COMS 511 - Homework 10

Due: April 21 11:59 PM

Guidelines

- When proofs are required, you should make them both clear and rigorous. Do not hand-waive.
- Your assignment needs to be submitted via Canvas.
  - You **must** type your solutions. Please submit a PDF version.
  - Please make sure that the file you submit is not corrupted and that its size is reasonable (e.g., roughly at most 10-11 MB).

    If we cannot open your file, your homework will not be graded.
- The following are examples of activities that are prohibited:
  - Sharing solutions or fragments of solutions (e.g., via email, whiteboard, handwritten, or printed copies).
  - Post solutions or fragments of solutions in a location accessible to others.
  - Using solutions or fragments of solutions provided by other students (including students who had taken the course in the past).
  - Using solutions or solution fragments obtained on the Internet or from solution manuals for textbooks.
  - Using material from textbooks, reference books, or research articles without properly acknowledging and citing the source.
- Concerns about grading should be expressed within one week of returning the homework.
- **No late homework is accepted** with the exception of at most one late submission up to 12 hours late.

Problem

Problem 1. (50 points)

A set of vertices S in a graph G is called *almost-independent-set* if by removing at most one edge to G, S becomes an independent set. Define

$$\text{Almost-IS} = \{\langle G, k\rangle | G \text{ is a graph on } n \text{ vertices that has an almost-independent set of size } k\}.$$

Show that Almost-IS is NP-complete. (Hint: reduce from Independent-Set.)

Problem 2. (**EXTRA CREDIT**) (50 points)

Charon needs to ferry n recently deceased people across the river Acheron into Hades. Certain pairs of these people are sworn enemies, who cannot be together on either side of the river unless Charon is also present. (If two enemies are left alone, one will steal the obol from the other's mouth, leaving them to wander the banks of the Acheron as a ghost for all eternity. Let's just say this is a Very Bad Thing.) The ferry can hold at most k passengers at a time, including Charon, and only Charon can pilot the ferry.

Prove that it is NP-hard to decide whether Charon can ferry all n people across the Acheron unharmed. The input for Charon's problem consists of the integers k and n and an n-vertex graph G describing the pairs of enemies. The output is either True or False.

Problem 3. (**EXTRA CREDIT**) (50 points)

The *string reversal problem* is defined as follows: given a permutation π of the set $\{1, \ldots, n\}$, find the minimum number of reversal-operations that transform π into the identity permutation. This minimum number is defined as the distance $d(\pi)$.

The *reversal-operation* $w(i, j)$ for a permutation $\pi = \pi_1 \dots \pi_{i-1}\pi_i\pi_{i+1} \dots \pi_{j-1}\pi_j\pi_{j+1} \dots \pi_n$ is defined as

$$\pi \; w(i, j) := \pi_1 \dots \pi_{i-1}\pi_j\pi_{j-1} \dots \pi_{i+1}\pi_i\pi_{j+1} \dots \pi_n, \text{where } 0 \leq i \leq j \leq n \text{ and } n \in \mathbb{N}.$$

Describe a polynomial time 2-approximation algorithm for the problem.