COMS 331: Theory of Computation
Summer 2023

Homework Assignment 2

Due: 11:59PM, June 2 (Friday).

Problem 1 (60 points)

Design regular expressions for four languages over $\{0, 1\}$ with the following patterns:

1. doesn't contain the substring 110
2. contains at least two 0s but not consecutive 0s
3. has at least 3 characters, and the third character is 0
4. consists of alternating 0s and 1s (i.e., ϵ, 1, 0, 01, 10, 010, 0101,....)

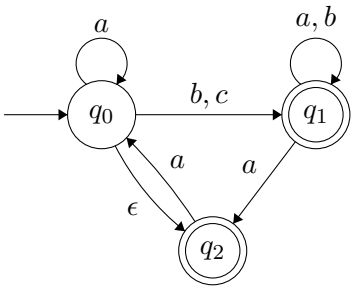
(Note that the answers are not unique. A language can be generated by different regular expressions.)

Problem 2 (20 points)

Convert the regular expression $0(0 + 1)^*1$ to an NFA.

Problem 3 (20 points)

Convert the following NFA to an equivalent regular expression.



Problem 4 (20 points)

Use the pumping lemma to prove that the language

$$L = \{a^p \mid p \text{ is a prime number}\}$$

is not regular.
(Recall that a *prime number* is a whole number greater than 1 whose only factors are 1 and itself. To avoid confusion, keep in mind that the smallest prime number is 2, i.e., 1 is not considered a prime number.)

Problem 5 (80 points)

Prove or disprove that the following languages are regular:

1. $L_1 = \{a^n \mid n \text{ is even or prime}\}$.
2. $L_2 = \{a^n b^m \mid n \text{ is even or } m \text{ is prime}\}$.
3. $L_3 = \{a^n b^m \mid n \text{ is even and } m \text{ is prime}\}$.
4. $L_4 = \{a^n a^m \mid n \text{ is even or } m \text{ is prime}\}$.

(Disproving a language is regular means you need to prove a language is not regular. Similar to the previous question, we consider the smallest prime number is 2, i.e., 1 is not considered a prime number.)