COMS 331: Theory of Computation
Summer 2023

Homework Assignment 6

Due: 11:59PM, July 1 (Saturday).

Problem 1 (30 points)

Prove

$$L = \{\langle M \rangle \mid M \text{ is a TM, } L(M) = \Sigma^* \}$$

is **NOT** Turing decidable.

Problem 2 (30 points)

Prove

$$L = \{\langle M_1, M_2, M_3 \rangle \mid M_1, M_2, M_3 \text{ are TMs, } L(M_1) = L(M_2) \cup L(M_3)\}$$

is **NOT** Turing acceptable.

Problem 3 [EXTRA CREDIT] (20 points)

Prove that TAL is closed under (1) concatenation, and (2) Kleene star.

Problem 4 [EXTRA CREDIT] (30 points)

Prove that the set of Turing-decidable languages is closed under reversal, i.e., if L is Turing-decidable, then

$$L^{\mathcal{R}} = \{x \in \Sigma^* \mid x^{\mathcal{R}} \in L, \text{ where } x^{\mathcal{R}} \text{ is the reverse of x}\}$$

is Turing decidable.

Problem 5 [EXTRA CREDIT] (30 points)

Prove

$$L = \{\langle M \rangle \mid M \text{ is a TM, } M \text{ accepts } \langle M \rangle\}$$

is **NOT** Turing decidable using diagonalization.

Problem 6 [EXTRA CREDIT] (30 points)

Prove that

$$L = \{\langle M, D \rangle \mid M \text{ is a TM, } D \text{ is a DFA, and } L(M) = L(D)\}$$

is not co-recognizable. That is, prove that $\overline{L}$ is **NOT** recognizable.

Problem 7 [EXTRA CREDIT] (30 points)

Consider the problem of determining whether a two-tape Turing machine ever writes a non-blank symbol on its second tape, i.e.

$$N = \{\langle M, w\rangle \mid M \text{ is a two-tape Turing machine which writes a non-blank symbol onto its second tape when it runs on } w\}.$$

Show that N is **NOT** decidable.
(Hint: Use a reduction from A_{TM}).