Q no 1

```
% Constants
P=zeros(1,step+1);
P(1) = 1;
Q = 0.1;
R =0.1;
U = 0;
A = 1;
B = 1;
H = 1;
step = 10;
x=zeros(1,step+1);
K=zeros(1,step+1);
z = [0, 0.5, 0.52, 0.49, 0.53, 0.45, 0.54, 0.48, 0.47, 0.46, 0.53];

for k=1:step
    xest = x(k) + B*U;                          % Project state ahead
    Pest = A*P(k)*transpose(A) + Q;             % Project error covariance ahead
    K(k+1) = Pest*transpose(H)*inv(H*Pest*transpose(H)+R); % Compute Kalman Gain
    x(k+1) = xest + K(k+1)*(z(k+1)-H*xest); % Update the estimate zk
    P(k+1) = (eye(size(K(k+1)*H,1))-K(k+1)*H)*Pest; % Update error covariance
end
```

Command Window

```
>> z(2:11)

ans =

    0.5000    0.5200    0.4900    0.5300    0.4500    0.5400    0.4800    0.4700    0.4600    0.5300

>> x(2:11)

ans =

    0.4583    0.4989    0.4933    0.5160    0.4752    0.5153    0.4935    0.4790    0.4672    0.5060
```

Q no 2

Question no: 2

$x_k = x_{k-1} + w_{k-1}$

$y_k = \cancel{y_k}\, x_k + v_k$

$Q = 0.001$, $R = 0.1$, $x_0 = 0$, $P_0 = 1000$

y_1	y_2	y_3	y_4
0.9	0.8	1.1	1.0

x_0	x_1	x_2	x_3	x_4
0	0.8999	0.8497	0.9345	0.9514

Iteration 1:

$\hat{x}_1 = x_0 + w_0 = 0$

$\hat{P}_1 = P_0 + Q = 1000 + 0.001$
$= 1000.001$

$K_1 = \hat{P}_1(\hat{P}_1 + R)^{-1} = 1000.001 \times \frac{1}{1000.101}$
$= 0.9999$

$x_1 = \hat{x}_1 + K_k(y_1 - \hat{x}_1) = 0.9999 \times (0.9 - 0)$
$= 0.8999$

$P_1 = \hat{P}_1 - \hat{P}_1 \times K_k = (1 - 0.9999) \times 1000.001$
$= 0.1$

After iteration 1, $x_1 = 0.8999$, $K_1 = 0.9999$

Iteration 2

$\hat{x}_2 = 0.8999$

$\hat{P}_2 = 0.1 + 0.001 = 0.101$

$K_2 = 0.101 \times (0.201)^{-1} = 0.5025$

$x_2 = 0.8999 + 0.5025(0.8 - 0.8999)$
$= 0.8497$

$P_2 = (1 - 0.5025) \times 0.101 = 0.0502$

$x_2 = 0.8497$, $K_2 = 0.5025$

Iteration 3

$\hat{x}_3 = 0.8497$

$\hat{P}_3 = 0.0502 + 0.001$
$= 0.0512$

$K_3 = 0.0512 \times (0.1512)^{-1} = 0.3386$

$x_3 = 0.8497 + 0.3386 \times (1.1 - 0.8497) = 0.9345$

$P_3 = (1 - 0.3386) \times 0.0512 = 0.0339$

$x_3 = 0.9345$, $P_3 = 0.0339$, $K_3 = 0.3386$

Iteration 4

$\hat{x}_4 = 0.9345$

$\hat{P}_4 = 0.0339 + 0.001$
$= 0.0349$

$K_4 = 0.0349 \times (0.1349)^{-1} = 0.2587$

$x_4 = 0.9345 + 0.2587(1 - 0.9345) = 0.9514$

$P_4 = (1 - 0.2587) \times 0.0349 = 0.0259$

$x_4 = 0.9514$, $K_4 = 0.2587$

Q no 3

Qno 3 $x_{k+1} = Ax_k + BU_k + w_k \sim N(0, Q)$

$Y_k = Cx_k + V_k \sim N(0, R)$.

Given, $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -0.5 \\ 1 \end{bmatrix}$, $C = [1 \;\; 0]$

$U_k = 1$

Y(k)	Y(k+1)	Y(k+2)
100	97.9	94.4

$X_{k|k} = \begin{bmatrix} 95 \\ 1 \end{bmatrix}$, $P_{k|k} = \begin{bmatrix} 10 & 0 \\ 0 & 1 \end{bmatrix}$, $Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $R = 1$

Solution:-

Step k+0

$$\hat{x}_1 = \overset{A}{\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}} \overset{x_0}{\begin{bmatrix} 95 \\ 1 \end{bmatrix}} + \overset{B}{\begin{bmatrix} -0.5 \\ 1 \end{bmatrix}} \overset{u}{[1]} = \begin{bmatrix} 95.5 \\ 2 \end{bmatrix}$$

$$\hat{P}_1 = \overset{A}{\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}} \overset{P_0}{\begin{bmatrix} 10 & 0 \\ 0 & 1 \end{bmatrix}} \overset{A^T}{\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}} + \overset{Q}{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}} = \begin{bmatrix} 12 & 1 \\ 1 & 2 \end{bmatrix}$$

$$K_1 = \overset{\hat{P}_1}{\begin{bmatrix} 12 & 1 \\ 1 & 2 \end{bmatrix}} * \overset{H^T}{[1 \;\; 0]^T} * \left(\overset{H}{[1 \;\; 0]} * \overset{\hat{P}_1}{\begin{bmatrix} 12 & 1 \\ 1 & 2 \end{bmatrix}} * \overset{H^T}{\begin{bmatrix} 1 \\ 0 \end{bmatrix}} + \overset{R}{1} \right)^{-1} = \begin{bmatrix} 0.9231 \\ 0.0769 \end{bmatrix}$$

$$x_1 = \hat{x}_1 + K_1 (y_k - H * \hat{x}_1) = \begin{bmatrix} 95.5 \\ 2 \end{bmatrix} + \begin{bmatrix} 0.9231 \\ 0.0769 \end{bmatrix} \left(100 - [1 \;\; 0] \begin{bmatrix} 95.5 \\ 2 \end{bmatrix} \right) = \begin{bmatrix} 99.6538 \\ 2.3462 \end{bmatrix}$$

$$P_1 = (I - K_1 * H) * \hat{P}_1 = \left[\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{bmatrix} 0.9231 \\ 0.0769 \end{bmatrix} * [1 \;\; 0] \right] * \begin{bmatrix} 12 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 0.9231 & 0.0769 \\ 0.0769 & 1.9231 \end{bmatrix}$$

$$x(k) = \begin{bmatrix} 99.653 \\ 2.346 \end{bmatrix} \quad \boxed{K = \begin{bmatrix} 0.9231 \\ 0.0769 \end{bmatrix}}$$

Step k+1

$$\hat{x}_2 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 99.6538 \\ 2.3462 \end{bmatrix} + \begin{bmatrix} -0.5 \\ 1 \end{bmatrix} [1] = \begin{bmatrix} 101.5 \\ 3.3462 \end{bmatrix}$$

$$\hat{P}_2 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0.9231 & 0.0769 \\ 0.0769 & 1.9231 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ 2 & 2.9231 \end{bmatrix}$$

$$K_2 = \begin{bmatrix} 4 & 2 \\ 2 & 2.9231 \end{bmatrix} * \begin{bmatrix} 1 \\ 0 \end{bmatrix} * \left([1 \;\; 0] * \begin{bmatrix} 4 & 2 \\ 2 & 2.9231 \end{bmatrix} * \begin{bmatrix} 1 \\ 0 \end{bmatrix} + 1 \right)^{-1} = \boxed{\begin{bmatrix} 0.8 \\ 0.4 \end{bmatrix}}$$

$$x_2 = \begin{bmatrix} 101.5 \\ 3.3462 \end{bmatrix} + \begin{bmatrix} 0.8 \\ 0.4 \end{bmatrix} \left(97.9 - [1 \;\; 0] \begin{bmatrix} 101.5 \\ 3.3462 \end{bmatrix} \right) = \begin{bmatrix} 98.1769 \\ 3.0692 \end{bmatrix}$$

$$P_2 = \left[\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{bmatrix} 0.8 \\ 0.4 \end{bmatrix} * [1 \;\; 0] \right] * \begin{bmatrix} 4 & 2 \\ 2 & 2.9231 \end{bmatrix} = \begin{bmatrix} 0.8 & 0.4 \\ 0.4 & 2.1231 \end{bmatrix}$$

Step k+2

$$x_3 = \begin{bmatrix} 94.882 \\ 3.5811 \end{bmatrix} \quad K_3 = \boxed{\begin{bmatrix} 0.8253 \\ 0.4409 \end{bmatrix}}$$

Kalman gain =

K	K+1	K+2
$\begin{bmatrix} 0.9231 \\ 0.0769 \end{bmatrix}$	$\begin{bmatrix} 0.8 \\ 0.4 \end{bmatrix}$	$\begin{bmatrix} 0.8253 \\ 0.4409 \end{bmatrix}$

```
% Constants
P = [10 0; 0 1];
Q = [1 0; 0 1];
R = 1;
x = [95;1];
A = [1 1; 0 1];
B = [-0.5 1];
H = [1 0];
U = 1;
z = [0 100 97.9 94.4];
step = 3;
KalmanGain=[0;0];

for k=1:step
    xest = A*x + B*U;                 % Project state ahead
    Pest = A*P*transpose(A)+ Q;       % Project error covariance ahead
    K = Pest*transpose(H)*inv(H*Pest*transpose(H)+R); % Compute Kalman Gain
    KalmanGain = [KalmanGain K];      % Kalman
    x = xest + K*(z(k+1)-H*xest);     % Update the estimate zk
    P = (eye(size(P,2))-K*H)*Pest;    % Update error covariance
end
disp("Kalman Gain:")
```

Command Window

```
    0.9231    0.8000    0.8253
    0.0769    0.4000    0.4409
```

Kalman gain for k, k+1 and k+2.

4(a) $\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} -1/2 & 1 \\ 1/2 & 0 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} w(k)$

$y(k) = \begin{bmatrix} 2 & 1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$

$A = \begin{bmatrix} -0.5 & 1 \\ 0.5 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C = H = \begin{bmatrix} 2 & 1 \end{bmatrix}, P = \begin{bmatrix} 10 & 0 \\ 0 & 1 \end{bmatrix}, Q = I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, R = 5$

Say $\begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, V_k = [1]$ (supposed)

Iteration 1

$\hat{x}_1 = \begin{bmatrix} -0.5 & 1 \\ 0.5 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} [1] = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$\hat{P}_1 = \begin{bmatrix} -0.5 & 1 \\ 0.5 & 0 \end{bmatrix} \begin{bmatrix} 10 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -0.5 & 0.5 \\ 1 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4.5 & -1.5 \\ -1.5 & 3.5 \end{bmatrix}$

k	1	2	3	4	5	6	7	8	9	10
z_k	0.5	0.52	0.49	0.53	0.45	0.54	0.48	0.47	0.49	0.53

10 values of measurement with variance ~~[illegible]~~ is assumed

$K_1 = \begin{bmatrix} 4.5 & -1.5 \\ -1.5 & 3.5 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} \left(\begin{bmatrix} 2 & 1 \end{bmatrix} \begin{bmatrix} 4.5 & -1.5 \\ -1.5 & 3.5 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} + 5 \right)^{-1} = \begin{bmatrix} 0.3659 \\ 0.0244 \end{bmatrix}$

$x_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 0.3659 \\ 0.0244 \end{bmatrix} \left[0.5 - \begin{bmatrix} 2 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right] = \begin{bmatrix} -0.1829 \\ 0.9878 \end{bmatrix}$

$P_1 = \left[\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} 0.3659 \\ 0.0244 \end{pmatrix} \begin{pmatrix} 2 & 1 \end{pmatrix} \right] \begin{bmatrix} 4.5 & -1.5 \\ -1.5 & 3.5 \end{bmatrix} = \begin{bmatrix} 1.7561 & -1.6829 \\ -1.6829 & 3.4878 \end{bmatrix}$

Iteration 2

$\hat{x}_2 = \begin{bmatrix} -0.5 & 1 \\ 0.5 & 0 \end{bmatrix} \begin{bmatrix} -0.1829 \\ 0.9878 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} [1] = \begin{bmatrix} 0.8963 \\ 1.0915 \end{bmatrix}$ ~~$\begin{bmatrix} 0.7660 \\ 0.9245 \end{bmatrix}$~~

$\hat{P}_2 = \begin{bmatrix} -0.5 & 1 \\ 0.5 & 0 \end{bmatrix} \begin{bmatrix} 1.75 & -1.6 \\ -1.68 & 3.4 \end{bmatrix} \begin{bmatrix} -0.5 & 0.5 \\ 1 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3.2439 & 1.4024 \\ 1.4024 & 1.4390 \end{bmatrix}$

$K_2 = \begin{bmatrix} 3.2439 & 1.4024 \\ 1.4024 & 1.4390 \end{bmatrix} \begin{bmatrix} 2 & 1 \end{bmatrix}^T \left(\begin{bmatrix} 2 & 1 \end{bmatrix} \begin{bmatrix} 3.2439 & 1.4024 \\ 1.4024 & 1.4390 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} + 5 \right)^{-1} = \begin{bmatrix} 0.3153 \\ 0.1696 \end{bmatrix}$

$x_2 = \begin{bmatrix} 0.8963 \\ 1.0915 \end{bmatrix} + \begin{bmatrix} 0.3153 \\ 0.1696 \end{bmatrix} \left(0.52 - \begin{bmatrix} 2 & 1 \end{bmatrix} \begin{bmatrix} 0.8963 \\ 1.0915 \end{bmatrix} \right) = \begin{bmatrix} 0.1509 \\ 0.6905 \end{bmatrix}$

$P_2 = \left[\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} 0.3153 \\ 0.1696 \end{pmatrix} \begin{pmatrix} 2 & 1 \end{pmatrix} \right] \begin{bmatrix} 3.2439 & 1.4024 \\ 1.4024 & 1.4390 \end{bmatrix} = \begin{bmatrix} 0.7561 & 0.0643 \\ 0.0643 & 0.7193 \end{bmatrix}$

Iteration 3

$\hat{x}_3 = \begin{bmatrix} -0.5 & 1 \\ 0.5 & 0 \end{bmatrix} \begin{bmatrix} 0.1509 \\ 0.6905 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} [1] = \begin{bmatrix} 0.7660 \\ 0.9295 \end{bmatrix}$

$\hat{P}_3 \begin{bmatrix} -0.5 & 1 \\ 0.5 & 0 \end{bmatrix} \cancel{\begin{bmatrix} 0.7660 \\ 0.9295 \end{bmatrix}} \begin{bmatrix} 0.756 & 0.064 \\ 0.064 & 0.719 \end{bmatrix} \begin{bmatrix} -0.5 & 0.5 \\ 1 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1.9726 & 0.7788 \\ 0.7788 & 1.189 \end{bmatrix}$

$K_3 = \begin{bmatrix} 1.972 & 0.778 \\ 0.778 & 1.189 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} \left(\begin{bmatrix} 2 & 1 \end{bmatrix} \begin{bmatrix} 1.972 & 0.778 \\ 0.778 & 1.118 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} + 5 \right)^{-1} = \begin{bmatrix} 0.2747 \\ 0.1597 \end{bmatrix}$

$x_3 = \begin{bmatrix} 0.2257 \\ 0.6104 \end{bmatrix}$

$P_3 = \begin{bmatrix} 0.6747 & 0.0242 \\ 0.0242 & 0.7503 \end{bmatrix}$

Q no 4a

Q no 4 a)

```
% Input Data
A = [0.5 1; -0.5 0];
B = [0;1];
C = [2 1];
Q = 1;
R = 5;
Y = [.50 .52 .49 .53 .45 .54 .48 .47 .46 .53];  % Assumed
U = 1;   % Assumed
P = [10 0; 0 1];  % Assumed
X = [0;0];
KG = [0;0];
xPredict = [0;0];
pPredict = [0 0; 0 0];
updatedStates = [0;0];
updatedCovariance = [0 0; 0 0];

for i=1:10
  % Make predictions
  X = A*X + B*U;
  xPredict = [xPredict X];
  P = A*P*transpose(A) + Q;
  pPredict = [pPredict P];
  % Calculate Kalman Gain
  K = P*transpose(C)*(inv(C*P*transpose(C)+R));
  KG = [KG K];
  % Update estimates
  X = X + K*(Y(i)-C*X);
  updatedStates = [updatedStates X];
  P=(eye(size(P,2))-K*C)*P;
  updatedCovariance = [updatedCovariance P];
end
```

```
Kalman Gain

ans =

    0.3659    0.3153    0.2747    0.2733    0.2726    0.2725    0.2725    0.2725    0.2725    0.2725
    0.0244    0.1696    0.1597    0.1630    0.1627    0.1628    0.1627    0.1627    0.1627    0.1627
```

Q no 4 b)

```
% Input Data
A = [0 1 0; .25 0 1; 0 0 0];
B = [2;0;1];
C = [1 0 0; 1 -1 1];
U = 1;
P = [10 0 0; 0 1 0; 0 0 1];  % Assumed
Q = 5;
R = [2 0; 0 3];
Y = [.50 .52 .49 .53 .45 .54 .48 .47 .46 .53];  % Assumed
X = [0; 0; 0];
KG = [0 0; 0 0; 0 0];
xPredict = [0 ;0 ;0];
pPredict = [0 0 0; 0 0 0; 0 0 0];
updatedStates = [0 ; 0; 0];
updatedCovariance = [0 0 0; 0 0 0; 0 0 0];
```

```
Kalman Gain

ans =

  Columns 1 through 14

    0.5663    0.2449    0.6800    0.2185    0.6496    0.2097    0.6512    0.2205    0.6551    0.2145    0.6524    0.2167    0.6537    0.2161
    0.6709   -0.0612    0.6060    0.1514    0.6871    0.0646    0.6414    0.0928    0.6593    0.0872    0.6545    0.0862    0.6550    0.0876
    0.4719    0.2041    0.2537    0.3783    0.3520    0.3378    0.3210    0.3385    0.3260    0.3443    0.3272    0.3403    0.3255    0.3421

  Columns 15 through 20

    0.6533    0.2162    0.6534    0.2162    0.6534    0.2162
    0.6555    0.0868    0.6551    0.0871    0.6553    0.0870
    0.3264    0.3415    0.3260    0.3416    0.3261    0.3416
```