COMS 511 - Homework 3

Due: February 17 11:59 PM

Guidelines

- When proofs are required, you should make them both clear and rigorous. Do not hand-waive.
- Your assignment needs to be submitted via Canvas.
  - You **must** type your solutions. Please submit a PDF version.
  - Please make sure that the file you submit is not corrupted and that its size is reasonable (e.g., roughly at most 10-11 MB).

    If we cannot open your file, your homework will not be graded.
- The following are examples of activities that are prohibited:
  - Sharing solutions or fragments of solutions (e.g., via email, whiteboard, handwritten, or printed copies).
  - Post solutions or fragments of solutions in a location accessible to others.
  - Using solutions or fragments of solutions provided by other students (including students who had taken the course in the past).
  - Using solutions or solution fragments obtained on the Internet or from solution manuals for textbooks.
  - Using material from textbooks, reference books, or research articles without properly acknowledging and citing the source.
- Concerns about grading should be expressed within one week of returning the homework.
- **No late homework is accepted** with the exception of at most one late submission up to 12 hours late.

Problems

Problem 1. (50 points)
Let $G = (V, E)$ be a flow network with source s and sink t. Suppose that G has a modified capacity function $c\colon V \times V \to \mathbb{R} \cup \{\infty\}$. Note that c can adopt negative values, which is different from our original definition. In such a network, a feasible flow need not exist.

Prove that if there is a feasible flow f in G, then there is a maximal flow with a value equal to that of the minimal cut. (This problem is motivated by a question in class about whether capacities can be negative.)

(One natural interpretation of a "negative" capacity is that this is a way to enforce a mandatory minimum flow. Suppose there is an edge between vertex x and y such that $c_{yx} = -3$. It means that the total flow from y to x must be at most -3; in other words, the total flow from x to y is at least 3. It is natural to consider a flow network with negative capacity; in plumbing, for example, you often want to ensure a minimal amount of flow through your pipes to stop them from bursting when it drops below freezing in the winter.)