COMS 511 - Homework 2

Due: February 10 11:59 PM

GUIDELINES

- When proofs are required, you should make them both clear and rigorous. Do not hand-waive.
- Your assignment needs to be submitted via Canvas.
  - You **must** type your solutions. Please submit a PDF version.
  - Please make sure that the file you submit is not corrupted and that its size is reasonable (e.g., roughly at most 10-11 MB).

    If we cannot open your file, your homework will not be graded.
- The following are examples of activities that are prohibited:
  - Sharing solutions or fragments of solutions (e.g., via email, whiteboard, handwritten, or printed copies).
  - Post solutions or fragments of solutions in a location accessible to others.
  - Using solutions or fragments of solutions provided by other students (including students who had taken the course in the past).
  - Using solutions or solution fragments obtained on the Internet or from solution manuals for textbooks.
  - Using material from textbooks, reference books, or research articles without properly acknowledging and citing the source.
- Concerns about grading should be expressed within one week of returning the homework.
- **No late homework is accepted** with the exception of at most one late submission up to 12 hours late.

PROBLEMS

Problem 1. (50 points) Here are two possible algorithms for computing MST. The input of these algorithms is a connected and undirected graph $G = (V, E)$ that is edge weighted by ω. The output is a set of edges $T \subseteq E$. For each algorithm, you need to determine whether the possible MST algorithm *works*, i.e., returns an MST for G. If you determine it works, then show the correctness of the algorithm. Otherwise, show that the algorithm does not compute a MST.

Algorithm 1 POSSIBLE-MST-1(G, w)

sort the edges in nonincreasing order based on their weight
$T \leftarrow E$
for each edge e, obtained in nonincreasing oreder by weight **do**
    if $T - \{e\}$ is a connected graph **then**
        $T \leftarrow T - \{e\}$
return T

Algorithm 2 POSSIBLE-MST-2(G, w)

$T \leftarrow \emptyset$
for each edge e, obtained in arbitrary order **do**
    if there is no cycle in $T \cup \{e\}$ **then**
        $T \leftarrow T \cup \{e\}$
return T

Problem 2. (50 points) Consider two stacks A and B manipulated using the following operations (n is the size of A and m the size of B):

- $PushA(x)$: Push element x on stack A.
- $PushB(x)$: Push element x on stack B.
- $MultiPopA(k)$: Pop $min\{k, n\}$ elements from A.
- $MultiPopB(k)$: Pop $min\{k, m\}$ elements from B.
- $Transfer(k)$: Repeatedly pop an element from A and push it on B, until either k elements have been moved or A is empty.

Assume that A and B are implemented using doubly-linked lists such that $PushA$ and $PushB$, as well as a single pop from A or B, can be performed in $O(1)$ time worst-case.

(a) What is the worst-case running time of the operations $MultiPopA$, $MultiPopB$ and $Transfer$?

(b) Define a potential function $\Phi(n, m)$ and use it to prove that the operations have amortized running time $O(1)$.