Homework 1 - Conversion and Logic Gates

Problem 1.1

Solution:

(a) 10100_2
$(1 \times 2^4) + (0 \times 2^3) + (1 \times 2^2) + (0 \times 2^1) + (0 \times 2^0)$
$= (1 \times 16) + (0 \times 8) + (1 \times 4) + (0 \times 2) + (0 \times 1)$
$= 16 + 0 + 4 + 0 + 0$
$= 20_{10}$

(b) 11011011_2
$(1 \times 2^7) + (1 \times 2^6) + (0 \times 2^5) + (1 \times 2^4) + (1 \times 2^3) + (0 \times 2^2) + (1 \times 2^1) + (1 \times 2^0)$
$= (1 \times 128) + (1 \times 64) + (0 \times 32) + (1 \times 16) + (1 \times 8) + (0 \times 4) + (1 \times 2) + (1 \times 1)$
$= 128 + 64 + 0 + 16 + 8 + 0 + 2 + 1$
$= 219_{10}$

(c) 001001001_2
$(0 \times 2^8) + (0\times 2^7) + (1 \times 2^6) + (0 \times 2^5) + (0 \times 2^4) + (1 \times 2^3) + (0 \times 2^2) + (0 \times 2^1) + (1 \times 2^0)$
$= (0 \times 256) + (0 \times 128) + (1 \times 64) + (0 \times 32) + (0 \times 16) + (1 \times 8) + (0 \times 4) + (0 \times 2) + (1 \times 1)$
$= 0 + 0 + 64 + 0 + 0 + 8 + 0 + 0 + 1$
$= 73_{10}$

(d) 111111111111_2
$(1 \times 2^11) + (1\times 2^10) + (1 \times 2^9) + (1 \times 2^8) + (1\times 2^7) + (1 \times 2^6) + (1 \times 2^5) + (1 \times 2^4) + (1 \times 2^3) + (1 \times 2^2) + (1 \times 2^1) + (1 \times 2^0)$
$= (1 \times 2048) + (1 \times 1024) + (1 \times 512) +(1 \times 256) + (1 \times 128) + (1 \times 64) + (1 \times 32) + (1 \times 16) + (1 \times 8) + (1 \times 4) + (1 \times 2) + (1 \times 1)$
$= 2048 + 1024 + 512 + 256 + 128 + 64 + 32 + 16 + 8 + 4 + 2 + 1$
$= 4095_{10}$

(e) 75077_8
$(7 \times 8^4) + (5 \times 8^3) + (0 \times 8^2) + (7 \times 8^1) + (7 \times 8^0)$
$= (7 \times 4096) + (5 \times 512) + (0 \times 64) + (7 \times 8) + (7 \times 1)$
$= 28{,}672 + 2{,}560 + 0 + 56 + 7$
$= 31{,}295_{10}$

(f) 12101_3
$(1 \times 3^4) + (2 \times 3^3) + (1 \times 3^2) + (0 \times 3^1) + (1 \times 3^0)$
$= (1 \times 81) + (2 \times 27) + (1 \times 9) + (0 \times 3) + (1\times 1)$
$= 81 + 54 + 9 + 0 + 1$
$= 145_{10}$

(g) 26601_7
$(2 \times 7^4) + (6 \times 7^3) + (6 \times 7^2) + (0 \times 7^1) + (1 \times 7^0)$
$= (2 \times 2401) + (6 \times 343) + (6 \times 49) + (0 \times 7) + (1\times 1)$
$= 4{,}802 + 2{,}058 + 294 + 0 + 1$
$= 7{,}155_{10}$

(h) 431021_5
$(4 \times 5^5) + (3 \times 5^4) + (1 \times 5^3) + (0 \times 5^2) + (2 \times 5^1) + (1 \times 5^0)$
$= (4 \times 3{,}125) + (3 \times 625) + (1 \times 125) + (0 \times 25) + (2\times 5) + (1 \times 1)$
$= 12{,}500 + 1{,}875 + 125 + 0 + 10 + 1$
$= 14{,}511_{10}$

Problem 1.2

Solution:

(a) Convert 4272_{10} to binary
$4272 \div 2 = 2136 + 0$
$2136 \div 2 = 1068 + 0$
$1068 \div 2 = 534 + 0$
$534 \div 2 = 267 + 0$
$267 \div 2 = 133 + 1$
$133 \div 2 = 66 + 1$
$66 \div 2 = 33 + 0$
$33 \div 2 = 16 + 1$
$16 \div 2 = 8 + 0$
$8 \div 2 = 4 + 0$
$4 \div 2 = 2 + 0$
$2 \div 2 = 1 + 0$
$1 \div 2 = 0 + 1$
Answer: 1000010110000_2

(b) Convert CBA_{16} to binary
C in hexadecimal is 12, B is 11 and A is 10. 12, 11, and 10 in binary are 1100, 1011, and 1010. Therefore, when combining all binary numbers, the answer is: 110010111010_2

(c) Convert $B8C_{16}$ to decimal
*Note that B in hexadecimal is 11 and C is 12
$(11 \times 16^2) + (8 \times 16^1) + (12 \times 16^0)$
$= (11 \times 256) + (8 \times 16) + (12 \times 1)$
$= 2{,}816 + 128 + 12$
$= 2956_{10}$

(d) Convert $29D8_{16}$ to decimal
*Note that D in hexadecimal is 13
$(2 \times 16^3) + (9 \times 16^2) + (13 \times 16^1) + (8 \times 16^0)$
$= (2 \times 4{,}096) + (9 \times 256) + (13 \times 16) + (8 \times 1)$
$= 8{,}192 + 2{,}304 + 208 + 8$
$= 10{,}712_{10}$

(e) Write down the next five hexadecimal numbers that follow $8CE_{16}$
The next hexadecimal number is simple, which is $8CF_{16}$ because all one had to do was add a number, which was from E to F. However, after F, there is no other alphabet, therefore what one can do is convert the hexadecimal into decimal and then convert back to hexadecimal. 8CF in decimal is 2255. We now want 2256, which is as follows:

$2256 \div 16 = 141$, remainder 0
$141 \div 16 = 8$, remainder 13 = D
$8 \div 16 = 0$, remainder 8

Hence, 2256 is 8D0 in hexidecimal. From now and forward, we can just increase the number, which is why we have the final answer of:
$8CF_{16}$, $8D0_{16}$, $8D1_{16}$, $8D2_{16}$, $8D3_{16}$

Problem 1.3

Solution:

(a) Convert 732_{10} to BCD

7 = 0111; 3 = 0011; 2 = 0010. Therefore, the answer is:

011100110010_{BCD}

(b) Write down all invalid BCD codes

Due to the fact that BCD codes only support numbers ranging from 0 to 9, this means that hexadecimal number system of 0 to F can't be fully supported. We compare BCD with hexadecimal because they follow the same binary format. However, the numbers 10(A) to 15(F) of hexadecimal cannot be written as follows (invalid BCD codes):

$10_{16} = 1010_{BCD}$
$11_{16} = 1011_{BCD}$
$12_{16} = 1100_{BCD}$
$13_{16} = 1101_{BCD}$
$14_{16} = 1110_{BCD}$
$15_{16} = 1111_{BCD}$

However, it should be as follows (the valid BCD codes):

$10_{16} = 0001\ 0000_{BCD}$
$11_{16} = 0001\ 0001_{BCD}$
$12_{16} = 0001\ 0010_{BCD}$
$13_{16} = 0001\ 0011_{BCD}$
$14_{16} = 0001\ 0100_{BCD}$
$15_{16} = 0001\ 0101_{BCD}$

(c) Convert to decimal $1001\ 0101\ 0110_{BCD}$

1001 = 9; 0101 = 5; 0110 = 6. Therefore, the answer is:

956_{10}

(d) The decimal ASCII code of the uppercase letter M is 77. What is the binary and hexadecimal representation of this letter?

Since the ASCII code of M is 77, we can easily convert 77 into binary, which is 01001101 because:

$(0 \times 2^7) + (1 \times 2^6) + (0 \times 2^5) + (0 \times 2^4) + (1 \times 2^3) + (1 \times 2^2) + (0 \times 2^1) + (1 \times 2^0)$
$= (0 \times 128) + (1 \times 64) + (0 \times 32) + (0 \times 16) + (1 \times 8) + (1 \times 4) + (0 \times 2) + (1 \times 1)$
$= 0 + 64 + 0 + 0 + 8 + 4 + 0 + 1$
$= 77$

The hexadecimal representation is 4D because:

$77 \div 16 = 4$, remainder 13 = D
$4 \div 16 = 0$, remainder 4

(e) The decimal ASCII code of the uppercase letter m is 109. What is the binary and hexadecimal representation of this letter?

Since the ASCII code of m is 109, we can easily convert 109 into binary, which is 01101101 because:

$(0 \times 2^7) + (1 \times 2^6) + (1 \times 2^5) + (0 \times 2^4) + (1 \times 2^3) + (1 \times 2^2) + (0 \times 2^1) + (1 \times 2^0)$
$= (0 \times 128) + (1 \times 64) + (1 \times 32) + (0 \times 16) + (1 \times 8) + (1 \times 4) + (0 \times 2) + (1 \times 1)$
$= 0 + 64 + 32 + 0 + 8 + 4 + 0 + 1$
$= 109$

The hexadecimal representation is 6D because:

$109 \div 16 = 6$, remainder 13 = D
$6 \div 16 = 0$, remainder 6

Problem 1.4

Solution:

(a) Which logic function provides a low output in response to one or more low inputs?
The answer is: (iii) AND

(b) Which logic function provides a low output only when all inputs are low?
The answer is: (i) OR

Problem 1.5

Solution:

Write down the truth table for an AND gate with three inputs

p	q	r	p∧q∧r
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

Problem 1.6

Solution:

Write down the truth table for an OR gate with four inputs

p	q	r	s	p∨q∨r∨s
0	0	0	0	0
0	0	0	1	1
0	0	1	0	1
0	0	1	1	1
0	1	0	0	1
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1