

COMS 511 - Homework 7

Due: March 31 11:59 PM

GUIDELINES

- When proofs are required, you should make them both clear and rigorous. Do not hand-waive.
- Your assignment needs to be submitted via Canvas.
 - You **must** type your solutions. Please submit a PDF version.
 - Please make sure that the file you submit is not corrupted and that its size is reasonable (e.g., roughly at most 10-11 MB).
If we cannot open your file, your homework will not be graded.
- The following are examples of activities that are prohibited:
 - Sharing solutions or fragments of solutions (e.g., via email, whiteboard, handwritten, or printed copies).
 - Post solutions or fragments of solutions in a location accessible to others.
 - Using solutions or fragments of solutions provided by other students (including students who had taken the course in the past).
 - Using solutions or solution fragments obtained on the Internet or from solution manuals for textbooks.
 - Using material from textbooks, reference books, or research articles without properly acknowledging and citing the source.
- Concerns about grading should be expressed within one week of returning the homework.
- **No late homework is accepted** with the exception of at most one late submission up to 12 hours late.

PROBLEMS

Problem 1. (50 points) Show that the class P is closed under (1) union, (2) intersection, (3) concatenation, (4) complement, and (5) Kleene star. That is, if $L_1, L_2 \in P$, then $L_1 \cup L_2 \in P$, $L_1 \cap L_2 \in P$, $L_1 L_2 \in P$, $\overline{L_1} \in P$, and $L_1^* \in P$.

Problem 2. (50 points) Let

$RECTANGLE := \{\langle R, k \rangle \mid R \text{ is a set of } n \text{ axis-aligned rectangles in the plane, where } k \leq n \text{ of them share a common point}\}$

Solve the following questions:

(1) Prove $RECTANGLE \leq_p CLIQUE$.

(2) Suppose $P \neq NP$. Decide whether from $RECTANGLE \leq_p CLIQUE$ follows that $CLIQUE$ is NP-hard? Provide rigorous proof for your answer.

Problem 3. (EXTRA CREDIT) (50 points) Let

$SET-SPLITTING := \{\langle S, C \rangle \mid S \text{ is a finite set and } C = \{C_1, \dots, C_k\} \text{ is a collection of subsets of } S, \text{ for some } k > 0, \text{ such that elements of } S \text{ can be colored red or blue so that no } C_i \text{ has all its elements colored with the same color.}\}$

Show that $SET-SPLITTING$ is NP-complete.