1.

KKT for SVM

Equation A: $\mathbf{w} = \sum_{k=1}^{K} \lambda_k y_k \mathbf{x_k}$

(w1 w2 w3) = λ1(0.5 0.25 0.125)+ λ2(0.4 0.15 0.225)- λ3(0.3 0.75 0.325)- λ4(0.2 0.65 0.425)

λ1 = 4.5, λ2 = 0, λ3 = 1.5, λ4 = 0

w1 = 4.5*0.5-1.5*0.3 = 1.8

w2 = 4.5*0.25-1.5*0.75 = 0

w3 = 4.5*0.125-1.5*0.325 = 0.075

w1 = 1.8, w2=0, w3 = 0.075

Equation B: $\frac{\partial L}{\partial w_b} = \sum_{k=1}^{K} \lambda_k y_k = 0$

λ1*1+ λ2*1+ λ3*(-1)+ λ4*(-1) ≠ 0 (our λ not satisfy KKT conditions)

Equation C:

λk >0 (satisfy KKT conditions)

Equation D (support gutters): $\mathbf{w}^T \mathbf{x_k} + w_b = \pm 1.$

λ1 [+1 ((w1 w2 w3) (0.5 0.25 0.125) + wb)-1]=0

λ2 [+1 ((w1 w2 w3) (0.4 0.15 0.225) + wb)-1]=0

λ3 [-1 ((w1 w2 w3) (0.3 0.75 0.325) + wb)-1]=0

λ4 [-1 ((w1 w2 w3) (0.2 0.65 0.425)+ wb)-1]=0

Hence: prediction $w^T x + W_b$

w1 = 1.8, w2=0, w3 = 0.075

$w^T x + w_b$ = 1.8*1 + 0*1 + 0.075*0 + 1 = **2.8 >0 (Class 1)**

2. **Gutters from Equation D**

$w^T xk + wb = \pm 1$

$(w1\ w2\ w3)^T x_k + 1 = 1$

$(w1\ w2\ w3)^T x_k + 1 = -1$

3. w1 = 1.8, w2=0, w3 = 0.075

prediction $w^T x + w_b$ =

Sample 1: $(1.8\ 0\ 0.075)^T (0.5\ 0.25\ 0.125) + w_b = 0.909375+1 = 1.909375$

Sample 2: $(1.8\ 0\ 0.075)^T (0.4\ 0.15\ 0.225) + w_b = 0.736875+1 = 1.736875$

Sample 3: $(1.8\ 0\ 0.075)^T (0.3\ 0.75\ 0.325) + w_b = 0.564375+1 = 1.564375$

Sample 4: $(1.8\ 0\ 0.075)^T (0.2\ 0.65\ 0.425) + w_b = 0.391875+1 = 1.391875$

A sample falls into the margin if it is between the two gutters

$-1 < w^T x + w_b < 1$

All of sample above do not fall into the margin

4. prediction per SVM $w^T x + wb$

If $w^T x + wb > 0$ then x belong to Class 1.

If $w^T x + wb < 0$ then x belong to Class 2.

The equation $w^T x + wb = 0$ is the hyperplane.

If we use hard margin SVM, $w^T x + wb > +1$ then x belong to Class 1.

Multiply the label y : $+1*(w^T x + wb)> +1$

But if it is misclassified, then: $+1*(w^T x + wb) <-1$

If an value <-1, it will less than 0 and also 1

So condition **2,4,6** will hold and vice versa, 1,3,5 will hold if sample in the Class 2 misclassified with Class 1.