COMS 511 - Homework 1

Due: February 3 11:59 PM

Guidelines

- When proofs are required, you should make them both clear and rigorous. Do not hand-waive.
- Your assignment needs to be submitted via Canvas.
  - You **must** type your solutions. Please submit a PDF version.
  - Please make sure that the file you submit is not corrupted and that its size is reasonable (e.g., roughly at most 10-11 MB).

    If we cannot open your file, your homework will not be graded.
- The following are examples of activities that are prohibited:
  - Sharing solutions or fragments of solutions (e.g., via email, whiteboard, handwritten, or printed copies).
  - Post solutions or fragments of solutions in a location accessible to others.
  - Using solutions or fragments of solutions provided by other students (including students who had taken the course in the past).
  - Using solutions or solution fragments obtained on the Internet or from solution manuals for textbooks.
  - Using material from textbooks, reference books, or research articles without properly acknowledging and citing the source.
- Concerns about grading should be expressed within one week of returning the homework.
- **No late homework is accepted** with the exception of at most one late submission up to 12 hours late.

Problems

Problem 1. (50 points) Asymptotic notation.

Question (1). (30 points) In each of the following situations, write **YES** or **NO** in the appropriate box, to indicate whether $f(n)$ is $O, o, \Omega, \Theta, \theta$ of $g(n)$. Note that $f(n)$ and $g(n)$ are functions that map positive integers to positive real numbers.
We define $f(n) \in \theta(g(n))$ *if and only if* for any real constant $c > 0$, there exist integer constants $n_0 > 0$ and $n'_0 > 0$ such that the following conditions are satisfied.

(1) $0 \leq f(n) < c \cdot g(n)$ for every integer $n \geq n_0$
(2) $0 \leq c \cdot g(n') < f(n')$ for every integer $n' \geq n'_0$.

$f(n)$	$g(n)$	o	O	ω	Ω	θ	Θ
$1/n + \sqrt{n}/2$	$\log(n)$						
2^{2n}	$3 \cdot 2^n$						
$n + n \log n$	$\sqrt{n} + n^2$						
$n \log(2/n)$	n^2						
$n^3 + n^2 \log\lceil n \rceil$	$n^3 + 5n^2\sqrt{n}$						

Question (2). (20 points) Prove or disprove the following:

(1) $7n + 8 = o(n^2)$
(2) $n^2/2 = O(n)$

Problem 2. (50 points) Suppose we modify the input of the comparison sort problem such that we assume that some elements of the input array are "nearly" at their correct position, as defined in the following.

Input: A sequence $A := (a_1, \ldots, a_n)$ of positive integers such that

- each element a_i of A, where i is divisible by 10, is in the sorted list of A at the correct position;
- each element a_i of A, where i is divisible by 7, is in the sorted list of A either (i) at the correct position or (ii) no more than one position away from its correct position.

For example, given a sequence $A := (a_1, \ldots, a_{35})$, the position of the element a_{30} in the sorted list of A is 30, and the possible positions of the element a_7 in the sorted list would be $6, 7, 8$.

Show that the $\Omega(n \log n)$ lower bound for the general comparison sort problem still holds for the constrained input described above.