COMS 511 - Homework 9

Due: April 14 11:59 PM

Guidelines

- When proofs are required, you should make them both clear and rigorous. Do not hand-waive.
- Your assignment needs to be submitted via Canvas.
  - You **must** type your solutions. Please submit a PDF version.
  - Please make sure that the file you submit is not corrupted and that its size is reasonable (e.g., roughly at most 10-11 MB).

    If we cannot open your file, your homework will not be graded.
- The following are examples of activities that are prohibited:
  - Sharing solutions or fragments of solutions (e.g., via email, whiteboard, handwritten, or printed copies).
  - Post solutions or fragments of solutions in a location accessible to others.
  - Using solutions or fragments of solutions provided by other students (including students who had taken the course in the past).
  - Using solutions or solution fragments obtained on the Internet or from solution manuals for textbooks.
  - Using material from textbooks, reference books, or research articles without properly acknowledging and citing the source.
- Concerns about grading should be expressed within one week of returning the homework.
- **No late homework is accepted** with the exception of at most one late submission up to 12 hours late.

Problems

Problem 1. (50 points)

(1) Prove that $E \in P$ if $E \leq_p \{1\}^* \circ \{0\}^*$. ($\circ$ means the concatenation of the strings.)

(2) Prove that the reduction relation is transitive on languages. That is, if $L_1 \leq_p L_2$ and $L_2 \leq_p L_3$, then $L_1 \leq_p L_3$.

(3) Suppose we have an NP-complete problem A. Prove or disprove that the complement of the problem A is co-NP-complete.

(4) Prove that if NP $\neq$ co-NP, then P $\neq$ NP.

Problem 2. (**EXTRA CREDIT**) (50 points) Let $U = \{a_1, ..., a_n\}$. We consider the following problem A: The inputs to the problem A is a family of $\{S_1, ..., S_m\}$, where $S_i \subseteq U$ for each $i \in \{1, ..., m\}$, and a non-negative integer k. We will search for a set $H \subseteq U$ of size k that intersects every S_i, if such a set H exists.

(1) Prove that the problem A is in NP.

(2) Prove that the problem A is NP-complete.

Problem 3. (**EXTRA CREDIT**) (50 points) Prove the following problems.

(1) If every literal in a 3-SAT instance appears at most once, then the problem is in P.

(2) An *independent set* of a graph $G = (V, E)$ is a subset of vertices $V' \subseteq V$ such that each edge in E is incident on at most one vertex in V'. The *Independent Set Problem* is to find a maximum-size independent set in G. Show that the *Independent Set Problem* remains NP-complete in the special case when all vertices in a graph have a degree of at most 4.

(3) A *dragon* is a graph $G = (V, E)$ where $|V| = 2n$, n of the vertices form a clique, and the remaining n nodes are connected in a tail that consists of a path joined to one of the vertices of the clique. Given a graph G and $r \in \mathbb{N}$, the *Dragon Problem* asks for a subgraph that is a dragon and contains $2r$ vertices. Prove that the *Dragon Problem* is NP-complete by showing $CLIQUE \leq_p DRAGON$.