COMS 331: Theory of Computation
Summer 2023

Homework Assignment 4

Due: 11:59PM, June 16 (Friday).

Problem 1 (25 points)
Define a NPDA for the language $L = \{a^n b^m | m, n \in N, n \geq 3, m > n\}$

Problem 2 (25 points)
Define a NPDA for the language $L = \{uv \in \{a, b\}^* || u| = |v| \text{ and } u \neq v^R\}$.
Another way to interpret L is that it is the set of all even length strings that are not a palindrome.

Problem 3 (25 points)
Consider the language

$$L = \{w \in a, b^* \mid \text{the longest run of } a\text{'s in } w \text{ is longer than any run of } b\text{'s in } w\}.$$

For example, $abbbbaabbbaaaaaa \in L$ because the longest run of b's in it has length four, while the longest run of a's has length six. Prove that L is not context-free.

Problem 4 (25 points)
Consider the language

$$L = \{xyy^R \in \{a, b\}^+ \mid |x| = |y|\}.$$

Is it regular? Context-free? Not even context-free? Justify your answer.

Problem 5 [EXTRA CREDIT] (60 points)
Prove or disprove the following statements:

1. If L_1 is context-free and L_2 is regular, $L_1 \backslash L_2$ is context-free.
2. If L_1 is regular and L_2 is context-free, $L_1 \backslash L_2$ is regular.
3. If L_1 is regular and L_2 is context-free, $L_1 \backslash L_2$ is context-free.

Problem 6 [EXTRA CREDIT] (30 points)

Let A and B be languages. We define

$$A\blacklozenge B = \{xy \mid x \in A, y \in B, |x| = |y|\}.$$

Prove that if A and B are both regular languages, then $A\blacklozenge B$ is a context-free language.
(Hint: Construct a NPDA for $A\blacklozenge B$ using the machines for A and B.)