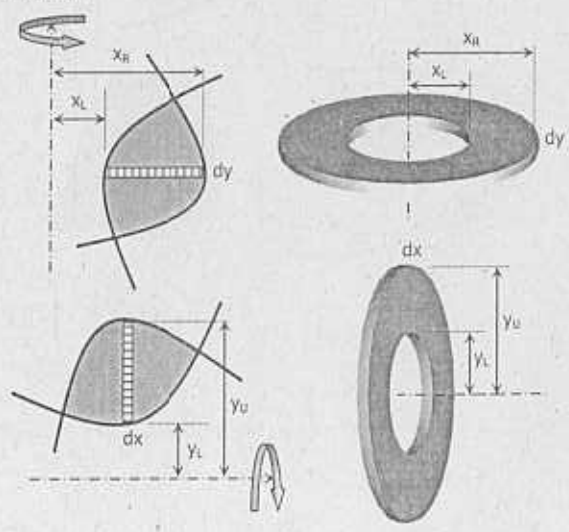
Triangle	Rectangle
	
$x_c = \frac{a+b}{3}$ $y_c = h/3$ Area = ½ b h $Ix = \frac{bh^3}{12}$ $I_{gx} = \frac{bh^3}{36}$	Area = bd $I_x = \frac{bd^3}{3}$ $I_y = \frac{db^3}{3}$ $I_{gx} = \frac{bd^3}{12}$ $I_{gy} = \frac{db^3}{12}$
Circle	**Quarter circle**
	
Area = $\pi r^2 = \frac{1}{4} \pi D^2$ $I_{gx} = I_{gy} = \frac{\pi r^4}{4} = \frac{\pi D^4}{64}$	Area = $\frac{1}{4} \pi r^2$ $x_c = y_c = \frac{4r}{3\pi}$ $I_x = I_y = \frac{\pi r^4}{16}$ $I_{gx} = I_{gy} = 0.055 r^4$

Semicircle	Ellipse
	
Area = $\frac{1}{2}\pi r^2$ $y_c = \frac{4r}{3\pi}$ $I_x = I_{gy} = \frac{\pi r^4}{8}$ $I_{gx} = 0.11\, r^4$	Area = $\pi a b$ $I_{gx} = \frac{\pi a b^3}{4}$ $I_{gy} = \frac{\pi b a^3}{4}$
Half ellipse	**Quarter ellipse**
	
Area = $\frac{1}{2} ab$ $y_c = \frac{4b}{3\pi}$ $I_x = \frac{\pi a b^3}{8}$ $I_{gx} = 0.11\, ab^3$ $I_{gy} = \frac{\pi b a^3}{8}$	Area = $\frac{1}{4} ab$ $x_c = \frac{4a}{3\pi}$ $y_c = \frac{4b}{3\pi}$ $I_x = \frac{ab^3}{16}$ $I_y = \frac{ba^3}{16}$ $I_{gx} = 0.055ab^3$ $I_{gy} = 0.055ba^3$
Sector of a circle	**Parabolic segment**
	

Area = $\frac{1}{2} r^2 (2\theta) = r^2 \theta$ $x_c = \frac{2}{3}\frac{r\sin\theta}{\theta}$ $I_x = \frac{r^4}{4}(\theta - \frac{1}{2}\sin 2\theta)$ $I_y = \frac{r^4}{4}(\theta + \frac{1}{2}\sin 2\theta)$	Area = $\frac{2}{3} bh$ $x_c = \frac{2}{5} h$ $y_c = \frac{3}{8} b$ $I_x = \frac{2}{15} hb^3$ $I_y = \frac{2}{7} bh^3$
SPANDREL	**Segment of arc**
	
Area = $\frac{1}{n+1} bh$ $x_c = \frac{1}{n+2} b$ $y_c = \frac{n+1}{4n+2} h$	Length of arc = $r(2\theta) = 2r\theta$ $x_c = \frac{r\sin\theta}{\theta}$ When $\theta = 90°$ (semicircle) $x_c = \frac{2r}{\pi}$

SOLIDS OF REVOLUTION

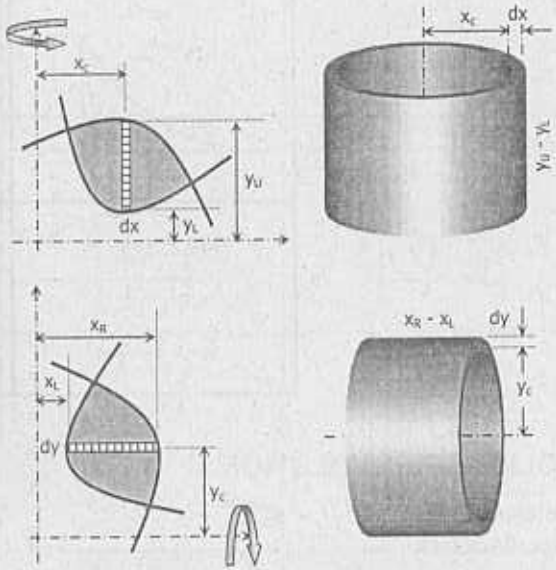
Volume Using Circular Disk



$$V = \pi \int_{y_1}^{y_2} \left(x_R^{\,2} - x_L^{\,2}\right) dy$$

$$\text{or} \quad V = \pi \int_{x_1}^{x_2} \left(y_U^{\,2} - y_L^{\,2}\right) dx$$

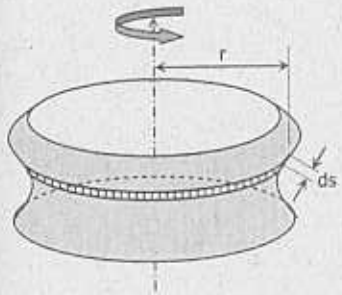
Volume Using Hallow Cylindrical Shell



$$V = \int_{x_1}^{x_2} 2\pi x_c (y_U - y_L)\, dx$$

$$\text{or} \quad V = \int_{y_1}^{y_2} 2\pi y_c (x_R - x_L)\, dy$$

Surface Area

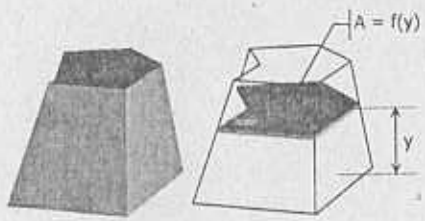


$$A_s = \int_{y_1}^{y_2} 2\pi r\, ds$$

$$ds = \sqrt{1 + (dx/dy)^2}$$

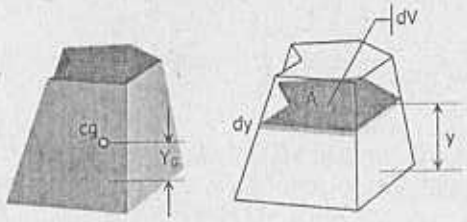
$$\text{or } ds = \sqrt{1 + (dy/dx)^2}$$

VOLUME OF OTHER SOLIDS WITH KNOWN CROSS SECTIONS



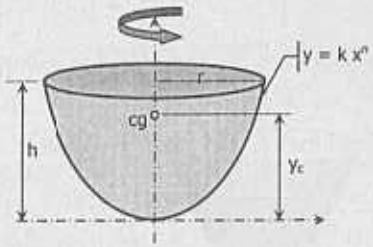
$$V = \int_{y_1}^{y_2} A\, dy$$

CENTROID OF VOLUME



$$V\, Y_G = \int_{y_1}^{y_2} y\, dV$$

where V = total volume of the body
$dV = A\, dy$
$A = f(y)$



$$\text{Volume} = k^{\frac{-2}{n}} \frac{\pi n}{2+n} h^{\frac{2+n}{n}}$$

$$y_c = \frac{2+n}{2+2n} h$$

WORK

Constant Force

The work done by a constant force F acting over a directed distance s along a straight line is:

$$\text{Work} = \text{Force} \times \text{distance} = F \times s$$

Work under Variable Force

$$\text{Work} = \int_{x_1}^{x_2} (dF)(x)$$

$$\text{or Work} = \int_{x_1}^{x_2} F(x)\,dx$$

Example

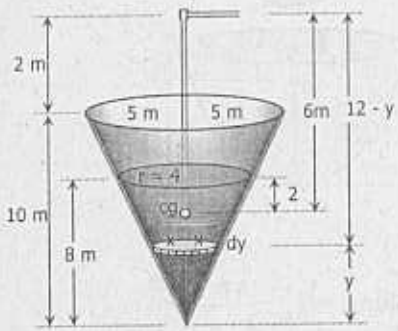
The conical tank shown contains liquid of unit weight γ. Find the work to empty the liquid at a point 2 m above the top of the tank.

By Integration:

$$\text{Work} = \int_{z_1}^{z_2} dF \times z$$

$$dF = dW = \gamma\, dV = \gamma\, \pi x^2\, dy$$

$$z = 12 - y$$



$$\frac{x}{y} = \frac{5}{10}$$

$x = 0.5y$

$dF = \gamma\,\pi\,(0.5y)^2\,dy = 0.25\,\pi\,\gamma\,y^2\,dy$

$$\text{Work} = \int_0^8 0.5\pi\gamma y^2\,dy(12-y)$$

$$\text{Work} = 0.25\pi\gamma \int_0^8 (12y^2 - y^3)dy$$

$$\text{Work} = 0.25\pi\gamma\left[4y^3 - \frac{y^4}{4}\right]_0^8 = 256\pi\gamma$$

By Formula:

Work = $F \times s$

F = force = total weight of liquid in the tank
s = distance from centroid of the liquid to point of exit

F = Weight = $\gamma \times$ Volume

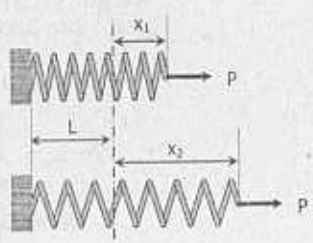
$$\text{Force} = \gamma \times \frac{1}{3}\pi(4)^2(8) = \frac{128\pi\gamma}{3}$$

$s = 6$ m

$$\text{Work} = \frac{128\pi\gamma}{3} \times 6 = 256\pi\gamma$$

Work required to Stretch a Spring

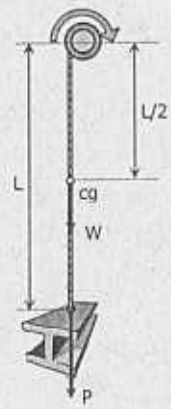
The work done in stretching a spring of natural length L from x_1 to x_2 is:



$$\text{Work} = \frac{1}{2}k\left(x_2^{\,2} - x_1^{\,2}\right)$$

where k = spring constant or stiffness in N/m

Work in Winding up a Load



If a load P is to be wind-up by a flexible rope or cable having a uniform mass and length L, the work done is:

$$\text{Work} = PL + W(L/2)$$

W = weight of the rope or cable
P = load at the end of the rope or cable

Problems - Set 3
Integration

PROBLEM 3 - 1
ME April 1997

Integrate: $(7x^3 + 4x^2)\,dx$

A. $\frac{7x^3}{3} + \frac{4x^2}{2} + C$
B. $\frac{7x^4}{4} + \frac{4x^2}{5} + C$
C. $\frac{7x^4}{4} + \frac{4x^3}{3} + C$
D. $7x^4 - \frac{4x^2}{2} + C$

PROBLEM 3 - 2
CE May 1999

Evaluate $\int \frac{4\,dx}{3x+2}$

A. $4 \ln (3x + 2) + C$
B. $\frac{4}{3} \ln (3x + 2) + C$
C. $\frac{1}{3} \ln (3x + 2) + C$
D. $2 \ln (3x + 2) + C$

PROBLEM 3 - 3
CE May 1994

Evaluate the integral of $e^{x^2+1}\ 2x\,dx$.

A. $\frac{e^{x^2+1}}{\ln 2} + C$
B. $e^{2x} + C$
C. $e^{x^2+1} + C$
D. $2x\,e^x + C$

PROBLEM 3 - 4
CE May 1995

What is the integral of $\cos 2x\ e^{\sin 2x}\,dx$?

A. $-e^{\sin 2x} + C$
B. $e^{\sin 2x} / 2 + C$
C. $e^{\sin 2x} + C$
D. $-e^{\sin 2x} / 2 + C$

PROBLEM 3 - 5
ME Oct. 1997

The integral of $\cos x$ with respect to x; $\int \cos x\,dx$ is:

A. $\sec x + C$
B. $\sin x + C$
C. $\cos x + C$
D. $-\sin x + C$

PROBLEM 3 - 6
ME April 1998

Integrate $x \cos (2x^2 + 7)\,dx$.

A. $\frac{1}{4} \sin (2x^2 + 7) + C$
B. $\sin (2x^2 + 7) + C$
C. $\frac{1}{4} \cos (2x^2 + 7)$
D. $\frac{1}{4} (\sin -\theta)\,(x^2 + 7) + C$

PROBLEM 3 - 7

Evaluate $\int \frac{\tan(\ln x)}{x}\,dx$.

A. $\ln \cos (\ln x) + C$
B. $\ln \sec (\ln x) + C$
C. $\frac{1}{2} \tan^2 (\ln x) + C$
D. $\tan (\ln x)^2 + C$

PROBLEM 3 - 8

Evaluate $\int (\cos x)(\ln \sin x)\,dx$.

A. $\sin x\,(1 - \ln \sin x) + C$
B. $\sin x\,(1 + \ln \sin x) + C$
C. $\sin x\,(\ln \sin x - 1) + C$
D. $\ln \sqrt{\sin x} + C$

PROBLEM 3 - 9
CE May 1992

Evaluate $\int \tan \theta \ln \sec \theta\,d\theta$.

A. $2(\ln \sec \theta)^2 + C$
B. $(\ln \sec \theta)^2 + C$
C. $\frac{1}{2} (\ln \sec \theta) + C$
D. $\frac{1}{2}(\ln \sec \theta)^2 + C$

PROBLEM 3 - 10

Evaluate $\int \frac{e^x\,dx}{1+e^{2x}}$.

A. $\frac{1}{2} \ln(1+e^{2x}) + C$
B. $\ln(1+e^{2x}) + C$
C. $\frac{1}{2} (1+e^{2x})^2 + C$
D. $\arctan (e^x) + C$

PROBLEM 3 - 11

Evaluate $\int \frac{dx}{\ln x^x \sqrt{(\ln x)^2 - 1}}$

A. $\operatorname{arcsec} (\ln x) + C$
B. $\frac{2}{3} [(\ln x)^2 - 1]^{3/2} + C$
C. $\ln \sqrt{(\ln x)^2 - 1} + C$
D. $\arcsin (\ln x) + C$

PROBLEM 3 - 12

Evaluate $\int \frac{x^3+1}{x+2}\,dx$.

A. $x^3 + x^2 + 4x + 7 \ln (x + 2) + C$
B. $x^3 - \frac{x^2}{2} + 4x - \ln (x + 2) + C$
C. $\frac{x^3}{3} - x^2 + 4x - 7 \ln (x + 2) + C$
D. $\frac{x^3}{3} + x^2 + 4x - \ln (x + 2) + C$

PROBLEM 3 - 13

Evaluate $\int \frac{e^{2x}}{e^x + 1}\,dx$.

A. $\frac{1}{2} e^x + \ln (e^x + 1) + C$
B. $e^x - \ln (e^x + 1) + C$
C. $\ln (e^x + 1) + C$
D. $e^x + \ln (e^x + 1) + C$

PROBLEM 3 - 14

Evaluate $\int \frac{3dx}{x(x+3)}$.

A. $3 \ln x(x+3) + C$
B. $\frac{1}{3} \ln \frac{x}{x+3}$
C. $\ln \frac{x}{x+3} + C$
D. $\ln x(x+3) + C$

PROBLEM 3 - 15

Evaluate the integral of $x \sin 2x\, dx$.

A. $-\frac{x}{4} \cos 2x + \frac{1}{2} \sin 2x + C$
B. $-\frac{x}{2} \cos 2x - \frac{1}{4} \sin 2x + C$
C. $\frac{x}{4} \cos 2x - \frac{1}{2} \sin 2x + C$
D. $-\frac{x}{2} \cos 2x + \frac{1}{4} \sin 2x + C$

PROBLEM 3 - 16

Evaluate $\int e^x \sin x\, dx$.

A. $\frac{e^x}{2}(\sin x - \cos x) + C$
B. $-e^x (\cos x + \sin x) + C$
C. $-e^x (\sin x - \cos x) + C$
D. $e^x (\cos x + \sin x) + C$

PROBLEM 3 - 17

Evaluate $\int \arctan x\, dx$.

A. $\arctan x - \ln \sqrt{1+x^2} + C$
B. $\arctan x + 2 \ln (1+x^2) + C$
C. $x \arctan x - \ln \sqrt{1+x^2} + C$
D. $x \arctan x - 2 \ln (1+x^2) + C$

PROBLEM 3 - 18
ECE Nov. 1998

Integrate the square root of $(1 - \cos x)\, dx$.

A. $\sqrt{2} \cos x + C$
B. $-2\sqrt{2} \cos \frac{x}{2} + C$
C. $-2 \sin \frac{x}{2} + C$
D. $-\sqrt{2} \cos \frac{x}{2} + C$

PROBLEM 3 - 19
CE Nov. 1999

Evaluate the integral of $x \cos 2x\, dx$ with limits from 0 to $\pi/4$.

A. 0.143
B. 0.258
C. 0.114
D. 0.186

PROBLEM 3 - 20
CE May 1997

Evaluate the integral of $x(x-5)^{12}\, dx$ with limits from 5 to 6.

A. 81/182
B. 82/182
C. 83/182
D. 84/182

PROBLEM 3 - 21
CE Nov. 1996

Evaluate the integral of $\frac{x dx}{(x+1)^8}$ if it has an upper limit of 1 and a lower limit of 0.

A. 0.022
B. 0.056
C. 0.043
D. 0.031

PROBLEM 3 - 22
CE May 1996

Find the integral of $12 \sin^5 x \cos^5 x\, dx$ if lower limit = 0 and upper limit = $\pi/2$.

A. 0.2
B. 0.8
C. 0.6
D. 0.4

PROBLEM 3 - 23
CE Nov. 1997
similar to
CE Nov. 1994

Using lower limit = 0 and upper limit = $\pi/2$, what is the integral of $15 \sin^7 x\, dx$?

A. 6.783
B. 6.857
C. 6.648
D. 6.539

PROBLEM 3 - 24
CE Nov. 1998

Evaluate the integral of 3 (sin x)^3 dx using lower limit of 0 and upper limit = $\pi/2$.

A. 2.0
B. 1.7
C. 1.4
D. 2.3

PROBLEM 3 - 25
CE May 1998,
similar to
CE Nov. 95 &
May 96

Evaluate the integral of $5 \cos^6 x \sin^2 x\, dx$ using lower limit = 0 and upper limit = $\pi/2$

A. 0.5046
B. 0.3068
C. 0.6107
D. 0.4105

PROBLEM 3 - 26
ECE April 1998

Evaluate the integral $\cos^8 3A\, dA$ from 0 to $\pi/6$.

A. $35\pi/768$
B. $23\pi/765$
C. $27\pi/363$
D. $12\pi/81$

PROBLEM 3 - 27
CE Nov. 1996

Evaluate the integral of $(3x^2 + 9y^2)\, dx\, dy$ if the interior limit has an upper limit of y and a lower limit of 0, and whose outer limit has an upper limit of 2 and lower limit of 0.

A. 10
B. 40
C. 30
D. 20

PROBLEM 3 - 28
CE May 1999

Evaluate $\int_1^2 \int_0^{2y} (x^2 + y^2)\, dx\, dy$.

A. 35/2
B. 19/2
C. 17/2
D. 37/2

PROBLEM 3 - 29
EE April 1997

Evaluate the double integral of $r \sin u \, dr \, du$, the limits of r is from 0 to $\cos u$ and the limits of u is from 0 to pi.

A. -1/6
B. 1/6
C. 1/3
D. 1/2

PROBLEM 3 - 30

Evaluate $\int_0^1 \int_0^z \int_0^y dx \, dy \, dz$.

A. 1/3
B. 1/4
C. 1/2
D. 1/6

PROBLEM 3 - 31
CE Nov. 2002

Determine the value of the integral of $\sin^5 3x \, dx$ with limits from 0 to $\pi/6$.

A. 0.324
B. 0.178
C. 0.275
D. 0.458

PROBLEM 3 - 32
CE Nov. 2001

Evaluate the integral of $x \cos(4x) \, dx$ with lower limit of 0 and upper limit of $\pi/4$.

A. 1/8
B. -1/8
C. 1/16
D. -1/16

PROBLEM 3 - 33

It is known that $f(x) = 4$ when $x = 2$. What is the value of $f(x)$ when $x = 0$.

$$f(x) = \int (3x^3 - 7x)\,dx$$

A. 2
B. 8
C. 0
D. 6

PROBLEM 3 - 34

Evaluate $\int_0^9 \frac{dx}{\sqrt{1+\sqrt{x}}}$

A. 6.667
B. 5.333
C. 4.667
D. 7.333

PROBLEM 3 - 35

A body moves such that its acceleration as a function of time is $a = 2 + 12t$, where "t" is in minutes and "a" is in m/min². Its velocity after 1 minute is 11 m/min. Find distance traveled after 4 minutes.

A. 135 m
B. 186 m
C. 121 m
D. 156 m

PROBLEM 3 - 36

The value of $\int_2^4 \frac{x+1}{x^2 - x} dx$ is closest to

A. 2.3
B. 1.9
C. 1.5
D. 3.2

ANSWER SHEET

1.	A	B	C	D	E	21.	A	B	C	D	E	41.	A	B	C	D	E
2.	A	B	C	D	E	22.	A	B	C	D	E	42.	A	B	C	D	E
3.	A	B	C	D	E	23.	A	B	C	D	E	43.	A	B	C	D	E
4.	A	B	C	D	E	24.	A	B	C	D	E	44.	A	B	C	D	E
5.	A	B	C	D	E	25.	A	B	C	D	E	45.	A	B	C	D	E
6.	A	B	C	D	E	26.	A	B	C	D	E	46.	A	B	C	D	E
7.	A	B	C	D	E	27.	A	B	C	D	E	47.	A	B	C	D	E
8.	A	B	C	D	E	28.	A	B	C	D	E	48.	A	B	C	D	E
9.	A	B	C	D	E	29.	A	B	C	D	E	49.	A	B	C	D	E
10.	A	B	C	D	E	30.	A	B	C	D	E	50.	A	B	C	D	E
11.	A	B	C	D	E	31.	A	B	C	D	E	51.	A	B	C	D	E
12.	A	B	C	D	E	32.	A	B	C	D	E	52.	A	B	C	D	E
13.	A	B	C	D	E	33.	A	B	C	D	E	53.	A	B	C	D	E
14.	A	B	C	D	E	34.	A	B	C	D	E	54.	A	B	C	D	E
15.	A	B	C	D	E	35.	A	B	C	D	E	55.	A	B	C	D	E
16.	A	B	C	D	E	36.	A	B	C	D	E	56.	A	B	C	D	E
17.	A	B	C	D	E	37.	A	B	C	D	E	57.	A	B	C	D	E
18.	A	B	C	D	E	38.	A	B	C	D	E	58.	A	B	C	D	E
19.	A	B	C	D	E	39.	A	B	C	D	E	59.	A	B	C	D	E
20.	A	B	C	D	E	40.	A	B	C	D	E	60.	A	B	C	D	E

Solutions to Set 3 Integration

SOLUTION 3 - 1
Ans: C

$$\int (7x^3 + 4x^2)\,dx = \frac{7x^4}{4} + \frac{4x^3}{3} + C$$

SOLUTION 3 - 2
Ans: B

$$\int \frac{4\,dx}{3x+2} = 4 \times \frac{1}{3} \int \frac{3\,dx}{3x+2} = \frac{4}{3} \ln(3x+2) + C$$

SOLUTION 3 - 3
Ans: C

$$\int e^{x^2+1}\, 2x\,dx$$

From $\int e^u\,du = e^u + C$

$u = x^2 + 1;\ du = 2x\,dx$ (OK)

$$\int e^{x^2+1}\, 2x\,dx = e^{x^2+1} + C$$

SOLUTION 3 - 4
Ans: B

$$\int (\cos 2x)\, e^{\sin 2x}\,dx$$

From $\int e^u\,du = e^u + C$

$u = \sin 2x;\ du = 2\cos 2x\,dx$ (OK)

$$\int (\cos 2x)\, e^{\sin 2x}\,dx = \tfrac{1}{2} \int e^{\sin 2x}(2\cos 2x\,dx)$$

$$= e^{\sin 2x} / 2 + C$$

SOLUTION 3 - 5
Ans: B

$$\int \cos x\,dx = \sin x + C$$

SOLUTION 3 - 6
Ans: A

$$\int x\cos(2x^2+7)\,dx = \frac{1}{4}\int \cos(2x^2+7)(4x\,dx)$$

$$= \tfrac{1}{4}\sin(2x^2+7) + C$$

SOLUTION 3 - 7
Ans: B

$$\int \frac{\tan(\ln x)}{x}\,dx \qquad \text{From } \int \tan u\,du = \ln(\sec u) + C$$

$u = \ln x,\ du = \dfrac{dx}{x}$ (OK)

$$\int \frac{\tan(\ln x)}{x}\,dx = \int \tan(\ln x)\frac{dx}{x} = \ln[\sec(\ln x)] + C$$

SOLUTION 3 - 8
Ans: C

$$\int (\cos x)(\ln \sin x)\,dx$$

From $\int \ln u\,du = u\ln u - u + C$

$u = \sin x;\ du = \cos x\,dx$ (OK)

$$\int (\cos x)(\ln \sin x)\,dx = \sin x \ln(\sin x) - \sin x + C$$

$$= \sin x(\ln \sin x - 1) + C$$

SOLUTION 3 - 9
Ans: D

$$\int \tan\theta \ln\sec\theta\,d\theta$$

From $\int u^n\,du = \dfrac{1}{n+1}u^{n+1}$

$u = \ln\sec\theta$

$du = \dfrac{\sec\theta\tan\theta\,d\theta}{\sec\theta} = \tan\theta\,d\theta$ (OK)

$$\int \tan\theta\ln\sec\theta\,d\theta = \int (\ln\sec\theta)\tan\theta\,d\theta = \tfrac{1}{2}(\ln\sec\theta)^2 + C$$

SOLUTION 3 - 10
Ans: D

$$\int \frac{e^x\,dx}{1+e^{2x}}$$

From $\int \dfrac{du}{1+u^2} = \arctan u + C$

$u = e^x;\ du = dx$ (OK)

$$\int \frac{e^x\,dx}{1+e^{2x}} = \arctan(e^x) + C$$

SOLUTION 3 - 11
Ans: A

$$\int \frac{dx}{\ln x^2 \sqrt{(\ln x)^2 - 1}} = \int \frac{dx}{x \ln x \sqrt{(\ln x)^2 - 1}}$$

$$= \int \frac{\frac{dx}{x}}{\ln x \sqrt{(\ln x)^2 - 1}}$$

From $\int \frac{du}{u\sqrt{u^2 - 1}} = \text{arcsec } u + C$

$u = \ln x$, $du = dx / x$ (OK)

$$\int \frac{\frac{dx}{x}}{\ln x \sqrt{(\ln x)^2 - 1}} = \text{arcsec } (\ln x) + C$$

SOLUTION 3 - 12
Ans: C

First we divide $x^3 + 1$ by $x + 2$:

$$\begin{array}{r} x^2 - 2x + 4 \\ x + 2 \overline{\big) x^3 + 1} \\ - \quad \underline{x^3 + 2x^2} \\ -2x^2 + 1 \\ - \quad \underline{-2x^2 - 4x} \\ 4x + 1 \\ - \quad \underline{4x + 8} \\ -7 \end{array}$$

$$\int \frac{x^3 + 1}{x + 2} dx = \int \left(x^2 - 2x + 4 - \frac{7}{x + 2} \right) dx$$

$$= \frac{x^3}{3} - x^2 + 4x - 7 \ln (x + 2) + C$$

SOLUTION 3 - 13
Ans: B

Divide e^{2x} by $(e^x + 1)$

$$\begin{array}{r} e^x \\ e^x + 1 \overline{\big) e^{2x}} \\ - \quad \underline{e^{2x} + e^x} \\ -e^x \end{array}$$

$$\int \frac{e^{2x}}{e^x + 1} dx = \int \left(e^x - \frac{e^x}{e^x + 1} \right) dx = e^x - \ln (e^x + 1) + C$$

SOLUTION 3 - 14
Ans: C

Resolve $\frac{3}{x(x+3)}$ into partial fraction.

$$\frac{3}{x(x+3)} = \frac{A}{x} + \frac{B}{x+3}$$

$3 = A(x + 3) + Bx$

Set $x = 0$: $3 = A(0 + 3) + B(0)$; $A = 1$
Set $x = -3$: $3 = A(-3 + 3) + B(-3)$; $B = -1$

$$\int \frac{3dx}{x(x+3)} = \int \left(\frac{1}{x} - \frac{1}{x+3} \right) dx = \ln x - \ln (x + 3)$$

$$= \ln \frac{x}{x+3} + C$$

SOLUTION 3 - 15
Ans: D

Using integration by parts:

$u = x$ $\quad dv = \sin 2x\, dx$
$du = dx$ $\quad v = -\frac{1}{2} \cos 2x$

$$\int x \sin 2x\, dx = x(-\tfrac{1}{2} \cos 2x) - \int -\frac{1}{2} \cos 2x\, dx$$

$$= -\frac{x}{2} \cos 2x + \frac{1}{4} \sin 2x + C$$

SOLUTION 3 - 16
Ans: A

Using integration by parts:

$u = e^x$ $\quad dv = \sin x\, dx$
$du = e^x\, dx$ $\quad v = -\cos x$

$$\int e^x \sin x\, dx = e^x (-\cos x) - \int -\cos x\, e^x\, dx$$

$$= -e^x \cos x + \int e^x \cos x\, dx$$

Applying integration by parts to $\int e^x \cos x\, dx$:

$u = e^x$ $\quad dv = \cos x\, dx$
$du = e^x\, dx$ $\quad v = \sin x$

$$\int e^x \sin x\, dx = -e^x \cos x + e^x \sin x - \int \sin x\, e^x\, dx$$

$$2\int e^x \sin x\,dx = -e^x \cos x + e^x \sin x$$

$$\int e^x \sin x\,dx = \frac{e^x}{2}(\sin x - \cos x) + C$$

SOLUTION 3 - 17
Ans: C

Using integration be parts:

$u = \arctan x \qquad dv = dx$

$du = \dfrac{dx}{1+x^2} \qquad v = x$

$$\int \arctan x\,dx = (\arctan x)\,x - \int x\frac{dx}{1+x^2}$$

$$\int \arctan x\,dx = x\arctan x - \tfrac{1}{2}\ln(1+x^2) + C$$

$$\int \arctan x\,dx = x\arctan x - \ln\sqrt{1+x^2} + C$$

SOLUTION 3 - 18
Ans: B

$$\int \sqrt{1-\cos x}\,dx$$

Note: $\sin(x/2) = \sqrt{(1-\cos x)/2}$

$\sqrt{1-\cos x} = \sqrt{2}\,\sin(x/2)$

$$\int \sqrt{1-\cos x}\,dx = \int \sqrt{2}\sin(x/2)\,dx$$

$$= \sqrt{2} \times 2\int \sin\frac{x}{2}\frac{dx}{2}$$

$$= -2\sqrt{2}\cos\frac{x}{2} + C$$

SOLUTION 3 - 19
Ans: A

$$\int_0^{\pi/4} x\cos 2x\,dx$$

Using integration by parts:

$u = x \qquad dv = \cos 2x\,dx$

$du = dx \qquad v = \tfrac{1}{2}\sin 2x$

$$\int_0^{\pi/4} x\cos 2x\,dx = \left[\frac{1}{2}x\sin 2x\right]_0^{\pi/4} x(\tfrac{1}{2}\sin 2x)$$

$$- \int_0^{\pi/4} \frac{1}{2}\sin 2x\,dx$$

$$= \left[\frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x\right]_0^{\pi/4} \qquad \text{Note: } \pi/4 = 45°$$

$$= \left[\frac{1}{2}\frac{\pi}{4}\sin(2\cdot 45°) + \frac{1}{4}\cos(2\cdot 45°)\right]$$

$$- \left[\frac{1}{2}(0)\sin(2\cdot 0°) + \frac{1}{4}\cos(2\cdot 0°)\right]$$

$$= 0.3927 - 0.25 = \mathbf{0.1427}$$

SOLUTION 3 - 20
Ans: C

Using integration by parts:

Let $u = x \qquad du = dx$

$dv = (x-5)^{12}\,dx \qquad v = \dfrac{1}{13}(x-5)^{13}$

$$\int u\,dv = uv - \int v\,du$$

$$\int_5^6 (x-5)^{12}x\,dx = x\frac{1}{13}(x-5)^{13} - \int_5^6 \frac{1}{12}(x-5)^{13}\,dx$$

$$= \frac{x}{13}(x-5)^{13} - \frac{1}{182}(x-5)^{14}\Big]_5^6$$

$$\int_5^6 (x-5)^{12}x\,dx = \left\{(x-5)^{13}\left[\frac{x}{13} - \frac{1}{182}(x-5)\right]\right\}_5^6$$

$$= \left[(x-5)\left(\frac{x}{14} + \frac{5}{182}\right)\right]_5^6$$

$$= \left[(6-5)\left(\frac{6}{14} + \frac{5}{182}\right) - (5-5)\left(\frac{5}{14} + \frac{5}{182}\right)\right]$$

$$= \frac{83}{182}$$

SOLUTION 3 - 21
Ans: A

$$\int_0^1 \frac{x}{(x+1)^8}dx$$

By algebraic substitution:

Let $u = x + 1$; $x = u - 1$

$dx = du$

Limits:

When $x = 1$; $u = 2$ and when $x = 0$; $u = 1$

$$\int_0^1 \frac{x}{(x+1)^8}dx = \int_1^2 \frac{u-1}{u^8}du = \int_1^2 (u^{-7} - u^{-8})du$$

$$= \left[\frac{1}{-6}u^{-6} - \frac{1}{-7}u^{-7}\right]_1^2$$

$$= \left[\frac{(2)^{-6}}{-6} - \frac{(2)^{-7}}{-7}\right] - \left[\frac{(1)^{-6}}{-6} - \frac{(1)^{-7}}{-7}\right]$$

$$= 0.0223$$

SOLUTION 3 - 22
Ans: A

By Wallis' Formula:

$$\int_0^{\pi/2} 12\sin^5 x\cos^5 x\,dx = 12\frac{[(4)(2)][(4)(2)]}{10(8)(6)(4)(2)} \times 1 = 0.2$$

SOLUTION 3 - 23
Ans: B

By Wallis' formula:

$$\int_0^{\pi/2} 15\sin^7 x\,dx = 15\frac{6(4)(2)}{7(5)(3)(1)} = 6.857$$

SOLUTION 3 - 24
Ans: A

$$\int_0^{\pi/2} 3\sin^3 x\,dx = 3\frac{2}{3(1)}(1) = 2$$

SOLUTION 3 - 25
Ans: B

By Wallis' Formula:

$$\int_0^{\pi/2} 5\cos^6 x\sin^2 x\,dx = 5\,\frac{(5)(3)(1)\times(1)}{(8)(6)(4)(2)} \times \frac{\pi}{2} = 0.3068$$

SOLUTION 3 - 26
Ans: A

$$\int_0^{\pi/6} \cos^8 3A\,dA\,; \quad \text{Let } \theta = 3A;\ d\theta = 3dA;\ dA = d\theta/3$$

Limits:

When $A = 0$, $\theta = 0$

When $A = \pi/6$, $\theta = \pi/2$

Then,

$$\int_0^{\pi/6} \cos^8 3A\,dA = \frac{1}{3}\int_0^{\pi/2} \cos^8\theta\,d\theta$$

By Wallis' formula:

$$\frac{1}{3}\int_0^{\pi/2} \cos^8\theta\,d\theta = \frac{1}{3}\frac{7(5)(3)(1)}{8(6)(4)(2)}\frac{\pi}{2} = 35\pi/768$$

SOLUTION 3 - 27
Ans: B

$$\int_0^2\int_0^y (3x^2 + 9y^2)dx\,dy = \int_0^2 \left[x^3 + 9y^2x\right]_0^y dy$$

$$= \int_0^2 (y^3 + 9y^3)\,dy = \int_0^2 10\,y^3\,dy$$

$$= 10\left[\frac{y^4}{4}\right]_0^2 = 2.5\,(2^4 - 0^4) = 40$$

SOLUTION 3 - 28
Ans: A

$$\int_1^2\int_0^{2y} (x^2 + y^2)dx\,dy = \int_1^2 \left[\frac{x^3}{3} + y^2x\right]_0^{2y} dy$$

$$= \int_1^2 \left[\frac{(2y)^3}{3} + y^2(2y)\right]_0^{2y} dy$$

$$= \frac{14}{3}\int_1^2 y^3\,dy = \frac{14}{3}\left[\frac{y^4}{4}\right]_1^2 = \frac{14}{3}\,\frac{2^4 - 1^4}{4} = \frac{35}{2}$$

SOLUTION 3 - 29
Ans: C

$$\int_0^{\pi}\int_0^{\cos u} r\sin u\,dr\,du = \int_0^{\pi}\left[\sin u\frac{r^2}{2}\right]_0^{\cos u} du$$

$$= \frac{1}{2}\int_0^{\pi} \cos^2 u\sin u\,du = -\frac{1}{2}\int_0^{\pi} \cos^2 u(-\sin u\,du)$$

$$= -\frac{1}{2}\left[\frac{\cos^3 u}{3}\right]_0^{\pi} \qquad \text{Note: } \pi = 180°$$

$$= -\frac{1}{6}\left(\cos^3 180° - \cos^3 0\right) = -\frac{1}{6}(-1 - 1) = \frac{1}{3}$$

SOLUTION 3 - 30
Ans: D

$$\int_0^1\int_0^z\int_0^y dx\,dy\,dz = \int_0^1\int_0^z \left[x\right]_0^y dy\,dz = \int_0^z\int_0^z y\,dy\,dz$$

$$= \int_0^1 \left[\frac{y^2}{2}\right]_0^z dz = \int_0^1 \frac{z^2}{2}dz = \left[\frac{z^3}{6}\right]_0^1 = \frac{1}{6}$$

SOLUTION 3 - 31
Ans: B

$$\int_0^{\frac{\pi}{6}} \sin^5 3x\,dx$$

Let $\theta = 3x$, $x = \frac{1}{3}\theta$

$dx = \frac{1}{3}d\theta$

Limits: when $x = 0$, $\theta = 0$

when $x = \frac{\pi}{6}$, $\theta = \frac{\pi}{2}$

$$\int_0^{\frac{\pi}{6}} \sin^5 3x\,dx = \int_0^{\frac{\pi}{2}} (\sin^5\theta)\left(\tfrac{1}{3}d\theta\right) = \tfrac{1}{3}\int_0^{\frac{\pi}{2}} \sin^5\theta\,d\theta$$

$$\tfrac{1}{3}\int_0^{\frac{\pi}{2}} \sin^5\theta\,d\theta = \tfrac{1}{3}\,\frac{4(2)}{5(3)(1)} \times 1 = 0.178$$

SOLUTION 3 - 32
Ans: B

$$\int_0^{\pi/4} x\cos 4x\,dx\,; \qquad \text{From } \int u\,dv = uv - \int v\,du$$

$u = x$; $du = dx$; $dv = \cos 4x\,dx$; $v = \frac{1}{4}\sin 4x$

$$\int_0^{\pi/4} x\cos 4x\,dx = x\,\frac{1}{4}\sin 4x - \int \frac{1}{4}\sin 4x\,dx$$

$$= \left[\frac{1}{4}x\sin 4x + \frac{1}{16}\cos 4x\right]_0^{\pi/4}; \text{ Note } \frac{\pi}{4} = 45°$$

$$= \left[\frac{1}{4}\frac{\pi}{4}\sin 180° + \frac{1}{16}\cos 180°\right] - \left[\frac{1}{4}(0)\sin 0° + \frac{1}{16}\cos 0°\right]$$

$= -1/8$

SOLUTION 3 - 33
Ans: D

$$f(x) = \int (3x^3 - 7x)\,dx = \tfrac{3}{4}x^4 - \tfrac{7}{2}x^2 + C$$

$f(2) = \frac{3}{4}(2)^4 - \frac{7}{2}(2)^2 + C = 4$; $C = 6$

$f(0) = \frac{3}{4}(0)^4 - \frac{7}{2}(0)^2 + 6 = 6$

SOLUTION 3 - 34
Ans: B

Let $u = \sqrt{1+\sqrt{x}}$

$u^2 = 1 + \sqrt{x}$; $\sqrt{x} = u^2 - 1$

$x = (u^2-1)^2$; $dx = 2(u^2-1)(2u)\,du = 4(u^3-u)du$

When $x = 0$; $u = 1$

When $x = 9$; $u = 2$

$$\int_0^9 \frac{dx}{\sqrt{1+\sqrt{x}}} = \int_1^2 \frac{4(u^3-u)}{u}du = 4\int_0^2 (u^2-1)\,du$$

$$= 4\left[\frac{u^3}{3} - u\right]_1^2 = 4\left[\frac{2^3-1^3}{3} - (2-1)\right] = 5.333$$

SOLUTION 3 - 35
Ans: D

$a = \frac{dv}{dt} = 2 + 12t \qquad dv = (2+12t)\,dt$

Integrate both sides: $v = 2t + 6t^2 + C$

When $t = 1$ min, $v = 11$ m/min:

$11 = 2(1) + 6(1)^2 + C$; $C = 3$

$v = \frac{dS}{dt} = 2t + 6t^2 + 3$; $dS = (2t + 6t^2 + 3)\,dt$

$$\int_0^S dS = \int_0^4 (2t + 6t^2 + 3)dt\,; \quad S = \left[t^2 + 2t^3 + 3t\right]_0^4 = 156 \text{ m}$$

SOLUTION 3 - 36
Ans: C

Decompose $\frac{x+1}{x^2-x}$ into partial fractions:

$$\frac{x+1}{x^2-x} = \frac{x+1}{x(x-1)} = \frac{A}{x} + \frac{B}{x-1}$$

$x + 1 = A(x-1) + Bx$

Setting $x = 1$; $B = 2$

Setting $x = 0$; $A = -1$

$$\text{Thus, } \int_2^4 \frac{x+1}{x^2-x}dx = \int_2^4 \left[\frac{-1}{x} + \frac{2}{x-1}\right]dx$$

$$= \left[-\ln(x) + 2\ln(x-1)\right]\Big|_2^4 = \left[\ln\frac{(x-1)^2}{x}\right]\Bigg|_2^4$$

$= \ln\frac{9}{4} - \ln\frac{1}{2} = 1.504$

Problems - Set 4
Plane Areas, Volumes, Surfaces, Centroid, Etc.

PROBLEM 4 - 1

Find the area under the curve $y = x^3 + 3x^2$ and the x-axis between $x = 1$ and $x = 3$.

A. 28 sq. units
B. 46 sq. units
C. 36 sq. units
D. 54 sq. units

PROBLEM 4 - 2
ECE April 1995

Find the area bounded by $y = (11 - x)^{1/2}$, the lines $3x = 2$ and $x = 10$, and the X-axis.

A. 19.456 sq. units
B. 20.567 sq. units
C. 22.567 sq. units
D. 21.478 sq. units

PROBLEM 4 - 3

Find the area of the region bounded by the curves $y = \dfrac{12x}{x^2 + 4}$, the x-axis, $x = 1$, and $x = 4$.

A. 4 ln 6
B. ln 24
C. 6 ln 15
D. 6 ln 4

PROBLEM 4 - 4
ECE Nov. 1996

Find the area bounded by the y-axis and $x = 4 - y^{2/3}$.

A. 25.6
B. 28.1
C. 12.8
D. 56.2

PROBLEM 4 - 5

Find the area of the region bounded by one loop of the curve $x^2 = y^4 (1 - y^2)$.

A. π sq. units
B. $(\pi/2)$ sq. units
C. $(\pi/4)$ sq. units
D. $(\pi/8)$ sq. units

PROBLEM 4 - 6
CE Nov. 1996, Nov. 1998

Find the area bounded by the curve $r^2 = a^2 \cos 2\theta$

A. $3a^2$
B. a^2
C. $4a^2$
D. $2a^2$

PROBLEM 4 - 7
CE Nov. 1997

What is the area within the curve $r^2 = 16 \cos \theta$?

A. 26
B. 28
C. 30
D. 32

PROBLEM 4 - 8
CE May 1999

Find the area enclosed by $r^2 = 2a^2 \cos \theta$

A. $2a^2$
B. a^2
C. $4a^2$
D. $3a^2$

PROBLEM 4 - 9

Find the curved surface (area) of the solid generated by revolving the part of the curve $y = x^2$ from (0, 0) to $(\sqrt{6}, 6)$ about the y-axis.

A. 62π sq. units
B. $62\pi/3$ sq. units
C. $62\pi/5$ sq. units
D. $5/62\pi$ sq. units

PROBLEM 4 - 10

Find the volume generated by rotating the region bounded by $y = x$, $x = 1$, and $y^2 = 4x$, about the x-axis.

A. π
B. 2π
C. 3π
D. 9π

PROBLEM 4 - 11
CE Nov. 1995

The area bounded by the curve $y^2 = 12x$ and the line $x = 3$ is revolved about the line $x = 3$. What is the volume generated?

A. 186
B. 179
C. 181
D. 184

PROBLEM 4 - 12
CE May 1995

Given is the area in the first quadrant bounded by $x^2 = 8y$, the line $x = 4$ and the x-axis. What is the volume generated by revolving this area about the y- axis?

A. 50.26
B. 52.26
C. 53.26
D. 51.26

PROBLEM 4 - 13
CE Nov. 1994

Given is the area in the first quadrant bounded by $x^2 = 8y$, the line $y - 2 = 0$ and the y-axis. What is the volume generated when this area is revolved about the line $y - 2 = 0$?

A. 28.41
B. 26.81
C. 27.32
D. 25.83

PROBLEM 4 - 14
CE May 1998

Find the length of the arc of $x^2 + y^2 = 64$ from $x = -1$ to $x = -3$, in the second quadrant.

A. 2.24
B. 2.61
C. 2.75
D. 2.07

PROBLEM 4 - 15

How far from the y-axis is the centroid of the area bounded by the curve $y = x^3$, the line $x = 2$, and the x-axis.

A. 1.2
B. 1.4
C. 1.6
D. 1.8

PROBLEM 4 - 16
CE May 1998

The area in the first quadrant, bounded by the curve $y = 2x^{1/2}$, the y-axis and the line $y - 6 = 0$ is revolved about the line $y = 6$. Find the centroid of the solid formed.

A. (2.2,6)
B. (1.6,6)
C. (1.8,6)
D. (2.0,6)

PROBLEM 4 - 17

A solid is formed by revolving about the y-axis, the area bounded by the curve $y = x^4$, the y-axis, and the line $y = 16$. Find its centroid.

A. (0, 9.6) C. (0, 8.3)
B. (0, 12.4) D. (0, 12.8)

PROBLEM 4 - 18
CE Nov. 1998

A solid is formed by revolving about the y-axis, the area bounded by the curve $x^3 = y$, the y-axis and the line $y = 8$. Find its centroid.
A. (0, 4.75) C. (0, 5.25)
B. (0, 4.5) D. (0, 5)

PROBLEM 4 - 19
CE Nov. 1995

Find the moment of inertia of the area bounded by the parabola $y^2 = 4x$, x- axis and the line $x = 1$, with respect to the x-axis.
A. 1.067 C. 0.968
B.1.244 D. 0.878

PROBLEM 4 - 20

Find the work done in stretching a spring of natural length 8 cm, from 10 cm to 13 cm. Assume a force of 6 N is needed to hold it at a length of 11 cm.
A. 21 N-m C. 0.21 N-m
B. 2.1 N-m D. 0.021 N-m

PROBLEM 4 - 21

A conical tank that is 5 meters high has a radius of 2 meters, and is filled with a liquid that weighs 800 kg per cubic meter. How much work is done in discharging all the liquid at a point 3 meters above the top of the tank?
A. 21,256 π kg-m C. 23,457 π kg-m
B. 21,896 π kg-m D. 22,667 π kg-m

PROBLEM 4 - 22

How much work is required to pump all the water from a right circular cylindrical tank, that is 8 feet in diameter and 9 feet tall, if it is emptied at a point 1 foot above the top of the tank?
A. 49,421π ft-lb. C. 54,448π ft-lb.
B. 52,316π ft-lb. D. 56,305π ft-lb.

PROBLEM 4 - 23

A 60-m cable that weighs 4 kg/m has a 500-kg weight attached at the end. How much work is done in winding-up the last 20 m of the cable?
A. 9,866 kg-m C. 12,500 kg-m
B. 10,800 kg-m D. 15,456 kg-m

PROBLEM 4 - 24

A uniform chain that weighs 0.50 kg per meter has a leaky 15-liter bucket attached to it. If the bucket is full of liquid when 30 meters of chain is out and half-full when no chain is out, how much work is done in winding the chain? Assume that the liquid leaks out at a uniform rate and weighs 1 kg per liter.

A. 356.2 kg-m C. 562.5 kg-m
B. 458.2 kg-m D. 689.3 kg-m

PROBLEM 4 - 25
ECE Nov. 1995

The velocity of a body is given by $v(t) = \sin(\pi t)$, where the velocity is given in meters per second and t is given in seconds. The distance covered in meters between $t = 1/4$ and $1/2$ second is close to:
A. 0.5221 m C. 0.2251 m
B. -0.2251 m D. -0.5221 m

PROBLEM 4 - 26
ECE Nov. 1995

The rate of change of a function of y with respect to x equals $2 - y$, and $y = 8$ when $x = 0$. Find y when $x = \ln(2)$.
A. 2 C. 5
B. -5 D. -2

PROBLEM 4 - 27
CE Nov. 2002

Find the length of one arc of the curve whose parametric equations are $x = 2\theta - 2\sin\theta$ and $y = 2 - 2\cos\theta$.
A. 16 C. 14
B. 18 D. 12

PROBLEM 4 - 28
CE May 2002

A conical tank 12 ft high and 10 ft across the top is filled with a liquid that weighs 62.4 pcf. How much work is done in pumping all the liquid at the top of the tank?
A. 58,811 ft-lb C. 59,475 ft-lb
B. 63,421 ft-lb D. 47,453 ft-lb

PROBLEM 4 - 29
CE Nov. 2001

Determine the moment of inertia of the area bounded by the curve $x^2 = 4y$, the line $x - 4 = 0$ and the x-axis, with respect to the y-axis.
A. 51.2 C. 52.1
B. 25.1 D. 21.5

PROBLEM 4 - 30
CE Nov. 2001

Determine the area bounded by the curves $x = 1/y$, $2x - y = 0$, $x = 6$, and the x-axis.
A. 2.138 C. 2.324
B. 2.328 D. 2.638

PROBLEM 4 - 31
CE May 2001

Find the area bounded by the curve $y = 4\sin x$ and the x-axis from $x = \pi/3$ to $x = \pi$.
A. 9 square units C. 8 square units
B. 12 square units D. 6 square units

PROBLEM 4 - 32
CE May 2001

A body moves such that its acceleration as a function of time is $a = 2 + 12t$, where t is in minutes and a is in m/min^2. Its velocity after 1 minute is 11 m/min. Find its velocity after 2 minutes.

A. 31 m/min C. 45 m/min
B. 23 m/min D. 18 m/min

PROBLEM 4 - 33 Determine the area bounded by the curve $y^2 = 9x/5$ and the line $y = x - 2$.
A. 6.86 C. 9.39
B. 5.75 D. 8.78

PROBLEM 4 - 34 An open-top hemispherical tank having a radius of 1.2 m, full of water, is being drained by a vertical pipe whose exit end is 0.8 m above the top of the tank. How much work is done in emptying the full content of the tank?
A. 32.54 kJ C. 75.42 kJ
B. 44.38 kJ D. 24.78 kJ

PROBLEM 4 - 35 A mine cable weighing 60 N/m is unwound from a cylindrical drum. If the lower end of the cable is initially 15 m below the drum, how much work is done by gravity as an additional 100 m are unwound?
A. 360,000 N-m C. 380,000 N-m
B. 390,000 N-m D. 410,000 N-m

PROBLEM 4 - 36 The area bounded by the curve $y = \cos 2x$ and the x-axis is revolved about the x -axis. What is the volume generated from $x = \pi/4$ to $x = \pi/2$?
A. 1.547 C. 1.498
B. 1.234 D. 2.587

PROBLEM 4 - 37 The length of arc of the function $g(x) = x^{2/3}$ from $x = 0$ to $x = 8$ is:
A. 8.765 C. 10.231
B. 9.073 D. 12.988

PROBLEM 4 - 38 The area formed by the boundaries $y = 1$, $x = 2$, and $y = e^{-x}$ is closest to:
A. 2.746 C. 1.586
B. 3.542 D. 1.135

PROBLEM 4 - 39 CE Nov. 2003 What is the perimeter of the curve $r = 4(1 - \sin\theta)$?
A. 32.47 C. 25.13
B. 30.12 D. 28.54

PROBLEM 4 - 40 CE Nov. 2003 Find the surface area generated by rotating the first quadrant portion of the curve $x^2 = 16 - 8y$ about the y-axis.
A. 58.41 C. 61.27
B. 64.25 D. 66.38

ANSWER SHEET

1.	A	B	C	D	E	21.	A	B	C	D	E	41.	A	B	C	D	E
2.	A	B	C	D	E	22.	A	B	C	D	E	42.	A	B	C	D	E
3.	A	B	C	D	E	23.	A	B	C	D	E	43.	A	B	C	D	E
4.	A	B	C	D	E	24.	A	B	C	D	E	44.	A	B	C	D	E
5.	A	B	C	D	E	25.	A	B	C	D	E	45.	A	B	C	D	E
6.	A	B	C	D	E	26.	A	B	C	D	E	46.	A	B	C	D	E
7.	A	B	C	D	E	27.	A	B	C	D	E	47.	A	B	C	D	E
8.	A	B	C	D	E	28.	A	B	C	D	E	48.	A	B	C	D	E
9.	A	B	C	D	E	29.	A	B	C	D	E	49.	A	B	C	D	E
10.	A	B	C	D	E	30.	A	B	C	D	E	50.	A	B	C	D	E
11.	A	B	C	D	E	31.	A	B	C	D	E	51.	A	B	C	D	E
12.	A	B	C	D	E	32.	A	B	C	D	E	52.	A	B	C	D	E
13.	A	B	C	D	E	33.	A	B	C	D	E	53.	A	B	C	D	E
14.	A	B	C	D	E	34.	A	B	C	D	E	54.	A	B	C	D	E
15.	A	B	C	D	E	35.	A	B	C	D	E	55.	A	B	C	D	E
16.	A	B	C	D	E	36.	A	B	C	D	E	56.	A	B	C	D	E
17.	A	B	C	D	E	37.	A	B	C	D	E	57.	A	B	C	D	E
18.	A	B	C	D	E	38.	A	B	C	D	E	58.	A	B	C	D	E
19.	A	B	C	D	E	39.	A	B	C	D	E	59.	A	B	C	D	E
20.	A	B	C	D	E	40.	A	B	C	D	E	60.	A	B	C	D	E

Solutions to Set 4
Plane Areas, Volumes, Surfaces, Centroid, Etc.

SOLUTION 4 - 1
Ans: B

$$A = \int_1^3 y\,dx = \int_1^3 (x^3 + 3x^2)dx = \left[\frac{x^4}{4} + x^3\right]_1^3$$

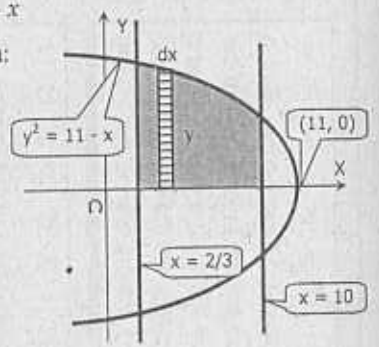
$A = \frac{1}{4}(3^4 - 1^4) + (3^3 - 1^3) = 46$ **sq. units**

SOLUTION 4 - 2
Ans: D

$y = (11 - x)^{1/2}$ or $y^2 = 11 - x$

Change to standard form:
$y^2 = -(x + 11)$

Vertex: (11, 0), opening to the left



$dA = y\,dx$
$dA = (11 - x)^{1/2}\,dx$

$$A = \int_{2/3}^{10} (11 - x)^{1/2}\,dx$$

$$A = \left[-\frac{2}{3}(11 - x)^{3/2}\right]_{2/3}^{10}$$

$A = 21.478$ **sq. units**

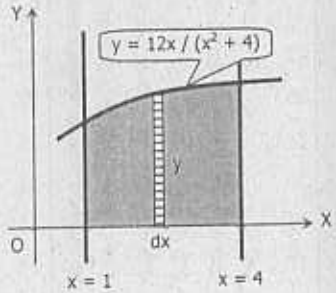
SOLUTION 4 - 3
Ans: D

$dA = y\,dx$

$$y = \frac{12x}{x^2 + 4}$$

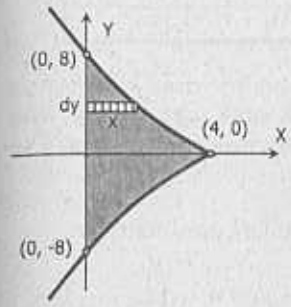
$$dA = \frac{12x}{x^2 + 4}\,dx$$

$$A = \int_1^4 \frac{12x}{x^2 + 4}\,dx$$



$A = 6\left[\ln(x^2 + 4)\right]_1^4 = 6\,[\ln 20 - \ln 5]$
$A = 6 \ln (20/5)$
$A = 6 \ln 4$

SOLUTION 4 - 4
Ans: A



$x = 4 - y^{2/3}$
when $x = 0$, $y = \pm 8$; $(0, \pm 8)$
when $y = 0$, $x = 4$; $(4, 0)$

$$A = \int_{-8}^{8} x\,dy = \int_{-8}^{8} (4 - y^{2/3})dy$$

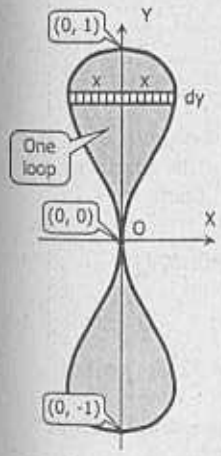
$$A = \left[4y - \tfrac{3}{5}y^{5/3}\right]_{-8}^{8}$$

$A = [4(8) - (3/5)(8)^{5/3}] - [4(-8) - (3/5)(-8)^{5/3}]$
$A = 25.6$ **sq. units**

SOLUTION 4 - 5
Ans: D

Equation of curve: $x^2 = y^4(1 - y^2)$ or $x = \pm y^2\sqrt{1 - y^2}$

y	0	1	-1	±0.5
x	0	0	0	±0.216



We notice that the curve is symmetrical with respect to the origin as shown.

$dA = 2x\,dy = 2y^2\sqrt{1 - y^2}\,dy$

$$A = 2\int_0^1 y^2\sqrt{1 - y^2}\,dy$$

Let $y^2 = \sin^2\theta$; $y = \sin\theta$
$dy = \cos\theta\,d\theta$

when $y = 0$, $\theta = 0$
when $y = 1$, $\theta = 90° = \pi/2$

$$A = 2\int_0^{\pi/2} \sin^2\theta\sqrt{1 - \sin^2\theta}\cos\theta\,d\theta$$

$$A = 2\int_0^{\pi/2} \sin^2\theta\sqrt{\cos^2\theta}\cos\theta\,d\theta = 2\int_0^{\pi/2} \sin^2\theta\cos^2\theta\,d\theta$$

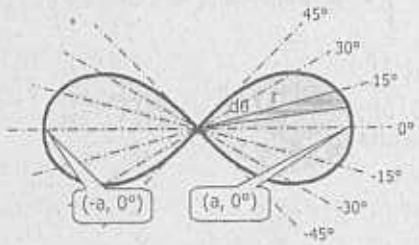
By Walli's Formula:

$A = 2\,\dfrac{(1)(1)}{4(2)}\,\dfrac{\pi}{2} = \pi/8$ **square units**

SOLUTION 4 - 6
Ans: B

$r^2 = a^2 \cos 2\theta$

θ	0°	± 15°	± 30°	± 45°	> 45°
r	± a	±0.9306	± 0.7071	0	imaginary



The curve is symmetrical with respect to the origin, and occurs only with values of θ from -45° to 45° (-π/4 to π/4).

The area by polar coordinates is:

$$A = 2 \times \int_{-\pi/4}^{\pi/4} \frac{1}{2} r^2 d\theta = 4 \times \int_{0}^{\pi/4} \frac{1}{2} a^2 \cos 2\theta d\theta$$

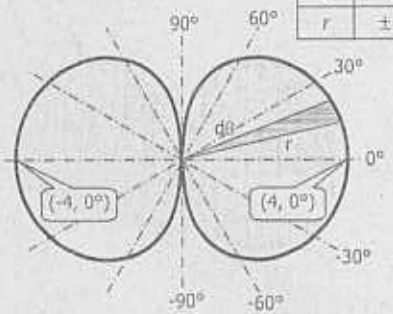
$$A = 2 \times a^2 \left[\frac{1}{2}\sin 2\theta\right]_0^{\pi/4}$$

$$A = a^2 \sin(\pi/2) - \sin(0) = a^2$$

SOLUTION 4 - 7
Ans: D

$r^2 = 16 \cos \theta$

θ	0°	± 30°	± 60°	± 90°	>90°
r	± 4	± 3.72	± 2.83	0	imaginary



The values in the table shows that the graph is symmetrical to the origin and θ ranges from -90° to 90°. Then,

$$A = 4 \int_0^{90°} \frac{1}{2} r^2 d\theta = 2 \int_0^{90°} 16 \cos\theta \, d\theta$$

$$A = 32 \left[\sin\theta\right]_0^{90°}$$

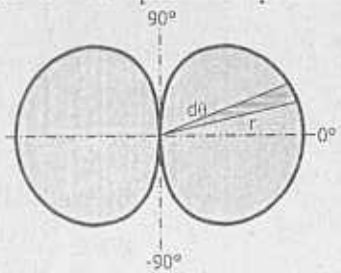
$$A = 32 [\sin 90° - \sin 0°] = \mathbf{32\ sq.\ units.}$$

SOLUTION 4 - 8
Ans: C

This problem is similar to the previous problem **(Problem 4-7)**

$$A = \tfrac{1}{2} \int_{\theta_1}^{\theta_2} r^2 d\theta$$

$$A = 4 \times \tfrac{1}{2} \int_0^{90°} 2a^2 \cos\theta \, d\theta$$



$$A = 4a^2 \left[\sin\theta\right]_0^{90°}$$

$$A = 4a^2 (\sin 90° - \sin 0) = 4a^2$$

SOLUTION 4 - 9
Ans: B

$dA = 2\pi x \, dS$

$$dS = \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \, dx$$

$$y = x^2, \ dy/dx = 2x$$

$$dS = \sqrt{1 + (2x)^2} \, dx$$

$$dA = 2\pi x \sqrt{1 + (2x)^2} \, dx = 2\pi \int_0^{\sqrt{6}} (1 + 4x^2)^{1/2} x \, dx$$

$$A = 2\pi \frac{1}{8}\left[\frac{2}{3}(1 + 4x^2)^{3/2}\right]_0^{\sqrt{6}}$$

$$A = \frac{\pi}{6}\left\{\left[1 + 4(\sqrt{6})^2\right]^{3/2} - [1 + 0]^{3/2}\right\} = \frac{62}{3}\pi$$

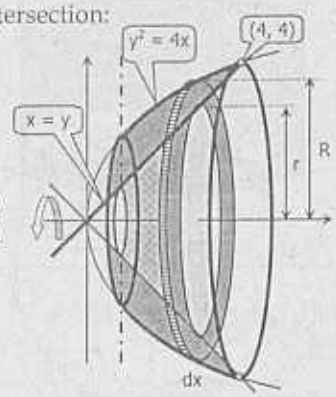
SOLUTION 4 - 10
Ans: D

Solving for the points of intersection: of $y^2 = 4x$ and $y = x$

$x^2 = 4x$ or $x^2 - 4x = 0$
$x(x - 4) = 0$
$x = 0$ and $x = 4$

when $x = 0$, $y = 0$; (0, 0)
when $x = 4$, $y = 4$; (4, 4)

$dV = (\pi y_u^2 - \pi y_l^2) \, dx$
$dV = \pi(y_u^2 - y_l^2) \, dx$
$dV = \pi\left[\left(2\sqrt{x}\right)^2 - (x)^2\right] dx$

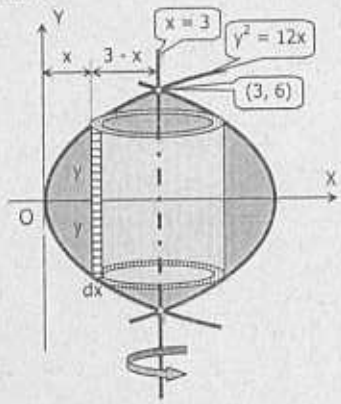


$$V = \pi \int_1^4 (4x - x^2) dx = \pi \left[2x^2 - \frac{x^3}{3}\right]_1^4$$

$$V = \pi \left\{\left[2(4)^2 - \frac{4^3}{3}\right] - \left[2(1)^2 - \frac{1^3}{3}\right]\right\}$$

$V = \mathbf{9\pi\ cubic\ units}$

SOLUTION 4 - 11
Ans: C



$dV = 2\pi(3 - x)(2y)\,dx$

$y^2 = 12x$

$y = \sqrt{12}\ x^{1/2}$

$dV = 2\pi(3 - x)(2\sqrt{12}\ x^{1/2})\,dx$

$dV = 4\sqrt{12}\,\pi\,(3x^{1/2} - x^{3/2})\,dx$

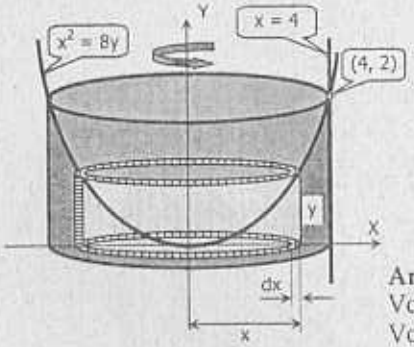
$$V = 4\sqrt{12}\,\pi \int_0^3 \left(3x^{1/2} - x^{3/2}\right)dx$$

$$V = 4\sqrt{12}\,\pi \left[2x^{3/2} - \frac{2}{5}x^{5/2}\right]_0^3$$

$$V = 4\sqrt{12}\,\pi \left\{\left[2(3)^{3/2} - \frac{2}{5}(3)^{5/2}\right] - 0\right\}$$

V = 180.96 cu. units

SOLUTION 4 - 12
Ans: A



$dV = 2\pi\, x\, y\, dx$

$x^2 = 8y$

$y = x^2 / 8$

$$dV = 2\pi\, x\,(x^2/8)\,dx = \frac{\pi}{4}x^3\,dx$$

$$V = \frac{\pi}{4}\int_0^4 x^3\,dx = \frac{\pi}{4}\left[\frac{x^4}{4}\right]_0^4$$

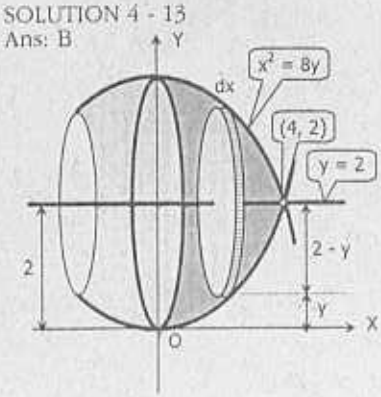
$$V = \frac{\pi}{16}(4^4 - 0^4) = 50.26 \text{ cu. units}$$

Another Solution

Volume = $V_{cylinder} - V_{paraboloid}$

Volume = $\pi(4)^2\,(2) - \frac{1}{2}\pi(4)^2\,(2) = 50.26$

SOLUTION 4 - 13
Ans: B



$dV = \pi(2 - y)^2\,dx$

$x^2 = 8y$

$y = x^2/8$

$dV = \pi\,(2 - x^2/8)^2 dx$

$$dV = \pi\left(\frac{16 - x^2}{8}\right)^2 dx$$

$$V = \frac{\pi}{64}\int_0^4 (256 - 32x^2 + x^4)\,dx$$

$$V = \frac{\pi}{64}\left[256x - \frac{32}{3}x^3 + \frac{x^5}{5}\right]_0^4$$

V = 26.81 cu. units

SOLUTION 4 - 14
Ans: D

The length of arc of the curve $y = f(x)$ is:

$$S = \int_{x_1}^{x_2} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx$$

Solve for dy/dx or y'

$x^2 + y^2 = 64$

$2x + 2yy' = 0$

$y' = -x/y = dy/dx$

$$S = \int_{-3}^{-1} \sqrt{1 + (-x/y)^2}\,dx = \int_{-3}^{-1} \sqrt{\frac{y^2 + x^2}{y^2}}\,dx$$

but $x^2 + y^2 = 64$ and $y^2 = 64 - x^2$, then

$$S = \int_{-3}^{-1} \sqrt{\frac{64}{64 - x^2}}\,dx = 8\int_{-3}^{-1} \frac{dx}{\sqrt{8^2 - x^2}}$$

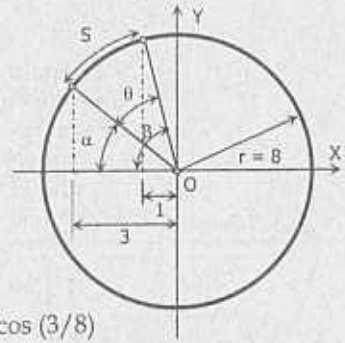
$$S = 8\left[\arcsin\left(\frac{x}{8}\right)\right]_{-3}^{-1}$$

$S = 8\,[\arcsin(-1/8) - \arcsin(-3/8)]_{radians}$

S = 2.07 units

Another solution:

Since the given curve is a circle, the length of arc, $S = \dfrac{\pi r\theta}{180°}$, where θ is the central angle of the arc in degrees.



From the figure shown:

$\theta = \beta - \alpha$

$\theta = \arccos(1/8) - \arccos(3/8)$

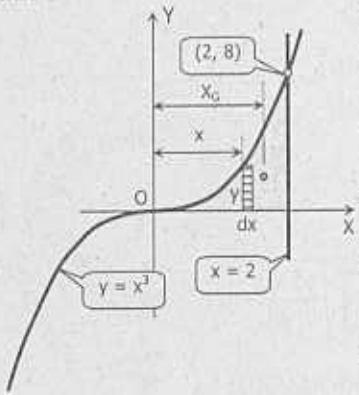
$\theta = 14.84°$

Then

$$S = \frac{\pi(8)(14.84°)}{180°}$$

S = 2.07 units

SOLUTION 4 - 15
Ans: C



$$A\,X_G = \int x\,dA \qquad \rightarrow (1)$$

$$dA = y\,dx = x^3\,dx \qquad \rightarrow (2)$$

$$A = \int_0^2 x^3\,dx = \left[\frac{x^4}{4}\right]_0^2 = \frac{1}{4}(2^4 - 0^4) = 4$$

Substitute $A = 4$ and Eq. (2) to Eq. (1)

$$4\,X_G = \int_0^2 x\left(x^3\,dx\right) = \int_0^2 x^4\,dx$$

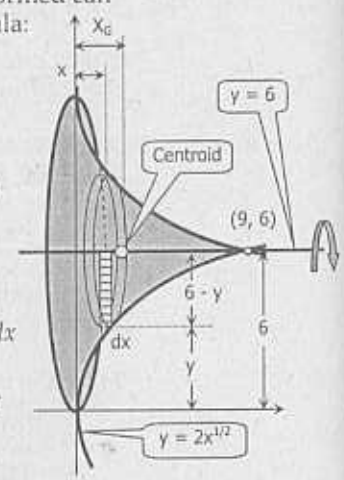
$$4\,X_G = \left[\frac{x^5}{5}\right]_0^2 = \frac{1}{5}(2^5 - 0^5);\ X_G = 1.6$$

SOLUTION 4 - 16
Ans: C

Solve for the point of intersection:
When $y = 6$
$6^2 = 4x;\ x = 9;\ (9, 6)$

The centroid of the solid formed can be computed by the formula:

$$V\,x_G = \int_{x_1}^{x_2} x\,dV \qquad \rightarrow (1)$$



Solving for the volume, V:
$dV = \pi(6 - y)^2\,dx$
$dV = \pi(36 - 12y + y^2)\,dx$
but $y = 2x^{1/2}$
and $y^2 = 4x$
$dV = \pi\,(36 - 24\,x^{1/2} + 4x)\,dx$

$$V = \pi\int_0^9 (36 - 24x^{1/2} + 4x)dx$$

$$V = \pi\left[36x - 16x^{3/2} + x^2\right]_0^9$$

$V = 169.646$ cu. units

In Eq. (1)

$$169.646\,X_G = \int_0^9 x\pi(36 - 24x^{1/2} + 4x)dx$$

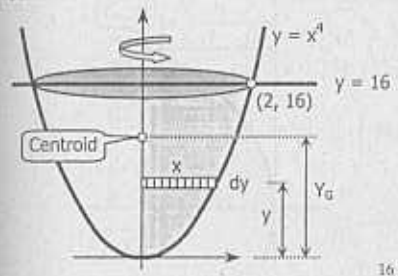
$$169.646\,X_G = \pi\int_0^9 (36x - 24x^{3/2} + 4x^2)dx$$

$$= \pi\left[18x^2 - \tfrac{48}{5}x^{5/2} + \tfrac{4}{3}x^3\right]_0^9$$

$X_G = 1.8$ units

Thus, the centroid is at **(1.8, 6)**

SOLUTION 4 - 17
Ans: A



$dV = \pi\,x^2\,dy$

$x = y^{1/4}$

$dV = \pi(\,y^{1/4})^2\,dy = \pi\,y^{1/2}\,dy$

$$V = \int_0^{16} \pi\,y^{1/2}\,dy = \frac{2}{3}\pi\left[y^{3/2}\right]_0^{16}$$

$$V = \frac{2}{3}\pi\,[(16)^{3/2} - 0\,] = \frac{128}{3}\pi \text{ cu. units}$$

$$V\,Y_G = \int_0^{16} y\,dV = \int_0^{16} y\left[\pi\,y^{1/2}\,dy\right] = \pi\int_0^{16} y^{3/2}\,dy$$

$$\frac{128}{3}\pi\,Y_G = \pi\left[\frac{2}{5}y^{5/2}\right]_0^{16} = \frac{2}{5}\pi\,[\,(16)^{5/2} - 0\,]$$

$Y_G = 9.6$ units

Thus, the centroid of the solid is at **(0, 9.6)**.

SOLUTION 4 - 18
Ans: D

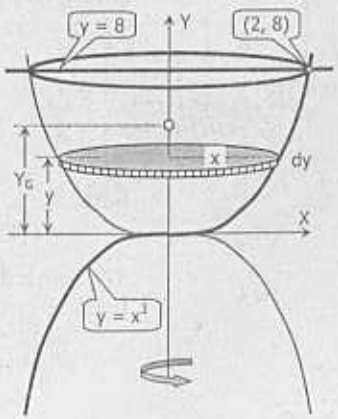
$$V\,Y_G = \int_0^8 y\,dV$$

$dV = \pi\,x^2\,dy$
$y = x^3;\ x = y^{1/3}$
$x^2 = y^{2/3}$
$dV = \pi\,y^{2/3}\,dy$

$$V = \pi\int_0^8 y^{2/3}\,dy$$

$$V = \pi\left[\frac{3}{5}y^{5/3}\right]_0^8$$

$V = \frac{3}{5}\pi\,[(8)^{5/3} - 0\,]$



$V = \frac{96}{5}\pi$

$$\frac{96}{5}\pi\, Y_G = \int_0^8 y(\pi y^{2/3}\,dy) = \pi\int_0^8 y^{5/3}\,dy = \pi\left[\frac{3}{8}y^{8/3}\right]_0^8$$

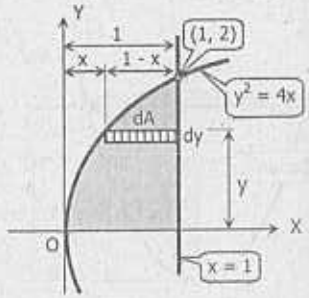
$$\frac{96}{5}\pi\, Y_G = \pi\tfrac{3}{8}(8^{8/3} - 0);\ Y_G = 5$$

Therefore the centroid is at **(0, 5)**

SOLUTION 4 - 19
Ans: A

$$I = \int y^2\,dA$$

$dA = (1 - x)\,dy$
$y^2 = 4x$
$x = y^2/4$
$dA = (1 - y^2/4)\,dy$



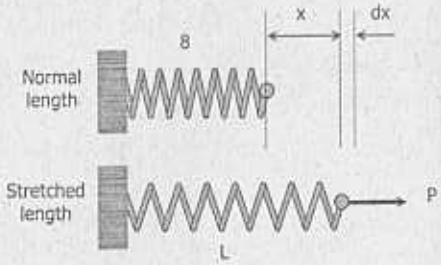
$$I = \int_0^2 y^2(1 - y^2/4)\,dy$$

$$I = \int_0^2\left(y^2 - \frac{y^4}{4}\right)dy$$

$$I = \left[\frac{y^3}{3} - \frac{y^5}{20}\right]_0^2 = \left\{\left[\frac{2^3}{3} - \frac{2^5}{20}\right] - 0\right\} = 1.067$$

SOLUTION 4 - 20
Ans: C

The work done in stretching a spring of stiffness k at length of x is, Work = ½ $k\,x^2$

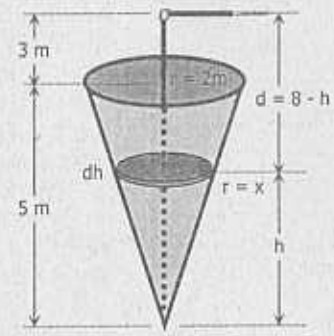


Solve for k, from $F = kx$
$6 = k(11 - 8);\ k = 2$ N/cm

The work done to stretch a spring of natural length 8 cm, from 10 cm ($x_1 = 2$) to 13 cm ($x_2 = 5$) is:
Work = ½ $k\,(x_2^2 - x_1^2)$ = ½ (2 N/cm)($5^2 - 2^2$) cm²
Work = 21 N-cm = 0.21 N-m

SOLUTION 4 - 21
Ans: D

Work = force × distance



$dW = dF \times d$
$dF = \gamma\,dV$
$dF = 800\,\pi x^2\,dh$
$d = 8 - h$

$x/h = 2/5$
$x = 2h/5$
$dF = 800\pi(2h/5)^2\,dh$
$dF = 128\pi\,h^2\,dh$

$dW = 128\pi\,h^2\,dh \times (8 - h)$
$dW = 128\pi\,(8h^2 - h^3)\,dh$

$$W = 128\pi\int_0^5 (8h^2 - h^3)\,dh = 128\pi\left[\frac{8h^3}{3} - \frac{h^4}{4}\right]_0^5$$

$$W = 128\pi\left[\frac{8(5)^3}{3} - \frac{5^4}{4}\right] = \mathbf{22{,}667\pi\ kg\text{-}m}$$

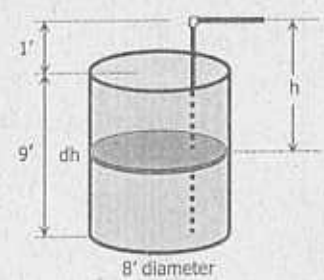
Another Solution:

Work = Force × Distance
Force = Weight = $\gamma\,V$
Force = 800 × $\frac{1}{3}\pi(2)^2(5)$ = 5333.33π Kg
Distance = 3 + ¼(5) = 4.25 m

Work = 5333.33π (4.25) = **22667π kg-m**

SOLUTION 4 - 22
Ans: A

$dW = dF \times d$
$dF = \gamma\,dV$
$dF = 62.4\,[\pi(4)^2 dh]$
$dF = 998.4\pi\,dh$



$dW = 998.4\pi\,dh\,(h)$

$$W = 998.4\pi\int_1^{10} h\,dh$$

$$W = 998.4\pi\left[\frac{h^2}{2}\right]_1^{10}$$

$W = 499.2\pi\,(10^2 - 1^2)$
$W = \mathbf{49{,}421\pi\ ft\text{-}lb}$

Another Solution:

Work = Force × distance

Force = $\gamma V = 62.4 \times \pi(4)^2 (9)$

Force = 8985.6π lbs.

Distance = $1 + \frac{1}{2}(9)$

Distance = 5.5 ft.

Work = $8985.6\pi\ (5.5)$ = **49421π ft-lb**

SOLUTION 4 - 23
Ans: B

$W = F \times d$

$dW = F\,dx$

F = Wt. of load + wt. of chain

$F = 500 + 4x$

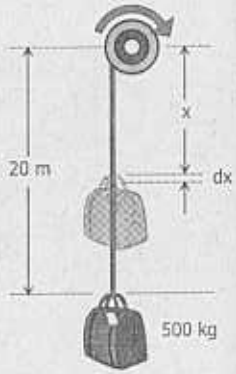
$dW = (500 + 4x)\,dx$

$$W = \int_0^{20} (500 + 4x)\,dx$$

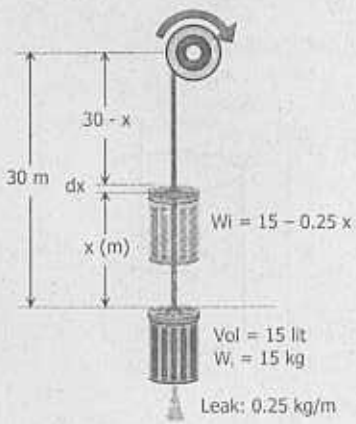
$$W = \left[500x + \frac{4x^2}{2}\right]_0^{20}$$

$W = [500(20) + 2(20)^2]$

W = **10,800 kg-m**



SOLUTION 4 - 24
Ans: C



$dW = F\,dx$

$Wt_{initial}$ = 15 lit (1 kg/lit) = 15 kg

Wt_{final} = 15(1/2) = 7.5 kg

Rate of leak = (7.5 kg/30 m)

Rate og leak = 0.25 kg/m

F = Wt. of chain + wt. of liquid

$F = (30 - x)(0.5) + (15 - 0.25x)$

$F = 15 - 0.5x + 15 - 0.25x$

$F = 30 - 0.75x$

$dW = (30 - 0.75x)\,dx$

$$W = \int_0^{30} (30 - 0.75x)\,dx$$

$$W = \left[30x - 0.75\frac{x^2}{2}\right]_0^{30}$$

$W = [30(30) - 0.75(30)^2/2]$

W = **562.5 kg-m**

SOLUTION 4 - 25
Ans: C

$dS = v\,dt = \sin(\pi t)\,dt$

$$S = \int_{1/4}^{1/2} \sin(\pi t)\,dt = \left[-\frac{1}{\pi}\cos(\pi t)\right]_{1/4}^{1/2}$$

$S = -(1/\pi)\,[\cos(\pi/2) - \cos(\pi/4)]$

$S = -(1/\pi)(\cos 90° - \cos 45°)$ = **0.2251**

SOLUTION 4 - 26
Ans: C

$$\frac{dy}{dx} = 2 - y; \quad \frac{dy}{2-y} = dx$$

$-\ln(2 - y) = x + C \qquad \rightarrow (1)$

When $x = 0$, $y = 8$ (Substitute to Eq. 1)

$-\ln(2 - 8) = 0 + C$; $C = -\ln(-6)$

When $x = \ln(2)$ (Substitute to Eq. 1)

$-\ln(2 - y) = \ln(2) - \ln(-6)$

$$\ln(2 - y)^{-1} = \ln\frac{2}{-6} = \ln\frac{-1}{3}$$

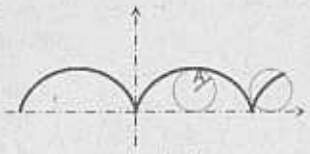
$$\frac{1}{2-y} = \frac{1}{3}$$

$-3 = 2 - y$; $y = 5$

SOLUTION 4 - 27
Ans: A

$x = 2\theta - 2\sin\theta$; $y = 2 - 2\cos\theta$

Note: This curve is a cycloid where 2 is the radius of rolling circle and θ is the angle of rotation (0° - 360° for one arc)



$x = a(\theta - \sin\theta)$
$y = a(1 - \cos\theta)$
a = radius of rolling circle
θ = angle of rotation
CYCLOID
Cycloid

$$\text{Length of arc, } S = \int \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\,dy$$

$dx = (2 - 2\cos\theta)\,d\theta$

$dy = 2\sin\theta\,d\theta$

$$S = \int_{0°}^{360°} \sqrt{1 + \left[\frac{(2 - 2\cos\theta)\,d\theta}{2\sin\theta\,d\theta}\right]^2}\ 2\sin\theta\,d\theta$$

$$S = 2\int_{0°}^{360°} \sqrt{1 + \frac{1 - 2\cos\theta + \cos^2\theta}{\sin^2\theta}}\ \sin\theta\,d\theta$$

$$S = 2\int_{0^\circ}^{360^\circ} \sqrt{\frac{\sin^2\theta + 1 - 2\cos\theta + \cos^2\theta}{\cancel{\sin^2\theta}}\,\cancel{\sin^2\theta}}\; d\theta$$

but $\sin^2\theta + \cos^2\theta = 1$

$$S = 2\int_{0^\circ}^{360^\circ} \sqrt{2 - 2\cos\theta}\, d\theta = 2\sqrt{2}\int_{0^\circ}^{360^\circ} \sqrt{1-\cos\theta}$$

but $\sqrt{1-\cos\theta} = \sqrt{2}\,\sin\frac{\theta}{2}$

$$S = 2\sqrt{2}\int_{0^\circ}^{360^\circ} \sqrt{2}\sin\frac{\theta}{2}\,d\theta = 4\int_{0^\circ}^{360^\circ} \sin\frac{\theta}{2}\,d\theta$$

$$S = 4\left[-2\cos\frac{\theta}{2}\right]_{0^\circ}^{360^\circ} = -8\left[\cos\frac{360^\circ}{2} - \cos\frac{0^\circ}{2}\right]$$

$S = 16$

> Note: The length of one arc of CYCLOID with parametric equations $x = a(\theta - \sin\theta)$ and $y = a(1 - \cos\theta)$ is $8a$.

SOLUTION 4 - 28
Ans: A

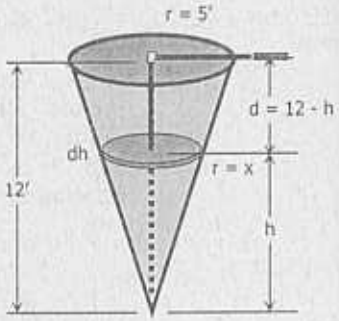
Work = force × distance

$dW = dF \times d$

$dF = \gamma\, dV$
$dF = 62.4\,\pi\, x^2\, dh$
$d = 12 - h$

In the figure shown:

$\frac{x}{h} = \frac{5}{12}$
$x = 5h/12$

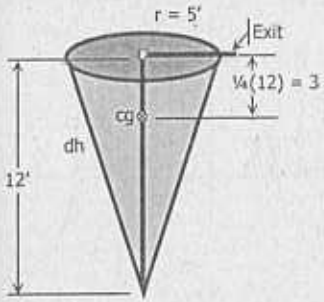


$dF = 62.4\pi(5h/12)^2\, dh$
$dF = 34.034\, h^2\, dh$

$dW = 34.034\, h^2\, dh \times (12 - h) = 34.034\,(12h^2 - h^3)\, dh$

$$W = 34.034\int_0^{12}(12h^2 - h^3)dh = 34.034\left[4h^3 - \frac{h^4}{4}\right]_0^{12}$$

$$W = 34.034\left[4(12)^3 - \frac{12^4}{4}\right] = \mathbf{58{,}810.75\ ft\text{-}lb}$$



Another Solution:
Work = Force × Distance

Force = Weight
Force = $\gamma\, V = 62.4 \times \frac{1}{3}\pi(5)^2(12)$
Force = 19,603.54 lbs

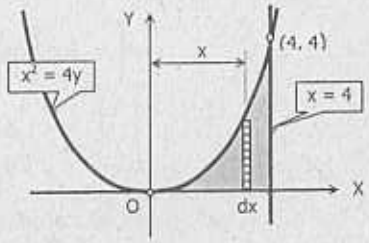
Distance = distance from centroid of volume to point of exit
Distance = ¼(12) = 3 m

Work = 19,603.54 (3) = **58,810.6 ft-lb**

SOLUTION 4 - 29
Ans: A

$$I_y = \int_{x_1}^{x_2} x^2\, dA$$

$dA = (y_U - y_L)\, dx$
$y_U = x^2/4$
$y_L = 0$
$dA = (x^2/4 - 0)\, dx$
$dA = \frac{1}{4}x^2\, dx$



$$I_y = \int_0^4 x^2\,\frac{1}{4}x^2 dx$$

$$I_y = \frac{1}{4}\left[\frac{x^5}{5}\right]_0^4 = \frac{1}{4}\,\frac{4^5 - 0^5}{5} = \mathbf{51.2}$$

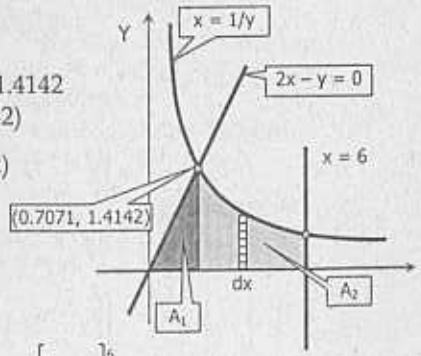
SOLUTION 4 - 30
Ans: D

Point of intersection:
$2x - y = 0;\ y = 2x$
$x = 1/y$ or $xy = 1$
$x(2x) = 1$
$x = 0.7071$ & $y = 1.4142$
P.I. (0.7071, 1.4142)

$A_1 = \frac{1}{2}(0.7071)(1.4142)$
$A_1 = 0.5$

$$A_2 = \int_{x_1}^{x_2}(y_U - y_L)dx$$

$y_U = 1/x;\ y_L = 0$



$$A_2 = \int_{0.7071}^{6}\left(\frac{1}{x} - 0\right)dx = \Big[\ln x\Big]_{0.7071}^{6}$$

$A_2 = \ln(6) - \ln(0.7071) = 2.138$

$A = A_1 + A_2 = \mathbf{2.638\ sq.\ units}$

SOLUTION 4 - 31
Ans: D

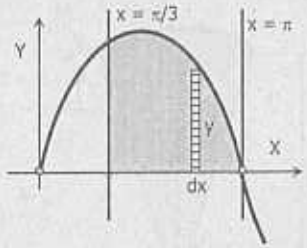
$$A = \int_{\pi/3}^{\pi} y\,dx$$

$$A = \int_{60°}^{180°} 4\sin x\,dx$$

$$A = 4\left[-\cos x\right]_{60°}^{180°}$$

$A = -4\,[\cos 180° - \cos 60°]$

$A =$ **6 square units**



SOLUTION 4 - 32
Ans: A

$$a = \frac{dv}{dt} = 2 + 12t;\ dv = (2 + 12t)\,dt$$

$$\int_{11}^{v} dv = \int_{1}^{2} (2 + 12t)dt$$

$$\left[v\right]_{11}^{v} = \left[2t + 6t^2\right]_{1}^{2}$$

$v - 11 = 2(2 - 1) + 6(2^2 - 1^2)$; $v =$ **31 m/min**

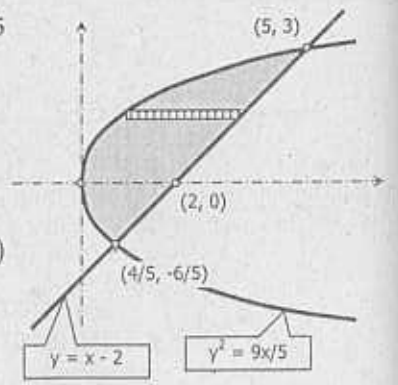
SOLUTION 4 - 33
Ans: A

Points of intersection:
Parabola: $y^2 = 9x/5$
Line: $x = y + 2$

$5y^2 = 9(y + 2)$
$5y^2 - 9y - 18 = 0$
$(5y + 6)(y - 3) = 0$

$y = -6/5$
$x = 4/5$; $(4/5, -6/5)$

$y = 3$
$x = 5$; $(5, 3)$



$$A = \int_{y_1}^{y_2} (x_R - x_L)dy$$

Line: $y = x_R - 2$
$x_R = y + 2$

Parabola: $y^2 = 9x_L/5$
$x_L = 5y^2/9$

$$A = \int_{-6/5}^{3} \left[(y+2) - (5y^2/9)\right]dy = \left[\frac{y^2}{2} + 2y - \frac{5y^3}{27}\right]_{-6/5}^{3}$$

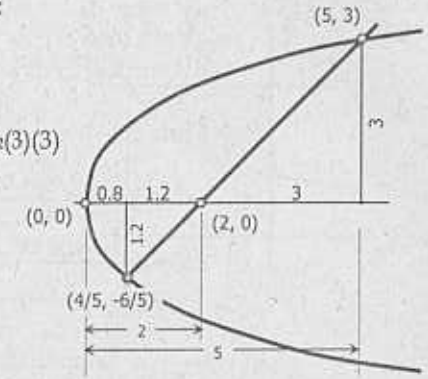
$$A = \frac{3^2 - (-6/5)^2}{2} + 2[3 - (-6/5)] - \frac{5[(3)^3 - (-6/5)^3]}{27}$$

$A = 3.78 + 8.4 - 5.32 =$ **6.86 sq. units**

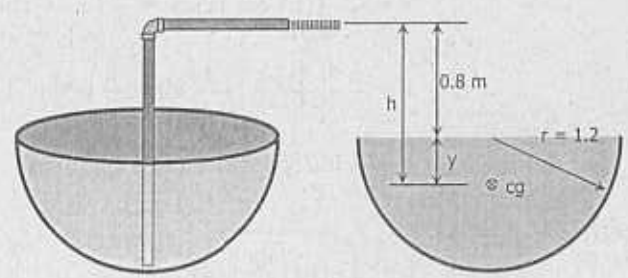
Another Solution:

$A = \frac{2}{3}(1.2)(0.8)$
$+ \frac{1}{2}(1.2)(1.2)$
$+ \frac{2}{3}(3)(5) - \frac{1}{2}(3)(3)$

$A =$ **6.86 sq. units**



SOLUTION 4 - 34
Ans: B



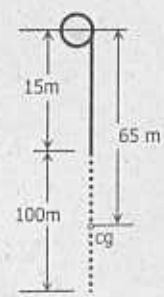
Work = Force × Distance
Force = Weight of water = γ_{water} × Volume

$$\text{Force} = (9.81)\left[\frac{1}{2} \times \frac{4}{3}\pi(1.2)^3\right] = 35.5035 \text{ kN}$$

Distance = $h = 0.8 + y$
y = centroid of hemisphere from the top
$y = 3r/8 = 3(1.2)/8 = 0.45$ m
Distance = 0.8 + 0.45 = 1.25 m
Work = 35.5035 × 1.25 = 44.38 kN-m = **44.38 kJ**

SOLUTION 4 - 35
Ans: B

Work = 60(100)(15 + 50)
Work = **390,000 N-m**



SOLUTION 4 - 36
Ans: B

$dV = \pi\, y^2\, dx$

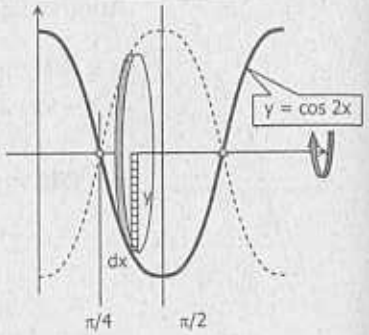
$dV = \pi\, (\cos 2x)^2\, dx$

$dV = \pi \cos^2 2x\, dx$

From $\cos^2\theta = \dfrac{1+\cos 2\theta}{2}$

$\cos^2 2x = \dfrac{1+\cos 4x}{2}$

$dV = \pi\, \dfrac{1+\cos 4x}{2}\, dx$



$$V = \frac{\pi}{2}\int_{\pi/4}^{\pi/2}(1+\cos 4x)dx = \frac{\pi}{2}\left[x+\frac{1}{4}\sin 4x\right]_{\pi/4}^{\pi/2}$$

$$V = \frac{\pi}{2}\left\{\left[\frac{\pi}{2}-\frac{\pi}{4}\right]+\frac{1}{4}\left[\sin\left(4\frac{\pi}{2}\right)-\sin\left(4\frac{\pi}{4}\right)\right]\right\}$$

$$V = \frac{\pi}{2}[0.7854+0] = \mathbf{1.234\ cu.\ units.}$$

SOLUTION 4 - 37
Ans: B

$$L = \int_0^8 \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$$

$y = x^{2/3}$

$dy/dx = y' = \frac{2}{3}\, x^{-1/3}$

$$L = \int_0^8 \sqrt{1+\left(\tfrac{2}{3}x^{-1/3}\right)^2}\, dx = \int_0^8 \sqrt{1+\frac{4}{9x^{2/3}}}\, dx$$

$$L = \int_0^8 \sqrt{\frac{9x^{2/3}+4}{9x^{2/3}}}\, dx = \frac{1}{3}\int_0^8 \left(4+9x^{2/3}\right)^{1/2} x^{-1/3}\, dx$$

$u = 4 + 9x^{2/3}$ $\quad n = ½$

$du = 6x^{-1/3}$

$$L = \frac{1}{3}\,\frac{1}{6}\left[\frac{2}{3}\left(4+9x^{2/3}\right)^{3/2}\right]_0^8 = \frac{1}{27}\left[\left(4+9x^{2/3}\right)^{3/2}\right]_0^8$$

$$L = \frac{1}{27}\left\{\left[4+9(8)^{2/3}\right]^{3/2}-\left[4+0\right]^{3/2}\right\} = \mathbf{9.073}$$

SOLUTION 4 - 38
Ans: D

$$A = \int_{x_1}^{x_2}(y_U - y_L)dx$$

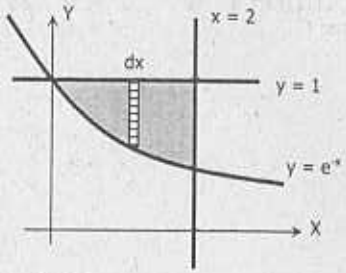
$y_U = 1;\ y_L = e^{-x}$

$x_1 = 0,\ x_2 = 2$

$$A = \int_0^2\left(1-e^{-x}\right)dx$$

$$A = \left[x+e^{-x}\right]_0^2$$

$A = [2-0] + [e^{-2} - e^{-0}] = e^{-1} = \mathbf{1.135}$



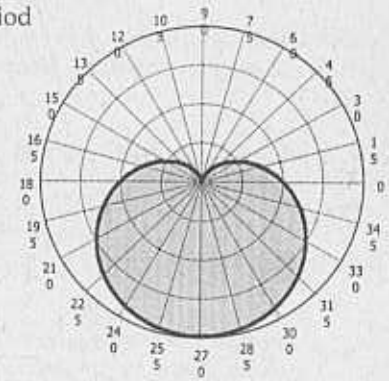
SOLUTION 4 - 39
Ans: C

$r = 4(1 - \sin\theta)$ cardiod

$$S = \int_{\theta_1}^{\theta_2} r\, d\theta$$

$$S = 2\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 4(1-\sin\theta)d\theta$$

$$S = 8\left[\theta+\cos\theta\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$



$S = 8\{[(\pi/2) + \cos(90°)] - [(-\pi/2) + \cos(-90°)]\}$

$S = 8\pi = \mathbf{25.133}$

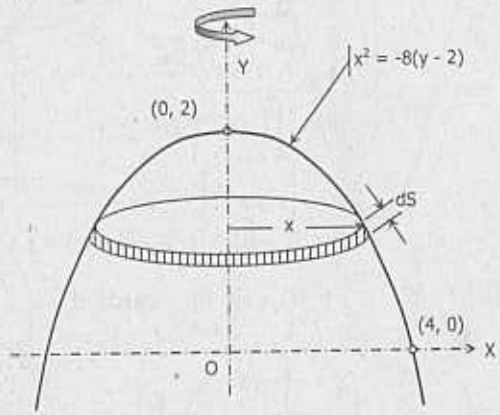
Note: For the following cardioid:

$r = a(1 - \sin\theta)$ $\quad r = a(1 - \cos\theta)$

$r = a(1 + \sin\theta)$ $\quad r = a(1 + \cos\theta)$

Area = $1.5\pi a^2$; Perimeter = $2\pi a$

SOLUTION 4 - 40 $x^2 = 16 - 8y$
Ans: C $x^2 = -8(y - 2)$ Vertex at (0, 2) open down



$$dA_s = 2\pi x\, dS$$

$$dS = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$y = 2 - \tfrac{1}{8}x^2$$

$$\frac{dy}{dx} = -\tfrac{1}{4}x$$

$$dS = \sqrt{1 + (-\tfrac{1}{4}x)^2}\, dx = \sqrt{1 + \tfrac{1}{16}x^2}\, dx$$

$$dA_s = 2\pi x \sqrt{1 + \tfrac{1}{16}x^2}\, dx$$

$$A_s = 2\pi \int_0^4 \sqrt{1 + \tfrac{1}{16}x^2}\, x\, dx$$

$$A_s = 2\pi(-8) \int_0^4 \left(1 + \tfrac{1}{16}x^2\right)^{1/2} (\tfrac{1}{8})x\, dx$$

$$A_s = 16\pi\, \frac{2}{3} \left[\left(1 + \tfrac{1}{16}x^2\right)^{3/2}\right]_0^4$$

$$A_s = \frac{32}{3}\pi \left[\left(1 + \tfrac{1}{16}(4)^2\right)^{3/2} - (1 + 0)^{3/2}\right]$$

$$A_s = \frac{32}{3}\pi\, [\, 2^{3/2} - 1] = \mathbf{61.27\ sq.\ units}$$

Part 3
DIFFERENTIAL EQUATION

DEFINITIONS

Differential equations are equations that contain differential coefficients. Example $dy/dx = 12x$ and $y'' + 2xy' - y = 0$.

Differential equations are classified according to the highest derivative that occurs in them. The differential equation $dy/dx = 12x$ is a **first order differential equation** and the equation $d^2y/dx^2 + 4dy/dx - 3y = 0$ is a **second order differential equation.**

A solution to a differential equation that contains one or more arbitrary constants of integration is called **general solution**. When additional information is given so that these constants may be calculated the **particular solution** of differential equation is obtained. The additional information is called **boundary conditions.**

VARIABLE SEPARABLE

A. Differential equations of type $dy/dx = f(x)$

Differential equations of type $dy/dx = f(x)$ can be solved by direct integration by writing it in the form

$$dy = f(x)\, dx$$

Example 3-1 Solve the differential equation $dy/dx = 2x + \sin 3x$.

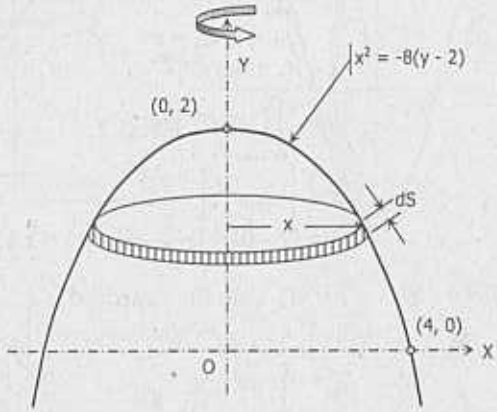
Solution

$dy = (2x + \sin 3x)\, dx$
$y = x^2 - (1/3)\cos 3x + C$ (general solution)

SOLUTION 4 - 40
Ans: C

$x^2 = 16 - 8y$

$x^2 = -8(y - 2)$ Vertex at (0, 2) open down



$$dA_s = 2\pi x\, dS$$

$$dS = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$y = 2 - \tfrac{1}{8}x^2$$

$$\frac{dy}{dx} = -\tfrac{1}{4}x$$

$$dS = \sqrt{1 + \left(-\tfrac{1}{4}x\right)^2}\, dx = \sqrt{1 + \tfrac{1}{16}x^2}\, dx$$

$$dA_s = 2\pi x \sqrt{1 + \tfrac{1}{16}x^2}\, dx$$

$$A_s = 2\pi \int_0^4 \sqrt{1 + \tfrac{1}{16}x^2}\, x\, dx$$

$$A_s = 2\pi(-8) \int_0^4 \left(1 + \tfrac{1}{16}x^2\right)^{1/2} \left(\tfrac{1}{8}\right) x\, dx$$

$$A_s = 16\pi \frac{2}{3}\left[\left(1 + \tfrac{1}{16}x^2\right)^{3/2}\right]_0^4$$

$$A_s = \frac{32}{3}\pi\left[\left(1 + \tfrac{1}{16}(4)^2\right)^{3/2} - (1 + 0)^{3/2}\right]$$

$$A_s = \frac{32}{3}\pi\,[\,2^{3/2} - 1] = \mathbf{61.27\ sq.\ units}$$

Part 3 DIFFERENTIAL EQUATION

DEFINITIONS

Differential equations are equations that contain differential coefficients. Example $dy/dx = 12x$ and $y'' + 2xy' - y = 0$.

Differential equations are classified according to the highest derivative that occurs in them. The differential equation $dy/dx = 12x$ is a **first order differential equation** and the equation $d^2y/dx^2 + 4dy/dx - 3y = 0$ is a **second order differential equation**.

A solution to a differential equation that contains one or more arbitrary constants of integration is called **general solution**. When additional information is given so that these constants may be calculated the **particular solution** of differential equation is obtained. The additional information is called **boundary conditions**.

VARIABLE SEPARABLE

A. Differential equations of type $dy/dx = f(x)$

Differential equations of type $dy/dx = f(x)$ can be solved by direct integration by writing it in the form

$$dy = f(x)\, dx$$

Example 3-1

Solve the differential equation $dy/dx = 2x + \sin 3x$.

Solution

$dy = (2x + \sin 3x)\, dx$

$y = x^2 - (1/3)\cos 3x + C$ (general solution)

B. Differential equation of type $dy/dx = f(y)$

Differential equation of type $dy/dx = f(y)$ can be solved by direct integration by writing it in the form

$$dx = \frac{dy}{f(y)}$$

Example 3-2

Solve the equation $(y^2 - 1)\, dy/dx = 3y$, given that $y = 1$ when $x = 13/6$.

Solution

Rearrange: $\int dx = \int \frac{y^2-1}{3y}\, dy = \int \left(\frac{y}{3} - \frac{1}{3y}\right) dy$

$$x = \frac{y^2}{6} - \frac{1}{3}\ln y + C \text{ (general solution)}$$

When $x = \frac{13}{16}$, $y = 1$ then:

$$\frac{13}{16} = \frac{1^2}{6} - \frac{1}{3}\ln(1) + C;\ C = 2$$

$$x = \frac{y^2}{6} - \frac{1}{3}\ln y + 2 \text{ (particular solution)}$$

C. Differential equation of type $dy/dx = f(x)\, g(y)$

Differential equation of type $dy/dx = f(x)\, g(y)$ can be solved by direct integration by writing it in the form

$$\frac{dy}{g(y)} = f(x)\, dx$$

D. Differential equation of type $dQ/dt = kQ$

The general solution of an equation of the form $dQ/dt = kQ$ is

$$Q = Ce^{kt}$$

where C is constant

Example 3-3

Solve the equation $dy/dx = 3y$

Solution

$Q = y$ $\quad dQ = dy$,
$t = x$ $\quad k = 3$
then $y = Ce^{3x}$

HOMOGENEOUS FIRST ORDER DIFFERENTIAL EQUATION

Some first order differential equation are not of the variable-separable type but can be made separable by changing the variable. Differential equations in the form $P\, dy/dx = Q$, where P and Q are functions of both x and y of the same degree throughout, is said to be **homogeneous** in x and y.

To solve these types of equation, the following procedures may be done:

1. Rearrange the equation into the form $dy/dx = Q/P$
2. Substitute $y = vx$ from where $dy = vdx + xdv$
3. Substitute both y and dy to the original equation and separate variables.
4. Solve using the previous methods and substitute $v = y/x$ to solve in terms of the original variables.

Example 3-4

Solve the equation; $(x^2 - xy + y^2)\, dx - xy\, dy = 0$.

Solution

The coefficients are of the second degree.

Let $y = vx$
$dy = v\, dx + x\, dv$

then $(x^2 - x^2v + v^2x^2)dx - x^2v(v\, dx + x\, dv) = 0$

Removing x^2

$(1 - v + v^2)\, dx - v(v\, dx + x\, dv) = 0$
$dx - v\, dx + v^2 dx - v^2 dx - vx\, dv = 0$
$(1 - v)\, dx - vx\, dv = 0$

Separating variables

$$\frac{dx}{x} + \frac{vdv}{v-1} = 0$$

$$\text{or } \frac{dx}{x} + \left(1 + \frac{1}{v-1}\right)dv = 0$$

Integrating:

$\ln x + v + \ln(v - 1) = \ln C$

$$\ln \frac{C}{x(v-1)} = v$$

$$\frac{c}{x(v-1)} = e^v$$

Finally, substituting $v = y/x$

$$x(y/x - 1)\, e^{y/x} = C \text{ or } (y - x)e^{y/x} = C$$

LINEAR FIRST ORDER DIFFERENTIAL EQUATION

The equation in the form $dy/dx + Py = Q$ where P and Q are functions of x only is called **linear differential equation** since y and its derivatives are of the first degree.

The solution for $dy/dx + Py = Q$ is obtained by multiplying throughout by an integrating factor $e^{\int P dx}$ to become

$$y\, e^{\int P dx} = e^{\int P dx}\, Q\, dx$$

Example 3-5

Solve the equation $dy + 4xy\, dx = 2x\, dx$.

Solution

Rearranging:
$dy/dx + 4xy = 2x$
$P = 4x$ and $Q = 2x$

Integrating factor $= e^{\int P dx}$

$\therefore \quad = e^{\int 4x dx} = e^{2x^2}$

then from $y\, e^{\int P dx} = e^{\int P dx}\, Q\, dx$

$$y\, e^{2x^2} = e^{2x^2}\, 2x\, dx$$

Integrate the right side to get

$$y\, e^{2x^2} = \frac{1}{2} e^{2x^2} + C$$

LINEAR SECOND ORDER DIFFERENTIAL EQUATION

Equations in the form $a(d^2y/dx^2) + b(dy/dx) + cy = 0$ where a, b, and c are constants, is called a **linear second order differential equation with constant coefficients.**

Setting $D = d/dx$ and $D^2 = d^2/dx^2$, the following procedures may be followed:

1. Write the equation in D-operator form $(aD^2 + bD + c)y = 0$, substitute m for D and solve the auxiliary equation $am^2 + bm + c = 0$ for m.

   A. If the roots are real **and different** ($b^2 > 4ac$) say $m = \alpha$ and $m = \beta$, then the general solution is

   $$y = A\, e^{\alpha x} + B\, e^{\beta x}$$

where A and B are constants.

   B. If the roots are **real and equal** ($b^2 = 4ac$), say $m = \alpha$ twice, the general solution is

   $$y = (Ax + B)e^{\alpha x}$$

   C. If the roots are imaginary ($b^2 < 4ac$), say $m = \alpha \pm \beta i$, the general solution is
   $$y = e^{\alpha x}\left[C \cos\beta x + D \sin\beta x\right]$$

Example 3-6

Solve the equation $2(d^2y/dx^2) + 5(dy/dx) - 3y = 0$

Solution

Writing in D-operator form: $(2D^2 + 5D - 3)y = 0$

Substituting m for D gives the auxiliary equation $2m^2 + 5m - 3 = 0$ which can be factored as $(2m - 1)(m + 3) = 0$, and the roots are $m = ½$ and $m = -3$.

Since the roots are real and different the general solution is $y = Ae^{\alpha x} + Be^{\beta x}$ with $\alpha = ½$ and $\beta = -3$ then the general solution is $y = Ae^{x/2} + Be^{-3x}$

EXACT DIFFERENTIAL EQUATION

Differential equations of form $M(x, y)dx + N(x, y)dy = 0$ are said to be exact if

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

Example 3-7

Solve the equation:
$(2x + 4y + 6)\, dx + (4x - 2y - 5)\, dy = 0.$

Solution

Check for exactness:

$M = 2x + 4y + 6 \qquad N = 4x - 2y - 5$

$\dfrac{\partial M}{\partial y} = 4 \qquad \dfrac{\partial N}{\partial x} = 4$

Since $\dfrac{\partial M}{\partial y} = \dfrac{\partial N}{\partial x}$, the equation is exact.

$(2x + 4y + 6)\, dx + (4x - 2y - 5)\, dy = 0$
$2x\, dx + 4y\, dx + 6\, dx + 4x\, dy - 2y\, dy - 5\, dy = 0$

$$2x\,dx - 2y\,dy + 6\,dx - 5\,dy + 4(y\,dx + x\,dy) = 0$$

Note: $y\,dx + x\,dy = d(xy)$

Integrate:

$$x^2 - y^2 + 6 - 5 + 4xy = C$$

BERNOULLI'S EQUATION TYPE

Equations of type $\frac{dy}{dx} + P(x)\,y = Q(x)\,y^n$ has a general solution of

$$ve^{(1-n)\int p dx} = (1-n)\int Q e^{(1-n)\int p dx}\,dx$$

where $v = y^{1-n}$

If $n = 1$, the solution is:

$$\ln y = \int (Q - P)dx + C$$

FINDING THE DIFFERENTIAL EQUATION FROM A GENERAL SOLUTION

To find the differential equation when the general solution is given, differentiate the general solution, differentiate the derived solution, differentiate the second derived solution, etc., until the number of derived equation is equal to the number of independent arbitrary constants; finally eliminate the constants from the derived equations.

Example 3-8 Find the differential equation of $x^2 + y^2 = cx$.

Solution

Isolate c: $c = \frac{x^2 + y^2}{x} = x + \frac{y^2}{x}$

Differentiate with respect to x:

$$0 = 1 + \frac{x(2yy') - y^2}{x^2}$$

$$0 = x^2 + 2xyy' - y^2$$

$$2xyy' = y^2 - x^2$$

Example 3-9 Find the differential equation of $(x - c_1)^2 + c_2 y = c_3$

Solution

Differentiate & isolate c_2:

$$2(x - c_1) + c_2 y' = 0$$

$$c_2 = 2(c_1 - x)/y'$$

Differentiate & isolate c_1:

$$0 = 2\frac{y'(0-1) - (c_1 - x)y''}{(y')^2}$$

$$0 = -y' - c_1 y'' + x y''; \quad c_1 = -y'/y'' + x$$

Differentiate:

$$0 = -\frac{y''(y'') - y'(y''')}{(y'')^2} + 1$$

$$0 = -(y'')^2 + y'(y''') + (y'')^2$$

$$y'(y''') = 0; \quad y''' = 0$$

Example 3-10 Find the differential equation of all lines through the origin.

Solution

The general solution of all lines through the origin is:

$$y = mx$$

$$m = y/x$$

Differentiate

$$0 = \frac{xy' - y}{x^2}; \quad xy' = y$$

Example 3-11 Find the differential equation of all circles through (0, 0) and (2, 0).

Solution

The standard equation of a circle is: $(x - h)^2 + (y - k)^2 = r^2$

at (0, 0): $(0 - h)^2 + (0 - k)^2 = r^2$

$r^2 = h^2 + k^2 \quad \rightarrow$ Eq. (1)

at (2, 0): $(2 - h)^2 + (0 - k)^2 = r^2$

but $r^2 = h^2 + k^2$

$$4 - 4h + h^2 + k^2 = h^2 + k^2; \quad h = 1$$

then

$$(x - 1)^2 + (y - k)^2 = r^2 = 1 + k^2$$

$$x^2 - 2x + 1 + y^2 - 2ky + k^2 = 1 + k^2$$

$$x^2 - 2x + y^2 - 2ky = 0$$

$$2k = \frac{x^2 - 2x + y^2}{y}$$

Differentiate:

$$0 = \frac{y[2x - 2 + 2yy'] - [x^2 - 2x + y^2]y'}{y'}$$

$$0 = 2xy - 2y + 2y^2y' - x^2y' + 2xy' - y^2y'$$

$$0 = y'(y^2 - x^2 + 2x) + 2y(x - 1)$$

Example 3-12

Find the differential equation of all circles of radius 1 and centers on line $x = y$.

Solution

The equation of the circle is $(x - h)^2 + (y - k)^2 = r^2$ where $h = k = c$ and $r = 1$, then

$$(x - c)^2 + (y - c)^2 = 1 \quad \rightarrow \text{Eq. (1)}$$

$$2(x - c)(1) + 2(y - c)y' = 0$$

$$x - c + yy' - cy' = 0$$

$$x + yy' = c + cy'; \quad c = \frac{x + yy'}{1 + y'}$$

In Eq. (1):

$$\left[x - \frac{x + yy'}{1 + y'}\right]^2 + \left[y - \frac{x + yy'}{1 + y'}\right]^2 = 1$$

$$\frac{(x + xy' - x - yy')^2}{(1 + y')^2} + \frac{(y + yy' - x - yy')^2}{(1 + y')^2} = 1$$

$$(xy' - yy')^2 + (y - x)^2 = (1 + y')^2$$

$$(y')^2(x - y)^2 + (y - x)^2 = (1 + y')^2$$

But $(x - y)^2 = (y - x)^2$

$$(y - x)^2[(y')^2 + 1] = (1 + y')^2$$

SOME APPLICATIONS OF DIFFERENTIAL EQUATION

Population Growth

The rate of population growth is proportional to the present population P.

$$\frac{dP}{dt} = kP$$

$$P = P_o e^{kt}$$

where P_o is the population at time $t = 0$, k is constant.

Exponential Growth and Decay

$$\frac{dQ}{dt} = kQ$$

$$Q = Q_o e^{kt}$$

Cooling and Heating Problems

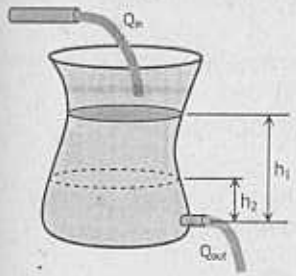
Newton's Law of Cooling: *The surface temperature of a cooling body changes at the rate proportional to the difference between the surface and ambient temperatures.*

$$\frac{dT}{dt} = -k(T - t_s)$$

$$T = t_s + (T_o - t_s)e^{-kt}$$

where T = temperature of the body at any time
t_s = ambient temperature

Flow Problems



$$\frac{dQ}{dt} = \text{Rate of inflow} - \text{Rate of outflow}$$

$$\int_{t_1}^{t_2} dt = \int_{h_1}^{h_2} \frac{dQ}{Q_{in} - Q_{out}}$$

where Q = Concentration or volume of liquid in the tank at any time

Continuous Compound Interest

$$\frac{dP}{dt} = rP$$

$$P = P_o e^{rt}$$

where P_o is the amount at $t = 0$, r is the nominal rate of interest, and P is the amount after time t.

Motion Problems

$$\text{Velocity, } v = \frac{ds}{dt}$$

$$\text{Acceleration, } a = \frac{dv}{dt}$$

$$v\,dv = a\,ds$$

Newton's Second Law of Motion

$$F = M\frac{dv}{dt}$$

where M = mass, F = force

Example 3-13

A tank contains 400 liters of brine holding 100 kg of salt in solution. Water containing 125 g of salt per liter flows into the tank at the rate of 12 liters per minute, and the mixture, kept uniform by stirring, flows out at the same rate. Find the amount of salt at the end of 90 minutes.

Solution

Let Q be the amount of salt in the tank at any time.
Rate of inflow of salt = 125 g/lit × 12 lit/min
Rate of inflow of salt = 1.5 kg/min
Rate of outflow of salt = $(Q/400) \times 12$
$= 0.03Q$ (kg/lit)

$$\frac{dQ}{dt} = \text{Rate of inflow} - \text{Rate of outflow}$$

$$\frac{dQ}{dt} = 1.5 - 0.03Q = -(0.03Q - 1.5)$$

When $t = 0$ min, $Q = 100$ kg
Find Q when $t = 90$ minutes

$$-\int_{100}^{Q} \frac{dQ}{0.03Q - 1.5} = \int_{0}^{90} dt$$

$$-\frac{1}{0.03}\Big[\ln(0.03Q - 1.5)\Big]_{100}^{Q} = \Big[t\Big]_{0}^{90}$$

$\ln[0.03Q - 1.5] - \ln[0.03(100) - 1.5] = -0.03(90 - 0)$
$\ln(0.03Q - 1.5) = -2.2945$
$0.03Q - 1.5 = e^{-2.2945} = 0.100808$
$Q = 53.36$ kg

Problems - Set 5 Differential Equation & Application

PROBLEM 5 - 1

Obtain the differential equation of the family of straight lines with slope and y-intercept equal.

A. $y\,dy + (x + 1)\,dx = 0$
B. $y\,dy - (x + 1)\,dx = 0$
C. $y\,dx + (x + 1)\,dy = 0$
D. $y\,dx - (x + 1)\,dy = 0$

PROBLEM 5 - 2

Obtain the differential equation of all straight lines with algebraic sum of the intercepts fixed as k.

A. $(1 + y')(xy' - y) = ky$
B. $(1 - y')(xy' + y) = ky$
C. $(1 - y')(xy' - y) = ky$
D. $(1 + y')(xy' + y) = ky$

PROBLEM 5 - 3

Obtain the differential equation of all straight lines at a fixed distance p from the origin.

A. $(xy' - y)^2 = p[1 + (y')^2]$
B. $(xy' + y)^2 = [1 + (y')^2]$
C. $(xy' - y)^2 = p[1 + (y')^2]$
D. $(xy' - y) = [1 + y']$

PROBLEM 5 - 4
CE May 1997

Determine the differential equation of the family of lines passing through the origin.

A. $x\,dy - y\,dx = 0$
B. $x\,dx - y\,dy = 0$
C. $x\,dy + y\,dx = 0$
D. $x\,dx + y\,dy = 0$

PROBLEM 5 - 5

Obtain the differential equation of all circles with center on line $y = -x$ and passing through the origin.

A. $(y^2 + 2xy - x^2)\,dx - (y^2 - 2xy + x^2)dy = 0$
B. $(y^2 - 2xy - x^2)\,dx - (y^2 - 2xy - x^2)dy = 0$
C. $(y^2 - 2xy - x^2)\,dx - (y^2 - 2xy - x^2)dy = 0$
D. $(y^2 + 2xy - x^2)\,dx + (y^2 - 2xy - x^2)dy = 0$

PROBLEM 5 - 6

Obtain the differential equation of all parabolas with axis parallel to the x-axis.

A. $3(y'')^2 - y'\,y''' = 0$
B. $3(y'')^2 + y'\,y''' = 0$
C. $2(y'')^2 - y'\,y''' = 0$
D. $2(y'')^2 - y''\,y''' = 0$

PROBLEM 5 - 7
CE May 1996

What is the differential equation of the family of parabolas having their vertices at the origin and their foci on the x-axis.

A. $2x\,dx - y\,dy = 0$
B. $y\,dx - 2x\,dy = 0$
C. $2x\,dx + y\,dy = 0$
D. $y\,dx + 2x\,dy = 0$

PROBLEM 5 - 8

Obtain the particular solution of $dr/dt = -4rt$ when $t = 0$, $r = r_0$.

A. $r = 2\, r_o\, e^{-2t^2}$ C. $r = r_o\, e^{2t^2}$
B. $r = r_o\, e^{-t^2}$ D. $r = r_o\, e^{-2t^2}$

PROBLEM 5 - 9 Obtain the general solution of the differential equation $xy\, dx - (x + 2)\, dy = 0$
A. $e^x = y(x + 2)^2$ C. $e^x = cy(x + 2)$
B. $e^x = cy(x + 2)^2$ D. $e^x = 2cy(x + 2)$

PROBLEM 5 - 10 Obtain the general solution of $y' = x\, y^2$.
A. $y(x^2 + c) + 2 = 0$ C. $(x^2 + c) + 2 = 0$
B. $y(x^2 + c) - 2 = 0$ D. $y(x + c) - 2 = 0$

PROBLEM 5 - 11 Solve the equation $xy\, dx - (x + 2y)2\, dy = 0$.
A. $e^{x/y} = c\, y^3\, (x\, y)$ C. $e^{x/y} = c\, y^3\, (x + y)$
B. $e^{x/y} = c\, y^3\, (x - y)$ D. $e^{x/y} = c\, y^4\, (x - 1)$

PROBLEM 5 - 12 Obtain the particular solution of $(3x^2 - 2y^2)y' = 2xy$; when $x = 0$, $y = -1$.
A. $x^2 = 2y^2(y + 1)$ C. $x^2 = 2y^2(y - 1)$
B. $x^2 = y^2(y + 1)$ D. $x^2 = y^2(y + 1)$

PROBLEM 5 - 13 Solve the equation $(2xy + y)\, dx + (x^2 - x)\, dy = 0$.
A. $y = cx(x + 1)^3 + 2x$ C. $y = 3x + cx(x - 1)^3$
B. $y = cx(x + 1)^{-3} - 2$ D. $y = cx(x - 1)^{-3}$

PROBLEM 5 - 14 Solve the equation $(6x + y^2)\, dx + y(2x - 3y)\, dy = 0$.
A. $xy^2 + 3x^2 - y^3 = c$ C. $xy^2 + 2x^2 - 3y^3 = c$
B. $3xy^2 - 3x^2 - y^3 = c$ D. $xy^2 + 3x^2 - y^2 = c$

PROBLEM 5 - 15 Solve the equation $(r + \sin\theta - \cos\theta)\, dr + r(\sin\theta + \cos\theta)\, d\theta = 0$.
A. $r^2 + r\,(\sin\theta + \cos\theta) = c$
B. $r^2 + 2r\,(\sin\theta - \cos\theta) = c$
C. $r - 2r\,(\sin\theta - \cos\theta) = c$
D. $r^3 + 2r\,(\sin\theta + \cos\theta) = c$

PROBLEM 5 - 16 Solve $(x^2 + y)\, dx + (y^3 + x)\, dy = 0$.
A. $\frac{x^3}{3} + xy + \frac{y^4}{4} = c$ C. $\frac{x^3}{3} + xy - \frac{y^4}{4} = c$
B. $\frac{x^3}{3} + x + y + \frac{y^4}{4} = c$ D. $\frac{x^3}{3} - xy^2 + \frac{y^4}{4} = c$

PROBLEM 5 - 17 Solve the equation $x\, dy - y\, dx = 2x^3\, dx$.
A. $y = x^2 + cx$ C. $y = x^3 + cx$
B. $y = x^3 + c$ D. $y = x^2 + cx^3$

PROBLEM 5 - 18 Solve the equation $x\, dy + y\, dx = 2x^2y\, dx$.
A. $\ln|xy| = x^3 + c$ C. $\ln|xy| = x^2 + c$
B. $\ln|x + y| = x^2 + c$ D. $\ln|x - y| = 2x^2 + c$

PROBLEM 5 - 19 Solve the equation $\frac{dy}{dx} + \frac{2}{x}y = 6x^3$.
A. $x^2\, y = 6x^5 + c$ C. $x^2\, y^2 = 6x^5 + c$
B. $x^2 = 5x^6 + c$ D. $x^2\, y = x^6 + c$

PROBLEM 5 - 20 Solve the equation $(x^5 + 3y)\, dx - x\, dy = 0$.
A. $2y = x^5 + cx^3$ C. $2y = x^3 + cx^2$
B. $y = x^5 + cx^3$ D. $y^2 = 5x^4 + cx^3$

PROBLEM 5 - 21 The differential equation $\frac{dy}{dx} = \frac{3x^2 + y^2}{2xy}$ can be made exact by using the integrating factor:
A. x^2 C. y^2
B. $1/x^2$ D. $1/y^2$

PROBLEM 5 - 22 What is not true for the differential equation $y' + y/x^2 = 1/x^2$?
A. it is linear
B. it is homogeneous
C. it is separable
D. it can be solved using the integrating factor $e^{-1/x}$

PROBLEM 5 - 23 A tank contains 400 liters of brine holding 100 kg of salt in solution. Water containing 125 g of salt per liter flows into the tank at the rate of 12 liters per minute, and the mixture, kept uniform by stirring, flows out at the same rate. Find the amount of salt at the end of 90 minutes.
A. 53.36 kg C. 53.63 kg
B. 0 D. 65.33 kg

PROBLEM 5 - 24 Under certain conditions, cane sugar in water is converted into dextrose at a rate proportional to the amount that is unconverted at any time. If, of 75 kg at time $t = 0$, 8 kg are converted during the first 30 minutes, find the amount converted in 2 hours.
A. 72.73 kg C. 27.23 kg
B. 23.27 kg D. 32.72 kg

PROBLEM 5 - 25 A thermometer reading 18°C is brought into a room where the temperature is 70 °C; 1 minute later the thermometer reading is 31°C. Determine the thermometer reading 5 minutes after it is brought into the room.

A. 62.33°C
B. 58.99°C
C. 56.55°C
D. 57.66°C

PROBLEM 5 - 26

Solve the equation $(4xy + 3y^2 - x)\,dx + x(x + 2y)\,dy = 0$.

A. $4xy + 4y^2 - x = c$
B. $x^3(4xy + 4y^2 - x) = c$
C. $x^2(4xy - 4y^2 - x) = c$
D. $x^3(4x^2y + 4y^2 + x^2) = c$

PROBLEM 5 - 27
ECE April 1998

The equation $y^2 = cx$ is the general equation of

A. $y' = 2x/y$
B. $y' = 2y/x$
C. $y' = y/2x$
D. $y' = x/2y$

PROBLEM 5 - 28
ME Oct. 1997

Given the following simultaneous differential equations.

$$2\,dx/dt - 3\,dy/dt + x - y = k$$
$$3\,dx/dt + 2\,dy/dt - x + \cos t = 0.$$

Solve for dy/dt.

A. $(2/9)\,[\cos t + (3/2)x - (5/2)y - (3/2)k]$
B. $(-1/6)\,[\sin t + (1/9)x + y^2 - (3/2)k]$
C. $(1/13)\,(5x - 3y - 3k - 2\cos t)$
D. $(2/13)\,[\cos t + (5/2)x - (3/2)y - (3/2)k]$

ANSWER SHEET

1.	A	B	C	D	E	11.	A	B	C	D	E	21.	A	B	C	D	E
2.	A	B	C	D	E	12.	A	B	C	D	E	22.	A	B	C	D	E
3.	A	B	C	D	E	13.	A	B	C	D	E	23.	A	B	C	D	E
4.	A	B	C	D	E	14.	A	B	C	D	E	24.	A	B	C	D	E
5.	A	B	C	D	E	15.	A	B	C	D	E	25.	A	B	C	D	E
6.	A	B	C	D	E	16.	A	B	C	D	E	26.	A	B	C	D	E
7.	A	B	C	D	E	17.	A	B	C	D	E	27.	A	B	C	D	E
8.	A	B	C	D	E	18.	A	B	C	D	E	28.	A	B	C	D	E
9.	A	B	C	D	E	19.	A	B	C	D	E	29.	A	B	C	D	E
10.	A	B	C	D	E	20.	A	B	C	D	E	30.	A	B	C	D	E

Solutions to Set 5 Differential Equation & Application

SOLUTION 5 - 1
Ans: D

The standard equation of the line in slope-intercept form is: $y = mx + b$

Since the slope and y-intercept are equal, $m = b = c$, then

$$y = cx + c$$

Isolating constant and differentiate:

$$c = \frac{y}{x+1}$$

$$0 = \frac{(x+1)dy - y\,dx}{(x+1)}$$

$$\mathbf{y\,dx - (x + 1)\,dy = 0}$$

SOLUTION 5 - 2
Ans: C

The equation of the line in intercept form is:

$$\frac{x}{a} + \frac{y}{b} = 1$$

where $a + b = k$ or $b = k - a$

$$\frac{x}{a} + \frac{y}{k-a} = 1 \qquad \rightarrow (1)$$

Differentiate

$$\frac{dx}{a} + \frac{dy}{k-a} = 0; \qquad \frac{dx}{a} = \frac{dy}{a-k}$$

$$a - k = ay'; \qquad a = \frac{k}{1-y'}$$

Substitute a to Eq. (1):

$$\frac{x}{\frac{k}{1-y'}} + \frac{y}{k - \frac{k}{1-y'}} = \frac{x}{\frac{k}{1-y'}} + \frac{y}{\frac{k-ky'-k}{1-y'}} = 1$$

$$\frac{x(1-y')}{k} - \frac{y(1-y')}{ky'} = 1$$

$$\frac{xy'(1-y') - y(1-y')}{ky'} = 1$$

$$\mathbf{(1 - y')(xy' - y) = ky}$$

SOLUTION 5 - 3
Ans: A

The equation of the line in normal form is:

$$x \cos \alpha + y \sin \alpha = p \qquad \rightarrow (1)$$

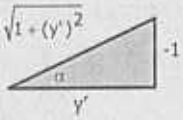
Where α = arbitrary constant
p = distance of the line from the origin.

Differentiate Eq. (1)

$$\cos \alpha + y' \sin \alpha = 0; \quad y' \sin \alpha = -\cos \alpha$$
$$\tan \alpha = -1/y'$$

From the right triangle shown:

$$\sin \alpha = -1/\sqrt{1+(y')^2}$$
$$\cos \alpha = y' / \sqrt{1+(y')^2}$$



Substitute sin α and cos α to Eq. (1):

$$x\, y' / \sqrt{1+(y')^2} + y\,[-1/\sqrt{1+(y')^2}\,] = p$$
$$xy' - y = p\sqrt{1+(y')^2}$$

Square both side:

$$(xy' - y)^2 = p[1 + (y')^2]$$

SOLUTION 5 - 4
Ans: A

The standard equation of the line (slope-intercept form) is: $y = mx + b$

Since the line passes through the origin, $b = 0$

$$y = mx\,;\ m = \frac{y}{x}$$

Differentiate: $\quad 0 = \dfrac{xdy - ydx}{x^2}$

Thus, the differential equation is: $x\,dy - y\,dx = 0$

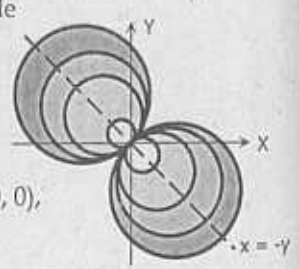
SOLUTION 5 - 5
Ans: D

The standard equation of a circle with center at (h, k) is:

$$(x - h)^2 + (y - k)^2 = r^2$$

Where $h = -k$

$$(x + k)^2 + (y - k)^2 = r^2$$



Since the line passes through (0, 0),

$$k^2 + k^2 = r^2\,;\quad r^2 = 2k^2$$

Thus;

$$(x + k)^2 + (y - k)^2 = 2k^2$$
$$x^2 + 2kx + k^2 + y^2 - 2ky + k^2 = 2k^2$$
$$x^2 + y^2 = 2ky - 2kx$$

Isolating $2k$ and differentiate:

$$2k = \frac{x^2 + y^2}{y - x}$$

$$\frac{(y-x)(2xdx + 2ydy) - (x^2 + y^2)(dy - dx)}{(y-x)^2} = 0$$

$$2xydx + 2y^2dy - 2x^2dx - 2xydy - x^2dy + x^2dx - y^2dy + y^2dx = 0$$

$$2xydx - 2x^2dx + x^2dx + y^2dx + 2y^2dy - 2xydy - x^2dy - y^2dy = 0$$

$$(y^2 + 2xy - x^2)\,dx + (y^2 - 2xy - x^2)dy = 0$$

SOLUTION 5 - 6
Ans: A

The general solution of all parabolas with axis parallel to x-axis is: $(y - k)^2 = 4a(x - h)$

The equation contains three arbitrary constants (h, k, and a) therefore this should be differentiated three times.

Isolate $4a$:

$$\frac{(y-k)^2}{x-h} = 4a$$

Differentiate and isolate h:

$$\frac{(x-h)[2(y-k)y'] - (y-k)^2(1)}{(x-h)^2} = 0$$
$$(x - h)[2y'(y - k)] = (y - k)^2$$
$$x - h = \frac{y-k}{2y'}; \qquad h = x - \frac{y-k}{2y'}$$

Differentiate and isolate k:

$$0 = 1 - \frac{2y'(y') - (y-k)(2y'')}{(2y')^2}$$
$$1 = \frac{2(y')^2 - 2y''(y-k)}{4(y')^2}$$
$$4(y')^2 = 2(y')^2 - 2y''(y - k)$$
$$2(y')^2 = -2y''(y - k)$$
$$y - k = -\frac{(y')^2}{y''}\,;\ k = y + \frac{(y')^2}{y''}$$

Differentiate:

$$0 = y' + \frac{y''[2(y')y''] - (y')^2 y'''}{(y'')^2}$$

$$0 = y'(y'')^2 + 2y'(y'')^2 - (y')^2 y'''$$

$$0 = 3y'(y'')^2 - (y')^2 y'''$$

$$0 = 3(y'')^2 - y' y'''$$

SOLUTION 5 - 7
Ans: B

The general solution of a parabola with vertex at the origin and focus at the x-axis is, $y^2 = 4ax$

$$4a = \frac{y^2}{x}$$

Differentiate

$$0 = \frac{x(2y\,dy) - y^2\,dx}{x^2}$$

$$0 = 2x\,dy - y\,dx \text{ or } 0 = y\,dx - 2x\,dy$$

SOLUTION 5 - 8
Ans: D

$dr/dt = -4rt$

By separation of variables:

$$\frac{dr}{r} = -4t\,dt$$

Integrating both side:

$$\int_0^r \frac{dr}{r} = -4\int_0^t t\,dt; \quad \Big[\ln r\Big]_{r_o}^{r} = -4\left[\frac{t^2}{2}\right]_0^t$$

$$\ln r - \ln r_o = -2t^2$$

$$\ln\frac{r}{r_o} = -2t^2; \quad \frac{r}{r_o} = e^{-2t^2}$$

$$r = r_o\, e^{-2t^2}$$

SOLUTION 5 - 9
Ans: B

$xy\,dx - (x + 2)\,dy = 0$

By separation of variables:

$$\frac{x\,dx}{x+2} = \frac{dy}{y}; \quad \left(1 - \frac{2}{x+2}\right)dx = \frac{dy}{y}$$

Integrating:

$$x - 2\ln(x + 2) = \ln y + \ln c$$

$$x = \ln y + \ln c + \ln(x + 2)^2$$

$$x = \ln[cy(x + 2)^2]$$

$$e^x = cy(x + 2)^2$$

SOLUTION 5 - 10
Ans: A

$y' = xy^2 = dy/dx$

By separation of variables:

$$x\,dx = \frac{dy}{y^2} = y^{-2}\,dy$$

$$\tfrac{1}{2}x^2 = (-1)y^{-1} + c$$

$$x^2 + c = -\frac{2}{y}; \quad y(x^2 + c) = -2$$

$$y(x^2 + c) + 2 = 0$$

SOLUTION 5 - 11
Ans: C

The given differential equation is homogeneous.

Let $x = vy$, then $dx = v\,dy + y\,dv$

$$vy\,y\,(v\,dy + y\,dv) = (vy + 2y)^2\,dy$$

$$v\,y^2(v\,dy + y\,dv) = y^2(v + 2)^2\,dy$$

$$v\,(v\,dy + y\,dv) = (v + 2)^2\,dy$$

$$v^2\,dy + vy\,dv = (v + 2)^2\,dy$$

$$vy\,dv = (v^2 + 4v + 4)\,dy - v^2\,dy$$

$$vy\,dv = 4(v + 1)dy$$

by separation of variables:

$$\frac{v\,dv}{v+1} = 4\frac{dy}{y}$$

$$\left(1 - \frac{1}{v+1}\right)dv = 4\frac{dy}{y}$$

Integrating:

$$v - \ln(v + 1) = 4\ln y + \ln c$$

$$v = \ln y^4 + \ln(v + 1) + \ln c = \ln[y^4(v + 1)c]$$

But $v = x / y$,

$$x/y = \ln[c\,y^4(x/y + 1)]$$

$$x/y = \ln[c\,y^3(x + y)]$$

$$e^{x/y} = c\,y^3(x + y)$$

SOLUTION 5 - 12
Ans: A

$$(3x^2 - 2y^2)\frac{dy}{dx} = 2xy$$

$$(3x^2 - 2y^2)\,dy = 2xy\,dx$$

The given equation is homogeneous:

Let $x = vy$

$dx = v\,dy + y\,dv$

Then,

$$[3(vy)^2 - 2y^2]\,dy = 2(vy)\,y\,(v\,dy + y\,dv)$$

$$y^2(3v^2 - 2)dy = 2vy^2\,(v\,dy + y\,dv)$$

$(3v^2 - 2)dy = 2v^2\,dy + 2vy\,dv$
$(3v^2 - 2)dy - 2v^2\,dy = 2vy\,dv;\quad (v^2 - 2)\,dy = 2vy\,dv$

Separating variables and integrate:

$$\frac{dy}{y} = \frac{2v\,dv}{v^2 - 2}$$

$$\ln y = \ln(v^2 - 2) + \ln c\,;\quad \ln\frac{y}{v^2 - 2} = \ln c$$

$$\frac{y}{v^2 - 2} = c;\qquad y = c(v^2 - 2)$$

Since $v = x/y$

$y = c\,[(x/y)^2 - 2];\ y^3 = c(x^2 - 2y^2)$

When $x = 0$, $y = -1$;

$(-1)^3 = c[(0)^2 - 2(-1)^2];\ c = ½$

Then;

$y^3 = ½(x^2 - 2y^2)$
$2y^3 = x^2 - 2y^2$
$x^2 = 2y^2(y + 1)$

SOLUTION 5 - 13
Ans: D

$(2xy + y)\,dx + (x^2 - x)\,dy = 0$

Separating variables:

$y(2x + 1)\,dx + (x^2 - x)\,dy = 0$

$$\frac{(2x+1)dx}{x(x-1)} + \frac{dy}{y} = 0$$

$$\frac{2x+1}{x(x-1)} = \frac{A}{x} + \frac{B}{x-1}$$

$2x + 1 = A(x - 1) + Bx$

when $x = 0$; $A = -1$
when $x = 1$; $B = 3$

$$\frac{2x+1}{x(x-1)} = \frac{3}{x-1} - \frac{1}{x}$$

Then,

$$\left(\frac{3}{x-1} - \frac{1}{x}\right)dx + \frac{dy}{y} = 0$$

$3\ln(x - 1) - \ln x + \ln y = \ln c$
$\ln y = \ln c + \ln x - 3\ln(x - 1)$
$\ln y = \ln[cx\,(x - 1)^{-3}]$
$y = cx(x - 1)^{-3}$

SOLUTION 5 - 14
Ans: B

$(6x + y^2)\,dx + y(2x - 3y)\,dy = 0$

The given equation cannot be separated and is non-homogeneous.

Checking for exactness:

$M = 6x + y^2$

$$\frac{\partial M}{\partial y} = 2y$$

$N = 2xy - 3y^2$

$$\frac{\partial N}{\partial x} = 2y = \frac{\partial M}{\partial y}$$ (therefore the equation is exact, $F = c$)

Thus;

$$\frac{\partial F}{\partial x} = M = 6x + y^2 \qquad \rightarrow (1)$$

$$\text{and } \frac{\partial F}{\partial y} = N = 2xy - 3y^2 \qquad \rightarrow (2)$$

Integrating Eq. (1):

$F = 3x^2 + xy^2 + f(y) \qquad \rightarrow (3)$

$$\frac{\partial F}{\partial y} = 0 + 2xy + f'(y) = 2xy - 3y^2$$

$f'(y) = -3y^2$
$f(y) = -y^3$

From Eq. (3)

$F = 3x^2 + xy^2 - y^3$
$3x^2 + xy^2 - y^3 = c$

Another Solution:

Knowing that the equation is exact, it can be rearranged as:

$(6x + y^2)\,dx + y(2x - 3y)\,dy = 0$
$(y^2\,dx + 2xy) + 6x\,dx - 3y^2\,dy = 0$

$xy^2 + 3x^2 - y^3 = c$

SOLUTION 5 - 15
Ans: B

$(r + \sin\theta - \cos\theta)\,dr + r(\sin\theta + \cos\theta)\,d\theta = 0$

Check for exactness:

$M = r + \sin\theta - \cos\theta$

$$\frac{\partial M}{\partial \theta} = \cos\theta + \sin\theta$$

$$N = r(\sin\theta + \cos\theta)$$

$$\frac{\partial N}{\partial r} = \sin\theta + \cos\theta = \frac{\partial M}{\partial \theta} \quad \text{(Exact equation)}$$

$$\frac{\partial F}{\partial r} = r + \sin\theta - \cos\theta \quad \rightarrow (1)$$

and $\dfrac{\partial F}{\partial \theta} = r(\sin\theta + \cos\theta) \quad \rightarrow (2)$

Integrate Eq. (2):

$$F = r(-\cos\theta + \sin\theta) + f(r) \quad \rightarrow (3)$$

then $\dfrac{\partial F}{\partial r} = (-\cos\theta + \sin\theta) + f'(r) = r + \sin\theta - \cos\theta$

$$f'(r) = r$$

$$f(r) = \tfrac{1}{2} r^2$$

In Eq. (3):

$$F = r(-\cos\theta + \sin\theta) + \tfrac{1}{2} r^2 = c$$

$$r^2 + 2r(\sin\theta - \cos\theta) = c$$

Another solution:

Since the equation is exact:

$$(r + \sin\theta - \cos\theta)\, dr + r(\sin\theta + \cos\theta)\, d\theta = 0$$

Rearranging:

$$r\, dr + (r\cos\theta\, d\theta + \sin\theta\, dr) + (r\sin\theta\, d\theta - \cos\theta\, dr) = 0$$

$$\tfrac{1}{2} r^2 + r\sin\theta - r\cos\theta = c$$

$$r^2 + 2r\sin\theta - 2r\cos\theta = c$$

$$r^2 + 2r(\sin\theta - \cos\theta) = c$$

SOLUTION 5 - 16
Ans: A

$$(x^2 + y)\, dx + (y^3 + x)\, dy = 0$$

Note that $d(xy) = x\, dy + y\, dx$

Rearrange:

$$x^2\, dx + (y\, dx + x\, dy) + y^3\, dy = 0$$

Integrate:

$$\frac{x^3}{3} + xy + \frac{y^4}{4} = c$$

SOLUTION 5 - 17
Ans: C

$$x\, dy - y\, dx = 2x^3\, dx$$

Notice that $d\left(\dfrac{y}{x}\right) = \dfrac{x\, dy - y\, dx}{x^2}$

Rearranging the equation and integrate:

$$\frac{x dy - y dx}{x^2} = 2x\, dx$$

$$\frac{y}{x} = x^2 + c$$

$$y = x^3 + cx$$

SOLUTION 5 - 18
Ans: C

$$x\, dy + y\, dx = 2x^2 y\, dx$$

Multiply both side by $\dfrac{1}{xy}$

$$\frac{x dy + y dx}{xy} = 2x\, dx$$

Integrate:

$$\ln|xy| = x^2 + c$$

SOLUTION 5 - 19
Ans: D

$$\frac{dy}{dx} + \frac{2}{x} y = 6x^3 \quad \rightarrow (1)$$

The given equation is linear in y and has the form,

$$\frac{dy}{dx} + Py = Q$$

$$\int P dx = \int \frac{2}{x} dx = 2\ln x = \ln x^2$$

Integrating factor $= e^{\int P dx} = e^{\ln x^2} = x^2$

Multiplying both side of Eq. (1) by x^2 and rearrange.

$$x^2\, dy + 2xy\, dx = 6x^5$$

Integrate:

$$x^2 y = x^6 + c$$

SOLUTION 5 - 20
Ans: A

$$(x^5 + 3y)\, dx - x\, dy = 0$$

The equation is linear in y.

$$x^5 + 3y - x\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} - \frac{3}{x} y = x^4$$

$$P = -\frac{3}{x}$$

$$\int P dx = \int -\frac{3}{x} dx = -3 \ln x = \ln x^{-3}$$

Integrating factor = $e^{\int P dx} = e^{\ln x^{-3}} = x^{-3}$

Multiply both side by x^{-3} and rearrange.

$x^{-3}\, dy - 3x^{-4} y\, dx = x$

Integrate:

$x^{-3} y = \frac{1}{2} x^{2} + c$

$2y = x^{5} + cx^{3}$

SOLUTION 5 - 21
Ans: B

$$\frac{dy}{dx} = \frac{3x^2 + y^2}{2xy}$$

$(3x^2 + y^2)\, dx - 2xy\, dy = 0$

$M = 3x^2 + y^2$; $N = -2xy$

$$\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = 2y - (-2y) = 4y$$

$$\frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = \frac{4y}{-2xy}$$

$$\frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = -\frac{2}{x}$$

The integrating factor is:

$$e^{\int -\frac{2}{x} dx} = e^{-2\ln x} = e^{\ln x^{-2}} = x^{-2}$$

$$e^{\int -\frac{2}{x} dx} = 1/x^2$$

SOLUTION 5 - 22
Ans: B

$y' + y/x^2 = 1/x^2$

$\frac{dy}{dx} + \frac{y}{x^2} = \frac{1}{x^2}$ multiply both side by $x^2\, dx$

$x^2\, dy + y\, dx = dx$

$x^2\, dy + (y - 1)\, dx = 0$

The equation is linear in x, separable, not homogeneous.

SOLUTION 5 - 23
Ans: A

Let Q = amount of salt in the solution at any time t, in kg

$Q/400 = 0.0025Q$ kg/liter = concentration at any time t

$\frac{dQ}{dt}$ = rate of change of the amount of salt

Amount of salt entering the solution per minute = 12(125)/1000 = 1.5 kg/min

Amount of salt removed = $(Q/400)(12) = 0.03Q$ (kg/min)

Then;

$$\frac{dQ}{dt} = 1.5 - 0.03Q$$

$$\frac{dQ}{1.5 - 0{,}03Q} = dt$$

When $t = 0$; $Q = 100$ kg

When $t = 90$; $Q = ?$

Integrate:

$$\int_{100}^{Q} \frac{dQ}{1.5 - 0.03Q} = \int_{0}^{90} dt$$

$$-\frac{1}{0.03} \ln(1.5 - 0.03Q)\Big]_{100}^{Q} = t\ \Big]_{0}^{90}$$

$\ln(1.5 - 0.03Q) - \ln[1.5 - 0.03(100)] = -0.03(90)$

$$\ln \frac{1.5 - 0.03Q}{1.5 - 3} = -2.7$$

$$\frac{1.5 - 0.03Q}{1.5 - 3} = e^{-2.7} = 0.0672055$$

$1.5 - 0.03Q = -0.100808$

Q = **53.36 kg**

SOLUTION 5 - 24
Ans: C

Let q = the amount converted into dextrose at any time t in minutes

$75 - q$ = amount unconverted

Then;

$$\frac{dq}{dt} = k\,(75 - q)$$

k = constant of proportionality

and $\dfrac{dq}{75-q} = k\,dt$

Integrate:

$$\int_{q_1}^{q_2} \frac{dq}{75-q} = \int_{t_1}^{t_2} k\,dt$$

$$\Big[-\ln(75-q)\Big]_{q_1}^{q_2} = \Big[kt\Big]_{t_1}^{t_2}$$

When $t = 0$; $q = 0$
When $t = 30$; $q = 8$
When $t = 120$; $q = ?$

$$\Big[-\ln(75-q)\Big]_0^8 = \Big[kt\Big]_0^{30}$$

$-[\ln(75-8) - \ln 75] = k(30-0)$

$k = 0.0037598$

Then;

$$\Big[-\ln(75-q)\Big]_0^q = \Big[0.0037598t\Big]_0^{120}$$

$-[\ln(75-q) - \ln 75] = 0.0037598(120-0)$

$\ln(75-q) = -0.4512 + \ln 75$

$\ln(75-q) = 3.866$

$75 - q = e^{3.866} = 47.766$

$q =$ **27.23 kg**

SOLUTION 5 - 25
Ans: D

According Newton's Law of cooling, "the temperature of a body changes at a rate that is proportional to the difference in temperature between the outside medium and the body itself".

Let u = temperature of the thermometer at any time t

then, $\dfrac{du}{dt} = k(u-70)$

Where k is a constant of proportionality and is taken as positive since the temperature is increasing.

Rearranging:

$$\frac{du}{u-70} = k\,dt$$

$$\frac{du}{70-u} = -k\,dt$$

Integrate:

$\ln(70-u) = -kt + C$

when $t = 0$, $u = 18°C$

$\ln(70-18) = -k(0) + C$; $C = \ln 52$

$\ln(70-u) = -kt + \ln 52$

when $t = 1$ min, $u = 31°C$

$\ln(70-31) = -k(1) + \ln 52$

$k = \ln 52 - \ln 39$

$k = 0.2877$

Thus;

$\ln(70-u) = -0.2877\,t + \ln 52$

$70 - u = e^{-0.2877t + \ln 52} = e^{-0.2877t}e^{\ln 52}$

$u = 70 - 52e^{-0.2877t}$ (general solution)

when $t = 5$ minutes:

$u = 70 - 52e^{-0.2877(5)}$

$u =$ **57.66°C**

SOLUTION 5 - 26
Ans: B

$(4xy + 3y^2 - x)\,dx + x(x+2y)\,dy = 0 \quad \rightarrow (1)$

Check for exactness:

$M = 4xy + 3y^2 - x$

$$\frac{\partial M}{\partial y} = 4x + 6y$$

$N = x(x+2y)$

$$\frac{\partial N}{\partial x} = 2x + 2y \neq \frac{\partial M}{\partial y} \text{ (not exact)}$$

Determine the integrating factor:

$$\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = 4x + 6y - (2x + 2y) = 2x + 4y$$

$$\frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = \frac{1}{x(x+2y)}(2)(x+2y)$$

$$\frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = \frac{2}{x}$$

The integrating factor is:

$$e^{\int \frac{2dx}{x}} = e^{2\ln x} = e^{\ln x^2} = x^2$$

Multiplying Eq. (1) by the integrating factor x^2 to make it exact:

$$(4x^3y + 3x^2y^2 - x^3)\,dx + (x^4 + 2x^3y)\,dy = 0$$

Rearranging:

$$(4x^3y\,dx + x^4\,dy) - x^3\,dx + (3x^2y^2\,dx + 2x^3y\,dy) = 0$$

Integrating:

$$x^4y + x^3y^2 - x^4/4 = c$$
$$x^3(4xy + 4y^2 - x) = c$$

SOLUTION 5 - 27
Ans: C

General solution: $y^2 = cx$

To get the differential equation, differentiate once, twice, ... until the constant c is eliminated. This can be established easily by writing it in the form:

$$c = \frac{y^2}{x}$$

Differentiate once:

$$0 = \frac{x(2yy') - y^2}{x^2}$$
$$0 = 2xyy' - y^2$$
$$y' = \frac{y^2}{2xy}$$
$$y' = y/2x$$

SOLUTION 5 - 28
Ans: C

$$2\,dx/dt - 3\,dy/dt + x - y = k \quad \rightarrow (1)$$
$$3\,dx/dt + 2\,dy/dt - x + \cos t = 0 \quad \rightarrow (2)$$

Subtract (2) × 2 - (1) × 3

$$\begin{array}{lll} & 6\,dx/dt + 4\,dy/dt - 2x + 2\cos t & = 0 \\ (-) & 6\,dx/dt - 9\,dy/dt + 3x \quad - 3y & = 3k \\ \hline & 13\,dy/dt - 5x + 3y + 2\cos t & = -3k \end{array}$$

$$13\,dy/dt = 5x - 3y - 3k - 2\cos t$$

$$dy/dt = (1/13)\,(5x - 3y - 3k - 2\cos t)$$

Part 4 ENGINEERING MECHANICS

DEFINITION

Engineering Mechanics is the science which considers the effects of forces on rigid bodies. The subject is divided into two parts: *statics* and *dynamics*. In *statics* we consider the effects and distribution of forces on rigid bodies which are and remain at rest. In *dynamics* we consider the motion of rigid bodies caused by the forces acting upon them.

Outline of Engineering Mechanics

ENGINEERING MECHANICS
- STATICS
  - *Force System*
    - Concurrent
    - Parallel
    - Non-Concurrent
  - *Applications*
    - Trusses
    - Centroids
    - Friction
- DYNAMICS
  - *Kinematics*
    - Translation
    - Rotation
    - Plane Motion
  - *Kinetics*
    - Translation
    - Rotation
    - Plane Motion

The integrating factor is:

$$e^{\int \frac{2dx}{x}} = e^{2\ln x} = e^{\ln x^2} = x^2$$

Multiplying Eq. (1) by the integrating factor x^2 to make it exact:

$$(4x^3y + 3x^2y^2 - x^3)\,dx + (x^4 + 2x^3y)\,dy = 0$$

Rearranging:

$$(4x^3y\,dx + x^4\,dy) - x^3\,dx + (3x^2y^2\,dx + 2x^3y\,dy) = 0$$

Integrating:

$$x^4y + x^3y^2 - x^4/4 = c$$
$$x^3(4xy + 4y^2 - x) = c$$

SOLUTION 5 - 27
Ans: C

General solution: $y^2 = cx$

To get the differential equation, differentiate once, twice, ... until the constant c is eliminated. This can be established easily by writing it in the form:

$$c = \frac{y^2}{x}$$

Differentiate once:

$$0 = \frac{x(2yy') - y^2}{x^2}$$
$$0 = 2xyy' - y^2$$
$$y' = \frac{y^2}{2xy}$$
$$y' = y/2x$$

SOLUTION 5 - 28
Ans: C

$$2\,dx/dt - 3\,dy/dt + x - y = k \quad \rightarrow (1)$$
$$3\,dx/dt + 2\,dy/dt - x + \cos t = 0 \quad \rightarrow (2)$$

Subtract (2) × 2 - (1) × 3

$$\begin{array}{lll} & 6\,dx/dt + 4\,dy/dt - 2x + 2\cos t & = 0 \\ (-) & 6\,dx/dt - 9\,dy/dt + 3x \qquad - 3y & = 3k \\ \hline & 13\,dy/dt - 5x + 3y + 2\cos t & = -3k \end{array}$$

$$13\,dy/dt = 5x - 3y - 3k - 2\cos t$$

$$dy/dt = (1/13)\,(5x - 3y - 3k - 2\cos t)$$

Part 4 ENGINEERING MECHANICS

DEFINITION

Engineering Mechanics is the science which considers the effects of forces on rigid bodies. The subject is divided into two parts: *statics* and *dynamics*. In *statics* we consider the effects and distribution of forces on rigid bodies which are and remain at rest. In *dynamics* we consider the motion of rigid bodies caused by the forces acting upon them.

Outline of Engineering Mechanics

ENGINEERING MECHANICS
- STATICS
  - *Force System*
    - Concurrent
    - Parallel
    - Non-Concurrent
  - *Applications*
    - Trusses
    - Centroids
    - Friction
- DYNAMICS
  - *Kinematics*
    - Translation
    - Rotation
    - Plane Motion
  - *Kinetics*
    - Translation
    - Rotation
    - Plane Motion

STATICS

Statics is a branch of mechanics which studies the effects and distribution of forces of rigid bodies which are and remain at rest.

FORCE SYSTEM

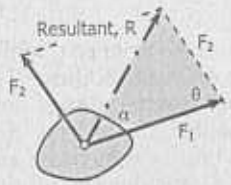
A force system is any arrangement where two or more forces act on a body or on a group of related bodies.

Resultant of Two Concurrent Coplanar Forces

Parallelogram Law

$$R^2 = F_1^2 + F_2^2 - 2F_1F_2\cos\theta$$

$$\frac{F_2}{\sin\alpha} = \frac{R}{\sin\theta}$$



Resultant of Two or More Concurrent Coplanar Forces

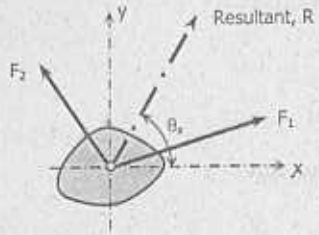
Analytical Solution

$$R_x = \Sigma F_x = F_{1x} + F_{2x} + \ldots$$

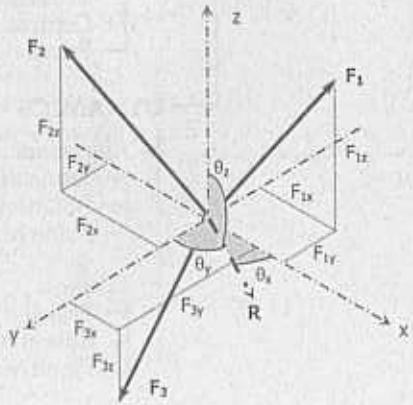
$$R_y = \Sigma F_y = F_{1y} + F_{2y} + \ldots$$

$$R = \sqrt{R_x^2 + R_y^2}$$

$$\tan\theta_x = \frac{R_y}{R_x}$$



Resultant of Concurrent Forces in Space



$$R_x = \Sigma F_x = F_{1x} + F_{2x} + F_{3x} + \ldots$$

$$R_y = \Sigma F_y = F_{1y} + F_{2y} + F_{3y} + \ldots$$

$$R_z = \Sigma F_z = F_{1z} + F_{2z} + F_{3z} + \ldots$$

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2}$$

$$\cos\theta_x = \frac{R_x}{R} \quad \cos\theta_y = \frac{R_y}{R} \quad \cos\theta_z = \frac{R_z}{R}$$

Example 4- 1

Given the three concurrent forces which passes through
(1, -3, 4) and the indicated points:

$F_1 = 150$ N (5, -6, 2)
$F_2 = 340$ N (4, 0, -3)
$F_3 = 280$ N (-1, 2, 6)

Determine the magnitude of the resultant force.

Solution

$$F_1 = 150\text{ N}, L_1 = \sqrt{(5-1)^2 + (-6+3)^2 + (2-4)^2} = 5.385$$

$$F_2 = 340\text{ N}, L_2 = \sqrt{(4-1)^2 + (0+3)^2 + (-3-4)^2} = 8.185$$

$$F_3 = 280\text{ N}, L_3 = \sqrt{(-1-1)^2 + (2+3)^2 + (6-4)^2} = 5.745$$

$$R_x = \Sigma F_x = F_{1x} + F_{2x} + F_{3x}$$

$$R_x = 150\frac{5-1}{5.385} + 340\frac{4-1}{8.185} + 280\frac{-1-1}{5.745} = 138.56\text{ N}$$

$$R_y = \Sigma F_y = F_{1y} + F_{2y} + F_{3y}$$

$$R_y = 150\frac{-6+3}{5.385} + 340\frac{0+3}{8.185} + 280\frac{2+3}{5.745} = 284.76\text{ N}$$

$$R_z = \Sigma F_z = F_{1z} + F_{2z} + F_{3z}$$

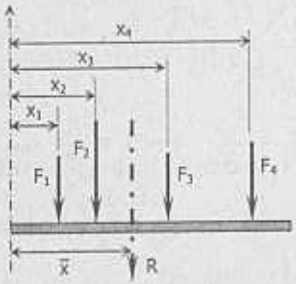
$$R_z = 150\frac{2-4}{5.385} + 340\frac{-3-4}{8.185} + 280\frac{6-4}{5.745} = -248.99\text{ N}$$

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2} = 402.84\text{ N}$$

Resultant of Parallel Forces

$$R = \sum F = F_1 + F_2 + F_3 + \ldots$$

$$R\bar{x} = \sum F x = F_1 x_1 + F_2 x_2 + F_3 x_3 + \ldots$$

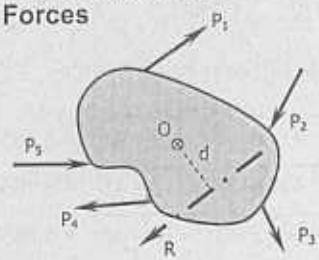


Resultant of Non-Concurrent Forces

$$R_x = \sum P_x \qquad R_y = \sum P_y$$

$$R = \sqrt{R_x^2 + R_y^2}$$

$$R \times d = \sum M_O = P_1 d_1 + P_2 d_2 + \ldots$$



EQUILIBRIUM OF FORCES

General Conditions of Equilibrium

$$\sum F_x = 0 \qquad \sum F_y = 0$$

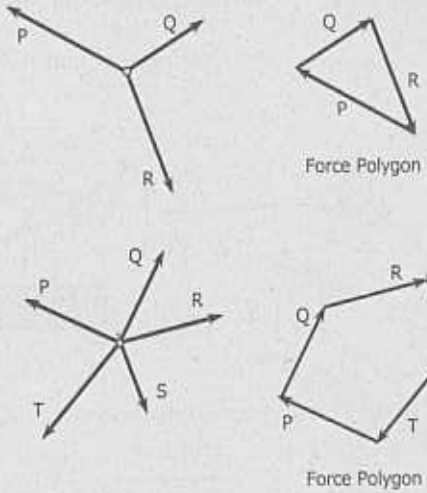
$$\sum M_O = 0$$

Equilibrium of Two Forces

Two forces are in equilibrium if they are equal, collinear, and oppositely directed.

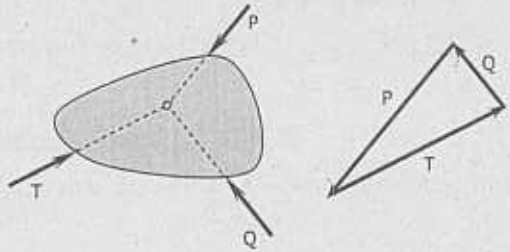
Equilibrium of Two or More Concurrent Forces

If two or more coplanar forces are in equilibrium, they must form a closed polygon.

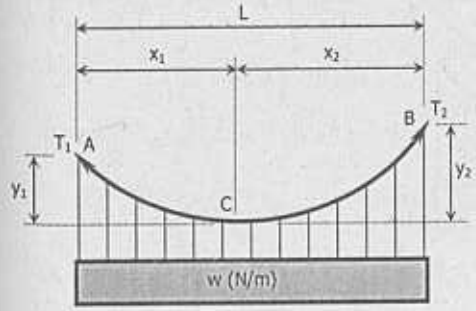


Equilibrium of Three Coplanar Forces

Three coplanar forces in equilibrium are **concurrent**.



PARABOLIC CABLES

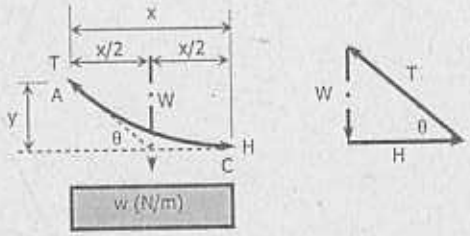


If a flexible wire cable is suspended over two supports and where most of the loads are distributed horizontally as in a suspension bridge, the cable assumes the shape of a parabolic segment.

$$\frac{x_1^2}{y_1} = \frac{x_2^2}{y_2}$$

The solution is to cut a segment from the support to the lowest point of the cable.

Cutting segment AC:



$$W = w\,x$$

$[\Sigma M_A = 0]$

$$H \times y = W \times \frac{x}{2}$$

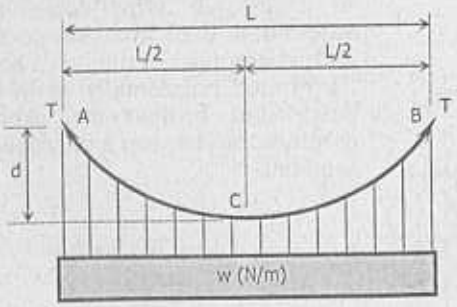
From the force polygon:

$$T^2 = W^2 + H^2$$

$$\tan\theta = \frac{W}{H} = \frac{y}{x/2}$$

$$T \sin\theta = W$$

Symmetrical Parabolic Cable



$$\text{Tension at } C = H = \frac{w\,L^2}{8d}$$

$$\text{Tension at } A = T = \sqrt{H^2 + \left(\frac{wL}{2}\right)^2}$$

Length of Cable S:

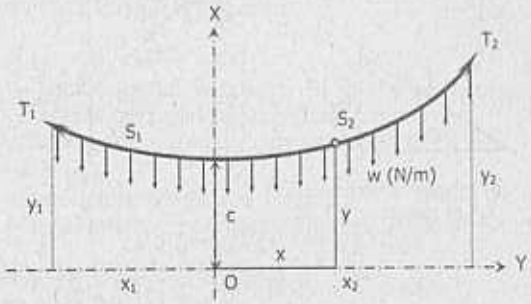
$$\text{Exact } S = \frac{L^2}{8d}\left[m\,k + \ln(m+k)\right]$$

$$m = \frac{4d}{L};\ k = \sqrt{1+m^2}$$

$$\text{Approximate } S = L + \frac{8d^2}{L} - \frac{32d^4}{5L^3}$$

CATENARY

When the cable sags due to its own weight, the cable assumes the shape of a *catenary* which is the graph of the equation $y = \cosh x$.



Tension at lowest point, $H = w \times c$

$$T_1 = w\,y_1 \qquad T_2 = w\,y_2$$

$$T_1 = \sqrt{H^2 + (w\,S_1)^2} \qquad T_2 = \sqrt{H^2 + (w\,S_2)^2}$$

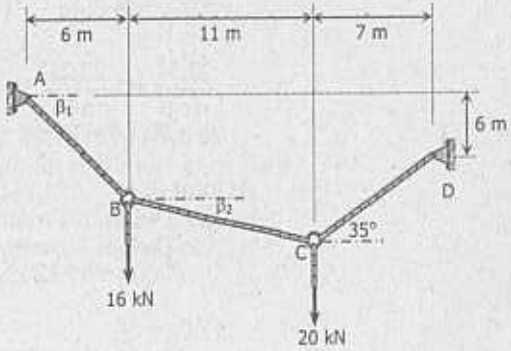
$$y_1^2 = S_1^2 + c^2 \qquad y_2^2 = S_2^2 + c^2$$

$$x_1 = c\ln\frac{S_1 + y_1}{c} \qquad x_2 = c\ln\frac{S_2 + y_2}{c}$$

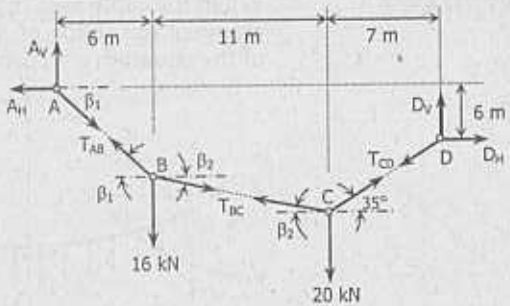
CABLES UNDER CONCENTRATED LOADS

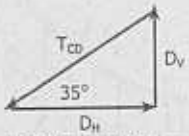
Example 4- 2

Determine the tension in each segment of the cable shown.



Solution



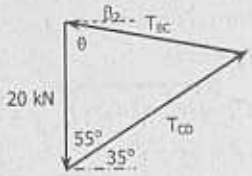


(b) Polygon of forces at Joint D

$[\Sigma M_A = 0]$

$16(6) + 20(17) = D_V(24) + D_H(6)$

$3D_H + 12D_V = 218 \quad \rightarrow$ Eq. (1)



(c) Polygon of forces at Joint D

From figure (*b*)

$\tan 35° = D_V/D_H$

$D_V = 0.7002\, D_H \rightarrow$ Eq. (2)

Substitute D_V of Eq. (2) to Eq. (1):

$3D_H + 12(0.7002\, D_H) = 218$

$D_H = 19.1186$ kN

$$\cos 35° = \frac{D_H}{T_{CD}} = \frac{19.1186}{T_{CD}}$$

$T_{CD} = \mathbf{23.34}$ **kN**

From figure (*c*):

$(T_{BC})^2 = (20)^2 + (23.34)^2 - 2(20)(23.34)\cos 55°$

$T_{BC} = \mathbf{20.23}$ **kN**

$$\frac{23.34}{\sin\theta} = \frac{20.23}{\sin 55°}; \quad \theta = 70.92°$$

$\beta_2 = 90 - \theta = 19.08°$

At joint *B*:

$[\Sigma F_H = 0]$

$(T_{AB})_H = T_{BC}\cos\beta_2 = 20.23\cos 19.08°$

$(T_{AB})_H = 19.12$ kN

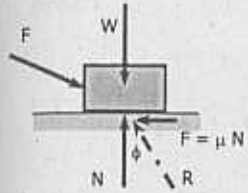
$[\Sigma F_V = 0]$

$(T_{AB})_V = 16 + 20.23\sin 19.08°$

$(T_{AB})_V = 22.613$ kN+

$$T_{AB} = \sqrt{(19.12)^2 + (22.613)^2} = \mathbf{29.61\ kN}$$

FRICTION



Friction is the contact resistance by one body when the second body moves or tends to move past the first body. *Friction force always acts opposite to the motion or to the tendency to move.*

Elements

N = total normal reaction. It is the sum of all forces perpendicular to the surface.

f = friction force (maximum available friction)

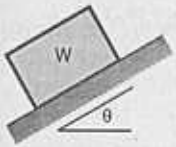
$f = \mu N$

μ = coefficient of friction = $\tan\phi$

R = total surface reaction. The resultant of f and R.

ϕ = angle of friction = $\arctan\mu$

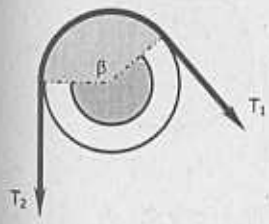
$$\tan\phi = \frac{f}{N} = \mu$$



The maximum angle that a plane may be inclined without causing the body to slide down is:

$$\theta = \phi = \arctan\mu$$

Belt Friction



$$\frac{T_1}{T_2} = e^{f\beta}$$

$$\ln\frac{T_1}{T_2} = f\beta$$

where:

f = coefficient of friction

β = angle of contact in radians

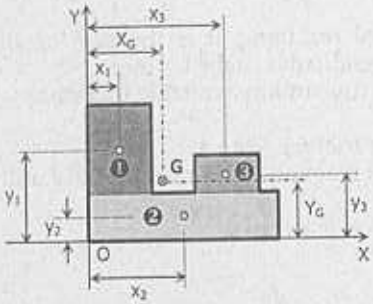
$e = 2.71828$

T_1 = tension in the tight side

T_2 = tension in the slack side

PROPERTIES OF SECTIONS

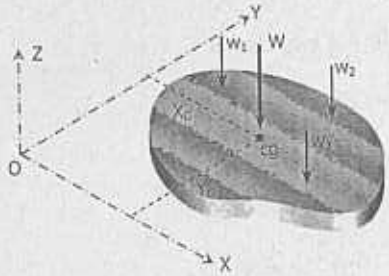
Centroid of Plane Area



$$A X_G = \Sigma a x = a_1 x_1 + a_2 x_2 + \ldots$$
$$A Y_G = \Sigma a y = a_1 y_1 + a_2 y_2 + \ldots$$

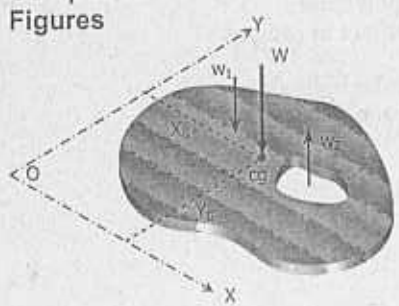
Center of Gravity of Flat Plates

$$W X_G = \Sigma w x = w_1 x_1 + w_2 x_2 + \ldots$$
$$W Y_G = \Sigma w y = w_1 y_1 + w_2 y_2 + \ldots$$

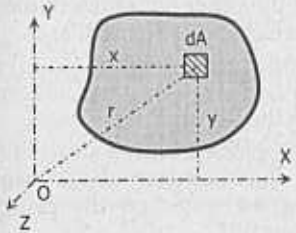


Centroids of Composite Figures

$$W X_G = \Sigma w x = w_1 x_1 - w_2 x_2$$
$$W Y_G = \Sigma w y = w_1 y_1 - w_2 y_2$$



Moment of Inertia (first moment of area)



$$I_x = \int y^2\, dA \qquad I_y = \int x^2\, dA$$

Polar Moment of Inertia

Referring to the figure above:

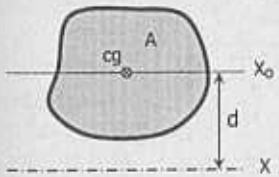
$$J = \int r^2\, dA = I_x + I_y$$

Radius of Gyration

$$r_x = \sqrt{\frac{I_x}{A}} \qquad r_y = \sqrt{\frac{I_y}{A}}$$

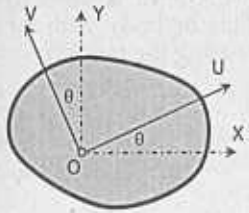
Transfer Formula for moment of Inertia

$$I_x = \Sigma (I_g + A d^2)$$



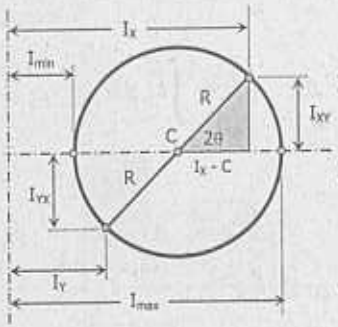
where I_g = centroidal moment of inertia

Moment of Inertia with respect to an Inclined axis



In some cases, it is necessary to determine the moment of inertia with respect to axes, which are inclined to the usual axes. The moment of inertia of such cases can be determined by the use of a formula and more conveniently by graphical solution using Mohr's Circle specially if the principal moments of inertia are known.

Mohr's Circle for Moment of Inertia



The following procedures applies to moments of inertia using Mohr's circle:

1. On a set of rectangular coordinate axes choose one axis on which to plot values of I_x and I_y and the other on which to plot I_{xy}. These axes area called principal axes.
2. With I_x, I_y, and I_{xy} known plot (I_x, I_{xy}) and (I_y, -I_{xy}) with I_x and I_y along the horizontal and I_{xy} along the vertical.
3. Join the points plotted with a straight line. This line is the diameter of the Mohr's circle having its center on the principal axis
4. The angle between any two radii on the Mohr's circle is double the actual angle between the usual axes. The rotational sense of this angle corresponds to the rotational sense of the actual angle between the axes.

The following relationships can be derived from the above procedures:

$$I_{yx} = -I_{xy}$$

$$C = \frac{I_x + I_y}{2} = \frac{I_{max} + I_{min}}{2}$$

$$R = I_{max} - C$$

$$R = \sqrt{(I_{xy})^2 + (I_x - C)^2}$$

$$I_{max} = C + R \qquad I_{min} = C - R$$

DYNAMICS

The branch of mechanics that deals with bodies in motion.

Branches of Dynamics

Kinematics
The geometry of motion. This term is used to define the motion of a particle or body without consideration of the forces causing the motion.

Kinetics
The branch of mechanics that relates the force acting on the body to its mass and acceleration.

KINEMATICS

Motion of Particles

I. Translation
The motion of a rigid body in which a straight line passing through any two of its particle always remain to be parallel to its initial position.

II. Rotation
The motion of a rigid body in which the particles move in circular paths with their centers on a fixed straight line called the axis of rotation.

III. Plane Motion
The motion of a rigid body in which all particles in the body remain at a constant distance from a fixed reference plane.

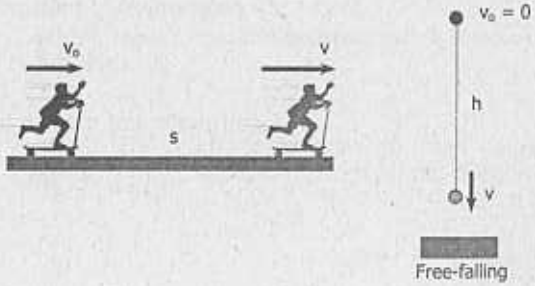
Notations

s = distance
x = horizontal displacement
y = vertical displacement
v = velocity at any time (final velocity)
v_o = initial velocity (velocity at time = 0)
a = acceleration
g = acceleration due to gravity (9.81 m/s², 32.2 ft/s²)
t = time

TRANSLATION

Rectilinear Translation

Motion along a straight line



A. Uniform motion (constant velocity)

$$s = v\,t$$

B. Variable acceleration

$$a = \frac{dv}{dt} \qquad v = \frac{ds}{dt}$$

$$v\,dv = a\,ds$$

where a may either be a function of v, t, or s, and v may either be a function of t or s.

C. Constant acceleration

$$v = v_o + at$$

$$s = v_o t + \frac{1}{2}at^2 \qquad v^2 = v_o^2 + 2as$$

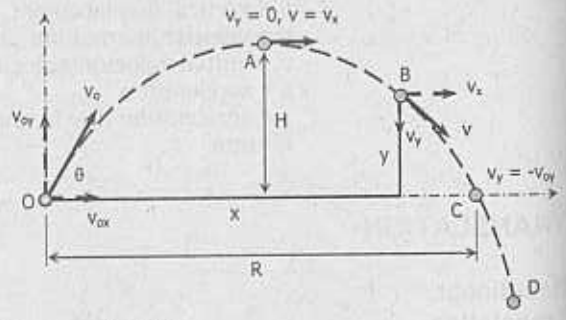
D. Free-falling body ($v_o = 0$, $s = h$)

$$v = gt$$

$$h = \frac{1}{2}gt^2 \qquad v^2 = 2gh$$

Curvilinear Translation (Projectile Motion)

Resolve the motion into x and y-components and use the formulas in rectilinear translation.



At any point B:

x – component of motion ($a_x = 0$)

$$v_x = v_{ox} \qquad x = v_{ox}t$$

y-component of motion ($a_y = -g$)

$$v_y = v_{oy} - gt \qquad y = v_{oy}t - \frac{1}{2}gt^2$$

$$v_y^2 = v_{oy}^2 - 2gy$$

$$y = x\tan\theta - \frac{gx^2}{2v_o^2\cos^2\theta}$$

At the summit A: ($v_y = 0$)

$$H = \frac{v_{oy}^2}{2g} \qquad t = \frac{v_{oy}}{g}$$

At point C:($y = 0$)

$$v_y = -v_{oy} \qquad v = v_o$$

$$R = \frac{v_o^2 \sin 2\theta}{g}$$

$$t = \frac{2v_{oy}}{g} = \frac{2v_o \sin\theta}{g}$$

Note: *θ is positive if the projectile directed upward and negative if directed downward.*

- At any point D below the origin O, the sign of y is negative.
- v_y is positive if directed upward and negative if directed downward

MOTION CURVES

Motion curves are plot of distance s, velocity v, and acceleration a versus time. The relationships between these curves are:

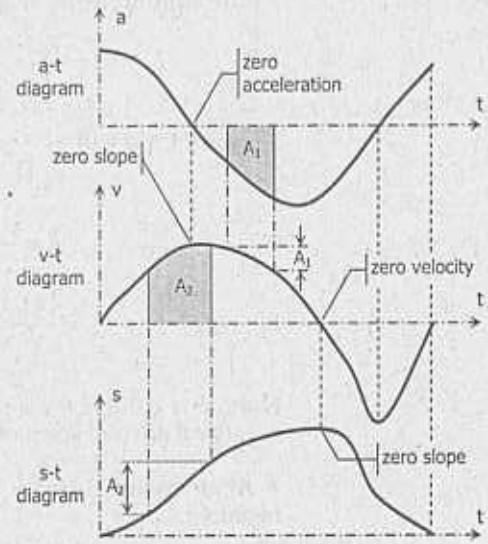
$$v = \frac{ds}{dt}$$

which means that the velocity is the slope of the s-t diagram, and,

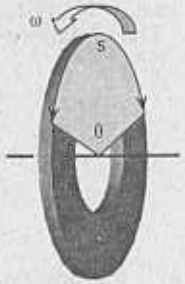
$$a = \frac{dv}{dt}$$

which means that the acceleration is the slope of the v-t diagram.

Note: The relationship between these curves is the same as the load, shear, and moment diagrams.



ROTATION



Notation

θ = angular displacement, radians
ω = angular speed, rad/sec
α = angular acceleration, rad/sec^2
t = time

Uniform Motion

$$\theta = \omega t$$

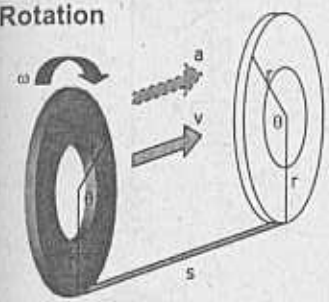
Uniform Acceleration

$$\omega = \omega_o + \alpha t$$

$$\theta = \omega_o t + \frac{1}{2}\alpha t^2$$

$$\omega^2 = \omega_o^2 + 2\alpha\theta$$

Relationship between Translation and Rotation



$$s = r\theta \qquad v = r\omega$$
$$a = r\alpha$$

where θ, ω, and α are expressed in radians.

KINETICS

Newton's Laws of Motion

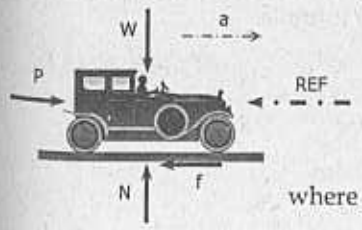
1. *A body at rest will remain to be at rest or in motion will remain in motion along a straight path unless acted upon by an unbalanced force.*
2. *A particle acted upon by an unbalanced force system has an acceleration in line with and directly proportional to the resultant of the force system and inversely proportional to its mass.*

$$a \propto \frac{F}{M} \quad \text{or} \quad a = k\frac{F}{M} \quad (\text{where } k = 1)$$
$$F = M \times a$$

3. *In every action there is always an equal and opposite reaction.*

D'Alembert's Principle

The resultant of the external forces applied to a body (rigid or non-rigid) composed of a system of particles is equivalent to the vector summation of the effective forces acting on all particles.

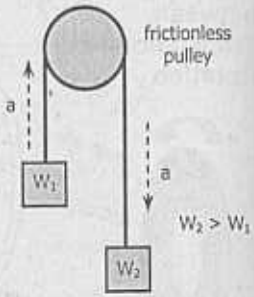


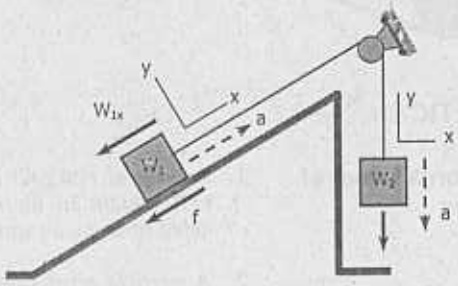
$$\text{REF} = P - f$$
$$\text{REF} = M a = \frac{W}{g} a$$

where M is the mass and W is the weight.

Problems Involving D'Alembert's Principle

$$\frac{W_1 + W_2}{g} a = W_2 - W_1$$



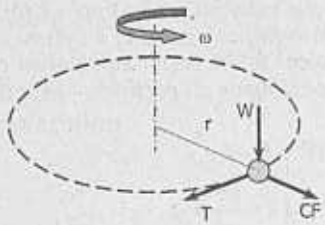


$$\frac{W_1 + W_2}{g} a = \overrightarrow{\text{Forces}} - \overleftarrow{\text{Forces}} = W_2 - f - W_{1x}$$

CENTRIFUGAL FORCE (Reversed Normal Effective Force)

Whenever a body rotates about an axis at a speed of ω, there exist a force called *centrifugal inertia force* directed away from the axis of rotation.

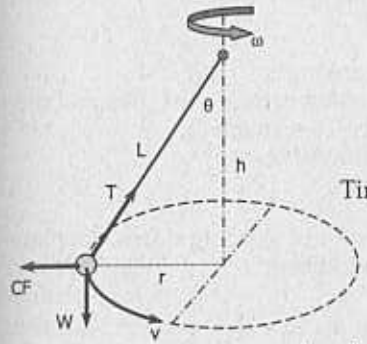
$$CF = M\omega^2 r = \frac{W}{g}\omega^2 r = \frac{WV^2}{gr}$$



The tangential inertia force (centripetal force), also known as *reversed tangential effective force* is given by the formula:

$$T = Ma_t = \frac{W}{g} r\alpha$$

Conical Pendulum

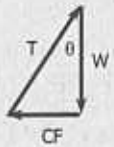


$$\tan\theta = \frac{CF}{W} = \frac{\omega^2 r}{g} = \frac{v^2}{gr}$$

$$\cos\theta = \frac{g}{\omega^2 L}$$

$$T = \frac{W}{\cos\theta}$$

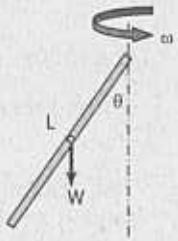
Time to complete one revolution, $t = 2\pi\sqrt{\frac{h}{g}}$



As the value of θ decreases, the value of h approaches the limiting value L so that the maximum time for a revolution is:

$$t_{max} = 2\pi\sqrt{\frac{L}{g}}$$

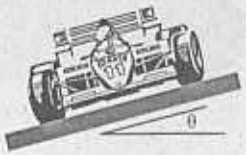
Rod of uniform mass of length L rotated about one end:



$$\cos\theta = \frac{2g}{\omega^2 L}, \text{ provided } \omega^2 L > 2g$$

BANKING OF CURVES

The maximum speed v that a car can round a highway curve without skidding is given by the relationship:



$$\tan(\theta + \phi) = \frac{v^2}{gR}$$

where θ = angle of banking
ϕ = angle of friction between the tires and the road surface ($\phi = \arctan \mu$)
R = radius of the curve, m
v = velocity in m/s

If the car is on the point of slipping down the plane because of insufficient speed:

$$\tan(\theta - \phi) = \frac{v^2}{gR}$$

Ideal Angle of Banking

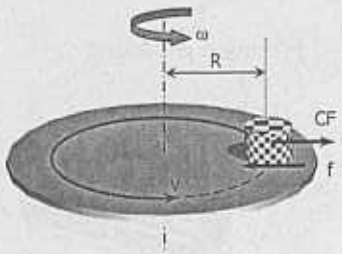
The ideal angle of banking is the angle θ such that the car has no tendency to slide up or down the road. With this angle, there will be no shearing stress (or friction does not work) at the tires of the car, hence $\phi = 0$, and thus;

$$\tan\theta = \frac{v^2}{gR}$$

The ratio $\frac{v^2}{gR}$ is also known as the *impact factor* or *centrifugal ratio.*

Horizontal Rotating Platform

The maximum speed that the platform may be rotated so that the block will not slide is given by the formula:



$$\tan\phi = \mu = \frac{\omega^2 R}{g} = \frac{v^2}{gR}$$

WORK AND ENERGY

Work-energy method is used particularly for solving problems involving *force, displacement,* and *velocity.*

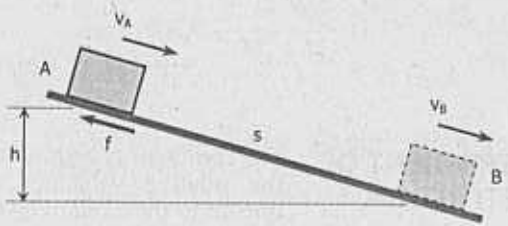
$$\text{Work} = \text{Force} \times \text{distance}$$

$$\text{Kinetic energy, KE} = \frac{1}{2}Mv^2 = \frac{1}{2}\frac{W}{g}v^2$$

$$\text{Potential energy, PE} = Mgh = Wh$$

Work-Energy Equation for Constant Forces

If the forces acting on a body are constant:



$$KE_A \pm \text{Work} \pm Wh = KE_B$$

IMPULSE AND MOMENTUM

Problems involving *force, velocity,* and *time* are conveniently solved by means of the *impulse-momentum method.*

$$\text{Impulse} = \text{Force} \times \text{time}$$

$$\text{Momentum} = Mv = \frac{W}{g}v$$

Impulse-Momentum Equation

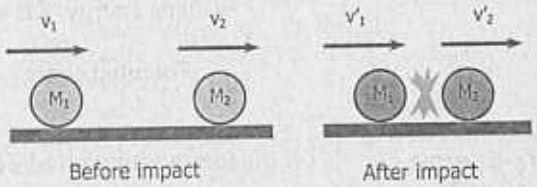
When a body of weight W moving with an initial velocity v_o changes its velocity to v_f over a period of t along a straight line,

$$\Sigma(+)\ \text{Impulse} - \Sigma(-)\ \text{Impulse} = \frac{W}{g}(v_f - v_o)$$

(+) Impulse = impulse in the same direction with the motion
(-) Impulse = impulse in the opposite direction with the motion

Law of Conservation of Momentum

If a system is composed of particles of mass M_1, M_2, etc., having velocities v_1, v_2, etc., and after mutual reaction between the particles they possess new velocities v'_1, v'_2, etc., the condition that the momentum of the system be constant may be expressed as:



$$+\rightarrow \quad M_1 v_1 + M_2 v_2 = M_1 v_1' + M_2 v_2'$$

COEFFICIENT OF RESTITUTION

The *coefficient of restitution* is defined as the ratio of the relative velocities of colliding bodies after impact to their relative velocities before impact. e is always positive.

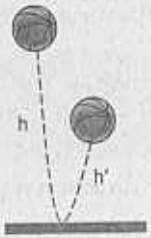
Coefficient of restitution, e

$$e = \frac{\text{Relative velocity after impact}}{\text{Relative velocity before impact}}$$

$$e = \frac{v'_2 \rightarrow v'_1}{v_1 \rightarrow v_2} = \frac{v_{\text{seppration}}}{v_{\text{approach}}}$$

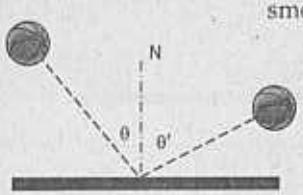
Note: $\rightarrow$ denotes vectorial subtraction.

where $0 > e < 1$ for elastic or inelastic collision
$e = 0$ for perfectly inelastic collision
$e = 1$ for perfectly elastic collision



If a ball is dropped from a height of h upon a pavement and rebounds to a height of h', the coefficient of restitution between the ball and the pavement is:

$$e = \sqrt{\frac{h'}{h}}$$



If a ball is thrown at an angle θ with the normal to a smooth surface and rebounds at an angle θ',

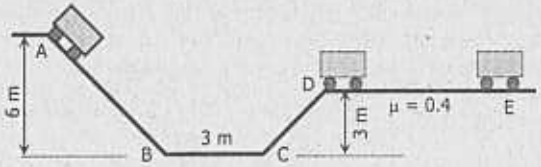
$$e = \frac{\tan\theta}{\tan\theta'}$$

If two masses M_1 and M_2 moving in opposite directions with velocities v_1 and v_2 collide, the energy loss in direct central impact is:

$$\text{Loss in KE} = \frac{1}{2}\frac{M_1 M_2}{M_1 + M_2}(1-e^2)(v_1 - v_2)^2$$

Example 4- 3

A 110-N car in Figure starting from rest rolls without friction down the left incline, then up the right incline to the horizontal surface.



(1) Determine the velocity of the car at B in m/s.
(2) Determine the velocity of the car at D in m/s.
(3) If the coefficient of friction along the plane DE is 0.4, determine the distance DE that the car will travel before it comes to rest.

Solution

Work-energy equation from A to B:

$$KE_A \pm F s \pm W h = KE_B$$

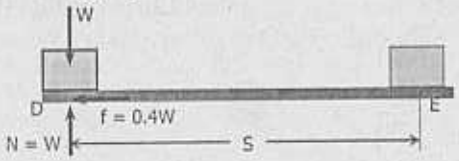
$$0 + 0 + W(6) = \frac{1}{2}\frac{W}{9.81}v_B^2; \quad v_B = \mathbf{10.85\ m/s}$$

Work-energy equation from A to D:

$$KE_A \pm F s \pm W h = KE_D$$

$$0 + 0 + W(6) - W(3) = \frac{1}{2}\frac{W}{9.81}v_D^2; \quad v_D = \mathbf{7.67\ m/s}$$

Work-energy equation from A to E:

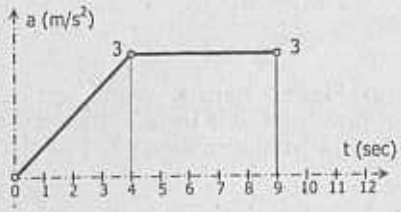


$KE_A \pm F\,s \pm W\,h = KE_E$

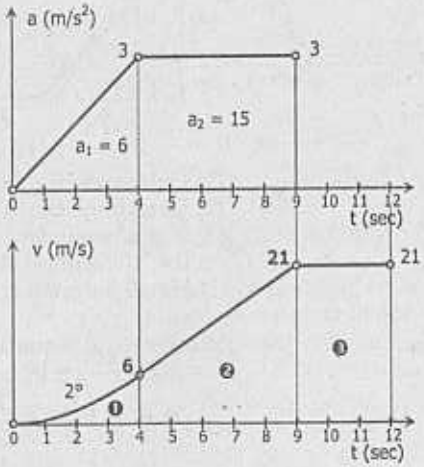
$0 + W(6) - W(3) - 0.4W(s) = 0;$ $\quad s = 7.5$ **m**

Example 4- 4

The acceleration of a body moving on a horizontal plane is as shown on the a-t diagram in Figure. Determine the total displacement of the body after 12 seconds.



Solution



From the v-t diagram shown:

$A_1 = (1/3)(4)(6) = 8$ m

$A_2 = ½(6 + 21)(5) = 67.5$ m

$A_3 = 21(3) = 63$ m

Total displacement = 8 + 67.5 + 63 = **138.5 m**

Problems - Set 6
Translation and Rotation

PROBLEM 6 - 1

The weight of a mass of 10 kilograms at a location where $g = 9.77$ m/s² is:

A. 79.7 N
B. 77.9 N
C. 97.7 N
D. 977 N

PROBLEM 6 - 2
ME April 1997

What is the resultant velocity of a point of x-component $V_x = t^3 - 1$, and y-component $V_y = t^2 - t$ at time $t = 4$?

A. 63.1326
B. 62.1326
C. 64.1326
D. 74.1326

PROBLEM 6 - 3

A boat has a speed of 8 mph in still water attempts to go directly across a river with a current of 3 mph. What is the effective speed of the boat?

A. 8.35 mph
B. 8.54 mph
C. 7.42 mph
D. 6.33 mph

PROBLEM 6 - 4

A ship moving North at 10 mph. A passenger walks Southeast across the deck at 5 mph. In what direction and how fast is the man moving, relative to the earth's surface.

A. N 28°40′W; 7.37 mph
B. N 61°20′E; 7.37 mph
C. N 61°20′W; 7.37 mph
D. N 28°40′E; 7.37 mph

PROBLEM 6 - 5

A man wishes to cross due west on a river which is flowing due north at the rate of 3 mph. If he can row 12 mph in still water, what direction should he take to cross the river?

A. S 14.47° W
B. S 75.52° W
C. S 81.36° W
D. S 84.36° W

PROBLEM 6 - 6
ECE Nov. 1997

A plane is headed due east with air speed of 240 kph. If a wind of 40 kph is blowing from the north, find the ground speed of the plane.

A. 243 kph
B. 423 kph
C. 200 kph
D. 240 kph

PROBLEM 6 - 7
ECE March 1996

Three forces 20N, 30N, and 40N are in equilibrium. Find the angle between the 30-N and 40-N forces.

A. 30° 15′ 25″
B. 28.96°
C. 40°
D. 25.97°

PROBLEM 6 - 8

A 10-kg weight is suspended by a rope from a ceiling. If a horizontal force of 5.80 kg is applied to the weight, the rope will make an angle with the vertical equal to:

A. 60° C. 45°
B. 30° D. 75°

PROBLEM 6 - 9

A 100 kN block slides down a plane inclined at an angle of 30° with the horizontal. Neglecting friction, find the force that causes the block to slide.

A. 86.6 kN C. 20 kN
B. 80 kN D. 50 kN

PROBLEM 6 - 10

What tension must be applied at the ends of a flexible wire cable supporting a load of 0.5 kg per horizontal meter in a span of 100 m if the sag is to be limited to 1.25 m?

A. 423.42 kg C. 500.62 kg
B. 584.23 kg D. 623.24 kg

PROBLEM 6 - 11

The allowable spacing of towers to carry an aluminum cable weighing 0.03 kg per horizontal meter if the maximum tension at the lowest point is not to exceed 1150 kg at sag of 0.50 m is:

A. 248 m C. 408 m
B. 390 m D. 422 m

PROBLEM 6 - 12

A wooden plank x meters long has one end leaning on top of a vertical wall 1.5 m high and the other end resting on a horizontal ground. Neglecting friction, find x if a force (parallel to the plank) of 100 N is needed to pull a 400 N block up the plank.

A. 6 m C. 4 m
B. 5 m D. 3 m

PROBLEM 6 - 13

A block of wood is resting on a level surface. If the coefficient of friction between the block and the surface is 0.30, how much can the plane be inclined without causing the block to slide down?

A. 16.7° C. 21.2°
B. 30.2° D. 33.3°

PROBLEM 6 - 14

A 500- kg block is resting on a 30° inclined plane with a $\mu = 0.3$. Find the required force P acting horizontally that will prevent the block from sliding.

A. 1020 N C. 4236 N
B. 1160 N D. 5205N

PROBLEM 6 - 15

A 500- kg block is resting on a 30° inclined plane with a $\mu = 0.3$. Find the required force P acting horizontally that will start the block up the plane.

A. 4236 N C. 5205 N
B. 1160 N D. 2570 N

PROBLEM 6 - 16
ME April 1996

What is the acceleration of the body that increases in velocity from 20 m/s to 40 m/s in 3 seconds? Answer in S.I. units.

A. 8 m/s² C. 5 m/s²
B. 6.67 m/s² D. 7 m/s²

PROBLEM 6 - 17
CE May 1996

From a speed of 75 kph, a car decelerates at the rate of 500 m/min² along a straight path. How far in meters, will it travel in 45 sec?

A. 795 C. 797
B. 791 D. 793

PROBLEM 6 - 18
CE Nov. 1997

With a starting speed of 30 kph at point A, a car accelerates uniformly. After 18 minutes, it reaches point B, 21 km from A. Find the acceleration of the car in m/s².

A. 0.126 m/s² C. 0.0206 m/s²
B. 0.0562 m/s² D. 3.42 m/s²

PROBLEM 6 - 19
CE Nov. 1996

A train upon passing point A at a speed of 72 kph accelerates at 0.75 m/s² for one minute along a straight path then decelerates at 1.0 m/s². How far in kilometers from point A will it be 2 minutes after passing point A.

A. 4.95 C. 4.85
B. 4.75 D. 4.65

PROBLEM 6 - 20

A car starting from rest moves with a constant acceleration of 10 km/hr² for 1 hour, then decelerates at a constant -5 km/hr² until it comes to a stop. How far has it traveled?

A. 10 km C. 12 km
B. 20 km D. 15 km

PROBLEM 6 - 21
ECE Nov. 1997

The velocity of an automobile starting from rest is given by

$$ds/dt = 90t / (t + 10) \text{ ft./sec.}$$

Determine its acceleration after an interval of 10 seconds (in ft/sec²).

A. 2.10 C. 2.25
B. 1.71 D. 2.75

PROBLEM 6 - 22
CE May 1998

A train running at 60 kph decelerated at 2 m/min² for 14 minutes. Find the distance traveled, in kilometers within this period.

A. 12.2 C. 13.8
B. 13.2 D. 12.8

PROBLEM 6 - 23
ECE Nov. 1997

An automobile accelerates at a constant rate of 15 mi/hr to 45 mi/hr in 15 seconds, while travelling in a straight line. What is the average acceleration?

A. 2 ft/s² C. 2.12 ft/s²
B. 2.39 ft/s² D. 2.93 ft/s²

PROBLEM 6 - 24
CE Nov. 1998

A car was traveling at a speed of 50 mph. The driver saw a road block 80 m ahead and stepped on the brake causing the car to decelerate uniformly at 10 m/s². Find the distance from the roadblock to the point where the car stopped. Assume perception reaction time is 2 seconds.

A. 12.48 m C. 10.28 m
B. 6.25 m D. 8.63 m

PROBLEM 6 - 25

A man driving his car at 45 mph suddenly sees an object in the road 60 feet ahead. What constant deceleration is required to stop the car in this distance?

A. -36.3 ft/s² C. -33.4 ft/s²
B. -45.2 ft/s² D. -42.3 ft/s²

PROBLEM 6 - 26
ECE March 1996

A mango falls from a branch 5 meters above the ground. With what speed in meters per second will it strike the ground? Assume $g = 10$ m/s².

A. 8 C. 10
B. 12 D. 14

PROBLEM 6 - 27

A man standing at a window 5 meters tall watches a falling stone pass by the window in 0.3 seconds. From how high above the top of the window was the stone released?

A. 12.86 m C. 9.54 m
B. 11.77 m D. 15.21 m

PROBLEM 6 - 28

A bullet is fired at an initial velocity of 350 m/s and an angle of 50° with the horizontal. Neglecting air resistance, what maximum height could the bullet rises?

A. 3,646 m C. 3,466 m
B. 4,366 m D. 3,664 m

PROBLEM 6 - 29

A bullet is fired at an initial velocity of 350 m/s and an angle of 50° with the horizontal. Neglecting air resistance, find its range on the horizontal plane through the point it was fired.

A. 12,298 km C. 12.298 m
B. 12.298 km D. 12,298 cm

PROBLEM 6 - 30

A bullet is fired at an initial velocity of 350 m/s and an angle of 50° with the horizontal. Neglecting air resistance, how long will the bullet travel before hitting the ground?

A. 54.66 min. C. 54.66 sec
B. 56.42 sec D. 56.42 min.

PROBLEM 6 - 31
ME Oct. 1997

The muzzle velocity of a projectile is 1,500 fps and the distance of the target is 10 miles. What must be the angle of elevation of the gun?

A. 25° 33' C. 24° 33'
B. 23° 34' D. 26° 34'

PROBLEM 6 - 32
ME Oct. 1997

A shot is fired at an angle of 45° with a horizontal and a velocity of 300 ft per sec. Find the height and the range of the projectile.

A. 600 ft and 2500 ft C. 1000 ft and 4800 ft
B. 700 ft and 2800 ft D. 750 ft and 3000 ft

PROBLEM 6 - 33
ECE April 1998

A baseball is thrown from a horizontal plane following a parabolic path with an initial velocity of 100 m/s at an angle of 30 degrees above the horizontal. How far from the throwing point will the ball attain its original level?

A. 890 m C. 883 m
B. 875 m D. 880 m

PROBLEM 6 - 34

What is the range of a projectile if the initial velocity is 30 m/s at an angle of 30° with the horizontal?

A. 100 m C. 79.45 m
B. 92 m D. 110 m

PROBLEM 6 - 35

A bullet is fired at an angle of 75° with the horizontal with an initial velocity of 420 m/s. How high can it travel after 2 seconds?

A. 840 m C. 750 m
B. 792 m D. 732 m

PROBLEM 6 - 36

A coin is tossed vertically upward from the ground at a velocity of 12 m/s. How long will the coin touch the ground?

A. 4.45 sec. C. 2.45 sec.
B. 3.45 sec. D. 1.45 sec.

PROBLEM 6 - 37
CE May 1997

A stone is projected from the ground with a velocity of 15 m/s at an angle of 30° with the horizontal ground. How high in m will it rise? Use $g = 9.817$ m/s².

A. 2.865 m C. 2.586 m
B. 2.685 m D. 8.652 m

PROBLEM 6 - 38
CE Nov. 1996

A ball is thrown from a tower 30 m. high above the ground with a velocity of 300 m/s directed at 20° from the horizontal. How long will the ball hit the ground?

A. 24.2 C. 21.2
B. 23.2 D. 22.2

PROBLEM 6 - 39
ECE Nov. 1996

A man in a hot-air balloon drops an apple at a height of 150 m. If the balloon is rising at 15 m/s, find the highest point reached by the apple.

A. 151.15 m C. 171.15 m
B. 161.15 m D. 141.15 m

PROBLEM 6 - 40

A balloon is ascending at the rate of 10 kph and is being carried horizontally by a wind at 20 kph. If a bomb is dropped from the balloon such that it takes 8 sec. to reach the ground, the balloon's altitude when the bomb was released is:

A. 336.14 m C. 252 m
B. 322.13 m D. 292 m

PROBLEM 6 - 41

A plane is flying horizontally 350 kph at an altitude of 420 m. At this instant, a bomb is released. How far horizontally from this point will the bomb hit the ground?

A. 625 m C. 785 m
B. 577 m D. 900 m

PROBLEM 6 - 42

A car whose wheels are 30 cm in radius is traveling with a velocity of 110 kph. If it is decelerated at a constant rate of 2 m/s², how many complete revolutions does it make before it comes to rest?

A. 121 C. 123
B. 122 D. 124

PROBLEM 6 - 43

The wheel of an automobile revolves at the rate of 700 rpm. How fast does it move, in km per hr., if the radius of its wheel is 250 mm?

A. 73.3 C. 5.09
B. 18.33 D. 34.67

PROBLEM 6 - 44
CE Nov. 1997

Using a constant angular acceleration, a water turbine is brought to its normal operating speed of 180 rev/min in 6 minutes. How many complete revolutions did the turbine make in coming to normal speed?

A. 550 C. 560
B. 530 D. 540

PROBLEM 6 - 45

A horizontal platform 6 m in diameter revolves so that a point on its rim moves 6.28 m/s. Find its angular speed in rpm.

A. 15 C. 25
B. 20 D. 12

PROBLEM 6 - 46
CE May 1998

A horizontal platform with a diameter of 6 m revolves about its center at 20 rpm. Find the tangential speed, in m/s of a point at the edge of the platform.

A. 6.34 C. 6.46
B. 6.28 D. 6.12

PROBLEM 6 - 47

A flywheel rotating at 500 rpm decelerates uniformly at 2 rad/sec². How many seconds will it take for the flywheel to stop?

A. 24.5 s C. 25.1 s
B. 28.4 s D. 26.2 s

PROBLEM 6 - 48
ME April 1996

A cyclist on a circular track of radius $r = 800$ feet is traveling at 27 ft/sec. His speed in the tangential direction increases at the rate of 3 ft/s². What is the cyclist's total acceleration?

A. 2.8 ft/s² C. -5.1 ft/s²
B. -3.12 ft/s² D. 3.13 ft/s²

PROBLEM 6 - 49
CE May 1998,
May 1996

The radius of the earth is 3,960 miles. The gravitational acceleration at the earth's surface is 32.16 ft/s². What is the velocity of escape from the earth in miles/s?

A. 6.94 C. 9.36
B. 8.62 D. 7.83

PROBLEM 6 - 50
CE May 1996

The radius of the moon is 1080 mi. The gravitational acceleration at the moon's surface is 0.165 times the gravitational acceleration at the earth's surface. What is the velocity of escape from the moon in miles/second?

A. 2.38 C. 3.52
B. 1.47 D. 4.26

ANSWER SHEET

1.	A	B	C	D	E	21.	A	B	C	D	E	41.	A	B	C	D	E
2.	A	B	C	D	E	22.	A	B	C	D	E	42.	A	B	C	D	E
3.	A	B	C	D	E	23.	A	B	C	D	E	43.	A	B	C	D	E
4.	A	B	C	D	E	24.	A	B	C	D	E	44.	A	B	C	D	E
5.	A	B	C	D	E	25.	A	B	C	D	E	45.	A	B	C	D	E
6.	A	B	C	D	E	26.	A	B	C	D	E	46.	A	B	C	D	E
7.	A	B	C	D	E	27.	A	B	C	D	E	47.	A	B	C	D	E
8.	A	B	C	D	E	28.	A	B	C	D	E	48.	A	B	C	D	E
9.	A	B	C	D	E	29.	A	B	C	D	E	49.	A	B	C	D	E
10.	A	B	C	D	E	30.	A	B	C	D	E	50.	A	B	C	D	E
11.	A	B	C	D	E	31.	A	B	C	D	E	51.	A	B	C	D	E
12.	A	B	C	D	E	32.	A	B	C	D	E	52.	A	B	C	D	E
13.	A	B	C	D	E	33.	A	B	C	D	E	53.	A	B	C	D	E
14.	A	B	C	D	E	34.	A	B	C	D	E	54.	A	B	C	D	E
15.	A	B	C	D	E	35.	A	B	C	D	E	55.	A	B	C	D	E
16.	A	B	C	D	E	36.	A	B	C	D	E	56.	A	B	C	D	E
17.	A	B	C	D	E	37.	A	B	C	D	E	57.	A	B	C	D	E
18.	A	B	C	D	E	38.	A	B	C	D	E	58.	A	B	C	D	E
19.	A	B	C	D	E	39.	A	B	C	D	E	59.	A	B	C	D	E
20.	A	B	C	D	E	40.	A	B	C	D	E	60.	A	B	C	D	E

Solutions to Set 6
Translation and Rotation

SOLUTION 6 - 1
Ans: C

$W = Mg = 10(9.77)$
$W = 97.7$ N

SOLUTION 6 - 2
Ans: C

Velocity is a vector quantity. The resultant is:

$$V = \sqrt{V_x^2 + V_y^2}$$

At t = 4:

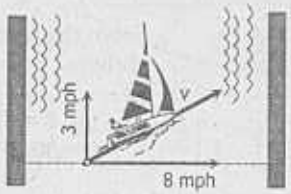
$$V_x = t^3 - 1 = 4^3 - 1 = 63$$
$$V_y = t^2 - t = 4^2 - 4 = 12$$

Then;

$$V = \sqrt{(63)^2 + (12)^2} = 64.1327$$

SOLUTION 6 - 3
Ans: B

$v = \sqrt{8^2 + 3^2}$
$v = 8.54$ mph



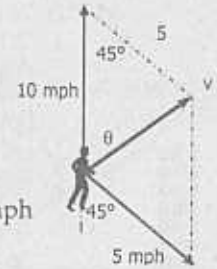
SOLUTION 6 - 4
Ans: D

From the figure shown:

$v^2 = 10^2 + 5^2 - 2(10)(5)\cos 45°$
$v = 7.37$ mph

$$\frac{7.37}{\sin 45°} = \frac{5}{\sin\theta};\ \theta = 28°\ 40'$$

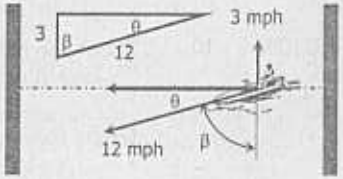
Therefore, the man moves at 7.37 mph in the direction N 28° 40′ E



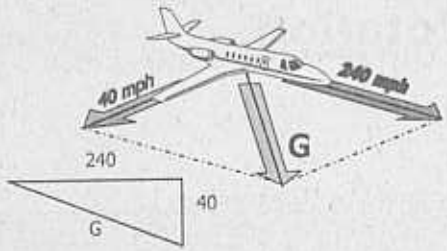
SOLUTION 6 - 5
Ans: B

$\cos\beta = 3/12$
$\beta = 75.52°$

Direction: S 75.52° W



SOLUTION 6 - 6
Ans: A

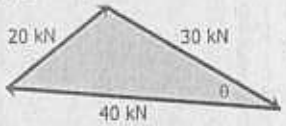


The ground speed is the resultant of the airspeed and the wind speed.

Since the two component vectors are perpendicular:

$$G = \sqrt{(240)^2 + (40)^2}$$
$$G = 243.31 \text{ kph}$$

SOLUTION 6 - 7
Ans: B



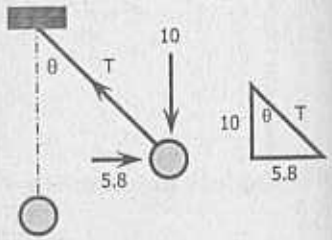
Three forces are in equilibrium if their force vector forms a closed polygon (a triangle)

$20^2 = 30^2 + 40^2 - 2(30)(40)\cos\theta$
$\cos\theta = 0.875$
$\theta = 28.96°$

SOLUTION 6 - 8
Ans: B

From the force polygon shown:

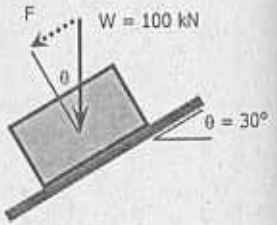
$\tan\theta = 5.8/10$
$\theta = 30.11°$



SOLUTION 6 - 9
Ans: D

The force causing the block the slide is the component of the weight parallel to the plane.

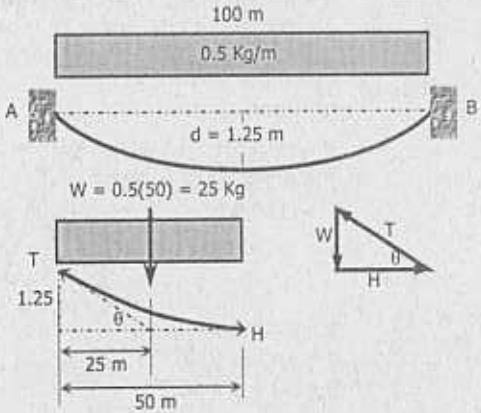
$F = W\sin\theta = 100\sin 30°$
$F = 50$ kN



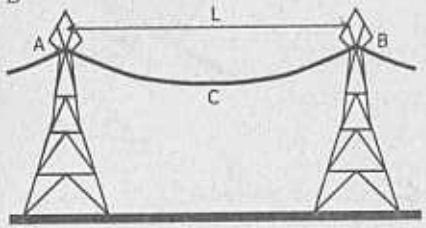
SOLUTION 6 - 10
Ans: C

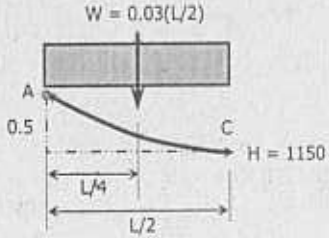
From the right triangle shown:
$\tan\theta = 1.25/25;\ \theta = 2.862°$

In the force polygon shown:
$T = W / \sin\theta$
$T = 25 / \sin 2.862°$
$T = 500.62$ kg



SOLUTION 6 - 11
Ans: B

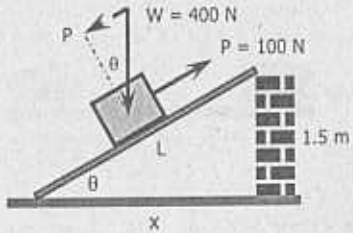




From the figure shown:
$\Sigma M_A = 0$
$1150(0.5) = 0.03(L/2)(L/4);\ \ L = 391.5$ m

SOLUTION 6 - 12
Ans: A

$\sin\theta = \dfrac{P}{100} = \dfrac{1.5}{L}$
$L = 6$ m



SOLUTION 6 - 13
Ans: A

Coefficient of friction, $\mu = 0.3$
Angle of friction $= \phi = \arctan\mu = 16.7°$

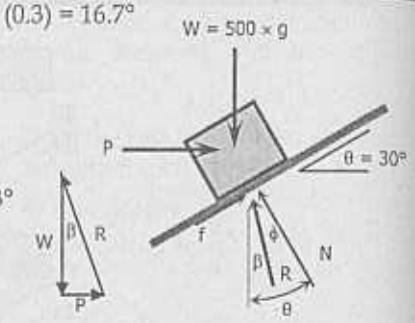
Note: The angle of inclination θ of the plane that the block impends to slide down equals the angle of friction between the block and the plane.
Thus, $\theta = \phi = 16.7°$

SOLUTION 6 - 14
Ans: B

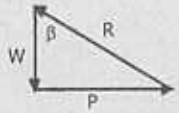
$\phi = \arctan \mu = \arctan (0.3) = 16.7°$
$\beta = 30 - \phi = 13.3°$

From the force polygon shown:

$P = W \tan \beta$
$P = 500(9.81) \tan 13.3°$
$P = 1159.5$ N



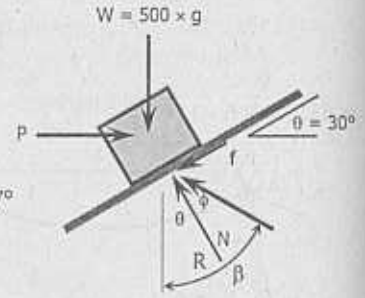
SOLUTION 6 - 15
Ans: C



$\phi = \arctan \mu$
$\phi = \arctan (0.3) = 16.7°$
$\beta = 30 + \phi = 46.7°$

From the force polygon shown:

$P = W \tan \beta$
$P = 500(9.81) \tan 46.7°$
$P = 5205$ N



SOLUTION 6 - 16
Ans: B

📖 16

From the formula:

$v = v_o + at$
$40 = 20 + a(3)$; $a = 6.67$ m/s²

SOLUTION 6 - 17
Ans: C

$S = v_o t + \frac{1}{2} a t^2$

$v_o = 75 \frac{\text{km}}{\text{hr}} \times \frac{1000\text{m}}{\text{km}} \times \frac{1\text{hr}}{3600\text{sec}}$

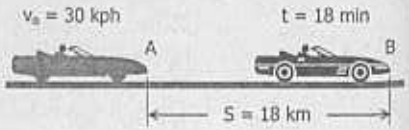
$v_o = 20.833$ m/s
$a = -500$ m/min² × (1 min/60 sec)²
$a = -0.1389$ m/s²

$S = (20.833)(45) + \frac{1}{2}(-0.1389)(45)^2$
$S = 796.87$ m

SOLUTION 6 - 18
Ans: C

$v_o = 30$ kph $= 8.33$ m/s
$t = 18$ minutes $= 1080$ sec.
$S = 21$ km $= 21{,}000$ m

Required: a



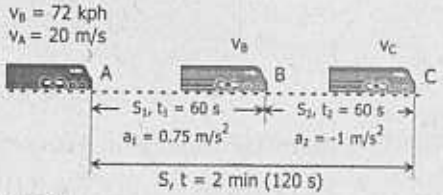
$S = v_o t + \frac{1}{2} a t^2$
$21000 = 8.33(1080) + \frac{1}{2} a (1080)^2$
$a =$ **0.0206 m/s²**

SOLUTION 6 - 19
Ans: D

From A to B:

$S_1 = v_A t_1 + \frac{1}{2} a_1 t_1^2$
$S_1 = 20(60) + \frac{1}{2}(0.75)(60)^2 = 2550$ m

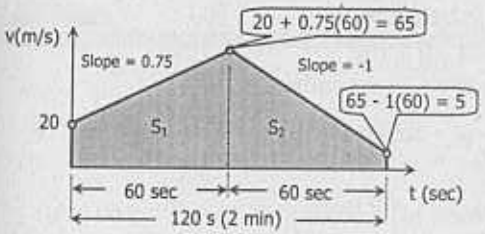
$V_B = V_A + a_1 t_1$
$V_B = 20 + 0.75(60) = 65$ m/s



From B to C:

$S_2 = V_B t_2 + \frac{1}{2} a_2 t_2^2$
$S_2 = 65(60) + \frac{1}{2}(-1)(60)^2 = 2100$ m
$S = S_1 + S_2 = 2550 + 2100 = 4650$ m $=$ **4.65 km**

Another Solution



Using the velocity diagram, plot the velocity of the train versus time.

The area under the diagram represents the distance traveled within the period.

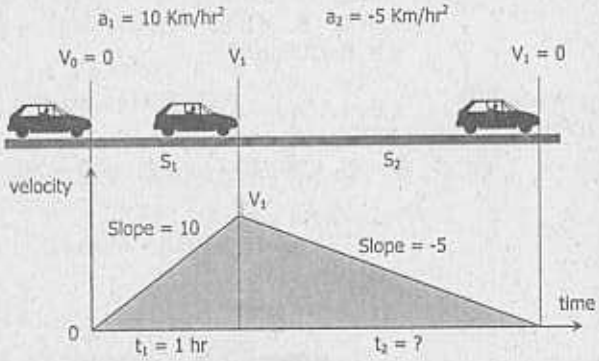
The velocity of the train at $t = 0$ is 72 kph (20 m/s).

Distance, S = Area
$S = S_1 + S_2$

$$S = \frac{20+65}{2}(60) + \frac{65+5}{2}(60)$$

$S = 2550 + 2100$
$S = 4650$ m $=$ **4.65 km**

SOLUTION 6 - 20
Ans: D



Velocity-time curve

From the velocity-time curve:

From 0 to 1: $V_1 = 0 + 10(1) = 10$ km/hr
From 1 to 2: $V_2 = 10 - 5\,t_2 = 0;\ t_2 = 2$ hrs.

S = Total area of the curve
$S = \frac{1}{2}(1 + 2)(10) = \mathbf{15\ km}$

SOLUTION 6 - 21
Ans: C

$$V = \frac{dS}{dt} = \frac{90t}{t+10}$$

$$a = \frac{dV}{dt} = \frac{(t+10)(90) - 90t(1)}{(t+10)^2} = \frac{900}{(t+10)^2}$$

when $t = 10$ seconds:

$$a = \frac{900}{(10+10)^2} = \mathbf{2.25\ ft/s^2}$$

SOLUTION 6 - 22
Ans: C

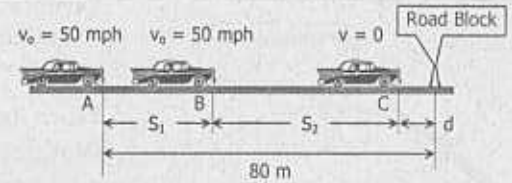
$S = v_o t + \frac{1}{2} a t^2$
$v_o = 60$ (km/hr) × (1000 m/km) × (1 hr/60 min)
$v_o = 1000$ m/min
$S = (1000)14) + \frac{1}{2}(-2)(14)^2$
$S = 13804$ m = **13.804 km**

SOLUTION 6 - 23
Ans: D

Using the formula $v = v_0 + at$, where v is the final velocity = 45 mph, v_0 is the initial velocity = 15 mph, and t is the time = 15 sec (15/3600 hr.)

Then $45 = 15 + a(15/3600)$
$a = 7200$ mi / hr² × (5280 ft/mi) × (1 hr / 3600 s)²
$a = \mathbf{2.933\ ft/s^2}$

SOLUTION 6 - 24
Ans: C



From A to B: ($t = 2$ sec)

$S_1 = v_o\, t_1$

$$v_o = 50\ \frac{\text{mi}}{\text{hr}} \times \frac{5280\text{ft}}{\text{mile}} \times \frac{1\text{m}}{3.28\text{ft}}$$

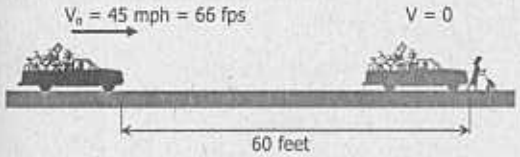
$v_o = 80487.805$ m/hr = 22.36 m/s

$S_1 = 22.36(2) = 44.72$ m

From B to C: ($v = 0$, $v_o = 22.36$ m/s, $a = -10$ m/s²)

$v^2 = v_o^2 + 2\,a\,S_2$
$0^2 = 22.36^2 + 2(-10)\,S_2;\ S_2 = 25$ m
$d = 80 - S_1 - S_2 = 80 - 44.72 - 25 = \mathbf{10.28\ m}$

SOLUTION 6 - 25
Ans: A



From $V^2 = V_o^2 + 2as$
$0^2 = 66^2 + 2a(60)$
$a = \mathbf{-36.3\ ft/s^2}$

SOLUTION 6 - 26
Ans: C

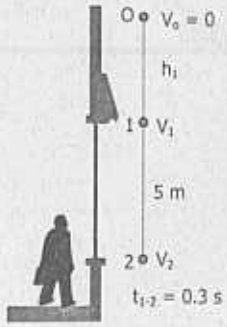
From $v^2 = v_o^2 + 2gh$
$v_o = 0$ (free fall); $h = 5$ m

$v^2 = 0^2 + 2(10)(5) = 100;\ \ v = \mathbf{10\ m/s}$

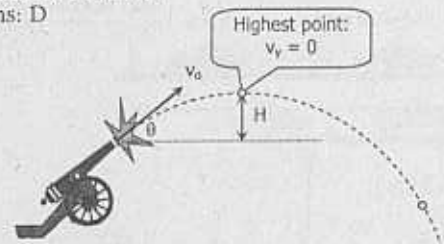
SOLUTION 6 - 27
Ans: B

For a free-falling body:
$h = \frac{1}{2} g\, t^2$

$t_2 - t_1 = 0.3$ s; $t_2 = t_1 + 0.3$
$h_2 - h_1 = 5$ m
$\frac{1}{2}(9.81)\,t_2^2 - \frac{1}{2}(9.81)\,t_1^2 = 5$
$t_2^2 - t_1^2 = 1.0194$
$(t_1 + 0.3)^2 - t_1^2 = 1.0194$
$t_1^2 + 0.6\,t_1 + 0.09 - t_1^2 = 1.0194$
$t_1 = 1.549$ seconds
$h_1 = \frac{1}{2}(9.81)(1.549)^2$
$h_1 = \mathbf{11.77\ m}$



SOLUTION 6 - 28
Ans: D

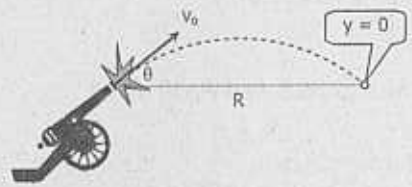


Given: $v_o = 350$ m/s, $\theta = 50°$
Required: $y = H$
(maximum height)

When the bullet is at its highest point, $v_y = 0$.

$v_y^2 = v_{oy}^2 - 2gy$
$0 = (350 \sin 50°)^2 - 2(9.81)H$
$H = \mathbf{3664\ m}$

SOLUTION 6 - 29
Ans: C



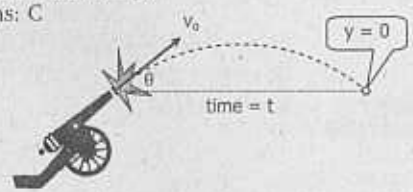
$v_o = 350$ m/s, $\theta = 50°$
Required: $x = R$ (horizontal range through the point it was fired, $y = 0$)

$$y = x \tan\theta - \frac{gx^2}{2v_o^2 \cos^2\theta}$$

$$0 = R \tan 50° - \frac{9.81R^2}{2(350)^2 \cos^2 50°}$$

$R = \mathbf{12{,}298\ m}$

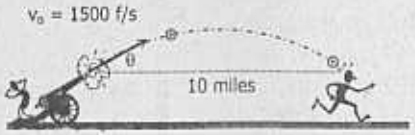
SOLUTION 6 - 30
Ans: C



$v_o = 350$ m/s, $\theta = 50°$
Required: total time of flight ($y = 0$)

$y = v_{oy}\, t - \frac{1}{2} g\, t^2$
$0 = (350 \sin 50°)\, t - \frac{1}{2}(9.81)\, t^2$
$t = \mathbf{54.66\ sec}$

SOLUTION 6 - 31
Ans: C



$$y = x \tan\theta - \frac{gx^2}{2v_o^2 \cos^2\theta}$$

$y = 0$

$$x = 10 \text{ miles} \times \frac{5280 \text{ ft}}{\text{mile}}$$

$x = 52800$ feet

$$0 = 52800 \tan\theta - \frac{32.2(52800)^2}{2(1500)^2 \cos^2\theta} \quad \rightarrow (1)$$

$$\tan\theta = \frac{0.3778}{\cos^2\theta} = \frac{\sin\theta}{\cos\theta}$$

$\sin\theta \cos\theta = 0.3778$; but $\sin\theta \cos\theta = \frac{1}{2} \sin 2\theta$

$\sin 2\theta = 0.7556$; $2\theta = 49.0778°$
$\theta = 24.54° = \mathbf{24° 32'}$

Note: The value of θ may also be found by substituting the choices to Eq. (1).

SOLUTION 6 - 32
Ans: B

At the highest point of the projectile $v_y = 0$.
Then from $v_y^2 = v_{oy}^2 - 2gy$;
$0^2 = (300 \times \sin 45°)^2 - 2(32.2)y$; $y = 698.76$ feet.

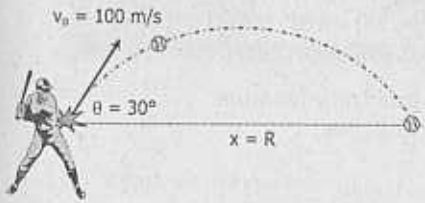
As the shot touches the ground (for the range), $y = 0$.

Then from $y = x \tan\theta - \dfrac{gx^2}{2v_o^2 \cos^2\theta}$;

$$0 = x \tan 45° - \frac{32.2x^2}{2(300)^2 \cos^2 45°}$$

$x = 0.0003577\, x^2$; $x = \mathbf{2795\ feet}$

SOLUTION 6 - 33
Ans: C



Using the formula:

$$y = x \tan\theta - \frac{gx^2}{2v_0^2 \cos^2\theta}$$

When the ball attain its original level, $y = 0$

$$0 = x \tan 30° - \frac{9.81x^2}{2(100)^2 \cos^2 30°}$$

$x = \mathbf{883\ m}$

SOLUTION 6 - 34
Ans: C

From $y = x \tan\theta - \dfrac{gx^2}{2v_o^2 \cos^2\theta}$

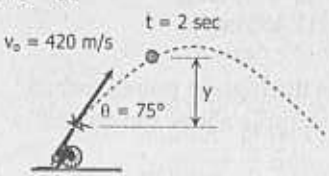
Where $v_o = 30$ m/s, $\theta = 30°$.

For the range on the horizontal plane through the point it was fired, $y = 0$.

$$0 = x \tan 30° - \frac{9.81x^2}{2(30)^2 \cos^2 30°}$$

$x = R = \mathbf{79.45\ m}$

SOLUTION 6 - 35
Ans: B



$y = v_{oy}\, t - \frac{1}{2} g\, t^2$
$y = (420 \sin 75°)(2) - \frac{1}{2}(9.81)(2)^2$
$y = \mathbf{791.76\ m}$

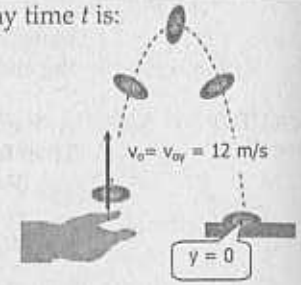
SOLUTION 6 - 36
Ans: C

The height "y" of the coin at any time t is:

$$y = v_{oy}\, t - \tfrac{1}{2} g\, t^2$$

When the coin touches the ground, $y = 0$.
Then

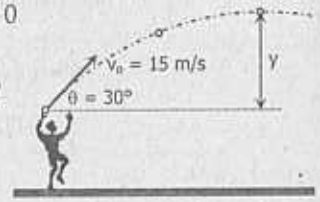
$$0 = 12t - \tfrac{1}{2}(9.81)t$$
$$t = \mathbf{2.45\ sec}$$



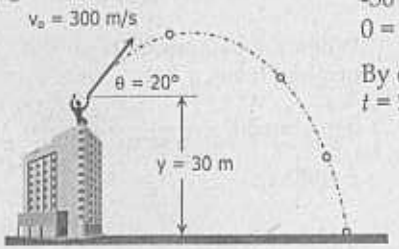
SOLUTION 6 - 37
Ans: A

At maximum height, $v_y = 0$

$$v_y^2 = v_{oy}^2 - 2gy$$
$$0 = (15 \sin 30°)^2 - 2(9.817)y$$
$$y = \mathbf{2.865\ m}$$



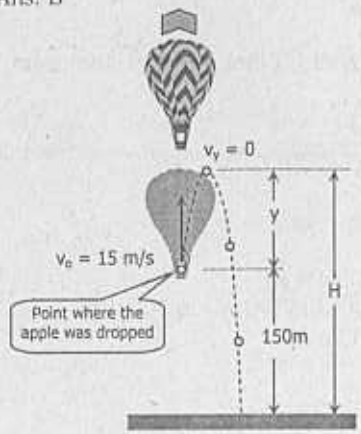
SOLUTION 6 - 38
Ans: C



$$y = v_{oy}\, t - \tfrac{1}{2} g\, t^2$$
$$-30 = 300 \sin 20°\, t - \tfrac{1}{2}(9.81)\, t^2$$
$$0 = 4.905\, t^2 - 102.61t - 30$$

By quadratic formula:
$t = \mathbf{21.21\ sec}$

SOLUTION 6 - 39
Ans: B



Since the apple is in the balloon, it possesses an initial velocity of 15 m/s.

The highest point reached by the apple from the balloon is given by:

$$v_y^2 = v_{oy}^2 - 2gy$$

where $v_y = 0$

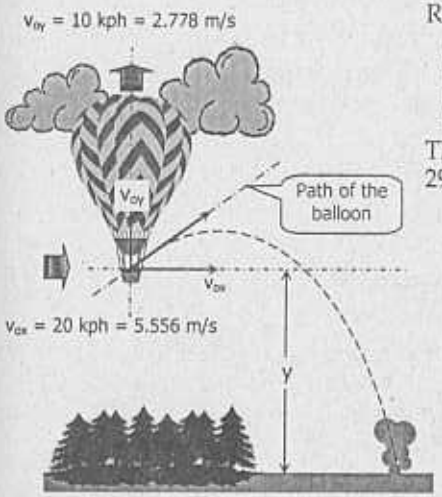
$$0 = 15^2 - 2(9.81)y$$
$$y = 11.468\ \text{m}$$

Thus, the highest point reached by the apple above the ground is

$$H = 150 + y = \mathbf{161.468\ m}$$

SOLUTION 6 - 40
Ans: D

v_{oy} = 10 km/hr = 2.778 m/s
v_{ox} = 20 km/hr = 5.556 m/s
t = 8 sec.

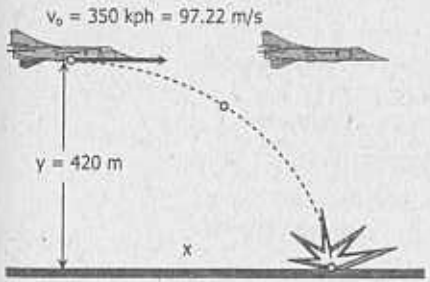


Required: y

$$y = v_{oy}\, t - \tfrac{1}{2} g\, t^2$$
$$y = (2.778)(8) - \tfrac{1}{2}(9.81)(8)^2$$
$$y = -292\ \text{m}$$

Therefore, the balloon's altitude is 292 m

SOLUTION 6 - 41
Ans: D



Since the bomb was dropped freely, its initial velocity and direction is the same as that of the plane.

v_o = 350 km/hr = 97.222 m/s
θ = 0°, and y = -420 m.

$$y = x \tan\theta - \frac{g x^2}{2 v_o^2 \cos^2\theta}$$

$$-420 = x \tan 0° - \frac{9.81 x^2}{2(97.222)^2 \cos^2 0°}$$

x = **899.6 m say 900 m**

SOLUTION 6 - 42
Ans: C

Initial velocity, v_o = 110 km/hr = 30.556 m/s
Final velocity, v = 0 (it comes to rest)
Acceleration, a = -2 m/s²

$$v^2 = v_o^2 + 2as$$
$$0^2 = (30.556)^2 + 2(-2)s;\ s = 233.417\ \text{m}$$

Circumference of the wheel = $2\pi r = 2\pi(0.3) = 0.6\pi$ m
Number of revolutions = 233.417 / (0.6π) = 123.8
Number of complete revolutions = **123**

SOLUTION 6 - 48
Ans:

$a^2 = a_t^2 + a_n^2$

$a_t = 3\ \text{ft/s}^2$

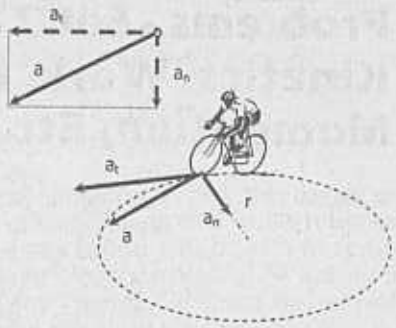
$a_n = v^2/r$

$a_n = (27)^2/800$

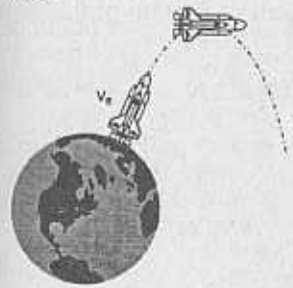
$a_n = 0.911\ \text{ft/s}^2$

$a^2 = (3)^2 + (0.911)^2$

$a = 3.135\ \text{ft/s}^2$ ☑ D



SOLUTION 6 - 49
Ans: A



The escape velocity is given by the formula $v_e = \sqrt{2gR}$, where R is the radius of the planet and g is the acceleration due to gravity of the planet.

$R_{earth} = 3960\ \text{mi} \times 5280\ \text{ft/mile}$

$R = 20{,}908{,}800\ \text{ft.}$

$v_e = \sqrt{2(32.16)(20{,}908{,}800)}$

$v_e = 36{,}672\ \text{ft/s} \times 1\ \text{mile}/5280\ \text{ft}$

$v_e = \mathbf{6.946\ mi/sec}$

SOLUTION 6 - 50
Ans: B

Escape velocity, $v = \sqrt{2gR}$

$g = 0.165(32.2) = 5.313\ \text{ft/sec}^2$

$R = 1080\ \text{mi} \times 5280\ \text{ft./mi.} = 5{,}702{,}400\ \text{ft.}$

$v = \sqrt{2(5.313)(5702400)} = 7784.196\ \text{ft/sec.}$

$v = 7784.196\ \text{ft/sec} \times 1\ \text{mi}/5280\ \text{ft}$

$v = \mathbf{1.47\ mi/sec.}$

Problems - Set 7
Kinetics, Work, Energy, Momentum, Etc.

PROBLEM 7 - 1
ME Oct. 1997

A 10-lbm object is acted upon by a 4-lb force. What is the acceleration in ft/min²?
A. 8.0 × 10 to the 4th power ft/min²
B. 9.2 × 10 to the 4th power ft/min²
C. 7.8 × 10 to the 4th power ft/min²
D. 4.637 × 10 to the 4th power ft/min²

PROBLEM 7 - 2

What horizontal force P can be applied to a 100-kg block in a level surface with coefficient of friction of 0.2, that will cause and acceleration of 2.50 m/s²?
A. 343.5 N C. 53.8 N
B. 224.5 N D. 446.2 N

PROBLEM 7 - 3

A skier wishes to build a rope tow to pull herself up a ski hill that is inclined at 15° with the horizontal. Calculate the tension needed to give the skier's 54-kg body an acceleration of 1.2 m/s². Neglect friction.
A. 202 N C. 106 N
B. 403 N D. 304 N

PROBLEM 7 - 4
ME April 1997

A pick-up truck is traveling forward at 25 m/s. The truck bed is loaded with boxes, whose coefficient of friction with the bed is 0.4. What is the shortest time that the truck can be brought to a stop such that the boxes do not shift?
A. 4.75 sec C. 5.45 sec
B. 2.35 sec D. 6.37 sec

PROBLEM 7 - 5
CE Nov. 1996

A 40-kg block is resting on an inclined plane making an angle of 20° from the horizontal. If the coefficient of friction is 0.60, determine the force parallel to the incline that must be applied to cause impending motion down the plane.
A. 77 C. 72
B. 82 D. 87

PROBLEM 7 - 6
ECE Nov. 1997

A 50-kilogram block of wood rest on the top of the smooth plane whose length is 3 m. and whose altitude is 0.8 m. How long will it take for the block to slide to the bottom of the plane when released?
A. 1.51 seconds C. 2.51 seconds
B. 2.41 seconds D. 2.14 seconds

PROBLEM 7 - 7
ME April 1998,
CE May 1999

A body weighing 40 lbs. starts from rest and slides down a plane at an angle of 30° with the horizontal for which the coefficient of friction μ = 0.3. How far will it move during the third second?
A. 19.99 ft C. 18.33 ft
B. 39.63 ft D. 34.81 ft

PROBLEM 7 - 8

A car and its load weighs 27 kN and the center of gravity is 600 mm from the ground and midway between the front and rear wheel which are 3 m apart. The car is brought to rest from a speed of 54 kph in 5 seconds by means of the brakes. Compute the normal force on each of the front wheels of the car.
A. 7.576 kN C. 5.478 kN
B. 9.541 kN D. 6 kN

PROBLEM 7 - 9
ME April 1998,
CE Nov. 1999

An elevator weighing 2,000 lb attains an upward velocity of 16 fps in 4 sec with uniform acceleration. What is the tension in the supporting cables?
A. 1,950 lb C. 2,495 lb
B. 2,150 lb D. 2,250 lb

PROBLEM 7 - 10
CE Nov. 1997

A block weighing 200 N rests on a plane inclined upwards to the right at a slope of 4 vertical to 3 horizontal. The block is connected to a cable initially parallel to the plane, passing through the pulley and connected to another block weighing 100 N moving vertically downward. The coefficient of kinetic friction between the 200 N block and the inclined plane is 0.10. Which of the following most nearly gives the acceleration (a) of the system?
A. a = 2.93 m/sec² C. a = 1.57 m/ sec²
B. a = 0.37 m/ sec² D. a = 3.74 m/ sec²

PROBLEM 7 - 11
ME Oct. 1997

A car travels on the horizontal unbanked circular track of radius r. Coefficient of friction between the tires and the track is 0.3. If the car's velocity is 10 m/s, what is the smallest radius it may travel without skidding?
A. 50 m C. 15 m
B. 60 m D. 34 m

PROBLEM 7 - 12

If a car travels at 15 m/s and the track is banked 5°, what is the smallest radius it can travel so that friction will not be necessary to resist sliding?
A. 262.16 m C. 278.14 m
B. 651.23 m D. 214.74 m

PROBLEM 7 - 13
CE May 1999

A vertical bar of length L with a mass of 40 kg is rotated vertically about its one end at 40 rpm. Find the length of the bar if it makes an angle of 45° with the vertical?

A. 1.58 m
B. 2.38 m
C. 3.26 m
D. 1.86 m

PROBLEM 7 - 14

The seats of a carousel are attached to a vertical rotating shaft by a flexible cable 8 m long. The seats have a mass of 75 kg. What is the maximum angle of tilt for the seats if the carousel operates at 12 rpm?

A. 30°
B. 35°
C. 45°
D. 39°

PROBLEM 7 - 15
CE Nov. 1998

A highway curve is superelevated at 7°. Find the radius of the curve if there is no lateral pressure on the wheels of a car at a speed of 40 mph.

A. 247.4 m
B. 265.6 m
C. 229.6 m
D. 285.3 m

PROBLEM 7 - 16
CE Nov. 1997

A 2-N weight is swung in a vertical circle of 1-m radius at the end of the cable that will break if the tension exceeds 500 N. Find the angular velocity of the weight when the cable breaks:

A. 49.4 rad/s
B. 37.2 rad/s
C. 24.9 rad/s
D. 58.3 rad/s

PROBLEM 7 - 17
ME April 1998

Traffic travels at 65 mi/hr around a banked highway curve with a radius of 3000 ft. What banking angle is necessary such that friction will not be required to resist the centrifugal force?

A. 5.4°
B. 18°
C. 3.2°
D. 2.5°

PROBLEM 7 - 18
ME April 1997

A concrete highway curve with a radius of 500 feet is banked to give a lateral pressure equivalent to $f = 0.15$. For what coefficient of friction will skidding impend for a speed of 60 mph?

A. < 0.360
B. < 0.310
C. > 0.310
D. > 0.360

PROBLEM 7 - 19
ME Oct. 1997

A 3500 lbf car is towing a 500 lbf trailer. The coefficient of friction between all tires and the road is 0.80. How fast can the car and trailer travel around an unbanked curve of radius 0.12 mile without either the car or trailer skidding?

A. 87 mph
B. 72 mph
C. 26 mph
D. 55 mph

PROBLEM 7 - 20
ME Oct. 1997

A cast-iron governor ball 3 inches in diameter has its center 18 inches from the point of support. Neglecting the weight of the arm itself, find the tension in the arm if the angle with the vertical axis is 60°.

A. 7.63 lb
B. 6.36 lb
C. 7.56 lb
D. 7.36 lb

PROBLEM 7 - 21

An object is placed 3 feet from the center of a horizontally rotating platform. The coefficient of friction is 0.3. The object will begin to slide off when the platform speed is nearest to:

A. 17 rpm
B. 12 rpm
C. 22 rpm
D. 26 rpm

PROBLEM 7 - 22
ME April 1998

A force of 200 lbf acts on a block at an angle of 28° with respect to horizontal. The block is pushed 2 feet horizontally. What is the work done by this force?

A. 320 J
B. 540 J
C. 480 J
D. 215 J

PROBLEM 7 - 23
ME April 1998

A 10-kg block is raised vertically 3 meters. What is the change in potential energy. Answer in SI units closest to:

A. 350 N-m
B. 294 J
C. 350 kg-m²/s²
D. 320 J

PROBLEM 7 - 24

At her highest point, a girl on the swing is 7 feet above the ground, and at her lowest point, she is 3 feet above the ground. What is her maximum velocity?

A. 10 fps
B. 12 fps
C. 14 fps
D. 16 fps

PROBLEM 7 - 25

An automobile has a power output of 1 hp. When it pulls a cart with a force of 300 N, what is the cart's velocity?

A. 249 m/s
B. 24.9 m/s
C. 2.49 m/s
D. 0.249 m/s

PROBLEM 7 - 26

A hunter fires a 50 gram bullet at a tiger. The bullet left the gun with a speed of 600 m/s. What is the momentum of the bullet?

A. 15 kg-m/s
B. 30 kg-m/s
C. 300 kg-m/s
D. 150 kg-m/s

PROBLEM 7 - 27

An elevator can lift a load of 5000 N from the ground level to a height of 20 meters in 10 seconds. Find the horsepower rating of the elevator.

A. 10000
B. 13400
C. 13.4
D. 1340

PROBLEM 7 - 28 The average horsepower required to raise a 150-kg box to a height of 20 meters over a period of one minute is:
A. 450 hp C. 2960 hp
B. 0.657 hp D. 785 hp

PROBLEM 7 - 29 What is the force of attraction between two 90-kg bodies spaced 40 m apart? Assume gravitational constant, $G = 6.67 \times 10^{-11}$ N-m²/kg².
A. 45.6×10^{-6} N C. 4.26×10^{-6} N
B. 3.38×10^{-6} N D. 33.8×10^{-6} N

PROBLEM 7 - 30 What is the efficiency of the pulley system, which lifts a 1 tonne load a distance of 2 m by the application of a force of 150 kg for a distance of 15 m?
A. 11% C. 75%
B. 46% D. 89%

PROBLEM 7 - 31 How much mass is converted to energy per day in a nuclear power plant operated at a level of 100 MW?
A. 9.6×10^{-4} kg C. 9.6×10^{-5} kg
B. 9.6×10^{-7} kg D. 9.6×10^{-6} kg

PROBLEM 7 - 32
ECE April 1998
What is the kinetic energy of a 4000-lb automobile, which is moving at 44 ft/s.
A. 2.10×10^5 ft-lb C. 1.12×10^5 ft-lb
B. 1.20×10^5 ft-lb D. 1.8×10^5 ft-lb

PROBLEM 7 - 33
ME Oct. 1997
A 4000-kg elevator starts from rest accelerates uniformly to a constant speed of 2.0 m/s and decelerates uniformly to stop 20 m above its initial position. Neglecting the friction and other losses, what work was done on the elevator?
A. 785×10^3 Joule C. 900×10^3 Joule
B. 700×10^3 Joule D. 685×10^3 Joule

PROBLEM 7 - 34 The brakes of a 1000-kg automobile exert 3000 N. How long will it take for the car to come to a complete stop from a velocity of 30 m/s?
A. 15 sec C. 5 sec
B. 10 sec D. 2 sec

PROBLEM 7 - 35
ME April 1997
A car weighing 40 tons is switched to a 2% upgrade with a velocity of 30 mph. If the train resistance is 10 lbs/ton, how far up the grade will it go?
A. 1124 ft C. 1204 ft
B. 2104 ft D. 1402 ft

PROBLEM 7 - 36
ME Oct. 1997
A body weighing 1000 lbs. fall 6 inches and strikes a 2000 lbs (per inch) spring. What is the deformation of the spring?
A. 3 inches C. 100 mm
B. 6 inches D. 2 inches

SITUATION:
PROBLEM 7 - 37 to 7 - 39:
CE Nov. 1999
A block weighing 100 kN is pushed by a horizontal force of 100 kN along a smooth horizontal plane from *A* to *B*, 10 m long. The velocity of the block at *A* is 10 m/s. The block is released at *B* and passes through an upward circular arc *BC* (tangent to *B*) of radius 10 m and central angle 45°, and along an inclined plane *CD* tangent to the arc at *C*.

PROBLEM 7 - 37
❶
Which of the following most nearly gives the velocity of the block at *B*?
A. 25 m/s C. 20 m/s
B. 23 m/s D. 17 m/s

PROBLEM 7 - 38
❷
Which of the following most nearly gives the velocity of the block at *C*?
A. 13.5 m/s C. 15.5 m/s
B. 11.5 m/s D. 10.5 m/s

PROBLEM 7 - 39
❸
Which of the following most nearly gives the distance along *CD* that the block will move before it comes to rest?
A. 17 m C. 14 m
B. 20 m D. 12 m

PROBLEM 7 - 40 A 16-gram mass is moving at 30 cm/s while a 4-gram mass is moving in an opposite direction at 50 cm/s. They collide head on and stick together. Their velocity after collision is:
A. 14 cm/s C. 13 cm/s
B. 15 cm/s D. 18 cm/s

PROBLEM 7 - 41 A 5-kg rifle fires a 15-g bullet at a muzzle velocity of 600 m/s. What is the recoil velocity of the rifle?
A. 1800 m/s C. 18 m/s
B. 180 m/s D. 1.80 m/s

PROBLEM 7 - 42 A 0.50-kg ball with a speed of 20 m/s strikes and sticks to a 70-kg block resting on a frictionless surface. Find the block's velocity.
A. 142 m/s C. 1.42 m/s
B. 14.2 m/s D. 0.142 m/s

PROBLEM 7 - 43
CE Nov. 2002

A body starting from point A is given an acceleration which uniformly increase from zero at A to 1.8 m/s² at B in 4 seconds. The initial velocity at A is 3 m/s. Find the distance travelled at the end of 4 seconds.

A. 16.8 m
B. 16.5 m
C. 16.2 m
D. 17.1 m

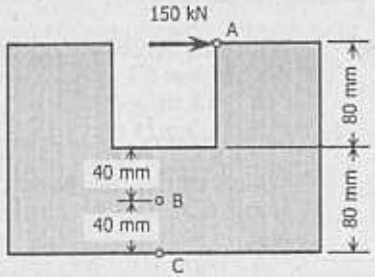
PROBLEM 7 - 44
CE May 2001

A 45 cm × 45 cm square plate $ABCD$ of uniform thickness is supported by three vertical strings. The first string is located at B, the second is located 15 cm from A along side AD, and the third is at the point midway of side CD. What percent of the load is carried by the first and second supports?

A. 40%
B. 60%
C. 50%
D. 30%

SITUATION
PROBLEM 7 – 45
to 7 – 47
(CE May 2003)

The concentrated load shown in the Figure acts at point A.



PROBLEM 7 - 45
❶

If the concentrated load acting at A is replaced by an equivalent force-couple acting at point B, which of the following most nearly gives the magnitude of the equivalent couple, in kiloNewton-meters:

A. 12
B. 18
C. 16
D. 22

PROBLEM 7 - 46
❷

If the concentrated load acting at A is replaced by two horizontal forces, one acting at point B and the other acting at point C, which of the following most nearly gives the magnitude of the force at B, in kiloNewtons:

A. 450
B. 500
C. 600
D. 550

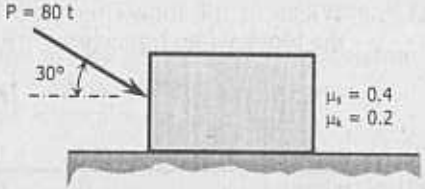
PROBLEM 7 - 47
❸

Which of the following most nearly gives the magnitude of the force at C, in kiloNewtons:

A. 550
B. 500
C. 600
D. 450

SITUATION
PROBLEM 7 – 48
to 7 – 49
(CE Nov. 2002)

The 300-Newton block shown in the Figure is at rest on the rough horizontal plane before the force P is applied at $t = 0$. The force $P = 80t$, where P is in Newtons and t is in seconds.



PROBLEM 7 - 48
❶

Which of the following most nearly gives the time, in seconds, when the block starts to slide?

A. 2.34
B. 2.25
C. 1.76
D. 3.21

PROBLEM 7 - 49
❷

Which of the following most nearly gives the velocity of the block, in meters per second, after five seconds?

A. 24.5
B. 12.1
C. 14.6
D. 18.3

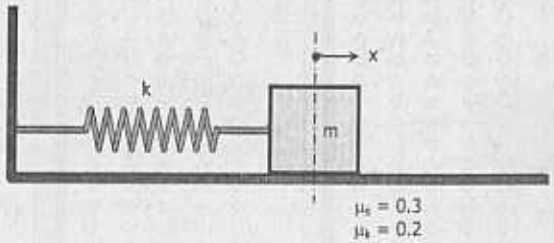
PROBLEM 7 - 50
❸

Which of the following most nearly gives the distance travelled by the block, in meters, after five seconds?

A. 16.6
B. 12.2
C. 18.5
D. 10.8

SITUATION
PROBLEM 7 – 51
to 7 – 53
(CE Nov. 2002)

The block of mass $m = 1.6$ kg slides along the horizontal rough plane as shown in Figure ME-35. The position coordinate x is measured from the undeformed position of the ideal spring, which has a stiffness of $k = 30$ N/m. At $x = 0$ the block is initially moving to the right with speed $v_1 = 6$ m/s.



PROBLEM 7 - 51

Which of the following most nearly gives the force in the spring when $x = 0.80$ m.

A. 28 N
B. 32 N
C. 36 N
D. 24 N

PROBLEM 7 - 52 Which of the following most nearly gives the value of x when the block first comes to rest.

A. 1.3 m
B. 1.1 m
C. 1.6 m
D. 0.9 m

PROBLEM 7 - 53 Which of the following most nearly gives the speed of the block when it reaches $x = 0$ for the second time.

A. 3 m/s
B. 6 m/s
C. 5 m/s
D. 4 m/s

ANSWER SHEET

1.	A	B	C	D	E	21.	A	B	C	D	E	41.	A	B	C	D	E
2.	A	B	C	D	E	22.	A	B	C	D	E	42.	A	B	C	D	E
3.	A	B	C	D	E	23.	A	B	C	D	E	43.	A	B	C	D	E
4.	A	B	C	D	E	24.	A	B	C	D	E	44.	A	B	C	D	E
5.	A	B	C	D	E	25.	A	B	C	D	E	45.	A	B	C	D	E
6.	A	B	C	D	E	26.	A	B	C	D	E	46.	A	B	C	D	E
7.	A	B	C	D	E	27.	A	B	C	D	E	47.	A	B	C	D	E
8.	A	B	C	D	E	28.	A	B	C	D	E	48.	A	B	C	D	E
9.	A	B	C	D	E	29.	A	B	C	D	E	49.	A	B	C	D	E
10.	A	B	C	D	E	30.	A	B	C	D	E	50.	A	B	C	D	E
11.	A	B	C	D	E	31.	A	B	C	D	E	51.	A	B	C	D	E
12.	A	B	C	D	E	32.	A	B	C	D	E	52.	A	B	C	D	E
13.	A	B	C	D	E	33.	A	B	C	D	E	53.	A	B	C	D	E
14.	A	B	C	D	E	34.	A	B	C	D	E	54.	A	B	C	D	E
15.	A	B	C	D	E	35.	A	B	C	D	E	55.	A	B	C	D	E
16.	A	B	C	D	E	36.	A	B	C	D	E	56.	A	B	C	D	E
17.	A	B	C	D	E	37.	A	B	C	D	E	57.	A	B	C	D	E
18.	A	B	C	D	E	38.	A	B	C	D	E	58.	A	B	C	D	E
19.	A	B	C	D	E	39.	A	B	C	D	E	59.	A	B	C	D	E
20.	A	B	C	D	E	40.	A	B	C	D	E	60.	A	B	C	D	E

Solutions to Set 7 Kinetics, Work, Energy, Momentum, Etc.

SOLUTION 7 - 1
Ans: D

By the second law of motion by Newton:

F = Mass × acceleration

$F = m \times a$

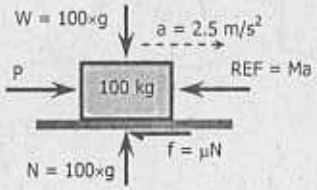
Note: 1 lbm = (1 / 32.2) slugs

$$4 = \frac{10}{32.2}a;\ a = 12.88\ \text{ft/s}^2 \times (60\ \text{s/min})^2$$

$a = 46{,}368\ \text{ft/min}^2 = \mathbf{4.6368 \times 10^4\ ft/min^2}$

SOLUTION 7 - 2
Ans: D

From the figure shown:



$P = \text{REF} + f$

$\text{REF} = Ma = 100(2.5)$

$\text{REF} = 250$

$f = \mu N = 0.2(100g)$

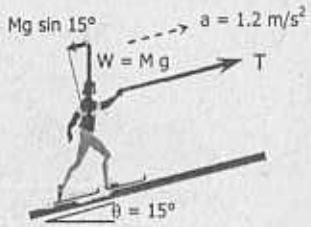
$f = 20g$

$P = 250 + 20(9.81)$

$P = \mathbf{446.2\ N}$

SOLUTION 7 - 3
Ans: A

From the figure shown:



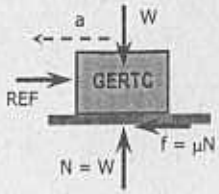
$T = \text{REF} + 54g \sin 15°$

$T = 54a + 54g \sin 15°$

$T = 54(1.2 + 9.81 \sin 15°)$

$T = \mathbf{202\ N}$

SOLUTION 7 - 4
Ans: D



$$\text{REF} = ma = \frac{W}{g}a$$

$\text{REF} = f = \mu N = \mu W$

$$0.40\,W = \frac{W}{g}a;\quad a = 0.4(9.81) = 3.924\ \text{m/s}^2$$

$v = v_o + a\,t$

$v = 0$

$v_o = 25$ m/s

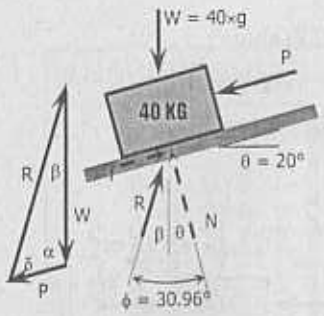
$a = -3.924$ m/s² (deceleration)

$0 = 25 + (-3.924)t$; $t = 6.37$ sec

SOLUTION 7 - 5
Ans: D

The angle of friction $\phi = \tan^{-1}\mu$

$\phi = \tan^{-1}(0.6) = 30.96°$



Since the angle of friction is greater than the angle of friction, the block will not slide down freely, hence required P to cause an impending motion downward must act downward.

From the figures shown:

$\beta = \phi - \theta = 10.96°$

$\alpha = 90° + 20° = 110°$

$\delta = 180° - 110° - 10.96° = 59.04°$

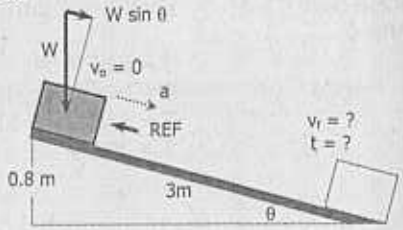
Applying sine law from the force polygon:

$$\frac{P}{\sin 10.96°} = \frac{W}{\sin 59.04°} = \frac{40(9.81)}{\sin 59.04°}$$

$P = 87$ N

SOLUTION 7 - 6
Ans: A

$\sin\theta = 0.8/3$; $\theta = 15.466°$



$REF = W\sin\theta$

$\frac{W}{g}a = W\sin 15.466°$

$a = 2.616$ m/s²

$S = v_o\,t + ½\,a\,t^2$

$3 = 0 + ½(2.616)\,t^2$

$t = 1.514$ seconds

Another solution

Since the surface is frictionless (smooth), the final velocity is the same as that of a free falling body through a height of 0.8 m.

From the relationship $v = \sqrt{2gh} = \sqrt{2(9.81)(0.8)}$

$v = 3.9618$ m/s

By Impulse Momentum Equation:

$\Sigma(+)$ impulse - $\Sigma(-)$ impulse = $M(v_f - v_o)$

$(W\sin 15.466°)\,t - 0 = \frac{W}{g}(3.9618 - 0)$

$t = 1.514$ sec

SOLUTION 7 - 7
Ans: D

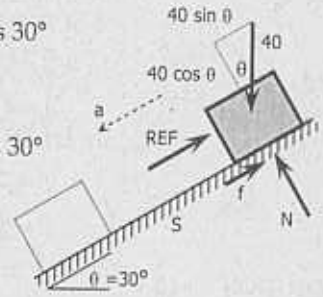
From the figure shown:

$\Sigma F_V = 0$

$N = 40\cos 30°$

$f = \mu N = 0.3 \times 40\cos 30°$

$f = 12\cos 30°$



$\Sigma F_H = 0$

$40\sin 30° = REF + f$

$20 = (40/32.2)\,a + 12\cos 30°$

$a = 7.734$ ft/s²

$S = v_o t + ½at^2$

$v_o = 0$ (starts from rest)

$a = 7.734$ ft/s²

$t = 3$ seconds

$S = 0(3) + ½(7.734)(3)^2 = 34.805$ ft.

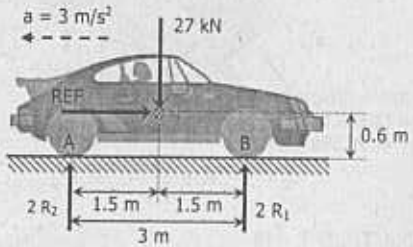
SOLUTION 7 - 8
Ans: A

Solve for the acceleration:

$V = V_o + a\,t$; $V_o = 54$ km/hr × 1000/3600 = 15 m/s

$0 = 15 + a(5)$

$a = -3$ m/s²



$REF = \frac{W}{g}a$

$REF = \frac{27}{9.81}(3)$

$REF = 8.257$ kN

$\Sigma M_A = 0$

$2\,R_1\,(3) = 27(1.5) + 8.257(0.6)$

$R_1 = 7.576$ kN

SOLUTION 7 - 9
Ans: D

From the figure shown:

$\Sigma F_V = 0$

$T = REF + W$

$T = \frac{W}{g}a + W \quad \rightarrow (1)$

Solve for the acceleration a:

Since the elevator (from rest) attains a final velocity v of 16 ft/s in 4 seconds.

$v = v_o + at$

$16 = 0 + a(4)$

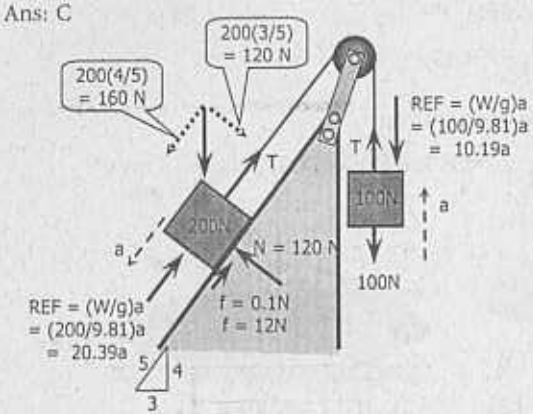
$a = 4 \text{ ft/s}^2$

In Eq. (1)

$T = \frac{2000}{32.2}(4) + 2000$

$T = \mathbf{2{,}248.45 \text{ lb}}$

SOLUTION 7 - 10
Ans: C



For the 100-N block:

$\Sigma F_V = 0$

$T = 100 + 10.19a$

For the 200-N block:

$\Sigma F// = 0$

$T + 20.39a = 160 - 12$

$100 + 10.19a + 20.39a = 148$

$30.58a = 48$

$a = \mathbf{1.57 \text{ m/s}^2}$

SOLUTION 7 - 11
Ans: D

$\tan(\theta + \phi) = \frac{v^2}{gR}, \theta = 0°, \mu = \tan\theta = 0.3$

$\tan(0° + \phi) = \tan\theta = \frac{10^2}{9.81R} = 0.3$

$R = \mathbf{33.98 \text{ m}}$

SOLUTION 7 - 12
Ans: D

$\tan\theta = \frac{V^2}{gR}; \tan 5° = \frac{(15)^2}{9.81R}$

$R = \mathbf{262.16 \text{ m}}$

SOLUTION 7 - 13
Ans: A

$\omega = 40 \frac{\text{rev}}{\text{min}}$

$\omega = 4.1888 \text{ rad/s}$

$CF = M\omega^2 r$

$CF = 40(4.1888)^2 r$

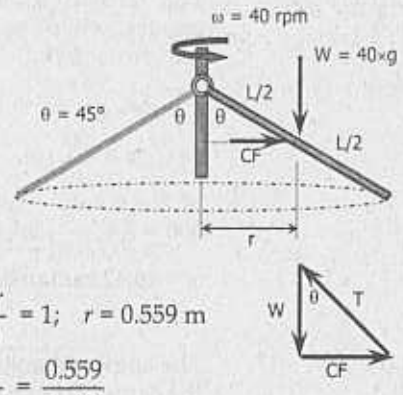
$CF = 701.838\, r$

$\tan\theta = \frac{CF}{W}$

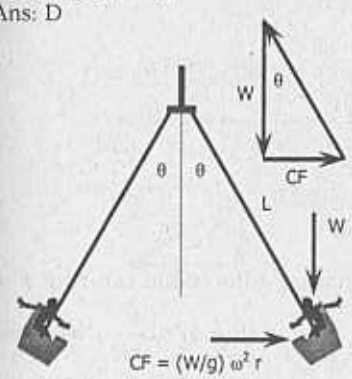
$\tan 45° = \frac{701.838r}{40(9.81)} = 1; \quad r = 0.559 \text{ m}$

$\sin 45° = \frac{r}{L/2} = \frac{0.559}{L/2}$

$L = \mathbf{1.58 \text{ m}}$



SOLUTION 7 - 14
Ans: D



$CF = \frac{W}{g}\omega^2 r$

$r = L\sin\theta$

$\omega = 12 \text{ rpm} = 1.257 \text{ rad/s}$

$CF = \frac{W}{9.81}(1.257)^2(L\sin\theta)$

$CF = 0.161\, WL\sin\theta$

From the force polygon:

$\tan\theta = CF/W$

$\tan\theta = \frac{0.161W(8)\sin\theta}{W}$

$\frac{\sin\theta}{\cos\theta} = 1.288\sin\theta$

$\cos\theta = 0.776; \theta = \mathbf{39°}$

SOLUTION 7 - 15
Ans: B

Ideal angle of banking, $\theta = 7°$

$\tan\theta = \frac{v^2}{gR}$

$v = 40 \frac{\text{miles}}{\text{hr}} \times \frac{5280 \text{ ft}}{\text{mile}} \times \frac{1 \text{ m}}{3.28 \text{ ft}} \times \frac{1 \text{ hr}}{3600 \text{ sec}}$

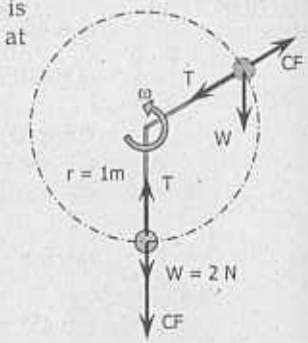
$v = 17.886 \text{ m/s}$

$\tan 7° = \frac{(17.886)^2}{9.81R}$

$R = \mathbf{265.6 \text{ m}}$

SOLUTION 7 - 16
Ans: A

The tension in the cable is greatest when the weight is at the lowest point.



$T = W + CF$

$CF = \frac{W}{g}\omega^2 r$

$500 = 2 + \frac{2}{9.81}\omega^2(1)$

ω = **49.42 radians/sec**

SOLUTION 7 - 17
Ans: A

The angle of banking required so that for a given speed v the car will move so that there will be no lateral pressure between the wheels and the road, is called the *ideal angle of banking* and is given by the formula:

$$\tan\theta = \frac{v^2}{gR}$$

R = the radius of the curve = 3000 feet

$$v = 65\ \frac{\text{mi}}{\text{hr}} \times \frac{5280\text{ ft}}{\text{mi}} \times \frac{1\text{ hr}}{3600\text{ s}} = 95.33\text{ ft/s}$$

$$\tan\theta = \frac{(95.33)^2}{32.2(3000)};\ \theta = \mathbf{5.37°}$$

SOLUTION 7 - 18
Ans: B

The maximum velocity that an automobile can round a curve of radius R is given by, $\tan(\theta + \phi) = \frac{v^2}{gR}$, where θ = angle of banking, ϕ = angle of friction between the wheel and the pavement, v = velocity in ft/sec, and g = acceleration due to gravity = 32.2 ft/s².

$\theta = \arctan f = \arctan(0.15) = 8.53°$

$$v = 60\ \frac{\text{mi}}{\text{hr}} \times \frac{5280\text{ft}}{\text{mi}} \times \frac{1\text{hr}}{3600\text{sec}} = 88\text{ ft/sec}$$

$$\tan(8.53° + \phi) = \frac{(88)^2}{32.2(500)} = 0.481$$

$8.53 + \phi = \arctan 0.481 = 25.687$

$\phi = 17.157;\ \mu = \tan\phi = 0.308$

Therefore, the coefficient of friction μ < **0.310**

SOLUTION 7 - 19
Ans: A

The maximum velocity that a car can round a curve banked at an angle of θ without skidding is given by $\tan(\theta + \phi) = \frac{v^2}{gR}$, where $\theta = 0°$ (unbanked) and ϕ = is the angle of friction.

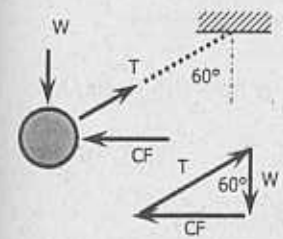
Coefficient of friction = $\tan\phi = 0.8$; $\phi = 38.66°$

$$R = 0.12\text{ mile} \times \frac{5280\text{ ft.}}{\text{mile}} = 633.6\text{ ft.}$$

$$\tan(0 + 38.66°) = \frac{v^2}{32.2(633.6)}$$

$$v = 127.756\ \frac{\text{ft}}{\text{sec}} \times \frac{1\text{ mile}}{5280\text{ ft}} \times \frac{3600\text{ sec}}{1\text{ hr}} = \mathbf{87.11\ mph}$$

SOLUTION 7 - 20
Ans: D



Note: unit weight of cast iron = 450 lb/ft³

$W = \gamma V = 450 \times \frac{4}{3}\pi(1.5/12)^3$

$W = 3.68$ lbs.

From the force polygon (triangle) shown:

$$T = \frac{W}{\cos 60°} = \frac{3.68}{\cos 60°}$$

T = **7.36 lbs.**

SOLUTION 7 - 21
Ans: A

From the formula: $\tan(\theta + \phi) = \frac{v^2}{gR}$; where angle of inclination (banking) which is zero (0), and ϕ is the angle of friction ($\tan\phi = \mu$).

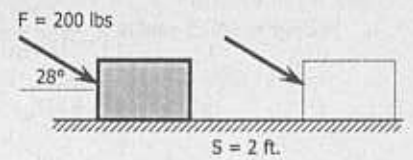
$$\tan\phi = \frac{v^2}{gR} = \mu;\ \frac{v^2}{32.2(3)} = 0.3$$

$v = 5.383$ ft/sec = R ω = 3 ω

$\omega = 1.794$ rad/sec × (1 rev / 2π rad) × (60 sec / min)

ω = **17.13 rpm**

SOLUTION 7 - 22
Ans: C



Work = force × distance
Work = (200 cos 28°) (2)
Work = 353.18 ft-lb

$$\text{Work} = 353.18\ \text{ft-lb} \times \frac{1\,\text{m}}{3.28\,\text{ft.}} \times \frac{1\,\text{kg}}{2.22\,\text{lb}} \times 9.81\ \text{m/s}^2$$

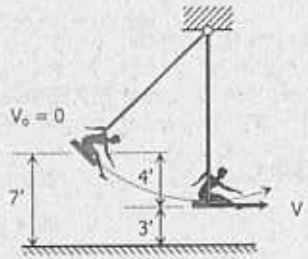
Work = 475.8 N-m = **475.8 J**

SOLUTION 7 - 23
Ans: B

Potential Energy = $Wh = Mgh$
Potential Energy = 10 × 9.81 × 3 = 294.3 N-m
Potential Energy = **294.3 J**

SOLUTION 7 - 24
Ans: D

$\Delta KE = \Delta PE$
$\frac{1}{2} Mv^2 = Mgh$
$\frac{1}{2} v^2 = g\,h = 32.2(4)$
v = **16 fps**



SOLUTION 7 - 25
Ans: C

$$\text{Power} = \frac{\text{Work}}{\text{time}} = \frac{F \times S}{t} = F\,v = 1\ \text{hp} \times 746\ \text{watts/hp}$$

300 N × v = 746 N-m/s
v = **2.49 m/s**

SOLUTION 7 - 26
Ans: B

Momentum = Mass × velocity = 0.05(600)
Momentum = **30 kg-m/s**

SOLUTION 7 - 27
Ans: C

$$\text{Power} = \frac{\text{work}}{\text{time}}$$

$$\text{Power} = \frac{5000(20)}{10} = 10{,}000\ \text{watts} \times 1\text{hp}/746\ \text{watts}$$

Power = **13.4 hp**

SOLUTION 7 - 28
Ans: B

Power = Work / time = force × distance / time

$$\text{Power} = \frac{150(9.81)\text{N} \times 20\text{m}}{60\,\text{sec}} = 490.5\ \text{watts}$$

$$\text{Power} = 490.5\ \text{watts} \times \frac{1\,\text{hp}}{746\,\text{watts}} = \mathbf{0.6575\ hp}$$

SOLUTION 7 - 29
Ans: B

The force of attraction between two bodies is:

$$F = \frac{m_1 m_2}{d^2} G$$

Where:
m_1 & m_2 = mass of the bodies in kg
d = distance between the bodies in m
G = gravitational constant in N-m²/kg²

Then;

$$F = \frac{90(90)}{(0.4)^2} \times 6.67 \times 10^{-11}$$

F = **3.38 × 10⁻⁶ N**

SOLUTION 7 - 30
Ans: D

$$\text{Efficiency, } \eta = \frac{\text{work output}}{\text{work input}} \times 100\%$$

$$\eta = \frac{1000(2)}{150(15)} \times 100\% = \mathbf{88.88\%}$$

Note: 1 ton = 2000 lbs (907.18 kg), also called *net ton* or *short ton*
1 long ton = 2240 lbs (1016.05 Kg)
1 tonne = 1000 kg (2205 lbs), also called metric ton

SOLUTION 7 - 31
Ans: C

Energy, $E = \frac{1}{2} m\,c^2$, where: m = mass,

c = speed of light = 3 × 10⁸ m/s

$E = \frac{1}{2} m\,(3 \times 10^8)^2 = 100 \times 10^6$
$m = 1.111 \times 10^{-9}$ kg/sec × 24(3600) sec/day
m = **9.6 × 10⁻⁵ kg/day**

SOLUTION 7 - 32
Ans: B

$$\text{Kinetic energy, KE} = \tfrac{1}{2} M\,v^2 = \tfrac{1}{2} \frac{W}{g} v^2$$

$$\text{KE} = \tfrac{1}{2} \frac{4000}{32.2} (44)^2 = 120248\ \text{ft-lb}$$

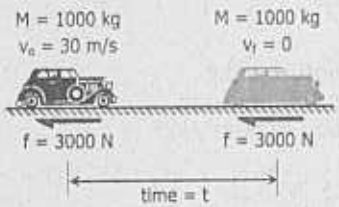
KE = **1.2025 × 10⁵ ft-lb**

SOLUTION 7 - 33
Ans: A

Work = Force × distance
Force = $W = Mg$ = 4000(9.81) = 39240 N
Distance, h = 20 m

Work = 39240 × 20 = 784800 N-m
Work = **784.8 × 10³ Joule**

SOLUTION 7 - 34
Ans: B



By impulse momentum Equation

Δ Impulse = Δ Momentum

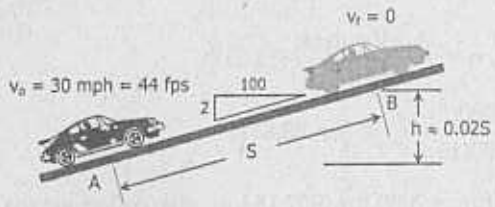
$\Sigma(+)\text{impulse} - \Sigma(-)\text{impulse} = M(v_f - v_o)$

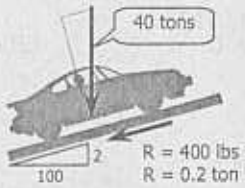
$\text{impulse} = F \times t$

$0 \times t - 3000 \times t = 1000(0 - 30)$

$t = \mathbf{10\ sec}$

SOLUTION 7 - 35
Ans: C





By work-energy equation:

$KE_A \pm \text{Work} \pm Wh = KE_B$

$\frac{1}{2}\frac{40}{32.2}(44)^2 - 0.2\,S - 40(0.02S) = 0$

$S = \mathbf{1202.5\ feet}$

SOLUTION 7 - 36
Ans: A

From the figure below:

Work done by the body = Work done by the spring

Force × Distance = $\frac{1}{2} k y^2$

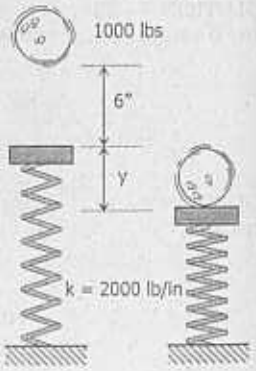
$1000(6 + y) = \frac{1}{2}(2000)\,y^2$

$6 + y = y^2$

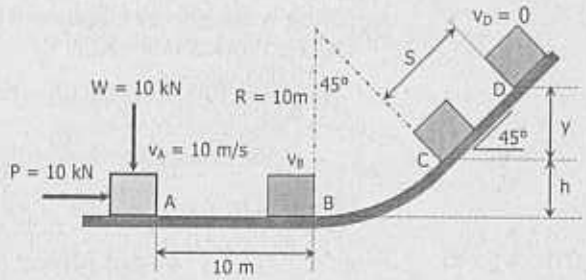
$y^2 - y - 6 = 0$

$y = \dfrac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-6)}}{2(1)}$

$y = \mathbf{3\ inches}$



SOLUTION 7 - 37
Ans: D



Work-energy equation from A to B:

$KE_A \pm \text{Work} \pm Wh = KE_B$

$\frac{1}{2}\frac{W}{g}v_A^2 + P(10) + 0 = \frac{1}{2}\frac{W}{g}v_B^2$

$\frac{1}{2}\frac{10{,}000}{9.81}(10)^2 + 10{,}000(10) + 0 = \frac{1}{2}\frac{10{,}000}{9.81}v_B^2$

$v_B = \mathbf{17.21\ m/s}$

SOLUTION 7 - 38
Ans: C

With reference to the figure in Solution 24-37:

$h = 10 - 10\cos 45° = 2.93\ \text{m}$

$KE_B \pm \text{Work} \pm Wh = KE_C$

$\frac{1}{2}\frac{10{,}000}{9.81}(17.21)^2 + 0 - 10{,}000(2.93) = \frac{1}{2}\frac{10{,}000}{9.81}v_C^2$

$v_C = \mathbf{15.45\ m/s}$

Another Solution: (without solving for v_B)
Applying the work-energy equation from A to C:

$KE_A \pm \text{Work} \pm Wh = KE_C$

$\frac{1}{2}\frac{10{,}000}{9.81}(10)^2 + 10{,}000(10) - 10(2.93) = \frac{1}{2}\frac{10{,}000}{9.81}v_C^2$

$v_C = \mathbf{15.45\ m/s}$

SOLUTION 7 - 39
Ans: A

With reference to the figure in Solution 24-37:

$KE_C \pm \text{Work} \pm Wh = KE_D$

$\frac{1}{2}\frac{10}{9.81}(15.45)^2 - 10(S\sin 45°) = \frac{1}{2}\frac{10}{9.81}(0)^2$

$S = \mathbf{17.2\ m}$

Another solution: (without solving v_C)

Applying work-energy equation from A to D:

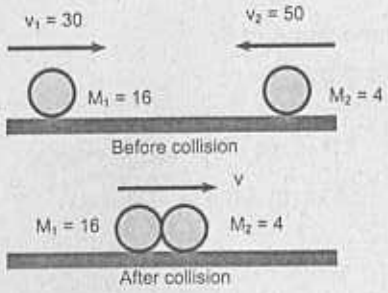
$$KE_A \pm \text{Work} \pm Wh = KE_D$$

$$\tfrac{1}{2}\frac{10{,}000}{9.81}(10)^2 + 10{,}000(10) - 10{,}000(2.93)$$

$$-10(S\sin 45°) = \tfrac{1}{2}\frac{10}{9.81}(0)^2$$

S = **17.2 m**

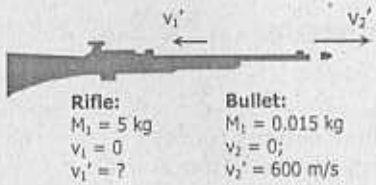
SOLUTION 7 - 40
Ans: A



$$M_1 v_1 + M_2 v_2 = M_1 v_1' + M_2 v_2'$$
$$v_1' = v_2' = v$$

$$16(30) + 4(-50) = 16(v) + 4(v)$$
$$v = \mathbf{14\ cm/s}$$

SOLUTION 7 - 41
Ans: D



By Law of Conservation of Momentum:

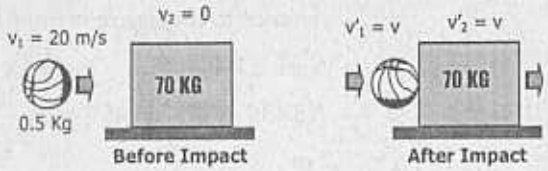
$$M_1v_1 + M_2v_2 = M_1v_1' + M_2v_2'$$

M_1 = mass of rifle, 5 kg
v_1 = initial velocity of rifle = 0
v_1' = recoil velocity of rifle
M_2 = mass of the bullet, 0.015 kg
v_2 = initial velocity of bullet, 0 m/s
v_2' = final velocity of bullet, 600 m/s

$$5(0) + 0.015(0) = 5(v'_1) + 0.015(600)$$
$$v'_1 = -1.8\ \text{m/s}$$

Thus, the recoil velocity is **1.8 m/s**

SOLUTION 7 - 42
Ans: D

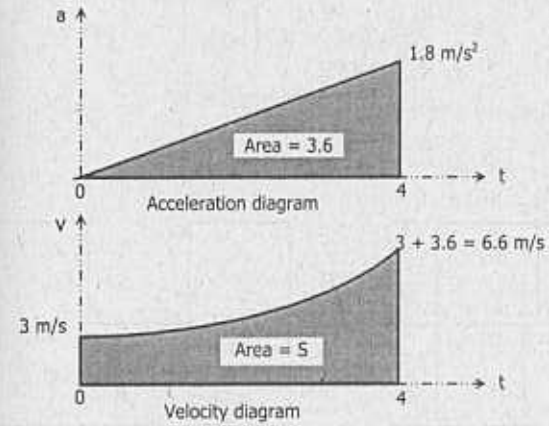


For the 0.50-kg ball, M_1 = 50 kg
Initial velocity = v_1 = 20 m/s
Final velocity = v_1' (same as the block)

For the 70-kg block, M_2 = 70 kg
Initial velocity, v_2 = 0 (at rest)
Final velocity, $v_2' = v$

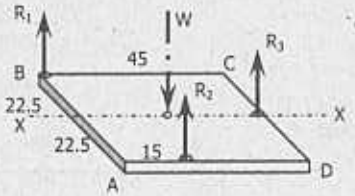
$$M_1 v_1 + M_2 v_2 = M_1 v_1' + M_2 v_2'$$
$$0.5(20) + 70(0) = 0.5v + 70v$$
$$v = \mathbf{0.142\ m/s}$$

SOLUTION 7 - 43
Ans: A



S = Area
$S = 4(6.6) - \frac{2}{3}(4)(3.6)$
S = **16.8 m**

SOLUTION 7 - 44
Ans: B



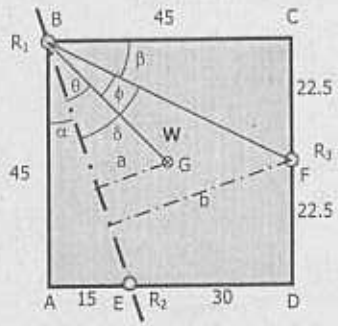
$[\Sigma M_{CD} = 0]$
$30R_2 + 45R_1 = W(22.5)$
$4R_2 + 6R_1 = 3W \rightarrow (1)$

$[\Sigma M_{XX} = 0]$
$R_1(22.5) = R_2(22.5)$
$R_1 = R_2$

In Eq. (1):
$4R_1 + 6R_1 = 3W;\ R_1 = 0.3W = R_2$

$R_1 + R_2 = 0.6W = 60\%W$

Thus, **60%** of the weight is carried by the first and the second supports.



Another solution:

$\tan\alpha = 15/45;\ \alpha = 18.435°$

$\alpha + \theta = 45°;\ \theta = 26.565°$

$BG = 31.8198$

$\sin\theta = a / BG;\ a = 14.23$ cm

$\tan\beta = 22.5/45;\ \beta = 26.565°$

$\beta + \phi = 45°;\ \phi = 18.435°$

$\delta = \theta + \phi = 45°$

$BF = 50.312$

$\sin\delta = b / BF$

$b = 35.576$ cm

$[\Sigma M_{BE} = 0]$

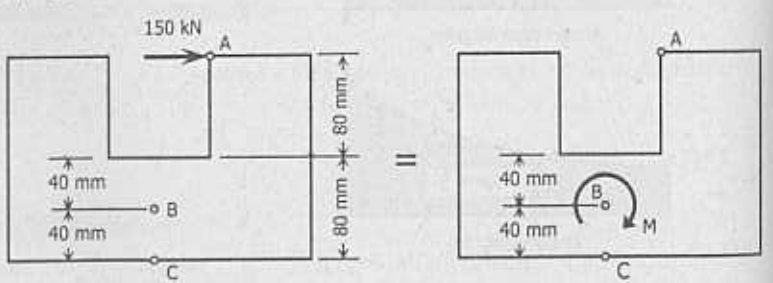
$R_3\,(b) = W\,(a)$

$R_3(35.576) = W(14.23)$

$R_3 = 0.4\,W$

$R_1 + R_2 = 0.6W = 60\%\ W$

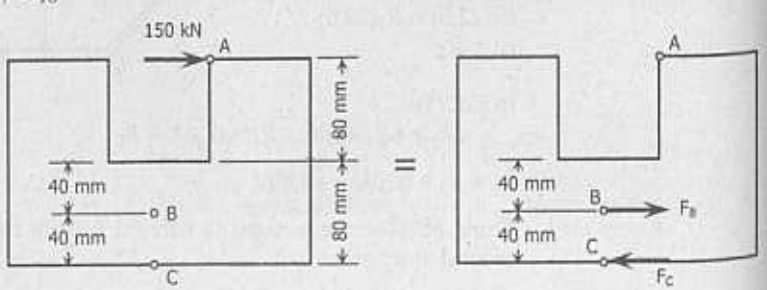
SOLUTION 7 - 45
Ans: B



M = Moment at B due to 150 kN force

$M = 150 \times \frac{80+40}{1000}$ = **18 kN-m**

SOLUTION 7 - 46
Ans: C



Moment at C due to 150 kN = Moment at C due to F_B

$150 \times \frac{160}{1000} = F_B \times \frac{40}{1000}$

F_B = **600 kN**

SOLUTION 7 - 47
Ans: D

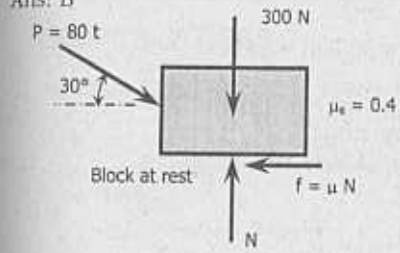
With reference to the Figure in Solution 24-46:

Moment at B due to 150 kN = Moment at B due to F_C

$150 \times \frac{120}{1000} = F_C \times \frac{40}{1000}$

F_C = **450 kN**

SOLUTION 7 - 48
Ans: B



Before the block starts to slide, $\mu = \mu_s = 0.4$

$N = 300 + P\sin 30° = 300 + (80\,t)\sin 30°$

$N = 300 + 40t$

$f = 0.4N = 120 + 16t$

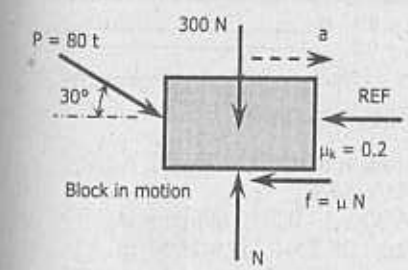
When the block starts to move,

$P\cos 30° = f$

$(80t)\cos 30° = 120 + 16t$

t = **2.252 seconds**

SOLUTION 7 - 49
Ans: C



When the block moves,

$\mu = \mu_k = 0.2$

$f = 0.2N = 0.2(300 + 40t)$

$f = 60 + 8t$

$[\Sigma F_H = 0]$

$\text{REF} + f = P\cos 30°$

$\frac{300}{9.81}a = (80t)\cos 30° - (60 + 8t)$

$a = 2.0039t - 1.962 = \frac{dv}{dt}$

$dv = (2.0039t - 1.962)\,dt$

Integrate both side:

$v = 1.00195\,t^2 - 1.962\,t + C$

When $t = 2.252$ s, $v = 0$

$0 = 1.00195(2.252)^2 - 1.962(2.252) + C$

$C = -0.663$

$v = 1.00195\,t^2 - 1.962\,t - 0.663$

When $t = 5$ sec

$v = 1.00195(5)^2 - 1.962(5) - 0.663$ = **14.58 m/s**

SOLUTION 7 - 50
Ans: A

With reference to Solution 24-49:

Distance traveled:

$$v = 1.00195\,t^2 - 1.962\,t - 0.663 = \frac{dS}{dt}$$

$$S = \int_{2.252}^{5} (1.00195\,t^2 - 1.962\,t - 0.663)\,dt$$

$$S = 0.334\,t^3 - 0.981\,t^2 - 0.663t \Big]_{2.252}^{5}$$

$$S = 0.334(5^3 - 2.252^3) - 0.981(5^2 - 2.252^2) - 0.663(5 - 2.252)$$

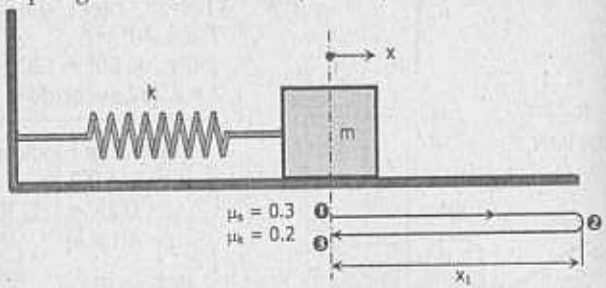
$$S = \mathbf{16.56\ m}$$

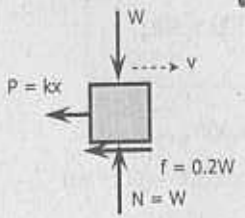
SOLUTION 7 - 51
Ans: D

Force in spring, $P = kx = 30(0.8) = \mathbf{24\ N}$

SOLUTION 7 - 52
Ans: A

Note: Work done in stretching a spring from x_1 to x_2 is $\frac{1}{2}k(x_2^2 - x_1^2)$





$W = m \times g = 1.6(9.81) = 15.696$ N

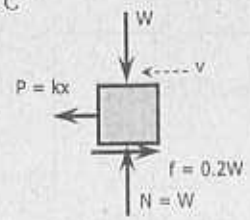
Work-energy equation from ❶ to ❷:

$$E_{❶} \pm \text{Work} \pm Wh = KE_{❷}$$

$$\tfrac{1}{2}(1.6)(6)^2 - \tfrac{1}{2}(30)x_1^2 - 0.2(15.696)x_1 = 0$$

$$15x_1^2 + 3.1392x_1 - 28.8 = 0;\ x_1 = \mathbf{1.285\ m}$$

SOLUTION 7 - 53
Ans: C



Work-energy equation from ❷ to ❶:

$$E_{❷} \pm \text{Work} \pm Wh = KE_{❶}$$

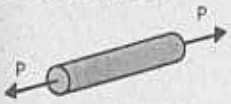
$$0 + \tfrac{1}{2}(30)(1.285)^2 + 0.2(15.696)(1.285) = \tfrac{1}{2}(1.6)v_2^2$$

$$v_2 = \mathbf{5.09\ m/s}$$

Part 5
STRENGTH OF MATERIALS

SIMPLE STRESS

Normal Stress



Bar in Tension

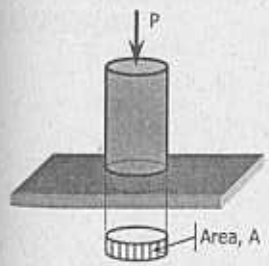
Bar in Compression

Stress is defined as the strength of a material per unit area or unit strength. It is the force on a member divided by the area, which carries the force, formerly expressed as psi, now in N/mm² or MPa.

$$\sigma = P/A$$

where P is the applied normal load in Newton and A is the area in mm². The maximum stress in tension or compression occurs over a section normal to the load.

Shearing Stress

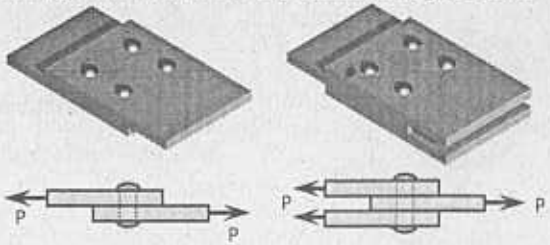


Punching Shear

Forces parallel to the area resisting the force cause shearing stress. It differs to tensile and compressive stresses, which are caused by forces perpendicular to the area on which they act. Shearing stress is also known as tangential stress.

$$\tau = V/A$$

Where V is the resultant shearing force which passes through the centroid of the area A being sheared.



Single Shear Double Shear

SOLUTION 7 - 50
Ans: A

With reference to Solution 24-49:

Distance traveled:

$$v = 1.00195\,t^2 - 1.962\,t - 0.663 = \frac{dS}{dt}$$

$$S = \int_{2.252}^{5} (1.00195\,t^2 - 1.962\,t - 0.663)\,dt$$

$$S = 0.334\,t^3 - 0.981\,t^2 - 0.663t \Big]_{2.252}^{5}$$

$$S = 0.334(5^3 - 2.252^3) - 0.981(5^2 - 2.252^2) - 0.663(5 - 2.252)$$

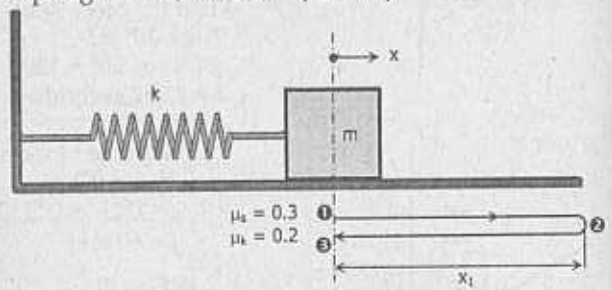
$$S = \mathbf{16.56\ m}$$

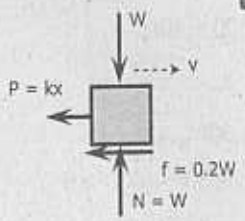
SOLUTION 7 - 51
Ans: D

Force in spring, $P = kx = 30(0.8) = \mathbf{24\ N}$

SOLUTION 7 - 52
Ans: A

Note: Work done in stretching a spring from x_1 to x_2 is $\frac{1}{2}k(x_2^2 - x_1^2)$





$W = m \times g = 1.6(9.81) = 15.696$ N

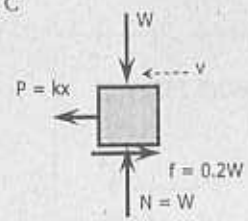
Work-energy equation from ❶ to ❷:

$$E_{❶} \pm \text{Work} \pm Wh = KE_{❷}$$

$$\tfrac{1}{2}(1.6)(6)^2 - \tfrac{1}{2}(30)x_1^2 - 0.2(15.696)x_1 = 0$$

$$15x_1^2 + 3.1392x_1 - 28.8 = 0;\ x_1 = \mathbf{1.285\ m}$$

SOLUTION 7 - 53
Ans: C



Work-energy equation from ❷ to ❶:

$$E_{❷} \pm \text{Work} \pm Wh = KE_{❶}$$

$$0 + \tfrac{1}{2}(30)(1.285)^2 + 0.2(15.696)(1.285) = \tfrac{1}{2}(1.6)v_2^2$$

$$v_2 = \mathbf{5.09\ m/s}$$

Part 5
STRENGTH OF MATERIALS

SIMPLE STRESS

Normal Stress

Bar in Tension

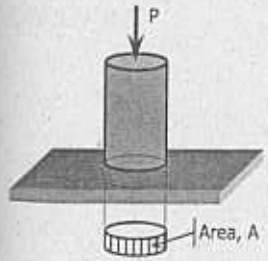
Bar in Compression

Stress is defined as the strength of a material per unit area or unit strength. It is the force on a member divided by the area, which carries the force, formerly expressed as psi, now in N/mm² or MPa.

$$\sigma = P/A$$

where P is the applied normal load in Newton and A is the area in mm². The maximum stress in tension or compression occurs over a section normal to the load.

Shearing Stress

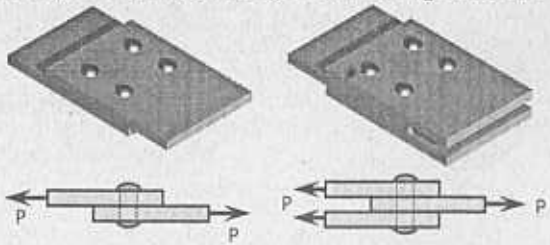


Punching Shear

Forces parallel to the area resisting the force cause shearing stress. It differs to tensile and compressive stresses, which are caused by forces perpendicular to the area on which they act. Shearing stress is also known as tangential stress.

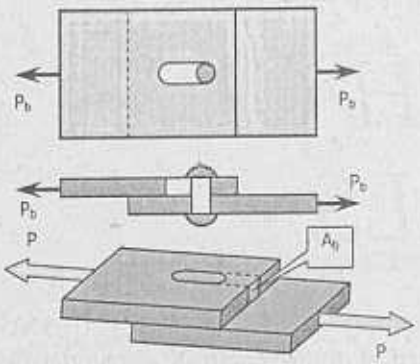
$$\tau = V/A$$

Where V is the resultant shearing force which passes through the centroid of the area A being sheared.



Single Shear Double Shear

Bearing Stress

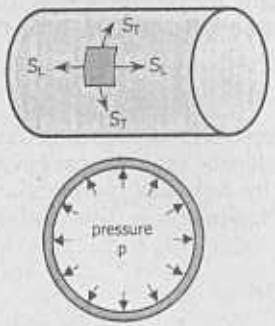


Bearing stress is the contact pressure between separate bodies. It differs from compressive stress, as it is an internal stress caused by compressive forces.

$$\sigma_b = \frac{P_b}{A_b}$$

THIN-WALLED PRESSURE VESSELS

Thin-Walled Cylindrical Vessel



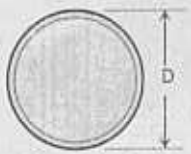
A tank or pipe carrying a fluid or gas under a pressure is subjected to tensile forces, which resist bursting, developed across longitudinal and transverse sections.

$$\text{Tangential stress, } S_T = \frac{pD}{2t}$$

$$\text{Longitudinal stress, } S_L = \frac{pD}{4t}$$

p = internal pressure – external pressure

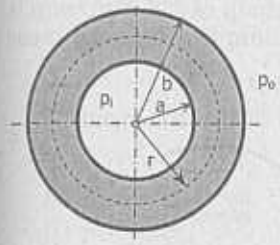
Spherical Shell



If a spherical tank of diameter D and thickness t contains gas under a pressure of p, the stress at the wall can be expressed as:

$$\text{Wall stress, } S = \frac{pD}{4t}$$

Thick-Walled Cylinder



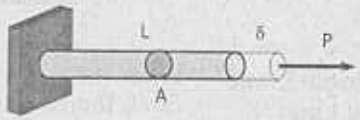
In thin-walled cylinders, the wall thickness is very small compared to the tank diameter. If in the case of thick-walls cylinders, the tangential stress S_T and radial stress S_R at any distance r from the center is given by the following equations:

$$S_R = \frac{a^2 p_i - b^2 p_o}{b^2 - a^2} - \frac{a^2 b^2 (p_i - p_o)}{(b^2 - a^2) r^2}$$

$$S_T = \frac{a^2 p_i - b^2 p_o}{b^2 - a^2} + \frac{a^2 b^2 (p_i - p_o)}{(b^2 - a^2) r^2}$$

SIMPLE STRAIN

Also known as a unit deformation, strain is the ratio of the change in length caused by an applied force, to the original length.



$$\varepsilon = \frac{\delta}{L}$$

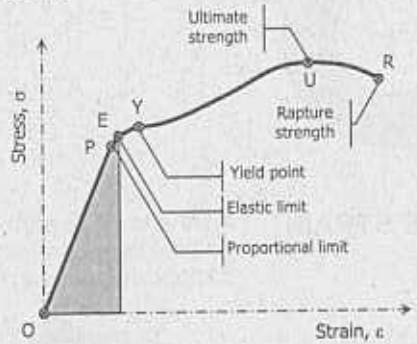
where δ is the elongation and L is the original length, thus ε is dimensionless.

STRESS-STRAIN DIAGRAM

Suppose that a metal specimen be placed in a tension-compression-testing machine. As the axial load is gradually increased in increments, the total elongation over the gage length is measured at each increment of the load and this is continued until failure of the specimen takes place. Knowing the original cross-sectional area and length of the specimen, the normal stress σ and the strain ε can be obtained. The graph of these quantities with the stress σ along the y-axis and the strain ε along the x-axis is called the stress-strain diagram. The stress-strain diagram differs in form for various materials. The diagram shown below is that for a medium-carbon structural steel.

Metallic engineering materials are classified as either ductile or brittle materials. A ductile material is one having relatively large tensile strains up to

the point of rapture like structural steel and aluminum, whereas brittle materials has a relatively small strain up to the point of rapture like cast iron and concrete. An arbitrary strain of 0.05 mm/mm is frequently taken as the dividing line between these two classes.



Proportional Limit (Hooke's Law)

From the origin, O to a point called proportional limit, the stress-strain curve is a straight line. This linear relation between elongation and the axial force causing it was first noticed by Sir Robert Hook in 1678 and is called Hooke's Law that within the proportional limit, the stress is directly proportional to strain or

$$\sigma \propto \varepsilon \text{ or } \sigma = K\varepsilon$$

The constant of proportionality K is called the Modulus of Elasticity E or Young's Modulus and is equal to the slope of the stress-strain diagram from O to P. Then

$$\sigma = E\varepsilon$$

Elastic Limit

The *elastic limit* is the limit beyond which the material will no longer go back to its original shape when the load is removed, or it is the maximum stress that may be developed such that there is no permanent or residual deformation when the load is entirely removed.

Elastic and Plastic Ranges

The region in the stress-strain diagram from O to P is called the *elastic range*. The region from P to R is called the *plastic range*.

Yield Point

The point at which the material will have an appreciable elongation or yielding without any increase of load.

Ultimate Strength

The maximum ordinate in the stress-strain diagram is the ultimate strength or tensile strength.

Rapture Strength

The strength of the material at rapture. This is also known as the *breaking strength*.

Modulus of Resilience

Modulus of resilience is the work done on a unit volume of material as the force is gradually increased from O to P, in N-m/m³. This may be calculated as the area under stress-strain curve from the origin O up to the elastic limit E (the shaded area in the figure). The resilience of a material is its ability to absorb energy without creating a permanent distortion.

Modulus of Toughness

Modulus of toughness is the work done on a unit volume of material as the force is gradually increased from O to R, in N-m/m³. This may be calculated as the area under the entire stress-strain curve (from O to R). The toughness of a material is its ability to absorb energy without causing it to break.

Working Stress, Allowable Stress, and Factor of Safety

Working stress is defined as the actual stress of a material under a given loading. The maximum safe stress that a material can carry is termed as the allowable stress. The allowable stress should be limited to values not exceeding the proportional limit. However, since proportional limit is difficult to determine accurately, the allowable stress is taken as either the yield point or ultimate strength divided by a factor of safety. The ratio of this strength (ultimate or yield strength) to the allowable strength is called the factor of safety.

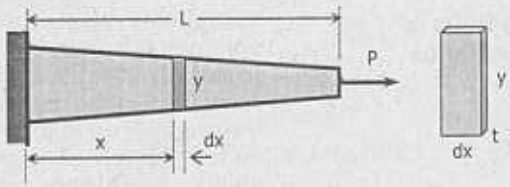
AXIAL DEFORMATION

In the linear portion of the stress-strain diagram, the *stress is proportional to strain* and is given by $\sigma = E\varepsilon$. Since $\sigma = P/A$ and $\varepsilon = \delta/L$, then $P/A = E\delta/L$, or

$$\delta = \frac{PL}{AE} = \frac{\sigma L}{E}$$

To use this formula, the load must be axial, the bar must have a uniform cross-sectional area, and the stress must not exceed the proportional limit.

If however, the cross-sectional area is not uniform, the axial deformation can be determined by considering a differential length and applying integration.



$$\delta = \frac{P}{E}\int_0^L \frac{dx}{A}$$

where $A = t\,y$ and y and t, if variable, must be expressed in terms of x.

For rod of unit mass ρ suspended vertically from one end, the total elongation of due to its own weight is

$$\delta = \frac{\rho g L^2}{2E} = \frac{MgL}{2AE}$$

where ρ is in kg/m³, L is the length of the rod in mm, M is the total mass of the rod in kg, A is the cross-sectional area of the rod in mm², and $g = 9.81$ m/s².

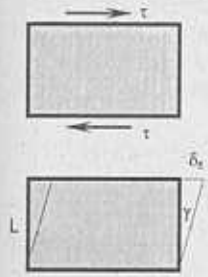
Stiffness, *k*

Stiffness is the ratio of the steady force acting on an elastic body to the resulting displacement. It has the unit of N/mm.

$$k = \frac{P}{\delta}$$

SHEARING DEFORMATION

Shearing forces cause shearing deformation. An element subject to shear does not change is length but undergoes a change in shape.



The change in angle at the corner of an original rectangular element is called the shear strain and is expressed as

$$\gamma = \frac{\delta_s}{L}$$

The ratio of the shear stress τ and the shear strain γ is called the *modulus of elasticity in shear* or *modulus of rigidity* and is denoted as G, in MPa.

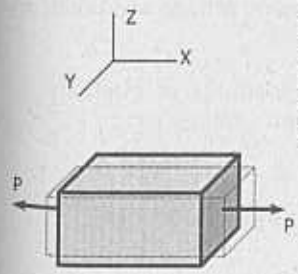
$$G = \frac{\tau}{\gamma}$$

The relationship between the shearing deformation and the applied shearing force is

$$\delta_s = \frac{VL}{A_s G} = \frac{\tau L}{G}$$

where V is the shearing force acting over an area A_s.

Poisson's Ratio



When a bar is subjected to a tensile loading there is an increase in length of the bar in the direction of the applied load, but there is also a decrease in the lateral dimension perpendicular to the load. The ratio of the sidewise deformation (or strain) to the longitudinal deformation (or strain) is called the Poisson's ratio and is denoted by μ. For most steel, it lies in the range 0.25 to 0.3, and 0.20 for concrete.

$$\mu = -\frac{\varepsilon_y}{\varepsilon_x} = -\frac{\varepsilon_z}{\varepsilon_x}$$

where ε_x is strain in the X-direction and ε_y and ε_z are the strains in the perpendicular direction. The negative sign indicates a decrease in the transverse dimension when ε_x is positive.

Biaxial Deformation

If an element is subjected simultaneously by tensile stresses, σ_x and σ_y, in the X and Y directions, the strain in the X direction is σ_x/E and the strain in the Y-direction is σ_y/E. Simultaneously, the stress in the Y direction will produce a lateral contraction on the X-direction of the amount $-\mu\,\varepsilon_y$ or $-\mu\,\sigma_y/E$. The resulting strain in the X-direction will be

$$\varepsilon_x = \frac{\sigma_x}{E} - \mu\frac{\sigma_y}{E} \text{ or } \sigma_x = \frac{(\varepsilon_x + \mu\varepsilon_y)E}{1-\mu^2}$$

and

$$\varepsilon_y = \frac{\sigma_y}{E} - \mu\frac{\sigma_x}{E} \text{ or } \sigma_y = \frac{(\varepsilon_y + \mu\varepsilon_x)E}{1-\mu^2}$$

Triaxial Deformation

If an element is subjected simultaneously by three mutually perpendicular normal stresses σ_x, σ_y and σ_z, which are accompanied by strains ε_x, ε_y, and ε_z, respectively,

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \mu(\sigma_y + \sigma_z)\right]$$
$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \mu(\sigma_x + \sigma_z)\right]$$
$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \mu(\sigma_x + \sigma_y)\right]$$

Tensile stresses and elongation are taken as positive. Compressive stresses and contractions are taken as negative.

Relationship between *E*, *G*, and μ

The relationship between modulus of elasticity E, shear modulus G and Poisson's ratio μ is:

$$G = \frac{E}{2(1+\mu)}$$

Bulk Modulus of Elasticity or Modulus of Volume Expansion, *K*

The bulk modulus of elasticity K is a measure of a resistance of a material to change in volume without change in shape or form. It is given as

$$K = \frac{E}{3(1-2\mu)} = \frac{\sigma}{\Delta V/V}$$

where V is the volume and ΔV is change in volume. The ratio $\Delta V/V$ is called volumetric strain and can be expressed as

$$\frac{\Delta V}{V} = \frac{\sigma}{K} = \frac{3(1-2\mu)\sigma}{E}$$

THERMAL STRESS

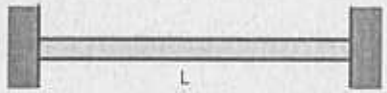
Temperature changes cause the body to expand or to contract. The amount of linear deformation, δ_T, is given by

$$\delta_T = \alpha\, L(T_f - T_i) = \alpha\, L\, \Delta T$$

where α is the coefficient of thermal expansion in m/m-°C, L is the length in meter, T_i & T_f is the initial and final temperatures, respectively in °C. For steel $\alpha = 11.25 \times 10^{-6}$ / °C.

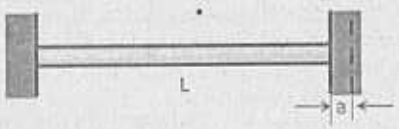
If temperature deformation is permitted to occur freely, no load or stress will be induced in the structure. In some cases where temperature deformation is not permitted, an internal stress is created. The internal stress created is termed as thermal stress.

For a homogeneous rod mounted between unyielding supports as shown, the thermal stress is computed as



Thermal stress, $\sigma = E\,\alpha\,\Delta T = E\,\alpha\,(T_i - T_f)$

If the wall yields a distance of "a", the stress is computed as follows:



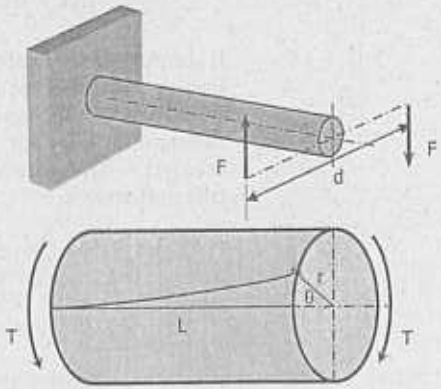
$$\alpha L \Delta T = a + \frac{\sigma L}{E}$$

where σ the thermal stress.

Take note that is the temperature rises above the normal, the stress in the rod is compression, and if the temperature drops below the normal, the rod is in tension.

TORSION

Consider a bar to be rigidly attached at one end and twisted at the other end by a torque or twisting moment T equivalent to $F \times d$, which is applied perpendicular to the axis of the bar, as shown in the figure. Such a bar is said to be in torsion.



Torsional Shearing Stress

For a solid or hallow circular shaft subject to a twisting moment T the torsional shearing stress S_s at a distance ρ from the center of the shaft is

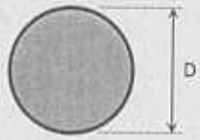
$$S_s = \frac{T\rho}{J} \quad \text{and} \quad S_{s\,max} = \frac{Tr}{J}$$

where J is the polar moment of inertia of the section and r is the outer radius.

For solid cylindrical shaft:

$$J = \frac{\pi}{32} D^4$$

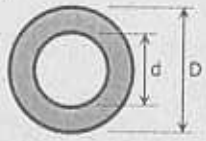
$$S_s = \frac{16T}{\pi D^3}$$



For hallow cylindrical shaft

$$J = \frac{\pi}{32}(D^4 - d^4)$$

$$S_s = \frac{16TD}{\pi(D^4 - d^4)}$$



Angle of Twist

The angle θ through which the bar of length L will twist is

$$\theta = \frac{TL}{JG} \quad \text{in radians}$$

where T is the torque in N-mm, L is the length of shaft in mm, G is shear modulus in MPa, J is the polar moment of inertia in mm⁴, D and d are diameter in mm, and r is the radius in mm.

Power Transmitted by the Shaft

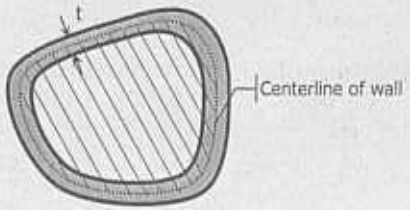
A shaft rotating with a constant angular velocity ω (in radians per second) is being acted on by a twisting moment T. The power transmitted by the shaft is

$$P = T\omega = 2\pi T n$$

where T is the torque in N-m, n is number of revolutions per second, and P is the power in watts

Torsion on Thin-Walled Tubes

The torque applied to thin-walled tubes is expressed as



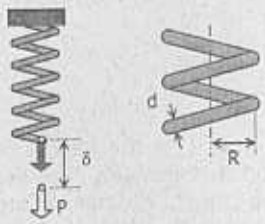
$$T = 2Aq$$

where T is the torque in N-mm, A is the area enclosed by the centerline of the tube (as shown in the stripe-filled portion) in mm², and q is the shear flow in N/mm.

The average shearing stress across any thickness t is

$$S_s = \frac{q}{t} = \frac{T}{2At}$$

HELICAL SPRINGS



When close-coiled helical spring, composed of a wire of round rod of diameter d wound into a helix of mean radius R with n number of turns, is subjected to an axial load P produces the following stresses and elongation:

$$S_s = \frac{16PR}{\pi d^3}\left(1 + \frac{d}{4R}\right)$$

This formula neglects the curvature of the spring. This is used for light springs where the ratio $d/4R$ is small.

The elongation of the bar is

$$\delta = \frac{64PR^3 n}{Gd^4}$$

Notice that the deformation δ is directly proportional to the applied load P. The ratio of P to δ is called the spring constant k is equal to

$$k = \frac{P}{\delta} = \frac{Gd^4}{64R^3 n} \quad \text{in N/mm}$$

A.M. Wahl Formula

For heavy springs and considering the curvature of the spring, a more precise formula is given by: (A.M. Wahl Formula)

$$S_s = \frac{16PR}{\pi d^3}\left(\frac{4m-1}{4m-4} + \frac{0.615}{m}\right)$$

where m is called the spring index and $(4m-1)/(4m-4)$ is the Wahl factor.

Springs in Series

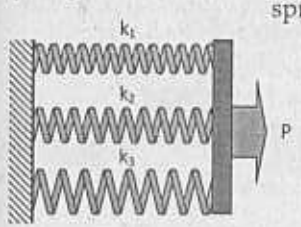
For two or more springs with spring laid in series, the resulting spring constant k is given by

$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2} + \ldots$$

where $k_1, k_2, \ldots$ are the spring constants for different springs.

Springs in Parallel

For two or more springs in parallel, the resulting spring constant is



$$k = k_1 + k_2 + \ldots$$

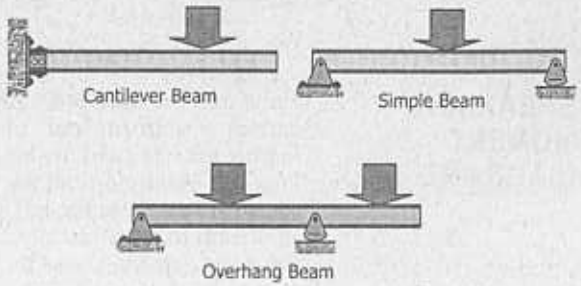
SHEAR AND MOMENT IN BEAMS

Definition of Beam

A beam is a bar subject to forces or couples that lie in a plane containing the longitudinal of the bar. According to determinacy, a beam may be determinate or indeterminate.

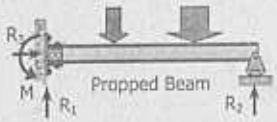
Statically Determinate Beams

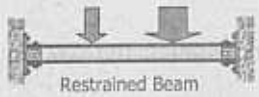
Statically determinate beams are those beams in which the reactions of the supports may be determined by the use of the equations of static equilibrium. The beams shown below are examples of statically determinate beams.



Statically Indeterminate Beams

If the number of reactions exerted upon a beam exceeds the number of equations in static equilibrium, the beam is said to be statically indeterminate. In order to solve the reactions of the beam, the statics equation must be supplemented by equations based upon the elastic deformations of the beam.

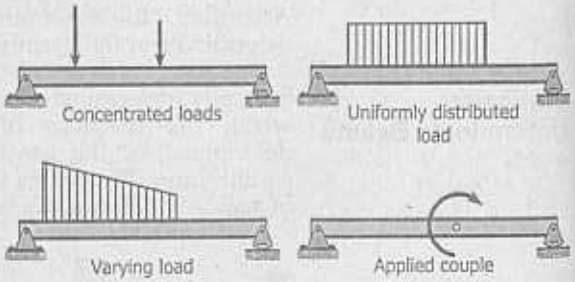




The degree of indeterminacy is taken as the difference between the *number of reactions* to the *number of equations in static equilibrium* that can be applied. In the case of the propped beam shown, there are four unknown reactions R_1, R_2, R_3, and M and three static equilibrium equations (ΣM, ΣF_H, and ΣF_V) can be applied, thus the beam is indeterminate to the first degree (4 - 3 = 1).

Types of Loading

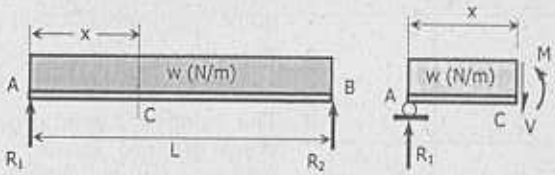
Loads applied to a beam may consist of a concentrated load (load applied at a point), uniform load, uniformly varying load, or an applied couple or moment. These loads are shown in the following figures.



SHEAR AND MOMENT DIAGRAMS

Consider the simple beam shown of length L that carries a uniform load of w (N/m) throughout its length and is held in equilibrium by reactions R_1 and R_2. Assume that the beam is cut at point C a distance of x from the left support and the portion of the beam to the right of C be removed. The portion removed must then be replaced by a vertical

shearing force V together with a couple M, to hold the left portion of the bar in equilibrium under the action of the force R_1 and wx.



$$R_1 = R_2 = wL/2$$

The couple M is called the *resisting moment* or *moment* and the force V is called the *resisting shear* or *shear*. The sign of V and M are taken to be positive if they have the senses indicated above.

Relationship between Load, Shear, and Moment

From the figure shown above, the shear and moment at point C is as follows:

$$V = \frac{wL}{2} - w\,x \qquad M = \frac{wL}{2}x - w\frac{x^2}{2}$$

If we differentiate M with respect to x:

$$\frac{dM}{dx} = \frac{wL}{2} - w\,x = V \text{ (shear)}$$

Therefore, the rate of change of the moment with respect to x is the shearing force, or *the shear is the slope of the moment diagram.*

If we differentiate V with respect to x.

$$\frac{dV}{dx} = -w \text{ (load)}$$

Therefore, the rate of change of the shear with respect to x is the load, or *the load is the slope of the shear diagram.*

Properties of Shear and Moment Diagrams

The following are some important properties of shear and moment diagrams:

1. The area of the shear diagram to the left or to the

right of the section is equal to the moment at that section,

2. The slope of the moment diagram at a given point is the shear at that point.
3. The slope of the shear diagram at a given point equals the load at that point.
4. The relative maximum moment occurs at the point of zero shears. This is in reference to property number 2, that when the shear (also the slope of the moment diagram) is zero, the tangent drawn to the moment diagram is horizontal.
5. When the shear diagram is increasing, the moment diagram is concave up.
6. When the shear diagram is decreasing, the moment diagram is concave down.

Sign Conventions

The customary sign conventions for shearing force and bending moment are represented by the figures below. A force that tends to bend the beam downward is said to produce a positive bending moment. A force that tends to shear the left portion of the beam upward with respect to the right portion is said to produce a positive shearing force.

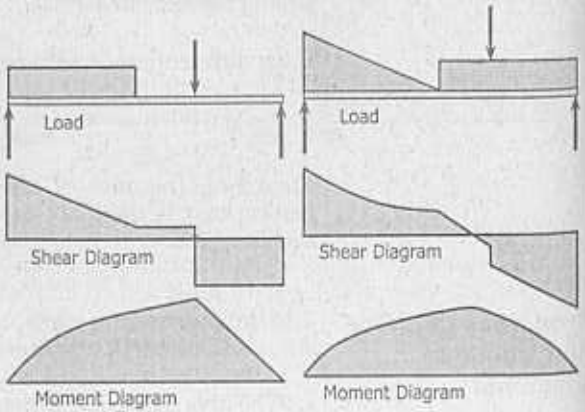
Positive bending

Positive shear

Negative bending

Negative shear

An easier way for determining the sign of the bending moment at any section is that upward forces always cause positive bending moments regardless of whether they act to the left or to the right of the exploratory section.



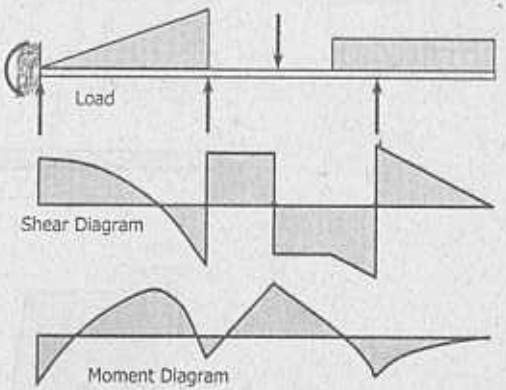


Figure: Shear and moment diagrams of beams

STRESSES IN BEAMS

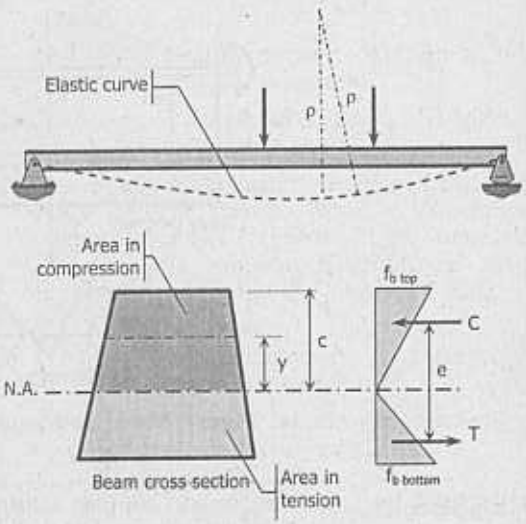
Forces and couples acting on a beam cause bending (flexural stresses) and shearing stresses on any cross section of the beam and deflection perpendicular to the longitudinal axis of the beam. If couples are applied to the ends of the beam and no forces act on it, the bending is said to be pure bending. If forces produce the bending, the bending is called ordinary bending.

Assumptions

In using the following formulas for flexural and shearing stresses, it is assumed that a plane section of the beam normal to its longitudinal axis prior to loading remains plane after the forces and couples have been applied, and that the beam is initially straight and of uniform cross section and that the moduli of elasticity in tension and compression are equal.

Notations

M = bending moment
f_b = bending stress
I = moment of inertia
S = section modulus
f_v = shearing stress
Q = static moment of area



Radius of Curvature

$$\rho = \frac{EI}{M} = \frac{Ec}{f_b}$$

Flexure Formula

The stresses caused by bending moment are known as bending or *flexural stresses*, and the relation between these stresses and the bending moment is expressed by the *flexure formula*.

At any fiber at distance y from the neutral axis, the bending (flexural) stress is:

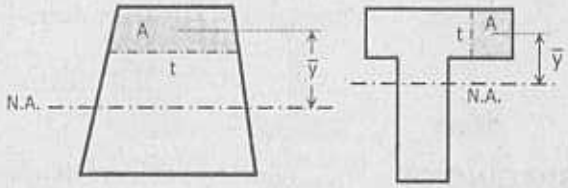
$$f_b = \frac{My}{I}$$

The maximum flexural stress occurs at the outermost fiber whose distance from the neutral axis is denoted as c.

$$f_{b\,max} = \frac{Mc}{I} = \frac{M}{S}$$

$$\text{Section modulus, } S = \frac{I}{c}$$

Shearing Stress Formula



$$f_v = \frac{VQ}{It}$$

$$Q = A\bar{y}$$

Shear Flow

If the shearing stress f_v is multiplied by the width t, we obtain a quantity q, known as the shear flow, which represents the longitudinal force per unit length transmitted across a section at a level y_1 from the neutral axis.

$$q = f_v\, t = \frac{VQ}{I}$$

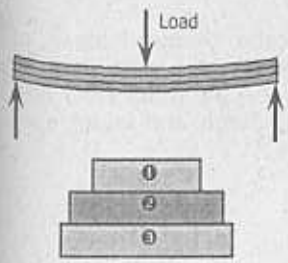
Application of Flexural and Shearing Stresses to Rectangular Section

For rectangular beam of width b and depth d:

$$f_{b\,max} = \frac{6M}{bd^2}$$

$$f_{v\,max} = \frac{3V}{2bd} = \frac{3V}{2A}$$

SUPERIMPOSED BEAMS



If a beam is composed of two or more thin layers placed on each other without any attachment, the separate layers would slide past each other and the total strength of the beam would be the sum of the strengths of the various layers. The strength of this beam is lesser than a solid beam having the same cross-sectional area.

Neglecting friction between any two adjacent layers, the following relationships may be used:

$$M = M_1 + M_2 + M_3$$

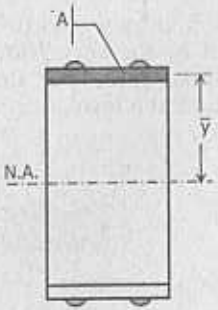
$$I = I_1 + I_2 + I_3$$

$$\text{Curvature, } \frac{M}{EI} = \text{constant}$$

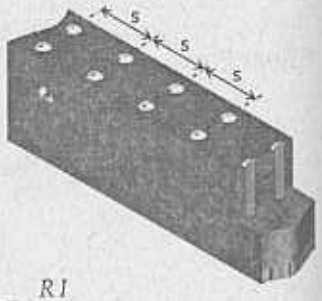
$$\frac{M}{EI} = \frac{M_1}{E_1 I_1} = \frac{M_2}{E_2 I_2} = \frac{M_3}{E_3 I_3}$$

SPACING OF RIVETS OR BOLT IN BUILT-UP BEAMS

When two or more thin layers of beams are fastened together with a bolt or a rivet so that they act as a unit to gain more strength, it is necessary to design the to size or spacing of these bolts or rivets so that it can carry the shearing force acting between each adjacent layers.



Consider the beam shown in the figure.



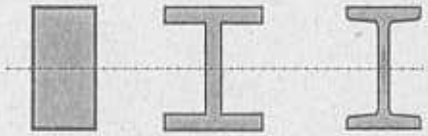
$$s = \frac{RI}{VQ}$$

where R is the total shearing force to be resisted by the bolts and is equal to the allowable shearing stress × area × number of bolts in the group. R may also be taken as the bearing capacity at the section.

ECONOMIC SECTIONS

From the flexure formula $f_b = M\,y/I$, it can be seen that the bending stress at the neutral axis, where $y = 0$, is zero and increases linearly outwards. This means that for a rectangular or circular section a large portion of the cross section near the middle section is under stressed.

For steel beams or composite beams, instead of adopting the rectangular shape, the area may be arranged to give more area on the outer fiber and maintaining the same overall depth, and saving a lot of weight.



(a) Rectangular (b) Wide flange (W Shape) (c) I beam (S Shape)

When using a wide flange or I-beam section for long beams, the compression flanges tend to buckle horizontally sidewise. This buckling is a column effect, which may be prevented by providing lateral support such as a floor system so that the full allowable stresses may be used, otherwise the stress should be reduced. The reduction of stresses for these beams will be discussed in steel design.

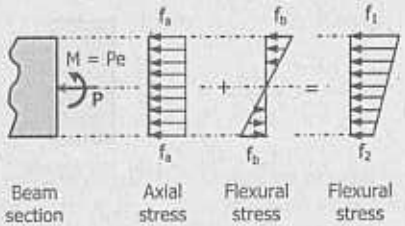
COMBINED STRESSES

In the previous sections, we studied the three basic types of stresses; the axial or normal stress, torsional shearing stress, and flexural stress, in which we assume that only one of these stresses acts on a member. However, these stresses may act simultaneously and there are four possible combinations of these stresses: (1) axial and flexure; (2) axial and torsional; (3) torsional and flexural; and (4) axial, torsional, and flexural.

Combined Axial and Flexure

The simplest of the four combinations is the axial and flexural because it combines only normal stresses, which can be added arithmetically.

A member subject to bending moment M and an axial load P causes a flexural stress of $f_b = My/I$ at any point from its neutral axis, and a normal stress of P/A which is uniformly distributed over its entire area, respectively. The combination of these stresses results if the member is eccentrically loaded as in a column or a prestressed beam. The combined stress developed is



$$f = f_a \pm f_b$$

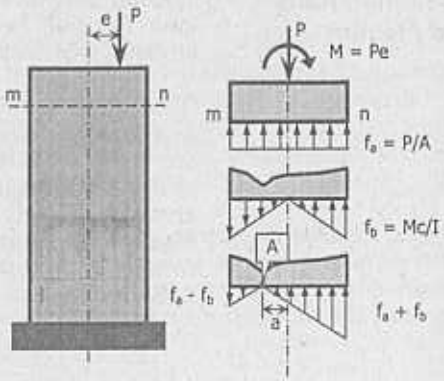
$$f = \begin{matrix}(+)\\(-)\end{matrix}\frac{P}{A} \pm \frac{My}{I}$$

where $M = Pe$, e is the eccentricity of P from the neutral axis.

Note that the axial stress may be pure tensile or compressive, this is the reason for the (+) and (-) signs before P/A because axial stress is uniform over the section while flexural stress vary with position.

Kern of a Section

When a member is eccentrically loaded so that the maximum flexural stress Mc/I is larger than the compressive stress P/A, the resulting stress diagram will be as shown.



The point of zero stress A can be found by computing a at which the direct compressive stress

equals the tensile stress

$$a = \frac{I}{Ae}$$

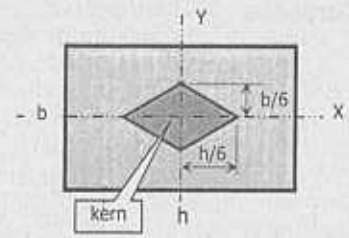
If the maximum flexural stress in tension just equals the direct compressive stress, there would be no tensile stress in any section of the member.

If P is applied at any point with respect the principal axes X and Y, there would be a certain area in which P should act so that there will be no tensile stress in the section. This area is called the kern of the cross-section. The stress at any section whose coordinates are x and y is

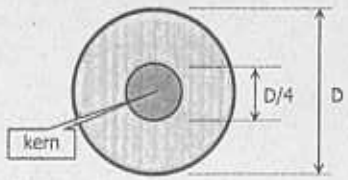
$$f = \begin{matrix}(+)\\(-)\end{matrix}\frac{P}{A} \pm \frac{M_x y}{I} \pm \frac{M_y x}{I}$$

$$f = \begin{matrix}(+)\\(-)\end{matrix}\frac{P}{A} \pm \frac{(Pe_y)y}{I} \pm \frac{(Pe_x)x}{I}$$

For a rectangular section of dimension b and h, the kern region is within a diamond-shaped area as shown.

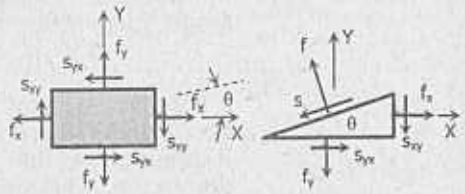


The kern of a circular section of diameter D is a circle whose diameter is $D/4$.



COMBINED AXIAL AND SHEARING STRESS

In general when a body is subject to combined loadings, every element will be subject to combined normal stresses f_x, f_y together with the shearing stress s_{xy} as shown. For normal stresses, tensile stresses are considered positive, compressive stresses negative, and shearing stresses positive if it creates clockwise rotation about the center.



Stresses on an Inclined Plane

If the stresses f_x, f_y, and s_{xy} are known, the normal and shearing stresses on a plane inclined at an angle θ to the x-axis can be determined using the following formulas or by Mohr's circle.

$$f = \frac{f_x + f_y}{2} - \frac{f_x - f_y}{2}\cos 2\theta + s_{xy}\sin 2\theta$$

$$s = \frac{s_x - s_y}{2}\sin 2\theta + s_{xy}\cos 2\theta$$

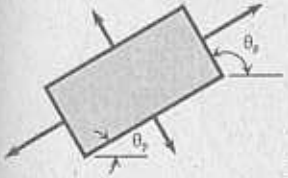
Principal Stresses

There are certain values of the angle θ that will lead to maximum and minimum values of f for a given set of stresses f_x, f_y, and s_{xy}. These maximum and minimum values that f may assume are termed principal stresses and are given by

$$f_{max} = \frac{f_x + f_y}{2} + \sqrt{\left(\frac{f_x - f_y}{2}\right)^2 + (s_{xy})^2}$$

$$f_{min} = \frac{f_x + f_y}{2} - \sqrt{\left(\frac{f_x - f_y}{2}\right)^2 + (s_{xy})^2}$$

Principal Planes



The angle θ_p between the x-axis and the planes on which the principal stresses occur are given by the equation

$$\tan 2\theta_p = \frac{-2s_{xy}}{f_x - f_y}$$

There are always two values of θ_p that will satisfy this equation. The maximum stress occurs on one of these planes, and the minimum stress occurs on the other. The planes defined by the angle θ_p are known as principal planes.

The shearing stress on the principal planes is always zero.

Maximum Shearing Stress

The maximum and minimum values of shearing stress are given by

$$s_{\substack{max\\min}} = \sqrt{\left(\frac{f_x - f_y}{2}\right)^2 + (s_{xy})^2}$$

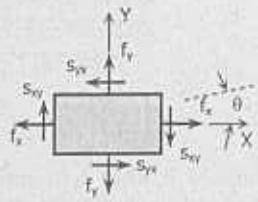
The angle θ between the x-axis and the planes on which the maximum and minimum shearing stresses occur are given by the equation

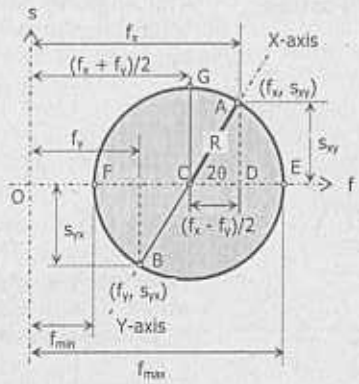
$$\tan 2\theta_s = \frac{f_x - f_y}{2s_{xy}}$$

MOHR'S CIRCLE

A visual interpretation of the formulas in the preceding sections, devised by the German engineer Otto Mohr in 1882, eliminates the necessity of remembering them. In this interpretation, a circle is used; accordingly, the construction is called Mohr's circle.

The following are the rules for applying Mohr's circle to combined stresses, given f_x, f_y, and s_{xy}.





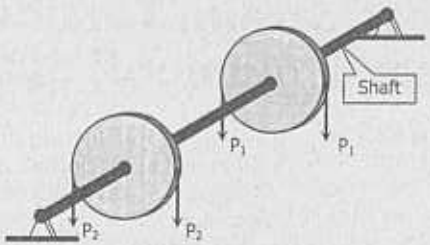
1. On rectangular f-s axes (f along x-axis and s along y-axis), plot points having the coordinates (f_x, s_{xy}) and (f_y, s_{yx}). In plotting these points, assume tension as plus, compression as minus, and shearing stress as plus when its moment about the center of the element is clockwise.

   Note that $s_{yx} = -s_{xy}$.

2. Join the points plotted by a straight line. This line is the diameter of the circle whose center is on the σ-axis.
3. The radius of the circle to any point on its circumference represents the axis directed normal to the plane whose stress components are given by the coordinates of that point.
4. The angle between the radii to selected points on Mohr's circle is twice the angle between the normals to the actual planes represented by these points, or twice the space angularity between the planes so represented. The rotational sense of this angle corresponds to the rotational sense of the actual angle between the normals to the planes. i.e. if N axis is actually at a counterclockwise angle θ from the X axis, then on Mohr's circle the N radius is laid off at a counterclockwise angle 2θ from the X-radius.

Combined Torsional and Flexural Stresses

A shaft loaded as shown is subject to combined torsional and flexural stresses.



The equivalent bending moment M_e and torque T_e due to applied loads is given by

$$T_e = \sqrt{M^2 + T^2}$$

$$M_e = \frac{1}{2}(M + T_e)$$

where M is the actual bending moment and T is the actual torque. Once M_e and T_e is computed, the formulas for flexural and torsional stresses can be used

$$\text{Max. } f_b = \frac{M_e c}{I} = \frac{4M_e}{\pi r^3}$$

$$\text{Max. } s = \frac{T\rho}{J} = \frac{16T_e}{\pi d^3}$$

The formulas given at the right end is for a circular shaft of radius r and diameter d.

Problems - Set 8
Stress, Strain, Torsion, Shear & Moment, etc.

PROBLEM 8 - 1

Determine the outside diameter of hallow steel tube that will carry a tensile load of 500 kN at a stress of 140 MPa. Assume the wall thickness to be one-tenth of the outside diameter.

A. 123 mm
B. 113 mm
C. 103 mm
D. 93 mm

PROBLEM 8 - 2
ME April 1998

A force of 10 Newtons is applied to one end of a 10 inches diameter circular rod. Calculate the stress.

A. 0.20 kPa
B. 0.05 kPa
C. 0.10 kPa
D. 0.15 kPa

PROBLEM 8 - 3

What force is required to punch a 20-mm diameter hole through a 10-mm thick plate. The ultimate strength of the plate material is 450 MPa.

A. 241 kN
B. 283 kN
C. 386 kN
D. 252 kN

PROBLEM 8 - 4

A steel pipe 1.5m in diameter is required to carry an internal pressure of 750 kPa. If the allowable tensile stress of steel is 140 MPa, determine the required thickness of the pipe in mm

A. 4.56
B. 5.12
C. 4.25
D. 4.01

PROBLEM 8 - 5

A spherical pressure vessel 400-mm in diameter has a uniform thickness of 6 mm. The vessel contains gas under a pressure of 8,000 kPa. If the ultimate tensile stress of the material is 420 MPa, what is the factor of safety with respect to tensile failure?

A. 3.15
B. 3.55
C. 2.15
D. 2.55

PROBLEM 8 - 6
CE Nov. 1996

A metal specimen 36-mm in diameter has a length of 360 mm. A force of 300 kN elongates the length by 1.20 mm. What is the modulus of elasticity?

A. 88.419 GPa
B. 92.564 GPa
C. 92.658 GPa
D. 95.635 GPa

PROBLEM 8 - 7

A steel wire 5-m long hanging vertically supports a weight of 1200 N. Determine the required wire diameter if the stress is limited to 140 MPa and the total elongation must not exceed 4 mm. Neglect the weight of the wire and assume E = 200 GPa.

A. 3.09 mm
B. 3.56 mm
C. 3.33 mm
D. 2.89 mm

PROBLEM 8 - 8

During a stress-strain test, the unit deformation at a stress of 35 MPa was observed to be 167×10^{-6} m/m and at a stress of 140 MPa it was 667×10^{-6} m/m. if the proportional limit was 200 MPa, what is the modulus of elasticity. What is the strain corresponding to t stress of 80 MPa?

A. E = 210,000 MPa; $\varepsilon = 381 \times 10^{-4}$ m/m
B. E = 200,000 MPa; $\varepsilon = 318 \times 10^{-6}$ m/m
C. E = 211,000 MPa; $\varepsilon = 318 \times 10^{-4}$ m/m
D. E = 210,000 MPa; $\varepsilon = 381 \times 10^{-6}$ m/m

PROBLEM 8 - 9

An axial load of 100 kN is applied to a flat bar 20 mm thick, tapering in width from 120 mm to 40 mm in a length of 10 m. Assuming E = 200 GPa, determine the total elongation of the bar.

A. 3.43 mm
B. 2.125 mm
C. 4.33 mm
D. 1.985 mm

PROBLEM 8 - 10

Steel bar having a rectangular cross-section 15mm × 20mm and 150m long is suspended vertically from one end. The steel has a unit mass of 7850 kg/m³ and a modulus of elasticity E of 200 GPa. If a loaf of 20 kN is suspended at the other end of the rod, determine the total elongation of the rod.

A. 43.5 mm
B. 54.3 mm
C. 35.4 mm
D. 45.3 mm

PROBLEM 8 - 11

A steel bar 50 mm in diameter and 2 m long is surrounded by a shell of cast iron 5 mm thick. Compute the load that will compress the bar a total of 1 mm in the length of 2 m. Use E_{steel} = 200 GPa and $E_{cast\ iron}$ = 100 GPa.

A. 200 kN
B. 240 kN
C. 280 kN
D. 320 kN

PROBLEM 8 - 12

A 20-mm diameter steel rod, 250 mm long is subjected to a tensile force of 75 kN. If the Poisson's ratio μ is 0.30, determine the lateral strain of the rod. Use E = 200 GPa.

A. $\varepsilon_y = 3.581 \times 10^{-4}$ mm/mm
B. $\varepsilon_y = -3.581 \times 10^{-4}$ mm/mm

C. $\varepsilon_y = -2.467 \times 10^{-4}$ mm/mm
D. $\varepsilon_y = 2.467 \times 10^{-4}$ mm/mm

PROBLEM 8 - 13

A solid aluminum shaft of 100-mm diameter fits concentrically in a hallow steel tube. Determine the minimum internal diameter of the steel tube so that no contact pressure exists when the aluminum shaft carries an axial compressive load of 600 kN. Assume Poisson's ratio $\mu = 1/3$ and the modulus of elasticity of aluminum E be 70 GPa.

A. 100.0364 mm
B. 100.0312 mm
C. 100.0303 mm
D. 100.0414 mm

PROBLEM 8 - 14
CE May 1996

The maximum allowable torque, in kN-m, for a 50-mm diameter steel shaft when the allowable shearing stress is 81.5 MPa is:

A. 3.0
B. 1.0
C. 4.0
D. 2.0

PROBLEM 8 - 15
CE May 1997

The rotation or twist in degrees of a shaft, 800 mm long subjected to a torque of 80 N-m, 20 mm in diameter and shear modulus G of 80,000 MPa is:

A. 3.03
B. 4.04
C. 2.92
D. 1.81

PROBLEM 8 - 16

Compute the value the shear modulus G of steel whose modulus of elasticity E is 200 GPa and Poisson's ratio μ is 0.30.

A. 72,456 MPa
B. 76,923 MPa
C. 79,698 MPa
D. 82,400 MPa

PROBLEM 8 - 17

Determine the length of the shortest 2-mm diameter bronze wire, which can be twisted through two complete turns without exceeding a stress of 70 MPa. Use G = 35 GPa.

A. 6.28 m
B. 5.23 m
C. 6.89 m
D. 8.56 m

PROBLEM 8 - 18

A hallow steel shaft 2540 mm long must transmit torque of 34 kN-m. The total angle of twist must not exceed 3 degrees. The maximum shearing stress must not exceed 110 MPa. Find the inside diameter and the outside diameter of the shaft that meets these conditions.
Use G = 83 GPa

A. D = 129 mm; d = 92 mm
B. D = 125 mm; d = 65 mm
C. D = 132 mm; d = 100 mm
D. D = 112 mm; d = 85 mm

PROBLEM 8 - 19

Determine the maximum shearing stress in a helical steel spring composed of 20 turns of 20-mm-diameter wire on a mean radius of 80 mm when the spring is supporting a load of 2 kN.

A. 110.6 MPa
B. 101.1 MPa
C. 120.6 MPa
D. 136.5 MPa

PROBLEM 8 - 20

A load P is supported by two springs arranged in series. The upper spring has 20 turns of 29-mm-diameter wire on a mean diameter of 150 mm. The lower spring consist of 15 turns of 10-mm-diameter wire on a mean diameter of 130 mm. Determine the value of P that will cause a total deflection of 80 mm. Assume G = 80 GPa for both springs.

A. 223.3 N
B. 228.8 N
C. 214.8 N
D. 278.4 N

PROBLEM 8 - 21

A 10-meter long simply supported beam carries a uniform load of 8 kN/m for 6meters from the left support and a concentrated load of 15 kN 2 meters from the right support. Determine the maximum shear and moment.

A. V_{max} = 33.2 kN; M_{max} = 85.92 kN-m
B. V_{max} = 31.3 kN; M_{max} = 81.74 kN-m
C. V_{max} = 36.6 kN; M_{max} = 83.72 kN-m
D. V_{max} = 41.8 KN; M_{max} = 92.23 kN-m

PROBLEM 8 - 22
ECE Nov. 1996

A simple beam, 10 m long carries a concentrated load of 500 kN at the midspan. What is the maximum moment of the beam?

A. 1250 kN-m
B. 1050 kN-m
C. 1520 kN-m
D. 1510 kN-m

PROBLEM 8 - 23
CE May 1997

A small square 5 cm by 5 cm is cut out of one corner of a rectangular cardboard 20 cm wide by 30 cm long. How far, in cm from the uncut longer side, is the centroid of the remaining area?

A. 9.56
B. 9.35
C. 9.48
D. 9.67

PROBLEM 8 - 24
ECE April 1998

What is the inertia of a bowling ball (mass = 0.5 kg) of radius 15 cm rotating at an angular speed of 10 rpm for 6 seconds?

A. 0.0045 kg-m^2
B. 0.001 kg-m^2
C. 0.005 kg-m^2
D. 0.002 kg-m^2

PROBLEM 8 - 25
ECE Nov. 1997

What is moment of inertia of a cylinder of radius 5 m and mass of 5 kg?

A. 62.5 kg - m²
B. 80 kg - m²
C. 72.5 kg - m²
D. 120 kg - m²

ANSWER SHEET

1.	A	B	C	D	E	11.	A	B	C	D	E	21.	A	B	C	D	E
2.	A	B	C	D	E	12.	A	B	C	D	E	22.	A	B	C	D	E
3.	A	B	C	D	E	13.	A	B	C	D	E	23.	A	B	C	D	E
4.	A	B	C	D	E	14.	A	B	C	D	E	24.	A	B	C	D	E
5.	A	B	C	D	E	15.	A	B	C	D	E	25.	A	B	C	D	E
6.	A	B	C	D	E	16.	A	B	C	D	E	26.	A	B	C	D	E
7.	A	B	C	D	E	17.	A	B	C	D	E	27.	A	B	C	D	E
8.	A	B	C	D	E	18.	A	B	C	D	E	28.	A	B	C	D	E
9.	A	B	C	D	E	19.	A	B	C	D	E	29.	A	B	C	D	E
10.	A	B	C	D	E	20.	A	B	C	D	E	30.	A	B	C	D	E

Solutions to Set 8 Stress, Strain, Torsion, Shear & Moment, etc.

SOLUTION 8 - 1
Ans: B

$$\sigma = \frac{P}{A}$$

$$A = A_{outside} - A_{inside}$$

$$A = \frac{\pi}{4}[(D^2 - (4D/5)^2] = 0.2827D^2$$

$$140 = \frac{500 \times 10^3}{0.2827D^2}; D = \mathbf{113\ mm}$$

SOLUTION 8 - 2
Ans: A

$$\text{Stress} = \sigma = \frac{P}{A} = \frac{P}{(\pi/4)D^2}$$

$$D = 10\text{ in} \times (2.54\text{ cm}/1\text{ in}) \times (1\text{ m}/100\text{ cm})$$

$$D = 0.254\text{ m}$$

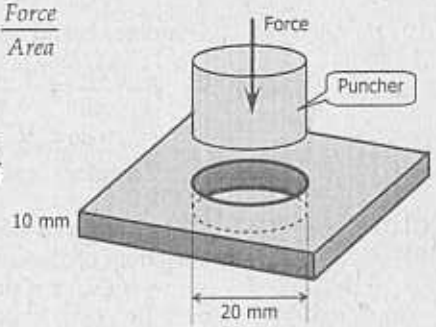
$$\sigma = \frac{10}{(\pi/4)(0.254)^2} = 197\text{ Pa} = \mathbf{0.197\ kPa}$$

SOLUTION 8 - 3
Ans: B

$$\text{Shearing stress} = \frac{Force}{Area}$$

$$450 = \frac{\text{Force}}{\pi(20)(10)}$$

Force = 282743 N
Force = **282.74 kN**



SOLUTION 8 - 4
Ans: D

$$\sigma_t = \frac{pD}{2t}$$

$$140 = \frac{\frac{750}{1000}(1500)}{2t}; t = \mathbf{4.01\ mm}$$

SOLUTION 8 - 5
Ans: A

$$\text{Factor of safety, F.S.} = \frac{\text{Ultimate stress}}{\text{Actual stress}}$$

For spherical shell:

$$\sigma_l = \frac{pD}{4t}$$

$$\sigma_l = \frac{8{,}000 \times \frac{1}{1000}(400)}{4(6)} = 133.33 \text{ MPa}$$

$$\sigma_{ult} = 420 \text{ MPa}$$

$$\text{F.S.} = \frac{420}{133.33} = \mathbf{3.15}$$

SOLUTION 8 - 6
Ans: A

$$\delta = \frac{PL}{AE}$$

$$1.20 = \frac{(300 \times 10^3)(360)}{\frac{\pi}{4}(36)^2 E}$$

$E = 88{,}419.4$ N/mm² or **88.419 GPa**

SOLUTION 8 - 7
Ans: C

Diameter based on allowable stress:

$$\sigma = \frac{P}{A};\ 140 = \frac{1200}{\frac{\pi}{4}D^2}$$

$D = 3.3$ mm

Diameter based on allowable elongation:

$$\delta = \frac{PL}{AE};\ 4 = \frac{1200(5 \times 10^3)}{\frac{\pi}{4}D^2(200 \times 10^3)}$$

$D = 3.09$ mm

Therefore, use $D =$ **3.3 mm**

SOLUTION 8 - 8
Ans: D

Since the given stresses are below the proportional limit, the stress is directly proportional to strain, then

$\sigma = E\,\varepsilon$; E = slope of the stress-strain diagram

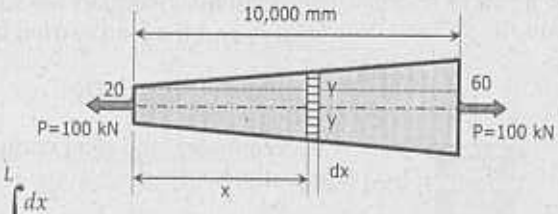
$$E = \frac{\Delta\sigma}{\Delta\varepsilon} = \frac{140 - 35}{667 \times 10^{-6} - 167 \times 10^{-6}};\ E = 210{,}000 \text{ MPa}$$

and $\sigma = E\,\varepsilon$

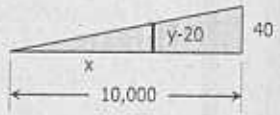
$80 = 210{,}000 \times \varepsilon$

$\varepsilon =$ **381×10^{-6} m/m**

SOLUTION 8 - 9
Ans: A



$$\delta = \frac{P}{E}\int_0^L \frac{dx}{A}$$



$A = (2y)(20) = 40y$

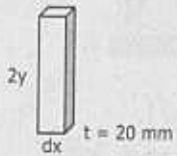
$$\frac{y - 20}{x} = \frac{40}{10000}$$

$y - 20 = 0.004x$

$y = 20 + 0.004x$

$A = 40(20 + 0.004x)$

$A = 800 + 0.16x$



$$\delta = \frac{100{,}000}{200{,}000}\int_0^{10000} \frac{dx}{800 + 0.16x}$$

$$\delta = 0.5\,\frac{1}{0.16}\Big[\ln(800 + 0.16x)\Big]_0^{10000}$$

$\delta = 3.125\,\{\ln[800 + 0.16(10{,}000)] - \ln(800)\}$

$\delta =$ **3.433 mm**

SOLUTION 8 - 10
Ans: B

The total elongation of the rod is the elongation due to its own weight δ_1 plus the elongation due to the suspended load δ_2.

$$\delta = \delta_1 + \delta_2 = \frac{\rho g L^2}{2E} + \frac{PL}{AE}$$

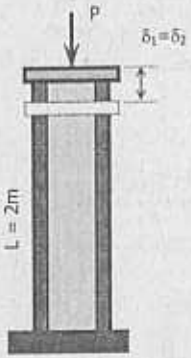
Note: the product ρg is the unit weight in N/m³ and $\rho g L^2$ with L in m is in N/m

$$\delta = \frac{7850(9.81)(150)^2 \times \frac{1}{1000}}{2(200{,}000)} + \frac{20{,}000 \times 150{,}000}{(15 \times 20)(200{,}000)}$$

$\delta = 4.33 + 50 =$ **54.33 mm**

SOLUTION 8 - 11
Ans: B

The total load P equals the sum of the load carried by the steel P_1 and the load carried by the cast iron shell P_2.



$$P = P_1 + P_2 \quad \rightarrow (1)$$

By symmetry, the steel core and cast iron shell deforms equally, hence

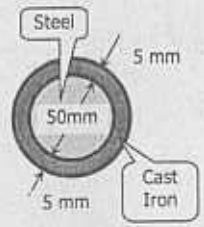
$$\delta_1 = \delta_2 = 1 \text{ mm}$$

$$\frac{P_1 L_1}{A_1 E_1} = \frac{P_2 L_2}{A_2 E_2} = 1$$

$$A_1 = \tfrac{\pi}{4}(50)^2 = 1963.5 \text{ mm}^2$$

$$A_2 = \tfrac{\pi}{4}(60)^2 - \tfrac{\pi}{4}(50)^2 = 864 \text{ mm}^2$$

$$P_1 = \frac{1(1963.5)(200\times10^3)}{2\times10^3} = 196{,}350 \text{ N}$$



$$P_2 = \frac{1(864)(100\times10^3)}{2\times10^3} = 43{,}200 \text{ N}$$

Substitute P_1 and P_2 to Eq. (1):

$$P = P_1 + P_2 = 239{,}550 \text{ N} = 239.55 \text{ kN} \approx \mathbf{240\ kN}$$

SOLUTION 8 - 12
Ans: B

Lateral strain, $\varepsilon_y = -\mu\,\varepsilon_x = -\mu\,\dfrac{\sigma_x}{E}$

$$\sigma_x = \frac{P}{A} = \frac{75\times10^3}{\frac{\pi}{4}(20)^2} = 238.73 \text{ MPa (tension)}$$

$$\varepsilon_y = -(0.30)\,\frac{238.73}{200\times10^3} = \mathbf{-3.581\times10^{-4}\ mm/mm}$$

SOLUTION 8 - 13
Ans: A

So that there can be no contact pressure between the shaft and the tube, the internal diameter of the tube must be sufficient to allow the lateral deformation of the shaft caused by the compressive force.

The axial compressive force on the shaft is:

$$\sigma_x = -\frac{600\times10^3}{\frac{\pi}{4}(100)^2} = -76.394 \text{ MPa (compression)}$$

The transverse strain is (uniaxial stress)

$$\varepsilon_y = -\mu\,\varepsilon_x = -\mu\,\frac{\sigma_x}{E} = -(1/3)\,\frac{-76.394}{70\times10^3}$$

$$\varepsilon_y = 0.000364 \text{ mm/mm}$$

The sidewise deformation is:

$$\delta_y = \varepsilon_y L = 0.000364 \times 100 = 0.0364 \text{ mm}$$

Therefore, the required tube diameter is:

100 mm + 0.0364 mm = **100.0364 mm**

SOLUTION 8 - 14
Ans: D

$$\tau = \frac{16T}{\pi D^3} = \frac{16T}{\pi(50)^3} = 81.5$$

T = 2,000,311 N-mm = **2 kN-m**

SOLUTION 8 - 15
Ans: C

The angle of twist in radians is given by:

$$\theta = \frac{TL}{JG}, \text{ where } J = \frac{\pi D^4}{32}$$

$$\theta = \frac{(80\times10^3)(800)}{\frac{\pi}{32}(20)^4(80000)} = 0.0509 \text{ radians}$$

$$\theta = 0.0509 \times (180°/\pi) = \mathbf{2.918°}$$

SOLUTION 8 - 16
Ans: B

From the formula

$$G = \frac{E}{2(1+\mu)}$$

$$G = \frac{200\times10^3}{2(1+0.30)} = \mathbf{76{,}923\ MPa}$$

SOLUTION 8 - 17
Ans: A

From the formulas

$$\theta = \frac{TL}{JG}$$

$$\tau = \frac{Tr}{J};\ T = \frac{J}{r}\tau$$

$$\theta = \frac{\frac{J}{r}\tau L}{JG} = \frac{\tau L}{Gr}$$

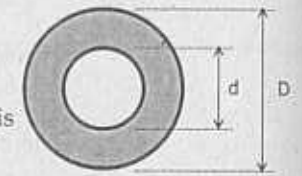
r = radius = 1 mm
θ = 2 turns = $(2\pi) = 4\pi$
L = ?

$$4\pi = \frac{70L}{35000(1)}\ ;\ L = 6283 \text{ mm} = \mathbf{6.283 \text{ m}}$$

SOLUTION 8 - 18
Ans: A

The angle of twist is given by

$$\theta = \frac{TL}{JG}$$



The maximum shearing stress is

$$\tau = \frac{Tr}{J} = \frac{TD}{2J}$$

Based on angle of twist:

$$\theta = \frac{(34\times10^6)(2540)}{J(83\times10^3)} = 3° \times (\pi/180°)$$

$$J = 19871741 \text{ mm}^4$$

Based on the allowable shearing stress

$$\tau = \frac{(34\times10^6)D}{2(19871741)} = 110$$

$$D = 128.6 \text{ mm}$$

For hallow circular shaft, $J = \frac{\pi}{32}(D^4 - d^4)$, then

$$J = \frac{\pi}{32}\left[(128.6)^4 - d^4\right] = 19871741$$

$$d = \mathbf{91.82 \text{ mm}}$$

SOLUTION 8 - 19
Ans: C

Wire diameter, d = 20 mm (heavy spring)
Mean radius of the helix, R = 80 mm
Load P = 2 kN = 2000 N
Using A.M. Wahl's formula:

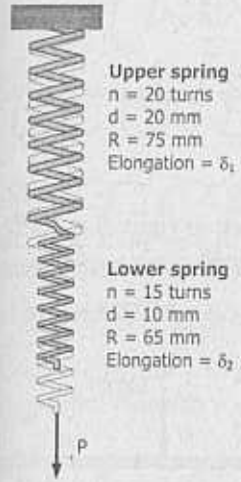
$$\tau_{max} = \frac{16PR}{\pi d^3}\left(\frac{4m-1}{4m-4} + \frac{0.615}{m}\right)$$

$$m = \frac{2R}{d} = 2(80)/20 = 8$$

$$\tau_{max} = \frac{16(2000)(80)}{\pi(20)^3}\left(\frac{4\times8-1}{4\times8-4} + \frac{0.615}{8}\right)$$

$$\tau_{max} = \mathbf{120.6 \text{ MPa}}$$

SOLUTION 8 - 20
Ans: A



For the springs in series, the total elongation is the sum of their individual elongation

$$\delta = \delta_1 + \delta_2$$

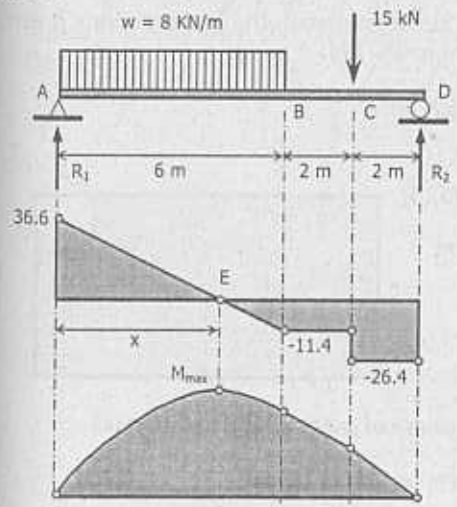
$$\delta = \frac{64PR^3n}{Gd^4}$$

$$\delta = \frac{64PR_1^3n_1}{Gd_1^4} + \frac{64PR_2^3n_2}{Gd_2^4}$$

$$80 = \frac{64P}{83000}\left[\frac{(75)^3(20)}{(20)^4} + \frac{(65)^3(15)}{(10)^4}\right]$$

$$P = \mathbf{223.3 \text{ N}}$$

SOLUTION 8 - 21
Ans: C



Solving for the reactions:

$[\Sigma M_A = 0]$
$(8)(6)(6/2) + 15(8) - 10R_2 = 0$
$R_2 = 26.4$ kN

$[\Sigma M_D = 0]$
$10R_1 - 15(2) + (8)(6)(4 + 6/2) = 0$
$R_1 = 36.6$ kN

Shear:
$V_A = R_1 = 36.6$ kN
$V_B = 36.6 - (8)(6) = -11.4$ kN
$V_C = -11.4$ kN $- 15 = -26.4$ kN
$V_{max} = \underline{36.6 \text{ kN}}$

Moment: (The maximum moment is at the point of zero shear, at E)

Solve for x by ratio and proportion:

$$\frac{x}{36.6} = \frac{6}{36.6+11.4}$$

$x = 4.575$

Another way to solve for x is to use the shear at E which is zero

$V_E = 36.6 - (8)x = 0$

$x = 4.575$

M_{max} = Area of shear diag. to the left or right of point E

$M_{max} = ½(36.6)(4.575) = 83.7225$ kN-m

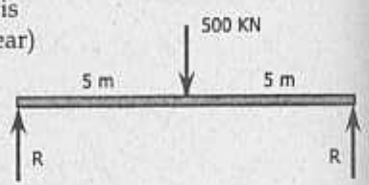
or

$M_{max} = 36.6(4.575) - 8(4.575)(4.575/2) =$ **83.7225 kN-m**

SOLUTION 8 - 22
Ans: A

The maximum moment is at the midspan (zero shear)

$M_{max} = R(5)$



Since the load is at the midspan, the end reactions R are equal and is equal to 250 kN.

$M_{max} = 250(5)$

$M_{max} =$ **1250 kN-m**

SOLUTION 8 - 23
Ans: D

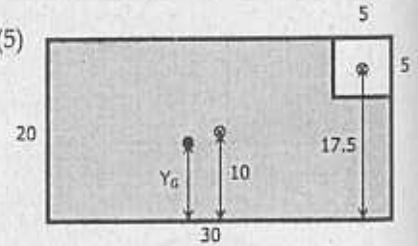
$A\,Y_G = \Sigma a\,y$

$A = 20(30) - (5)(5)$

$A = 575$ cm²

$575\,Y_G = 20(30)(10) - (5)(5)(17.5)$

$Y_G =$ **9.674 cm**



SOLUTION 8 - 24
Ans: A

The inertia of a sphere of mass M and radius r is:

$I = (2/5)Mr^2$

$I = (2/5)(0.5)(0.15)^2 =$ **0.0045 kg-m²**

SOLUTION 8 - 25
Ans: A

The mass moment of inertia of a cylinder of radius r is:

$I_Z = ½\,m\,r^2$

$I_Z = ½\,(5)(5)^2 =$ **62.5 kg - m²**

Part 6 FLUID MECHANICS

PROPERTIES OF FLUID

Unit Weight or Specific Weight, γ

The weight per unit volume of a fluid.

$$\gamma = \frac{Weight\ of\ Fluid}{Volume}$$

For water, $\gamma = 9810$ N/m³ = 62.4 lb/ft³

Mass Density or Density, ρ

The mass of fluid per unit of volume.

$$\rho = \frac{Mass\ of\ Fluid}{Volume}$$

for water at 4°C, $\rho = 1000$ kg/m³

Density of Gases

$$\rho = \frac{p}{RT}$$

where:

p = absolute pressure of gas in kPa

R = gas constant in Joule/kg-°K

For air, $R = 287$ J/kg-°K

T = absolute temperature in degree Kelvin

°K = °C + 273

Specific Volume, V_s

$$V_s = \frac{1}{\rho}$$

Solve for x by ratio and proportion:

$$\frac{x}{36.6} = \frac{6}{36.6+11.4}$$

$x = 4.575$

Another way to solve for x is to use the shear at E which is zero

$V_E = 36.6 - (8)x = 0$

$x = 4.575$

M_{max} = Area of shear diag. to the left or right of point E

$M_{max} = ½(36.6)(4.575) = 83.7225$ kN-m

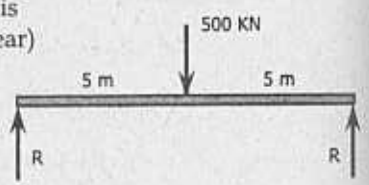
or

$M_{max} = 36.6(4.575) - 8(4.575)(4.575/2) =$ **83.7225 kN-m**

SOLUTION 8 - 22
Ans: A

The maximum moment is at the midspan (zero shear)

$M_{max} = R(5)$



Since the load is at the midspan, the end reactions R are equal and is equal to 250 kN.

$M_{max} = 250(5)$

$M_{max} =$ **1250 kN-m**

SOLUTION 8 - 23
Ans: D

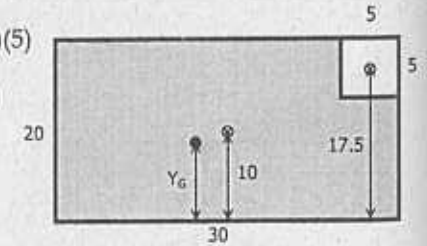
$A\,Y_G = \Sigma a\,y$

$A = 20(30) - (5)(5)$

$A = 575$ cm²

$575\,Y_G = 20(30)(10) - (5)(5)(17.5)$

$Y_G =$ **9.674 cm**



SOLUTION 8 - 24
Ans: A

The inertia of a sphere of mass M and radius r is:

$I = (2/5)Mr^2$

$I = (2/5)(0.5)(0.15)^2 =$ **0.0045 kg-m²**

SOLUTION 8 - 25
Ans: A

The mass moment of inertia of a cylinder of radius r is:

$I_Z = ½\,m\,r^2$

$I_Z = ½\,(5)(5)^2 =$ **62.5 kg - m²**

Part 6 FLUID MECHANICS

PROPERTIES OF FLUID

Unit Weight or Specific Weight, γ

The weight per unit volume of a fluid.

$$\gamma = \frac{Weight\ of\ Fluid}{Volume}$$

For water, $\gamma = 9810$ N/m³ = 62.4 lb/ft³

Mass Density or Density, ρ

The mass of fluid per unit of volume.

$$\rho = \frac{Mass\ of\ Fluid}{Volume}$$

for water at 4°C, $\rho = 1000$ kg/m³

Density of Gases

$$\rho = \frac{p}{RT}$$

where:

p = absolute pressure of gas in kPa

R = gas constant in Joule/kg-°K

For air, $R = 287$ J/kg-°K

T = absolute temperature in degree Kelvin

°K = °C + 273

Specific Volume, V_s

$$V_s = \frac{1}{\rho}$$

Solve for x by ratio and proportion:

$$\frac{x}{36.6} = \frac{6}{36.6 + 11.4}$$

$x = 4.575$

Another way to solve for x is to use the shear at E which is zero

$V_E = 36.6 - (8)x = 0$

$x = 4.575$

M_{max} = Area of shear diag. to the left or right of point E

$M_{max} = ½(36.6)(4.575) = 83.7225$ kN-m

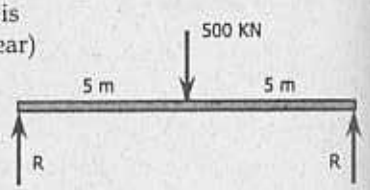
or

$M_{max} = 36.6(4.575) - 8(4.575)(4.575/2) =$ **83.7225 kN-m**

SOLUTION 8 - 22
Ans: A

The maximum moment is at the midspan (zero shear)

$M_{max} = R(5)$



Since the load is at the midspan, the end reactions R are equal and is equal to 250 kN.

$M_{max} = 250(5)$

$M_{max} =$ **1250 kN-m**

SOLUTION 8 - 23
Ans: D

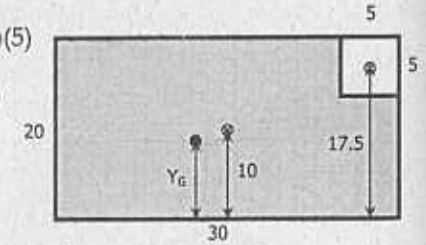
$A\,Y_G = \Sigma a\,y$

$A = 20(30) - (5)(5)$

$A = 575$ cm²

$575\,Y_G = 20(30)(10) - (5)(5)(17.5)$

$Y_G =$ **9.674 cm**



SOLUTION 8 - 24
Ans: A

The inertia of a sphere of mass M and radius r is:

$I = (2/5)Mr^2$

$I = (2/5)(0.5)(0.15)^2 =$ **0.0045 kg-m²**

SOLUTION 8 - 25
Ans: A

The mass moment of inertia of a cylinder of radius r is:

$I_Z = ½\, m\, r^2$

$I_Z = ½\,(5)(5)^2 =$ **62.5 kg - m²**

Part 6
FLUID MECHANICS

PROPERTIES OF FLUID

Unit Weight or Specific Weight, γ

The weight per unit volume of a fluid.

$$\gamma = \frac{Weight\ of\ Fluid}{Volume}$$

For water, $\gamma = 9810$ N/m³ = 62.4 lb/ft³

Mass Density or Density, ρ

The mass of fluid per unit of volume.

$$\rho = \frac{Mass\ of\ Fluid}{Volume}$$

for water at 4°C, $\rho = 1000$ kg/m³

Density of Gases

$$\rho = \frac{p}{RT}$$

where:

p = absolute pressure of gas in kPa

R = gas constant in Joule/kg-°K

For air, $R = 287$ J/kg-°K

T = absolute temperature in degree Kelvin

°K = °C + 273

Specific Volume, V_s

$$V_s = \frac{1}{\rho}$$

Specific Gravity, *s*

$$s = \frac{\gamma_{fluid}}{\gamma_{water}} = \frac{\rho_{fluid}}{\rho_{water}}$$

Viscosity

The property of a fluid which determines the amount of its resistance to shearing forces. A perfect fluid would have no viscosity.

Dynamic or Absolute Viscosity, μ (mu)

$$\mu = \frac{\sigma}{dV/dy} \text{ (Pascal-second or poise)}$$

Note: 1 poise = 0.01 Pa/s

Kinematic Viscosity, ν (nu)

$$\nu = \frac{\mu}{\rho} \text{ (m}^2\text{/s or stoke)}$$

Note: 1 stoke = 1 cm²/s = 0.0001 m²/s

Surface Tension σ (sigma)

The surface tension of a fluid is the work that must be done to bring enough molecules from inside the liquid to the surface to form a new unit area of that surface in ft-lb/ft² or N-m/m².

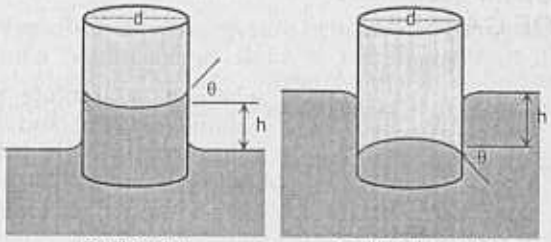
Pressure inside a droplet of liquid

$$p = \frac{4\sigma}{d}$$

where:
σ = surface tension in N/m
d = diameter of the droplet in m
p = gage pressure in Pascal

Capillarity

The rise or fall or a fluid in a capillary tube which is caused by surface tension and depends on the relative magnitudes of the cohesion of the liquid and the adhesion of the liquid to the walls of the containing vessel. Liquids rise in tubes they wet (adhesion > cohesion) and fall in tubes they do not wet (cohesion > adhesion). Capillary is important when using tubes smaller than about 3/8 inch (9.5 mm) in diameter.



Capillary rise Capillary depression

$$h = \frac{4\sigma\cos\theta}{\gamma d}$$

Use θ = 140° for mercury on clean glass

For complete wetting, as with water on clean glass, the angle θ is 0°. Hence the formula becomes

$$h = \frac{4\sigma}{\gamma d}$$

Where:
h = capillary rise or depression
γ = unit weight
d = diameter of the tube
σ = surface tension

Bulk Modulus of Elasticity, E

The bulk modulus of elasticity of the fluid expresses the compressibility of the fluid. It is the ratio of the change in unit pressure to the corresponding volume change per unit of volume.

$$E = \frac{dp'}{-dv/v} \text{ (lb/in}^2\text{ or Pa)}$$

Where:
dp' = change in pressure
dv = change in volume
v = volume

COMPRESSION OF GASSES

For a perfect gas:

$$pv^n = p_1 v_1^n = \text{constant}$$

where p is absolute pressure, v is the specific volume ($v = 1/\rho$), and n may have any nonnegative value from zero to infinity, depending upon the process to which the gas is subjected. If the process is at constant temperature (*isothermal*), $n = 1$.

$$pv = p_1 v_1$$

If there is no heat transfer to and from the gas, the process is known as *adiabatic*.

$$p_1 v_1^k = p_2 v_2^k$$

A frictionless adiabatic process is called an *isentropic* process and n is denoted by k, where $k = c_p/c_v$, the ratio of specific heat at constant pressure to that at constant volume.

Boyle's Law (perfect gas)

If the temperature of a given mass of gas remains constant, the absolute pressure of the gas varies inversely with the volume.

$$p = \frac{k}{V} \text{ or } pV = k$$

$$p_1 V_1 = p_2 V_2$$

Charle's or Guy-Lussac's Law (perfect gas)

If a given mass of gas can expand or contract with the pressure remaining constant, the volume V of the gas varies directly as the absolute temperature T, i.e. V/T is constant.

Pressure Disturbances

Pressure disturbances imposed on a fluid move in waves. The velocity or celerity is expressed as:

$$c = \sqrt{\frac{E_B}{\rho}} \text{ (m/s or ft/s)}$$

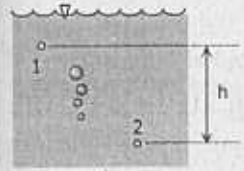
Where:

c = celerity or velocity of pressure wave in m/s or ft/s

E_B = bulk modulus of elasticity of the fluid in Pa or lb/ft²

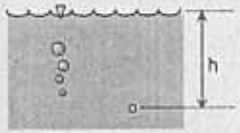
UNIT PRESSURE

Variations in Pressure



The difference in pressure between any two points in a homogeneous fluid at rest is equal to the product of the unit weight of the fluid and the vertical distance between the points.

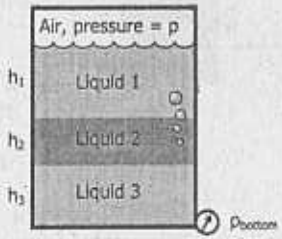
$$p_2 - p_1 = \gamma h$$



The pressure at any point below the free surface of a liquid equals the product of the unit weight of the liquid and the depth of the point.

$$p = \gamma h$$

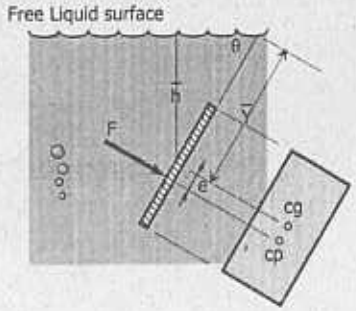
Pressure below layers of different liquids



$$p_{bottom} = \sum \gamma h + p = \gamma_1 h_1 + \gamma_2 h_2 + \gamma_3 h_3 + p$$

TOTAL HYDROSTATIC PRESSURE

Total Pressure on Plane Surface

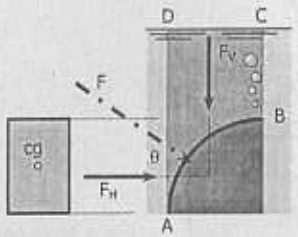


$$F = p_{cg} \times A \quad \text{or} \quad F = \gamma \bar{h} A$$

$$e = \frac{I_g}{A\bar{Y}} \qquad \bar{Y} = \frac{\bar{h}}{\sin\theta}$$

where
p_{cg} = pressure at the centroid of the plane
I_g = centroidal moment of inertia of the plane
A = area of the plane surface
θ = angle that the plane makes with the horizontal

Total Pressure on Curved Surface



$$F_H = p_{cg} A$$

$$F_V = \gamma V_{ABCD}$$

$$F = \sqrt{F_H^2 + F_V^2}$$

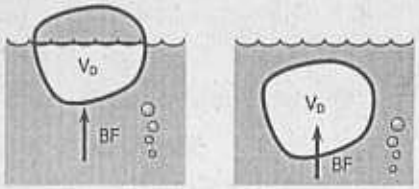
$$\tan\theta = F_V / F_H$$

where F_H = total force acting on the vertical projection of the curved surface.
F_V = weight of imaginary or real fluid directly above the curved surface

Note: For cylindrical and spherical surfaces, the total force F always passes to the center of the circle defined by its surface.

BUOYANCY

Archimedes' Principle – Any body immersed in a fluid is acted upon by an unbalanced upward force called the buoyant force, which is equal to the weight of the fluid displaced.



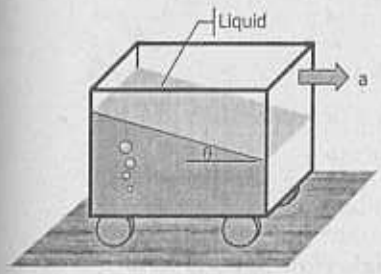
$BF = \gamma_f V_D$

For homogeneous body floating on a homogeneous liquid, the volume displaced is:

$$V_D = \frac{\gamma_{body}}{\gamma_{liquid}} V_{body} = \frac{S_{body}}{S_{liquid}} V_{body}$$

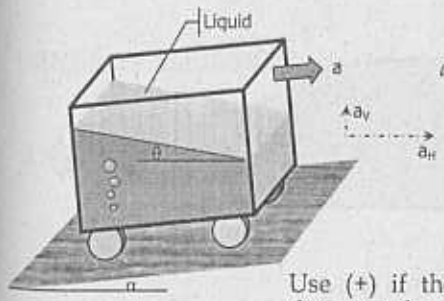
RELATIVE EQUILIBRIUM OF LIQUIDS

Horizontal Motion



$$\tan\theta = \frac{a}{g}$$

Inclined Motion

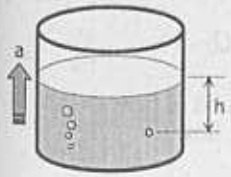


$$\tan\theta = \frac{a_h}{g \pm a_v}$$

$a_H = a\cos\alpha$; $a_V = a\sin\alpha$

Use (+) if the acceleration is upward and (-) if downwards.

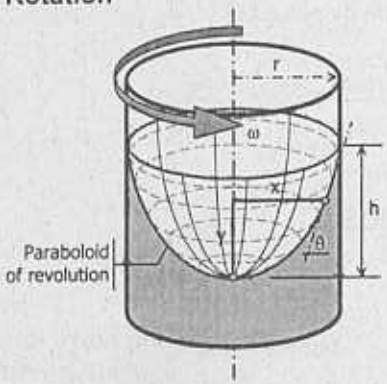
Vertical Motion



$$p = \gamma h\left(1 \pm \frac{a}{g}\right)$$

Use (+) for upward motion and (-) for downward motion.

Rotation



$$y = \frac{\omega^2 x^2}{2g}$$

$$\tan\theta = \frac{dy}{dx} = \frac{\omega^2 x}{g}$$

Volume of Paraboloid

$$V = \tfrac{1}{2}\pi r^2 h$$

FLUID FLOW AND PIPES

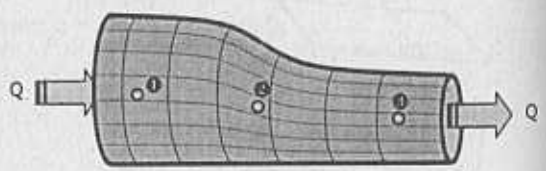
Flow Rate

Volume Flow Rate, $Q = Av$

Mass Flow Rate, $M = \rho Q$

Weight Flow Rate, $W = \gamma Q$

Continuity Equation



Incompressible fluid

$$Q_1 = Q_2 = Q_3 \ldots$$

$$A_1 v_1 = A_2 v_2 = A_3 v_3 \ldots$$

Compressible fluid

$$\rho_1 Q_1 = \rho_2 Q_2 \ldots$$

where A = cross-sectional area of flow
v = mean velocity of flow

Reynold's Number (for pipes)

Reynolds Number R is the ratio of inertia forces to viscous forces.

$$R = \frac{vD\rho}{\mu} = \frac{vD}{\nu}$$

where v = mean velocity of flow, m/s
D = pipe diameter, m
μ = (mu) dynamic viscosity (Pa-s)
ν = (nu) kinematic viscosity (m^2/s) = μ/ρ
ρ = density, kg/m^3

For non-circular pipes, use $D = 4R$, where R is the hydraulic radius, $R = A/P$

For R < 2000, the flow is laminar.

Laminar flow in circular pipes can be maintained up to values of R as high as 50,000. However, in such cases this type of flow is inherently unstable, and the least disturbance will transform it instantly into turbulent flow. On the other hand, it is practically impossible for turbulent flow in a straight pipe to persist at values of R much below 2000, because any turbulence that is set up will be damped out by viscous friction.

ENERGY EQUATION

Total Energy of Flow:

E = Kinetic Energy + Potential Energy

$$E = \frac{v^2}{2g} + \frac{p}{\gamma} + Z$$

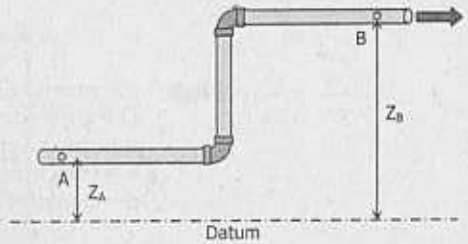
$\frac{v^2}{2g}$ = velocity head (K.E.)

$\frac{p}{\gamma}$ = pressure head (P.E.)

Z = elevation head (P.E.)

BERNOULLI'S ENERGY THEOREM

Between any two points (1 and 2) along the stream:



$$E_1 + HA - HE - HL = E_2$$

where E_1 = Total energy (head) at section 1
HA = head added (by the pump)
HE = head extracted (by turbine or any other device)
HL = total head lost

HEAD LOST IN PIPE FLOW

Major Head Lost (Frictional Losses)

Darcy-Weisbach Formula

$$h_f = \frac{fL}{D}\frac{v^2}{2g} \text{ in ft or meter}$$

For Laminar flow, $f = \frac{64}{R}$

For non-circular pipe, use $D = 4R$

For circular pipes (S.I.)

$$\frac{v^2}{2g} = \frac{8Q^2}{\pi^2 g D^4}$$

$$h_f = \frac{fL}{D}\frac{8Q^2}{\pi^2 g D^4}$$

For S.I. units, $h_f = \frac{0.0826\, f L Q^2}{D^5}$

Manning's Formula (S.I.)

$$v = \frac{1}{n}R^{2/3}S^{1/2}$$

where R = hydraulic radius = A/P
S = slope of EGL = h_f/L

$$h_f = \frac{6.35 n^2 L v^2}{D^{4/3}}$$

Use $D = 4R$ for non-circular pipes

For circular pipes (S.I.)

$$h_f = \frac{10.29 n^2 L Q^2}{D^{16/3}}$$

Hazen-William's Formula (S.I.)

$$Q = 0.2785 C_1 D^{2.63} S^{0.54}$$

where C_1 = Hazen-William's coefficient
S = slope of EGL = h_f/L

$$h_f = \frac{10.67 L Q^{1.85}}{C_1^{1.85} D^{4.87}}$$

Minor Head Lost

Minor losses are due to changes in direction and velocity of flow, and is expressed in terms of the velocity head at the smaller section of the pipe in case of constrictions.

$$h_m = K\frac{v^2}{2g}$$

where K = coefficient of minor loss

Head Lost through Nozzles

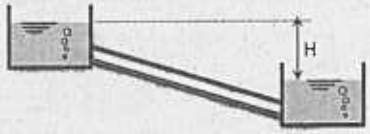
$$h_n = \left(\frac{1}{C_v^2} - 1\right)\frac{v_n^2}{2g}$$

Other Cases

For a horizontal pipe with uniform diameter, the head lost between any two points is equal to the difference in pressure head between the points.

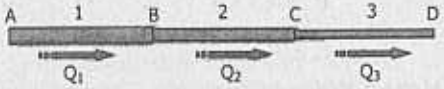
$$HL = \frac{p_2 - p_1}{\gamma}$$

For a pipe or system of pipes connecting two reservoirs, the total head lost is equal to the difference in water surface elevation of the reservoirs.



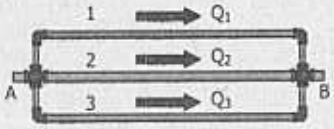
$$HL = H$$

Pipes in Series



$$Q_1 = Q_2 = Q_3$$
$$HL = h_{L1} + h_{L2} + h_{L3}$$

Pipes in parallel



$$Q = Q_1 + Q_2 + Q_3$$
$$HL = h_{L1} = h_{L2} = h_{L3}$$

Equivalent Pipe

For a pipe or system of pipes (O), the equivalent single pipe (E) is must satisfy the following conditions.

$$Q_E = Q_O$$
$$\text{and } HL_E = HL_O$$

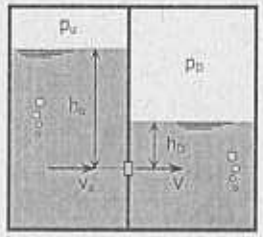
ORIFICE AND TUBES

The velocity and discharge through an orifice is given by:

$$v = C_V\sqrt{2gH}$$
$$Q = C\,A_o\sqrt{2gH}$$
$$C = C_c \times C_v$$

where:
C_V = coefficient of velocity
C = coefficient of discharge
C_c = coefficient of contraction
H = total head in meter or feet of the fluid flowing

Value of H

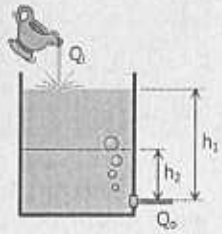


H = head upstream – head downstream

$$H = h_u + \frac{v_u^2}{2g} + \frac{p_u}{\gamma} - h_D - \frac{p_D}{\gamma}$$

where v_u = velocity of approach
p_u = pressure at the upstream side
p_D = pressure at the downstream side

Unsteady Flow (Variable Head)



If water flows into a tank at the rate of Q_i and at the same time leaves at Q_o, the time for the level to change from h_1 to h_2 is:

$$t = \int_{h_1}^{h_2} \frac{A_s\,dh}{Q_i - Q_o}$$

If $Q_i = 0$

$$t = \int_{h_2}^{h_1} \frac{A_s\,dh}{Q_o}$$

If the flow is through an orifice under a variable head H:

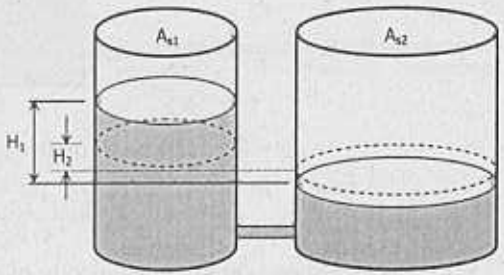
$$Q_o = C\,A_o\sqrt{2gH}$$

If the cross-sectional area A_s is constant and the flow is through an orifice, the formula becomes

$$t = \frac{2A_s}{CA_o\sqrt{2g}}\left(\sqrt{H_1} - \sqrt{H_2}\right)$$

where H_1 = initial head (at level 1)
H_2 = final head (at level 2)

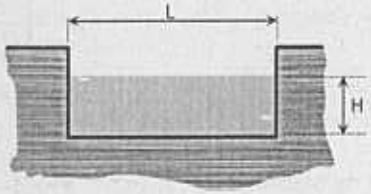
If water flows through the pipe connecting the two tanks shown, the time for the head to change from H_1 to H_2 is:



$$t = \frac{A_{s1}A_{s2}}{A_{s1}+A_{s2}}\,\frac{2}{CA_o\sqrt{2g}}\left(\sqrt{H_1} - \sqrt{H_2}\right)$$

WEIR

Rectangular Weir (Suppressed)



General Formula

$$Q = \frac{2}{3}C\sqrt{2g}\,L\left[(H+h_v)^{3/2} - h_v^{3/2}\right]$$

$$\text{or } Q = C_w L\left[(H+h_v)^{3/2} - h_v^{3/2}\right]$$

where

$$h_v = \frac{v_a^2}{2g} \quad \text{velocity head of approach}$$

C = coefficient of discharge
C_w = weir factor

Neglecting v_a:

$$Q = \frac{2}{3}C\sqrt{2g}\,LH^{3/2}$$

$$\text{or } Q = C_w LH^{3/2}$$

Francis Formula (C_w = 1.84) S.I.

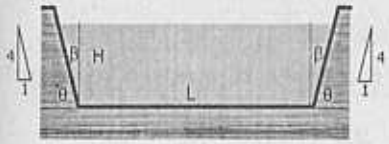
Considering v_a:

$$Q = 1.84L\left[(H+h_v)^{3/2} - h_v^{3/2}\right]$$

Neglecting v_a:

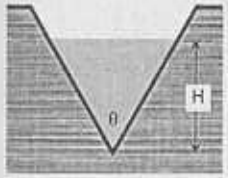
$$Q = 1.84\,LH^{3/2}$$

Cipolletti Weir



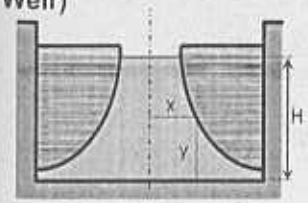
$$Q = 1.875\,LH^{3/2}$$
$$\theta = 75.9637° = 75°\,57'\,50''$$
$$\beta = 14.0363° = 14°\,2'\,10''$$

Triangular V-notch weir



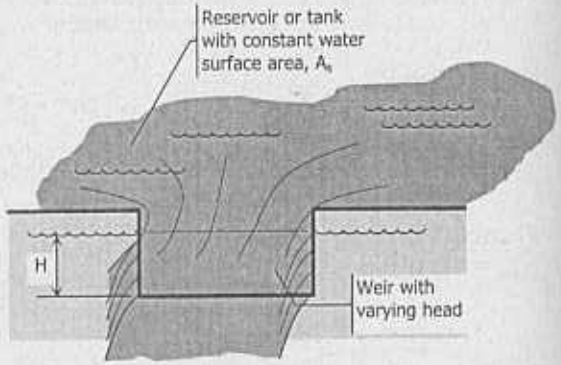
$$Q = \frac{8}{15}C\sqrt{2g}\tan\frac{\theta}{2}H^{5/2}$$

Suttro Weir (Proportional Flow Weir)



$$Q = C\pi K\sqrt{2g}\,H$$
$$K = x\sqrt{y}$$

Unsteady flow Weir(Variable Head)



$$t = \int_{H_2}^{H_1} \frac{A_s\, dH}{Q_o}$$

If the flow is through a suppressed rectangular weir:

$$t = \frac{2A_s}{C_w L}\left[\frac{1}{\sqrt{H_2}} - \frac{1}{\sqrt{H_1}}\right]$$

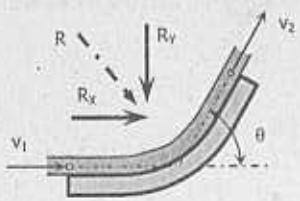
where C_w = weir factor, H_1 = initial head, H_2 = final head

HYDRODYNAMICS

Force against a fixed flat plate held normal to the jet

$$F = \frac{Q\gamma}{g} v = \rho Q v$$

Force against a fixed curved vane



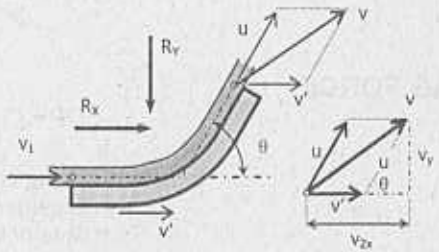
$$F_x = \frac{Q\gamma}{g}(v_{1x} - v_{2x})$$

$$F_y = \frac{Q\gamma}{g}(v_{1y} - v_{2y})$$

$$F = \sqrt{F_x^2 + F_y^2}$$

where
v_1 = velocity of the jet before hitting the vane
v_2 = velocity of the jet as it leaves the vane

Force Against a Moving Vane

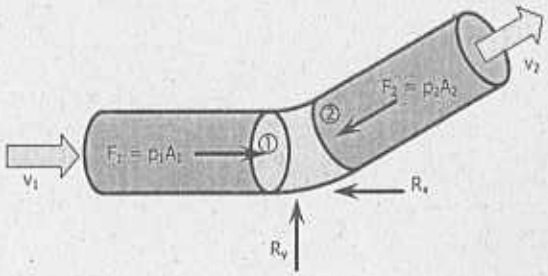


$$F_x = \frac{Q'\gamma}{g}(v_{1x} - v_{2x}); \quad F_y = \frac{Q'\gamma}{g}(v_{1y} - v_{2y})$$

$$Q' = A\,u \qquad u = v_1 - v'$$

u = relative velocity of the jet as it moves along the vane
Q' = amount of fluid deflected by the vane

Force on Bends in Pressure Conduits



Dynamic Force

$$F_x = \frac{Q\gamma}{g}(v_{1x} - v_{2x})$$

$$F_y = \frac{Q\gamma}{g}(v_{1y} - v_{2y})$$

Total Force

$$\Sigma F_x = \frac{Q\gamma}{g}(v_{2x} - v_{1x}) = F_{1x} - R_x - F_{2x}$$

$$\Sigma F_y = \frac{Q\gamma}{g}(v_{2y} - v_{1y}) = F_{1y} + R_y - F_{2y}$$

DRAG FORCE

$$DF = C_D \rho A \frac{v^2}{2} = C_D \gamma A \frac{v^2}{2g}$$

where C_D = drag coefficient
ρ = density of the fluid
A = area normal to the direction of motion

A. 6.5 kN/m³ C. 3.2 kN/m³
B. 5.1 kN/m³ D. 8.5 kN/m³

PROBLEM 9 - 9
The weight density of a mud is given by $w = 10 + 0.5h$, where w is in kN/m³ and h is in meters. Determine the pressure, in kPa, at a depth of 5 m.
A. 89.36 kPa C. 62.5 kPa
B. 56.25 kPa D. 78.54 kPa

PROBLEM 9 - 10
ME April 1997
What is the resulting pressure when one pound of air at 15 psia and 200°F is heated at constant volume to 800°F?
A. 28.6 psia C. 36.4 psia
B. 52.1 psia D. 15 psia

PROBLEM 9 - 11
ECE Nov. 1997
The volume of a gas under standard atmospheric pressure 76 cm Hg is 200 in³. What is the volume when the pressure is 80 cm Hg, if the temperature is unchanged?
A. 190 in³ C. 110 in³
B. 90 in³ D. 30.4 in³

PROBLEM 9 - 12
A two-meter square plane surface is immersed vertically below the water surface. The immersion is such that the two edges of the square are horizontal. If the top of the square is 1 meter below the water surface, what is the total water pressure exerted on the plane surface?
A. 43.93 kN C. 64.76 kN
B. 52.46 kN D. 78.48 kN

PROBLEM 9 - 13
Find the total water pressure on a vertical circular gate, 2 meters in diameter, with its top 3.5 meters below the water surface.
A. 138.7 kN C. 169.5 kN
B. 107.9 kN D. 186.5 kN

PROBLEM 9 - 14
CE Board
An iceberg having specific gravity of 0.92 is floating on salt water of sp. gr. 1.03. If the volume of ice above the water surface is 1000 cu. m., what is the total volume of the ice?
A. 8523 m³ C. 9364 m³
B. 7862 m³ D. 6325 m³

PROBLEM 9 - 15
A block of wood requires a force of 40 N to keep it immersed in water and a force of 100 N to keep it immersed in glycerin (sp. gr. = 1.3). Find the weight and sp. gr. of the wood.
A. 0.7 C. 0.9
B. 0.6 D. 0.8

PROBLEM 9 - 16
ME April 1998
Reynolds number may be calculated from:
A. diameter, density, and absolute viscosity
B. diameter, velocity, and surface tension
C. diameter, velocity, and absolute viscosity
D. characteristic length, mass flow rate per unit area, and absolute viscosity

PROBLEM 9 - 17
ME April 1998
The sum of the pressure head, elevation head, and the velocity head remains constant, this is known as:
A. Bernoulli's Theorem C. Archimedes' Principle
B. Boyle's Law D. Torrecelli's Theorem

PROBLEM 9 - 18
ME Oct. 1997
What is the expected head loss per mile of closed circular pipe (17-in inside diameter, friction factor of 0.03) when 3300 gal/min of water flow under pressure?
A. 38 ft C. 3 580 ft
B. 0.007 ft D. 64 ft

PROBLEM 9 - 19
What is the rate of flow of water passing through a pipe with a diameter of 20 mm and speed of 0.5 m/sec?
A. 1.24×10^{-4} m³/s C. 1.57×10^{-4} m³/s
B. 2.51×10^{-4} m³/s D. 1.87×10^{-4} m³/s

PROBLEM 9 - 20
An orifice has a coefficient of discharge of 0.62 and a coefficient of contraction of 0.63. Determine the coefficient of velocity for the orifice.
A. 0.98 C. 0.97
B. 0.99 D. 0.96

PROBLEM 9 - 21
The theoretical velocity of flow through an orifice 3 m below the surface of water in a tall tank is:
A. 8.63 m/s C. 5.21 m/s
B. 9.85 m/s D. 7.67 m/s

PROBLEM 9 - 22
Water having kinematic viscosity $v = 1.3 \times 10^{-6}$ m²/s flows in a 100-mm diameter pipe at a velocity of 4.5 m/s. The Reynolds Number is:
A. 346,150 C. 387,450
B. 258,250 D. 298,750

PROBLEM 9 - 23
Oil having specific gravity of 0.869 and dynamic viscosity of 0.0814 Pa-s flows through a cast iron pipe at a velocity of 1 m/s. The pipe is 50 m long and 150 mm in diameter. Find the head lost due to friction.
A. 0.73 m C. 0.68 m
B. 0.45 m D. 1.25 m

PROBLEM 9 - 24 What commercial size of new cast iron pipe shall be used to carry 4490 gpm with a lost of head of 10.56 feet per mile? Assume $f = 0.019$.

A. 625 mm
B. 576 mm
C. 479 mm
D. 352 mm

PROBLEM 9 - 25 Assume that 57 liters per second of oil ($\rho = 860$ kg/m³) is pumped through a 300 mm diameter pipeline of cast iron. If each pump produces 685 kPa, how far apart can they be placed? (Assume $f = 0.031$)

A. 23.7 m
B. 32.2 m
C. 12.6 m
D. 19.8 m

PROBLEM 9 - 26 A 20-mm-diameter commercial steel pipe, 30 m long is used to drain an oil tank. Determine the discharge when the oil level in the tank is 3 m above the exit of the pipe. Neglect minor losses and assume $f = 0.12$.

A. 0.000256 m³/s
B. 0.000179 m³/s
C. 0.000113 m³/s
D. 0.000869 m³/s

ANSWER SHEET

1.	A	B	C	D	E	11.	A	B	C	D	E	21.	A	B	C	D	E
2.	A	B	C	D	E	12.	A	B	C	D	E	22.	A	B	C	D	E
3.	A	B	C	D	E	13.	A	B	C	D	E	23.	A	B	C	D	E
4.	A	B	C	D	E	14.	A	B	C	D	E	24.	A	B	C	D	E
5.	A	B	C	D	E	15.	A	B	C	D	E	25.	A	B	C	D	E
6.	A	B	C	D	E	16.	A	B	C	D	E	26.	A	B	C	D	E
7.	A	B	C	D	E	17.	A	B	C	D	E	27.	A	B	C	D	E
8.	A	B	C	D	E	18.	A	B	C	D	E	28.	A	B	C	D	E
9.	A	B	C	D	E	19.	A	B	C	D	E	29.	A	B	C	D	E
10.	A	B	C	D	E	20.	A	B	C	D	E	30.	A	B	C	D	E

Solutions to Set 9 Fluid Mechanics

SOLUTION 9 - 1
Ans: A

Density, ρ = Mass / Volume

$\rho = 350 / (3 \times 5 \times 20) = \mathbf{1.167\ kg/m^3}$

SOLUTION 9 - 2
Ans: D

$$\text{Sp. gr.} = \frac{\rho_{liquid}}{\rho_{water}}$$

$$\rho_{liquid} = \frac{M_{liquid}}{V_{liquid}};\ V_{liquid} = \frac{M(\text{in kg})}{\rho(\text{in kg/m}^3)}$$

$$\rho_{liquid} = \frac{0.1 + 0.15}{\frac{0.1}{1000} + \frac{0.15}{790}} = 862.445\ \text{kg/m}^3$$

$$\text{Sp. gr.} = \frac{862.445}{1000} = \mathbf{0.862}$$

SOLUTION 9 - 3
Ans: B

Specific volume $= \frac{1}{\rho}$, where ρ is the density.

$$\rho = \frac{\text{Mass}}{\text{Volume}},\ \text{Volume} = \text{Mass}/\rho$$

Mass = 100g + 150g = 250 g
Volume = $V_{water} + V_{alcohol}$

$$\text{Volume} = \frac{100\text{g}}{1\text{g/cm}^3} + \frac{150\text{g}}{0.79\text{g/cm}^3} = 289.87\ \text{cm}^3$$

$$\rho = \frac{250}{289.87} = 0.862\ \text{g/cm}^3$$

$$\text{Specific volume} = \frac{1}{0.862} = \mathbf{1.159\ cm^3/g}$$

SOLUTION 9 - 4
Ans: D

$$\text{Density, } \rho = \frac{M}{V} = \frac{350}{3(5)(20)} = \mathbf{1.167\ kg/m^3}$$

SOLUTION 9 - 5
Ans: C

The pressure at a depth h in a liquid is $p = \gamma h$.

For salt water, $\gamma = 9.81 \times 1.03 = 10.1043$ kN/m³

$p = 10.1043\ \text{kN/m}^3 \times 34\ \text{m} = \mathbf{343.55\ kPa}$

SOLUTION 9 - 6
Ans: A

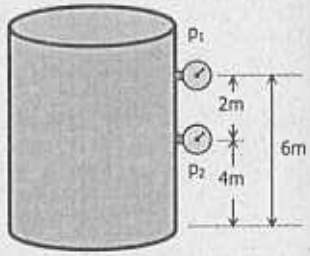
$p_{abs} = p_{atmospheric} + p_{gage}$
$100 = p_{atmospheric} + 10$
$p_{atm} = 90$ **kPa**

SOLUTION 9 - 7
Ans: C

$p_2 - p_1 = \gamma h$
$p_2 = p_1 + \gamma h = 60 + 9.81(1.03)(27)$
$p_2 = 332.82$ **kPa**

SOLUTION 9 - 8
Ans: A

$p_2 - p_1 = \gamma h$
$\gamma(2) = 103 - 90$
$\gamma = 6.5$ **kN/m³**



SOLUTION 9 - 9
Ans: B

$dp = w\,dh = (10 + 0.5\,h)dh$

$$\int_0^p dp = \int_0^5 (10 + 0.5h)\,dh$$

$$p = \left[10h + 0.25h^2\right]_0^5 = 10(5) + 0.25(5)^2 = \mathbf{56.25\ kPa}$$

SOLUTION 9 - 10
Ans: A

$$\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2}$$

$V_1 = V_2$

T_1 and T_2 are absolute temperature in °Rankine (°F + 460)

$$\frac{15}{200 + 460} = \frac{p_2}{800 + 460}$$

$p_2 = 28.64$ **psia**

SOLUTION 9 - 11
Ans: A

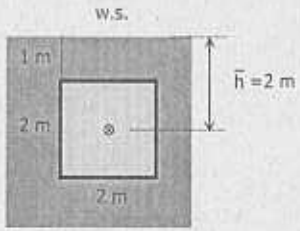
$$\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2}$$

For isothermal conditions (equal temperature)

$p_1\,V_1 = p_2\,V_2$
$200(76) = 80\,V_2$
$V_2 = 190$ **in³**

SOLUTION 9 - 12
Ans: D

$F = \gamma\bar{h}\,A$
$F = 9.81\ \text{kN/m}^3 \times 2\text{m} \times 4\text{m}^2$
$F = 78.48$ **kN**

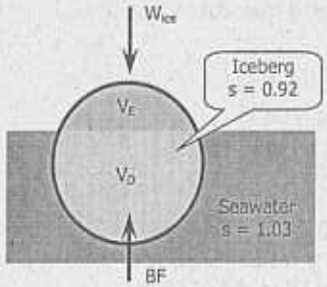


SOLUTION 9 - 13
Ans: A

$F = \gamma\bar{h}\,A$
$F = 9.81\ \text{kN/m}^3 \times (3.5 + 1) \times \frac{\pi}{4}(2)^2\ \text{m}^2$
$F = 138.68$ **kN**

SOLUTION 9 - 14
Ans: C

Let V = total volume of ice
V_D = volume displaced
$V_D = V - 1000$

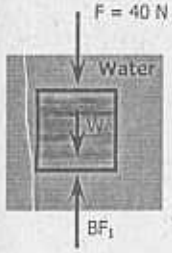


$W_{ice} = \gamma_{ice}\,V = 9.81(0.92)(V)$
$W_{ice} = 9.0252\,V$
$BF = \gamma_{seawater}\,V_D$
$BF = 9.81(10.3)\,(V - 1000)$
$BF = 10.1043(V - 1000)$

$\Sigma F_V = 0$
$W_{ice} = BF$
$9.0252\,V = 10.1043(V - 1000)$
$1.0791V = 10104.3$
$V =$ **9364 cu. m.**

SOLUTION 9 - 15
Ans: D

Let V = volume of wood



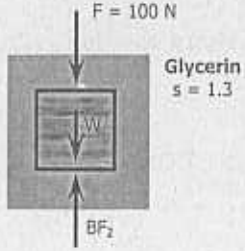
In water:
$\Sigma F_V = 0$
$BF_1 - W - F = 0$
$9810V - W = 40$

$$V = \frac{40 + W}{9810}$$

In glycerin:
$\Sigma F_V = 0$
$BF_2 - W - F = 0$
$9810(1.3)V - W = 100$

$$9810(1.3)\left[\frac{40 + W}{9810}\right] - W = 100$$

$52 + 1.3W - W = 100$



$W = 160$ N

$$V = \frac{40 + 160}{9810} = 0.0204 \text{ m}^3$$

$$\text{Unit weight, } \gamma = \frac{W}{V} = \frac{160}{0.0204}$$

$$\gamma = 7843 \text{ N/m}^3$$

$$\text{Sp. gr., } s = \frac{\gamma_{wood}}{\gamma_{water}} = \frac{7843}{9810} = \mathbf{0.8}$$

SOLUTION 9 - 16
Ans: D

Reynolds number is given by the formula $R = \frac{VD\rho}{\mu}$

$$R = \frac{\rho QD}{A\mu} = \frac{M}{A}\frac{D}{\mu}$$

Thus, it ca be calculated from characteristic length, mass flow rate per unit area, and absolute viscosity.

SOLUTION 9 - 17
Ans: A

Bernoulli's Theorem

SOLUTION 9 - 18
Ans: A

$$h_L = \frac{fL}{D}\frac{v^2}{2g}$$

f = friction factor = 0.03
L = length of pipe = 1 mile = 5280 ft.
D = pipe diameter = 17 in = 17/12 ft.

$$v = \frac{Q}{A} = \frac{3300/7.48}{(\pi/4)(17/12)^2}$$

$v = 279.89$ ft/min = 4.665 ft/sec

$$h_L = \frac{0.03(5280)}{17/12}\frac{(4.665)^2}{2(32.2)} = \mathbf{37.78 \text{ ft.}}$$

SOLUTION 9 - 19
Ans: C

Flow rate, $Q = Av$

$Q = \frac{\pi}{4}(0.02)^2(0.5) = \mathbf{1.57 \times 10^{-4} \text{ m}^3/\text{sec}}$

SOLUTION 9 - 20
Ans: A

$C = C_c \times C_v$
$0.62 = 0.63 \times C_v$
$C_v = \mathbf{0.984}$

SOLUTION 9 - 21
Ans: D

$$v = \sqrt{2gH} = \sqrt{2(9.81)(3)} = \mathbf{7.67 \text{ m/s}}$$

SOLUTION 9 - 22
Ans: A

$$R = \frac{vD\rho}{\mu} = \frac{vD}{\nu}$$

$$R = \frac{4.5\frac{\text{m}}{\text{s}}(0.1\text{m})}{1.3\times10^{-6}\text{m}^2/\text{s}} = \mathbf{346{,}154}$$

SOLUTION 9 - 23
Ans: C

$$R = \frac{vD\rho}{\mu} = \frac{(1)(0.15)[1000(0.869)]}{0.0814} = 1601 \text{ (laminar)}$$

$$f = \frac{64}{R} = \frac{64}{1601} = 0.04$$

$$h_f = \frac{fL}{D}\frac{V^2}{2g} = \frac{0.04(50)}{0.15}\frac{(1)^2}{2(9.81)} = \mathbf{0.68 \text{ m}}$$

SOLUTION 9 - 24
Ans: B

$$h_f = \frac{0.0826 fLQ^2}{D^5}$$

$$Q = 4490\frac{\text{gal}}{\text{min}} \times \frac{3.79 \text{ lit}}{\text{gal}} \times \frac{1 \text{ min}}{60 \text{ sec}} = 284 \text{ lit/sec}$$

$Q = 0.284$ m³/s

$$L = 1 \text{ mile} \times \frac{5280 \text{ ft}}{\text{mile}} \times \frac{1 \text{ m}}{3.28 \text{ ft}} = 1609.76 \text{ m}$$

$$h_f = 10.56 \text{ ft} \times \frac{1 \text{ m}}{3.28 \text{ ft}} = 3.22 \text{ m}$$

then;

$$3.22 = \frac{0.0826(0.019)(1609.76)(0.284)^2}{D^5}$$

$D = 0.576$ m = **576 mm**

SOLUTION 9 - 25
Ans: A

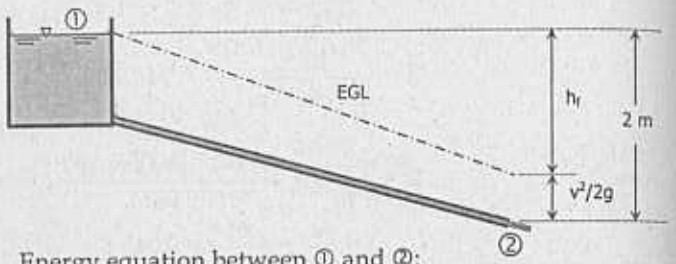
Each pump should be spaced such that the head lost between any two
pumps is equal to the pressure head produced by each.

$$h_f = \frac{0.0826 fLQ^2}{D^5} = \frac{p}{\gamma} = \frac{685(1000)}{860(9.81)} = 81.2 \text{ m}$$

$$h_f = \frac{0.0826(0.031)\,L\,(0.057)^2}{(0.3)^5} = 81.2$$

$L = 23718$ m
$L = \mathbf{23.718 \text{ km}}$

SOLUTION 9 - 26
Ans: B



Energy equation between ① and ②:

$$E_1 - h_f = E_2$$

$$\frac{v_1^2}{2g} + \frac{p_1}{\gamma} + Z_1 = \frac{v_2^2}{2g} + \frac{p_2}{\gamma} + Z_2 + h_f$$

$$0 + 0 + 3 = \frac{v^2}{2g} + 0 + 0 + \frac{fL}{D}\frac{v^2}{2g}$$

$$\frac{v^2}{2g}\left[1 + \frac{0.12(30)}{0.02}\right] = 3$$

$$v = 0.57 \text{ m/s}$$

$$Q = Av = \frac{\pi}{4}(0.02)^2(0.57)$$

$$Q = 0.000179 \text{ m}^3/\text{s}$$

Part 7 ENGINEERING ECONOMICS

INTEREST

Interest is the amount of money earned by a given capital. From the borrower's viewpoint, interest is the amount of money paid for the use of a borrowed capital. From the lender's viewpoint, it is the income generated by the capital that was lent.

CASH FLOW DIAGRAMS

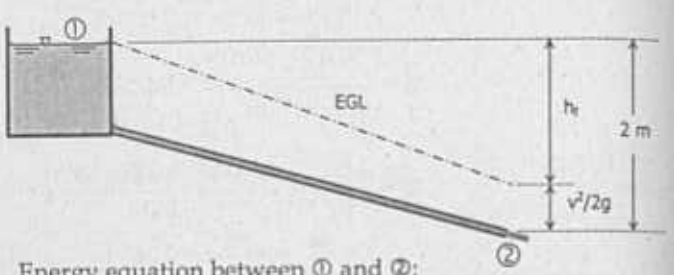
Cash flow diagrams may be drawn to help visualize and simplify problems having diverse receipts and disbursements.

Conventions used in cash flow diagram

- The horizontal (time) axis is marked off in equal increments, one per period, up to the duration or horizon of the project.
- All disbursement and receipts (cash flow) are assumed to take place at the end of the year in which they occur. This is known as the year-end convention. The exception of the year-end convention is the initial cost (purchase cost) which occur at $t = 0$.
- To or more transfers in the same period placed end-to-end may be combined into one.
- Expenses incurred before $t = 0$ are called sunk cost and are not relevant to the problem.
- Receipts and disbursement are represented by arrows on opposite sides of the horizontal time axis.

Consider the following example: An electronic equipment costs P30,000. Maintenance cost is P3,000 each year. The device will generate revenues of P15,000 each year for 5 years after which the salvage value is expected to be P12,000. Draw and simplify the cash flow diagram.

SOLUTION 9 - 26
Ans: B



Energy equation between ① and ②:

$$E_1 - h_f = E_2$$

$$\frac{v_1^2}{2g} + \frac{p_1}{\gamma} + Z_1 = \frac{v_2^2}{2g} + \frac{p_2}{\gamma} + Z_2 + h_f$$

$$0 + 0 + 3 = \frac{v^2}{2g} + 0 + 0 + \frac{fL}{D}\frac{v^2}{2g}$$

$$\frac{v^2}{2g}\left[1 + \frac{0.12(30)}{0.02}\right] = 3$$

$$v = 0.57 \text{ m/s}$$

$$Q = Av = \tfrac{\pi}{4}(0.02)^2(0.57)$$

$$Q = 0.000179 \text{ m}^3/\text{s}$$

Part 7
ENGINEERING ECONOMICS

INTEREST

Interest is the amount of money earned by a given capital. From the borrower's viewpoint, interest is the amount of money paid for the use of a borrowed capital. From the lender's viewpoint, it is the income generated by the capital that was lent.

CASH FLOW DIAGRAMS

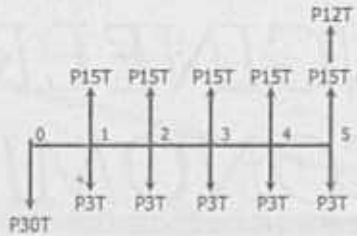
Cash flow diagrams may be drawn to help visualize and simplify problems having diverse receipts and disbursements.

Conventions used in cash flow diagram

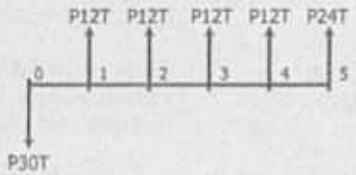
- The horizontal (time) axis is marked off in equal increments, one per period, up to the duration or horizon of the project.
- All disbursement and receipts (cash flow) are assumed to take place at the end of the year in which they occur. This is known as the year-end convention. The exception of the year-end convention is the initial cost (purchase cost) which occur at $t = 0$.
- To or more transfers in the same period placed end-to-end may be combined into one.
- Expenses incurred before $t = 0$ are called sunk cost and are not relevant to the problem.
- Receipts and disbursement are represented by arrows on opposite sides of the horizontal time axis.

Consider the following example: An electronic equipment costs P30,000. Maintenance cost is P3,000 each year. The device will generate revenues of P15,000 each year for 5 years after which the salvage value is expected to be P12,000. Draw and simplify the cash flow diagram.

The following shows the cash flow diagram for each transaction:



The simplified cash flow diagram is as follows:



SIMPLE INTEREST

In simple interest, the interest earned by the principal is computed at the end of the investment period, and thus, it varies directly with time.

Ordinary and Exact Simple Interest

In *ordinary simple interest*, the interest is computed on the basis of one banker's year.

1 banker's year = 12 months
(30 days each month) = 360 days

In *exact simple interest*, the interest is based on the exact number of days of the year, where there are 365 days for an ordinary year and 366 years for leap years.

Leap year occurs every four years for years that is exactly divisible by four, except century marks (1800, 1900, etc.) but not including those that are exactly divisible by 400 (2000, 2400, etc.)

Elements of Simple Interest

P = principal or present worth
I = interest earned
F =future worth

$$F = P + I$$

r = simple interest rate (per year)
t = time in years or fraction of a year

Note: P may stand for the amount borrowed or invested, while F may stand for the amount to be paid or amount accumulated.

$$I = Prt$$

$$F = P + I = P + Prt$$

$$F = P(1 + rt)$$

Value of t

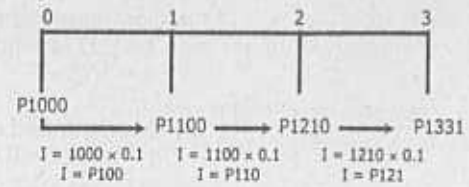
Example:

- 4 years; $t = 4$
- 3 months; $t = 3/12 = ¼$
- 90 days
  - Ordinary simple interest, $t = 90/360$
  - Exact simple interest, $t = 90/365$
    or 90/366 for leap years
- 2 years & 4 months;
  $t = 2 + 4/12 = 2.333$

COMPOUND INTEREST

In compound interest, the interest is computed every end of each interest period (compounding period), and the interest earned for that period is added to the principal (interest plus principal).

To demonstrate this, consider an investment of P1000 to earn 10% per year for three years. The following diagram shows how the money grows.



Elements of Compound Interest

P = present worth or principal
F = future worth or compound amount
i = effective interest per compounding period (per interest period)

$$i = r/m$$

n = total number of compoundings

$$n = t \times m$$

I = interest earned

$$I = F - P$$

r = nominal interest rate
ER = effective interest
t = number of years of investment
m = number of compoundings per year

Future worth of P

After n periods, the compound amount F is:

$$F = P(1+i)^n$$

The term $(1+i)^n$, also denoted as $(F/P, i, n)$ is called the *single payment compound-amount factor.*

Present worth of F

The present worth of F is:

$$P = \frac{F}{(1+i)^n}$$

The term $\frac{1}{(1+i)^n}$, also denoted as $(P/F, i, n)$ is called the *single payment present-worth factor.*

Values of i and n

The following examples show how to get the values of i and n.

Nominal interest rate, $r = 12\%$
Number of years of investment, $t = 5$ years

- Compounded **annually** ($m = 1$)
  $i = 0.12/1 = 0.12$
  $n = 5(1) = 5$

- Compounded **semi-annually** ($m = 2$)
  $i = 0.12/2 = 0.06$
  $n = 5(2) = 10$

- Compounded **quarterly** ($m = 4$)
  $i = 0.12/4 = 0.03$
  $n = 5(4) = 20$

- Compounded **monthly** ($m = 12$)
  $i = 0.12/12 = 0.01$
  $n = 5(12) = 60$

- Compounded **bi-monthly** ($m = 6$)
  $i = 0.12/6 = 0.02$
  $n = 5(6) = 30$

Continuous Compounding ($m \to \infty$)

Interest may be compounded daily, hourly, per minute, etc. As a limit, interest may be considered to be compounded an infinite number of times per year ($m \to \infty$).

The future worth of P at an interest rate of r compounded continuously for t years is:

$$F = Pe^{rt}$$

Nominal and Effective Rates of Interest

Nominal rate is the rate quoted in describing a given variety of compound interest. Consider a bank deposit of P1000 to earn 6% compounded quarterly. After one year, the compound amount F is:

$$F = P(1+i)^n = 1000(1 + 0.06/4)^{1 \times 4}$$
$$F = \text{P } 1061.36$$

Notice that the interest earned is P61.36 representing 6.136% of P1000 (not 6% of P1000). For this case, 6% (compounded quarterly) is called the nominal rate and 6.136% is the effective rate.

Thus the effective rate of interest (ER) is the actual interest earned in one year period. This can be computed by either of the following:

$$ER = \frac{\text{Interest earned in one year}}{\text{Principal at the beginning of the year}}$$

$$ER = \left(1 + \frac{r}{m}\right)^m - 1$$

Thus, the effective rate of 6% compounded quarterly is, $ER = (1 + 0.06/4)^4 - 1 = 0.06136$ or 6.136% as computed previously.

The effective rate of r (%) compounded continuously is:

$$ER = e^r - 1$$

Equivalent Nominal Rates

From the previous discussion, we see that 6% compounded quarterly is not the same as 6% compounded monthly, for the reason that they have *different* effective rates.

Two nominal rates are equal if they have the same effective rates.

Consider a nominal interest rate of 10% compounded quarterly. The equivalent nominal rate compounded monthly is:

$$ER_M = ER_Q$$
$$(1 + r/12)^{12} - 1 = (1 + 0.10/4)^4 - 1$$
$$r = 0.09918 = 9.918\%$$

Thus, 10% compounded quarterly will have the same interest as 9.918% compounded monthly.

ANNUITY

Annuity is a series of uniform payments made at equal intervals of time.

Annuities are established for the following purposes:

1. As payment of a debt by a series of equal payment at equal time intervals, also known as amortization.
2. To accumulate a certain amount in the future by depositing equal amounts at equal time intervals. These amounts are called sinking fund.
3. As a substitute periodic payment for a future lump sum payment.

Elements of Annuity

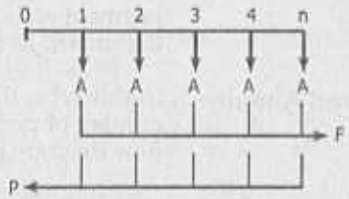
A = periodic payment
P = present worth of all periodic payments
F or S = future worth or sum of all the periodic payments after the last payment is made.

i = interest rate per payment
n = number of payments

TYPES OF ANNUITY

A. Ordinary Annuity

In ordinary annuity, the payment is made at the end of each period starting from the first period, as in the diagram shown below.



Future worth of A

The future worth if A is:

$$F = \frac{A[(1+i)^n - 1]}{i}$$

The factor $\frac{(1+i)^n - 1}{i}$ is called *equal-payment-series compound-amount factor* and is denoted as $(F/A, i, n)$

Value of A with known F (sinking-fund)

The value of A if F is known is:

$$A = \frac{Fi}{(1+i)^n - 1}$$

The factor $\frac{i}{(1+i)^n - 1}$, is known as *equal-payment sinking-fund factor* and is denoted as $(A/F, i, n)$

Present worth of A

The present worth of A is:

$$P = \frac{F}{(1+i)^n} = \frac{A[(1+i)^n - 1]}{(1+i)^n i}$$

The factor $\frac{(1+i)^n - 1}{(1+i)^n i}$, is known as *equal-payment-series present-worth factor* and is designated as $(P/A, i, n)$

Value of A with known P (capital recovery)

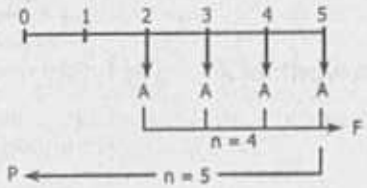
The value of A with known P is:

$$A = \frac{P(1+i)^n i}{(1+i)^n - 1}$$

The factor $\frac{(1+i)^n i}{(1+i)^n - 1}$, is known as the equal-payment-series-capital-recovery factor and is designated as $(A/P, i, n)$

B. Deferred Annuity

In this type, the first payment is deferred a certain number of periods after the first. Consider the cash flow diagram below.



For the cash flow diagram shown above, the following calculations can be made for solving P and F.

So solve for the future worth F:

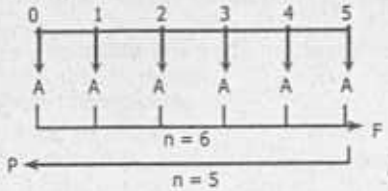
$$F = \frac{A[(1+i)^4 - 1]}{i}$$

To solve for the present worth P:

$$P = \frac{F}{(1+i)^5} = \frac{A[(1+i)^4 - 1]}{(1+i)^5 i}$$

C. Annuity Due

If the payment is made at the beginning of each period starting from the first period, the annuity is called *annuity due*.



From the diagram shown:

$$F = \frac{A[(1+i)^6 - 1]}{i}$$

$$P = \frac{F}{(1+i)^5} = \frac{A[(1+i)^6 - 1]}{(1+i)^5 i}$$

From the previous examples, the formula for P can be generalized as:

$$P = \frac{A[(1+i)^n - 1]}{(1+i)^{n'} i}$$

where n is the number of payments and n' is the number of periods from zero (0) period up to the last payment.

PERPETUITY

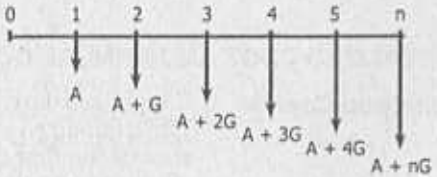
Perpetuity is an annuity where the payment periods extend forever or the periodic payments continue indefinitely.

If the payment is made at the end of each period starting from the first period, the present worth of perpetuity is:

$$P = \frac{A}{i}$$

UNIFORM GRADIENT

Arithmetic Gradient



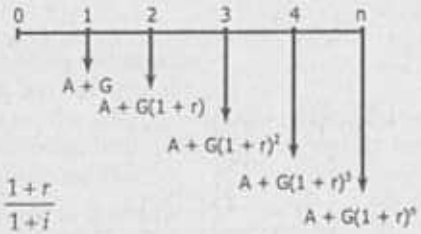
The present worth is:

$$P = \frac{A[(1+i)^n - 1]}{(1+i)^n i} + \frac{G}{i}\left[\frac{(1+i)^n - 1}{i(1+i)^n} - \frac{n}{(1+i)^n}\right]$$

The future worth is:

$$F = P(1+i)^n = \frac{A[(1+i)^n - 1]}{i} + \frac{G}{i}\left[\frac{(1+i)^n - 1}{i} - n\right]$$

Geometric Gradient



Let $w = \dfrac{1+r}{1+i}$

Present worth

If $w \neq 1$

$$P = \frac{A[(1+i)^n - 1]}{(1+i)^n i} + \frac{G}{1+i}\left[\frac{1-w^n}{1-w}\right]$$

If $w = 1$ (for $r = i$)

$$P = \frac{A[(1+i)^n - 1]}{(1+i)^n i} + \frac{Gn}{1+r}$$

The future worth is

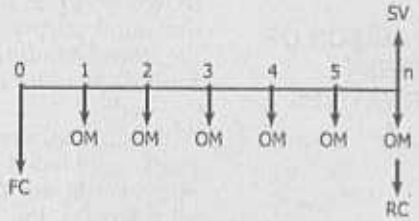
$$F = P(1+i)^n$$

CAPITALIZED COST AND ANNUAL COST

Capitalized Cost, *K*

Capitalized cost is an application of perpetuity. The capitalized cost of a project or structure is the sum of the first cost (*FC*) and the present worth of all future payments and replacements which is assumed to continue forever.

If a project requires a first cost of *FC*, annual operation and maintenance of *OM* for *n* years, a salvage value of *SV* after every *n* years, and a replacement cost of *RC* after every end of *n* years, then the capitalized cost is:



Capitalized cost, *K*

$$K = FC + \frac{OM}{i} + \frac{RC - SV}{(1+i)^n - 1}$$

If *RC* is not specified, use *RC* = *FC*.

Capitalized cost may also be defined as the first cost plus the present worth of annual maintenance and operation cost plus the present worth of depreciation assumed to continue forever.

Annual Cost, *AC*

The annual cost (*AC*) of a project is:

AC = Annual interest on investment +
Annual operation and maintenance +
Annual depreciation cost

In relation to capitalized cost, *AC* is:

Annual cost, $AC = K\,i$

$$AC = (FC)i + OM + \frac{(RC - SV)i}{(1+i)^n - 1}$$

Example 7 - 1

A machine costs P300,000.00 new, and must be replaced at the end of each 15 years. If the annual maintenance required is P5,000.00, find the capitalized cost, if money is worth 5% and the final salvage value is P50,000.

Solution

$$K = FC + \frac{OM}{i} + \frac{RC - SV}{(1+i)^n - 1}$$

$$K = 300{,}000 + \frac{5{,}000}{0.05} + \frac{300{,}000 - 50{,}000}{(1+0.05)^{15} - 1}$$

K = **P631,711.44**

COST COMPARISON OF DIFFERENT ALTERNATIVES

If two or more different articles are available for the same purpose, they are equally economical if the corresponding present worth, annual cost or capitalized costs are the same.

Example 7 - 2

A certain equipment costs P150,000.00, lasts for 6 years, and has a salvage value of P30,000. How much could an investor afford to pay for another machine for the same purpose, whose life is 10 years and salvage value is P40,000, if money is worth 5%?.

Solution

For the first machine:

$$K = FC + \frac{OM}{i} + \frac{RC - SV}{(1+i)^n - 1}$$

$$K = 150{,}000 + \frac{150{,}000 - 30{,}000}{(1+0.05)^6 - 1}$$

$$K = \text{P}502{,}841.92$$

For the other machine:

$$K = FC + \frac{FC - 40000}{(1+0.05)^{10} - 1} = 502{,}841.92$$

$$FC = \textbf{P218,696.41}$$

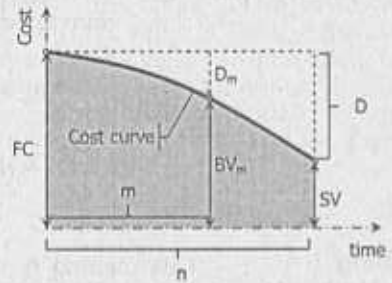
DEPRECIATION

Depreciation refers to the decrease in the value of an asset, due to usage of passage of time. An asset may depreciate physically or functionally.

Elements of Depreciation

FC = first cost
SV = salvage value or trade-in value
d = depreciation charge
n = economic life of the property in years
m = any time before n
BV_m = book value after m years
D_m = total depreciation for m years

The following diagram shows the cost of the property plotted versus time.



Book Value

The book value of the property at any time m is:

$$BV_m = FC - D_m$$

METHODS OF COMPUTING DEPRECIATION

A. Straight Line Depreciation (SLD)

This is the most common method used in computing depreciation. In this method, the cost of the property is assumed to vary linearly with time. The following formulas are used.

$$d = \frac{FC - SV}{n}$$

$$D_m = d \times m$$

In some cases, the life n may be the number of outputs. An example of this is the copying machine (XEROX) where the life span is dependent on the number of copies made.

B. Sinking Fund Method

In this method, an imaginary fund d called a singking fund is invested yearly at a rate of i to amount to $(FC - SV)$ at the end of the life of the property.

$$d = \frac{(FC - SV)i}{(1+i)^n - 1}$$

$$D_m = \frac{d[(1+i)^m - 1]}{i}$$

C. Sum of the Years Digit Method (SOYD)

The depreciation charge in this method is assumed to vary directly to the number of years and inversely to the sum of the year's digit.

$$\text{Sum of year's digit, SUM} = \frac{n}{2}(1+n)$$

$$d_m = (FC - SV)\frac{n-m+1}{\text{SUM}}$$

$$D_m = (FC - SV)\frac{m(2n-m+1)}{2\times\text{SUM}}$$

D. Declining Balance Method (Constant percentage method)

This method is based on the compound interest formula $F = P(1+i)^n$, where P is the first cost, FC, F is the book value at any time, and i is the depreciation rate and is equal to $-K$. Then

$$BV_m = FC(1-K)^m$$

$$\text{and } SV = FC(1-K)^n$$

$$\text{Constant percentage, } K = 1 - \sqrt[n]{\frac{SV}{FC}}$$

$$d_m = FC(1-K)^{m-1}K$$

E. Double Declining Balance Method

$$\text{Depreciation charge to date} = \frac{2}{n} \times BV \text{ at the beginning of the year.}$$

$$BV_m = FC\left(1-\frac{2}{n}\right)^m \geq SV$$

$$d_m = \frac{2}{n}BV_m$$

CAPITAL RECOVERY (DEPLETION)

If you invest FC now and desires a rate of return of r for n periods, and if you can deposit to an account earning an interest of i for n periods to recover an amount of RC, and will also receive a salvage value of SV from your invested property at the end of n periods, then the periodic dividend or income D required is:

$$D = (FC)\,r + \frac{(RC-SV)i}{(1+i)^n - 1}$$

Note: If RC is not specified, $RC = FC$.

Example 7 - 3

A mine costs P21M, and will last for 20 years. Its plant has a salvage value of P1M, at the end of the time. The mine will yield an equal dividend at the end of each year. What is the annual dividend, if it is sufficient to pay interest annually at the rate of 6% on the original investment and to accumulate a replacement fund, invested at 4%?

Solution

$$D = (FC)\,r + \frac{(RC-SV)i}{(1+i)^n - 1}$$

FC = P21,000,000 $\quad r = 6\% = 0.06$
SV = P1,000,000 $\quad i = 4\% = 0.04$

$$D = (21{,}000{,}000)(0.06) + \frac{(21{,}000{,}000 - 1{,}000{,}000)(0.04)}{(1+0.04)^{20} - 1}$$

D = **P1,931,635.00**

BOND

A bond is a written contract to pay a certain redemption value C on a specified redemption date and to pay equal dividends D periodically.

Elements of Bond

F = face value or par value of the bond
C = redemption value on a specified redemption date
r = bond rate or dividend rate
D = periodic dividend

$$D = F \times r$$

i = investor's interest rate of return
P = price of the bond at a given interest i

- A bond is said to be redeemable at par if the redemption value C equals the face value F.
- A bond is said to be redeemable at a premium if $C > F$.
- A bond is said to be redeemable at a discount if $C < F$.

Price of a bond at a given i

$$P = \frac{C}{(1+i)^n} + \frac{D[(1+i)^n - 1]}{(1+i)^n i}$$

Example 7 - 4

A P100,000.00, 6% bond, pays dividends semi-annually and will be redeemed at 110% on July 1,

1999. Find its price if bought on July 1, 1996, to yield an investor 4%, compounded semi-annually.

Solution

Face value of the bond, F = P100,000.00
Redemption value, C = 10%(100,000) = P110,000.00
Bond rate, $r = 0.06/2 = 0.03$
Periodic dividend, $D = F \times r = 100{,}000(0.03)$
D = P3,000.00
Investor's rate of return (per semi-annual),
$i = 0.04/2 = 0.02$
Number of dividends = $n = 3(2) = 6$

$$P = \frac{C}{(1+i)^n} + \frac{D[(1+i)^n - 1]}{(1+i)^n i}$$

$$P = \frac{110{,}000}{(1+0.02)^6} + \frac{3000[(1+0.02)^6 - 1]}{(1+0.02)^6(0.02)}$$

P = **P114,481.14**

BREAK-EVEN ANALYSIS

Break-even analysis is a method of determining when costs exactly equal revenue. If the manufactured quantity is less than the break-even quantity, a loss is incurred. If the manufactured quantity is greater than the break-even quantity, a profit is incurred.

Elements and Formulas

f = fixed cost which does not vary with production
a = an incremental cost which is the cost to produce one additional item. It may also be called the marginal cost or differential cost.
N = break-even point or quantity produced and sold for break-even
p = incremental revenue or selling price per unit
R = total revenue

$$R = pN$$

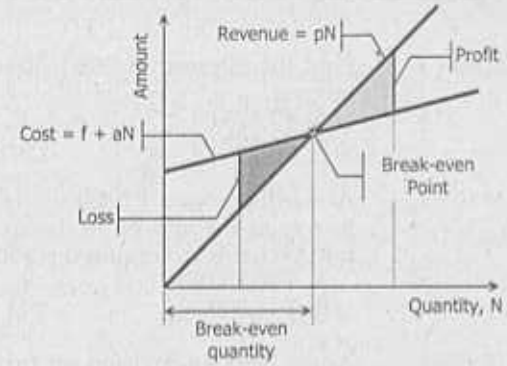
C = total cost

$$C = f + aN$$

Assuming there is no change in inventory, the break-even point can be found from:

$$\text{Cost, } C = \text{Revenue, } R$$
$$f + aN = pN$$

$$N = \frac{f}{p-a}$$



Example 7 - 5

The cost of producing a computer diskette is as follows: Material cost is P7.00 each, labor cost is P2.00 each, and other expense is P1.50 each. If the fixed expenses is P69,000.00 per month, how many diskettes must be produced each month for break-even if each diskettes is worth P45.00?

Solution

Given f = fixed cost = P69,000 per month
a = marginal cost = P7.00 + P2.00 + P1.50
a = P 10.50
p = marginal revenue = P45.00

$$N = \frac{f}{p-a}$$

$$N = \frac{69{,}000}{45 - 10.5}$$

N = **2000 diskettes per month**

Problems - Set 10
Simple and Compound Interest

PROBLEM 10 - 1

Find the interest on P6800.00 for 3 years at 11% simple interest.

A. P1,875.00 C. P2,144.00
B. P1,987.00 D. P2,244.00

PROBLEM 10 - 2

A man borrowed P10,000.00 from his friend and agrees to pay at the end of 90 days under 8% simple interest rate. What is the required amount?

A. P10,200.00 C. P9,500.00
B. P11,500.00 D. P10,700.00

PROBLEM 10 - 3
EE Board

Annie buys a television set from a merchant who offers P 25,000.00 at the end of 60 days. Annie wishes to pay immediately and the merchant offers to compute the required amount on the assumption that money is worth 14% simple interest. What is the required amount?

A. P20,234.87 C. P24,429.97
B. P19,222.67 D. P28,456.23

PROBLEM 10 - 4

What is the principal amount if the amount of interest at the end of 2½ year is P4500 for a simple interest of 6% per annum?

A. P 35,000.00 C. P 40,000.00
B. P 30,000.00 D. P 45,000.00

PROBLEM 10 - 5

How long must a P40,000 note bearing 4% simple interest run to amount to P 41,350.00?

A. 340 days C. 304 days
B. 403 days D. 430 days

PROBLEM 10 - 6

If P16,000 earns P480 in 9 months, what is the annual rate of interest?

A. 1% C. 3%
B. 2% D. 4%

PROBLEM 10 - 7

A man lends P6000 at 6% simple interest for 4 years. At the end of this time he invests the entire amount (principal plus interest) at 5% compounded annually for 12 years. How much will he have at the end of the 16-year period?

A. P 13,361.20 C. P 13,333.20
B. P 13,633.20 D. P 16,323.20

PROBLEM 10 - 8
CE May 1997

A time deposit of P110,000 for 31 days earns P890.39 on maturity date after deducting the 20% withholding tax on interest income. Find the rate of interest per annum.

A. 12.5% C. 12.25%
B. 11.95% D. 11.75%

PROBLEM 10 - 9
ME April 1998

A bank charges 12% simple interest on a P300.00 loan. How much will be repaid if the load is paid back in one lump sum after three years?

A. P408.00 C. P415.00
B. P551.00 D. P450.00

PROBLEM 10 - 10
CE May 1999

The tag price of a certain commodity is for 100 days. If paid in 31 days, there is a 3% discount. What is the simple interest paid?

A. 12.15% C. 22.32%
B. 6.25% D. 16.14%

PROBLEM 10 - 11

Accumulate P5,000.00 for 10 years at 8% compounded quarterly.

A. P12,456.20 C. P10,345.80
B. P13,876.50 D. P11,040.20

PROBLEM 10 - 12

Accumulate P5,000.00 for 10 years at 8% compounded semi-annually.

A. P10,955.61 C. P9,455.67
B. P10,233.67 D. P11,876.34

PROBLEM 10 - 13

Accumulate P5,000.00 for 10 years at 8% compounded monthly.

A. P15,456.75 C. P14,768.34
B. P11,102.61 D. P12,867.34

PROBLEM 10 - 14

Accumulate P5,000.00 for 10 years at 8% compounded annually.

A. P10,794.62 C. P10,987.90
B. P8,567.98 D. P7,876.87

PROBLEM 10 - 15

How long will it take P1,000 to amount to P1,346 if invested at 6% compounded quarterly?

A. 3 years C. 5 years
B. 4 years D. 6 years

PROBLEM 10 - 16

How long will it take for an investment to double its amount if invested at an interest rate of 6% compounded bi-monthly?

A. 10 years C. 13 years
B. 12 years D. 14 years

PROBLEM 10 - 17 If the compound interest on P3,000.00 in 2 years is P500.00, then the compound interest on P3,000.00 in 4 years is:
A. P956.00 C. P1,125.00
B. P1,083.00 D. P1,526.00

PROBLEM 10 - 18 The salary of Mr. Cruz is increased by 30% every 2 years beginning January 1, 1982. Counting from that date, at what year will his salary just exceed twice his original salary?
A. 1988 C. 1990
B. 1989 D. 1991

PROBLEM 10 - 19 If you borrowed P10,000 from a bank with 18% interest per annum, what is the total amount to be repaid at the end of one year?
A. P11,800.00 C. P28,000.00
B. P19,000.00 D. P10,180.00

PROBLEM 10 - 20 What is the effective rate for an interest rate of 12% compounded continuously?
A. 12.01% C. 12.42%
B. 12.89% D. 12.75%

PROBLEM 10 - 21 How long will it take for an investment to fivefold its amount if money is worth 14% compounded semiannually?
A. 11 C. 13
B. 12 D. 14

PROBLEM 10 - 22 An interest rate of 8% compounded semiannually is how many percent if compounded quarterly?
A. 7.81% C. 7.92%
B. 7.85% D. 8.01%

PROBLEM 10 - 23 A man is expecting to receive P450,000.00 at the end of 7 years. If money is worth 14% compounded quarterly, how much is it worth at present?
A. P125,458.36 C. P162,455.63
B. P147,456.36 D. P171,744.44

PROBLEM 10 - 24 A man has a will of P650,000.00 from his father. If his father deposited an amount of P450,000.00 in a trust fund earning 8% compounded annually, after how many years will the man receive his will?
A. 4.55 years C. 5.11 years
B. 4.77 years D. 5.33 years

PROBLEM 10 - 25 Mr. Adam deposited P120,000.00 in a bank who offers 8% interest compounded quarterly. If the interest is subject to a 14% tax, how much will he receive after 5 years?
A. P178,313.69 C. P170.149.77
B. P153,349.77 D. 175,343.77

PROBLEM 10 - 26 What interest compounded monthly is equivalent to an interest rate of 14% compounded quarterly?
A. 1.15% C. 10.03%
B. 13.84% D. 11.52%

PROBLEM 10 - 27
ME April 1996 What is the present worth of two P100.00 payments at the end of the third and the fourth year? The annual interest rate is 8%.
A. P152.87 C. P187.98
B. P112.34 D. P176.67

PROBLEM 10 - 28
ME April 1996 Consider a deposit of P600.00, to be paid up in one year by P700.00. What are the conditions on the rate of interest, i% per year compounded annually, such that the net present worth of the investment is positive? Assume $i \geq 0$.
A. $0 \leq i < 16.7\%$ C. $12.5\% \leq i < 14.3\%$
B. $0 \leq i < 14.3\%$ D. $16.7\% \leq i \leq 100\%$

PROBLEM 10 - 29
ME April 1996 A firm borrows P2000.00 for 6 years at 8%. At the end of 6 years, it renews the loan for the amount due plus P2000 more for 2 years at 8%. What is the lump sum due.
A. P5,679.67 C. P6,034.66
B. P6,789.98 D. P5,888.77

PROBLEM 10 - 30 At an annual rate of return of 8%, what is the future worth of P1000 at the end of 4 years?
A. P1388.90 C. P1765.56
B. P1234.56 D. P1360.50

PROBLEM 10 - 31
ME Oct. 1997 A student has money given by his grandfather in the amount of P20,000.00. How much money in the form of interest will he get if the money is put in a bank that offers 8% rate compounded annually, at the end of 7 years?
A. P34,276.48 C. P36,276.40
B. P34,270.00 D. P34,266.68

PROBLEM 10 - 32
ME Oct. 1997 If the interest rate on an account is 11.5% compounded yearly, approximately how many years will it take to triple the amount?

A. 11 years
B. 10 years
C. 9 years
D. 12 years

PROBLEM 10 - 33
ME Oct. 1997

The nominal interest rate is 4%. How much is my P10,000.00 worth in 10 years in a continuously compounded account?
A. P13,620.10
B. P13,650.20
C. P14,918.25
D. P13,498.60

PROBLEM 10 - 34
ME Oct. 1997

How much must be invested on January 1, year 1, in order to accumulate P2,000.00 on January 1, year 6 at 6%.
A. P1,295.00
B. P1,695.00
C. P1,495.00
D. P1,595.00

PROBLEM 10 - 35
ME April 1998

If P5000.00 shall accumulate for 10 years at 8% compounded quarterly. Find the compounded interest at the end of 10 years.
A. P6,005.30
B. P6,040.20
C. P6,000.00
D. P6,010.20

PROBLEM 10 - 36
ME April 1998

A sum of P1,000.00 is invested now and left for eight years, at which time the principal is withdrawn. The interest that has accrued is left for another eight years. If the effective annual interest rate is 5%, what will be the withdrawal amount at the end of the 16th year?
A. P706.00
B. P774.00
C. P500.00
D. P799.00

PROBLEM 10 - 37
ME April 1998

It is the practice of almost all bank in the Philippines that when they grant a loan, the interest for one year is automatically deducted from the principal amount upon release of money to a borrower. Let us therefore assume that you applied for a loan with a bank and the P80,000.00 was approved at an interest rate of 14% of which P11,200.00 was deducted and you were given a check of P68,800.00. Since you have to pay the amount of P80,000.00 one year after, what then will be the effective interest rate?
A. 15.90%
B. 16.30%
C. 16.28%
D. 16.20%

PROBLEM 10 - 38
ME April 1998

The amount of P1,500.00 was deposited in a bank account, 20 years ago. Today is worth P3,000.00. Interest is paid semi-annually. Determine the interest rate paid on this account?
A. 3.5%
B. 2.5%
C. 2.9%
D. 4%

PROBLEM 10 - 39
ME April 1998

If the nominal interest rate is 3%, how much is P5,000.00 worth in 10 years in a continuously compounded account?
A. P5750.00
B. P7500.00
C. P6750.00
D. P6350.00

PROBLEM 10 - 40
ME April 1998

A merchant puts in his P2,000.00 to a small business for a period of six years. With a given interest rate on the investment of 15% per year, compounded annually, how much will he collect at the end of the sixth year?
A. P4,400.00
B. P4,200.00
C. P4,390.00
D. P4,626.00

PROBLEM 10 - 41

A person invests P4500 to be collected in 8 years. Given that the interest rate on the investment is 14.5% per year compounded annually, what sum will be collected in 8 years?
A. P13, 678.04
B. P13,294.02
C. P14,888.05
D. P14,234.03

PROBLEM 10 - 42

The following schedule of funds is available to form a sinking fund.

current year (n)	50,000.00
$n+1$	40,000.00
$n+2$	30,000.00
$n+3$	20,000.00

At the end of the fourth year, equipment costing P250,000.00 will have to be purchased as a replacement for old equipment. Money is valued at 20% by the company. At the time of purchase, how much money will be needed to supplement the sinking fund?
A. P12,000.00
B. P11,000.00
C. P10,000.00
D. P9,000.00

PROBLEM 10 - 43
ME Oct. 1995

In year zero, you invest P10,000 in a 15% security for 5 years. During that time, the average annual inflation is 6%. How much, in terms of year zero pesos, will be in the account at the maturity?
A. P15,386.00
B. P15,030.00
C. P13,382.00
D. P 6,653.00

PROBLEM 10 - 44
ME Oct. 1995

A company invests P10,000 today to be repaid in five years in one lump sum at 12% compounded annually. If the rate of inflation is 3% compounded annually, how much profit in present day pesos is realized over five years?
A. P5,626.00
B. P7,623.00
C. P3,202.00
D. P5,202.00

PROBLEM 10 - 45 Compute the effective rate for an interest rate of 16% compounded annually.
A. 16% C. 16.98%
B. 16.64% D. 17.03%

PROBLEM 10 - 46 Compute the effective rate for an interest rate of 16% compounded quarterly.
A. 16% C. 16.98%
B. 16.64% D. 17.03%

PROBLEM 10 - 47 Convert 12% compounded semi-annually to x% compounded quarterly.
A. 11.83% C. 11.23%
B. 11.71% D. 11.12%

PROBLEM 10 - 48 Convert 12% compounded semi-annually to x% compounded monthly.
A. 11.83% C. 11.23%
B. 11.71% D. 11.12%

PROBLEM 10 - 49
ME Oct. 1995
A bank is advertising 9.5% accounts that yield 9.84% annually. How often is the interest compounded?
A. daily C. bi-monthly
B. monthly D. quarterly

PROBLEM 10 - 50
ECE Nov. 1995, Nov. 1998
By the conditions of a will, the sum of P25,000 is left to a girl to be held in a trust fund by her guardian until it amount to P45,000. When will the girl receive the money if the fund is invested at 8% compounded quarterly?
A. 7.42 years C. 7 years
B. 7.25 years D. 6.8 years

PROBLEM 10 - 51
ECE April 1995
A man expects to receive P20,000 in 10 years. How much is that money worth now considering interest at 6% compounded quarterly?
A. P11,025.25 C. P15,678.45
B. P17,567.95 D. P12,698.65

PROBLEM 10 - 52
ECE March 1996
What is the effective rate corresponding to 16% compounded daily? Take 1 year = 360 days.
A. 16.5% C. 17.35%
B. 16.78% D. 17.84%

PROBLEM 10 - 53 What amount will be accumulated by a present investment of P17,200 in 6 years at 2% compounded quarterly?

A. P 19,387.15 C. P19,856.40
B. P 20,456.30 D. P19,232.30

PROBLEM 10 - 54 What rate of interest compounded annually must be received if an investment of P54,000 made now will result in a receipt of P72,000 5 years hence?
A. 5.12% C. 5.92%
B. 5.54% D. 6.24%

PROBLEM 10 - 55 With interest at 6% compounded annually, how much is required 7 years hence to repay an P8 M loan made today?
A. P 12,456,789 C. P 12,029,042
B. P 12,345,046 D. P 12,567,000

PROBLEM 10 - 56 If money is worth 6% compounded annually, what payment 12 years from now is equivalent to a payment of P7000 9 years from now?
A. P8765.10 C. P8337.10
B. P8945.20 D. P8234.60

PROBLEM 10 - 57 If money is worth 6% compounded annually, how much can be loaned now if P6000 will be repaid at the end of 8 years?
A. P3567.30 C. P3456.34
B. P3444.44 D. P3764.50

PROBLEM 10 - 58 A person invests P4500 to be collected in 8 years. Given that the interest rate on the investment is 14.5% per year, compounded annually, what sum, in pesos, will be collected eight years hence?
A. P4504 C. P13294
B. P9720 D. P10140

PROBLEM 10 - 59
CE Nov. 1996
If P500,000 is deposited at a rate of 11.25% compounded monthly, determine the compounded interest after 7 years and 9 months.
A. 660,592 C. 680,686
B. 670,258 D. 690,849

PROBLEM 10 - 60
CE May 1996
P200,000 was deposited on Jan. 1, 1988 at an interest rate of 24% compounded semi-annually. How much would the sum on Jan. 1, 1993?
A. P421,170 C. P401,170
B. P521,170 D. P621,170

ANSWER SHEET

1.	A	B	C	D	E	21.	A	B	C	D	E	41.	A	B	C	D	E
2.	A	B	C	D	E	22.	A	B	C	D	E	42.	A	B	C	D	E
3.	A	B	C	D	E	23.	A	B	C	D	E	43.	A	B	C	D	E
4.	A	B	C	D	E	24.	A	B	C	D	E	44.	A	B	C	D	E
5.	A	B	C	D	E	25.	A	B	C	D	E	45.	A	B	C	D	E
6.	A	B	C	D	E	26.	A	B	C	D	E	46.	A	B	C	D	E
7.	A	B	C	D	E	27.	A	B	C	D	E	47.	A	B	C	D	E
8.	A	B	C	D	E	28.	A	B	C	D	E	48.	A	B	C	D	E
9.	A	B	C	D	E	29.	A	B	C	D	E	49.	A	B	C	D	E
10.	A	B	C	D	E	30.	A	B	C	D	E	50.	A	B	C	D	E
11.	A	B	C	D	E	31.	A	B	C	D	E	51.	A	B	C	D	E
12.	A	B	C	D	E	32.	A	B	C	D	E	52.	A	B	C	D	E
13.	A	B	C	D	E	33.	A	B	C	D	E	53.	A	B	C	D	E
14.	A	B	C	D	E	34.	A	B	C	D	E	54.	A	B	C	D	E
15.	A	B	C	D	E	35.	A	B	C	D	E	55.	A	B	C	D	E
16.	A	B	C	D	E	36.	A	B	C	D	E	56.	A	B	C	D	E
17.	A	B	C	D	E	37.	A	B	C	D	E	57.	A	B	C	D	E
18.	A	B	C	D	E	38.	A	B	C	D	E	58.	A	B	C	D	E
19.	A	B	C	D	E	39.	A	B	C	D	E	59.	A	B	C	D	E
20.	A	B	C	D	E	40.	A	B	C	D	E	60.	A	B	C	D	E

Solutions to Set 10
Simple and Compound Interest

SOLUTION 10 - 1
Ans: D

$I = Prt = 6800(0.11)(3) =$ **P2,244.00**

SOLUTION 10 - 2
Ans: A

$F = P(1 + rt)$

$P =$ P10,000; $t = \frac{90}{360} = 0.25$ year; $r = 8\% = 0.08$

$F = 10000(1 + 0.08\times0.25) =$ **P10,200.00**

SOLUTION 10 - 3
Ans: C

$F = P(1 + rt)$

$F =$ P25,000.00; $r = 14\% = 0.14$; $t = \frac{60}{360} = 1/6$

$25{,}000 = P[1 + 0.14\times(1/6)]$; $P =$ **P24,429.97**

SOLUTION 10 - 4
Ans: B

$I = Prt$

$4500 = P(0.06)(2.5)$; $P =$ **P 30,000**

SOLUTION 10 - 5
Ans: C

$F = P(1 + rt)$

$41350 = 40000[1 + 0.04t]$; $t = 0.84375$ year = **303.75**

SOLUTION 10 - 6
Ans: D

$I = Prt$

$480 = 16000 \times r \times (9/12)$; $r = 0.04 = 4\%$

SOLUTION 10 - 7
Ans: A

For the simple interest:

$F = P(1 + rt) = 6000[1 + 0.06(4)] =$ P7,440.00

For the compound interest:

$F = P(1+i)^n = 7440(1 + 0.05)^{12} =$ **13,361.2**

SOLUTION 10 - 8
Ans: D

$I = Prt$; $I - 20\%I = 890.39$; $I =$ P1,112.99

$1{,}112.99 = 110{,}000(r)(31/360)$; $r = 0.1175 =$ **11.75%**

SOLUTION 10 - 9
Ans: A

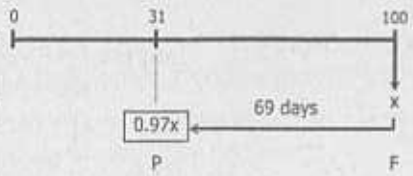
$F = P(1 + rt) = 300[1 + 0.12(3)] =$ **P408.00**

SOLUTION 10 - 10
Ans: D

$F = P(1 + rt)$

$x = 0.97x\left[1 + r\frac{69}{360}\right]$

$r = 0.1614 =$ **16.14%**



SOLUTION 10 - 11
Ans: D

$F = P(1+i)^n$

$P = P5{,}000.00;\ i = 0.08/4 = 0.02;\ n = 10(4) = 40$

$F = 5000(1 + 0.02)^{40} =$ **P11,040.20**

SOLUTION 10 - 12
Ans: A

$F = P(1+i)^n$

$P = P5{,}000.00;\ n = 10(2) = 20;\ i = 0.08/2 = 0.04$

$F = 5000(1 + 0.04)^{20} =$ **P10,955.62**

SOLUTION 10 - 13
Ans: B

$F = P(1+i)^n$

$P = P5{,}000.00;\ n = 10(12) = 120;\ i = 0.08/12 = 0.00667$

$F = 5000(1 + 0.00667)^{120} =$ **P11,102.61**

SOLUTION 10 - 14
Ans: A

$F = P(1+i)^n$

$P = P5{,}000.00;\ n = 10;\ i = 0.08$

$F = 5000(1 + 0.08)^{10} =$ **P10,794.62**

SOLUTION 10 - 15
Ans: C

Let x be the number of years required.

$F = P(1+i)^n$

$F = P1346;\ P = P1000$

$i = 0.06/4 = 0.015;\ n = 4x$

$1346 = 1000(1 + 0.015)^{4x}$

$1.015^{4x} = 1.346$

$\ln 1.015^{4x} = \ln 1.346 = 4x \ln 1.015$

$x = 4.99$ years $\approx$ **5 years**

SOLUTION 10 - 16
Ans: B

$P = P(1+i)^n$, $F = 2P$ (double the amount)

Let x be the number of years,
then $n = 6x$, and $i = 0.06/6 = 0.01$

$2P = P(1 + 0.01)^{6x}$

$2 = 1.01^{6x}$ taking the logarithm on both side

$\ln 2 = \ln 1.01^{6x} = 6x \ln 1.01$

$x = 11.61$ years say **12 years**

SOLUTION 10 - 17
Ans: B

Interest, $I = F - P = P(1+i)^n - P$

$500 = 3000(1 + i)^2 - 3000$

$(1 + i)^2 = 1.1667;\ i = 0.08012$

Then $I_4 = 3000(1 + 0.08012)^4 - 3000 =$ **P1,083.00**

SOLUTION 10 - 18
Ans: A

Let P be the original salary (1982)

This problem can be solved using the compound amount formula $F = P(1+i)^n$, where $F = 2P$ and $i = 0.3$, n = number of times his salary was increased by 30% (every two years)

$2P = P(1 + 0.3)^n$

$2 = 1.03^n$ taking the logarithm on both sides

$\ln 2 = \ln 1.3^n = n \ln 1.3;\ \ n = 2.64$ say 3

Thus, his salary was changed 3 times or for 3(2) = 6 years. Thus his salary exceed twice his original salary in 1988

SOLUTION 10 - 19
Ans: A

$F = P(1+i)^n = 10{,}000(1 + 0.18)^1 =$ **P11,800.00**

SOLUTION 10 - 20
Ans:

$ER = e^r - 1 = (2.71828)^{0.12} - 1 = 0.1275 =$ **12.75%**

SOLUTION 10 - 21
Ans: B

Let x be the number of years, then $n = 2x$

$F = P(1+i)^n$ where $F = 5P$ and $i = 0.14/2 = 0.07$

$5P = P(1 + 0.07)^{2x}$

$5 = 1.07^{2x}$ taking the logarithm on both sides

$\ln 5 = \ln 1.07^{2x} = 2x \ln 1.07$

$x = 11.89$ years say **12 years**

SOLUTION 10 - 22
Ans: C

$ER_Q = ER_S$

$(1 + r/4)^4 - 1 = (1 + 0.08/2)^2 - 1;\ \ r = 0.0792 =$ **7.92%**

SOLUTION 10 - 23
Ans: D

$$P = \frac{F}{(1+i)^n} = \frac{450{,}000}{(1+0.14/4)^{7(4)}} = \textbf{P171,744.44}$$

SOLUTION 10 - 24
Ans: B

Let x be the number of years, then $n = x$

$F = P(1+i)^n;\ i = 0.08$

$650000 = 450000(1 + 0.08)^x$

$1.444 = 1.08^x$ taking the logarithm on both sides

$\ln 1.444 = \ln 1.08^x = x \ln 1.08;\ \ x =$ **4.77 years**

SOLUTION 10 - 25
Ans: C

Interest $= F - P = P(1+i)^n - P$

Interest $= 120000(1 + 0.08/4)^{5(4)} - 120000 =$ P58,313.69

Net interest = P58,313.69 - 14%(58,313.69) = P50,149.77

Net amount after 5 years = 120000 + P50,149.77

Net amount after 5 years = **P170,149.77**

SOLUTION 10 - 26
Ans: B

$ER_{monthly} = ER_{quarterly}$

$(1 + r/12)^{12} - 1 = (1 + 0.14/4)^4 - 1;\ \ r = 0.1384 = \mathbf{13.84\%}$

SOLUTION 10 - 27
Ans: A

$P = P_3 + P_4$

Present worth, $P = F/(1+i)^n$

$$P = \frac{100}{(1+0.08)^3} + \frac{100}{(1+0.08)^4} = \mathbf{P152.87}$$

SOLUTION 10 - 28
Ans: A

Net present Worth, $P = P_{of\ 700} - 600 > 0$

$$\frac{700}{(1+i)^1} - 600 > 0$$

$7/6 > 1 + i$ or $i < 16.7\%$

Therefore; for $P > 0$, $0 \le i < \mathbf{16.7\%}$

SOLUTION 10 - 29
Ans: C

Future worth of P2000 for 6 years at 8% = $2000(1 + 0.08)^6$
= P3173.75

New capital after 6 years = 3173.75 + 2000 = 5173.75

Lump sum due after 2 years at 8%:

$F = 5173.75(1 + 0.08)^2 = \mathbf{6034.66}$

SOLUTION 10 - 30
Ans: D

$F = P(1+i)^n = 1000(1 + 0.08)^4 = \mathbf{P1360.50}$

SOLUTION 10 - 31
Ans: A

$F = P(1+i)^n = 20000\,(1 + 0.08)^7 = \mathbf{P34,276.48}$

SOLUTION 10 - 32
Ans: B

The future worth F of an amount P at an interest rate i for n periods is:

$F = P(1 + i)^n$

where $i = 0.115/1$, $n = t \times 1 = t$ and $F = 3P$ (triple)

$3P = P(1 + 0.115)^t$

$1.115^t = 3$

$\log(1.115)^t = \log(3)$

$t \times \log(1.115) = \log(3)$; $t = \mathbf{10.09\ years}$

SOLUTION 10 - 33
Ans: C

The future worth of P at a nominal interest rate r compounded continuously for t years is:

$F = P e^{rt}$

$F = 10{,}000 \times e^{(0.04)(10)}$

$F = \mathbf{P14,918.25}$

SOLUTION 10 - 34
Ans: C

The amount required is $P = \dfrac{F}{(1+i)^n}$

Where F = P2,000, $i = 0.06$ and $n = 6 - 1 = 5$.

$$P = \frac{2{,}000}{(1+0.06)^5} = \mathbf{P1,494.52}$$

SOLUTION 10 - 35
Ans: B

The future worth of P5000 for 10 years is:

$F = P(1 + i)^n$

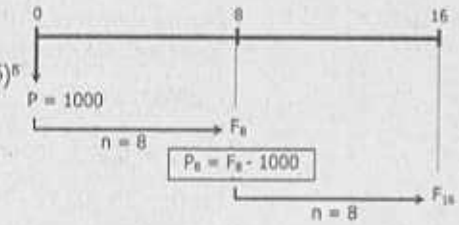
$F = 5000(1 + 0.08/4)^{10(4)} = $ P11,040.2

The compound interest is:

$I = F - P = $ P11,040.2 - P5,000 = **P6,040.2**

SOLUTION 10 - 36
Ans: A

The conditions in the problem is shown in the cash flow diagram below.



$F_8 = P(1 + i)^n$

$F_8 = 1000(1 + 0.05)^8$

$F_8 = $ P1,477.455

$P_8 = F_8 - 1000$

$P_8 = $ P477.455

$F_{16} = P_8(1 + i)^n$

$F_{16} = 477.455(1 + 0.05)^8 = \mathbf{P705.42}$

SOLUTION 10 - 37
Ans: C

The amount F to be paid at the end of one year is P80,000
The total principal amount P is P80,000 - P11,200 = P68,800.00

$F = P(1 + i)^n$

$80{,}000 = 68{,}800(1 + i)^1$

$1.1628 = 1 + i$

$i = 0.1628 = \mathbf{16.28\%}$

SOLUTION 10 - 38
Ans: A

Setting 20 years ago as present time, then;

$F = P(1 + i)^n$; $n = 20(2) = 40$, $i = r/2$

$3000 = 1500(1 + i)^{40}$

$i = 0.0174 = r/2$

$r = 0.0349 = \mathbf{3.49\%}$

SOLUTION 10 - 39
Ans: C

The future worth of P at an interest rate of r compounded continuously for t years is given by the formula $F = P e^{rt}$, where $e = 2.71828\ldots$

$F = 5000 \times e^{(0.03)(10)}$

$F = \mathbf{P6749.30}$

SOLUTION 10 - 40
Ans: D

$F = P(1 + i)^n = 2{,}000(1 + 0.15)^6 = \text{P}4{,}626.00$

SOLUTION 10 - 41
Ans: B

$F = P(1+i)^n = 4500(1 + 0.145)^8 = \textbf{P13,294.02}$

SOLUTION 10 - 42
Ans: C

Future worth of all funds (on the fourth year)

$F = 50000(1+0.2)^4 + 40000(1+0.2)^3 + 30000(1+0.2)^2 + 20000(1+0.2)^1$

$F = \text{P}240{,}000.00$

Additional amount needed = 250,000 - 240,000

Additional amount needed = **P10,000.00**

SOLUTION 10 - 43
Ans: B

Future worth of P10,000 without inflation:

$F = 10000(1 + 0.15)^5 = \text{P}20{,}113.57$

Present worth of P20,113.57 considering 6% inflation

$P = 20113.57/(1 + 0.06)^5 = \textbf{P15,030.03}$

SOLUTION 10 - 44
Ans: D

Future worth of P10,000:

$F = 10000(1 + 0.12)^5 = \text{P}17{,}623.42$

Present worth of P17,623.42 considering 3% inflation rate:

$P = 17{,}623.42/(1 + 0.03)^5 = \text{P}15{,}202.11$

Profit = 15,202.11 - 10,000 = **P5,202.11**

SOLUTION 10 - 45
Ans: A

$ER = \left(1 + \frac{0.16}{1}\right)^1 - 1 = 0.16 = \textbf{16\%}$

SOLUTION 10 - 46
Ans: C

$ER = (1 + 0.16/4)^4 - 1 = 0.1698 = \textbf{16.98\%}$

SOLUTION 10 - 47
Ans: A

Two nominal rates are equal if their effective rates are equal

$ER_{\text{semiannually}} = ER_{\text{quarterly}}$

$(1 + 0.12/2)^2 - 1 = (1 + x/4)^4 - 1$

$(1 + 0.25x)^4 = 1.06^2;\ \ x = 0.1183 = \textbf{11.83\%}$

SOLUTION 10 - 48
Ans: B

$ER_{\text{semiannually}} = ER_{\text{monthly}}$

$(1 + 0.12/2)^2 - 1 = (1 + x/12)^{12} - 1$

$(1 + 0.08333x)^{12} = 1.06^2$

$x = 0.1171 = \textbf{11.71\%}$

SOLUTION 10 - 49
Ans: D

$ER = \left(1 + \frac{r}{m}\right)^m - 1$

Nominal rate, $r = 9.5\%$; effective rate = 9.84%

$0.0984 = (1 + 0.095/m)^m - 1$

By trial and error using the choices, $m = 4$ **(quarterly)**

SOLUTION 10 - 50
Ans: A

$F = P(1+i)^n$

Let x = number of years required:

$F = \text{P}45{,}000;\ i = 0.08/4 = 0.02;\ P = \text{P}25{,}000;\ n = 4x$

$45{,}000 = 25{,}000(1 + 0.02)^{4x}$

$1.02^{4x} = 1.8$

$\ln(1.02)^{4x} = \ln(1.8);\quad 4x \times \ln(1.02) = \ln(1.8)$

$4x = \frac{\ln(1.8)}{\ln(1.02)};\ x = \textbf{7.42 years}$

SOLUTION 10 - 51
Ans: A

$F = P(1+i)^n$

$20000 = P(1 + 0.06/4)^{10(4)};\ \ P = \textbf{P11,025.25}$

SOLUTION 10 - 52
Ans: C

$ER = (1 + r/m)^m - 1$

$ER = (1 + 0.16/360)^{360} - 1 = 0.1735 = \textbf{17.35\%}$

SOLUTION 10 - 53
Ans: A

$F = P(1+i)^n = 17200(1 + 0.02/4)^{6(4)} = \textbf{P19,387.15}$

SOLUTION 10 - 54
Ans: C

$F = P(1+i)^n;\ \ 72000 = 54000(1 + i)^5$

$(1 + i)^5 = 1.333;\ i = 0.0592 = \textbf{5.92\%}$

SOLUTION 10 - 55
Ans: C

$F = P(1+i)^n = 8000000(1 + 0.06)^7 = \textbf{P12,029,042}$

SOLUTION 10 - 56
Ans: C

$P = F/(1+i)^n;\ \ P(P/F,6\%,12) = P(P/F,6\%,9)$

$F/(1 + 0.06)^{12} = 7000/(1 + 0.06)^9 = \textbf{P8337.10}$

SOLUTION 10 - 57
Ans: D

$P = F/(1+i)^n = 6000/(1 + 0.06)^8 = \textbf{P3764.50}$

SOLUTION 10 - 58
Ans: C

$F = F(P/F,i,n) = P(1 + i)^n = 4500(1 + 0.145)^8$

$F = \textbf{P13294.00}$

SOLUTION 10 - 59
Ans: D

Compound interest = $I = F - P$

$I = P(1 + i)^n - P$

$n = 7(12) + 9 = 93;\ i = 0.1125/12 = 0.009375$

$I = 500000(1 + 0.009375)^{93} - 500000 = \textbf{P690,848.73}$

SOLUTION 10 - 60
Ans: D

Present worth, $P = \text{P}200{,}000.00$

Future worth, $F = P(1 + i)^n$

$F = 200000(1 + 0.24/2)^{5(2)} = \textbf{P621169.64}$

Problems - Set 11
Compound Interest, Annuity

PROBLEM 11 - 1
ECE Nov. 1996

Find the nominal rate that if converted quarterly could be used instead for 15% compounded semiannually?

A. 14.93% C. 15.56%
B. 14.73% D. 15.90%

PROBLEM 11 - 2
CE Nov. 1999

Which of the following has the least effective annual interest rate?

A. 12% compounded quarterly
B. 11.5% compounded monthly
C. 11.7% compounded semi-annually
D. 12.2% compounded annually

PROBLEM 11 - 3
CE Nov. 1998

One hundred thousand pesos was placed in a time deposit that earns 9% compounded quarterly, tax free. After how many years would it be able to earn a total interest of fifty thousand pesos?

A. 4.56 C. 4.32
B. 4.47 D. 4.63

PROBLEM 11 - 4
ECE Nov. 1996

The amount of P2,825.00 in 8 years at 5% compounded quarterly is:

A. P4,166.77 C. P4,188.56
B. P4,397.86 D. P4,203.97

PROBLEM 11 - 5

The amount of P2,825.00 in 8 years at 5% compounded continuously is:

A. P4,166.77 C. P4,397.86
B. P4,188.56 D. P4,214.40

PROBLEM 11 - 6
CE May 1998

What rate (%) compounded quarterly is equivalent to 6% compounded semi-annually?

A. 5.93 C. 5.96
B. 5.99 D. 5.9

PROBLEM 11 - 7
ECE April 1998

The amount of P12,800 in 4 years at 5% compounded quarterly is:

A. P 15,614.59 C. P 16,311.26
B. P 14,785.34 D. P 15,847.33

PROBLEM 11 - 8
ECE Nov. 1997

Fifteen percent (15%) when compounded semi-annually will have effective rate of:

A. 15.93% C. 16.02%
B. 18.78% D. 15.56%

PROBLEM 11 - 9
ECE Nov. 1997

What rate of interest compounded annually is the same as the rate of interest of 8% compounded quarterly?

A. 8.24% C. 6.88%
B. 8.42% D. 7.90%

PROBLEM 11 - 10
ECE Nov. 1997

How long will it take the money to triple itself if invested at 10% compounded semi-annually?

A. 13.3 years C. 11.9 years
B. 11.3 years D. 12.5 years

PROBLEM 11 - 11
ECE Nov. 1997

What is the accumulated amount after three (3) years of P6,500.00 invested at the rate of 12% per year compounded semi-annually?

A. P9,500.00 C. P9,221.00
B. P9,321.00 D. P9,248.00

PROBLEM 11 - 12
ECE Nov. 1997

What interest rate, compounded monthly is equivalent to 10% effective rate?

A. 9.75% C. 9.68%
B. 9.50% D. 9.57%

PROBLEM 11 - 13
ECE Nov. 1997

A man wishes his son to receive P500,000.00 ten years from now. What amount should he invest now if it will earn interest of 12% compounded annually during the first 5 years and 15% compounded quarterly during the next 5 years?

A. P135,868.19 C. P123,433.23
B. P134,678.90 D. P145,345.34

PROBLEM 11 - 14
ME Oct. 1997

A savings association pays 4% interest quarterly. What is the effective annual interest rate?

A. 18.045% C. 16.985%
B. 17.155% D. 17.230%

PROBLEM 11 - 15
ME Oct. 1997

A bank offers 1.2% effective monthly interest. What is the effective annual rate with monthly compounding?

A. 15.4% C. 14.4%
B. 8.9% D. 7.9%

PROBLEM 11 - 16
ME Oct. 1997

What is the present worth of P27,000.00 due in 6 years if money is worth 13% and is compounded semi-annually?

A. P12,681.00 C. P15,250.00
B. P13,500.00 D. P21,931.00

PROBLEM 11 - 17
ME Oct. 1997

A student deposits P1,500.00 in a 9% account today. He intends to deposit another P3,000.00 at the end of two years. He plans to purchase in five years his favorite shoes worth P5,000.00. Calculate the money that will be left in his account one year after the purchase.

A. P1,280.00 C. P1,300.00
B. P1,250.00 D. P1,260.00

PROBLEM 11 - 18

If money is worth 4% compounded monthly, what payment at the end of each quarter will replace payments of P 500.00 monthly?

A. P 1,500.00 C. P 1,505.00
B. P 1,525.00 D. P 1,565.00

PROBLEM 11 - 19

What amount would have to be invested at the end of each year for the next 8 years at 4% compounded semi-annually in order to have P5,000.00 at the end of the time?

A. P 541.86 C. P 542.64
B. P 553.82 D. P 548.23

PROBLEM 11 - 20

A contractor bought a concrete mixer at P120,000.00 if paid in cash. The mixer may also be purchased by installment to be paid within 5 years. If money is worth 8%, the amount of each annual payment, if all payments are made at the beginning of each year, is:

A. P27,829.00 C. P31,005.00
B. P29,568.00 D. P32,555.00

PROBLEM 11 - 21

A contract calls for semiannual payments of P40,000.00 for the next 10 years and an additional payment of P250,000.00 at the end of that time. Find the equivalent cash value of the contract at 7% compounded semiannually?

A. P444,526.25 C. P694,138.00
B. P598,589.00 D. P752,777.00

PROBLEM 11 - 22

Mr. Rankine bought a piece of property for P100,000.00 down payment and 10 deferred semiannual payments of P8,000.00 starting 3 years from now. If the interest rate is 12% compounded semiannually, what is the present worth of the property?

A. P143,999.08 C. P152,444.05
B. P148,555.02 D. P166,888.01

PROBLEM 11 - 23
ECE Nov. 1996

If money is worth 8%, determine the present value of a perpetuity of P1,000 payable annually, with the first payment due at the end of 5 years.

A. P9,187.87 C. P9,272.64
B. P9,178.87 D. P9,272.46

PROBLEM 11 - 24

A man is left with an inheritance from his father. He has an option to receive P2 M at the end of 10 years, however he wishes to receive the money at the end of each year for 5 years. If interest rate is 8%, how much would he receive every year?

A. P400,000.00 C. P 232,020.00
B. P352,533.00 D. P 200,000.00

PROBLEM 11 - 25
CE Nov. 1999

To maintain its newly acquired equipment, the company needs P40,000 per year for the first five years and P60,000 per year for the next five years. In addition, an amount of P140,000 would also be needed at the end of the fifth and the eighth years. At 6%, what is the present worth of these costs?

A. P689,214 C. P549,812
B. P512,453 D. P586,425

PROBLEM 11 - 26

A man receives P125,000.00 credit for his old car when buying a new model costing P375,000.00. What cash payment will be necessary so that the balance can be liquidated by payments of P 12,500.00 at the end of each month for 18 months when interest is charged at the rate of 6% compounded monthly?

A. P23,400.00 C. P33,650.00
B. P28,750.00 D. P35,340.00

PROBLEM 11 - 27

Determine the present worth of an annual payment of P2500.00 at the end of each year for 12 years at 8% compounded annually.

A. P18,840.20 C. P15,000.00
B. P30,000.00 D. P17,546.04

PROBLEM 11 - 28

A man borrowed P200,000.00 from a bank at 12% compounded monthly, which is payable monthly for 10 years (120 payments). If the first payment is to be made after 3 months, how much is the monthly payment?

A. P2,869.42 C. P3,013.10
B. P2,927.10 D. P3,124.12

PROBLEM 11 - 29

What is the present worth of a P1000.00 annuity over a 10-year period, if interest rate is 8%?

A. P7896.00 C. P6234.80
B. P8976.00 D. P6710.00

PROBLEM 11 - 30
ME Oct. 1995

How much money must you invest today in order to withdraw P 1000 per year for 10 years if interest rate is 12%?

A. P5650.00 C. P5560.00
B. P6550.00 D. P7550.00

PROBLEM 11 - 31

A machine is under consideration for investment. The cost of the machine is P25,000. Each year it operates, the machine will generate a savings of P15,000. Given an effective annual interest of 18%, what is the discounted payback period, in years, on the investment in the machine?

A. 1.566 C. 2.155
B. 2.233 D. 2.677

PROBLEM 11 - 32
ME April 1996

What is the present worth of a P100 annuity starting at the end of the third year and continuing to the end of the fourth year, if the annual interest rate is 8%.

A. 153.44 C. 154.99
B. 152.89 D. 156.33

PROBLEM 11 - 33

Consider a project which involves the investment of P100,000 now and P100,000 at the end of one year. Revenues of P150,000 will be generated at the end of years 1 and 2. What is the net present value of this project if the effective annual interest rate is 10%

A. P65,421.50 C. P68,421.50
B. P67,421.50 D. P69,421.50

PROBLEM 11 - 34

An investment of x pesos is made at the end of each year for three years, at an interest rate of 9% per year compounded annually. What will be the value of the investment upon the deposit of the third payment?

A. $3.278x$ C. $3.728x$
B. $3.287x$ D. $3.782x$

PROBLEM 11 - 35
ME Oct. 1995

If P500 is invested at the end of each year for 6 years, at an effective annual interest rate of 7%. What is the total amount available upon the deposit of the 6th payment?

A. P3455.00 C. P3577.00
B. P3544.00 D. P3688.00

PROBLEM 11 - 36

You need a P40,000 per year for four years to go to college. Your father invested P50,000 in a 7% account for your education when you were born. If you withdraw the P40,000 at the end of your 17th, 18th, 19th, and 20th years, how much money will be left in the account at the end of your 21st year?

A. P16,000 C. P18,000
B. P17,000 D. P19,000

PROBLEM 11 - 37

If money is worth 10% yearly, compounded quarterly, what monthly savings is required monthly in order to have P200,000 at the end of 10 years?

A. P918.82 C. P866.99
B. P878.98 D. P981.28

PROBLEM 11 - 38

How much money must you deposit today to an account earning 12% so that you can withdraw P25,000 yearly indefinitely starting at the end of the 10th year?

A. P125,000 C. P73,767
B. P89,456 D. P75,127

PROBLEM 11 - 39
ME April 1996

In five years, P18,000 will be needed to pay for a building renovation. In order to generate this sum, a sinking fund consisting of three annual payments is established now. For tax purposes, no further payments will be made after three years. What payment is necessary if money is worth 15% per annum?

A. P3,345.65 C. P3,919.53
B. P3,789.34 D. P3,878.56

PROBLEM 11 - 40

An investment of P40,000.00 has a revenue of x pesos at the end of the first and second year. Given a discount rate of 15% compounded annually, find x so that the net present worth of the investment is zero.

A. P33,789.54 C. P24,604.65
B. P27,789.78 D. P21,879.99

PROBLEM 11 - 41

Mr. Jones borrowed P150,000 two years ago. The terms of the loan are 10% interest for 10 years with uniform payments. He just made his second annual payment. How much principal does he still owe?

A. P130,235.20 C. P132,456.20
B. P134,567.30 D. P129,456.78

PROBLEM 11 - 42

Given that the discount rate is 15%, what is the equivalent uniform annual cash flow of the following stream of cash flows?

year 0	P 100,000.00
year 1	200,000.00
year 2	50,000.00
year 3	75,000.00

A. P158,124.60 C. P157,345.98
B. P158,897.50 D. P155,789.34

PROBLEM 11 - 43 Mr. Bean borrowed P100,000 at 10% effective annual interest rate. He must pay back the loan over 30 years with uniform monthly payments due on the first day of each month. What does he pay each month?
A. P768.67 C. P856.30
B. P987.34 D. P839.20

PROBLEM 11 - 44
ECE Nov. 1995
An employee obtained a loan of P10,000 at the rate of 6% compounded annually in order to repair a house. How much must he pay monthly to amortize the loan within a period of ten years?
A. P198.20 C. P110.22
B. P150.55 D. P112.02

PROBLEM 11 - 45 What is the accumulated value of a payment of P12,500 at the end of each year for 9 years with interest at 5% compounded annually?
A. P 138,738.05 C. P 178,338.50
B. P 137,832.05 D. P 187,833.50

PROBLEM 11 - 46 What is the accumulated value of a payment of P 6,000 every six months for 16 years with interest at 7% compounded semiannually?
A. P 312,345.00 C. P 347,898.00
B. P 345,678.00 D. P 344,007.00

PROBLEM 11 - 47 A mining property is offered for sale for P5.7 M. On the basis of estimated production, an annual return of P800,000 is foreseen for a period of 10 years. After 10 years, the property will be worthless. What annual rate of return is in prospect?
A. 6.7% C. 5.6%
B. 6.1% D. 5.2%

PROBLEM 11 - 48 If a down payment of P600,000 is made on a house and P80,000 a year for the next 12 years is required, what was the price of the house if money is worth 6% compounded annually?
A. P1,270,707 C. P1,345,555
B. P1,130,450 D. P1,678,420

PROBLEM 11 - 49 What annuity over a 10-year period at 8% interest is equivalent to a present worth of P100,000?
A. P14,903 C. P13,803
B. P15,003 D. P12,003

PROBLEM 11 - 50 Deposits of P1500 each were made in a sinking fund at the end of each year. If the fund earned interest at the rate of 6% per annum compounded quarterly, what was the principal in the fund immediately after the seventh deposit was made?
A. P16,243.23 C. P14,243.00
B. P12,591.00 D. P13,643.23

PROBLEM 11 - 51
CE Nov. 1996,
Nov. 1994
A man inherited a regular endowment of P100,000 every end of 3 months for 10 years. However, he may choose to get a single lump sum payment at the end of 4 years. How much is this lump sum if the cost of money is 14% compounded quarterly?
A. P3,502,546 C. P3,702,939
B. P3,802,862 D. P3,602,431

PROBLEM 11 - 52
CE May 1996
A service car whose cash price was P540,000 was brought with a down payment of P162,000 and monthly of P10,874.29 for 5 years. What was the rate of interest if compounded monthly?
A. 20% C. 24%
B. 26% D. 22%

PROBLEM 11 - 53
CE May 1998
The present value of an annuity of R pesos payable annually for 8 years, with the first payment at the end of 10 years, is P187,481.25. Find the value of R if money is worth 5%.
A. P45,000 C. P42,000
B. P44,000 D. P43,000

PROBLEM 11 - 54
ECE April 1998
How much money must you invest today in order to withdraw P2,000 annually for 10 years if the interest rate is 9%?
A. P 12,385.32 C. P 12,835.32
B. P 12,853.32 D. P 12,881.37

PROBLEM 11 - 55
ECE April 1998
Money borrowed today is to be paid in 6 equal payments at the end of each of 6 quarters. If the interest is 12% compounded quarterly. How much was initially borrowed if quarterly payment is P2000.00?
A. P 10,382.90 C. P 10,834.38
B. P 10,200.56 D. P 10,586.99

PROBLEM 11 - 56
ME Oct. 1997
A car was bought on installment basis with a monthly installment of P10,000.00 for 60 months. If interest is 12% compounded annually calculate the cash price of the car.
A. P455,875.00 C. P678,519.75
B. P567,539.75 D. P345,539.75

PROBLEM 11 - 57
ME Oct. 1997

A steel mill estimates that one of its furnaces will require maintenance P20,000.00 at the end of 2 years, P40,000.00 at the end 4 years and P80,000.00 at the end of 8 years. What uniform semi-annual amounts could it set aside over the next eight years at the end of each period to meet these requirements of maintenance cost if all the funds would earn interest at the rate of 6% compounded semi-annually?

A. P7,897.35 C. P8,897.35
B. P9,397.35 D. P6,897.35

PROBLEM 11 - 58
ME April 1998

A house and lot can be acquired at a down payment of P500,000.00 and a yearly payment of P100,000.00 at the end of each year for a period of 10 years, starting at the end of 5 years from the date of purchase. If money is worth 14% compounded annually, what is the cash price of the property?

A. P810,100.00 C. P808,836.00
B. P801,900.00 D. P805,902.00

PROBLEM 11 - 59
ME April 1998

How much must be deposited at 6% each year beginning on January 1, year 1, in order to accumulate P5,000.00 on the date of the last deposit, January 1, year 6?

A. P751.00 C. P717.00
B. P715.00 D. P725.00

PROBLEM 11 - 60
ME April 1998

A piece of machinery can be bought for P10,000.00 cash, or for P2,000.00 down and payments of P750.00 per year for 15 years. What is the annual interest rate for the time payments?

A. 4.61% C. 3.81%
B. 5.71% D. 11.00%

PROBLEM 11 - 61
ME April 1998

An instructor plans to retire in exactly one year and want an account that will pay him P25,000.00 a year for the next 15 years. Assuming a 6% annual effective interest rate, what is the amount he would need to deposit now? (The fund will be depleted after 15 years.)

A. P249,000.00 C. P242,806.00
B. P248,500.00 D. P250,400.00

PROBLEM 11 - 62

A man invested P 1,000.00 per month on a bank that offers 6% interest. How much can he get after 5 years?

A. P 60,000.00 C. P 72,540.00
B. P 69,770.00 D. P 69,491.00

PROBLEM 11 - 63
CE Nov. 1995

Find the present value in pesos, of a perpetuity of P15,000 payable semi-annually if money is worth 8%, compounded quarterly.

A. P371,287 C. P392,422
B. P386,227 D. P358,477

PROBLEM 11 - 64
CE May 1999,
May 1995

A man paid 10% down payment of P200,000 for a house and lot and agreed to pay the balance on monthly installments for 60 months at an interest rate of 15% compounded monthly. Determine the required monthly payment.

A. P4,282.00 C. P58,477.00
B. P42,822.00 D. P5,848,00

PROBLEM 11 - 65
CE Nov. 1998

A debt of x pesos, with interest rate of 7% compounded annually will be retired at the end of 10 years through the accumulation of deposit in the sinking fund invested at 6% compounded semi-annually. The deposit in the sinking fund every end of six months is P 21,962.68. What is the value of x?

A. P 300,000 C. P 350,000
B. P 250,000 D. P 400,000

ANSWER SHEET

1.	A	B	C	D	E
2.	A	B	C	D	E
3.	A	B	C	D	E
4.	A	B	C	D	E
5.	A	B	C	D	E
6.	A	B	C	D	E
7.	A	B	C	D	E
8.	A	B	C	D	E
9.	A	B	C	D	E
10.	A	B	C	D	E
11.	A	B	C	D	E
12.	A	B	C	D	E
13.	A	B	C	D	E
14.	A	B	C	D	E
15.	A	B	C	D	E
16.	A	B	C	D	E
17.	A	B	C	D	E
18.	A	B	C	D	E
19.	A	B	C	D	E
20.	A	B	C	D	E
21.	A	B	C	D	E
22.	A	B	C	D	E
23.	A	B	C	D	E
24.	A	B	C	D	E
25.	A	B	C	D	E
26.	A	B	C	D	E
27.	A	B	C	D	E
28.	A	B	C	D	E
29.	A	B	C	D	E
30.	A	B	C	D	E
31.	A	B	C	D	E
32.	A	B	C	D	E
33.	A	B	C	D	E
34.	A	B	C	D	E
35.	A	B	C	D	E
36.	A	B	C	D	E
37.	A	B	C	D	E
38.	A	B	C	D	E
39.	A	B	C	D	E
40.	A	B	C	D	E
41.	A	B	C	D	E
42.	A	B	C	D	E
43.	A	B	C	D	E
44.	A	B	C	D	E
45.	A	B	C	D	E
46.	A	B	C	D	E
47.	A	B	C	D	E
48.	A	B	C	D	E
49.	A	B	C	D	E
50.	A	B	C	D	E
51.	A	B	C	D	E
52.	A	B	C	D	E
53.	A	B	C	D	E
54.	A	B	C	D	E
55.	A	B	C	D	E
56.	A	B	C	D	E
57.	A	B	C	D	E
58.	A	B	C	D	E
59.	A	B	C	D	E
60.	A	B	C	D	E
61.	A	B	C	D	E
62.	A	B	C	D	E
63.	A	B	C	D	E
64.	A	B	C	D	E
65.	A	B	C	D	E

Solutions to Set 11
Compound Interest, Annuity

SOLUTION 11 - 1
Ans: B

Let r be the equivalent nominal rate compounded quarterly.

$ER_Q = ER_S \quad (1 + r/4)^4 - 1 = (1 + 0.15/2)^2 - 1$

$r = 0.14729 = \mathbf{14.729\%}$

SOLUTION 11 - 2
Ans: C

Choice A: $ER = (1 + 0.12/4)^4 - 1 = 0.1225 = 12.55\%$

Choice B: $ER = (1 + 0.115/12)^{12} - 1 = 0.12126$

$ER = 12.126\%$

Choice C: $ER = (1 + 0.117/6)^6 - 1 = 0.1204 = 12.04\%$

Choice D: $ER = 12.2\%$

Thus, 11.7% compounded semi-annually has the least effective rate

SOLUTION 11 - 3
Ans: A

Let x be the required number of years:

Interest = $F - 100{,}000 = 50{,}000$

$F = 100{,}000(F/P, 0.09/4, 4x) = 150{,}000$

$100{,}000(1 + 0.09/4)^{4x} = 150{,}000$

$1.0225^{4x} = 1.5$

$(4x) \log 1.0225 = \log 1.5;\ x = \mathbf{4.56}$

SOLUTION 11 - 4
Ans: D

$F = P(1 + i)^n$

$F = 2825(1 + 0.05/4)^{8(4)} = \mathbf{P4203.97}$

SOLUTION 11 - 5
Ans: D

$F = Pe^{rt} = 2825\, e^{(0.05)(8)} = \mathbf{P4214.4}$

SOLUTION 11 - 6
Ans: C

Two nominal rates are equal if they have the same effective rate, then

$ER_{\text{quarterly}} = ER_{\text{semiannually}}$

$$\left(1+\frac{r}{4}\right)^4 - 1 = \left(1+\frac{0.06}{2}\right)^2 - 1$$

$(1 + 0.25r)^4 = 1.0609$

$1 + 0.25r = 1.01489;\ r = 0.0596 = \mathbf{5.96\%}$

SOLUTION 11 - 7
Ans: A

$F = P(1 + i)^n$

$F = 12{,}800\,[1 + 0.05/4]^{4(4)} = \mathbf{P15{,}614.59}$

SOLUTION 11 - 8
Ans: D

Effective rate, $ER = (1 + r/m)^m - 1$, where r is the nominal rate = 15%, and m is the number of compoundings per year = 2 (semi-annually)

$ER = [\,1 + 0.15/2\,]^2 - 1 = 0.1556 =$ **15.56%**

SOLUTION 11 - 9
Ans: A

The rate of interest compounded annually is also the effective rate.

$ER = (\,1 + r/m\,)^m - 1$

$r = 8\%;\ m = 4$ (compounded quarterly)

$ER = [\,1 + 0.08/4\,]^4 - 1 = 0.0824 =$ **8.24%**

SOLUTION 11 - 10
Ans: B

$F = P(1 + i)^n$ where $F = 3P$

t = number of years, then $n = 2t$

$3P = P[\,1 + 0.10 / 2\,]^{2t}$

$3 = (1.05)^{2t}$ Taking the logarithm on both sides

$\ln 3 = \ln (1.05)\ 2t = 2t \ln (1.05);\ \ t =$ **11.258 years**

SOLUTION 11 - 11
Ans: C

Accumulated amount:

$F = P(1 + i)^n = 6{,}500\,[\,1 + (0.12/2)\,]^{3(2)} =$ **P9,220.4**

SOLUTION 11 - 12
Ans: D

Effective rate, $ER = (1 + r/m)^m - 1$

$0.10 = [\,1 + r / 12\,]^{12} - 1$

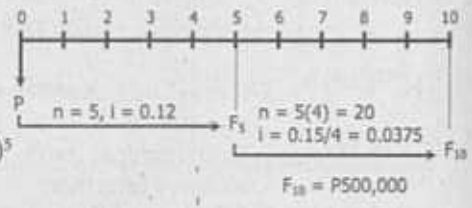
$1.1 = (1 + r/12)^{12}$

$1 + r/12 = \sqrt[12]{1.1} = 1.00797$

$r/12 = 0.00797;\ r = 0.0957 =$ **9.57%**

SOLUTION 11 - 13
Ans: A

From the cash flow diagram shown:



$F_5 = P(1 + i)^n$

$F_5 = P(1 + 0.12)^5$

$F_5 = 1.7623P$

$F_{10} = F_5(1 + i)^n$

$500{,}000 = 1.7623P\,(1 + 0.0375)^{20};\ P =$ **P135,868.19**

SOLUTION 11 - 14
Ans: C

Effective rate, $ER = (1 + r/m)^m - 1$

Note: The interest rate is 4% quarterly (not compounded quarterly). Meaning, every quarter they pay 4%, then the periodic interest $i = r/m = 4\%$

$ER = [\,1 + 0.04\,]^4 - 1 = 0.16985 =$ **16.985%**

SOLUTION 11 - 15
Ans: A

$ER = (1 + r/m)^m - 1$

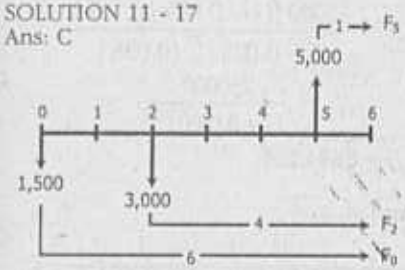
$r/m = 1.2\% = 0.012;\ \ m = 12$

$ER = (1 + 0.012)^{12} - 1 = 0.1539 =$ **15.39%**

SOLUTION 11 - 16
Ans: A

$P = F / (1+i)^n = 27000 / (1 + 0.13/2)^{6(2)} =$ **P12,681.44**

SOLUTION 11 - 17
Ans: C



The cash flow diagram is as follows:

The balance at the end of the 6th year is $F_0 + F_2 - F_5$

Balance = $1500(1 + 0.09)^6$
$+ 3000(1 + 0.09)^4$
$- 5000(1 + 0.09)^1$

Balance = **P1300.39**

SOLUTION 11 - 18
Ans: C

The equivalent quarterly payment is the future worth of three monthly payments of P500.

$$F = \frac{A[(1+i)^n - 1]}{i} = \frac{500[(1 + 0.04/12)^3 - 1]}{0.04/12}$$

$F =$ **P1,505.00**

SOLUTION 11 - 19
Ans: A

$$F = \frac{A[(1+i)^n - 1]}{i} = \text{P5,000.00}$$

Solve for i:

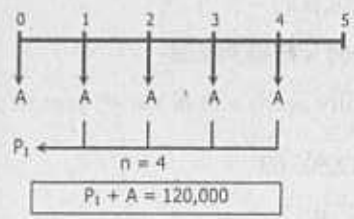
$ER_A = ER_S$

$(1 + i)^1 - 1 = (1 + 0.04/2)^2 - 1;\ i = 0.0404$

$$F = \frac{A[(1+0.0404)^8 - 1]}{0.0404} = 5000$$

$A =$ **P541.87**

SOLUTION 11 - 20
Ans: A



From the diagram shown:

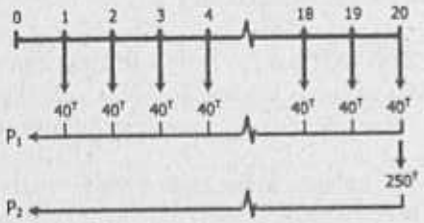
$P_1 + A = 120{,}000$

$$\frac{A[(1+0.08)^4 - 1]}{(1+0.08)^4(0.08)} + A = 120{,}000$$

$3.312A + A = 120{,}000$

$A =$ **P27,829.00**

SOLUTION 11 - 21
Ans: C



Present worth, $P = P_1 + P_2$

$$P = \frac{A[(1+i)^n - 1]}{(1+i)^n i} + \frac{F}{(1+i)^n}$$

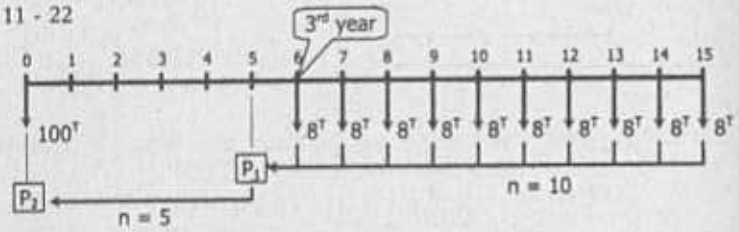
$i = 0.07/2 = 0.035$

$n = 10(2) = 20$

$$P = \frac{40000\,[(1+0.035)^{20} - 1]}{(1+0.035)^{20}(0.035)} + \frac{250000}{(1+0.035)^{20}}$$

P = **P694,138**

SOLUTION 11 - 22
Ans: A



Present worth = P_2 + 100,000

$i = 0.12/2 = 0.06$

$$P_1 = \frac{8000[(1+0.06)^{10} - 1]}{(1+0.06)^{10}(0.06)} = \text{P58,881.00}$$

$$P_2 = \frac{58{,}881}{(1+0.06)^5} = \underline{\text{P43,999.08}}$$

Another way to solve P_2:

$$P_2 = \frac{8000(F/A, 0.06, 10)}{(1+0.06)^{15}} = \frac{\dfrac{8000[(1+0.06)^{10} - 1]}{0.06}}{(1+0.06)^{15}}$$

P_2 = <u>P43,999.08</u>

P = P43,999.08 + 100,000.00 = **P143,999.08**

SOLUTION 11 - 23
Ans: A

The worth of the perpetuity at the end of the 4th year is:

$$P' = \frac{A}{i} = \frac{1000}{0.08} = \text{P12,500.00}$$

The present worth at of P' is:

$$P = \frac{P'}{(1+i)^4} = \frac{12500}{(1+0.08)^4} = \textbf{P 9,187.87}$$

SOLUTION 11 - 24
Ans: C

The present worth of P2M at the end of 10 years is:

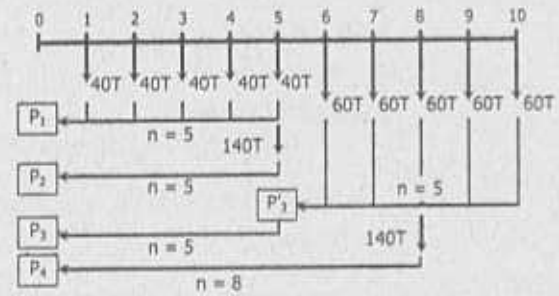
P_{10} = P2,000,000 / $(1 + 0.08)^{10}$ = P926,386.98

The present worth of the annuity of A at the end of each year for 5 years should be equal to P926,386.98.

$$P = \frac{A[(1+0.08)^5 - 1]}{(1+0.08)^5(0.08)} = \text{P926,386.98}$$

A = **P232,019.60**

SOLUTION 11 - 25
Ans: C



$$P = P_1 + P_2 + P_3 + P_4 = P_1 + P_2 + \frac{P'_3}{(1+i)^5} + P_4$$

P = 40000(P/A, 0.06, 5) + 140000(P/F, 0.06, 5) + $\dfrac{60000(P/A, 0.06, 5)}{(1+0.06)^5}$ + 140000(P/F, 0.06, 8)

$$P = \frac{40000[(1+0.06)^5 - 1]}{(1+0.06)^5(0.06)} + \frac{140000}{(1+0.06)^5} + \frac{\dfrac{60000[(1+0.06)^5 - 1]}{(1+0.06)^5(0.06)}}{(1+0.06)^5} + \frac{140000}{(1+0.06)^8}$$

P = P168,494.55 + P104,616.14 + P188,863.39 + P87,837.73

P = **P549,811.81**

SOLUTION 11 - 26
Ans: D

Amount payable (Balance) = P375,000 - P125,000

Amount payable (Balance) = P250,000.00

Present worth of P12,500 payment at the end of each month for 18 months:

$$P = \frac{12500[(1+0.06/12)^{18} - 1]}{(1+0.06/12)^{18}(0.06/12)} = \text{P214,660.00}$$

Cash payment = P250,000 - P214,660 = **P35,340.00**

SOLUTION 11 - 27
Ans: A

$$P = \frac{A[(1+i)^n - 1]}{(1+i)^n i} = \frac{2500[(1+0.08)^{12} - 1]}{(1+0.08)^{12}(0.08)} = \text{P18,840.20}$$

SOLUTION 11 - 28
Ans: B

Since the payment is to be made after three months, the amount of P200,000 at the beginning of the third month (end of second) is $200{,}000(1 + 0.12/12)^2 = \underline{\text{P204,020}}$

$$P = \frac{A[(1+i)^n - 1]}{(1+i)^n i}; \quad i = 0.12/12 = 0.01$$

$$204020 = \frac{A[(1+0.01)^{120} - 1]}{(1+0.01)^{120}(0.01)}; A = \text{P2,927.10}$$

Another Solution

$$200{,}000(F/P, 0.01, 122) = A(F/A, 0.01, 120)$$

$$200{,}000(1 + 0.01)^{122} = \frac{A[(1+0.01)^{120} - 1]}{0.01}$$

$$A = \text{P2,927.10}$$

SOLUTION 11 - 29
Ans: D

$$P = \frac{A[(1+i)^n - 1]}{(1+i)^n i} = \frac{1000[(1+0.08)^{10} - 1]}{(1+0.08)^{10}(0.08)} = \text{P6710.00}$$

SOLUTION 11 - 30
Ans: A

$$P = \frac{A[(1+i)^n - 1]}{(1+i)^n i} = \frac{1000[(1+0.12)^{10} - 1]}{(1+0.12)^{10}(0.12)} = \text{P5650.00}$$

SOLUTION 11 - 31
Ans: C

The discounted payback period is the period where the present worth of all income equals the investment

$P = 25{,}000.00$ (investment) $\quad i = 18\% = 0.18$
$A = 15{,}000$ (income per year)

$$P = \frac{A[(1+i)^n - 1]}{(1+i)^n i}; \quad 25000 = \frac{15000[(1+0.18)^n - 1]}{(1+0.18)^n(0.18)}$$

$$\frac{5}{3} = \frac{1.18^n - 1}{1.18^n 0.18}$$

$1.18^n - 1 = 0.3(1.18)^n$ $\quad \log(1.18)^n = \log 1.4286$
$0.7(1.18)^n = 1$ $\quad n \log 1.18 = \log 1.4286$
$1.18^n = 1.4286$ $\quad n = \mathbf{2.155\ years}$

SOLUTION 11 - 32
Ans: B

$$P = 100(P/F, 0.08, 3) + 100(P/F, 0.08, 4)$$

$$P = \frac{100}{(1+0.08)^3} + \frac{100}{(1+0.08)^4} = \text{P152.89}$$

SOLUTION 11 - 33
Ans: D

Present worth of all revenues:

$$P_r = \frac{A[(1+i)^n - 1]}{(1+i)^n i} = \frac{150000[(1+0.1)^2 - 1]}{(1+0.1)^2(0.1)} = \text{P260,330.58}$$

Present worth of all investment

$$P_i = 100000 + \frac{100000}{(1+0.1)^1} = \text{P190,909.09}$$

Net present worth = 260330.58 - 190090.09 = **P69,421.50**

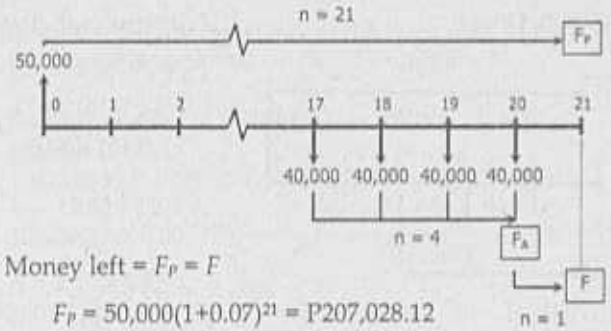
SOLUTION 11 - 34
Ans: A

$$F = x(F/A, 0.09, 3) = \frac{x[(1+0.09)^3 - 1]}{0.09} = \mathbf{3.278}x$$

SOLUTION 11 - 35
Ans: C

$$F = 500(F/A, 0.07, 6) = \frac{500[(1+0.07)^6 - 1]}{0.07} = \mathbf{P3577.00}$$

SOLUTION 11 - 36
Ans: B



Money left = F_P = F

$$F_P = 50{,}000(1+0.07)^{21} = \text{P207,028.12}$$

$$F = F_A(1 + 0.07)^1 = \frac{40000[(1+0.07)^4 - 1]}{0.07}(1 + 0.07)^1$$

$$F = \text{P190,029.56}$$

Money Left = P207,028.12 - P190,029.56 = **P16,998.56**

SOLUTION 11 - 37
Ans: D

F = P200,000; $n = 10(12) = 120$

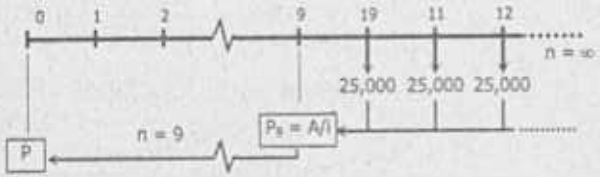
Solve for i (convert 10% compounded quarterly to compounded monthly)

$ER_m = ER_q$
$(1 + i)^{12} - 1 = (1 + 0.1/4)^4 - 1$
$i = 0.00826$

$$F = \frac{A[(1+i)^n - 1]}{i}$$

$$200000 = \frac{A(1+0.00826)^{120} - 1}{0.00826}; A = \text{P981.28}$$

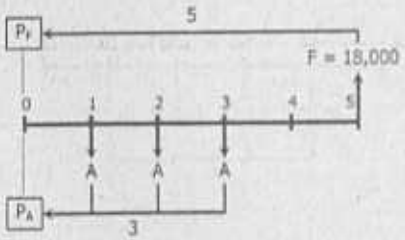
SOLUTION 11 - 38
Ans: D



$$P_9 = \frac{A}{i} = \frac{25000}{0.12} = \text{P208333.33}$$

$$P = \frac{P_9}{(1+i)^n} = \frac{208,333.33}{(1+0.12)^9} = \textbf{P75,127.09}$$

SOLUTION 11 - 39
Ans: C



From the cash flow diagram:

$P_A = P_F$

$$\frac{A[(1+0.15)^3 - 1]}{(1+0.15)^3(0.15)} = \frac{18,000}{(1+0.15)^5}$$

A = P3,919.53

SOLUTION 11 - 40
Ans: C

Present worth = $x(P/A,15\%,2)$ - 40,000 = 0

$$0 = \frac{x[(1+0.15)^2 - 1]}{(1+0.15)^2(0.15)} - 40,000$$

x = **P24,604.65**

SOLUTION 11 - 41
Ans: A

Annual payment, A: from $P = \dfrac{A[(1+i)^n - 1]}{(1+i)^n i}$

$$150,000 = A\,\frac{(1+0.1)^{10} - 1}{(1+0.1)^{10}(0.1)}; A = \text{P24,411.81}$$

Balance = 150000(F/P,10%,2) - 24,411.81(F/A,10%,2)

$$= 150000(1+0.1)^2 - \frac{24411.81[(1+0.1)^2 - 1]}{0.1}$$

Balance = **P130,235.20**

SOLUTION 11 - 42
Ans: A

Present worth of all payments:

$$P = 100000 + \frac{200,000}{(1+0.15)^1} + \frac{50,000}{(1+0.15)^2} + \frac{75,000}{(1+0.15)^3}$$

P = P361,034.00

Equivalent uniform annual payment:

$$P = \frac{A[(1+i)^n - 1]}{(1+i)^n i}$$

$$361034 = \frac{A[(1+0.15)^3 - 1]}{(1+0.15)^3(0.15)}; A = \text{P158,124.60}$$

SOLUTION 11 - 43
Ans: D

Solving for i (based on monthly compoundings)

$ER_m = ER_a$

$(1+i)^{12} - 1 = (1+0.1)^1 - 1; i = 0.007974$

Total number of payments = 30(12) = 360

Since the payment is made at the beginning of each year, the payments will end on the 359th month.

Future worth of P100,000 = Future worth of 360 payments of A

100,000(F/P, 0.007974, 359) = A(F/A, 0.007974, 360)

$$100,000(1+0.007974)^{359} = \frac{A[(1+0.007974)^{360} - 1]}{0.007974}$$

A = **P P839.20**

SOLUTION 11 - 44
Ans: C

$$P = \frac{A[(1+i)^n - 1]}{(1+i)^n i}$$

Solve for i based on monthly compounding:

$ER_M = ER_A$

$(1+i)^{12} - 1 = (1+0.06)^1 - 1; \quad i = 0.004867$

$$10000 = \frac{A[(1+0.004867)^{120} - 1]}{(1+0.004867)^{120}(0.004867)}; A = \text{P110.22}$$

SOLUTION 11 - 45
Ans: B

$$F = \frac{A[(1+i)^n - 1]}{i} = \frac{12500[(1+0.05)^9 - 1]}{0.05} = \textbf{P137,832.05}$$

SOLUTION 11 - 46
Ans: D

$$F = \frac{A[(1+i)^n - 1]}{i} = \frac{6000[(1+0.07/2)^{16(2)} - 1]}{0.07/2}$$

F = **P344,007.00**

SOLUTION 11 - 47
Ans: A

Capital = present worth of all annual income

$$P = \frac{A[(1+i)^n - 1]}{(1+i)^n i}$$

$$5{,}700{,}000 = \frac{800{,}000[(1+i)^{10} - 1]}{(1+i)^{10} i}$$

$$7.125 = \frac{(1+i)^{10} - 1}{(1+i)^{10} i}$$

Solve for i by trial and error using the choices,
$i = 0.067 =$ **6.7%**

SOLUTION 11 - 48
Ans: A

Price = Down payment + Present worth of 12 yearly payments of P80,000

$P = 600000 + 80{,}000(P/A, 6\%, 12)$

$$P = 600000 + \frac{80{,}000[(1+0.06)^{12} - 1]}{(1+0.06)^{12}\, 0.06} = \textbf{P 1,270,707}$$

SOLUTION 11 - 49
Ans: A

$$P = \frac{A[(1+i)^n - 1]}{(1+i)^n i}$$

$$100000 = \frac{A[(1+0.08)^{10} - 1]}{(1+0.08)^{10}(0.08)}; A = \textbf{P14903}$$

SOLUTION 11 - 50
Ans: B

$F = 1500(F/A, 0.06, 7)$

$$F = \frac{1{,}500[(1+0.06)^7 - 1]}{0.06} = \textbf{P12,591.00}$$

SOLUTION 11 - 51
Ans: C

The present worth of all P100,000 payment is:

$$P = \frac{A[(1+i)^n - 1]}{(1+i)^n i}; i = 0.14/4 = 0.035; n = 10(4) = 40$$

$$P = \frac{100000[(1+0.035)^{40} - 1]}{(1+0.035)^{40}(0.035)} = \text{P2,135,507.23}$$

The present worth of the lump sum after four years must be equal to P2,135,507.23.

$P = F(P/F, 0.035, 16) = 2{,}135{,}507.23$

$$\frac{F}{(1+0.035)^{16}} = 2{,}135{,}507.23$$

F = **P3,702,939.73**

SOLUTION 11 - 52
Ans: C

Balance to be amortized, P = 540,000 - 162,000
Balance to be amortized, P = P378,000

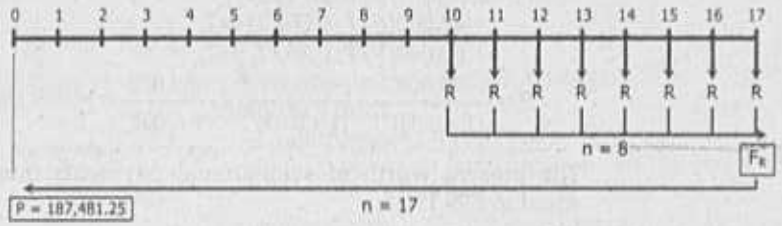
$P = 10{,}874.29(P/A, i, 5\times12)$

$$378{,}000 = \frac{10874.29[(1+i)^{60} - 1]}{(1+i)^{60} i}$$

Solve for i by trial and error using the choices,
Use $r = 24\%$ ($i = 0.24/12 = 0.02$)

SOLUTION 11 - 53
Ans: A

The cash flow diagram is as shown:



$$F_R = R(F/R, 0.05, 8) = \frac{R[(1+0.05)^8 - 1]}{0.05} = 9.5491R$$

$P = F_R(P/F, 0.05, 17)$

$$187{,}481.25 = \frac{9.5491R}{(1+0.05)^{17}}; R = \textbf{P45,000}$$

SOLUTION 11 - 54
Ans: C

A = P2,000; $i = 0.09$; $n = 10$
$P = 2000(P/A, 0.09, 10)$

$$P = \frac{2{,}000[(1+0.09)^{10} - 1]}{(1+0.09)^{10}(0.09)} = \textbf{P12,835.32}$$

SOLUTION 11 - 55
Ans: C

A = P2,000; $n = 6$; $i = 0.12/4 = 0.03$
$P = 2000(P/A, 0.03, 6)$

$$P = \frac{2{,}000[(1+0.03)^6 - 1]}{(1+0.03)^6(0.03)} = \textbf{P10,834.38}$$

SOLUTION 11 - 56
Ans: A

The price of the car is the present worth of all monthly installments of P10,000 for 60 months at 12% compounded annually.

$$P = A(P/A, i, n) = \frac{A[(1+i)^n - 1]}{(1+i)^n i}$$

A = P10,000; n = 60

Solve for i based on monthly compounding:

$ER_M = ER_A$

$(1 + i)^{12} - 1 = 0.12$; $i = 0.009489$

$$P = \frac{10{,}000[(1+0.009489)^{60} - 1]}{(1+0.009489)^{60}(0.009489)} = \text{P455,875}$$

SOLUTION 11 - 57
Ans: A

The present worth of all the maintenance cost is:

$P = 20{,}000(P/F, 0.06/2, 2\times2)$
$\quad + 40{,}000(P/F, 0.06/2, 4\times2)$
$\quad + 80{,}000(P/F, 0.06/2, 8\times2)$

$$P = \frac{20{,}000}{(1+0.03)^4} + \frac{40{,}000}{(1+0.03)^8} + \frac{80{,}000}{(1+.03)^{16}} = \text{P99,199.5}$$

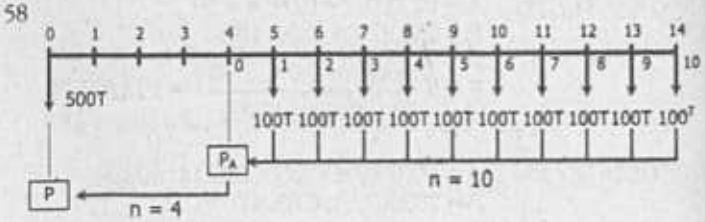
The present worth of semi-annual payments must be equal to P99,199.5.

$$P = \frac{A[(1+i)^n - 1]}{(1+i)^n i};\ i = 0.06/2 = 0.03;\ n = 8\times2 = 16$$

$$99{,}199.5 = \frac{A[(1+0.03)^{16} - 1]}{(1+0.03)^{16}0.03}$$

A = P7,897.35

SOLUTION 11 - 58
Ans: C



Cash price = P500,000 + P

$$P = \frac{P_A}{(1+i)^4}$$

$$P_A = \frac{A[(1+i)^{10} - 1]}{(1+i)^{10} i} = \frac{100{,}000[(1+0.14)^{10} - 1]}{(1+0.14)^{10}(0.14)}$$

P_A = P521,611.56

$$P = \frac{521{,}611.56}{(1+0.14)^4} = \text{P308,836.00}$$

Cash price = P500,000 + P308,836 = **P808,836.00**

Another way to solve for P:

$$P = \frac{100{,}000(F/A, 0.14, 10)}{(1+0.14)^{14}} = \frac{100{,}000[(1+0.14)^{10} - 1]}{0.14(1+0.14)^{14}}$$

P = P308,836.00

SOLUTION 11 - 59
Ans: C

$$F = A(F/A, 0.06, 6) = \frac{A[(1+0.06)^6 - 1]}{0.06} = 5{,}000$$

A = **P716.81**

SOLUTION 11 - 60
Ans: A

Cash price of machinery = P10,000.00
Installment price:
Down payment = P2,000.00
Annual payment (amortization) = P750.00 for 15 yrs

The present worth of the installment must be equal to the cash price:

Cash price = Down payment + $A(P/A, i, n)$

$$10{,}000 = 2{,}000 + \frac{750[(1+i)^{15} - 1]}{(1+i)^{15} i}$$

$$\frac{(1+i)^{15} - 1]}{(1+i)^{15} i} = 10.67$$

Solving for i by trial and error using the choices:
$i = 0.0461 = 4.61\%$

SOLUTION 11 - 61
Ans: C

The required amount now at 6% in order to withdraw P25,000 a year is:

$$P = \frac{A[(1+i)^n - 1]}{(1+i)^n i};\ A = \text{P25,000},\ i = 6\%,\ n = 15$$

$$P = \frac{25{,}000[(1+0.06)^{15} - 1]}{(1+0.06)^{15}(0.06)} = \text{P242,806.00}$$

SOLUTION 11 - 62
Ans: D

$F = 1{,}000(F/A, i, 5\times12) = 1{,}000(F/A, i, 60)$

Solve for i based on monthly compounding:

$ER_M = ER_A$

$(1 + i)^{12} - 1 = (1 + 0.06)^1 - 1 = 0.06, i = 0.00487$

$$F = \frac{1000[(1+0.00487)^{60} - 1]}{0.00487} = \mathbf{P69{,}491.00}$$

SOLUTION 11 - 63
Ans: A

$$P = \frac{A}{i}$$

Solve for i based on semiannual compounding:

$ER_S = ER_Q$

$(1 + i)^2 - 1 = (1 + 0.08/4)^4 - 1; i = 0.0404$

$$P = \frac{15{,}000}{0.0404} = \mathbf{P371{,}287}$$

SOLUTION 11 - 64
Ans: B

Let x be the cash price of the house and lot.
Down payment = 10% of x = P200,000; x = P2,000,000

Balance to be amortized, P = 2,000,000 - 200,000
Balance to be amortized, P = P1,800,000

$P = A(P/A, 0.15/12, 60) = 1{,}800{,}000$

$$\frac{A[(1+0.15/12)^{60} - 1]}{(1+0.15/12)^{60}(0.15/12)} = 1{,}800{,}000$$

$A = \mathbf{P42{,}822}$

SOLUTION 11 - 65
Ans: A

The future worth of x must be equal to the future worth of semi-annual payments of P21,962.68 for 10 years (20 payments)

$x(F/P, 0.07, 10) = 21{,}962.68(F/A, 0.06/2, 20)$

$$x(1 + 0.07)^{10} = \frac{21{,}962.68[(1+0.03)^{20} - 1]}{0.03}$$

$x = \mathbf{P300{,}000}$

Problems - Set 12
Depreciation, Capitalized Cost, Bonds, Etc.

PROBLEM 12 - 1

What is the value of an asset after 8 years of use if it depreciates from its original value of P120,000.00 to its salvage value of 3% in 12 years?

A. P44,200.00
B. P44,020.00
C. P44,200.00
D. P42,400.00

PROBLEM 12 - 2

A man bought an equipment which cost P524,000.00. Freight and installation expenses cost him P31,000.00. If the life of the equipment is 15 years with an estimated salvage value of P120,000.00, find its book value after 8 years.

A. P323,000.00
B. P244,000.00
C. P259,000.00
D. P296,000.00

PROBLEM 12 - 3

An equipment costing P250,000 has an estimated life of 15 years with a book value of P30,000 at the end of the period. Compute the depreciation charge and its book value after 10 years using straight-line method.

A. d = P14,666.67; BV = P103,333.30
B. d = P14,666.67; BV = P105,666.67
C. d = P13,333.33; BV = P103,333.30
D. d = P13,333.33; BV = P105,666.67

PROBLEM 12 - 4

An equipment costing P250,000 has an estimated life of 15 years with a book value of P30,000 at the end of the period. Compute the depreciation charge and its book value after 10 years using sinking fund method assuming i = 8%.

A. d = P8,102.50; BV = P103,333.30
B. d = P6,686.67; BV = P125,666.67
C. d = P7,567.50; BV = P138,567.60
D. d = P8,102.50; BV = P132,622.60

PROBLEM 12 - 5

An equipment costing P250,000 has an estimated life of 15 years with a book value of P30,000 at the end of the period. Compute the depreciation charge and its book value after 10 years using declining balance method.

A. d = P9,456.78; BV = P67,456.98
B. d = P8,987.45; BV = P60,832.80
C. d = P9,235.93; BV = P60,832.80
D. d = P9,235.93; BV = P59,987.34

PROBLEM 12 - 6

An equipment costing P250,000 has an estimated life of 15 years with a book value of P30,000 at the end of the period. Compute the depreciation charge and its book value after 10 years using the sum of year's digit method.

A. d = P11,000; BV = P67,500
B. d = P10,500; BV = P58,000
C. d = P11,500; BV = P60,000
D. d = P11,000; BV = P57,500

PROBLEM 12 - 7

An asset costing P50,000 has a life expectancy of 6 years and an estimated salvage value of P8000. Calculate the depreciation charge at the end of the fourth period using fixed-percentage method.

A. P7144.20 C. P3878.40
B. P5264.00 D. P2857.60

PROBLEM 12 - 8
CE May 1996

A machine costing P45,000 is estimated to have a salvage value of P4,350 when retired at the end of 6 years. Depreciation cost is computed using a constant percentage of the declining book value. What is the annual rate of depreciation in %?

A. 33.25% C. 35.25%
B. 32.25% D. 34.25%

PROBLEM 12 - 9
CE Nov. 1997, Nov. 1994

An engineer bought an equipment for P500,000. Other expenses including installation amounted to P30,000. At the end of its estimated useful life of 10 years, the salvage value will be 10% of the first cost. Using straight line method of depreciation, what is the book value after 5 years?

A. P281,500.00 C. P301,500.00
B. P291,500.00 D. P271,500.00

PROBLEM 12 - 10
ECE Nov. 1997

A machine costs P 8,000.00 and an estimated life of 10 years with a salvage value of P500.00. What is its book value after 8 years using straight-line method?

A. P2,500.00 C. P3,000.00
B. P4,000.00 D. P2,000.00

PROBLEM 12 - 11
ME Oct. 1997

A factory equipment has an initial cost of P200,000.00. Its salvage value after ten years is P20,000.00. As a percentage of the initial cost, what is the straight-line depreciation rate of the equipment?

A. 5% C. 9%
B. 6% D. 8%

PROBLEM 12 - 12
ME Oct. 1997

An asset is purchased for P120,000.00. Its estimated economic life is 10 years, after which it will be sold for P12,000.00. Find the depreciation for the first year using the sum-of-the-year's digit, (SOYD).

A. P20,000.00 C. P21,080.00
B. P18,400.00 D. P19,636.00

PROBLEM 12 - 13
ME April 1998

An asset is purchased for P9,000.00. Its estimated life is 10 years, after which it will be sold for P1,000.00. Find the book value during the third year if sum-of-the-year's digit (SOYD) depreciation is used.

A. P6,100.00 C. P4,500.00
B. P5,072.00 D. P4,800.00

PROBLEM 12 - 14
ME April 1998

An asset is purchased for P500,000.00. The salvage value in 25 yeas is P100,000.00. What is the total depreciation in the first three years using straight line method?

A. P48,000.00 C. P24,000.00
B. P32,000.00 D. P16,000.00

PROBLEM 12 - 15
ME April 1998

A machine has an initial cost of P50,000.00 and a salvage value of P10,000.00 after 10 years. What is the book value after five years using straight-line depreciation?

A. P35,000.00 C. P25,000.00
B. P15,500.00 D. P30,000.00

PROBLEM 12 - 16
ME April 1998

A company purchased an asset for P10,000.00 and plans to keep it for 20 years. If the salvage value is zero at the end of the 20th year, what is the depreciation in the third year? Use sum-of-the years digits depreciation.

A. P1000.00 C. P857.00
B. P937.00 D. P747.00

PROBLEM 12 - 17
ME April 1998

An asset is purchased for P9,000.00. Its estimated life is 10 years, after which it will be sold for P1,000.00. Find the book value during the first year if sum-of-the-year's digit (SOYD) depreciation is used.

A. P8,000.00 C. P6,500.00
B. P7,545.00 D. P6,000.00

PROBLEM 12 - 18
CE Nov. 1998

A machine having a first cost of P 60,000.00 will be retired at the end of 8 years. Depreciation cost is computed using a constant percentage of the declining book value. What is the total cost of depreciation, in pesos, up to the time the machine is retired if the annual rate of depreciation is 28.72%?

A. 56,000 C. 58,000
B. 57,000 D. 59,000

PROBLEM 12 - 19
ECE Nov. 1998

XYZ Corporation makes it a policy that for any new equipment purchased, the annual depreciation cost should not exceed 20% of the first cost at any time with no salvage value. Determine the length of service life necessary if the depreciation used is the sum-of-the-year's-digit (SOYD) method.

A. 7 years
B. 8 years
C. 9 years
D. 6 years

PROBLEM 12 - 20

A large profitable corporation has purchased a luxury van for use by its executives. The cost of the plane is P5 million. It has a useful life of 5 years. The estimated resale value at the end of five years is P1 million. Using the sum-of-years-digit method of depreciation, what is the book value of the van at the end of 3 years?

A. P2.2M
B. P1.2M
C. P3.6M
D. P1.8M

PROBLEM 12 - 21

Determine the capitalized cost of an equipment costing P2M with and annual maintenance of P200,000.00 if money is worth 20% per annum.

A. P 2.5 M
B. P 2.75 M
C. P 3M
D. P 3.5M

PROBLEM 12 - 22
CE Nov. 1996

At 6%, find the capitalized cost of a bridge whose cost is P200M and life is 20 years, if the bridge must be partially rebuilt at a cost of P100M at the end of each 20 years.

A. 245.3
B. 215
C. 210
D. 220

PROBLEM 12 - 23
ME Oct. 1997

An item is purchased for P100,000.00. Annual costs are P18,000.00. Using 8%, what is the capitalized cost of perpetual service?

A. P350,000.00
B. P335,000.00
C. P320,000.00
D. P325,000.00

PROBLEM 12 - 24
CE May 1997

A company uses a type of truck which costs P2M, with life of 3 years and a final salvage value of P320,000. How much could the company afford to pay for another type of truck for the same purpose, whose life is 4 years with a final salvage value of P400,000, if money is worth 4%?

A. P2,565,964.73
B. P2,855,964.73
C. P2,585,964.73
D. P2,585,864.73

PROBLEM 12 - 25

A P100,000, 6% bond, pays dividend semi-annually and will be redeemed at 110% on July 1, 1999. Find its price if bought on July 1, 1996, to yield an investor 4%, compounded semi-annually.

A. P100,000.00
B. P112,786.65
C. P113,456.98
D. P114,481.14

PROBLEM 12 - 26

A community wishes to purchase an existing utility valued at P500,000 by selling 5% bonds that will mature in 30 years. The money to retire the bond will be raised by paying equal annual amounts into a sinking fund that will earn 4%. What will be the total annual cost of the bonds until they mature?

A. P44,667.98
B. P37,345.78
C. P34,515.05
D. P33,915.05

PROBLEM 12 - 27

A man paid P110,000 for a P100,000 bond that pays P4000 per year. In 20 years, the bond will be redeemed for P105,000. What net rate of interest will the man obtain on his investment?

A. 3.37%
B. 3.47%
C. 3.56%
D. 3.40%

PROBLEM 12 - 28
ECE Nov. 1996

A man wants to make 14% nominal interest compounded semi-annually on a bond investment. How much should the man be willing to pay now for a 12%, P10,000 bond that will mature in 10 years and pays interest semi-annually?

A. P2,584.19
B. P3,118.05
C. P8,940.50
D. P867.82

PROBLEM 12 - 29
ME April 1998

A company issued 50 bonds of P1,000.00 face value each, redeemable at par at the ends of 15 years to accumulate the funds required for redemption, the firm established a sinking fund consisting of annual deposits, the interest rate of the fund being 4%. What was the principal in the fund at the end of the 12th year?

A. P35,983.00
B. P41,453.00
C. P38,378.00
D. P37,519.00

PROBLEM 12 - 30

It is estimated that a timber tract will yield an annual profit of P100,000 for 6 years, at the end of which time the timber will be exhausted. The land itself will then have an anticipated value of P40,000. If a prospective purchaser desires a return of 8% on his investment and can deposit money in a sinking fund at 4%, what is the maximum price he should pay for the tract.

A. P459,480.00
B. P467,456.00
C. P578,987.00
D. P589,908.00

PROBLEM 12 - 31

A mine is purchased for P1,000,000.00 and it is anticipated that it will be exhausted at the end of 20 years. If the sinking-fund rate is 4%, what must be the annual return from the mine to realize a return of 7% on the investment?

A. P108,350 C. P130,850
B. P150,832 D. P103,582

PROBLEM 12 - 32

A syndicate wishes to purchase an oil well which, estimates indicate, will produce a net income of P2M per year for 30 years. What should the syndicate pay for the well if, out of this net income, a return of 10% of the investment is desired and a sinking fund is to be established at 3% interest to recover this investment.

A. P16,526,295 C. P12,566,295
B. P15,626,245 D. P16,652,245

PROBLEM 12 - 33
CE May 1995

An investor pays P1,100,000 for a mine which will yield a net income of P200,000 at the end of each year for 10 years and then will become useless. He accumulates a replacement fund to recover his capital by annual investments at 4.5%. At what rate (%) does he receive interest on his investment at the end of each year?

A. 10.04 C. 11.5
B. 8.5 D. 17.5

PROBLEM 12 - 34
CE May 1997

Machine cost = $15,000; Life = 8 years; Salvage Value = $3,000. What minimum cash return would the investor demand annually from the operation of this machine if he desires interest annually at the rate of 8% on his investment and accumulates a capital replacement fund by investing annual deposits at 5%?

A. $5246.66 C. $2456.66
B. $2546.66 D. $4256.66

PROBLEM 12 - 35
ME April 1995

The purchase of a motor for P6000 and a generator for P4000 will allow a company to produce its own energy. The configuration can be assembled for P500. The service will operate for 1600 hours per year for 10 years. The maintenance cost is P300 per year and the cost to operate is P0.85 per hour for fuel and other related cost. Using straight-line depreciation, what is the annual cost for the operation? There is P400 in salvage value for the system at the end of 10 years.

A. P2,670.00 C. P2,567.00
B. P2,760.00 D. P2,657.00

PROBLEM 12 - 36

A company purchases P200,000 of equipment in year zero. It decides to use a straight-line depreciation over the expected 20-year life of the equipment. The interest rate is 14%. If its overall tax rate is 40%, what is the

present worth of the depreciation tax shield?

A. P27,678.78 C. P26,492.52
B. P26,876.45 D. P26,098.78

PROBLEM 12 - 37
ME Oct. 1996

A company must relocate one of its factories in three years. Equipment for the loading duck is being considered for purchase. The original cost is P20,000, the salvage value after three years is P8,000. The company's rate of return (i) on money is 10%. Determine the capital recovery rate per year.

A. P5987.67 C. P5625.38
B. P5897.56 D. P5578.35

PROBLEM 12 - 38
ME April 1998

XYZ Corporation manufacturers bookcases that it sells for P65.00 each. It costs XYZ P35,000.00 per year to operate its plant. This sum includes rent, depreciation charges on equipment, and salary payments. If the cost to produce one bookcase is P50.00, how many cases must be sold each year for XYZ to avoid taking a loss?

A. 2334 cases C. 539 cases
B. 750 cases D. 2333 cases

PROBLEM 12 - 39
ME April 1998

A Company that manufactures electric motors has a production capacity of 200 motors a month. The variable costs are P150.00 per motor. The average selling price of the motors is P275.00. Fixed costs of the company amount to P20,000.00 per month which includes taxes. The number of motors that must be sold each month to break even is closest to:

A. 40 C. 150
B. 80 D. 160

PROBLEM 12 - 40
CE May 1998,
similar to May 1999

Determine the break-even point in terms of number of units produced per month using the following data: (the costs are in pesos per unit)

Selling price per unit ..... 600 pesos
Total monthly overhead expenses ..... 428,000 pesos
Labor cost ..... 115 pesos
Cost of materials ..... 76 pesos
Other variable cost ..... 2.32 pesos

A. 1036 C. 1044
B. 1053 D. 1025

PROBLEM 12 - 41
ME Oct. 1997

A fixture that costs P700.00 will save P0.06 per item produced. Maintenance will be P40.00 annually, 3500 units are produced annually. What is the payback period at 10%?

A. 4.12 years C. 5.57 years
B. 4.65 years D. 4.85 years

PROBLEM 12 - 42
ME Oct. 1997

A company puts P25,000.00 down and will pay P5,000.00 every year for the life of a machine for 10 years. If the salvage value is zero and the interest rate is 10% compounded annually, what is the present value of the machine?

A. P56,200.00
B. P55,723.00
C. P55,100.00
D. P55,300.00

PROBLEM 12 - 43
ME October 1997

The balance sheet of Oriental Services Inc. is as follows:

Asset		Liabilities	
Cash	P 10,000.00	Payables	P17,000.00
Receivables	12,000.00	Notes due	6,000.00
Inventory	7,000.00	Long term debt	3,000.00
Capital equipt.	20,000.00	Owner's equity	23,000.00
Total:	P 49,000.00	Total:	P 49,000.00

What is the acid ratio

A. 0.846
B. 1.115
C. 0.592
D. 0.385

PROBLEM 12 - 44
ME Oct. 1997

A plastic manufacturing company produces a product from polystyrene. Two mixing processes, batch and continuous are available with the following data for each process:

	Batch	Continuous
Initial cost	P 50,000.00	P 100,000.00
Maintenance cost/year	10,000.00	36,000.00
Salvage value	6,000.00	12,000.00
Life time	5 years	5 years
Capacity (units/year)	1,000,000	2,000,000
Depreciation, D_1	$\frac{50000 - 60000}{5}$ = P 8,800.00	$\frac{100000-12000}{5}$ = P 17,600.00

The company uses straight-line depreciation, pays 48% of its net income as income tax, and has an after-tax minimum attractive rate of return of 15%. The company has an infinite market and can sell all of the products it produces at P0.50 per unit.

Which of the given comparative figures give the closest evaluation for the best investment?

A. P1,498,672.25 — P1,559,944.92
B. P1,553,403.94 — P1,614,676.61
C. P1,735,408.75 — P1,796,681.42
D. P1,345,890.45 — P1,407,163.12

PROBLEM 12 - 45
ME April 1998

A manufacturing firm wishes to give each 80 employees a holiday bonus. How much is needed to invest monthly for a year at 12% nominal interest rate, compounded monthly, so that each employee will receive a P2,000.00 bonus?

A. P12,608.00
B. P12,600.00
C. P12,610.00
D. P12,300.00

ANSWER SHEET

1.	A	B	C	D	E	21.	A	B	C	D	E	41.	A	B	C	D	E
2.	A	B	C	D	E	22.	A	B	C	D	E	42.	A	B	C	D	E
3.	A	B	C	D	E	23.	A	B	C	D	E	43.	A	B	C	D	E
4.	A	B	C	D	E	24.	A	B	C	D	E	44.	A	B	C	D	E
5.	A	B	C	D	E	25.	A	B	C	D	E	45.	A	B	C	D	E
6.	A	B	C	D	E	26.	A	B	C	D	E	46.	A	B	C	D	E
7.	A	B	C	D	E	27.	A	B	C	D	E	47.	A	B	C	D	E
8.	A	B	C	D	E	28.	A	B	C	D	E	48.	A	B	C	D	E
9.	A	B	C	D	E	29.	A	B	C	D	E	49.	A	B	C	D	E
10.	A	B	C	D	E	30.	A	B	C	D	E	50.	A	B	C	D	E
11.	A	B	C	D	E	31.	A	B	C	D	E	51.	A	B	C	D	E
12.	A	B	C	D	E	32.	A	B	C	D	E	52.	A	B	C	D	E
13.	A	B	C	D	E	33.	A	B	C	D	E	53.	A	B	C	D	E
14.	A	B	C	D	E	34.	A	B	C	D	E	54.	A	B	C	D	E
15.	A	B	C	D	E	35.	A	B	C	D	E	55.	A	B	C	D	E
16.	A	B	C	D	E	36.	A	B	C	D	E	56.	A	B	C	D	E
17.	A	B	C	D	E	37.	A	B	C	D	E	57.	A	B	C	D	E
18.	A	B	C	D	E	38.	A	B	C	D	E	58.	A	B	C	D	E
19.	A	B	C	D	E	39.	A	B	C	D	E	59.	A	B	C	D	E
20.	A	B	C	D	E	40.	A	B	C	D	E	60.	A	B	C	D	E

Solutions to Set 12 Depreciation, Capitalized Cost, Bonds, Etc.

SOLUTION 12 - 1
Ans: D

By straight-line depreciation, $BV_8 = FC - D_8$

$$d = \frac{FC - SV}{n} = \frac{120{,}000 - 3\%(120{,}000)}{12} = \text{P}9{,}700/\text{year}$$

$D_8 = d \times 8 = \text{P}9700 \times 8 = \text{P}77{,}600.00$

$BV_8 = \text{P}120{,}000 - \text{P}77{,}600 = \mathbf{P42{,}400.00}$

SOLUTION 12 - 2
Ans: A

$BV_8 = FC - D_8$; $FC = \text{P}524{,}000 + \text{P}31{,}000 = \text{P}555{,}000.00$

$$d = \frac{FC - SV}{n}$$

$$d = \frac{555{,}000 - 120{,}000}{15} = \text{P}29{,}000.00/\text{year}$$

$D_8 = \text{P}29{,}000 \times 8 = \text{P}232{,}000$

$BV_8 = \text{P}555{,}000 - \text{P}232{,}000 = \mathbf{P323{,}000.00}$

SOLUTION 12 - 3
Ans: A

$$d = \frac{FC - SV}{n} = \frac{250000 - 30000}{15} = \text{P}14{,}666.67$$

$BV_{10} = FC - D_{10} = 250{,}000 - 14{,}666.67(10) = \mathbf{P103{,}333.30}$

SOLUTION 12 - 4
Ans: D

$$d = \frac{(FC - SV)i}{(1+i)^n - 1} = \frac{(250000 - 30000)(0.08)}{(1 + 0.08)^{15} - 1}$$

$d = \text{P}8{,}102.50$ (depreciation charge)

$$D_{10} = \frac{d[(1+i)^m - 1]}{i} = \frac{8102.50\left[(1 + 0.08)^{10} - 1\right]}{(0.08)}$$

$D_{10} = \text{P}117{,}377.4$

$BV_{10} = FC - D_{10} = 250{,}000 - 117{,}377.4 = \mathbf{P132{,}622.60}$

SOLUTION 12 - 5
Ans: C

$$K = 1 - \sqrt[n]{\frac{SV}{FC}} = 1 - \sqrt[15]{\frac{30000}{250000}} = 0.1318$$

$BV_{10} = FC(1 - K)^m = 250000(1 - 0.1318)^{10}$

$BV_{10} = \text{P}60{,}832.80$ (book value)

$d_{10} = BV_9 - BV_{10} = 250000[(1 - 0.1318)^9 - (1 - 0.1318)^{10}]$

$d_{10} = \mathbf{P9{,}235.93}$ (depreciation charge)

SOLUTION 12 - 6
Ans: D

$$\text{Sum} = \frac{15}{2}(1 + 15) = 120$$

$FC - SV = \text{P}220{,}000.00$

$d = K(FC - SV)$

Year	Factor, K	$d = (FC - SV)\,K$	BV
1	15/120	27,500.00	222,500.00
2	14/120	25,666.67	196,833.33
3	13/120	23,833.33	173,000.00
4	12/120	22,000.00	151,000.00
5	11/120	20,166.67	130,833.33
6	10/120	18,333.33	112,500.00
7	9/120	16,500.00	96,000.00
8	8/120	14,666.67	81,333.33
9	7/120	12,833.33	68,500.00
10	6/120	11,000.00	57,500.00

Another solution

$$d_m = (FC - SV)\frac{n - m + 1}{Sum} = 220{,}000\,\frac{15 - 10 + 1}{120} = \text{P}11{,}000$$

$$D_m = (FC - SV)\frac{m(2n - m + 1)}{2(Sum)}$$

$$D_{10} = (220{,}000)\frac{\frac{10}{2}[2(15) - 10 + 1]}{120} = \text{P}192{,}500.00$$

$BV_{10} = FC - D_{10} = 250{,}000 - 192{,}500 = \mathbf{P57{,}500.00}$

SOLUTION 12 - 7
Ans: B

$d = FC(1 - k)^{m-1}\,k$

$$k = 1 - \sqrt[n]{\frac{SV}{FC}} = 1 - \sqrt[6]{\frac{8000}{50000}} = 0.263$$

$d = 50000(1 - 0.263)^{4-1}(0.263) = \mathbf{P5264.00}$

SOLUTION 12 - 8
Ans: B

The annual rate of depreciation, k is:

$$k = 1 - \sqrt[n]{\frac{SV}{FC}} = 1 - \sqrt[6]{\frac{4350}{45000}} = 0.3225 = \mathbf{32.25\%}$$

SOLUTION 12 - 9
Ans: B

$FC = \text{P}500{,}000 + \text{P}30{,}000 = \text{P}530{,}000$

$SV = 10\%(530{,}000) = \text{P}53{,}000$

$n = 10$; $m = 5$

$BV_m = FC - D_m$

By straight line depreciation;

$D_m = d \times m$

$d = (FC - SV)/n = (530{,}000 - 53{,}000)\ /\ 10$

$d = \text{P}47{,}700$

$D_5 = 47{,}700\ (5) = \text{P}238{,}500.00$

$BV_5 = FC - D_5 = 530{,}000 - 238{,}500 = \mathbf{P291{,}500.00}$

SOLUTION 12 - 10
Ans: D

$D_m = d \times m$

$$d = \frac{FC - SV}{n} = \frac{8000 - 500}{10} = \text{P}750.00$$

$BV_8 = FC - D_m = 8000 - 750(8) = \mathbf{P2{,}000.00}$

SOLUTION 12 - 11
Ans: C

Depreciation rate, $d = \frac{FC - SV}{n} = \frac{200{,}000 - 20{,}000}{10}$

d = P18,000

Percentage = $\frac{18{,}000}{200{,}000} \times 100\% = 9\%$

SOLUTION 12 - 12
Ans: D

$d_m = (FC - SV)\frac{n - m + 1}{\text{Sum}}$

$\text{Sum} = \frac{n}{2}(n+1) = \frac{10}{2}(10+1) = 55$

$d_1 = (120{,}000 - 12{,}000)\frac{10 - 1 + 1}{55}$ = **P19,636.36**

SOLUTION 12 - 13
Ans: B

$BV_m = FC - D_m$

$D_m = (FC - SV)\frac{m(2n - m + 1)}{2\,\text{Sum}}$

$\text{Sum} = \frac{n}{2}(1+n) = \frac{10}{2}(1+10) = 55$

$D_3 = (9000 - 1000)\frac{3[2(10) - 3 + 1]}{2(55)}$ = P3,927.27

or $D_3 = (9000 - 1000)\frac{10 + 9 + 8}{55}$ = P3,927.27

BV_3 = 9,000 – 3,927.27 = **P5072.72**

SOLUTION 12 - 14
Ans: A

$D = d \times m$

$d = \frac{FC - SV}{n} = \frac{500{,}000 - 100{,}000}{25}$ = P16,000 per yr

D = P16,000 × 3 = **P48,000.00**

SOLUTION 12 - 15
Ans: D

Book value = First Cost – Total depreciation

$BV_5 = FC - D_5$

$D_5 = d \times 5$

$d = \frac{FC - SV}{n} = \frac{50{,}000 - 10{,}000}{10}$ = P4,000.00

D_5 = P4,000 × 5 = P20,000.00

BV_5 = P50,000.00 – P20,000.00 = **P30,000.00**

SOLUTION 12 - 16
Ans: C

$d_m = (FC - SV)\frac{n - m + 1}{\text{Sum}}$

$\text{Sum} = \frac{n}{2}(1+n) = \frac{20}{2}(1+20) = 210$

$d_5 = (10{,}000 - 0)\frac{20 - 3 + 1}{210}$ = **P857.00**

SOLUTION 12 - 17
Ans: B

$BV_m = FC - D_m$

$D_m = (FC - SV)\frac{m(2n - m + 1)}{2(Sum)}$

$\text{Sum} = \frac{n}{2}(1+n) = \frac{10}{2}(1+10) = 55$

$D_1 = (9{,}000 - 1{,}000)\frac{1[2(10) - 1 + 1]}{2(55)}$ = P1,454.54

BV_1 = P9,000.00 – P1,454.54 = **P7545.00**

SOLUTION 12 - 18
Ans: A

Annual rate of depreciation, K:

$K = 1 - \sqrt[n]{\frac{SV}{FC}}$; $0.2872 = 1 - \sqrt[8]{\frac{SV}{60{,}000}}$

$\sqrt[8]{\frac{SV}{60000}} = 0.7128$ raise both sides to 8

$\frac{SV}{60{,}000} = 0.06664$; SV = P3998.5

Total cost of Depreciation, $D = FC - SV$ = 60,000 – 3998.5

Total cost of Depreciation, D = **P56,001.5**

SOLUTION 12 - 19
Ans: C

The depreciation rate at any period using SOYD is:

$d = (FC - SV)\frac{n - m + 1}{Sum}$

We see that d is largest when $m = 1$ (first year).

With $SV = 0$ and $d = 0.2\,FC$;

$0.2\,FC = FC\,\frac{n - 1 + 1}{Sum}$

$\text{Sum} = 5n$

$\frac{n}{2}(1+n) = 5n$

$1 + n = 10$; n = **9 years**

SOLUTION 12 - 20
Ans: D

By SOYD method, The total depreciation at the end of m years is:

$D_m = (FC - SV)\frac{m(2n - m + 1)}{2\,Sum}$

$\text{Sum} = \frac{n}{2}(1+n) = \frac{5}{2}(1+5) = 15$

$D_3 = (5 - 1)\frac{3[2(5) - 3 + 1]}{2(15)}$ = P 3.2 million

Book value = $FC - D_m$ = 5 – 3.2 = **P1.8**

SOLUTION 12 - 21
Ans: C

Capitalized Cost = $FC + \frac{OM}{i}$

FC = P 2,000,000.00; OM = 200,000.00

Capitalized Cost = $2{,}000{,}00 + \frac{200000}{0.2}$ = **P 3,000,000**

SOLUTION 12 - 22
Ans: A

First Cost, FC = P200M
Renewal Cost, RC = P100M
Life, n = 20 years

Capitalized Cost, $K = FC + \frac{OM}{i} + \frac{RC - SV}{(1+i)^n - 1}$

Where $SV = 0$ and $OM = 0$

$K = 200 + \frac{100 - 0}{(1+0.06)^{20} - 1}$ = **P245.31 M**

SOLUTION 12 - 23
Ans: D

$K = FC + OM/i$
$K = 100{,}000 + 18{,}000 / 0.08$ = **P325,000**

SOLUTION 12 - 24
Ans: C

Two alternatives are said to be economically the same if their corresponding annual cost or capitalized cost are equal.

Using the capitalized cost: $K = FC + \frac{FC - SV}{(1+i)^n - 1} + \frac{OM}{i}$

For the first truck:

$K_1 = 2{,}000{,}000 + \frac{2000000 - 320000}{(1+0.04)^3 - 1} + 0$

K_1 = P 15,454,638.65

For the second truck:

$K_2 = FC + \frac{FC - 400000}{(1+0.04)^4 - 1} + 0 = 15{,}454{,}638.65$

$15{,}454{,}638.65 = FC + 5.887FC - 2354900$
FC = **P2,585,964.73**

SOLUTION 12 - 25
Ans: D

Face value of bond, F = P100,000
Bond rate, $r = 0.06/2 = 0.03$ (semi-annually)
Redemption value, C = 110%(100,000) = P110,000
Investor's rate (semiannually) = 0.04 /2 = 0.02
Dividend period, $n = 3(2) = 6$
Dividend = 100,000(0.03) = P3,000.00 (semiannually)

Price of the bond, P:

$$P = \frac{C}{(1+i)^n} + \frac{D[(1+i)^n - 1]}{(1+i)^n i}$$

$$P = \frac{110000}{(1+0.02)^6} + \frac{3000[(1+0.02)^6 - 1]}{(1+0.02)^6\,(0.02)}$$

P = **114,481.14**

SOLUTION 12 - 26
Ans: D

Annual cost = Annual Bond Dividend (D)
+ Annual Deposit (A)
Annual dividend, $D = F \times r$ = 500,000(0.05) = P25,000
Annual Deposit:
$F = A(F/A, 0.04, 30) = 500{,}000$

$\frac{A[(1+0.04)^{30} - 1]}{0.04} = 500{,}000$

A = P8,915.05
Annual Cost = P25,000 + P8,915.05
Annual Cost = **P33,915.05**

SOLUTION 12 - 27
Ans: B

Face value of bond = P100,000.00
Annual Dividend, D = P4000.00
Redemption Value, C = P105,000
Buying price, P = P110,000

$$P = \frac{C}{(1+i)^n} + \frac{D[(1+i)^n - 1]}{(1+i)^n i}$$

$$110000 = \frac{105000}{(1+i)^{20}} + \frac{4{,}000[(1+i)^{20} - 1]}{(1+i)^{20} i}$$

Solve i by trial and error using the choices,
$i = 0.0341 = 3.47\%$

SOLUTION 12 - 28
Ans: C

Face value, F = P10,000
Bond rate, $r = 0.12/2 = 0.06$ (semi-annually)
Redemption value, $C = F$ =P10,000
Investor's interest rate, $i = 0.14/2 = 0.07$
Number of payments = 10(2) = 20

The price of the bond at a given interest is:

$$P = \frac{D[(1+i)^n - 1]}{(1+i)^n i} + \frac{C}{(1+i)^n}$$

$D = F \times r = 10000(0.06)$
$D = 600$ (dividend semi-annually)

$$P = \frac{600[(1+0.07)^{20} - 1]}{(1+0.07)^{20}(0.07)} + \frac{10000}{(1+0.07)^{20}}$$

P = **P 8,940.598**

SOLUTION 12 - 29
Ans: D

"Redeemable at par" means that the redemption value C equals the face value F.

$C = 50(1000) = P50,000$

The required annual deposits A to raise P50,000 at the end of 15 years is:

$$F = C = \frac{A[(1+i)^n - 1]}{i}$$

$$50,000 = \frac{A[(1+0.04)^{15} - 1]}{0.04}; \quad A = P2,497.055$$

The principal in the fund at the end of 12 years is:

$$F = \frac{A[(1+i)^n - 1]}{i}$$

$$F = \frac{2,497.055[(1+0.04)^{12} - 1]}{0.04} = \mathbf{P37,520.3}$$

SOLUTION 12 - 30
Ans: A

Annual Dividend, D = P100,000
Salvage Value, SV = P40,000
Investor's rate of return, r = 8%
Reinvestment Rate, i = 4%
Life of investment, n = 6 years
Price of the investment, FC = ?

$$D = FC(r) + \frac{(FC - SV)i}{(1+i)^n - 1}$$

$$100,000 = FC(0.08) + \frac{(FC - 40,000)(0.04)}{(1+0.04)^6 - 1}$$

FC = **P459,480.00**

SOLUTION 12 - 31
Ans: D

Capital, FC = P1,000,000
Salvage Value = 0
Investor's Rate of Return, r = 7%
Reinvestment Rate, i = 4%
Life of investment, n = 20

$$D = (FC)r + \frac{(FC - SV)i}{(1+i)^n - 1}$$

$$D = 1000000(0.07) + \frac{(1000000 - 0)(0.04)}{(1+0.04)^{20} - 1}$$

D = **P103,582.00**

SOLUTION 12 - 32
Ans: A

Capital, FC = ?
Annual dividend = P2,000,000
Salvage Value = 0

Investor's Rate of Return, r = 10%
Reinvestment Rate, i = 3%
Life of investment, n = 30

$$D = (FC)r + \frac{(FC - SV)i}{(1+i)^n - 1}$$

$$2000000 = FC(0.10) + \frac{(FC - 0)(0.03)}{(1+0.03)^{30} - 1}$$

$2000000 = 0.121FC$
FC = **P 16,526,295**

SOLUTION 12 - 33
Ans: A

Capital, FC = P1,100,000
Annual dividend, D = P200,000
Reinvestment Rate, i = 4.5%
n = 10 years
Investor's rate of return, r = ?

$$D = (FC)r + \frac{(FC - SV)i}{(1+i)^n - 1}$$

$200,000 = 1,100,000\, r$
$r = 0.1004$ = **10.04%**

SOLUTION 12 - 34
Ans: C

Capital, FC = \$15,000
Salvage Value, SV = \$3,000
Annual Dividend (Cash Return), D = ?
Investor' rate of return, r = 8%
Reinvestment rate, i = 5%

$$D = (FC)r + \frac{(FC - SV)i}{(1+i)^n - 1}$$

$$D = (15,000)(0.08) + \frac{(15000 - 3000)(0.05)}{(1+0.05)^8 - 1}$$

D = **\$2456.66**

SOLUTION 12 - 35
Ans: A

$FC = 6000 + 4000 + 500 = 10500$
$SV = 400$; $n = 10$

Annual cost of operation = Annual operation & maintenance + Annual deprectiation

Annual Depreciation, $d = \frac{10500 - 400}{10}$ = P1010

Annual Maintenance and Operation:
Maintenance = 300
Operation cost = 1600(0.85) = P1360

Annual Cost = 300 + 1360 + 1010 = **P2670.00**

SOLUTION 12 - 36
Ans: C

$$d = \frac{FC - SV}{n} = \frac{200000 - 0}{20} = \text{P}10{,}000$$

Present worth of depreciation:

$$P = d(P/A, 0.14, 20) = \frac{10{,}000[(1+0.14)^{20} - 1]}{(1+0.14)^{20}(0.14)}$$

$$P = 66{,}231.3$$

Present worth depreciation tax shield

$P_t = P \times 40\% = 66{,}231.3(0.4) =$ **P26,492.52**

SOLUTION 12 - 37
Ans: C

Capital recovery = Annual depreciation + annual interest

$$\text{Capital recovery} = \frac{(FC - SV)i}{(1+i)^n - 1} + (FC)i$$

$$\text{Capital recovery} = \frac{(20{,}000 - 8{,}000)(0.1)}{(1+0.1)^3 - 1} + 20000(0.1)$$

Capital recovery = **P5625.38**

SOLUTION 12 - 38
Ans: A

For break-even, Cost = Revenue
Let N = break-even quantity
Fixed cost + Variable Cost = Revenue
$f + aN = pN$
$35000 + 50N = 65N$
$N = 2333.33$
Below 2333.33, a loss is incurred.
Therefore, to avoid losses, N must be greater than 2333.33 and the nearest answer is **2334.**

SOLUTION 12 - 39
Ans: D

For break-even, COST = REVENUE
Fixed Cost + Variable Cost = Revenue
$f + aN = pN$
$20{,}000 + 150\,N = 275\,N$
$N =$ **160**

SOLUTION 12 - 40
Ans: B

For break-even, Cost = Revenue
Let N = break-even quantity

Cost = Revenue
$f + aN = pN$
$f =$ P428,000; $a = 115 + 76 + 2.32$
$f =$ P193.32
$p =$ P600
$428{,}000 + 193.32\,N = 600\,N$
$N = 1052.42$ say **1053 units**

SOLUTION 12 - 41
Ans: C

The payback period is the period where the future worth of the investment (P700) equals the future worth of all the annual savings.
Investment, P = P700.00
Annual savings = 0.06 × 3500 = P210.00
Annual maintenance = P40.00
Net annual savings, A = P210 - P40 = P170.00

$$700(F/P, 0.10, n) = 170\,(F/A, 0.10, n)$$

$$700(1 + 0.10)^n = \frac{170[(1+0.10)^n - 1]}{0.10}$$

$0.41176(1.10)^n = (1.10)^n - 1$
$0.58823(1.10)^n = 1$
$(1.10)^n = 1.7$
$n \times \log(1.10) = \log(1.70)$
$n =$ **5.567 years**

SOLUTION 12 - 42
Ans: B

The present worth of the machine is equal to the first cost (P25,000) plus the present worth of the annual payment of P5,000 for 10 years.

$$P = FC + \frac{A[(1+i)^n - 1]}{(1+i)^n i}$$

$$P = 25{,}000 + \frac{5{,}000[(1+0.10)^{10} - 1]}{(1+0.10)^{10}(0.10)} = \textbf{P55,722.84}$$

SOLUTION 12 - 43
Ans: D

$$\text{Acid-test ratio or quick ratio} = \frac{\text{quick assets}}{\text{Current liabilities}}$$

Quick assets = Cash = P10,000.00
Current liabilities = Payables + notes due + long term debt
Current liabilities = 17,000 + 6,000 + 3,000 = P26,000

$$\text{Acid-test ratio} = \frac{10000}{26000} = 0.385$$

OTHER FINANCIAL RATIOS:
Equity ratio = Total stockholders' equity / Total assets
Debt ratio = total liabilities / Total assets
Current ratio = Current assets / Current liabilities
Inventory turnover = Cost of goods sold / Average inventory
Accounts receivable turnover = Net sales on credit / Average receivables
Asset turnover = Annual sales / Ave. of assets used in producing these sales
Price-earnings ratio = Market price per share / Earnings per share
Dividend yield = Dividend per share / Market price per share
Operating expense ratio = Operating expense / Net sales

SOLUTION 12 - 44
Ans: B

BATCH PROCESS:
Initial cost = P50,000.00
Annual depreciation = P8,800
Annual income = P1,900,000(0.50) = P950,000.00
Annual maintenance = P40,000.00
Annual net income = 950,000 - 40,000 - 8,800
Annual net income = P 901,200
Annual income tax = P901,200(0.48) = P432,576
Total annual income = P468,624

Present worth of all annual income @ 15%:

$$P_{income} = \frac{468{,}624[(1+0.15)^5 - 1]}{(1+0.15)^5 0.15}$$

$$P_{income} = \underline{\text{P1,570,900.33}}$$

CONTINUOUS PROCESS:
Initial cost = P100,000.00
Annual depreciation = P17,600
Annual income = P2,000,000(0.50) = P1,000,000.00
Annual maintenance = P36,000.00
Annual net income = 1,000,000 - 36,000 - 17,600
Annual net income = P 946,400.00
Annual income tax = P946,400(0.48) = P454,272.00
Total annual income = P492,128.00

Present worth of all annual income @ 15%:

$$P_{income} = \frac{492{,}128[(1+0.15)^5 - 1]}{(1+0.15)^5 0.15}$$

$$P_{income} = \underline{\text{P1,649,689.38}}$$

The closest evaluation for best investment is choice *B*

SOLUTION 12 - 45
Ans: C

The total amount needed after one year is 80×2000 = P160,000.00

The required monthly savings, *A* is:

$$F = \frac{A[(1+i)^n - 1]}{i}$$

$$160000 = \frac{A[(1+0.12/12)^{12} - 1]}{0.12/12}$$

A = P12,615.81 ≈ **P12,610.00**

Part 8
ELEMENTS & MISCELANEOUS PROBLEMS

Problems - Set 13
Algebra, Trigonometry, Geometry

PROBLEM 13 - 1
If equals are added to equals, the sum is equal.
A. theorem
B. postulate
C. axiom
D. corollary

PROBLEM 13 - 2
Any number multiplied by _____ equals unity.
A. infinity
B. itself
C. its reciprocal
D. zero

PROBLEM 13 - 3
If every element of a column (or row) of a square matrix is multiplied by m, the determinant of the matrix will be:
A. Unchanged
B. Multiplied by m
C. It depends
D. None of these

PROBLEM 13 - 4
In probability theory, the set of possible outcomes of an experiment is ternted as:
A. a sample space
B. a set of random events
C. a set of random variables
D. a fuzzy set

PROBLEM 13 - 5
Which of the following is not a property of probability:
A. If events E_1 & E_2 are mutually exclusive, then the probability that both events can happen is zero.
B. The probability that an event can happen is always positive and is less than one or equal to one.

SOLUTION 12 - 44
Ans: B

BATCH PROCESS:
Initial cost = P50,000.00
Annual depreciation = P8,800
Annual income = P1,900,000(0.50) = P950,000.00
Annual maintenance = P40,000.00
Annual net income = 950,000 - 40,000 - 8,800
Annual net income = P 901,200
Annual income tax = P901,200(0.48) = P432,576
Total annual income = P468,624

Present worth of all annual income @ 15%:

$$P_{income} = \frac{468{,}624[(1+0.15)^5 - 1]}{(1+0.15)^5 0.15}$$

$$P_{income} = \underline{\text{P1,570,900.33}}$$

CONTINUOUS PROCESS:
Initial cost = P100,000.00
Annual depreciation = P17,600
Annual income = P2,000,000(0.50) = P1,000,000.00
Annual maintenance = P36,000.00
Annual net income = 1,000,000 - 36,000 - 17,600
Annual net income = P 946,400.00
Annual income tax = P946,400(0.48) = P454,272.00
Total annual income = P492,128.00

Present worth of all annual income @ 15%:

$$P_{income} = \frac{492{,}128[(1+0.15)^5 - 1]}{(1+0.15)^5 0.15}$$

$$P_{income} = \underline{\text{P1,649,689.38}}$$

The closest evaluation for best investment is choice *B*

SOLUTION 12 - 45
Ans: C

The total amount needed after one year is 80×2000 = P160,000.00

The required monthly savings, *A* is:

$$F = \frac{A[(1+i)^n - 1]}{i}$$

$$160000 = \frac{A[(1+0.12/12)^{12} - 1]}{0.12/12}$$

A = P12,615.81 ≈ **P12,610.00**

Part 8
ELEMENTS & MISCELANEOUS PROBLEMS

Problems - Set 13
Algebra, Trigonometry, Geometry

PROBLEM 13 - 1 If equals are added to equals, the sum is equal.
A. theorem
B. postulate
C. axiom
D. corollary

PROBLEM 13 - 2 Any number multiplied by _____ equals unity.
A. infinity
B. itself
C. its reciprocal
D. zero

PROBLEM 13 - 3 If every element of a column (or row) of a square matrix is multiplied by m, the determinant of the matrix will be:
A. Unchanged
B. Multiplied by m
C. It depends
D. None of these

PROBLEM 13 - 4 In probability theory, the set of possible outcomes of an experiment is termed as:
A. a sample space
B. a set of random events
C. a set of random variables
D. a fuzzy set

PROBLEM 13 - 5 Which of the following is not a property of probability:
A. If events E_1 & E_2 are mutually exclusive, then the probability that both events can happen is zero.
B. The probability that an event can happen is always positive and is less than one or equal to one.

C. If E_o is an event which cannot occur in the sample space, the probability of E_o is zero.
D. If events E_1 & E_2 are mutually exclusive, then $P(E_1 + E_2) = P(E_1) + P(E_2) - P(E_1 E_2)$

PROBLEM 13 - 6 An angle greater that a straight angle and less than two straight angles is called:
A. right angle C. reflex angle
B. obtuse angle D. acute angle

PROBLEM 13 - 7 A line segment joining two points on a circle is called:
A. arc C. sector
B. tangent D. chord

PROBLEM 13 - 8 All circles having the same center but with unequal radii are called
A. encircle C. concyclic
B. tangent circles D. concentric circles

PROBLEM 13 - 9 A triangle having three sides equal is called:
A. equilateral triangle C. isosceles triangle
B. scalene triangle D. right triangle

PROBLEM 13 - 10 In a regular polygon, the perpendicular line drawn from the center of the inscribed circle to any one of the sides is called:
A. radius C. median
B. altitude D. apothem

PROBLEM 13 - 11 A quadrilateral with two and only two sides of which are parallel, is called:
A. parallelogram C. quadrilateral
B. trapezoid D. rhombus

PROBLEM 13 - 12 A polygon with fifteen sides is called:
A. dodecagon C. pentedecagon
B. decagon D. nonagon

PROBLEM 13 - 13 A rectangle with equal sides is called:
A. rhombus C. square
B. trapezoid D. parallelogram

PROBLEM 13 - 14 The sum of the sides of a polygon is termed as:
A. circumference C. apothem
B. altitude D. perimeter

PROBLEM 13 - 15 A line that meets a plane but not perpendicular to it, in relation to the plane, is:
A. parallel C. coplanar
B. collinear D. oblique

PROBLEM 13 - 16 A quadrilateral whose opposite sides are equal is generally termed as:
A. a square C. a rhombus
B. a rectangle D. a parallelogram

PROBLEM 13 - 17 A part of a line included between two points on the line is called:
A. a tangent C. a sector
B. a secant D. a segment

PROBLEM 13 - 18 The section of the sphere cut by a plane through its center is termed as:
A. small circle C. big circle
B. incircle D. great circle

PROBLEM 13 - 19 Lines that pass through a common point are called:
A. collinear C. concurrent
B. coplanar D. congruent

PROBLEM 13 - 20 Points which lie on the same plane, are called:
A. collinear C. concurrent
B. coplanar D. congruent

PROBLEM 13 - 21 In two intersecting lines, the angles opposite to each other are termed as:
A. opposite angle C. horizontal angle
B. vertical angles D. inscribed angle

PROBLEM 13 - 22 A normal to a given plane is:
A. perpendicular to the plane
B. lying on the plane
C. parallel to the plane
D. oblique to the plane

PROBLEM 13 - 23 The chord passing through the focus of the parabola and perpendicular to its axis is termed as:
A. directrix C. latus rectum
B. translated axis D. axis

PROBLEM 13 - 24 The locus of the point which move so the sum of its distances between two fixed points is constant is called known as:
A. a parabola C. an ellipse
B. a circle D. a hyperbola

PROBLEM 13 - 25 A tangent to a conic is a line
A. which is parallel to the normal
B. which touches the conic at only one point

C. which passes inside the conic
D. all of the above

PROBLEM 13 - 26 The locus of a point that move so that its distance from a fixed point and a fixed line is always equal, is known as:
A. a parabola
B. a circle
C. an ellipse
D. a hyperbola

PROBLEM 13 - 27 The locus of a point, which moves so that it is always equidistant from a fixed point, is known as:
A. a parabola
B. a circle
C. an ellipse
D. a hyperbola

PROBLEM 13 - 28 In polar coordinate system, the polar angle is positive when:
A. measured clockwise
B. measured counterclockwise
C. measured at the terminal side of θ
D. none of these

PROBLEM 13 - 29 The plane rectangular coordinate system is divided into four parts which are known as:
A. coordinates
B. octants
C. quadrants
D. axis

PROBLEM 13 - 30 The rectangular coordinate system in space is divided into eight compartments, which are known as:
A. quadrants
B. octants
C. axis
D. coordinates

PROBLEM 13 - 31 A conic section whose eccentricity is less than one (1) is known as:
A. a parabola
B. an ellipse
C. a circle
D. a hyperbola

PROBLEM 13 - 32 A conic section whose eccentricity is equal to one (1) is known as:
A. a parabola
B. an ellipse
C. a circle
D. a hyperbola

PROBLEM 13 - 33 In polar coordinate system, the distance from a point to the pole is known as:
A. polar angle
B. x-coordinate
C. radius vector
D. y-coordinate

PROBLEM 13 - 34 The curve represented by the equation $r = 6$ is:
A. a parabola
B. a line
C. an ellipse
D. a circle

PROBLEM 13 - 35 When two lines are perpendicular, the slope of one is:
A. equal to the other
B. equal to the negative of the other
C. equal to the reciprocal of the other
D. equal to the negative reciprocal of the other

PROBLEM 13 - 36 The axis of the hyperbola, which is parallel to its directrices, is known as:
A. conjugate axis
B. transverse axis
C. major axis
D. minor axis

PROBLEM 13 - 37 The axis of the hyperbola through its foci is known as:
A. conjugate axis
B. transverse axis
C. major axis
D. minor axis

PROBLEM 13 - 38 A polygon is _____ if no side, when extended, will pass through the interior of the polygon.
A. convex
B. equilateral
C. isoperimetric
D. congruent

PROBLEM 13 - 39 Which of the following statements is correct?
A. all equilateral triangles are similar
B. all right-angled triangles are similar
C. all isosceles triangle are similar
D. all rectangles are similar

PROBLEM 13 - 40 The volume of any solid of revolution is equal to the generating area times the circumference of the circle described by the centroid of the area. This is commonly known as:
A. First proposition of Pappus
B. Second proposition of Pappus
C. Cavalier's Principle
D. Simpson's Rule

PROBLEM 13 - 41
ECE Nov. 1995
If the product of the slopes of any two straight lines is negative 1, one of these lines are said to be:
A. parallel
B. skew
C. perpendicular
D. non-intersecting

PROBLEM 13 - 42
ECE April 1995, March 1996
When two planes intersect with each other, the amount of divergence between the two planes is expressed be measuring the:
A. dihedral angle
B. plane angle
C. polyhedral angle
D. reflex angle

PROBLEM 13 - 43
ECE Nov. 1995
The angle which the line of sight to the object, makes with the horizontal, which is above the eye of the

observer is called:
A. angle of depression C. acute angle
B. angle of elevation D. bearing

PROBLEM 13 - 44
ECE April 1995
The median of a triangle is the line connecting a vertex and the midpoint of the opposite side. For a given triangle, these medians intersects at a point which is called the:
A. orthocenter C. circumcenter
B. incenter D. centroid

PROBLEM 13 - 45
ECE April 1995
The altitudes of the side of a triangle intersect at the point known as:
A. orthocenter C. centroid
B. circumcenter D. incenter

PROBLEM 13 - 46
ECE April 1995
The angular bisector of the sides of a triangle intersect at the point which is known as:
A. orthocenter C. centroid
B. circumcenter D. incenter

PROBLEM 13 - 47
ECE April 1995
The arc length equal to the radius of the circle is called:
A. 1 radian C. π radian
B. 1 quarter circle D. 1 grad

PROBLEM 13 - 48
ECE April 1995
A five-pointed star is also known as:
A. pentagon C. pentagram
B. pentatron D. quintagon

PROBLEM 13 - 49
ECE April 1995
The area bounded by two concentric circles is called:
A. ring C. annulus
B. disk D. sector

PROBLEM 13 - 50
ECE March 1996
The line passing through the focus and perpendicular to the directrix of a parabola is called:
A. latus rectum C. tangent line
B. axis of parabola D. secant line

PROBLEM 13 - 51
ECE March 1996
The altitudes of the sides of a triangle intersect at the point known as:
A. orthocenter C. centroid
B. circumcenter D. incenter

PROBLEM 13 - 52
ME April 1996
The length of time during which the property may be operated at a profit is called:
A. life C. physical life
B. length of time D. economic life

PROBLEM 13 - 53
ME April 1997
What is the graph of the equation $Ax^2 + Bx + Cy^2 + Dy + E = 0$?

A. Circle C. Parabola
B. Ellipse D. Hyperbola

PROBLEM 13 - 54
ECE Nov. 1996
Prisms are classified according to their _____.
A. diagonals C. vertices
B. sides D. bases

PROBLEM 13 - 55
ECE Nov. 1996
It is a polyhedron of which two faces are equal polygons in parallel planes and the other faces are parallelograms.
A. tetrahedron C. frustum
B. prism D. prismatoid

PROBLEM 13 - 56
ECE Nov. 1996
In Plain Geometry, two circular arcs that together make up a full circle are called:
A. coterminal arcs C. half arcs
B. conjugate arcs D. congruent arcs

PROBLEM 13 - 57
ECE Nov. 1996
It represents the distance of a point from the y-axis.
A. ordinate C. abscissa
B. coordinate D. polar distance

PROBLEM 13 - 58
ECE Nov. 1996
Polygons are classified according to the number of:
A. vertices C. diagonals
B. sides D. angles

PROBLEM 13 - 59
EE April 1994
In a conic section, if the eccentricity $e > 1$, the locus is :
A. an ellipse C. a parabola
B. a hyperbola D. a circle

PROBLEM 13 - 60
The family of curves which intersect a given family of curves at an angle less than 90° are called:
A. orthogonal trajectories C. isogonal trajectories
B. intersecting curves D. acute angle

PROBLEM 13 - 61
A line perpendicular to the x-axis has a slope of:
A. Zero C. Infinity
B. Unity D. None of these

PROBLEM 13 - 62
The locus of points generated when a circle is made to roll externally along the circumference of another circle.
A. Cissoid of Diocles C. Epicycloid
B. Folium of Descartes D. Cardioid

PROBLEM 13 - 63
It is the surface generated by moving a straight line (called the generator) which is always parallel to a fixed line and which always intersect a fixed plane curve (called the directrix) is:

A. Cylindrical surface
B. Locus of a point
C. Spherical surface
D. Paraboloid

PROBLEM 13 - 64 How many faces have an icosahedron?
A. 16
B. 18
C. 20
D. 22

PROBLEM 13 - 65 Each of the faces of a regular hexahedron is a:
A. Square
B. Triangle
C. Hexagon
D. Circle

PROBLEM 13 - 66 An arc length, which is equal to the radius of the circle, is called:
A. 1 degree
B. 2 π radians
C. 1 radian
D. 1 π radian

PROBLEM 13 - 67 Polygons with all interior angles less than 180° are called:
A. Concave polygon
B. Convex polygon
C. Acute polygon
D. Supplemental polygon

PROBLEM 13 - 68 To cut a right circular cone in order to reveal a parabola, it must be cut
A. perpendicular to the axis of symmetry
B. at any acute angle to the axis of symmetry
C. parallel to an element of a cone and intersecting the axis of symmetry
D. parallel to the axis of symmetry

PROBLEM 13 - 69 To find the angles of a triangle, given only the lengths of the sides, one would use
A. the law of cosines
B. the law of tangents
C. the law of sines
D. the inverse square law

PROBLEM 13 - 70 In finding the distance between two points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$, the most direct procedure is to use:
A. the law of cosines
B. the slope of the line
C. the translation of the axes
D. the Pythagorean Theorem

PROBLEM 13 - 71 In finding the distance between two points $P_1(r_1, \theta_1)$ and $P_2(r_2, \theta_2)$, the most direct procedure is to use:
A. the law of cosines
B. the slope of the line
C. the translation of the axes
D. the Pythagorean Theorem

PROBLEM 13 - 72
ECE April 1998
The area of the region bounded by two concentric circles is called:
A. washer
B. ring
C. annulus
D. circular disk

PROBLEM 13 - 73
ECE April 1998
It can be defined as the set of all points in the plane the sum of whose distance from two fixed points is a constant
A. circle
B. ellipse
C. hyperbola
D. parabola

PROBLEM 13 - 74
ECE April 1998
If the equation is unchanged by the substitution of $-x$ for x, its curve is symmetric with respect to the:
A. y-axis
B. x-axis
C. origin
D. line 45 degrees with the axis

PROBLEM 13 - 75
ECE Nov. 1997
A line which is perpendicular to the x-axis has a slope equal to:
A. zero
B. either
C. one
D. infinity

PROBLEM 13 - 76
ECE Nov. 1997
In an ellipse, a chord which contains a focus and is in a line perpendicular to the major axis is a:
A. latus rectum
B. minor
C. focal width
D. conjugate axis

PROBLEM 13 - 77
ME Oct. 1997
In general triangles the expression $(\sin A)/a = (\sin B)/b = (\sin C)/c$ is called:
A. Euler's formula
B. Law of cosines
C. the Law of sines
D. Pyhthagorean theorem

PROBLEM 13 - 78
ME Oct. 1997
What type of curve is generated by a point which moves in uniform circular motion about an axis, while travelling at a constant speed, v, parallel to the axis?
A. helix
B. spiral of Archimedes
C. hypocycloid
D. cycloid

PROBLEM 13 - 79
ME Oct. 1997
An angle more than π radian but less than 2π radians is:
A. straight angle
B. obtuse angle
C. related angle
D. reflex angle

PROBLEM 13 - 80
ME April 1998
The sum of the sides of a polygon:
A. perimeter
B. square
C. hexagon
D. circumference

PROBLEM 13 - 81
ME April 1998

A plane closed curve, all points of which are the same distance from a point within, called the center:
A. arc
B. circle
C. radius
D. chord

PROBLEM 13 - 82
ME April 1998

One-fourth of a great circle:
A. cone
B. quadrant
C. circle
D. sphere

PROBLEM 13 - 83
ME April 1998

Points that lie in the same plane:
A. coplanar
B. oblique
C. collinear
D. parallel

PROBLEM 13 - 84
ME April 1998

The study of the properties of figures of three dimensions:
A. physics
B. plane geometry
C. solid geometry
D. trigonometry

PROBLEM 13 - 85
ME April 1998

The volume of a circular cylinder is equal to the product of its base and altitude.
A. postulate
B. theorem
C. corollary
D. axiom

ANSWER KEY:

1. A	16. D	31. B	46. D	61. C	76. A
2. C	17. D	32. A	47. A	62. C	77. C
3. B	18. D	33. C	48. C	63. A	78. D
4. A	19. C	34. D	49. C	64. C	79. D
5. D	20. B	35. D	50. B	65. A	80. A
6. C	21. B	36. A	51. A	66. C	81. B
7. D	22. A	37. B	52. D	67. B	82. B
8. D	23. C	38. A	53. B	68. C	83. A
9. A	24. C	39. A	54. D	69. A	84. C
10. D	25. B	40. B	55. B	70. D	85. B
11. B	26. A	41. C	56. B	71. A	
12. C	27. B	42. A	43. B	72. C	
13. A	28. B	43. B	57. C	73. B	
14. D	29. C	44. D	58. B	74. A	
15. D	30. B	45. A	59. B	75. D	
			60. C		

Problems - Set 14
Calculus, Physics, Mechanics, Economics

PROBLEM 14 - 1
ECE Nov. 1995, March 1996

A point on the curve where the second derivative of a function is equal to zero is called:
A. maxima
B. minima
C. point of inflection
D. point of intersection

PROBLEM 14 - 2
ECE Nov. 1995

The point on the curve where the first derivative of a function is zero and the second derivative is positive is called:
A. maxima
B. minima
C. point of inflection
D. point of intersection

PROBLEM 14 - 3
ECE Nov. 1996

At the minimum point, the slope of the tangent line is:
A. negative
B. infinity
C. positive
D. zero

PROBLEM 14 - 4
ECE Nov. 1996

At the point of inflection where $x = a$,
A. $f''(a)$ = is not equal to zero
B. $f''(a) = 0$
C. $f''(a) > 0$
D. $f''(a) < 0$

PROBLEM 14 - 5
ECE April 1998

Point of the derivatives, which do not exist (and so equals zero) are called:
A. stationary point
B. maximum points
C. maximum and minimum point
D. minimum point

PROBLEM 14 - 6
ECE Nov. 1997

If the second derivative of the equation of a curve is equal to the negative of the equation of that same curve, the curve is:
A. a cissoid
B. a paraboloid
C. a sinusoid
D. an exponential

PROBLEM 14 - 7

It is defined as the motion of a rigid body in which a straight line passing through any two of its particles always remains parallel to its initial position.
A. translation
B. rotation
C. plane motion
D. kinetics

PROBLEM 14 - 8 Which of the following is not a vector quantity?
A. mass
B. torque
C. displacement
D. velocity

PROBLEM 14 - 9 The product of force and the time during which it acts is known as:
A. impulse
B. momentum
C. work
D. impact

PROBLEM 14 - 10 The property of the body which measures its resistance to changes in motion.
A. acceleration
B. weight
C. mass
D. rigidity

PROBLEM 14 - 11 The study of motion without reference to the forces which causes motion is known as:
A. kinetics
B. dynamics
C. statics
D. kinematics

PROBLEM 14 - 12 A branch of physical science that deals with state of rest or motion of bodies under the action of forces is known as:
A. mechanics
B. kinetics
C. kinematics
D. statics

PROBLEM 14 - 13 In physics, work is defined in terms of the force acting through a distance. The rate at which the work is done is called:
A. force
B. energy
C. power
D. momentum

PROBLEM 14 - 14 The point through which the resultant of the distributed gravity force passes regardless of the orientation of the body in space is known as:
A. center of inertia
B. center of gravity
C. center of attraction
D. moment of inertia

PROBLEM 14 - 15 The specific gravity of the substance is the ratio of the density of the substance to the density of water. Another term for specific gravity is:
A. specific weight
B. unit weight
C. relative density
D. density

PROBLEM 14 - 16 The momentum of a moving object is the product of its mass (M) and velocity (v). Newton's Second Law of Motion says that the rate of change of momentum with respect to time is:
A. power
B. energy
C. momentum
D. force

PROBLEM 14 - 17 The acceleration due to gravity in the English System or ft/s² is:
A. 20.2
B. 32.2
C. 15.2
D. 62.4

PROBLEM 14 - 18
ECE Nov. 1995
Ivory soap floats in water because:
A. all matter has mass
B. the density of ivory soap is unity
C. the specific gravity of ivory soap is greater than that of water
D. the specific gravity of ivory soap is less than that of water

PROBLEM 14 - 19
ECE April 1995
One (1) gram of ice at 0°C is placed on a container containing 2,000,000 cu. m. of water at 0 °C. Assuming no heat loss, what will happen?
A. ice will become water
B. some part of ice will not change
C. the volume of ice will not change
D. all of the above

PROBLEM 14 - 20
ECE March 1996
When two waves of the same frequency, speed and amplitude traveling in opposite directions superimposed,
A. destructive interference always results
B. constructive interference always results
C. standing waves are produced
D. the phase difference is always zero

PROBLEM 14 - 21
ECE Nov. 1995, March 1996
Any two points along a streamline in an ideal fluid in steady flow, the sum of the pressure, the potential energy per unit volume, and the kinetic energy per unit volume has the same value. This concept is known as the:
A. Pascal's theorem
B. Bernoulli's energy theorem
C. Fluid theory
D. Hydraulic theorem

PROBLEM 14 - 22
ECE March 1996
Whenever a net force acts on a body, it produces an acceleration in the direction of the resultant force, an acceleration which is directly proportional to the resultant force and inversely proportional to the mass of the body. This theory is popularly known as:
A. Newton's First Law of Motion
B. Newton's Second Law of Motion
C. Faraday's Law of Forces
D. Hooke's Law of Equilibrium

PROBLEM 14 - 23
ME April 1996

Kinematic viscosity in SI derived unit is described as:
A. watt per meter Kelvin
B. sq. m. per second
C. Pascal-second
D. Newton per meter

PROBLEM 14 - 24
ME April 1996

In a cantilever beam with a concentrated load at the free end, the moment is:
A. constant along the beam
B. maximum at the wall
C. 1/4 maximum halfway out on the beam
D. maximum at the free end

PROBLEM 14 - 25
ME April 1996

What is the name for the vector that represents the sum of two vectors?
A. scalar
B. tangent
C. tensor
D. resultant

PROBLEM 14 - 26
ME April 1996

The loss of weight of a body submerged in a fluid is:
A. proportional to the weight of the body
B. proportional to the depth of submergence
C. equal to the weight of the fluid displaced
D. independent of the volume of the body

PROBLEM 14 - 27
ME April 1997

A leak from a faucet comes out in separate drops. Which of the following is the main cause of this phenomenon?
A. gravity
B. air resistance
C. viscosity of the fluid
D. surface tension

PROBLEM 14 - 28
ECE Nov. 1996

Inelastic collision is a collision in which the total kinetic energy after collision is _____ before collision.
A. equal to zero
B. equal
C. less than
D. greater than

PROBLEM 14 - 29
ECE Nov. 1996

The property by virtue of which a body tends to return to its original size or shape after a deformation and when the deforming forces have been removed.
A. Elasticity
B. Malleability
C. Ductility
D. Plasticity

PROBLEM 14 - 30
ECE Nov. 1996

A flowerpot falls off the ledge of a fifth-floor window. Just as it passes the third-floor window someone accidentally drops a glass of water from the window. Which of the following is true?
A. The flowerpot hits the ground at the same time as the glass.
B. The glass hits the ground before the flowerpot.

C. The flowerpot hits the ground first and with a higher speed than the glass.
D. The flowerpot and the glass hit the ground at the same instant.

PROBLEM 14 - 31
EE April 1994

One Joule of work is done by a force of one Newton acting through a distance of:
A. one centimeter
B. one inch
C. one meter
D. one foot

PROBLEM 14 - 32
EE April 1994

Kinetic energy equals:
A. ½ mass × velocity
B. mass × velocity
C. mass × acceleration
D. ½ mass × velocity^2

PROBLEM 14 - 33
EE April 1994

In an ideal gas where p = pressure, V = volume, and T = absolute temperature in degrees Kelvin, which of the following is constant?
A. pV/T
B. pT/V
C. pVT
D. p/VT

PROBLEM 14 - 34
EE April 1994

The path of the projectile is:
A. a parabola
B. an ellipse
C. a part of a circle
D. a hyperbola

PROBLEM 14 - 35

One mole of gas at standard temperature and pressure (STP) conditions occupies a volume equal to:
A. 22.4 liters
B. 9.81 liters
C. 332 liters
D. 2274.5 liters

PROBLEM 14 - 36

"Equal volume of all gases under the same conditions of temperature and pressure contain the same number of molecules". This hypothesis is popularly known as:
A. Dalton's hypothesis
B. Avogadro's hypothesis
C. Debye-Sear's hypothesis
D. Compton's hypothesis

PROBLEM 14 - 37

The ratio of the uniform triaxial stress, to the change in volume at equal stress in all directions is:
A. Modulus of flexure
B. Modulus of rapture
C. Bulk Modulus of elasticity
D. Coefficient of restitution

PROBLEM 14 - 38

According to the laws of Johannes Kepler, "The orbit of satellite is an ellipse, the radius vector sweeps equal areas in equal intervals of time and the square of the periods of revolution with respect to both the satellite

and planet is proportional to the cubes of their mean distance from each other." The shape of the ellipse depends upon its:
A. Eccentricity
B. Lengths of latera recta
C. Apogee and perigee
D. Ascending and descending nodes

PROBLEM 14 - 39
This implies the resistance to shock or difficulty of breaking and expresses the work per unit volume required to fracture a material.
A. Toughness
B. malleability
C. Hardness
D. Ductility

PROBLEM 14 - 40
The reciprocal of bulk modulus of elasticity of any fluid is called:
A. Compressibility
B. Volume strain
C. Volume stress
D. Shape factor

PROBLEM 14 - 41
"The resultant of the external force applied to an object composed of a system of particles, is equal to the vector summation of the effective forces acting on all particles". This principle is known as:
A. Archimede's Principle
B. Bernoulli's Principle
C. D'Alembert's Principle
D. Gauss-Jordan principle

PROBLEM 14 - 42
Calorie is the amount of heat required to increase the temperature of _____ of water by one degree centigrade.
A. 1 kg
B. 1 lb
C. 1 mg
D. 1 gram

PROBLEM 14 - 43
ECE April 1998
It describes the luminous flux incidence per unit area and is expressed in lumens per square meter.
A. Luminous intensity
B. Illuminance
C. Radiance
D. Luminance

PROBLEM 14 - 44
ECE April 1998
The moment of inertia of a plane figure:
A. is zero at the centroidal axis
B. increase as distance of the axis moves farther from the centroid
C. decrease as the distance of the axis moves farther from the centroid
D. is maximum at the centroidal axis

PROBLEM 14 - 45
ECE April 1998
The distance that the top surface is displaced in the direction of the force divided by the thickness of the body is known as:

A. longitudinal strain
B. shear strain
C. volume strain
D. linear strain

PROBLEM 14 - 46
ECE April 1998
To maximize the horizontal range of the projectile, which of the following applies?
A. maximize the angle of elevation
B. maximize velocity
C. maximize the angle of elevation and velocity
D. the tangent function of the angle of trajectory must be equal to one

PROBLEM 14 - 47
ECE Nov. 1997
According to this law, "The force between two charges varies directly as the magnitude of each charge and inversely as the square of the distance between them.
A. Law of Universal Gravitation
B. Newton's Law
C. Coulomb's Law
D. Inverse Square Law

PROBLEM 14 - 48
ME Oct. 1997
Formation of bubbles in a low-pressure area in a centrifugal pump and later their sudden collapse, is called:
A. compression
B. corrosion
C. explosion
D. cavitation

PROBLEM 14 - 49
ME Oct. 1997
The hardness of steel may be increased by heating to approximately 1500° F and quenching in oil or water if
A. the carbon content is above 3.0%
B. the carbon content is from 0.2% to 2.0%
C. the carbon content is below 0.2%
D. the steel has been hot rolled instead of cast.

PROBLEM 14 - 50
ME Oct. 1997
Galvanized iron is a term referring to iron coated with:
A. magnesium
B. aluminum
C. zinc
D. tin

PROBLEM 14 - 51
ME Oct. 1997
A process of welding metals in a molten or in vaporous state without application of mechanical pressure or blow. Such welding may be accomplished by the oxy-acetylene or by hydrogen flame or by electric arc. It is called:
A. fusion welding
B. TIG welding
C. MIG welding
D. cold welding

PROBLEM 14 - 52
ME Oct. 1997
A chemical method of feed water treatment wherein water is passed through a bed of sodium zeolite Nesub2Z which reacts with calcium and magnesium salts:

A. demineralization process
B. ion exchange treatment
C. lime soda treatment
D. thermal treatment

PROBLEM 14 - 53
ME Oct. 1997

Used as a guide to selecting the most efficient centrifugal pump:
A. Specific Speed
B. Impeller type
C. Bernoulli's Equation
D. Overall Efficiency

PROBLEM 14 - 54
ME Oct. 1997

The impulse and momentum principle is mostly useful for solving problems involving:
A. velocity, acceleration, and time
B. force, acceleration, and time
C. force, velocity, and time
D. force, velocity, and acceleration

PROBLEM 14 - 55
ME Oct. 1997

Which of the following is not true regarding the Blasius boundary layer solution?
A. It permits one to calculate the skin friction on a flat plate
B. It is valid for laminar flow
C. It is an approximate solution
D. It is valid only for potential flow

PROBLEM 14 - 56
ME Oct. 1997

The greatest unit pressure the soil can continuously withstand:
A. point of rapture
B. bearing strength
C. ultimate strength
D. yield point

PROBLEM 14 - 57
ME Oct. 1997

Heat transmission carried by the movement of heated fluids away from a hot body, as in the heating of water by a hot surface:
A. radiation
B. convection
C. conduction
D. absorption

PROBLEM 14 - 58
ME Oct. 1997

The type of cooler extensively used for medium and large size diesel engines:
A. radiator cooler
B. shell and tube cooler
C. disk cooler
D. plate cooler

PROBLEM 14 - 59
ME Oct. 1997

A closed vessel intended for use in heating water or for application of heat to generate steam or other vapor to be used externally to itself is called:
A. Unfired pressure vessel
B. Steam Generator
C. Boiler or Steam Generator
D. Boiler

PROBLEM 14 - 60
ME Oct. 1997

The sum of the three types of energy at any point in the system is called:
A. Bernoulli's Theorem
B. enthalpy
C. internal energy
D. pressure heads

PROBLEM 14 - 61
ME Oct. 1997

In energy transformation process in which the resultant condition lacks the driving potential needed to reverse the process, the measure of this loss is expressed as:
A. enthalpy increase of the system
B. specific bent ratio of the moment
C. entropy increase of the system
D. entropy decrease of the system

PROBLEM 14 - 62
ME Oct. 1997

The system is safe to be in thermodynamics equilibrium:
A. if it has no tendency to undergo further chemical reaction
B. when there is no tendency towards spontaneous change
C. when the system is not accelerating
D. when all its parts are at the same temperature

PROBLEM 14 - 63
ME Oct. 1997

An instrument for measuring high temperature gases:
A. plenometer
B. manometer
C. anemometer
D. pyrometer

PROBLEM 14 - 64
ME Oct. 1997

The power output of the engine is increased through:
A. turbo-charging
B. scavenging
C. all of these
D. super-charging

PROBLEM 14 - 65
ME Oct. 1997

The equilibrium temperature that a regular thermometer measures if exposed to atmospheric air is:
A. dry bulb temperature
B. °C
C. wet bulb temperature
D. dew point

PROBLEM 14 - 66
ME Oct. 1997

On the hoist or load block or some equality visible space of every hoist designed to lift its load vertically shall be legibly marked:
A. its electrical voltage
B. its brand and model
C. its rated load capacity
D. its motor hp or kW

PROBLEM 14 - 67
ME Oct. 1997

The hardness of water is given in ppm (parts per million, i.e., pounds per million pounds of water). This hardness is
A. the total number of pounds of dissolved solids in the water per million pounds of water.
B. the total number of pounds of calcium and magnesium bicarbonate in the water.

C. the total number of pounds of sodium carbonate in the water per million pounds of water.
D. the total number of pounds of salt (sodium chloride) in the water per million pounds of water

PROBLEM 14 - 68
ME Oct. 1997

Momentum = Force × _____.
A. time
B. velocity
C. velocity^2
D. ½ velocity

PROBLEM 14 - 69
ME April 1998

An instrument for measuring specific gravity of fluids:
A. hygrometer
B. flowmeter
C. psycrometer
D. hydrometer

PROBLEM 14 - 70

The recorded current value of an asset is known as:
A. scrap value
B. book value
C. salvage value
D. present worth

PROBLEM 14 - 71

The ratio of the interest payment to the principal for a given unit of time and is usually expressed as a percentage of the principal is known as:
A. investment
B. nominal interest
C. interest
D. interest rate

PROBLEM 14 - 72

A method of depreciation whereby the amount to recover is spread over the estimated life of the asset in terms of the periods or units of output is called:
A. SOYD method
B. declining balance method
C. straight line method
D. sinking fund method

PROBLEM 14 - 73

The interest rate at which the present worth of cash flow on a project is zero, or the interest earned by an investment.
A. rate of return
B. effective rate
C. nominal rate
D. yield

PROBLEM 14 - 74

The lessening of the value of an asset due to the decrease in the quantity available. This refers to the natural resources such as coal, oil, and timber in the forest.
A. depreciation
B. depletion
C. inflation
D. incremental cost

PROBLEM 14 - 75

The method of depreciation where a fixed sum of money is regularly deposited at compound interest in a real or imaginary fund in order to accumulate an amount equal to the total depreciation of an asset at the end of the asset's estimated life is known as:

A. straight line method
B. SYD method
C. declining balance method
D. sinking fund method

PROBLEM 14 - 76

The term used to express the series of uniform payments occurring at equal interval of time is:
A. compound interest
B. annuity
C. perpetuity
D. depreciation

PROBLEM 14 - 77

The profit derived from a project or business enterprise without consideration of obligations to financial contributors and claims of others based on profit is known as:
A. yield
B. earning value
C. economic return
D. expected yield

PROBLEM 14 - 78

As applied to capitalized asset, the distribution of the initial cost by periodic changes to operation as in depreciation or the reduction of the depth by either periodic or irregular prearranged program is called:
A. amortization
B. annuity
C. depreciation
D. capital recovery

PROBLEM 14 - 79

Those funds that are required to make the enterprise or project a going concern.
A. banking
B. accumulated amount
C. working capital
D. principal or present worth

PROBLEM 14 - 80
ECE Nov. 1995

These are product or services that are desired by humans and will be purchased if money is available after the required necessities have been obtained.
A. utilities
B. necessities
C. luxuries
D. producer goods and services

PROBLEM 14 - 81
ECE Nov. 1995

These are product or services that are required to support human life and activities, that will be purchased in somewhat the same quantity even though the price varies considerably.
A. utilities
B. necessities
C. luxuries
D. producer goods and services

PROBLEM 14 - 82
ECE Nov. 1995

A condition where only few individuals produce a certain product and that any action of one will lead to almost the same action of the others.
A. oligopoly
B. semi-monopoly
C. monopoly
D. perfect competition

PROBLEM 14 - 83
ECE Nov. 1995

This occurs in a situation where a commodity or service is supplied by a number of vendors and there is nothing to prevent additional vendors entering the market.
A. perfect competition
B. monopoly
C. oligopoly
D. elastic demand

PROBLEM 14 - 84
ECE March 1996

It is the amount that a willing buyer will pay to a willing seller for a property where each has equal advantage and is under no compulsion to buy or sell.
A. fair value
B. use value
C. market value
D. book value

PROBLEM 14 - 85
ECE March 1996

It is defined to be the capacity of a commodity to satisfy human want.
A. discount
B. luxuries
C. utility
D. necessity

PROBLEM 14 - 86
ME April 1996

A form of summary of assets, liabilities and net worth:
A. balance method
B. break-even point
C. balance sheet
D. production

PROBLEM 14 - 87
ME April 1996

The worth of a property, which is equal to the original cost less depreciation, is known as:
A. earning value
B. scrap value
C. book value
D. face value

PROBLEM 14 - 88
ME April 1996

When using net present worth calculations to compare two projects, which of the following could invalidate the calculations?
A. mutually exclusive projects
B. evaluation over different periods
C. non-conventional cash flows
D. difference in the magnitude of the projects

PROBLEM 14 - 89
ME April 1996

Which of the following is a form of business/company ownership?
A. partnership
B. corporation
C. single proprietorship
D. all of these

PROBLEM 14 - 90

What must two investments with the same present worth and unequal lives have?

A. identical salvage value
B. different salvage values
C. identical equivalent uniform annual cash flows
D. different equivalent annual cash flows

PROBLEM 14 - 91

What is the formula for the straight-line depreciation rate?

A. $\dfrac{100\% - \%\text{netsalvagevalue}}{\text{estimatedservicelife}}$

B. $\dfrac{\%\text{netsalvagevalue}}{\text{estimatedservicelife}}$

C. $\dfrac{100\%\text{netsalvagevalue}}{\text{estimatedservicelife}}$

D. $\dfrac{\text{average netsalvagevalue}}{\text{estimatedservicelife}}$

PROBLEM 14 - 92

The amount received from the sale of an additional unit of a product is termed as:
A. marginal cost
B. marginal utility
C. marginal unit
D. marginal revenue

PROBLEM 14 - 93

An accounting book where the original record of all transactions is ordinarily recorded.
A. journal
B. credit entry
C. debit entry
D. transaction record

PROBLEM 14 - 94

An interest-earning fund in which equal deposits are made at equal intervals of time for the purpose of gradually accumulating a specific sum of money required at some future date.
A. amortization
B. sinking fund
C. annuity
D. capitalized cost

PROBLEM 14 - 95
ME April 1997

What is work-in-process classified as?
A. an asset
B. a liability
C. an expense
D. owner's equity

PROBLEM 14 - 96
ME April 1997

Which is a form of business/company ownership?
A. partnership
B. corporation
C. single proprietorship
D. all of the list

PROBLEM 14 - 97
ME April 1997

The decrease in value of a physical property due to the passage of time:
A. inflation
B. depletion
C. recession
D. depreciation

PROBLEM 14 - 98
ECE Nov. 1996

We may classify an interest rate, which specifies the actual rate on interest on the principal for one year as:
A. Nominal rate
B. Rate of return
C. Exact interest rate
D. Effective rate

PROBLEM 14 - 99

This law states that "When the use of one of the factors of production is limited, either in increasing cost or by absolute quantity, a point will be reached beyond which an increase in the variable factors will result in less than proportionate increase in output."
A. Law of Supply and Demand
B. Law of Diminishing Return
C. Law of Gravity
D. Law of Diminishing Utility

PROBLEM 14 - 100

The difference between the present worth and the worth at some future time is:
A. Discount
B. Profit
C. Credit
D. Debit

PROBLEM 14 - 101

The act of charging two or more buyers or sellers different prices for the same product where the elasticities of supply or demand at each price level differ among markets, is known as:
A. Price optimization
B. Price discrimination
C. Price manipulation
D. Price war

PROBLEM 14 - 102

A formal organization of producers within an industry forming a perfect collision purposely formed to increase profit and block newcomers from the industry, is termed as:
A. Oligopoly
B. Monopoly
C. Corporation
D. Cartel

PROBLEM 14 - 103

A situation in which rival firms drive prices down through attempts to undercut each other's prices is called:
A. Price war
B. Price discrimination
C. Price leadership
D. Oligopolistic monopoly

PROBLEM 14 - 104

The price that the quantity per unit time that the buyers want to buy is just equal to the quantity the sellers want to sell is called:
A. Break-even price
B. Equilibrium price
C. Minimum price
D. Maximum price

PROBLEM 14 - 105

The length of time during which a property is capable of performing the function for which it was designed and manufactured, is termed as:
A. Economic life
B. Life span
C. Eternal life
D. Physical life

PROBLEM 14 - 106

It is a distinct legal entity, separate from individuals who own it and which can engage in practically any business transaction which a real person can do.
A. Partnership
B. Single proprietorship
C. Corporation
D. Competition

PROBLEM 14 - 107
ECE Nov. 1997

A market whereby there is only one buyer of an item for which there are no goods substitute.
A. Monopoly
B. Oligopoly
C. Oligopsony
D. Monopsony

PROBLEM 14 - 108
ME Oct. 1997

This form of business ownership is a legal entity possessing many of the legal powers of individuals. It can hold title to property, sue and be sued in its own name. It is called:
A. corporation
B. company
C. general partnership
D. individual proprietorship

PROBLEM 14 - 109
ME Oct. 1997

The balance sheet is a statement showing the financial status of the company at any given time. Which of the statements is not a part of the balance sheet?
A. liabilities
B. asset
C. cost of goods sold
D. net worth

PROBLEM 14 - 110
ME Oct. 1997

Refers to cost of merchandise which excludes freight and insurance costs:
A. freight-on-board
B. sunk cost
C. debentures
D. book value

PROBLEM 14 - 111
ME Oct. 1997

A fund into which annual deposits of A are made in order to accumulate fund F at n years in the future:
A. Amortization
B. Annuity
C. Depreciation
D. Sinking fund

PROBLEM 14 - 112
ME Oct. 1997

Cash money credit necessary to establish and operate an enterprise:
A. funds
B. capital
C. liabilities
D. assets

PROBLEM 14 - 113
ME Oct. 1997

The worth of property which is equal to the original cost less depreciation:
A. scrap value
B. earning value
C. book value
D. face value

PROBLEM 14 - 114
ME Oct. 1997

Intangible assets of a corporation or company
A. dacion-en-pago
B. investment
C. patents
D. equity

PROBLEM 14 - 115
ME Oct. 1997

Additional information to prospective bidders on contract documents issued prior to bidding date:
A. escalatory clause
B. delicts
C. technological assessment
D. bid bulletin

PROBLEM 14 - 116
ME Oct. 1997

A civil wrong committed by one person causing damage to another person or his property or reputation:
A. tort
B. negligence
C. material breach
D. fraud

PROBLEM 14 - 117
ME April 1998

What is an annuity?
A. A series of equal payments at equal time periods.
B. The cost of manufacturing a product.
C. An investment that yields an equal amount of interest each y ear.
D. The overhead cost per unit of production.

PROBLEM 14 - 118
ME April 1998

Consist of the actual counting or determination of the actual quantity of the materials on hands as of a given date:
A. physical inventory
B. technological assessment
C. material update
D. material count

PROBLEM 14 - 119
ME April 1998

An artificial expense that spreads the purchase price of an asset or other property over a number of years.
A. Depreciation
B. Amnesty
C. Sinking fund
D. Bond

PROBLEM 14 - 120
ME April 1998

Estimated value at the end of the useful life:
A. salvage value
B. economic life
C. balance sheet
D. compounded annually

ANSWER KEY

1. C	21. B	41. C	61. A	81. B	101. B
2. B	22. B	42. D	62. B	82. A	102. D
3. D	23. B	43. B	63. D	83. A	103. A
4. B	24. B	44. B	64. D	84. C	104. B
5. A	25. C	45. B	65. A	85. C	105. D
6. C	26. C	46. D	66. C	86. C	106. C
7. A	27. D	47. C	67. B	87. C	107. D
8. A	28. C	48. D	68. A	88. B	108. A
9. A	29. A	49. B	69. D	89. D	109. C
10. C	30. C	50. C	70. B	90. C	110. D
11. D	31. C	51. A	71. D	91. A	111. D
12. A	32. D	52. B	72. C	92. D	112. B
13. C	33. A	53. A	73. A	93. A	113. C
14. B	34. A	54. C	74. B	94. B	114. C
15. C	35. A	55. D	75. D	95. A	115. D
16. D	36. B	56. B	76. B	96. D	116. A
17. B	37. C	57. B	77. C	97. D	117. A
18. D	38. A	58. A	78. A	98. D	118. A
19. C	39. D	59. C	79. C	99. D	119. A
20. C	40. A	60. A	80. C	100. A	120. A

Problems - Set 15
Miscellaneous Problems

PROBLEM 15 - 1
CE Nov. 2000

A line in a map was drawn at a scale of 1:25000. An error of 0.02 mm in the drawing is equivalent to how many meters in actual?

A. 5 m
B. 0.5 m
C. 0.05 m
D. 50 m

PROBLEM 15 - 2
ME Oct. 2000

One day a Celsius thermometer and a Fahrenheit thermometer registered exactly the same numerical value for the temperature. What was the temperature that day?

A. -20
B. 20
C. 40
D. -40

PROBLEM 15 - 3
CE May 2000

Convert 405° to mils.

A. 2,800 mils
B. 10,200 mils
C. 7,200 mils
D. 6,200 mils

PROBLEM 15 - 4
CE May 2000

Rationalize the following: $\dfrac{\sqrt{a}}{\sqrt{a}-\sqrt{ab}}$.

A. $\dfrac{1-\sqrt{b}}{1+b}$
B. $\dfrac{a+a\sqrt{b}}{a-b}$
C. $\dfrac{a-a\sqrt{b}}{a-b}$
D. $\dfrac{1+\sqrt{b}}{1-b}$

PROBLEM 15 - 5
CE Nov. 2000

Solve for B in the given partial fraction:

$$\frac{x^4-15x^3-32x^2-12x-14}{(x+1)(x-2)(x+3)(x^2+2)} = \frac{A}{x+1}+\frac{B}{x-2}+\frac{C}{x+3}+\frac{Dx+E}{x^2+2}$$

A. 1
B. -3
C. -4
D. 2

PROBLEM 15 - 6
ME Oct. 2000

Solve for x from the given equation, $7.4 \times 10^{-4} = e^{-9.7x}$

A. 0.7423
B. 0.7342
C. 0.7243
D. 0.4732

PROBLEM 15 - 7
CE May 2000

$\text{Log}_8\, 975 = x$. Find x

A. 3.31
B. 4.12
C. 5.17
D. 2.87

A. 1/2310 C. 1/3810
B. 1/5620 D. 1/1860

PROBLEM 15 - 16
ME Oct. 2000

What is the angle between two vectors A and B?

$$A = 4i + 12j + 6k$$
$$B = 24i - 8j + 6k$$

A. 175.4° C. 84.3°
B. -84.9° D. 86.3°

PROBLEM 15 - 17
ME Oct. 2000

The expression $[\tan\theta \sec\theta (1 - \sin^2\theta)] / (\cos\theta)$ simplifies to:

A. $\sin\theta$ C. $\tan\theta$
B. $\sec\theta$ D. $\cos\theta$

PROBLEM 15 - 18
CE Nov. 2000

Given that $\tan A = 4/5$, what is the value of $\dfrac{4\sin A - \cos A}{3\cos A + \sin A}$?

A. 0.579 C. 0.654
B. 0.752 D. 0.925

PROBLEM 15 - 19
CE Nov. 2000

A flagpole 3 m high stands at the top of a pedestal 2 m high located at one side of a pathway. At the opposite side of the pathway directly facing the flagpole, the flagpole subtends the same angle as the pedestal. What is the width of the pathway?

A. 4.47 m C. 6.28 m
B. 3.21 m D. 8.41 m

PROBLEM 15 - 20
CE May 2000

Find the area in sq. m. of a spherical triangle of whose angles are 123°, 84°, and 73°. The radius of the sphere is 30 m.

A. 1863.3 C. 1958.6
B. 1570.8 D. 1480.2

PROBLEM 15 - 21
CE May 2000

Two sides of a triangle measure 18 cm and 6 cm. The third side may be:

A. 12 C. 10
B. 13 D. 11

PROBLEM 15 - 22
CE May 2000

The perimeter of an ellipse is 28.448 units. If the major axis is 10 units, what is length of the minor axis?

A. 9 C. 8
B. 7 D. 6

PROBLEM 15 - 23
CE Nov. 2000

A right regular hexagonal prism is inscribed in a right circular cylinder whose height is 20 cm. The difference between the circumference of the circle and the perimeter of the hexagon is 4 cm. Determine the volume of the prism.

A. 9756 cc C. 10857 cc
B. 114752 cc D. 10367 cc

PROBLEM 15 - 24
ME Oct. 2000

Find the area bounded by the x-axis, the line $x = 4$, and the parabola $y^2 = 4x$.

A. 64/2 C. 32/4
B. 32/3 D. 32/2

PROBLEM 15 - 25
CE Nov. 2000

What is the area bounded by the curves $y^2 = 4x$ and $x^2 = 4y$?

A. 6.0 C. 6.666
B. 7.333 D. 5.333

PROBLEM 15 - 26
CE Nov. 2000

Given a regular hexagonal with consecutive corners $ABCDEF$. If the bearing of side AB is N 25° E, what is the bearing of side FA?

A. N 15° W C. N 35° W
B. N 45° W D. N 5° W

PROBLEM 15 - 27
CE Nov. 2000

The perimeter of a triangle is 58 cm and its area is 144 sq. cm. What is the radius of the inscribed circle?

A. 4.97 cm C. 5.52 cm
B. 9.65 cm D. 3.12 cm

PROBLEM 15 - 28
ME Oct. 2000

What is the area bounded by the curves $y = 8 - x^2$ and $y = -2 + x^2$?

A. 22.4 C. 44.7
B. 26.8 D. 29.8

PROBLEM 15 - 29

A solid sphere of radius 20 cm was placed on top of hallow circular cylinder of radius 10 cm. What volume of the sphere was inside the cylinder?

A. 431 cc C. 325 cc
B. 568 cc D. 542 cc

PROBLEM 15 - 30

A trough is formed by nailing together, edge to edge, two boards 130 cm in length, so that the right section is a right triangle. If 3500 cc of water is poured into the trough and if the trough is held level so that right section of the water is an isosceles right triangle, how deep is the water?

A. 6.32 cm C. 4.21 cm
B. 5.19 cm D. 6.93 cm

PROBLEM 15 - 31
CE May 2000

The lateral area of a right circular cone of radius 4 cm is 100.53 sq. cm. Determine the slant height.

A. 8 cm
B. 9 cm
C. 6 cm
D. 10 cm

PROBLEM 15 - 32
CE May 2000

The frustum of a regular triangular pyramid has equilateral triangles for its bases and has an altitude of 8 m. The lower base edge is 9m. If the volume is 135 cu.m., what is the upper base edge?

A. 2 m
B. 5 m
C. 4 m
D. 3 m

PROBLEM 15 - 33
CE May 2000

A cylinder of radius 6 m has its axis along the x-axis. A second cylinder of the same radius has its axis along the y-axis. Find the volume, in the first octant, common to the two cylinders.

A. 866 m^3
B. 1152 m^3
C. 144 m^3
D. 288 m^3

PROBLEM 15 - 34
CE May 2000

Find the volume of a right circular cylinder whose lateral area is 25.918 m^2 and base area of 7.068 m^2.

A. 19.44 m^3
B. 15.69 m^3
C. 20.53 m^3
D. 18.12 m^3

PROBLEM 15 - 35
CE Nov. 2000

A solid has a circular base of base radius 20 cm. Find the volume of the solid if every plane section perpendicular to a certain diameter is an isosceles right triangle with one leg in the plane of the base.

A. 21333 cc
B. 24155 cc
C. 18667 cc
D. 20433 cc

PROBLEM 15 - 36
CE Nov. 2000

The base diameter of a cone is 18 cm and its axis is inclined 60° with the base. If the axis is 20 cm long, what is the volume of the cone?

A. 1524 cc
B. 1469 cc
C. 1245 cc
D. 1689 cc

PROBLEM 15 - 37
ME Oct. 2000

The equation $9x^2 + 16y^2 + 54x - 64y = -1$ describes:

A. a circle
B. a parabola
C. a hyperbola
D. an ellipse

PROBLEM 15 - 38
CE May 2000

Two vertices of a triangle are (6, -1) and (7, -3). Find the ordinate of the third vertex such that the centroid of the triangle will lie at the origin.

A. -13
B. 4
C. 13
D. -4

PROBLEM 15 - 39
CE May 2000

Determine the equation of the directrix of the curve $x^2 = 16y$.

A. $x + 4 = 0$
B. $x - 4 = 0$
C. $y - 4 = 0$
D. $y + 4 = 0$

PROBLEM 15 - 40
CE Nov. 2000

Find the area of the curve $x^2 + y^2 + 6x - 12y + 9 = 0$.

A. 125 sq. units
B. 113 sq. units
C. 92 sq. units
D. 138 sq. units

PROBLEM 15 - 41
CE Nov. 2000

Find the distance between the foci of the curve $9x^2 + 25y^2 - 18x + 100y - 116 = 0$.

A. 7
B. 6
C. 8
D. 12

PROBLEM 15 - 42
CE Nov. 2000

What is the equivalent rectangular coordinate of a point whose polar coordinate is (7, 38°).

A. (3.56, 4.31)
B. 4.31, 5.52)
C. (5.52, 4.31)
D. (4.31, 3.56)

PROBLEM 15 - 43

The chords of the parabola $y^2 = 16x$ having equal slope of 2 is bisected by its diameter. What is the equation of the diameter?

A. $y = 16$
B. $y = 8$
C. $y = 2$
D. $y = 4$

PROBLEM 15 - 44

Find the slope of the line whose parametric equation is $x = 3t^2 - 6$ and $y = 4t^2 + 7$.

A. ¾
B. ½
C. 3
D. 4/3

PROBLEM 15 - 45
ME Oct. 2000

The first derivative with respect to y of the function $f(y) = \sqrt[3]{9}$ is:

A. $3\sqrt{9/2}$
B. 0
C. 9
D. $3\sqrt{9}$

PROBLEM 15 - 46
ME Oct. 2000

Find the derivative of $f(x)$ = [x to the 3rd power – (x – 1) to the 3rd power] to the 3rd power?

A. $3x^3 - 3(x - 1)$
B. 9[x to the 3rd power – (x – 1) to the 3rd power]^2 × [$x^2 - (x - 1)^2$]
C. 9[x to the 3rd power – (x – 1) to the 3rd power] × [$x^2 - (x - 1)^2$]
D. 3[x to the 3rd power – (x – 1) to the 3rd power]

PROBLEM 15 - 47

The derivative of $\sin(2x^2 + 1)$ with respect to x is:

A. $4x \cos(2x^2 + 1)$
B. $x \sin(2x^2 + 1)$
C. $\cos(2x^2 + 1)$
D. $x \cos(2x^2 + 1)$

PROBLEM 15 - 48

Find the dy/dx: $y = x^{x^2}$.

A. $x^{x^2+1}(1 + \ln x^2)$
B. $x^{x^2+1}(1 + \ln x)$
C. $x^{x^2}(1 + \ln x^2)$
D. $x^{x^2+2}(2 + \ln x^2)$

PROBLEM 15 - 49

What is the second derivative of $y = 4x^2 - 2x + 1$ at $x = 2$.

A. 8
B. 0
C. 1
D. not defined

PROBLEM 15 - 50
CE May 2000

At what value of x will the slope of the curve $x^3 - 9x - y = 0$ be 18?

A. 2
B. 4
C. 5
D. 3

PROBLEM 15 - 51
CE Nov. 2000

The slope of the curve at any point is given as $6x - 2$ and the curve passes through (5, 3). Determine the equation of the curve.

A. $3x^2 + 2x - y + 62 = 0$
B. $2x^2 - 3x + y + 62 = 0$
C. $2x^2 + 3x - y - 62 = 0$
D. $3x^2 - 2x - y - 62 = 0$

PROBLEM 15 - 52
CE May 2000

The total surface area of a closed cylindrical tank is 153.94 square meter. If the volume is to be maximum, what is its height in meters.

A. 6.8 m
B. 5.7 m
C. 3.6 m
D. 4.5.m

PROBLEM 15 - 53
CE Nov. 2000

A closed cylindrical tank having a volume of 71.57 m³ is to be constructed. If the surface area is to be minimum, what is the required diameter of the tank?

A. 4 m
B. 5.5 m
C. 5 m
D. 4.5 m

PROBLEM 15 - 54

Two posts, one 16 feet and the other 24 feet are 30 feet apart. If the posts are to be supported by a cable running from the top of the first post to a stake on the ground and then back to the top of the second post, find the distance from the lower post to the stake to use the least amount of wire.

A. 6 feet
B. 9 feet
C. 15 feet
D. 12 feet

PROBLEM 15 - 55
CE Nov. 2000

The motion of a body moving vertically upwards is expressed as $h = 100t - 16.1\ t^2$, where h is the height in feet and t is the time in seconds. What is the velocity of the body when $t = 2$ seconds?

A. 21.7 fps
B. 28.7 fps
C. 24.1 fps
D. 35.6 fps

PROBLEM 15 - 56
ME Oct. 2000

Water from the filtering facility is pouring into a swimming pool. After n hours, there are $n + \sqrt{n}$ gallons in the pool. At what rate is the water pouring into the pool when $n = 16$ hours.

A. ½ gph
B. 9/8 gph
C. 1 gph
D. 7/6 gph

PROBLEM 15 - 57
CE May 2000

A lighthouse is 2 km off a straight shore. A search light at the lighthouse focuses to a car moving along the shore. When the car is 1 km from the point nearest to the lighthouse, the searchlight rotates 0.25 rev/hour. Find the speed of the car in kph.

A. 3.93
B. 2.56
C. 2.92
D. 3.87

PROBLEM 15 - 58
CE May 2000

Evaluate $\int_0^{\pi/2} 3e^{3\sin\theta} \cos\theta\, d\theta$

A. 15.421
B. 19.086
C. 17.048
D. 20.412

PROBLEM 15 - 59

Determine the area enclosed by the curve $r^2 = 4a^2 \cos 3\theta$.

A. a^2
B. $4a^2$
C. $8a^2$
D. $16a^2$

PROBLEM 15 - 60
CE May 2000

Determine the moment of inertia about the x-axis, of the area bounded by the curve x^2 = $4y$, the line $x = -4$, and the x-axis.

A. 9.85
B. 13.24
C. 10.17
D. 12.19

PROBLEM 15 - 61
CE Nov. 2000

Evaluate the integral of $x\, dx\ /\ (x^2 + 2)$ with limits from 0 to 1.

A. 0.322
B. 0.108
C. 0.203
D. 0.247

PROBLEM 15 - 62
CE Nov. 2000

The area bounded by the curve $y = \sin x$ from $x = 0$ to $x = \pi$ is revolved about the x-axis. What is the volume generated?

A. 2.145 cu. units
B. 4.935 cu. units
C. 3.452 cu. units
D. 5.214 cu. units

PROBLEM 15 - 63
ME Oct. 2000

If you borrow money from your friend with simple interest of 12%, find the present worth of P20,000 which is due at the end of nine months.

A. P18,688.20
B. P18,691.50
C. P18,518.50
D. P18,348.60

PROBLEM 15 - 64
ME Oct. 2000

A businessman needs to have P100,000 in five years. How much must he put into his 10% account in the bank now?

A. P72,085.6 C. P70,654.1
B. P62,092.1 D. P60.345.2

PROBLEM 15 - 65
ME Oct. 2000

What is the present worth of P54,000.00 due in five years if money is worth 11% and is compounded semi-annually?

A. P30,367.12 C. P31,613.25
B. P28,654.11 D. P34,984.32

PROBLEM 15 - 66
CE May 2000

How long will it take for money to quadruple itself if invested at 20% compounded quarterly?

A. 10.7 years C. 9.5 years
B. 6.3 years D. 7.1 years

PROBLEM 15 - 67
ME Oct. 2000

The interest on an account is 13% compounded annually. How many years approximately will it take to triple the amount?

A. 8 years C. 9.5 years
B. 8.5 years D. 9 years

PROBLEM 15 - 68
ME Oct. 2000

When will an investment of P4000 double if the effective rate is 8% per annum?

A. 8.4 C. 9.01
B. 8.3 D. 10.2

PROBLEM 15 - 69
ME Oct. 2000

A savings association pays 1.5% interest quarterly. What is the effective annual interest rate?

A. 6.14% C. 7.32%
B. 8.54% D. 6.45%

PROBLEM 15 - 70
ME Oct. 2000

A bank offers 0.5% effective monthly interest. What is the effective annual rate with monthly compounding?

A. 6.2% C. 7.2%
B. 6% D. 7%

PROBLEM 15 - 71
ME Oct. 2000

What nominal rate converted quarterly could be used instead of 12% compounded semi-annually?

A. 10.76% C. 11.82%
B. 11.43% D. 11.97%

PROBLEM 15 - 72
CE Nov. 2000

P1,000,000 was invested to an account earning 8% compounded continuously. What is the amount after 20 years?

A. P4,452,796.32 C. P5,365,147.25
B. P4,953,032.42 D. P3,456,254.14

PROBLEM 15 - 73
ME Oct. 2000

A sum of money is deposited now in a savings account. The effective annual interest rate is 12%. How much money must be deposited to yield P500.00 at the end of 11 months?

A. P153.00 C. P446.00
B. P144.00 D. P451.00

PROBLEM 15 - 74
ME Oct. 2000

What amount of money would have to be invested to have P40,000 at the end of three years at a 10% compounded interest rate?

A. P30,052.59 C. P32,871.12
B. P28,456.79 D. P34,908.33

PROBLEM 15 - 75
CE May 2000

Five thousand pesos is deposited at the end of each year for 15 years into an account earning 7.5% compounded continuously. Find the amount after 15 years.

A. P133,541.3 C. P148,365.9
B. P152,754.2 D. P112,854.1

PROBLEM 15 - 76
ME Oct. 2000

A sinking fund consist of 15 annual deposits of P1,000 each, with interest earned at the rate of 4% compounded annually. What is the principal in the fund at its terminal date?

A. P25,115.00 C. P29,546.37
B. P20,023.59 D. P28,105.00

PROBLEM 15 - 77
CE Nov. 2000

A man made a year-end payment of P100,000 to an account earning 8% annually for 10 years. How much is in the account after 20 years?

A. P3,127,540.18 C. P3,327,452.36
B. P4,075,458.99 D. P3,247,111.25

PROBLEM 15 - 78
ME Oct. 2000

A P10,000 loan is to be paid off in 10 equal annual payments. The annual interest rate is 15%. How much interest will be paid in the first two years?

A. P2,926.12 C. P2,915.87
B. P2,970.11 D. P2,950.62

PROBLEM 15 - 79 A piece property is purchased for P1,000,000 and yields P100,000 yearly profit. If the property is sold after five years, what is the minimum price to break-even, with interest at 6%?

A. P774,516 C. P500,000
B. P568,254 D. P650,253

PROBLEM 15 - 80 Annual maintenance cost for a particular section of a highway pavement are P200,000. The placement of a new surface would reduce the annual maintenance cost to P50,000 per year for the first five years and to P100,000 for the next five years. The annual maintenance after ten years would again be P200,000. If maintenance cost are the only saving, what maximum investment can be justified for the new surface? Assume interest at 4%.

A. P1,523,256 C. P1,033,681
B. P1,254,004 D. P1,800,450

PROBLEM 15 - 81 A man made a ten annual end-of-year purchases of P100,000 of common stock. At the end of the tenth year he sold the stock for P1.2M. What interest rate did he obtain on his investment?

A. 4% C. 4.5%
B. 3.5% D. 2.5%

PROBLEM 15 - 82
CE May 2000 A machine costing P480,000 has a life expectancy of 12 years with a salvage value of 10% of the first cost. What is the book value after five years using the declining balance method.

A. P183,892 C. P196,432
B. P152,758 D. P214,785

PROBLEM 15 - 83
CE May 2000 A certain copier machine cost P150,000 with a trade-in value of P15,000 after making 800,000 copies. Using the declining balance method, what is the book value when the machine had made 300,000 copies?

A. P68,111 C. P62,531
B. P64,896 D. P63,254

PROBLEM 15 - 84 A generator costing P350,000 is guaranteed to have a useful operating life of 60,000 hours, after which it has a salvage value of 10% of its first cost. During its first year of service, it was used for 15,000 hours. What is its book value after 1 year?

A. P271,250 C. P256,350
B. P285,650 D. P198,650

PROBLEM 15 - 85 Company X has a fixed expense of P1.5M monthly and each unit is produced at P20. Company Y has a fixed expense of P500,000 per month and can produce the same unit at P60. At what number of units of monthly production will company X have the same overall cost as company Y?

A. 20000 C. 30000
B. 25000 D. 15000

PROBLEM 15 - 86
ME Oct. 2000 A factory is running at 80% efficiency with a fixed cot of P78,000.00, variable cost per unit of P130.00, selling price per unit of P416.00, and production capacity of 5000 units. What is the current profit of the factory if all product manufactured are sold?

A. P1,066,000 C. P1,213,000
B. P1,456,000 D. P1,146,000

PROBLEM 15 - 87
CE Nov. 2000 The cost of producing a certain commodity consist of P45.00 per unit for labor and material cost and P15.00 per unit for other variable cost. The fixed cost per month amounts to P450,000. If the commodity is sold at P250.00 each, what is the break-even quantity?

A. 2014 C. 2589
B. 2178 D. 2367

SITUATION:
Problem 15 – 88 to 15 – 89
CE May 2000 The three concurrent forces shown in the figure are in equilibrium.

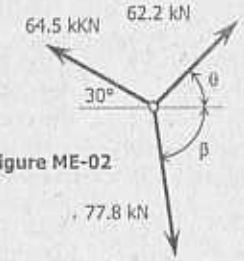


Figure ME-02

PROBLEM 15 - 88 ❶ Which of the following most nearly gives the value of angle θ.

A. 52° C. 38°
B. 42° D. 46°

PROBLEM 15 - 89 ❷ Which of the following most nearly gives the value of angle β.

A. 85° C. 75°
B. 81° D. 76°

PROBLEM 15 - 90 ❸ Which of the following most nearly gives the vertical component of the 77.8-kN force.

A. 76.8 kN C. 73.2 kN
B. 70.2 kN D. 65.3 kN

SITUATION: Problem 15 – 91 to 15 – 93 CE May 2000

The system shown in Figure ME-08 consist of rigid bars *AB*, *AC*, *BD*, & *CD* hinged at points *A*, *B*, *C*, and *D*. When *P* is zero, points *B* and *C* are coincident (i.e. *A*, *B*, *C*, & *D* lies on a straight line) and the tension on *AB* is 200 N. For the given arrangement in the figure:

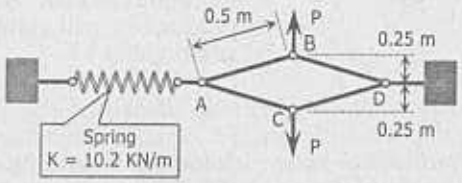


FIGURE ME-08

PROBLEM 15 - 91

❶ Which of the following most nearly gives the tension on member *AB* in N.

A. 860 C. 980
B. 1130 D. 1020

PROBLEM 15 - 92

❷ Which of the following most nearly gives the value of *P* in N.

A. 840 C. 860
B. 1020 D. 1540

PROBLEM 15 - 93

❸ Which of the following most nearly gives the spring constant *K* for system *ABCD* in N/m.

A. 3450 C. 2040
B. 1870 D. 2560

PROBLEM 15 - 94
ME Oct. 2000

A 56-lb mass rests on a horizontal surface. The force needed to move it along the surface is 20 lb. Determine the coefficient of friction.

A. 0.01 C. 0.12
B. 0.21 D. 0.34

PROBLEM 15 - 95
ME Oct. 2000

A car travels with an initial velocity of 10 m/s or 36 km/hr. If it is decelerating at a rate of 3 m/sec-sec, how far, in meters, does it travel before stopping?

A. 15.1 m C. 17.3 m
B. 13.9 m D. 16.7 m

PROBLEM 15 - 96

Cars *A* and *B* approached each other on a straight road from points 450 m apart. Car *A* has an initial velocity of 70 kph and decelerates at the rate of 0.4 m/s². Car *B* has an initial velocity of 20 kph and accelerates at the rate of 0.3 m/s². After how many seconds will the two cars meet?

A. 18.7 s C. 8.1 s
B. 12.3 s D. 25.8 s

PROBLEM 15 - 97

An object starting from rest moves along a straight line such that, after *t* seconds, its distance from its starting point is:

$$S = 4t^2 + 20/(t^2 - 1) \text{ meters.}$$

Finds its acceleration (in m/s²) after 4 seconds.

A. 8.581 C. 9.632
B. 8.77 D. 7.853

PROBLEM 15 - 98

An elevator starting from rest at the first floor of a tall building accelerates uniformly at 0.5 m/s² for 4 seconds, then move at a constant speed for 10 seconds, then decelerate uniformly for 6 seconds until it stopped. If the floors are 3 m apart, at what floor did it stop?

A. 9th C. 10th
B. 11th D. 12th

PROBLEM 15 - 99

A stone is dropped from the top of a building to a pool on the ground. A man at the top of the building heard the "splash" 5 seconds after the stone was dropped. How tall is the building? Assume speed of sound = 300 m/s.

A. 92 m C. 106 m
B. 85 m D. 134 m

PROBLEM 15 - 100
CE May 2000

A projectile is fired at an angle of 20° with the horizontal at the top of a 30-m high building. The muzzle velocity is 300 m/s. What is the total time of flight?

A. 32.6 s C. 21.2 s
B. 25.7 s D. 18.5 s

PROBLEM 15 - 101
ME Oct. 2000

A baseball is thrown straight upward with a velocity of 20 m/s. Compute the time elapsed for the baseball to return to the ground. Assume zero drag.

A. 2.04 s C. 4.08 s
B. 3.45 s D. 6.9 s

PROBLEM 15 - 102
ME Oct. 2000

Determine the superelevation of the outer rail of a 4-ft wide railroad track on a 10° curve to give equal soil pressure on a car when moving with a speed of 45 mph. (A 10° curve is one which a chord 100 ft long subtends an angle of 10° at the center.)

A. 2.7 feet C. 1.2 feet
B. 0.9 feet D. 2.6 feet

PROBLEM 15 - 103

A winch lifted a mass of 1600 kg through a height of 25 m in 30 seconds. Calculate the input power in kW if the efficiency of the winch is 60%.

A. 7.85 kW C. 13.1 kW
B. 18.2 kW D. 21.8 kW

SITUATION: Problem 15 – 104 to 15 – 106 CE Nov. 2000

5-kg block resting on a smooth surface is pushed horizontally by a force *P* as shown in Figure ME-11. The graph of force *P* versus time is also shown in the figure.

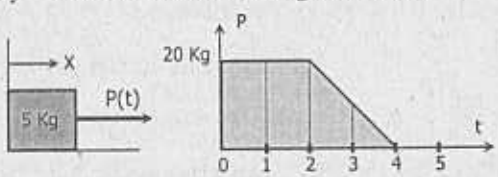


Figure ME-11

PROBLEM 15 - 104 ❶ Which of the following most nearly gives the acceleration of the block during the first two seconds in meters per second per second.

A. 39 C. 28
B. 45 D. 32

PROBLEM 15 - 105 ❷ Which of the following most nearly gives the velocity of the block after five seconds in meters per second.

A. 113 C. 118
B. 102 D. 129

PROBLEM 15 - 106 ❸ Which of the following most nearly gives the total distance traveled by the block in five seconds in meters.

A. 395 C. 420
B. 380 D. 405

PROBLEM 15 - 107 ME Oct. 2000

Compute for the torsional deflection in degrees of a 110-mm diameter, 1.4 m long shaft subjected to a twisting moment of 3×10^6 N-mm. The torsional modulus of elasticity is 83,000 N/mm².

A. 0.1° C. 0.2°
B. 0.3° D. 0.4°

PROBLEM 15 - 108 ME Oct. 2000

Find the torsional moment in Newton-mm) developed when the shaft delivers 20 kW at 200 rpm.

A. 2×10^6 N-mm C. 0×10^6 N-mm
B. 1×10^6 N-mm D. 0.5×10^6 N-mm

PROBLEM 15 - 109 ME Oct. 2000

Determine the load capacity in kN on a 25-mm diameter × 1200 mm long steel shaft if its maximum elongation shall not exceed 1 mm.

A. 78.9 kN C. 85.2 kN
B. 65.3 kN D. 81.8 kN

PROBLEM 15 - 110 ME Oct. 2000

A cylindrical tank with 10-inch inside diameter contains oxygen gas at 2500 psi. Calculate the required wall thickness in millimeters is the stress shall not exceed 28,000 psi.

A. 12.01 C. 11.34
B. 9.87 D. 10.76

ANSWER SHEET

1.	A	B	C	D	E	21.	A	B	C	D	E	41.	A	B	C	D	E
2.	A	B	C	D	E	22.	A	B	C	D	E	42.	A	B	C	D	E
3.	A	B	C	D	E	23.	A	B	C	D	E	43.	A	B	C	D	E
4.	A	B	C	D	E	24.	A	B	C	D	E	44.	A	B	C	D	E
5.	A	B	C	D	E	25.	A	B	C	D	E	45.	A	B	C	D	E
6.	A	B	C	D	E	26.	A	B	C	D	E	46.	A	B	C	D	E
7.	A	B	C	D	E	27.	A	B	C	D	E	47.	A	B	C	D	E
8.	A	B	C	D	E	28.	A	B	C	D	E	48.	A	B	C	D	E
9.	A	B	C	D	E	29.	A	B	C	D	E	49.	A	B	C	D	E
10.	A	B	C	D	E	30.	A	B	C	D	E	50.	A	B	C	D	E
11.	A	B	C	D	E	31.	A	B	C	D	E	51.	A	B	C	D	E
12.	A	B	C	D	E	32.	A	B	C	D	E	52.	A	B	C	D	E
13.	A	B	C	D	E	33.	A	B	C	D	E	53.	A	B	C	D	E
14.	A	B	C	D	E	34.	A	B	C	D	E	54.	A	B	C	D	E
15.	A	B	C	D	E	35.	A	B	C	D	E	55.	A	B	C	D	E
16.	A	B	C	D	E	36.	A	B	C	D	E	56.	A	B	C	D	E
17.	A	B	C	D	E	37.	A	B	C	D	E	57.	A	B	C	D	E
18.	A	B	C	D	E	38.	A	B	C	D	E	58.	A	B	C	D	E
19.	A	B	C	D	E	39.	A	B	C	D	E	59.	A	B	C	D	E
20.	A	B	C	D	E	40.	A	B	C	D	E	60.	A	B	C	D	E

ANSWER SHEET (Continuation..)

61.	A	B	C	D	E	81.	A	B	C	D	E	101.	A	B	C	D	E
62.	A	B	C	D	E	82.	A	B	C	D	E	102.	A	B	C	D	E
63.	A	B	C	D	E	83.	A	B	C	D	E	103.	A	B	C	D	E
64.	A	B	C	D	E	84.	A	B	C	D	E	104.	A	B	C	D	E
65.	A	B	C	D	E	85.	A	B	C	D	E	105.	A	B	C	D	E
66.	A	B	C	D	E	86.	A	B	C	D	E	106.	A	B	C	D	E
67.	A	B	C	D	E	87.	A	B	C	D	E	107.	A	B	C	D	E
68.	A	B	C	D	E	88.	A	B	C	D	E	108.	A	B	C	D	E
69.	A	B	C	D	E	89.	A	B	C	D	E	109.	A	B	C	D	E
70.	A	B	C	D	E	90.	A	B	C	D	E	110.	A	B	C	D	E

71.	A	B	C	D	E	91.	A	B	C	D	E	111.	A	B	C	D	E
72.	A	B	C	D	E	92.	A	B	C	D	E	112.	A	B	C	D	E
73.	A	B	C	D	E	93.	A	B	C	D	E	113.	A	B	C	D	E
74.	A	B	C	D	E	94.	A	B	C	D	E	114.	A	B	C	D	E
75.	A	B	C	D	E	95.	A	B	C	D	E	115.	A	B	C	D	E
76.	A	B	C	D	E	96.	A	B	C	D	E	116.	A	B	C	D	E
77.	A	B	C	D	E	97.	A	B	C	D	E	117.	A	B	C	D	E
78.	A	B	C	D	E	98.	A	B	C	D	E	118.	A	B	C	D	E
79.	A	B	C	D	E	99.	A	B	C	D	E	119.	A	B	C	D	E
80.	A	B	C	D	E	100.	A	B	C	D	E	120.	A	B	C	D	E

Solutions to Set 15 Miscellaneous Problems

SOLUTION 15 - 1
Ans: B

Actual error = 0.02 × 25,000 = 500 mm = **0.5 m**

SOLUTION 15 - 2
Ans: D

The relationship between Celsius (C) and Fahrenheit (F) is:

$C = (F - 32)(5/9)$

For $F = C$

$F = (F - 32)(5/9)$

$9F = 5F - 160$

$4F = 160;\ \ F = -40 = C$

SOLUTION 15 - 3
Ans: C

$$405° \times \frac{1600\text{ mils}}{90°} = \mathbf{7{,}200\ mils}$$

SOLUTION 15 - 4
Ans: D

$$\frac{\sqrt{a}}{\sqrt{a}-\sqrt{ab}} \times \frac{\sqrt{a}+\sqrt{ab}}{\sqrt{a}+\sqrt{ab}} = \frac{a+a\sqrt{b}}{a-ab} = \frac{1+\sqrt{b}}{1-b}$$

SOLUTION 15 - 5
Ans: A

$$\frac{x^4-15x^3-32x^2-12x-14}{(x+1)(x-2)(x+3)(x^2+2)} = \frac{A}{x+1} + \frac{B}{x-2} + \frac{C}{x+3} + \frac{Dx+E}{x^2+2}$$

$x^4 - 15x^3 - 32x^2 - 12x - 14 = A(x-2)(x+3)(x^2+2) + B(x+1)(x+3)(x^2+2) + C(x+1)(x-2)(x^2+2) + (Dx+E)(x+1)(x-2)(x+3)$

Set $x = 2$:

$(2)^4 - 15(2)^3 - 32(2)^2 - 12(2) - 14 = 0 + B(3)(5)(6) + 0 + 0$

$-270 = 90B;\ \ B = \mathbf{-3}$

SOLUTION 15 - 6
Ans: A

$7.4 \times 10^{-4} = e^{-9.7x}$, taking the ln of both sides

$\ln(7.4 \times 10^{-4}) = \ln(e^{-9.7x})$

$-7.2089 = -9.7x$

$x = \mathbf{0.7432}$

SOLUTION 15 - 7
Ans: A

$\log_8 975 = x$

$x = \log 975 / \log 8;\ \ x = \mathbf{3.31}$

SOLUTION 15 - 8
Ans: A

$a_{11} = a_1 + (n-1)d$

$18 = 6 + (11-1)d$

$d = \mathbf{1.2}$

SOLUTION 15 - 9
Ans: A

3, ___, ___, ___, ___, 729

$a_1 = 3;\ a_6 = 729$

$a_n = a_1\, r^{n-1}$

$729 = 3\, r^{6-1};\ r = 3$

$a_4 = 3(3)^{4-1}$

$a_4 = \mathbf{81}$

SOLUTION 15 - 10
Ans: C

In the series a1, a_2, and a_3, a_2 is called the mean.

Geometric mean, $a_3 = 8$

$$\frac{8}{a_1} = \frac{a_3}{8};\quad a_1 a_3 = 64 \quad \rightarrow (1)$$

Arithmetic mean, $a_2 = 17$

$17 - a_1 = a_3 - 17$

$a_1 + a_3 = 34;\quad a_3 = 34 - a_1 \quad \rightarrow (2)$

In Eq. (1): $a_1\,(34 - a_1) = 64$

$a_1^2 - 34\, a_1 + 64 = 0$

$a_1 = \mathbf{32}$ and 2

SOLUTION 15 - 11
Ans: C

Required number of man-days to finish the job = 28(60)
= 1680 man-days

28 persons worked for 15 days, 28 - 5 = 23 persons worked for 44 - 15 = 29 days, and 23 + 10 = 33 persons worked for t days until completion.

$28(15) + 23(29) + 33t = 1680;\ t = 17.97$ days

Number of days the job was completed = 15 + 29 + 17.97
= 61.97 days

Number of days delayed = **1.97 days**

SOLUTION 15 - 12
Ans: B

No. of man-days required to finish the job, N

$N = 20(30) = 600$ man-days

$25(18) + (25 - 10)\, t = 600;\ t = 10$ days

Total number of days = 18 + 10 = **28 days**

SOLUTION 15 - 13
Ans: D

Required man-days to finish the job = 12(20) = 240

Let x be the required additional workers:

$6(6) + (6 + x)(12) = 240$

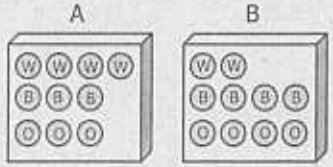
$x = \mathbf{11}$ **workers**

SOLUTION 15 - 14
Ans: C

Case I:

Box A orange & Box B not orange:

$$P_I = \frac{3}{10} \times \frac{6}{10} = \frac{9}{50}$$



Case II:

Box A not orange,

Box B orange:

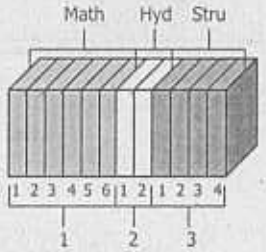
$$P_{II} = \frac{7}{10} \times \frac{4}{10} = \frac{14}{50}$$

$$P = P_I + P_{II} = \frac{9}{50} + \frac{14}{50} = \frac{23}{50}$$

SOLUTION 15 - 15
Ans: A

$$P = \frac{\textit{No of ways books of the same kind are all together}}{\textit{No of ways of arranging the twelve books in any order}}$$

No. of ways that books of the same kind are together:



$N = (6! \times 2! \times 4!) \times 3!$

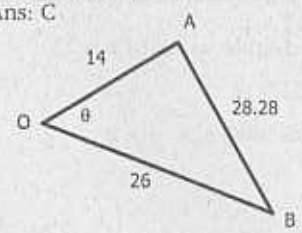
$N = 207360$ ways

No. of ways of arranging the twelve books in any order:

$N = 12! = 479001600$ ways

$$\text{Probability} = \frac{207{,}360}{479{,}001{,}600} = \mathbf{1/2310}$$

SOLUTION 15 - 16
Ans: C



$$OA = \sqrt{4^2 + 12^2 + 6^2} = 14$$

$$OB = \sqrt{24^2 + (-8)^2 + 6^2} = 26$$

$$AB = \sqrt{(4-24)^2 + (12+8)^2 + (6-6)^2}$$

$AB = 28.28$

By cosine law:

$28.28^2 = 14^2 + 26^2 - 2(14)(26)\cos\theta$

$\theta = \mathbf{84.30°}$

SOLUTION 15 - 17
Ans: C

$[\tan\theta \sec\theta\,(1 - \sin^2\theta)] \,/\, (\cos\theta)$

$$= \frac{\sin\theta}{\cos\theta}\,\frac{1}{\cos\theta}\,(\cos^2\theta) \,/\, \cos\theta = \frac{\sin\theta}{\cos\theta}$$

$= \tan\theta$

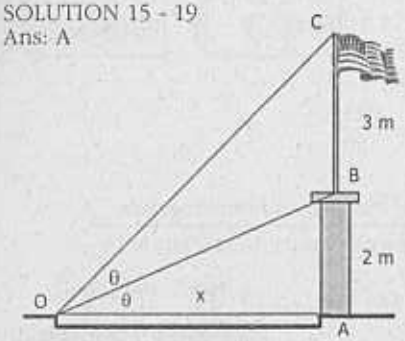
SOLUTION 15 - 18
Ans: A

$\tan A = 4/5,\ A = 38.66°$

$$\frac{4\sin A - \cos A}{3\cos A + \sin A} = \frac{4\sin 38.66° - \cos 38.66°}{3\cos 38.66° + \sin 38.66°}$$

$$\frac{4\sin A - \cos A}{3\cos A + \sin A} = \mathbf{0.579}$$

SOLUTION 15 - 19
Ans: A



In triangle OAB: $\tan\theta = 2/x$

In triangle OAC: $\tan 2\theta = 5/x$

$$\tan 2\theta = \frac{2\tan\theta}{1 = \tan^2\theta};\quad \frac{5}{x} = \frac{2\frac{2}{x}}{1 - \frac{4}{x^2}}$$

$$5 = \frac{4}{\frac{x^2-4}{x^2}} = \frac{4x^2}{x^2-4}$$

$5x^2 - 20 = 4x^2$

$x^2 = 20;\ x = \mathbf{4.47\ m}$

SOLUTION 15 - 20
Ans: B

$$\text{Area} = \frac{\pi R^2 E}{180°}$$

$E = A + B + C - 180°$

$E = 123° + 84° + 73° - 180° = 100°$

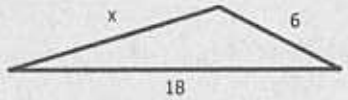
$$\text{Area} = \frac{\pi(30)^2(100°)}{180°} = \mathbf{1570.8\ sq.\ m.}$$

SOLUTION 15 - 21
Ans: B

From the figure shown,

$6 + x > 18$

$x > 12$



Thus, among the choices the possible value of x is **13**

SOLUTION 15 - 22
Ans: C

$$P = 2\pi\sqrt{\frac{a^2+b^2}{2}} = 2\pi\sqrt{\frac{5^2+b^2}{2}} = 28.448;\ b = 4$$

Minor axis = $2b = \mathbf{8}$

SOLUTION 15 - 23
Ans: D

$P_{cricle} - P_{hexagon} = 4$ cm

$2\pi r - 6r = 4;\ r = 14.125$ cm

$V = A_b \times h$

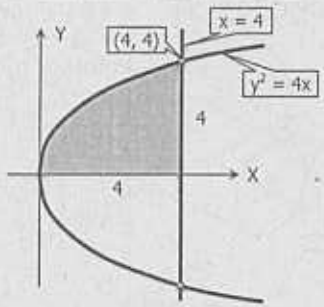
$A_b = ½(14.125)(14.125)\sin 60° \times 6$

$A_b = 518.36\ \text{cm}^2$

$V = 518.36 \times 20 = \mathbf{10367.13\ cm^3}$

SOLUTION 15 - 24
Ans: B

$A = \frac{1}{3}(4)(4) = 32/3$



SOLUTION 15 - 25
Ans: D

Points of intersection:

$y^2 = 4x$ square both side

$y^4 = 16x^2$

$y^4 = 16(4y) = 64y$

$y^4 - 64y = 0$

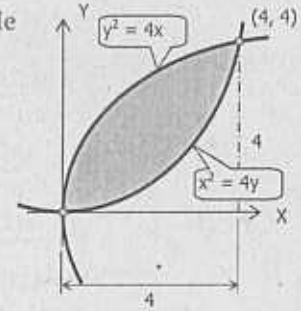
$y(y^3 - 64) = 0$

$y = 0$ and

$y = 4$

When $y = 0,\ x = 0$ (0, 0)

When $y = 4,\ x = 4$ (4, 4)



Using the formula:

Area = $A_{parabola} - A_{spandrel}$

Area = $\frac{2}{3}(4)(4) - \frac{1}{3}(4)(4) = \mathbf{5.333\ sq.\ units}$

SOLUTION 15 - 26
Ans: C

The sum of the interior angles of the lot is $180°(n - 2) = 180°(6 - 2) = 720°$

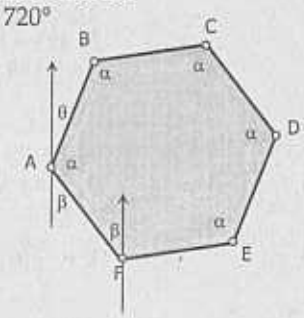
$\alpha = 720°/6 = 120°$

$\theta = 25°$

$\theta + \alpha + \beta = 180°$

$\beta = 180° - 25° - 120° = 35°$

Bearing of FA = **N 35° W**



SOLUTION 15 - 27
Ans: A

$A_T = rs$

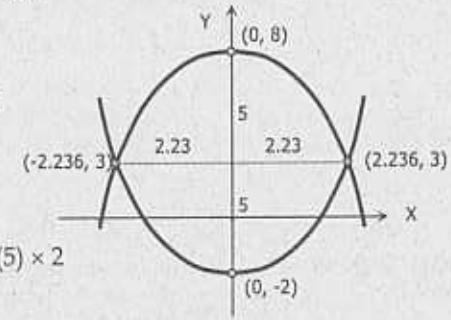
$$a = \text{semi-perimeter} = \frac{a+b+c}{2} = \frac{58}{2} = 29$$

$144 = r(29)$

$r = 4.97$ cm

SOLUTION 15 - 28
Ans: D

$y = 8 - x^2$ or $x^2 = -(y - 8)$ [vertex (0, 8) open down]
$y = -2 + x^2$ or $x^2 = y + 2$ [vertex (0, -2) open up]
Points of intersection:
$y = y$
$8 - x^2 = -2 + x^2$
$x^2 = 10$
$x = \pm\sqrt{5} = \pm 2.236$
$y = 8 - (\sqrt{5})^2$
$y = 3$



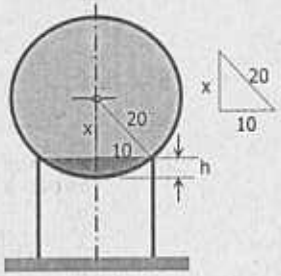
$A = \frac{2}{3}(2 \times 2.236)(5) \times 2$
$A = \mathbf{29.81}$

SOLUTION 15 - 29
Ans: A

$x = \sqrt{20^2 - 10^2} = 17.32$
$h = 20 - x = 2.68$ cm

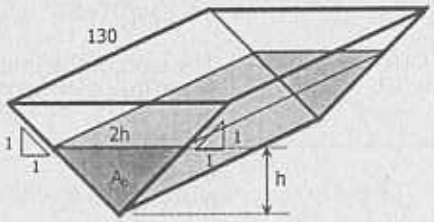
$V = \dfrac{\pi h^2}{3}(3R - h)$

$V = \dfrac{\pi(2.68)^2}{3}[3(20) - 2.68]$
$V = \mathbf{431.13\ cm^3}$



SOLUTION 15 - 30
Ans: B

$V_w = A_b \times h$
$3500 = A_b \times 130$
$A_b = 26.923$ cm²
$A_b = ½(2h)h = h^2$
$26.923 = h^2$
$h = \mathbf{5.189\ cm}$



SOLUTION 15 - 31
Ans: A

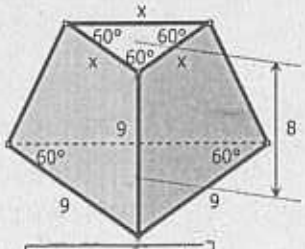
$A_L = \pi r L$; $100.53 = \pi(4)L$; $L = \mathbf{8\ cm}$

SOLUTION 15 - 32
Ans: D

$V = \dfrac{h}{3}\left[A_1 + A_2 + \sqrt{A_1 A_2}\right]$

$A_1 = ½(9)(9)\sin 60°$
$A_1 = 35.074$ m²

$A_2 = ½\, x^2 \sin 60°$
$A_2 = 0.433\, x^2$

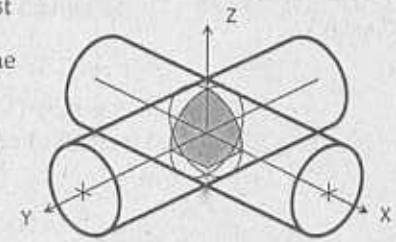


$135 = \dfrac{8}{3}\left[35.074 + 0.433x^2 + \sqrt{35.074(0.433x^2)}\right]$
$0.433\, x^2 + 3.897x - 15.551 = 0$; $x = \mathbf{3\ m}$

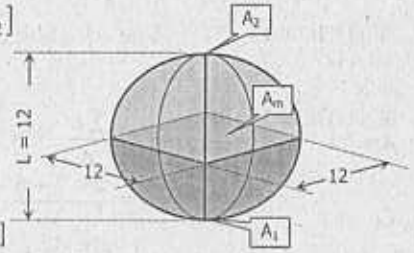
SOLUTION 15 - 33
Ans: C

The volume in the first octant is 1/8 of the volume common to the two cylinders.

The volume common to the two cylinders is a prismatoid.



$V = \dfrac{L}{6}\left[A_1 + 4A_m + A_2\right]$
$L = 12$
$A_1 = 0$
$A_2 = 0$
$A_m = 12(12)$
$A_m = 144$



$V = \frac{12}{6}[0 + 4(144) + 0]$
$V = 1152$ m³
Required volume = $V/8 = \mathbf{144\ m^3}$

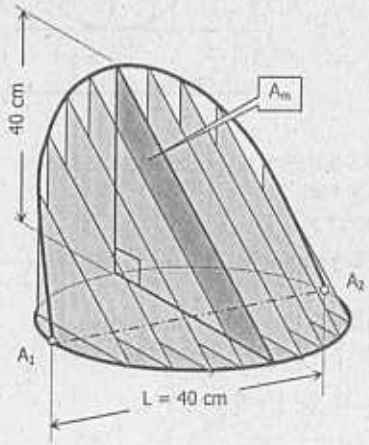
SOLUTION 15 - 34
Ans: A

$A_b = \pi r^2 = 7.068$; $r = 1.5$ m
$A_L = 2\pi r h = 2\pi(1.5)h = 25.918$; $h = 2.75$ m

$V = A_b \times h = 7.068(2.75)$
$V = \mathbf{19.437\ m^3}$

SOLUTION 15 - 35
Ans: A

The solid is a prismatoid with the figure shown.



$V = \dfrac{L}{6}(A_1 + 4A_m + A_2)$
$A_1 = A_2 = 0$
$A_m = ½(40)(40) = 800$ cm²

$V = \frac{40}{6}[0 + 4(800) + 0]$
$V = \mathbf{21,333.3\ cc}$

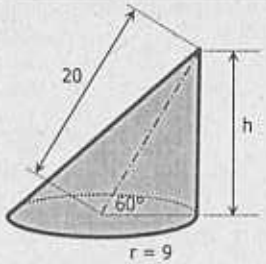
SOLUTION 15 - 36
Ans: B

$h = 20 \sin 60° = 17.32$ cm

$V = \frac{1}{3} A_b h$

$V = \frac{1}{3} \pi(9)^2 (17.32)$

$V =$ **1469.13 cm³**



SOLUTION 15 - 37
Ans: D

The equation describes an ellipse.

SOLUTION 15 - 38
Ans: B

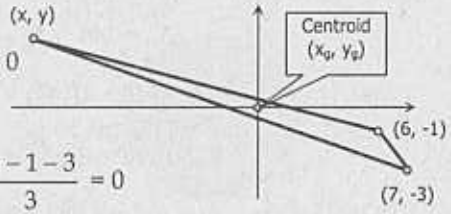
$x_g = \frac{\sum x}{n}$

$x_g = \frac{x + 7 + 6}{3} = 0$

$x = -13$

$y_g = \frac{\sum y}{n}$ $y_g = \frac{y - 1 - 3}{3} = 0$

$y = 4$

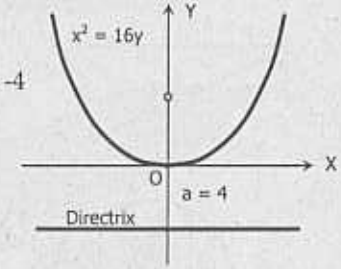


SOLUTION 15 - 39
Ans: D

$x^2 = 16\,y$

$4a = 16,\ a = 4$

Equation of directrix = $y = -4$
or $y + 4 = 0$



SOLUTION 15 - 40
Ans: B

Reduce $x^2 + y^2 + 6x - 12y + 9 = 0$ to standard form:

$x^2 + 6x + 9 + y^2 - 12y + 36 = -9 + 9 + 36 = 36$

$(x + 3)^2 + (y - 6)^2 = 6^2; r = 6$

Area $= \pi r^2 = \pi (6)^2 =$ **113.097 sq. units**

SOLUTION 15 - 41
Ans: C

$9x^2 + 25y^2 - 18x + 100y - 116 = 0$ (ellipse)

$9(x^2 - 2x + 1) + 25(y^2 + 4y + 4) = 116 + 9(1) + 25(4) = 225$

$\frac{(x-1)^2}{25} + \frac{(y+2)^2}{9} = 1;\ a = 5, b = 3$

$a^2 = b^2 + c^2; c = \sqrt{5^2 - 3^2} = 4$

Distance between foci = $2c =$ **8 units**

SOLUTION 15 - 42
Ans: C

For a point whose polar coordinate is (r, θ), $x = r \cos \theta$, $y = r \sin \theta$

$x = 7 \cos 38° = 5.52$

$y = 7 \sin 38° = 4.31$

Rectangular coordinate of the point = **(5.52, 4.31)**

SOLUTION 15 - 43
Ans: D

$y^2 = 16x$

$2y\,y' = 16;$ y' = slope of chord = ½

$2y(2) = 16;$ $y = 4$

SOLUTION 15 - 44
Ans: D

$x = 3t^2 - 6;\ dx = 6t$

$y = 4t^2 + 7,\ dy = 8t$

Slope $= \frac{dy}{dx} = \frac{8t}{6t} =$ **4/3**

SOLUTION 15 - 45
Ans: B

$f(y) = \sqrt[3]{9}$

$f'(y) = 0$

SOLUTION 15 - 46
Ans: B

$f(x) = [\,x^3 - (x - 1)^3]^3$

$f'(x) = 3[x^3 - (x - 1)^3]^2\,[3x^2 - 3(x - 1)^2]$

$f'(x) = 9[x^3 - (x - 1)^3]^2\,[x^2 - (x - 1)^2]$

SOLUTION 15 - 47
Ans: A

$\frac{d}{dx} \sin (2x^2 + 1) = \cos (2x^2 + 1)\, 4x$

$\frac{d}{dx} \sin (2x^2 + 1) =$ **$4x \cos (2x^2 + 1)$**

SOLUTION 15 - 48
Ans: A

$y = x^{x^2}$ taking the natural logarithm on both sides:

$\ln y = \ln x^{x^2} = x^2 \ln x$

$\frac{y'}{y} = x^2 \frac{1}{x} + 2x \ln x$

$dy/dx = y' = y\,(x + 2x \ln x) = x^{x^2}\,(x + 2x \ln x)$

$dy/dx = x^{x^2+1}\,(1 + \ln x^2)$

SOLUTION 15 - 49
Ans: A

$y = 4x^2 - 2x + 1$

$y' = 8x - 2$

$y'' = 8$

SOLUTION 15 - 50
Ans: D

$y = x^3 - 9x$

Slope $= dy/dx = 3x^2 - 9 = 18$

$3x^2 = 27;\ x = 3$

SOLUTION 15 - 51
Ans: D

Slope $\frac{dy}{dx} = 6x - 2$; $dy = 6x\,dx - 2\,dx$

$y = 3x^2 - 2x + C$

@ (5, 3): $3 = 3(5)^2 - 2(5) + C$; $C = -62$

$y = 3x^2 - 2x - 62$; $\mathbf{3x^2 - 2x - y - 62 = 0}$

SOLUTION 15 - 52
Ans: B

For a given total surface area, the volume is a maximum when the diameter (D) equals the height (H)

$A_s = \frac{\pi}{4} D^2 \times 2 + \pi DH$

$A_s = 2\frac{\pi}{4} H^2 + \pi(H)(H) = 153.94$; $H = \mathbf{5.72\ m}$

SOLUTION 15 - 53
Ans: D

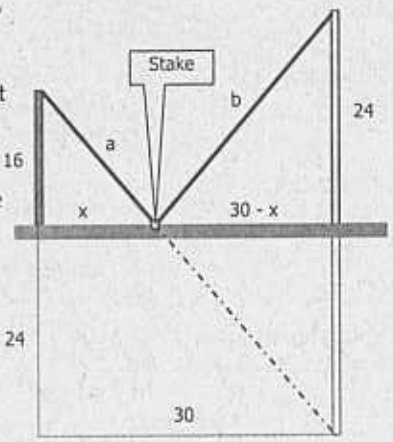
For minimum surface area, $D = H$

$V = A_b \times H = \frac{\pi}{4} D^2 D = 71.57$; $D = \mathbf{4.5\ m}$

SOLUTION 15 - 54
Ans: D

This problem is usually solved by calculus.

Based from the fact that *the shortest distance between two points is a straight line*, we can make a reflection of the other post as shown in the figure, thus making the wire straight. The by proportion of similar triangles:



$$\frac{x}{16} = \frac{30}{24 + 16}$$

$x = \mathbf{12\ feet}$

SOLUTION 15 - 55
Ans: D

$h = 100\,t - 16.1\,t^2$

Velocity, $v = \frac{dh}{dt} = 100 - 32.2\,t = 100 - 32.2(2)$

Velocity, $v = \mathbf{35.6\ ft/sec}$

SOLUTION 15 - 56
Ans: B

$V = n + n^{1/2}$

Rate of flow $= \frac{dV}{dn} = 1 + \frac{1}{2}n^{-1/2}$

At $n = 16$ hours:

Rate of flow $= 1 + \frac{1}{2}(16)^{-1/2}$

Rate of flow $= \mathbf{1.125\ gph}$

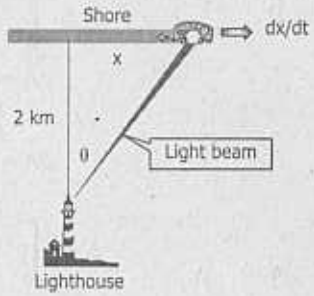
SOLUTION 15 - 57
Ans: A

$d\theta/dt = 0.25$ rev/hour

$d\theta/dt = 0.5\pi$ rad/hour

$\tan\theta = x/2 = 0.5\,x$

$\sec^2\theta\,\frac{d\theta}{dt} = 0.5\frac{dx}{dt}$



When $x = 1$; $\theta = 26.565°$

$\sec^2 26.565°\,(0.5\,\pi) = 0.5\frac{dx}{dt}$

$\frac{dx}{dt} = \mathbf{3.93\ km/hr}$

SOLUTION 15 - 58
Ans: B

$$\int_0^{\pi/2} 3e^{3\sin\theta}\cos\theta\,d\theta$$

From $\int e^u\,du = e^u + C$

$u = 3\sin\theta$; $du = 3\cos\theta\,d\theta$

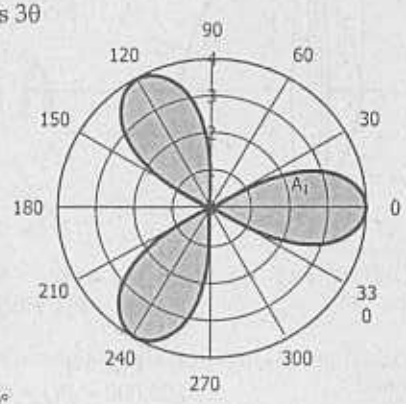
$$\int_0^{\pi/2} e^{3\sin\theta}(3\cos\theta\,d\theta) = \left[e^{3\sin\theta}\right]_0^{\pi/2} \quad \text{Note: } \pi/2 = 90°$$

$$= e^{3\sin 90°} - e^{3\sin 0°}$$

$$= \mathbf{19.086}$$

SOLUTION 15 - 59
Ans: B

The curve $r^2 = 4a^2\cos 3\theta$ is as shown



$A = 6\,A_1$

$$A = 6\int_0^{30°}\frac{1}{2}r^2\,d\theta$$

$$A = 3\int_0^{30°}4a^2\cos 3\theta\,d\theta$$

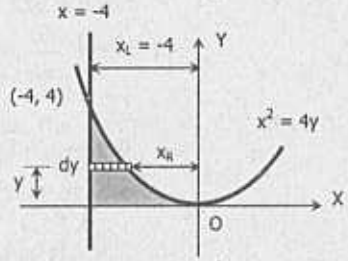
$$A = 12\,a^2\int_0^{30°}\cos 3\theta\,d\theta$$

$$A = 12a^2\left[\frac{1}{3}\sin 3\theta\right]_0^{30°} = \mathbf{4a^2}$$

SOLUTION 15 - 60
Ans: D

$$I_x = \int_0^4 y^2\, dA$$

$dA = (x_R - x_L)\, dy$
$x_L = -4$
$x_R = -2\, y^{1/2}$
$dA = (-2y^{1/2} + 4)\, dy$



$$I_x = \int_0^4 y^2(-2y^{1/2} + 4)dy$$

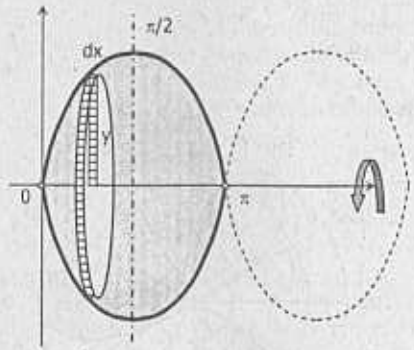
$$I_x = \int_0^4 (-2y^{5/2} + 4y^2)\, dy = \left[-\frac{4}{7}y^{7/2} + \frac{4}{3}y^3\right]_0^4 = \mathbf{12.19}$$

SOLUTION 15 - 61
Ans: C

$$\int_0^1 \frac{x\,dx}{x^2+2} = \left[\frac{1}{2}\ln(x^2+2)\right]_0^1$$

$$\int_0^1 \frac{x\,dx}{x^2+2} = ½\,[\ \ln(3) - \ln(2)\ ] = \mathbf{0.2027}$$

SOLUTION 15 - 62
Ans: B



$dV = \pi\, y^2\, dx = \pi\, (\sin x)^2\ dx$
$dV = \pi \sin^2 x\, dx$

$$V = \pi \int_0^{\pi} \sin^2 x\, dx$$

$$\text{or } V = 2\,\pi \int_0^{\pi/2} \sin^2 x\, dx$$

Using Wallis Formula:

$$V = 2\pi\ ½ \times \frac{\pi}{2}$$

V = **4.935 cu. units**

SOLUTION 15 - 63
Ans: D

$F = P(1 + rt)$
$20{,}000 = P[1 + 0.12(9/12)]$; P = **18,348.62**

SOLUTION 15 - 64
Ans: B

$F = P(1 + i)^n$
$100{,}000 = P(1 + 0.10)^5$; P = **P62,092.13**

SOLUTION 15 - 65
Ans: C

$F = P(1 + i)^n$
$54{,}000 = P(1 + 0.11/2)^{5(2)}$; P = **P31,613.25**

SOLUTION 15 - 66
Ans: D

Let t be the number of years:
$F = P(1 + i)^n$
$4P = P(1 + 0.2/4)^{4t}$
$1.05^{4t} = 4$; $4t\ (\ln 1.05) = \ln 4$
t = **7.1 years**

SOLUTION 15 - 67
Ans: D

$F = P(1 + i)^n$ where $F = 3P$
$3P = P(1 + 0.13)^t$
$3 = 1.13^t$; $\ln 3 = t \ln 1.13$
t = **8.99 years**

SOLUTION 15 - 68
Ans: C

$F = P(1 + i)^n$
$8000 = 4000(1 + 0.08)^n$
$2 = 1.08^n$; $\log 2 = n \log 1.08$
n = **9.01 years**

SOLUTION 15 - 69
Ans: A

$$ER = \left(1 + \frac{r}{m}\right)^m - 1; \quad m = 4,\ r/m = 0.015$$

$ER = (1 + 0.015)^4 - 1 = 0.0614 =$ **6.14%**

SOLUTION 15 - 70
Ans: A

$ER = (1 + 0.005)^{12} - 1 = 0.0617$
$ER =$ **6.17%**

SOLUTION 15 - 71
Ans: C

$ER_{Quarterly} = ER_{Semi\text{-}annually}$
$(1 + r/4)^4 - 1 = (1 + 0.12/2)^2 - 1$
$(1 + r/4)^4 = (1.06)^2$
$r = 0.1182 =$ **11.82%**

SOLUTION 15 - 72
Ans: B

$F = P\, e^{rt} = 1{,}000{,}000\ e^{0.08(20)}$
F = **P4,953,032.42**

SOLUTION 15 - 73
Ans: D

$F = P(1 + i)^n$
$500 = P(1 + 0.12)^{11/12}$; P = **P450.7**

SOLUTION 15 - 74
Ans: A

$$P = \frac{F}{(1+i)^n} = \frac{40{,}000}{(1+0.10)^3} = \mathbf{P30{,}052.59}$$

SOLUTION 15 - 75
Ans: A

$$F = \frac{A[(1+i)^n - 1]}{i}$$

Convert 7.5% compounded continuously to annually:
$ER_A = ER_C$
$(1 + i)^1 - 1 = e^r - 1 = e^{0.075} - 1$
$i = 0.07788$

$$F = \frac{5000[(1+0.07788)^{15} - 1]}{0.07788} = \mathbf{P133{,}541.3}$$

SOLUTION 15 - 76
Ans: B

$$F = \frac{A[(1+i)^n - 1]}{i} = \frac{1000[(1+0.04)^{15} - 1]}{0.04} = \text{P20,023.59}$$

SOLUTION 15 - 77
Ans: A

Let F be the money in the account after 20 years, then:
Present worth of F = Present worth of annuity for 10 years.

$$\frac{F}{(1+0.08)^{20}} = \frac{100,000[(1+0.08)^{10} - 1]}{(1+0.08)^{10}(0.08)}$$

F = **P3,127,540.18**

SOLUTION 15 - 78
Ans: A

The annual payment, A is:

$$P = \frac{A[(1+i)^n - 1]}{(1+i)^n i}; \quad 10,000 = \frac{A[(1+0.15)^{10} - 1]}{(1+0.15)^{10}(0.15)}$$

$A = 1992.52$

The balance after 2 years is:
Balance = F of P10,000 – F of A

$$\text{Balance} = 10,000(1 + 0.15)^2 - \frac{1992.52[(1+0.15)^2 - 1]}{0.15}$$

Balance = P8,941.082

Amount paid to the principal = 10,000 – 8,941.082
Amount paid to the principal = 1058.918
Interest paid = 1992.52(2) – 1058.918 = **P2,926.12**

SOLUTION 15 - 79
Ans: A

Break even price = Future worth of P1M after five years – Future worth of all profit after five years.

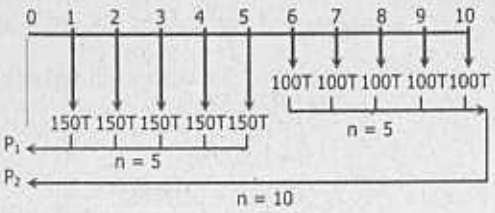
$$\text{Price} = 1,000,000(1 + 0.06)^5 - \frac{100,000[(1+0.06)^5 - 1]}{0.06}$$

Price = **P774,516.28**

SOLUTION 15 - 80
Ans: C

The maximum investment is the present worth of all savings in maintenance.

Annual savings during the first five years = P150,000
Annual savings during the next five years = P100,000



$$P_1 = \frac{150,000[(1+0.04)^5 - 1]}{(1+0.04)^5(0.04)} = \text{P667,773.35}$$

$$P_2 = \frac{100,000[(1+0.04)^5 - 1]}{(1+0.04)^{10}(0.04)} = \text{P365,907.34}$$

$P = P_1 + P_2$ = **P1,033,680.69**

SOLUTION 15 - 81
Ans: A

$$F = \frac{100,000[(1+i)^{10} - 1]}{i} = 1,200,000$$

$$\frac{(1+i)^{10} - 1}{i} = 12$$

By trial and error using the choices, $i = 0.04 = 4\%$

SOLUTION 15 - 82
Ans: A

$BV_m = FC(1 - K)^m$

$$K = 1 - \sqrt[n]{\frac{SV}{FC}} = 1 - \sqrt[12]{\frac{0.10\,FC}{FC}} = 0.1746$$

$BV_5 = 480,000(1 - 0.1746)^5$ = **P183,892**

SOLUTION 15 - 83
Ans: D

$$K = 1 - \sqrt[n]{\frac{SV}{FC}} = 1 - \sqrt[800,000]{\frac{15,000}{150,000}} = 0.00000287823$$

$BV_m = FC(1 - K)^m$
$BV_{300,000} = 150,000(1 - 0.00000287823)^{300,000}$
$BV_{300,000}$ = **P63,254.5**

SOLUTION 15 - 84
Ans: A

$BV_m = FC - D_m$ (m = 15,000 hours)

$$d = \frac{FC - SV}{n} = \frac{350,000 - 10\%(350,000)}{60,000}$$

d = P5.25 per hour
$D_{15000} = 5.25 \times 15000$ = P78,750
$BV = 350,000 - 78,750$ = **P271,250**

SOLUTION 15 - 85
Ans: B

Cost of $X = 1,500,000 + 20N$
Cost of $Y = 500,000 + 60N$
Cost of X = Cost of Y
$1,500,000 + 20N = 500,000 + 60N$; N = **25,000**

SOLUTION 15 - 86
Ans: A

Profit = Revenue – Cost
Revenue = 80%(5000)(416) = P1,664,000
Cost:
Variable cost = 80%(5000)(130) = P520,000
Fixed cost = P78,000

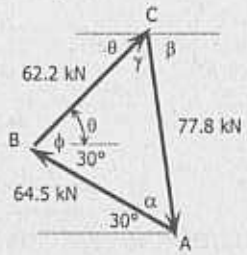
Cost = 520,000 + 78,000 = P598,000
Profit = 1,664,000 – 598,000 = **P1,066,000**

SOLUTION 15 - 87
Ans: D

For break-even, Cost = Revenue
Fixed cost + Variable cost = Revenue
$450{,}000 + (45 + 15)\,N = 250\,N$
$N = 2368.42$ say **2367 units**

SOLUTION 15 - 88
Ans: D

For the forces to be in equilibrium they must for a close polygon (as shown)



By cosine law:
$77.8^2 = 62.2^2 + 64.5^2 - 2(62.2)(64.5)\cos\phi$
$\phi = 75.74°$

By sine law:
$$\frac{62.2}{\sin\alpha} = \frac{64.5}{\sin\gamma} = \frac{77.8}{\sin 75.74°}$$
$\alpha = 50.79°$; $\gamma = 53.47°$
$\phi = \theta + 30° = 75.74°$; $\theta = \mathbf{45.74°}$

SOLUTION 15 - 89
Ans: B

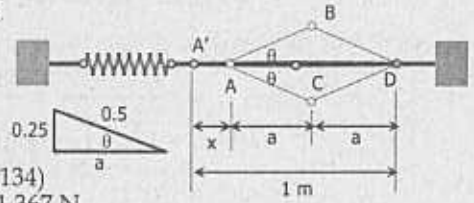
$\theta + \gamma + \beta = 180°$
$45.75° + 53.47° + \beta = 180°$
$\beta = \mathbf{80.78°}$

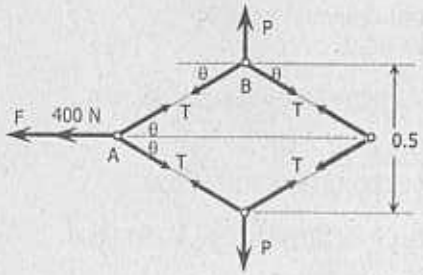
SOLUTION 15 - 90
Ans: A

Vertical component of 77.8 kN force = 77.8 sin β
Vertical component of 77.8 kN force = **76.8 kN**

SOLUTION 15 - 91
Ans: D

$\sin\theta = 0.25/0.5$
$\theta = 30°$
$a = 0.5\cos\theta$
$a = 0.433$
$x + 2a = 1$
$x = 0.134$ m
$F = k\,x = 10.2(0.134)$
$F = 1.367$ kN = 1,367 N





From the FBD shown:
At joint A:
$F + 200 = 2(T\cos\theta)$
$1367 + 400 = 2T\cos 30°$
$T = \mathbf{1020.2\ N}$

SOLUTION 15 - 92
Ans: B

At joint B:
$P = 2T\sin\theta = 2(1020.2)\sin 30° = \mathbf{1020.2\ N}$

SOLUTION 15 - 93
Ans: C

Spring constant, k_{ABC}
$P = k_{ABC}\,x = k_{ABC}\,(0.5) = 1020.2$; $k_{ABC} = \mathbf{2040.4\ N/m}$

SOLUTION 15 - 94
Ans: A

$P = f = \mu N$; $N = W = Mg$
$20 = \mu\,(56 \times 32.2)$; $\mu = \mathbf{0.011}$

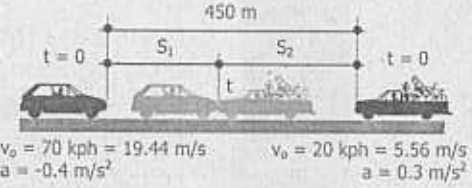
SOLUTION 15 - 95
Ans: D

$v^2 = v_o^2 + 2aS$
$0^2 = 10^2 + 2(-3)S$; $S = \mathbf{16.7\ m}$

SOLUTION 15 - 96
Ans: A

$S = v_o\,t + \frac{1}{2}a\,t^2$

$S_1 + S_2 = 450$ m



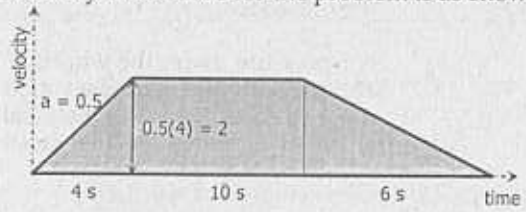
$19.44\,t + \frac{1}{2}(-0.4)\,t^2 + 5.56\,t + \frac{1}{2}(0.3)\,t^2 = 450$
$0.05\,t^2 - 25\,t + 450 = 0$; $t = \mathbf{18.7\ seconds}$

SOLUTION 15 - 97
Ans: A

$S = 4t^2 + 20/(t^2 - 1)$
$$v = \frac{dS}{dt} = 8\,t - \frac{20(2t)}{(t^2-1)^2} = 8t - \frac{40t}{(t^2-1)^2}$$
$$a = \frac{dv}{dt} = 8 - \frac{(t^2-1)^2(40) - (40t)[2(t^2-1)(2t)]}{(t^2-1)^4}$$
$$a = 8 - \frac{-120t^2 - 40}{(t^2-1)^3};\ \text{At } t = 4 \text{ seconds:}\quad a = \mathbf{8.581\ m/s^2}$$

SOLUTION 15 - 98
Ans: B

The velocity-time curve for the problem is as shown.



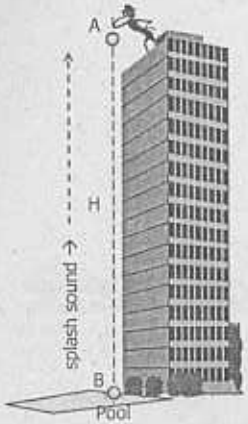
Velocity-time curve

The distance traveled by the elevator is the area under the curve
$$S = \frac{10 + 20}{2}(2) = 30\ \text{m}$$

No of floors the elevator did pass = 30/3 = 10 floors

Since the elevator started at the first floor, it stopped at the **11th floor**

SOLUTION 15 - 99
Ans: C



$(t_{AB})_{stone} + (t_{BA})_{sound} = 5$

For the stone (free fall)
$H = \frac{1}{2} g\,(t_{AB})^2$
$t_{AB} = 0.4515\,H^{1/2}$

For the sound (uniform motion)
$H = 300\,(t_{BA})$
$t_{BA} = H/300$

$0.4515\,H^{1/2} + H/300 = 5$
Let $H^{1/2} = u$; $H = u^2$

$0.4515\,u + u^2/300 = 5$
$(1/300)\,u^2 + 0.4515\,u - 5 = 0$

$$u = \frac{-0.4515 \pm \sqrt{(0.4515)^2 - 4(1/300)(-5)}}{2(1/300)}$$

$u = 10.295 = H^{1/2}$
$H =$ **106 m**

SOLUTION 15 - 100
Ans: C

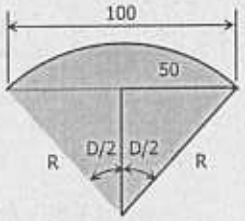
$y = v_{oy}\,t - \frac{1}{2} g\,t^2$
$-30 = (300 \sin 20°)t - \frac{1}{2}(9.81)\,t^2$
$t^2 - 20.92\,t - 6.12 = 0$; $t =$ **21.2 seconds**

SOLUTION 15 - 101
Ans: C

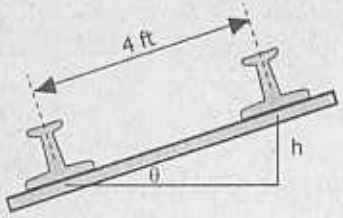
$y = v_{oy}\,t - \frac{1}{2} g\,t^2$
$0 = 20\,t - \frac{1}{2}(9.81)\,t^2$; $t =$ **4.08 sec**

SOLUTION 15 - 102
Ans: B

$\sin(D/2) = 50/R$
$\sin(10/2) = 50/R$
$R = 573.68$ feet



For pressure under the wheels to be uniform, the superelevation must be equal to that of the ideal angle of banking for the given speed.



$$\tan\theta = \frac{v^2}{gR}$$

$v = 45$ mi/hr × 5280 ft/mi × 1 hr/3600 s
$v = 66$ ft/s

$$\tan\theta = \frac{66^2}{32.2(573.68)};\ \theta = 13.27°$$

$\sin\theta = h/3.8$
$h = 4 \sin 13.27 = 0.918$ feet

SOLUTION 15 - 103
Ans: D

$$\text{Power} = \frac{\text{Work}}{\text{time}} = \frac{(\text{Force})(\text{Distance})}{\text{time}} = \frac{(1600 \times 9.81)(25)}{30}$$

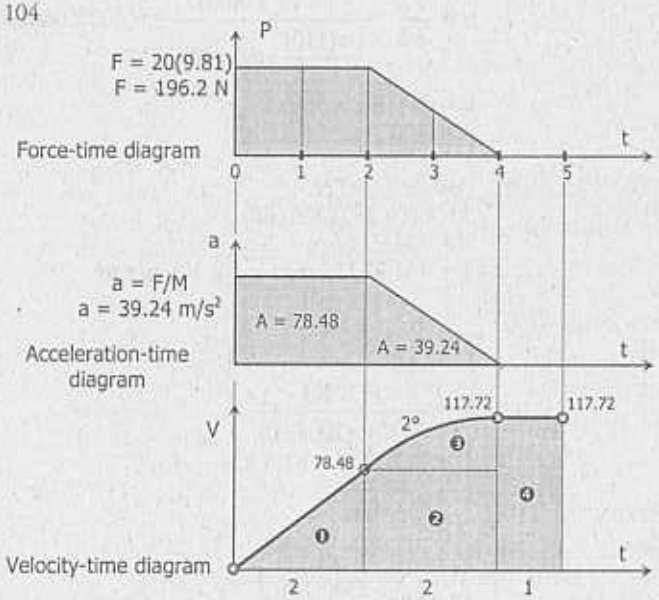
Power = 13,080 Watts = 13.08 kW (output power)

$$\text{Efficiency, } \eta = \frac{\text{Output}}{\text{Input}}$$

$$\text{Power input} = \frac{13.08}{60\%}$$

Power input = **21.8 kW**

SOLUTION 15 - 104
Ans: A



The given graph is that of Force (P) versus time. But since acceleration (a) = Force / Mass, the acceleration-time diagram can easily be constructed as shown above.

The area under the acceleration-time curve between any two points represents the change in velocity between the points. Since the block starts from rest, the initial velocity is zero as shown in the velocity-time diagram. The area under the velocity-time curve represents the distance traveled.

As shown in the diagrams:
Acceleration during the first 2 seconds = **39.24 m/s²**

SOLUTION 15 - 105
Ans: C

Velocity after 5 seconds = **117.72 m/s**

SOLUTION 15 - 106
Ans: D

Distance traveled:

$S = A_1 + A_2 + A_3 + A_4$

$S = ½(2)(78.48) + 78.48(2) + (2/3)(2)(117.72 - 78.48) + 117.72(1)$

$S =$ **405.48 m**

SOLUTION 15 - 107
Ans: C

$$\theta = \frac{TL}{JG} = \frac{3\times10^6(1400)}{\frac{\pi(110)^4}{32}(83{,}000)} = 0.00352 \text{ radians}$$

$\theta = 0.00352 \times 180/\pi$

$\theta =$ **0.202°**

SOLUTION 15 - 108
Ans: B

Power $= 2\pi TN$

$20{,}000 = 2\pi T(200/60)$

$T = 954.93$ N-m

$T = 954{,}930$ N-mm $\approx$ **1×10^6 N-mm**

SOLUTION 15 - 109
Ans: D

$$\delta = \frac{PL}{AE};\ E = 200{,}000 \text{ MPa}$$

$$1 = \frac{P(1200)}{\frac{\pi}{4}(25)^2(200{,}000)}$$

$P = 81{,}812$ N = **81.8 kN**

SOLUTION 15 - 110
Ans: C

$$\sigma = \frac{pD}{2t}$$

$$28{,}000 = \frac{2500(10)}{2t}$$

$t = 0.4464$ in $\times$ 25.4 mm/in

$t =$ **11.34 mm**

Problems - Set 16
Miscellaneous Problems

PROBLEM 16 - 1
ECE April 2004

Evaluate the following limit: $\lim_{x\to0} \frac{\sin x^3}{\sin x^2}$

A. -1
B. 2π
C. -2
D. 0

PROBLEM 16 - 2
CE May 2004

Given the function $y = x(x + 1)^3$. Determine the value of its second derivative at $x = 1$.

A. 36
B. 34
C. 35
D. 33

PROBLEM 16 - 3
ECE April 2004

A hotel is to be constructed in a rectangular lot with a perimeter of 800 m. What is the largest area that can be enclosed by fencing the perimeter?

A. 4 hectares
B. 5 hectares
C. 6.2 hectares
D. 3.5 hectares

PROBLEM 16 - 4
CE May 2004

Triangle ABC have sides measuring $AB = 7$ m, $BC = 5$ m, and $AC = 9$ m. What is the width of the largest rectangle that can be inscribed in it, with the longer side of the rectangle along the 9-m side?

A. 2.475 m
B. 1.514 m
C. 1.934 m
D. 4.5 m

PROBLEM 16 - 5
ECE April 2004

A statue 3.2 m high stands on a pedestal such that its foot is 0.4 m above the observer's eye level. How far from the statue must the observer stand in order that the angle subtended by the statue will be maximum?

A. 1.2 m
B. 1.13 m
C. 1.5 m
D. 1.37 m

PROBLEM 16 - 6
ECE april 2004

In how many equal parts can a wire 50 inches long be cut such that the product of its parts is a maximum?

A. 12
B. 11
C. 15
D. 19

PROBLEM 16 - 7
ME October 2003

A hot-air balloon rising straight up from a level ground is tracked by a range finder 500 feet from the point of lift off. At the moment the lift finder's elevation is 45°, the angle is increasing at the rate of 0.14 radian per minute. How fast is the balloon rising?

A. 150 fpm
B. 160 fpm
C. 120 fpm
D. 140 fpm

PROBLEM 16 - 8
CE May 2004

At 5:00 am, boat Q left the port in the direction N 45° E at 26 knots. At 5:30 am, boat R left the same port in the direction S 30° E at 32 knots. How fast are the two boats separating at 6:00 am. Express your answer in knots.

A. 55.31 C. 38.54
B. 44.61 D. 24.58

PROBLEM 16 - 9
ECE April 2004

A boy is using a straw to drink cola from a right cylindrical shaped glass, at the rate of 6 cm³ per minute. The glass is 12 cm tall and 6 cm in diameter. How fast is the level of the cola decreasing in cm per minute?

A. 2.145 C. 1.632
B. 0.586 D. 0.212

PROBLEM 16 - 10
ECE April 2004

A conical vessel 12 cm deep with radius of 6 cm at the top is filled with water. If the rate at which the water rises is 2 cm/sec when the water is 4 cm deep, how fast is the volume increasing?

A. 4π C. 16π
B. 2π D. 8π

PROBLEM 16 - 11
ECE April 2004

Evaluate the integral of $(2x^2 + 3x - 4)\,dx$.

A. $\frac{2}{3}x^3 + \frac{3}{2}x^2 - 4x + C$ C. $\frac{2}{3}x^3 + \frac{3}{4}x^2 - 4x + C$
B. $\frac{3}{2}x^3 + \frac{2}{3}x^2 - 4x + C$ D. $\frac{4}{3}x^3 + \frac{3}{4}x^2 - 4x + C$

PROBLEM 16 - 12
ECE April 2004

Evaluate the integral of: (square root of x + cube root of x) dx.

A. $\frac{2}{3}x^{3/2} + \frac{3}{4}x^{4/3} + C$
B. $\frac{3}{2}x^{3/2} + \frac{4}{3}x^{4/3} + C$
C. $\frac{2}{3}x^{2/3} + \frac{3}{4}x^{3/4} + C$
D. $\frac{3}{2}x^{2/3} + \frac{4}{3}x^{3/4} + C$

PROBLEM 16 - 13
ECE April 2004

What is the integral of $\sin x / \tan x\, dx$?

A. $\cos x + C$ C. $\sin x + \cos x + C$
B. $\sin x + C$ D. $\sec x + C$

PROBLEM 16 - 14
CE May 2004

Evaluate the integral of $\cos^7 3x\, dx$ with limits from 0 to $\pi/6$.

A. 0.417 C. 0.152
B. 0.304 D. 0.218

PROBLEM 16 - 15
CE May 2004

At t years from now, one investment plan will generate profit at the rate of $50 + t^2$ thousand pesos per year, while a second investment plan will generate $200 + 5t$ thousand pesos per year. What is the net excess profit if you invest in the second plan instead of the first plan up to time the two plans yield equal profits?

A. P1,532,200 C. P1,486,200
B. P1,687,500 D. P1,349,100

PROBLEM 16 - 16
ME April 2004

Find the area of the region bounded by the curve $y = x^3$ and the lines $y = 1$ and $x = 0$.

A. 3/4 C. 4/3
B. 2/3 D. 3/2

PROBLEM 16 - 17
CE November 2003

What is the area bounded by the curve $y = 6 \cos x$ and the X-axis from $x = \pi/6$ to $x = \pi/2$?

A. 2 C. 4
B. 3 D. 1

PROBLEM 16 - 18
CE May 2004

Find the volume in the first octant, bounded by the following surfaces:

$$x = 1$$
$$x^2 = y + 2z$$

A. 0.06 C. 0.07
B. 0.08 D. 0.05

PROBLEM 16 - 19
ECE April 2004

Determine the position of the centroid of the area in the first quadrant bounded by the curve $x^2 = -(y - 4)$, the X-axis, and the Y-axis.

A. (4/3, 2/5) C. (8/5, 3/4)
B. (2/5, 4/3) D. (3/4, 8/5)

PROBLEM 16 - 20
ECE April 2004

Two force vectors A and B act on the same point. If A = 50 kN, 30° and B = 40 kN, 120°, what is the equilibrant force?

A. 64 kN, 69° C. 98 kN, 214°
B. 64 kN, 249° D. 98 kN, 47°

PROBLEM 16 - 21
ECE April 2004

A grenade is fired from a launcher with a velocity of 50 m/s at an angle of 30° with the horizontal. Determine the maximum height that it could reach.

A. 38.52 m C. 28.63 m
B. 31.86 m D. 23.87 m

PROBLEM 16 - 22
ECE April 2004

A golf ball after being struck by a golf club at ground level, departs at an angle of 45° with the horizontal fairway with a velocity of 100 feet per second. How far in yards will the ball land from the point of impact?

A. 103.5 C. 142.3
B. 310.5 D. 247.2

PROBLEM 16 - 23
ECE April 2004

A ball is shot vertically upward with an initial velocity of 200 fps, followed by a second ball 5 seconds later. What must be the speed of the second ball so that it will meet the first ball 300 feet from the origin on its way up while the first ball is on its way down?

A. 126 fps
B. 158 fps
C. 144 fps
D. 134 fps

PROBLEM 16 - 24
ECE April 2004

An automobile tire is 30 inches in diameter. How fast does the wheel turn on the axle when the automobile maintains a speed of 50 miles per hour?

A. 560.2 rpm
B. 531.2 rpm
C. 480.6 rpm
D. 547.8 rpm

PROBLEM 16 - 25
CE May 2004

A 1.2-ton car moving at 30 kilometers per hour bumps the rear of a 1.8-ton car moving in the same direction with a velocity of 20 kilometers per hour. The bumpers got locked after the collision. What is the velocity of the cars immediately after impact?

A. 26
B. 29
C. 28
D. 24

PROBLEM 16 - 26
CE May 2004

An iron ball is projected upward with a velocity of 300 m/s at an angle of 30° from the level ground. When will the ball fall back to the ground?

A. 45.8 sec
B. 30.6 sec
C. 58.3 sec
D. 1.25 min

PROBLEM 16 - 27
ME April 2004

A hoist with a 100 hp engine is capable of lifting a 10,000-pound load a height of 20 feet in 10 seconds. The efficiency of the machine is:

A. 33.78%
B. 28.69%
C. 41.21%
D. 36.36%

PROBLEM 16 - 28
ME April 2004

If the gage pressure of a medium is 30 kPa (vacuum) and the atmospheric pressure is 101.3 kPa, the absolute pressure will be:

A. 131.3 kPa
B. 71.3 kPa
C. -131.3 kPa
D. -71.3 kPa

PROBLEM 16 - 29
ECE April 2004

A water tank has an orifice 8 cm² in its vertical side 3 m below the level surface of water. What is the rate of discharge assuming there is no energy wasted, in m/s?

A. 8.63
B. 6.68
C. 7.67
D. 5.89

PROBLEM 16 - 30

The kinetic energy K, in Joules, provided by the mass of aparticle of mass m in kilograms, with a velocity v meters per second, is given by the equation $K = \frac{1}{2} mv^2$. if a particle has a velocity of 4 meters per second and kinetic energy of 144 Joules, then the mass, in kilograms, of this particle is:

A. 16
B. 17
C. 19
D. 18

PROBLEM 16 - 31
ECE April 2004

A man buys an electric fan from a merchant that charges P1,500 at the end of 90 days. The man wishes to pay cash. What is the cash price if money is worth 10% simple interest?

A. P1,512.54
B. P1,463.41
C. P1,614.22
D. P1,452.33

PROBLEM 16 - 32
ECE April 2004

P20,000.00 was invested at simple interest, part at 11% for 2 years, and the rest at 8% for 3 years. The total interest earned was P4,690. How much was invested at 11%?

A. P4,500
B. P2,500
C. P5,500
D. P6,500

PROBLEM 16 - 33
ME April 2004

If P25,000 is invested now, P35,000 two years from now, and P45,000 four years from now, what will be the total amount in 10 years? Interest rate is 4%

A. P141,150
B. P141,845
C. P142,245
D. P140,845

PROBLEM 16 - 34
ME April 2004

What amount should be deposited at 6% in order to withdraw P20,000 at the end of each year for seven years, and leave nothing in thefund at the end?

A. P111,141
B. P112,256
C. P110,872
D. P111,648

PROBLEM 16 - 35
CE May 2004

An equipment costing P200,000 has an estimate scrap value of P25,000 at the end of its economic life of 10 years. Using the Double-Declining Balance Method of depreciation, what is its book value after eight years of service?

A. P33,554
B. P32,952
C. P34,758
D. P35,712

PROBLEM 16 - 36
CE May 2004

A shoes factory has a production capacity of 9000 units per month. Thefixed and variable cost are as follows:

Fixed monthly operating cost = P800,000.00
Material and labor cost = P200.00 per unit
Selling price per unit = P500.00

What is the profit per month if the factory has an efficiency of 80%?

A. P1,450,000
B. P1,360,000
C. P1,280,000
D. P1,540,000

PROBLEM 16 - 37
CE May 2004

The present value of an annuity of *R* pesos payable annually for 10 years, with the first payment at the end of 10 years is P100,701.30. If money is worth 8%, what is the value of *R*?

A. P15,000
B. P25,000
C. P20,000
D. P30,000

PROBLEM 16 - 38
ECE April 2004

A heavy-duty copying machine was procured forP100,000 with an estimated salvage value of P10,000 after 10 years. What is the book value after five years. Use straight-line depreciation.

A. P45,000
B. P35,000
C. P55,000
D. P65,000

PROBLEM 16 - 39
ECE April 2004

Based on its purchase price, a machine is expected to depreciate at uniform rate of 18% annually until it has zero salvage value. What is the useful life of the machine using the SLD method.

A. 8
B. 7
C. 6
D. 4

PROBLEM 16 - 40

A local firm is establishing a sinking fund for the purpose of accumulating sufficient capital to retire its outstanding bonds at maturity. The funds are redeemable in 10 years, and their maturity value is P150,000. How much should be deposited each year if the fund pays interest at the rate of 3%

A. P14,125.32
B. P13,084.57
C. P16,478.52
D. P12,563.32

ANSWER SHEET

| | | |

Solutions to Set 16 Miscellaneous Problems

SOLUTION 16 - 1
Ans: D

$$\lim_{x\to 0}\frac{\sin x^3}{\sin x^2} = \frac{0}{0} \qquad \text{(we may apply L'Hospitals rule)}$$

$$\lim_{x\to 0}\frac{\sin x^3}{\sin x^2} = \lim_{x\to 0}\frac{\cos x^3(3x^2)}{\cos x^2(2x)} = \lim_{x\to 0}\frac{3x}{2}\frac{\cos x^3}{\cos x^2}$$

$$= \frac{3(0)}{2}\frac{1}{1} = 0$$

By Gillesania's rule:
Set $x = 0.0001°$

$$\frac{\sin(0.0001)^3}{\sin(0.0001)^2} = 0.0001 \approx 0$$

SOLUTION 16 - 2
Ans: A

$y = x(x+1)^3 = x(x^3 + 3x^2 + 3x + 1)$
$y = x^4 + 3x^3 + 3x^2 + x$
$y' = 4x^3 + 9x^2 + 6x + 1$
$y'' = 12x^2 + 18x + 6$
at $x = 1$:
$y'' = 12(1)^2 + 18(1) + 6 = \mathbf{36}$

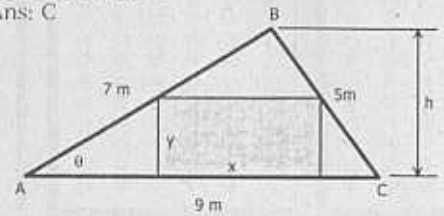
SOLUTION 16 - 3
Ans:

Note: For a given perimeter of a plane area, a regular shape has the largest area.

The problem requires a rectangular shape lot. A regular rectangle is a square.

If the side of the square is x, then $4x = 800$ and $x = 200$
Therefore, Area = x^2 = 40,000 m² = **4 hectares**

SOLUTION 16 - 4
Ans: C



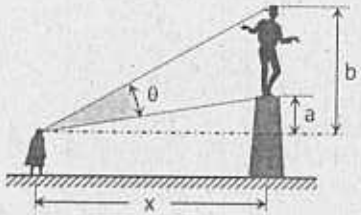
This is a maxima-minima problem, and is similar to CE Board November 2003.

By principle, the rectangle will be largest if $x = ½(9)$ and $y = ½\,h$

Solving for h:
$5^2 = 7^2 + 9^2 - 2(7)(9)\cos\theta;\ \theta = 33.557°$
$h = 7\sin\theta = 7\sin 33.557°$
$h = 3.8694$ m

Thus, $y = ½(3.8694) = \mathbf{1.934\ m}$ → **width**
$x = ½(9) = 4.5$ m → length

SOLUTION 16 - 5
Ans:



In Page 10 of this book, θ is maximum if
$x = \sqrt{ab}$
$a = 0.4$ m
$b = 3.2 + 0.4 = 3.6$ m

$x = \sqrt{0.4(3.6)} = \mathbf{1.2\ m}$

SOLUTION 16 - 6
Ans: D

Let x be the required number of parts.

Length of each part = $\dfrac{50}{x}$

Product of x parts, $P = \left(\dfrac{50}{x}\right)^x$

$$\ln P = x\ln\frac{50}{x} = x(\ln 50 - \ln x) = x\ln 50 - x\ln x$$

$$\frac{dP/dx}{P} = \ln 50(1) - \left(x\frac{1}{x} + \ln x(1)\right)$$

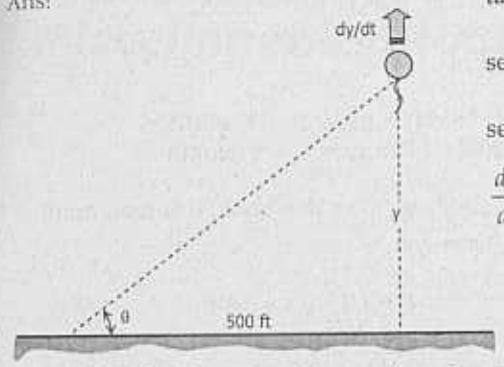
set dP/dx to 0

$$\frac{0}{P} = \ln 50 - 1 - \ln x$$

$\ln x = 2.912023$
$x = e^{2.912023} = \mathbf{18.39\ parts\ (choose\ 19)}$

Formula: $x = e^{\ln N - 1}$, where N is the number (in this problem, the length), and x is the number of equal parts.

SOLUTION 16 - 7
Ans:



$$\tan\theta = \frac{y}{500}$$

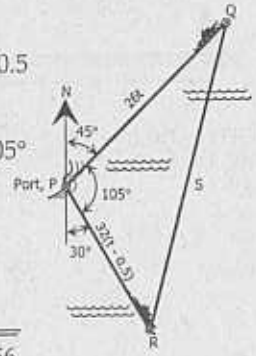
$$\sec^2\theta\frac{d\theta}{dt} = \frac{1}{500}\frac{dy}{dt}$$

$$\sec^2 45°(0.14) = \frac{1}{500}\frac{dy}{dt}$$

$$\frac{dy}{dt} = \mathbf{140\ ft/min}$$

SOLUTION 16 - 8
Ans: B

Let t be any time after 5:00 am
Time of travel of boat $Q = t$
Time of travel of boat $R = t - 0.5$



$$S^2 = (26t)^2 + [32(t-0.5)]^2 - 2(26t)[32(t-0.5)]\cos 105°$$

$$S^2 = 676t^2 + 1024t^2 - 1024t + 256 + 430.67t^2 - 215.34t$$

$$S^2 = 2130.67t^2 - 1239.34t + 256$$

$$S = \sqrt{2130.67t^2 - 1239.34t + 256}$$

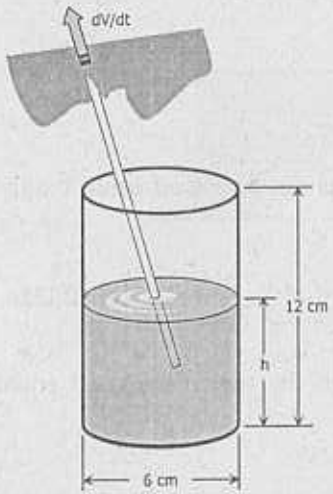
$$\frac{dS}{dt} = \frac{4261.34t - 1239.34}{2\sqrt{2130.67t^2 - 1239.34t + 256}}$$

At 6 am, $t = 1$

$$\frac{dS}{dt} = \frac{4261.34(1) - 1239.34}{2\sqrt{2130.67(1)^2 - 1239.34(1) + 256}}$$

$$\frac{dS}{dt} = \mathbf{44.609\ knots}$$

SOLUTION 16 - 9
Ans: D



Volume of cola at any instant:

$$V = A_b \times h$$

$$V = \frac{\pi}{4}(6)^2 h = 9\pi h$$

$$\frac{dV}{dt} = 9\pi\frac{dh}{dt}$$

$$\frac{dV}{dt} = -6\ \text{cm}^3/\text{min}$$

$$-6 = 9\pi\frac{dh}{dt}$$

$$\frac{dh}{dt} = -0.212\ \text{cm/min}$$

Thus, the level is decreasing at the rate of **0.212 cm/min.**

SOLUTION 16 - 10
Ans: D

$$V = \frac{1}{3}\pi x^2 h$$

$$\frac{x}{h} = \frac{6}{12};\ x = 0.5h$$

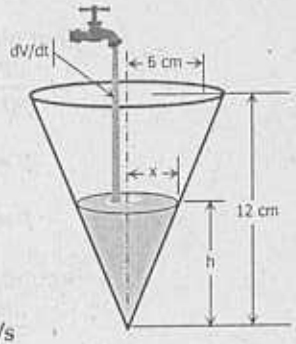
$$V = \frac{1}{3}\pi(0.5h)^2 h$$

$$V = \frac{0.25}{3}\pi h^3$$

$$\frac{dV}{dt} = \frac{0.25}{3}\pi 3h^2\frac{dh}{dt}$$

when $h = 4$, $\frac{dh}{dt} = 2$

$$\frac{dV}{dt} = \frac{0.25}{3}\pi 3(4)^2(2) = \mathbf{8\pi\ cm^3/s}$$



SOLUTION 16 - 11
Ans: A

$$\int(2x^2 + 3x - 4)dx = \frac{2}{3}x^3 + \frac{3}{2}x^2 - 4x + C$$

SOLUTION 16 - 12
Ans: A

$$\int\left[\sqrt{x} + \sqrt[3]{x}\right]dx = \int(x^{1/2} + x^{1/3})dx$$

$$= \frac{2}{3}x^{3/2} + \frac{3}{4}x^{4/3} + C$$

SOLUTION 16 - 13
Ans: B

$$\int \frac{\sin x}{\tan x}dx = \int \cos x\,dx = \sin x + C$$

SOLUTION 16 - 14
Ans: C

$$\int_0^{\frac{\pi}{6}} \cos^7 3x\,dx$$

Since $\frac{\pi}{6} \times 3 = \frac{\pi}{2}$, this can be solved using Wallis' Formula.

$$\int_0^{\frac{\pi}{6}} \cos^7 3x\,dx = \frac{1}{3}\int_0^{\frac{\pi}{2}} \cos^7 \theta\,d\theta = \frac{1}{3}\,\frac{6(4)(2)}{7(5)(3)(1)} \times 1 = \mathbf{0.152}$$

SOLUTION 16 - 15
Ans: B

Solving for the time when the two plans yield equal profit:

$50 + t^2 = 200 + 5t$
$t^2 - 5t - 150 = 0$
$(t + 10)(t - 15) = 0$
$t = 15$ years → time when they generate equal profit

First plan:

$$\frac{dP}{dt} = 50 + t^2;\ dP = (50 + t^2)\,dt$$

$$P = \int_0^{15}(50 + t^2)dt = \left[50t + \frac{1}{3}t^3\right]_0^{15}$$

$P = 1875$ (thousand pesos)

Second plan:

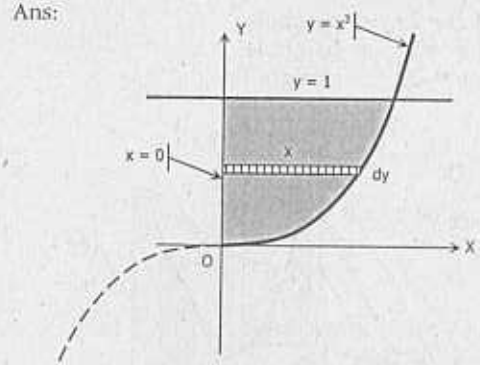
$$\frac{dP}{dt} = 200 + 5t;\ dP = (200 + 5t)\,dt$$

$$P = \int_0^{15}(200 + 5t)dt = \left[200t + \frac{5}{2}t^2\right]_0^{15}$$

$P = 3562.5$ (thousand pesos)

Excess profit = 3562.5 - 1875 = 1687.5 thousand pesos
Excess profit = **P1,687,500**

SOLUTION 16 - 16
Ans:



$$A = \int_{y_1}^{y_2}(x_R - x_L)dy$$

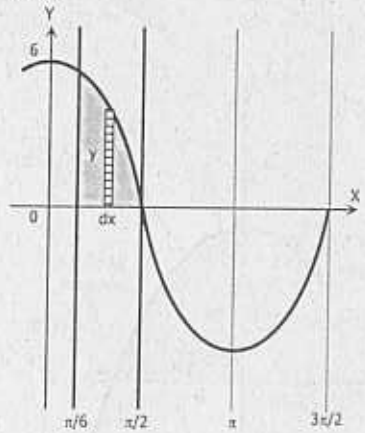
$x_R = y^{1/3}$
$x_L = 0$
$y_1 = 0$
$y_2 = 1$

$$A = \int_0^1 (y^{1/3} - 0)dy$$

$$A = \left[\frac{3}{4}y^{4/3}\right]_0^1$$

$A = \frac{3}{4}[1^{4/3} - 0]$
$A = 3/4$

SOLUTION 16 - 17
Ans: B



$$A = \int_{x_1}^{x_2}(y_U - y_L)dx$$

$y_U = 6\cos x$
$y_L = 0$
$x_1 = \pi/6$
$x_2 = \pi/2$

$$A = \int_{\pi/6}^{\pi/2} 6\cos x\,dx$$

$$A = \Big[6\sin x\Big]_{\pi/6}^{\pi/2} = \Big[6\sin x\Big]_{30°}^{90°}$$

$A = 6(\sin 90° - \sin 30°)$
$A = 3$ **sq. units**

SOLUTION 16 - 18
Ans: D

For the surface $x^2 = y + 2z$:

When $y = 0$; $x^2 = 2z$ → parabola
When $z = 0$; $x^2 = y$ → parabola
When $x = 0$; $y = -2z$ → straight line

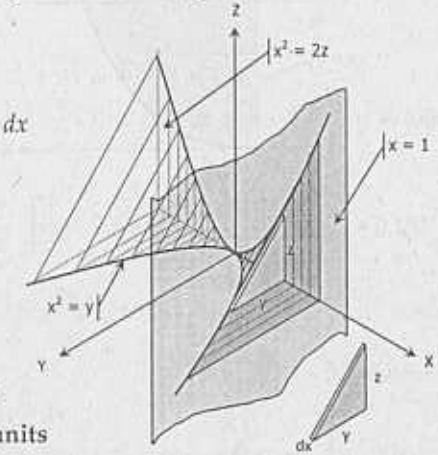
$dV = \frac{1}{2} y\, z\, dx$
$y = x^2$
$z = \frac{1}{2} x^2$
$dV = \frac{1}{2}(x^2)(\frac{1}{2}x^2)\, dx$
$dV = \frac{1}{4} x^4\, dx$

$$V = \frac{1}{4} \int_0^1 x^4\, dx$$

$$V = \frac{1}{4} \left[\frac{x^5}{5}\right]_0^1$$

$$V = \frac{1}{20}[1^5 - 0^5]$$

V = **0.05 cubic units**



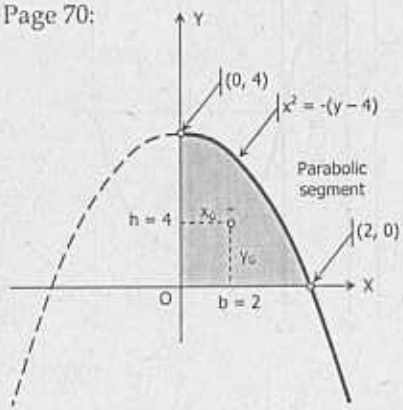
SOLUTION 16 - 19
Ans: D

From the formula in Page 70:

$y_G = \frac{2}{5} h = \frac{2}{5}(4)$
$y_G = 8/5$

$x_G = \frac{3}{8} b = \frac{3}{8}(2)$
$x_G = 3/4$

Centroid **(3/4, 8/5)**

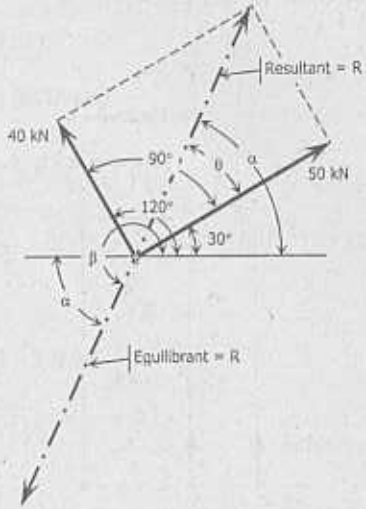


SOLUTION 16 - 20
Ans: B

$R = \sqrt{50^2 + 40^2}$
$R = 64.03$kN

$\tan\theta = \frac{40}{50}$
$\theta = 38.66°$
$\alpha = 30 + \theta = 68.66°$
$\beta = 180 + \alpha$
$\beta = 248.66°$

Equilibrant force:
64.03 kN, 248.66°



SOLUTION 16 - 21
Ans: B

$v_y^2 = v_{oy}^2 - 2gy$
At the highest point, $v_y = 0$
$0 = (50 \sin 30°)^2 - 2(9.81)y$; y = **31.86 m**

SOLUTION 16 - 22
Ans: A

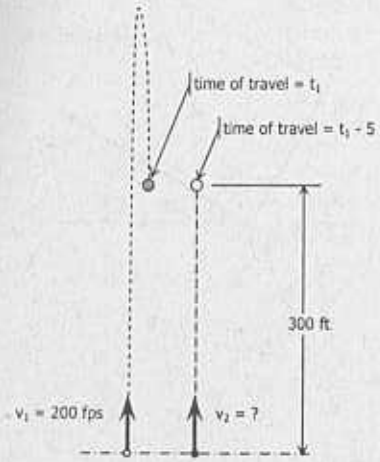
$$y = x \tan\theta - \frac{gx^2}{2v_o^2 \cos^2\theta};\ v_o = 100 \text{ fps},\ \theta = 45°$$

When theball touches the ground (level with the point of departure, $y = 0$)

$$0 = x \tan 45° - \frac{32.2x^2}{2(100)^2 \cos^2 45°}$$

$0 = x(1 - 0.00322x)$
$x = 310.56$ feet = **103.52 yards**

SOLUTION 16 - 23
Ans: C



For the first ball:

$y = v_{oy}t - ½gt^2$
$300 = 200t_1 - ½(32.2)(t_1)^2$
$16.1t_1^2 - 200t_1 + 300 = 0$
$t_1 = 1.745$ sec & 10.677 sec

For the second ball:

$t_2 = 10.677 - 5 = 5.677$ sec
$y = v_{oy}t - ½gt^2$
$300 = v_2(5.677) - ½(32.2)(5.677)^2$
$v_2 =$ **144.24 ft/sec**

SOLUTION 16 - 24
Ans: A

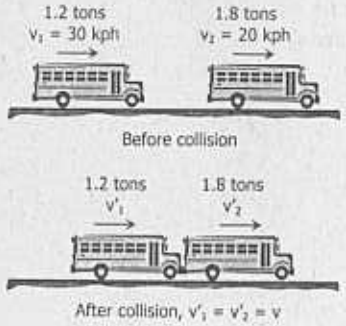
Linear speed, $v = R \times \omega$

$$v = 50\,\frac{\text{miles}}{\text{hr}} \times \frac{5280\,\text{feet}}{\text{mile}} \times \frac{1\,\text{hr}}{60\,\text{min}} \times \frac{12\,\text{in}}{\text{ft}}$$

$v = 52{,}800$ in/min
$R = 30/2 = 15$ in

$v = R\,\omega$
$52800 = 15\,\omega$
$\omega = 3520$ rad/min × 1 rev/(2π rad) = **560.2 rpm**

SOLUTION 16 - 25
Ans: D



By law of conservation of momentum:

$m_1 v_1 + m_2 v_2 = m_1 v'_1 + m_2 v'_2$
$1.2(30) + 1.8(20) = 1.2\,v + 1.8\,v$
$v =$ **24 kph**

SOLUTION 16 - 26
Ans: B

$y = v_{oy}\,t - ½\,g\,t^2$; where $y = 0$
$0 = (300 \sin 30°)t - ½(9.81)t^2$
$150t - 4.905t^2 = 0$
$t(150 - 4.905t) = 0$; $t = 30.58$ sec

SOLUTION 16 - 27
Ans:

Power input = 100 hp
Power output = $Fd/t = 10{,}000(20)/10$
Power output = 20,000 ft-lb/sec ÷ 550 = 36.364 hp
Efficiency, $\eta = (P_{out}/P_{in}) \times 100\% = (36.364/100) \times 100\%$
Efficiency, $\eta =$ **36.36%**

SOLUTION 16 - 28
Ans:

$p_{abs} = p_{gage} + p_{atm}$
$p_{abs} = -30 + 101.3 =$ **71.3 kPa**

SOLUTION 16 - 29
Ans: C

$v = \sqrt{2gH} = \sqrt{2(9.81)(3)} =$ **7.672 m/s**

SOLUTION 16 - 30
Ans: D

$K = ½\,m\,v^2$
$144 = ½\,m\,(4)^2$; $m =$ **18 kg**

SOLUTION 16 - 31
Ans: B

$F = P(1 + rt)$
$1{,}500 = P(1 + 0.10 \times \frac{90}{360})$; $P =$ **P1,463.41**

SOLUTION 16 - 32
Ans: C

Let x be the amount invested at 11% and y at 8%.

$x + y = 20{,}000$
$y = 20{,}000 - x$ → Eq. (1)

Interest, $I = Prt$

$x(0.11)(2) + y(0.08)(3) = 4{,}690$
$0.22x + (20{,}000 - x)(0.24) = 4{,}690$
$0.22x + 4800 - 0.24x = 4{,}690$
$x =$ **P5,500.00**

SOLUTION 16 - 33
Ans: B

$F = F_{25000} + F_{35000} + F_{45000}$
$F = 25{,}000(1 + 0.04)^{10-0} + 35{,}000(1 + 0.04)^{10-2}$
$\quad + 45{,}000(1 + 0.04)^{10-4}$
$F =$ **P141,845.38**

SOLUTION 16 - 34
Ans: D

$$P = \frac{A[(1+i)^n - 1]}{(1+i)^n\,i}$$

$$P = \frac{20{,}000[(1+0.06)^7 - 1]}{(1+0.06)^7(0.06)} = \text{P111,647.63}$$

SOLUTION 16 - 35
Ans: A

$FC = P200{,}000; \quad SV = P25{,}000$
$n = 10; \quad m = 8$

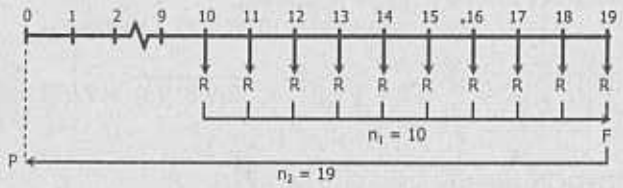
$$BV_m = FC\left(1 - \frac{2}{n}\right)^m \geq SV$$

$$BV_8 = 200{,}000\left(1 - \frac{2}{10}\right)^8 = \mathbf{P33{,}554.43}$$

SOLUTION 16 - 36
Ans: B

Production per month = 9000 × 80% = 7200 shoes
Profit = Total sales - Total cost
Profit = 7200(500) - [800,000 + 200(7200)] = **P1,360,000**

SOLUTION 16 - 37
Ans: D



$$P = \frac{F}{(1+i)^{19}}; F = \frac{R[(1+i)^{10} - 1]}{i}$$

$$P = \frac{R[(1+i)^{10} - 1]}{(1+i)^{19} i}$$

$$100{,}701.30 = \frac{R[(1+0.08)^{10} - 1]}{(1+0.08)^{19} 0.08}; R = \mathbf{P30{,}000.00}$$

SOLUTION 16 - 38
Ans: C

$$\text{Depreciation charge } d = \frac{FC - SV}{n} = \frac{100{,}000 - 10{,}000}{10}$$

Depreciation charge d = P9,000 per year
Book value, $BV_m = FC - D_m = FC - d \times m$
Book value, BV_5 = 100,000 - 9,000(5) = **P55,000.00**

SOLUTION 16 - 39
Ans: C

$$d = \frac{FC - SV}{n}; \; d = 0.18FC$$

$$0.18(FC) = \frac{FC - 0}{n}; n = 5.55$$

SOLUTION 16 - 40
Ans: B

$$F = \frac{A[(1+i)^n - 1]}{i}$$

$$150{,}000 = \frac{A[(1+0.03)^{10} - 1]}{0.03}; \; A = \mathbf{P13{,}084.57}$$

INDEX

A.M. Wahl formula, 228
Acceleration, 14
  Constant, 160
  Variable, 160
Adhesion, 258
Adiabatic, 260
Algebraic substitution, 62
Allowable stress, 221
Angle of twist, 227
Angular speed, 14
Annual cost, 294, 295
Annuity, 290
  Deferred, 292
  Due, 292
  Ordinary, 291
  Types of, 291
Arc, length of, 66
Archimedes' principle, 262
Arithmetic gradient, 293
Axial deformation, 221

Banking of curves, 165
Bearing stress, 218
Belt friction, 155
Bernoulli's energy theorem, 266
Bernoulli's equation, 124
Biaxial deformation, 224
Bond, 299
Book value, 297
Boundary conditions, 119
Boyle's law, 260
Break-even analysis, 300
Brittle, 220
Bulk modulus of elasticity, 224, 259
Buoyancy, 262

Cables
  Catenary, 153
  Parabolic, 151
  Under concentrated loads, 153
Capillarity, 258
Capital recovery, 292
Capitalized cost, 294
Cash flow diagram, 285
Catenary, 153
Celerity, 260
Centrifugal force, 164
Centroid
  of common solids, 74
  of plane areas, 66
  of volume, 73
Charle's law, 260
Cipolletti weir, 271
Circular disk, 71
Cohesion, 258
Combined stresses, 237
  Axial and flexure, 237
  Axial and shearing, 240
  Torsion and flexure, 243
Composite figures, 159
Compound interest, 127, 287
Compression of gases, 260
Concurrent, 148
Contact, angle of, 155
Continuity equation, 264
Continuous compound interest, 127, 289
Cooling problems, 127
Coplanar, 148, 150, 151
Cost comparison, 296
Curvature, 6
  Circle of, 6
  Radius of, 6
Curvilinear translation, 160

D'Alemberts principle, 163, 164
Darcy-Weisbach, 266
Decay, 127
Declining balance, 298
Density, 257
  of gases, 257
Depletion, 298
Depreciation, 296
  Declining balance, 298
  Double declining balance, 298
  Sinking-fund, 297
  Straight-line, 297
  Sum-of-years digit, 297
Determinate beams, 229
Differential equation, 119
  Bernoulli's equation, 124
  Exact, 123
  First order, 122
  Homogenous first order, 121

Linear second order, 122
Differentiation formulas, 3
Algebraic functions, 3
Exponential functions, 3
Hyperbolic functions, 4
Inverse hyperbolic functions, 5
Inverse trigonometric functions, 4
Logarithmic functions, 3
Trigonometric functions, 4
Discharge, 14, 268
Double declining balance, 298
Double integration, 61
Double shear, 217
Drag force, 274
Ductile, 219
Dynamic force, 273
Dynamics, 147, 158

Economic sections, 236
Effective rate, 289
Elastic collision, 168
Elastic limit, 220
Elastic range, 220
Energy, 167
Kinetic, 167
Potential, 167
Energy equation, 167, 265
Engineering economics, 285
Engineering mechanics, 147
Equal-payment-series capital recovery factor, 292
Equal-payment-series compound-amount factor, 291
Equal-payment-series present worth factor, 291
Equal-payment sinking-fund factor, 291
Equilibrium, 150
Equivalent pipe, 268
Exact differential equation, 123
Exact simple interest, 287
Exponential growth and decay, 127

Face value, 299
Factor of safety, 221
First moment of area, 67
First order differential equation, 119
Flat plates, 159
Flexure formula, 234
Flow problems, 127
Flow rate, 264
Fluid mechanics, 257
Fluid properties, 257
Force system, 148
Francis formula, 271
Free-falling body, 160
Friction, 155
Angle of, 155
Belt, 155
Coefficient of, 155

General solution, 119
Geometric gradient, 294
Gradient, 293
Graph of a function, 7
Guy-Lussac's law, 260

Hallow cylindrical shell, 72
Hazen Williams, 267
Head lost, 266
Heating problems, 127
Helical spring, 228
Homogenous first order DE, 121
Hooke's law, 220
Horizontal strip, 65
Hydrodynamics, 272

Ideal angle of banking, 166
Impulse, 167
Indeterminate beams, 230
Inelastic collision, 168
Inflection, points of, 8
Integration Formulas, 57
Algebraic functions, 57
Algebraic substitution, 62
By parts, 60, 62
Exponential functions, 57
Hyperbolic functions, 59
Inverse trigonometric functions, 58
Logarithmic functions, 57
Trigonometric functions, 57
Trigonometric substitution, 60
Wallis' Formula, 61
Interest, 285
Isentropic, 260
Isothermal, 260

Kern, 238
Kinematics, 158, 159
Kinetics, 158, 163

L'Hospital's rule, 1
Law of conservation of momentum, 168
Leap year, 286
Limits, 1
L'Hospital's rule, 1
Theorems of, 1
Linear first order DE, 122
Linear second order DE, 122
Longitudinal stress, 218

Mass moment of inertia, 68
Manning, 267
Maxima and minima, 8
Variable relations for, 9
Modulus of elasticity, 220, 223,224
Modulus of resilience, 221
Modulus of rigidity, 223
Modulus of toughness, 221
Modulus of volume expansion, 224
Mohr's circle, 157, 158, 240, 241
Moment in beams, 230
Moment of inertia, 67, 157
Inclined axis, 157
Momentum, 167
Motion curves, 161
Motion problems, 127

Newton's law of cooling, 127
Newton's law of motion, 128, 163
Nominal rate, 127, 289
Normal stress, 217
Nozzles, 267

Ordinary simple interest, 287
Orifice, 268

Parabolic segment, centroid of, 67
Parallel, 147
Parallelogram law, 148
Partial fraction, integration by, 64
Particular solution, 119, 120
Parts, integration by, 60, 62
Pendulum, 165
Perfect gas, 260
Perpetuity, 393
Pipes in parallel, 268
Pipes in series, 268
Plane areas, 64
Vertical strip, 64
Horizontal strip, 65
Polar coordinate, 65
Plane motion, 159
Plastic range, 220
Poisson's ratio, 223
Polar coordinate, 65
Polar moment of inertia, 68, 157
Population growth, 126
Power, 227
Pressure, 261
Below layers of different liquids, 261
Variations in, 261
Pressure disturbances, 260
Principal planes, 241
Principal stresses, 240
Product of inertia, 68
Projectile motion, 160
Properties
of common shapes, 69
of sections, 159
Proportional limit, 220
Punching, 217

Radius of curvature, 6, 234
Radius of gyration, 157
Rapture strength, 220, 221
Rate of change, 6, 15
Rectilinear translation, 159, 160
Redemption value, 299
Relative equilibrium, 263
Restitution coefficient of, 168
Resultant of forces, 148
of parallel forces, 150
Reversed effective force, 163
Reynolds number, 265
Rotation, 147, 159, 163

Second order differential equation, 119
Section modulus, 233, 234
Shear and moment in beams, 229
Diagram, 230
Properties of, 231
Shear flow, 228, 235
Shear modulus, 224
Shearing deformation, 223

Shearing stress, 217
  in beams, 235
Simple interest, 286
Simple stress, 217
Single-payment compound-amount factor, 288
Single-payment present-worth factor, 288
Single shear, 217
Sinking fund, 291
Slack side, 155
Slope, 6
Solids of revolution, 71
Spacing of rivets or bolts, 236
Spandrel, 67, 71
Specific gravity, 258
Specific volume, 257
Specific weight, 257
Spherical shell, 68, 218
Spring, 76, 77, 228
  Index, 229
  Parallel, 229
  Series, 229
Statics, 147, 148
Stiffness, 77, 222
Stress-strain diagram, 219
Stresses in beams, 233
Stresses on an inclined plane, 240
Straight-line depreciation, 297
Strain, 219
Strength of materials, 217
Sum-of-years digit, 297
Superimposed beams, 235
Surface area, 73
Surface tension, 258
Suttro weir, 271

Tangential stress, 217, 218, 219
Thermal stress, 225
Thick-walled cylinder, 219
Thin-walled pressure vessels, 218
Tight side, 155
Time rates, 13
Torsion, 226
  on thin-walled tubes, 227
  Shearing stress, 226
Total hydrostatic pressure, 261
Transfer formula, 157
Translation, 159
Triaxial deformation, 224
Trigonometric substitution, 60, 63
Triple integration, 62

Ultimate strength, 220
Uniform motion, 159
Unit weight, 257
Unsteady flow, 269, 272

Variable separable, 119, 121
Velocity, 14
Vertical strip, 64
Viscosity, 258
  Absolute, 258
  Dynamic, 258
  Kinematic, 258
Volumetric strain, 225

Wahl factor, 229
Wallis' Formula, 61
Weir, 270
Work, 75, 167
  Constant force, 75
  Spring, 76
  Variable force, 75
Work and energy, 167
Working stress, 221

Yield point, 221
Young's modulus of elasticity, 220