Computer Architecture and Programming Languages
Jacobs University Bremen
Jasmine Dayinta

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Homework 2 - Logic Circuits and Boolean Expressions

Problem 2.1

Solution:

Note that for this problem, I will be converting everything to decimal, increment by one, and then converting it back to the respective number system

(a) 777_8
$(7 \times 8^2) + (7 \times 8^1) + (7 \times 8^0)$
$= 448 + 56 + 7$
$= 511_{10}$
Increment 1, which results 512_{10}
$512 \div 8 = 64 + 0$
$64 \div 8 = 8 + 0$
$8 \div 8 = 1 + 0$
$1 \div 8 = 0 + 1$

Answer: 1000_8

(b) 888_{16}
$(8 \times 16^2) + (8 \times 16^1) + (8 \times 16^0)$
$= 2048 + 128 + 8$
$= 2184_{10}$
Increment 1, which results 2185_{10}
$2185 \div 16 = 136 + 9$
$136 \div 16 = 8 + 8$
$8 \div 16 = 0 + 8$

Answer: 889_{16}

(c) 32007_8
$(3 \times 8^4) + (2 \times 8^3) + (0 \times 8^2) + (0 \times 8^1) + (7 \times 8^0)$
$= 12288 + 1024 + 0 + 0 + 7$
$= 13319_{10}$
Increment 1, which results 13320_{10}
$13320 \div 8 = 1665 + 0$
$1665 \div 8 = 208 + 1$
$208 \div 8 = 26 + 0$
$26 \div 8 = 3 + 2$
$3 \div 8 = 0 + 3$

Answer: 32010_8

(d) 32108_{16}
$(3 \times 16^4) + (2 \times 16^3) + (1 \times 16^2) + (0 \times 16^1) + (8 \times 16^0)$
$= 196608 + 8192 + 256 + 0 + 8$
$= 205064_{10}$
Increment 1, which results 205065_{10}
$205065 \div 16 = 12816 + 9$
$12816 \div 16 = 801 + 0$
$801 \div 16 = 50 + 1$
$50 \div 16 = 3 + 2$
$3 \div 16 = 0 + 3$

Answer: 32109_{16}

(e) $8BFF_{16}$
$(8 \times 16^3) + (11 \times 16^2) + (15 \times 16^1) + (15 \times 16^0)$
$= 32768 + 2816 + 240 + 15$
$= 35839_{10}$
Increment 1, which results 35840_{10}
$35840 \div 16 = 2240 + 0$
$2240 \div 16 = 140 + 0$
$140 \div 16 = 8 + 12$ $(= C)$
$8 \div 16 = 0 + 8$

Answer: $8C00_{16}$

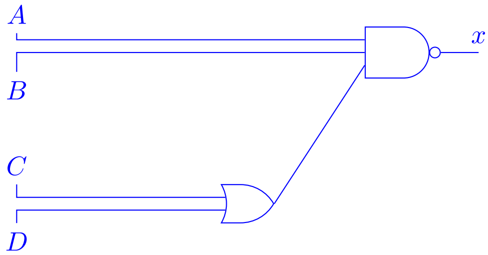
(f) 1219_{16}
$(1 \times 16^3) + (2 \times 16^2) + (1 \times 16^1) + (9 \times 16^0)$
$= 4096 + 512 + 16 + 9$
$= 4633_{10}$
Increment 1, which results 4634_{10}
$4634 \div 16 = 289 + 10$ $(= A)$
$289 \div 16 = 18 + 1$
$18 \div 16 = 1 + 2$
$1 \div 16 = 0 + 1$

Answer: $121A_{16}$

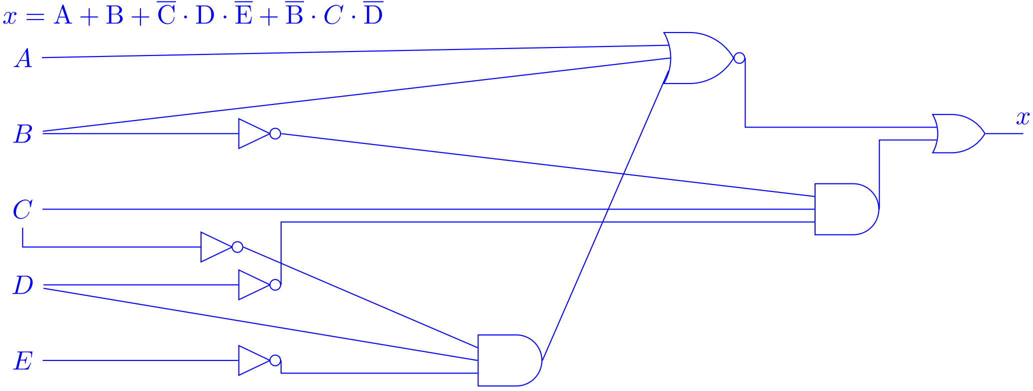
Problem 2.2

Solution:

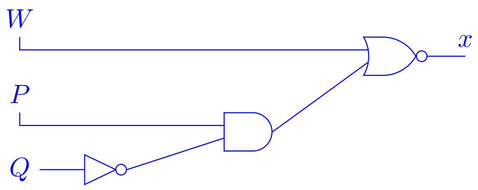
a) $x = \overline{\mathrm{A} \cdot \mathrm{B} \cdot (\mathrm{C} + \mathrm{D})}$



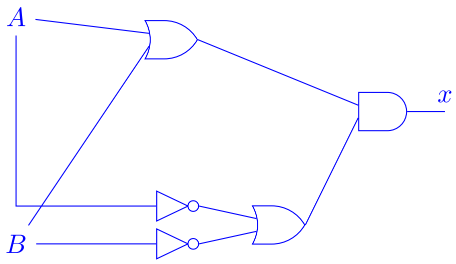
b) $x = \overline{\mathrm{A} + \mathrm{B} + \overline{\mathrm{C}} \cdot \mathrm{D} \cdot \overline{\mathrm{E}}} + \overline{\mathrm{B}} \cdot C \cdot \overline{\mathrm{D}}$



c) $x = \overline{\mathrm{W} + \mathrm{P} \cdot \overline{\mathrm{Q}}}$



d) $x = (A + B) \cdot (\overline{\mathrm{A}} + \overline{\mathrm{B}})$



Problem 2.3

Solution:

The boolean expression is: $\overline{\overline{(\mathrm{M} \wedge \mathrm{N} \wedge \mathrm{Q})} \wedge \overline{(\mathrm{M} \wedge \overline{\mathrm{N}} \wedge \mathrm{Q})} \wedge \overline{(\overline{\mathrm{M}} \wedge \mathrm{N} \wedge \mathrm{Q})}}$

M	N	Q	$\overline{\overline{(\mathrm{M} \wedge \mathrm{N} \wedge \mathrm{Q})} \wedge \overline{(\mathrm{M} \wedge \overline{\mathrm{N}} \wedge \mathrm{Q})} \wedge \overline{(\overline{\mathrm{M}} \wedge \mathrm{N} \wedge \mathrm{Q})}}$
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

$\overline{\overline{(\overline{\mathrm{M}} \wedge \mathrm{N} \wedge \mathrm{Q})} \wedge (\mathrm{M} \wedge \overline{\mathrm{N}} \wedge \mathrm{Q}) \wedge \overline{(\mathrm{M} \wedge \mathrm{N} \wedge \mathrm{Q})}}$

$= \overline{(\mathrm{M} \vee \overline{\mathrm{N}} \vee \overline{\mathrm{Q}}) \wedge (\overline{\mathrm{M}} \vee \mathrm{N} \vee \overline{\mathrm{Q}}) \wedge (\overline{\mathrm{M}} \vee \overline{\mathrm{N}} \vee \overline{\mathrm{Q}})}$ **(De Morgan's rule)**

= $(\overline{\mathrm{M}} \wedge N \wedge Q) \vee (M \wedge \overline{\mathrm{N}} \wedge Q) \vee M \wedge N \wedge Q$ **(De Morgan's rule)**

= Q $\wedge(\overline{\mathrm{M}} \wedge N \vee \overline{\mathrm{N}} \wedge M \vee N \wedge M)$

Problem 2.4

Solution:

As said on the slides, if the expression contains both AND and OR gates, the AND operation will be evaluated first.

$X + \overline{\mathrm{X}} \cdot Y = X + Y$

X	Y	$\overline{\mathrm{X}}$	$\overline{\mathrm{X}}\wedge$Y	$\overline{\mathrm{X}}\wedge$Y$\vee$X	X$\vee$Y
0	0	1	0	0	0
0	1	1	1	1	1
1	0	0	0	1	1
1	1	0	0	1	1

(b) $\bar{X} +$ X $\times$ Y $= \bar{X} +$ Y

X	Y	$\overline{\mathrm{X}}$	X $\wedge$ Y	$\overline{\mathrm{X}}\vee$ Y $\wedge$ X	$\bar{X}\vee$ Y
0	0	1	0	1	1
0	1	1	0	1	1
1	0	0	0	0	0
1	1	0	1	1	1

As you can see, the last two columns show the exact same numbers, thus proving that the boolean expression does equal to each other

Problem 2.5

Solution:

a) $A + 1 = A$

b) $A \cdot A = A$

c) $B \cdot \overline{\mathrm{B}} = 0$

d) $C + C = C$

e) $x \cdot 0 = 0$

f) $D \cdot 1 = D$

g) $D + 0 = D$

h) $C + \overline{\mathrm{C}} = 1$

i) $G + G \cdot F = G$

j) $y + \overline{\mathrm{w}} \cdot y = y$

Problem 2.6

Solution:

The DeMorgan's Theorem states two things:
1) When the OR sum of two variables is inverted, it is equivalent to inverting each variable individually and ANDing them

2)When the AND product of two variables is inverted, it is equivalent to inverting each variable individually and ORing them

This can otherwise be written as the following boolean expressions and truth table:

1) $\overline{\mathrm{X} \vee \mathrm{Y}} = \overline{\mathrm{X}} \wedge \overline{\mathrm{Y}}$

X	Y	$\overline{\mathrm{X}}$	$\overline{\mathrm{Y}}$	$\mathrm{X} \vee Y$	$\overline{\mathrm{X} \vee \mathrm{Y}}$	$\overline{\mathrm{X}} \wedge \overline{\mathrm{Y}}$
0	0	1	1	0	1	1
0	1	1	0	1	0	0
1	0	0	1	1	0	0
1	1	0	0	1	0	0

2) $\overline{X \wedge Y} = \overline{\mathrm{X}} \vee \overline{\mathrm{Y}}$

X	Y	$\overline{\mathrm{X}}$	$\overline{\mathrm{Y}}$	$\mathrm{X} \wedge Y$	$\overline{\mathrm{X} \wedge \mathrm{Y}}$	$\overline{\mathrm{X}} \vee \overline{\mathrm{Y}}$
0	0	1	1	0	1	1
0	1	1	0	0	1	1
1	0	0	1	0	1	1
1	1	0	0	1	0	0

As you can see, the last two columns produce the same outputs, thus proving DeMorgan's Theorem

Problem 2.7

Solution:

In order to determine the truth table, we must look at the rows that resulting 1 (true) and create the boolean expression:

$(\overline{A} \wedge \overline{B} \wedge \overline{C} \wedge D) \vee (\overline{A} \wedge \overline{B} \wedge C \wedge D) \vee (\overline{A} \wedge B \wedge \overline{C} \wedge D) \vee (\overline{A} \wedge B \wedge C \wedge D) \vee (A \wedge \overline{B} \wedge \overline{C} \wedge \overline{D}) \vee (A \wedge B \wedge \overline{C} \wedge D) \vee (A \wedge B \wedge C \wedge D)$

= B $\wedge C \wedge D \wedge (A \vee \overline{A}) \vee B \wedge \overline{C} \wedge D \wedge (A \vee \overline{A}) \vee \overline{A} \wedge \overline{B} \wedge D \wedge (C \vee \overline{C}) \vee A \wedge \overline{B} \wedge \overline{C} \wedge \overline{D}$ **(Distributive rule)**

$= B \wedge C \wedge D \vee B \wedge \overline{C} \wedge D \vee \overline{A} \wedge \overline{B} \wedge D \vee A \wedge \overline{B} \wedge \overline{C} \wedge \overline{D}$ **(Complement rule)**

$= B \wedge D \wedge (\overline{C} \vee C) + \overline{A} \wedge \overline{B} \wedge D \vee A \wedge \overline{B} \wedge \overline{C} \wedge \overline{D}$ **(Distributive rule)**

$= B \wedge D + \overline{A} \wedge \overline{B} \wedge D \vee A \wedge \overline{B} \wedge \overline{C} \wedge \overline{D}$ **(Complement rule)**

Problem 2.8

Solution:

Similar to the previous problem, we must evaluate the rows that result in 1 (true) and combine them to get the following:

$(\overline{A} \wedge \overline{B} \wedge \overline{C} \wedge D) \vee (\overline{A} \wedge \overline{B} \wedge C \wedge D) \vee (\overline{A} \wedge B \wedge \overline{C} \wedge D) \vee (\overline{A} \wedge B \wedge C \wedge D) \vee (A \wedge \overline{B} \wedge \overline{C} \wedge \overline{D}) \vee (A \wedge B \wedge \overline{C} \wedge D) \vee (A \wedge B \wedge C \wedge D)$

We can then create the K-map, which yields the following:

	$\overline{C \wedge D}$	$\overline{C} \wedge D$	$C \wedge D$	$C \wedge \overline{D}$
$\overline{A \wedge B}$	0	1	1	0
$\overline{A} \wedge B$	0	1	1	0
$A \wedge B$	0	1	1	0
$A \wedge \overline{B}$	1	0	0	0

We put 1 in the table when the combination of ABCD matches with the boolean expression above and this way, we could group them into the 6 1s in the middle and then there's one outlier on the bottom left