COMS 331: Theory of Computation
Summer 2023

Additional Assignment

Problem 1 (30 points)
Draw a minimized DFA that accepts the language L of the natural numbers that are multiple of 5, written in base 10. For example, 0 and 435 belong to L, but 00, 0435, and 5328 do not. Please explicitly draw the trap state, if needed, in your DFA.

Problem 2 (30 points)
Draw a minimized DFA that recognizes the language

$$L = \{w \in \{0,1\}^* \mid w \text{ ends with 000 or contains 111, but not both}\}.$$

Problem 3 (30 points)
Build a three-state NFA that accepts the language $(abc + bc + dc)^*$. Note: arcs labels must be one or more single symbols of the alphabet or ϵ, but not sequences of symbols (i.e., "abc" is not a legal label).

Problem 4 (35 points)
Consider the n-bit binary representation of a natural number x:

$$\text{the binary representation of } x \text{ is } (x_{n-1}x_{n-2}...x_1x_0)_2 \Leftrightarrow x = \sum_{i=0}^{n-1} x_i 2^i$$

where each bit x_i is a binary digit, either 0 or 1. For example, $(00000101)_2$ is the 8-bit binary representation of the number 5, since $0 \cdot 2^7 + 0 \cdot 2^6 + 0 \cdot 2^5 + 0 \cdot 2^4 + 0 \cdot 2^3 + 1 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0 = 4 + 1 = 5$. This is the format normally employed by digital computers to store nonnegative integers.
Consider the language

$$L = \{a_{n-1}b_{n-1}...a_0b_0 \mid n \text{ is a nonnegtive integer} \\ \wedge (a_{n-1}...a_0)_2 > (b_{n-1}...b_0)_2\}$$

For example, since $5 = (000101)_2$, $3 = (000011)_2$, and $5 > 3$, then $000000100111 \in L$. Design a minimized DFA that accepts L.

Problem 5 (35 points)
Prove or disprove: every finite language is recognized by some FA.

Problem 6 (100 points)
Prove or disprove each of the following three statements.

1. If L_1 is regular and L_2 is nonregular, $L_1 \cup L_2$ must be nonregular.
2. If L_1 is regular and L_2 is nonregular, $L_1 \cap L_2$ must be nonregular.
3. If L_1 is regular and L_2 is nonregular, $L_1 \cdot L_2$ must be nonregular.
4. If L_1 and L_2 are nonregular, $L1 \cup L_2$ must be nonregular.