Homework 4 - Logic Circuits and Number Representations

Problem 4.1

Solution:

A	B	C	$A \oplus B$	$B \odot C$	X
0	0	0	0	1	0
0	0	1	0	0	0
0	1	0	1	0	0
0	1	1	1	1	1
1	0	0	1	1	0
1	0	1	1	0	0
1	1	0	0	0	0
1	1	1	0	1	0

As shown in the truth table above, we can see that the input condition needed to produce X as 1 is A as 0, B as 1, and C as 1. After computing the XOR and XNOR gates, we just compare those two columns with the C values and use the AND gate to produce the results.

Problem 4.2

Solution:

a.

A	B	C	$A \wedge B$	$B \vee C$	$(B \vee C) \wedge \overline{A}$	$\overline{(A \wedge B) \vee C}$	Y
0	0	0	0	0	0	1	1
0	0	1	0	1	1	0	1
0	1	0	0	1	1	1	0
0	1	1	0	1	1	0	1
1	0	0	0	0	0	1	1
1	0	1	0	1	0	0	0
1	1	0	1	1	0	0	0
1	1	1	1	1	0	0	0

$Y = ((B \vee C) \wedge \overline{\mathrm{A}}) \oplus \overline{(\mathrm{A} \wedge \mathrm{B}) \vee \mathrm{C}}$

b.

$Y = \overline{\mathrm{ABC}} \vee \overline{\mathrm{AB}}C \vee \overline{\mathrm{A}}BC \vee A\overline{\mathrm{BC}}$
$= \overline{\mathrm{AB}} \wedge (\overline{\mathrm{C}} \vee C) \vee \overline{\mathrm{A}}BC \vee A\overline{\mathrm{BC}}$
$= \overline{\mathrm{AB}} \vee 1 \vee \overline{\mathrm{A}}BC \vee A\overline{\mathrm{BC}}$

Problem 4.3

Solution:

a. $+27_{10}$

$27 \div 2 = 13 + 1$
$13 \div 2 = 6 + 1$
$6 \div 2 = 3 + 0$
$3 \div 2 = 1 + 1$
$1 \div 2 = 0 + 1$

Due to the fact that we always have to use 8 bits, the answer is 00011011_2

b. $+66_{10}$

$66 \div 2 = 33 + 0$
$33 \div 2 = 16 + 1$
$16 \div 2 = 8 + 0$
$8 \div 2 = 4 + 0$
$4 \div 2 = 2 + 0$
$2 \div 2 = 1 + 0$
$1 \div 2 = 0 + 1$

The answer is: 01000010_2

c. -18_{10}

$18 \div 2 = 9 + 0$
$9 \div 2 = 4 + 1$
$4 \div 2 = 2 + 0$
$2 \div 2 = 1 + 0$
$1 \div 2 = 0 + 1$

+18_{10} = 00010010_2
-18_{10} = 11101101_2 (One's Compliment)
-18_{10} = 11101110_2 (Two's Compliment)

d. 127_{10}

$127 \div 2 = 63 + 1$
$63 \div 2 = 31 + 1$
$31 \div 2 = 15 + 1$
$15 \div 2 = 7 + 1$
$7 \div 2 = 3 + 1$
$3 \div 2 = 1 + 1$
$1 \div 2 = 0 + 1$

The answer is: 01111111_2

e. -127_{10}

+127_{10} = 01111111_2
-127_{10} = 10000000_2 (One's Compliment)
-127_{10} = 10000001_2(Two's Compliment)

f. +131_{10}

$131 \div 2 = 65 + 1$
$65 \div 2 = 32 + 1$
$32 \div 2 = 16 + 0$
$16 \div 2 = 8 + 0$
$8 \div 2 = 4 + 0$
$4 \div 2 = 2 + 0$
$2 \div 2 = 1 + 0$
$1 \div 2 = 0 + 1$

The answer is: 10000011_2

g. -7_{10}

$7 \div 2 = 3 + 1$
$3 \div 2 = 1 + 1$
$1 \div 2 = 0 + 1$

7_{10} = 00000111_2
-7_{10} = 11111000_2 (One's Compliment)
-7_{10} = 11111001_2 (Two's Compliment)

Problem 4.4

Solution:
a. 00011000_2
$(0 \times 2^7) + (0 \times 2^6) + (0 \times 2^5) + (1 \times 2^4) + (1 \times 2^3) + (0 \times 2^2) + (0 \times 2^1) + (0 \times 2^0)$
$= 8 + 16$
$= 24_{10}$

b. 11110101_2
We must now subtract one from the binary number since we are going backwards
$= 11110100_2$ (One's Compliment)

$= 00001011_2$ (Invert)
$= (0 \times 2^7) + (0 \times 2^6) + (0 \times 2^5) + (0 \times 2^4) + (1 \times 2^3) + (0 \times 2^2) + (1 \times 2^1) + (1 \times 2^0)$
$= 8 + 2 + 1$
$=$ -11_{10}

c. 01011011_2
$(0 \times 2^7) + (1 \times 2^6) + (0 \times 2^5) + (1 \times 2^4) + (1 \times 2^3) + (0 \times 2^2) + (1 \times 2^1) + (1 \times 2^0)$
$= 64 + 16 + 8 + 2 + 1$
$= 91_{10}$

d. 10110110_2
$= 10110101_2$ (One's Compliment)
$= 01001010_2$ (Invert)
$= (0 \times 2^7) + (1 \times 2^6) + (0 \times 2^5) + (0 \times 2^4) + (1 \times 2^3) + (0 \times 2^2) + (1 \times 2^1) + (0 \times 2^0)$
$= 64 + 8 + 2$
$=$ -74_{10}

e. 11111111_2
$= 11111110_2$ (One's Compliment)
$= 00000001_2$ (Invert)
$= (0 \times 2^7) + (0 \times 2^6) + (0 \times 2^5) + (0 \times 2^4) + (0 \times 2^3) + (0 \times 2^2) + (0 \times 2^1) + (1 \times 2^0)$
$=$ -1_{10}

f. 01101111_2
$(0 \times 2^7) + (1 \times 2^6) + (1 \times 2^5) + (0 \times 2^4) + (1 \times 2^3) + (1 \times 2^2) + (1 \times 2^1) + (1 \times 2^0)$
$= 64 + 32 + 8 + 4 + 2 + 1$
$= 111_{10}$

g. 10000001_2
$= 10000000_2$ (One's Compliment)
$= 01111111_2$ (Invert)
$= (0 \times 2^7) + (1 \times 2^6) + (1 \times 2^5) + (1 \times 2^4) + (1 \times 2^3) + (1 \times 2^2) + (1 \times 2^1) + (1 \times 2^0)$
$= 64 + 32 + 16 + 8 + 4 + 2 + 1$
$=$ -127_{10}

h. 10000000_2
$= 01111111_2$ (One's Compliment)
$= 10000000_2$ (Invert)
$= (1 \times 2^7) + (0 \times 2^6) + (0 \times 2^5) + (0 \times 2^4) + (0 \times 2^3) + (0 \times 2^2) + (0 \times 2^1) + (0 \times 2^0)$
$=$ -128_{10}

Problem 4.5

Solution:
a. $(27 + 36)_{BCD}$
$= 0010\ 0111 + 0011\ 0110$
$= 0110\ 1101$ (The binary on the right creates a number greater than 9, which is why 6 (or 0110) is going to be added)
$= 0110\ (1101 + 0110)$
$= 0110\ 0100_{BCD}$

b. $(73 + 29)_{BCD}$
$= 0111\ 0011 + 0010\ 1001$
$= 1001\ 1100$ (Right side creates a number greater than 9, so 0110 is added)
$= 1001\ (1100 + 0110)$
$= 1010\ 0010$ (However, now the left side is greater than 9 after 0110 was added to the right side, therefore now 0110 has to be added to the left side as well
$= 0001\ 0000\ 0010_{BCD}$

Problem 4.6

Solution:
a. The range of unsigned decimal numbers that can be represented by using 8 bits is between 0 and 255.
b. The range of signed decimal numbers that can be represented by using 8 bits is between -128 and 127.
c. The range of unsigned decimal numbers that can be represented by using 11 bits is between 0 and 2407.
d. The range of signed decimal numbers that can be represented by using 11 bits is between -1024 and 1023.
e. The range of signed decimal numbers that can be represented by using 16 bits is between -32768 and 23767.