COMS 511 - Homework 4

Due: February 24 11:59 PM

Guidelines

- When proofs are required, you should make them both clear and rigorous. Do not hand-waive.
- Your assignment needs to be submitted via Canvas.
  - You **must** type your solutions. Please submit a PDF version.
  - Please make sure that the file you submit is not corrupted and that its size is reasonable (e.g., roughly at most 10-11 MB).

    If we cannot open your file, your homework will not be graded.
- The following are examples of activities that are prohibited:
  - Sharing solutions or fragments of solutions (e.g., via email, whiteboard, handwritten, or printed copies).
  - Post solutions or fragments of solutions in a location accessible to others.
  - Using solutions or fragments of solutions provided by other students (including students who had taken the course in the past).
  - Using solutions or solution fragments obtained on the Internet or from solution manuals for textbooks.
  - Using material from textbooks, reference books, or research articles without properly acknowledging and citing the source.
- Concerns about grading should be expressed within one week of returning the homework.
- **No late homework is accepted** with the exception of at most one late submission up to 12 hours late.

Problems

Problem 1. (50 points) Suppose $G = (V, E)$ is a directed graph network with a source vertex s and a sink vertex t, and $c_e = 1$ for any $e \in E$. Given an integer parameter k, your need to delete k edges from G in order to reduce the maximum $s - t$ flow as much as possible. Describe an polynomial-time algorithm to solve this problem. You need to prove the correctness of the algorithm and formally analyze the runtime. (You can consider this task as deleting a subset of E consisting of k edges to make the maximum flow in the new graph as small as possible.)

Problem 2. (50 points) In a certain region, a zoonotic disease has broken out, affecting a group of n individuals who need immediate medical attention. These patients must be taken to hospitals within a 20-minute driving distance from their current location. The region has p hospitals, and a team of paramedics connected by a cell phone network is attempting to determine whether it is feasible to assign each patient to a hospital in a manner that balances the patient load across the hospitals. Specifically, each hospital should receive no more than $\lceil n/p \rceil$ patients. Design a polynomial-time algorithm with that can determine whether this is possible given the information about the peoples' location. You need to prove the correctness of the algorithm and formally analyze the runtime.