COMS 331: Theory of Computation
Summer 2023

Homework Assignment 5

Due: 11:59PM, June 23 (Friday).

Problem 1 (30 points)
Design a TM that takes as input a number i and adds 1 to it in binary. The initial tape contains $\vdash$ followed by i (in binary). The tape head initially scanning $\vdash$ in the initial state s. The TM should halt with $i+1$, in binary, on its tape, in state f. If necessary, you may overwrite the $\vdash$ symbol.
(E.g., $\vdash 110$ turns to $\vdash 111$, $\vdash 11$ turns to 100)

Problem 2 (35 points)
Give a Turing machine with input alphabet $\{a, b\}$ that on input w halts with w^R written on its tape.
(E.g., $\vdash abbb$ turns to $\vdash bbba$)

Problem 3 (35 points)
Consider a variant of Turing machine called *2-cell Turing machine* which has a finite set of states, a semi-infinite left-ended tape (just as a standard Turing machine), a read/write tape head that has access to 2 tape cells, i.e., in a single step, the head can simultaneously read the tape cell under it and the tape cell to the right of it, and can write to those two cells. Show how you can simulate these *2-cell Turing machines* with standard Turing machines.

Problem 4 (30 points)
Prove the language

$$L = \{\langle M \rangle \mid M \text{ is a TM and } |L(M)| > 5\}$$

is Turing acceptable.
(Hint: To determine the size of $|L(M)|$, your machine need to check M on each possible string over Σ.)

Problem 5 (35 points)
Prove the language

$$L = \{\langle M, w \rangle 01^n 0 \mid M\text{'s head uses at most } n \text{ tape cells when running on input } w\}$$

is Turing decidable.
(Hint: M may loop on w while still using at most n tape cells. How can your machine detect this

situation and still halt to make a decision? Remember, M has a finite number of states($|Q|$ states) and a finite alphabet($|\Gamma|$ symbols). How many distinct configurations does there exist if M can use at most n tape cells?)

Problem 6 [EXTRA CREDIT] (35 points)

Consider a Turing machine model that uses a 2-dimensional tape, corresponding to the upper right quadrant of the plane. The head of such a Turing machine could move to the right, left, up or down. Show that the 2D-TM is equivalent to the standard TM.