COMS 511 - Homework 8

Due: April 7 11:59 PM

Guidelines

- When proofs are required, you should make them both clear and rigorous. Do not hand-waive.
- Your assignment needs to be submitted via Canvas.
  - You **must** type your solutions. Please submit a PDF version.
  - Please make sure that the file you submit is not corrupted and that its size is reasonable (e.g., roughly at most 10-11 MB).

    If we cannot open your file, your homework will not be graded.
- The following are examples of activities that are prohibited:
  - Sharing solutions or fragments of solutions (e.g., via email, whiteboard, handwritten, or printed copies).
  - Post solutions or fragments of solutions in a location accessible to others.
  - Using solutions or fragments of solutions provided by other students (including students who had taken the course in the past).
  - Using solutions or solution fragments obtained on the Internet or from solution manuals for textbooks.
  - Using material from textbooks, reference books, or research articles without properly acknowledging and citing the source.
- Concerns about grading should be expressed within one week of returning the homework.
- **No late homework is accepted** with the exception of at most one late submission up to 12 hours late.

Problems

Problem 1. (**EXTRA CREDIT**) (50 points) The *Patience Game* consists of an $n \times m$ grid of squares, where each square may be empty, occupied by a red object, or occupied by a blue object. The game is *solvable* if it is possible to remove some of the given objects such that the remaining objects satisfy the following two conditions.

(i) Every row contains at least one stone.
(ii) No column contains stones of both colors.

To analyze the time complexity of this game we define the language

$$\text{PatienceGame} := \{\langle M \rangle \mid M \text{ is a solvable game}\}.$$

Prove that it is NP-hard to determine, given a Patience Game with an initial configuration of red and blue objects, whether this game is solvable.