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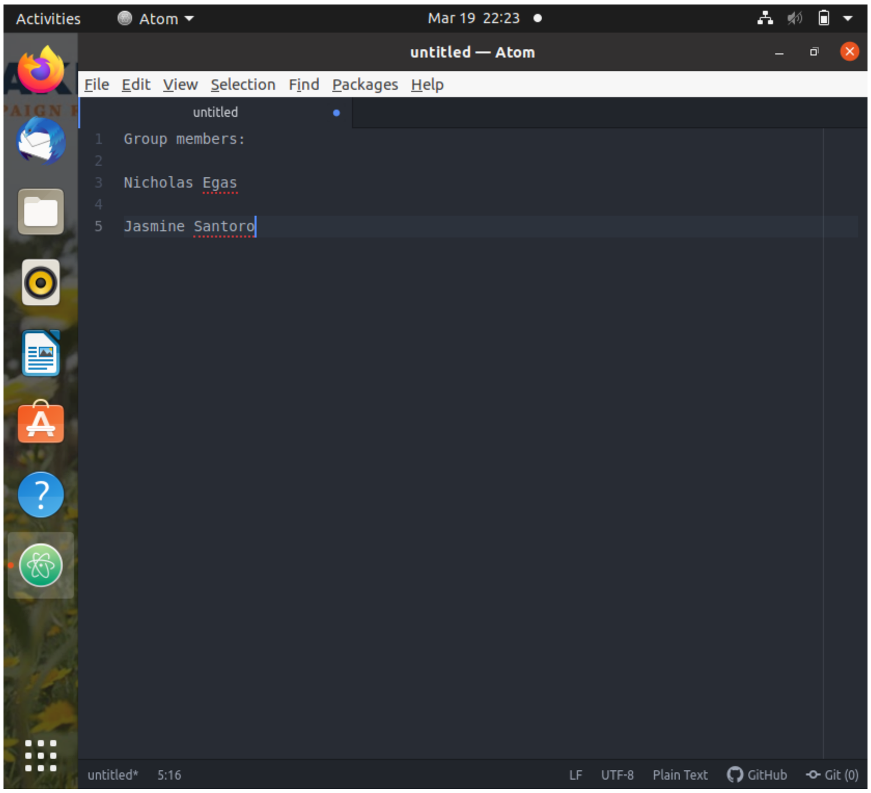
Project 1 Report



Lawnmower Algorithm Pseudocode:

Input: n = 4

```
int m = 0; → number of swaps
For i = 1 to n/2 do
        For j = 1 to 2n do
                if (array[ j ] == dark)
                        swap(array[ j ], array[ j+1])
                        m++;
                endif
        endfor
        For t = 2n down to 1 do
                if(array[ t ] == light)
                        swap(array[ t ], array [ t - 1])
                        m++;
                endif
```

endfor
Return m;

<u>Step Count for Lawnmower Algorithm:</u>

Lawnmower

I/P: $n=4$

m=0; //# of swaps → 1 tu

For i=1 to n/2 $\xrightarrow{}2$ do $\rightarrow 8$ → $S.C. = (n/2) - 1 + 1 = n/2$ tu

For j=1 to 2n do → $S.C = 2n - \cancel{1} + \cancel{1} = 2n$ tu

if (array[j] == dark) → 1 tu

swap(array[j], array[j+1]) → 1 tu

m++; → 1 tu

end if

endfor

For t = 2n $\rightarrow 8$ down to 1 do → $S.C. = \left(\frac{1-2n}{-1}\right) + 1 \rightarrow 2n - \cancel{1} + \cancel{1} = 2n$ tu

if (array[t] == light) → 1 tu

swap(array[t], array[t-1]) → 1 tu

m++; → 1 tu

endif

endfor

Return m; → 0 tu

$$S.C = \underbrace{1 + (n/2)}_{\text{outer loop}} * (\underbrace{2n(1+1+1)}_{\text{first inner loop}} + \underbrace{2n(1+1+1)}_{\text{second inner loop}})$$

$$= 1 + (n/2) * (2n(3) + 2n(3))$$
$$= 1 + (n/2) * (6n + 6n)$$
$$= 1 + (n/2) * 12n$$

$$\frac{\overset{6}{\cancel{12}}n}{1} * \frac{n}{\cancel{2}_1}$$

$$S.C. = 6n^2 + 1$$
$$O(n^2)$$

Proof of Time Complexity for Lawnmower Algorithm:

By Limits Theorem

$$\underset{n\to\infty}{Lt}\ \frac{f(n)}{g(n)} = L \begin{cases} \to L \geq 0 \text{ and a const} \Rightarrow f(n) \in O(g(n)) \\ \to \infty \Rightarrow f(n) \notin O(g(n)) \end{cases}$$

$$6n^2+1 \in O(n^2)$$

By def.

$$\underset{n\to\infty}{Lt}\ \frac{6n^2+1}{n^2} = \underset{n\to\infty}{Lt}\ \overset{6}{\cancel{\frac{6n^2}{n^2}}} + \underset{n\to\infty}{Lt}\ \overset{0}{\cancel{\frac{1}{n^2}}} = 6$$

$$\boxed{\text{By L.T. } 6n^2+1 \in O(n^2)}$$

Alternate Algorithm Pseudocode:

Input: n = 4

```
int m = 0  → number of swaps
For j = 1 to n+1
        For i = 0 to 2n-1
                If array[i] == dark && array[i+1] == light
                        Swap i and i+1
                        m++
                endif
                i+=2
        End for
        For i=1 to 2n-2
                If array[i] == dark && array[i+1] == light
                        Swap i and i+1
                        m++
                i+=2
                Endif
        End for
End for
Return m
```

Step Count for Alternate

$m = 0 \rightarrow \underline{1}$ tu   // # of swaps

for $j = 1$ to $j = n+1 \rightarrow SC = n + 1 - \cancel{1} + \cancel{1} = \underline{n+1}$

for $i = 0$ to $2n-1 \longrightarrow SC = 2n - \cancel{1} - 0 + \cancel{1} = \underline{2n}$

if array[i] == dark && array[i+1] == light $\rightarrow \underline{3}$

swap i and i+1 $\rightarrow \underline{1}$

m++ $\rightarrow \underline{1}$ tu

endif

i += 2 $\rightarrow \underline{1}$

endfor

for $i = 1$ to $2n-2 \longrightarrow SC = (2n-2) - \cancel{1} + \cancel{1} = \underline{2n-2}$ $\rightarrow \underline{3}$ tu

if array[i] == dark && array[i+1] == light

swap i and i+1 $\rightarrow \underline{1}$

m++ $\rightarrow \underline{1}$ tu

endif

i += 2 $\rightarrow \underline{1}$ tu

endfor

endfor

return m $\rightarrow 0$

$$1 + \underbrace{(n+1)}_{\text{outer loop}} * \overbrace{\left(\underbrace{2n(3+1+1+1)}_{\text{first inner loop}} + \underbrace{(2n-2)(3+1+1+1)}_{\text{second for loop}}\right)}^{\text{inner loop}}$$

$$1 + (n+1) * (2n(6) + (2n-2)(6))$$
$$1 + (n+1) * (12n + 12n - 12)$$
$$1 + (n+1) * (24n - 12)$$
$$1 + 24n^2 - 12n + 24n - 12$$
$$1 + 24n^2 + 12n - 12$$
$$SC = 24n^2 + 12n - 11$$
$$O(n^2)$$

Proof of Time Complexity for Alternate

By definition theorem,

$f(n) \in O(g(m))$

$24n^2 + 12n - 11 \in O(n^2)$

by def.

$24n^2 + 12n - 11 \le c \cdot (n^2) \ \forall n \ge n_0$

$c = 36 \qquad n_0 = 1$

$\cancel{24n^2} + 12\cancel{n} - 11 \le \cancel{24n^2} + 12n^{\cancel{2}}, \ n \ge 1$

$12 - 11 \le 12n$

$1 \le 12n, \ n \ge 1$

$\boxed{\text{Hence, } 24n^2 + 12n - 11 \in O(n^2)}$