COMS 331: Theory of Computation
Summer 2023

Homework Assignment 1

Due: 11:59PM, May 26 (Friday).

Problem 1 (20 points)

Draw a DFA that recognizes the language $L = \{a^i b^j \mid (i + j) \bmod 3 = 0\}$. Explicitly draw the trap state, if needed, in the DFA.
(If a transition goes to a state from which it can never escape, such a state is called a *trap state*.)

Problem 2 (30 points)

Consider the n-bit binary representation of a natural number x:

$$\text{the binary representation of } x \text{ is } (x_{n-1}x_{n-2}...x_1x_0)_2 \Leftrightarrow x = \sum_{i=0}^{n-1} x_i 2^i$$

where each bit x_i is a binary digit, either 0 or 1. For example, $(00000101)_2$ is the 8-bit binary representation of the number 5, since $0 \cdot 2^7 + 0 \cdot 2^6 + 0 \cdot 2^5 + 0 \cdot 2^4 + 0 \cdot 2^3 + 1 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0 = 4 + 1 = 5$. This is the format normally employed by digital computers to store nonnegative integers.
Consider the language

$$\begin{aligned} L = \{a_{n-1}b_{n-1}c_{n-1}...a_0b_0c_0 \mid\ & n \text{ is a nonnegtive integer} \\ & \wedge \forall i, 0 \leq i \leq n, a_i \in \{0,1\}, b_i \in \{0,1\}, c_i \in \{0,1\} \\ & \wedge (a_{n-1}...a_0)_2 + (b_{n-1}...b_0)_2 = (c_{n-1}...c_0)_2\} \end{aligned}$$

For example, since $5 + 3 = 8$, $5 = (000101)_2$, $3 = (000011)_2$, and $8 = (001000)_2$, then

$$000\ 000\ 001\ 100\ 010\ 110 \in L$$

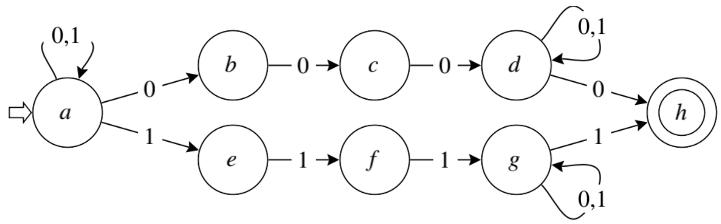
(the string is spaced every three digits for readability's sake only).
Design a DFA that accepts L.

Problem 3 (20 points)

Let $\Sigma = \{a, b, c\}$ and let $L = \{w \in \Sigma^* \mid \text{some character in } \Sigma \text{ appears at most twice in } w\}$. Design an NFA for L.

Problem 4 (50 points)

Consider the following NFA N:



1. Describe in English, as succinctly as you can, the essential characteristics of the language accepted by this NFA.
2. Using the methods described in class, derive an equivalent (non-minimized) DFA M.
3. Using the algorithm described in class, minimize M and obtain the minimal DFA M'.

Problem 5 (60 points)

Let L_1 and L_2 be regular languages. Show that the following languages are also regular:

1. The difference $L_1 \backslash L_2 = \{w \mid w \in L_1 \text{ and } x \notin L_2\}$
2. The symmetric difference $L_1 \bigoplus L_2 = (L_1 \backslash L_2) \cup (L_2 \backslash L_1)$.
3. The reversal $L_1^{\mathcal{R}} = \{w^{\mathcal{R}} \mid w \in L_1\}$.

Problem 6 (20 points)

Suppose that language A is recognized by an NFA N, and language B is the collection of strings not accepted by some DFA M. Prove that AB is a regular language.