

Homework 3 - Logic Circuits and Flip-Flops

Problem 3.1

Solution:

$$\begin{aligned}
 (a) \quad x &= (M + N)(\overline{M} + P)(\overline{N} + \overline{P}) \\
 &= (M \cdot \overline{M} + M \cdot P + N \cdot \overline{M} + N \cdot P)(\overline{N} + \overline{P}) && \text{Distributive Law} \\
 &= (0 + M \cdot P + N \cdot \overline{M} + N \cdot P)(\overline{N} + \overline{P}) && \text{Distributive Law} \\
 &= (M \cdot P + N \cdot \overline{M} + N \cdot P)(\overline{N} + \overline{P}) && \text{Complement Law} \\
 &= (MP\overline{N} + MPP + N\overline{M}\overline{N} + NMP + NP\overline{N} + NPP) && \text{Distributive Law} \\
 &= (MP\overline{N} + 0 + 0 + NMP + 0 + 0) && \text{Complement Law} \\
 &= (MP\overline{N} + NMP)
 \end{aligned}$$

$$\begin{aligned}
 (b) \quad z &= \overline{A}B\overline{C} + A\overline{B}\overline{C} + B\overline{C}D \\
 &= B(\overline{A}C + A\overline{C} + \overline{C}D) && \text{Distributive Law} \\
 &= B(\overline{A} + \overline{C} + A\overline{C} + \overline{C}D) && \text{DeMorgan's Law} \\
 &= B\overline{C}(\overline{A} + A\overline{C} + \overline{C}D) && \text{Distributive Law} \\
 &= B\overline{C}(1 + D) && \text{Distributive Law} \\
 &= B\overline{C}
 \end{aligned}$$

$$\begin{aligned}
 (c) \quad x &= \overline{(M + N + P)Q} \\
 &= \overline{MQ + NQ + PQ} && \text{Distributive Law} \\
 &= (\overline{MQ})(\overline{NQ})(\overline{PQ}) && \text{DeMorgan's Law} \\
 &= (\overline{M} + \overline{Q})(\overline{N} + \overline{Q})(\overline{P} + \overline{Q}) && \text{DeMorgan's Law} \\
 &= \overline{Q}(\overline{M} + \overline{N} + \overline{P}) && \text{Distributive Law}
 \end{aligned}$$

$$\begin{aligned}
 (d) \quad z &= \overline{ABC + DEF} \\
 &= (\overline{ABC})(\overline{DEF}) && \text{DeMorgan's Law} \\
 &= (\overline{A} + \overline{B} + \overline{C})(\overline{D} + \overline{E} + \overline{F}) && \text{DeMorgan's Law}
 \end{aligned}$$

$$\begin{aligned}
 (e) \quad z &= \overline{\overline{A}B + C\overline{D} + EF} \\
 &= \overline{\overline{A}B C \overline{D} E F} && \text{DeMorgan's Law} \\
 &= (\overline{A} + B)(\overline{C} + D)(\overline{E} + \overline{F}) && \text{DeMorgan's Law}
 \end{aligned}$$

$$\begin{aligned}
 (f) \quad z &= \overline{\overline{A + B\overline{C}} + D(\overline{E + F})} \\
 &= \overline{\overline{A}(\overline{B + C}) + D\overline{E}F} && \text{DeMorgan's Law} \\
 &= \overline{\overline{A}B + \overline{A}C + D\overline{E}F} && \text{Distributive Law} \\
 &= \overline{\overline{A}B \overline{A}C \overline{D} E F} && \text{DeMorgan's Law} \\
 &= (A + B)(A + \overline{C})(\overline{D} + E + \overline{F}) && \text{DeMorgan's Law}
 \end{aligned}$$

Problem 3.2

Solution:

The output expression of the circuit is: $\overline{A}B\overline{C} + A\overline{B}\overline{C} + \overline{A}B\overline{D}$

The corresponding K-Map is as follows:

| | $\overline{C}\overline{D}$ | $\overline{C}D$ | CD | $C\overline{D}$ |
|-----------------|----------------------------|-----------------|------|-----------------|
| $\overline{A}B$ | 0 | 0 | 0 | 0 |
| $\overline{A}B$ | 0 | 0 | 1 | 1 |
| AB | 0 | 0 | 0 | 0 |
| AB | 1 | 0 | 1 | 1 |

One can see that each expression only has 3 variables while the K-Map has all 4 variables. This is why we have to take into consideration for all possibilities i.e. for $\overline{A}B\overline{C}$, we should put 1 for when it is $\overline{A}B\overline{C}\overline{D}$ or $\overline{A}B\overline{C}D$. From the table, we can then group the values, which will give us the expression: $x = \overline{A}B\overline{C} + \overline{A}B(D + \overline{C}\overline{D})$

Problem 3.3**Solution:**

| S | R | Clock | Q(initial) | Q(result) |
|---|---|-------|------------|-----------|
| 1 | 0 | ↑ | 1 | 1 |
| 1 | 0 | ↑ | 1 | 1 |
| 0 | 0 | ↑ | 1 | 1 |
| 1 | 0 | ↑ | 1 | 1 |

Problem 3.4**Solution:**

| S | R | Clock | Q(initial) | Q(result) |
|---|---|-------|------------|-----------|
| 1 | 0 | ↓ | 1 | 1 |
| 0 | 1 | ↓ | 1 | 0 |
| 0 | 0 | ↓ | 0 | 0 |
| 0 | 0 | ↓ | 0 | 0 |

Problem 3.5**Solution:**

| J | K | Clock | Q(initial) | Q(result) |
|---|---|-------|------------|-----------|
| 0 | 0 | ↑ | 1 | 1 |
| 0 | 0 | ↑ | 1 | 1 |
| 1 | 1 | ↑ | 1 | 0 |
| 0 | 1 | ↑ | 0 | 1 |
| 1 | 0 | ↑ | 1 | 0 |
| 1 | 0 | ↑ | 0 | 1 |

Problem 3.6**Solution:**

| J | K | Clock | Q(initial) | Q(result) |
|---|---|-------|------------|-----------|
| 0 | 0 | ↓ | 0 | 0 |
| 0 | 0 | ↓ | 0 | 1 |
| 1 | 1 | ↓ | 1 | 0 |
| 0 | 1 | ↓ | 0 | 1 |
| 1 | 0 | ↓ | 1 | 0 |
| 1 | 0 | ↓ | 0 | 0 |

Problem 3.7**Solution:**

- a) Y will go HIGH only when J is HIGH, J is HIGH when X is HIGH and X is HIGH when A is HIGH.
b) The START pulse is needed because we need a HIGH signal to start the flip flop in order for it to work and then a LOW signal to clear.