

CH5120: Assignment 2 – State Space Model

Note

- Submit the assignment on or before **September 15th 2022**
- Submission link for assignment 2 is open in Moodle.
- Ensure the filename is in the format **<Rollno.pdf>**
- Attach the codes and results for the respective questions, if MATLAB or any software is used.

1. Consider the single-input system dynamics given by $\dot{x} = Ax + Bu$ and $y = Cx$ and choose the correct statement(s) from the following statements:
 - (i) The system is stable in an absolute sense if all eigenvalues of A have non-negative real parts.
 - (ii) The poles of the system is given by the eigenvalues of A.

2. Identify the transfer function representation of the state space model and find the right coefficient array of the numerator, given:

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0; & 0 & 0 & 1; & 1 & 4 & 3 \end{bmatrix} x + \begin{bmatrix} 0; & 0; & 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} x + \begin{bmatrix} 1 \end{bmatrix} u$$

3. Obtain the transfer function form from the given state space representation and find the correct coefficients of the denominator of the transfer function.

$$A = \begin{bmatrix} -1 & -1 & -1; & 0 & 1 & -1; & 1 & -1 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 0; & 1; & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 \end{bmatrix}$$

4. Obtain the state equation in the phase variable canonical form from the 3rd order given differential equation coefficients. Given the coefficient matrix of the differential equation $[a_3 \ a_2 \ a_1 \ a_0]$ is $[4 \ -6 \ 1 \ 7]$. The coefficient of $u(t)$ is 2. And find the correct representation of the state matrix (A) in its simplified form.

Note: The representation of the phase variable form is

$$\frac{d^n y}{dt^n} + a_{n-1} \frac{d^{n-1} y}{dt^{n-1}} + \dots + a_1 \frac{dy}{dt} + a_0 y = u(t)$$

5. Discretize the given continuous-time state space system with a sample time of .1s and what is the value of A matrix in the discrete state space model given below?

$$\text{Given: } A_c = \begin{bmatrix} -3 & -6 \\ 2 & 5 \end{bmatrix} \quad B_c = \begin{bmatrix} -5 \\ 8 \end{bmatrix} \quad C_c = \begin{bmatrix} -4 & 5 \end{bmatrix}$$

Note:

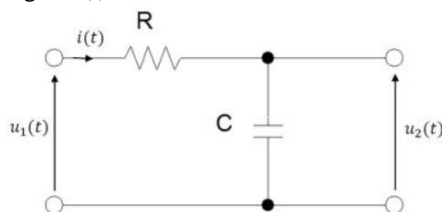
$$\text{Continuous-time State Equation} : \frac{d(x(t))}{dt} = A_c x(t) + B_c u(t)$$

$$\text{Continuous-time Output Equation} : y(t) = C_c x(t)$$

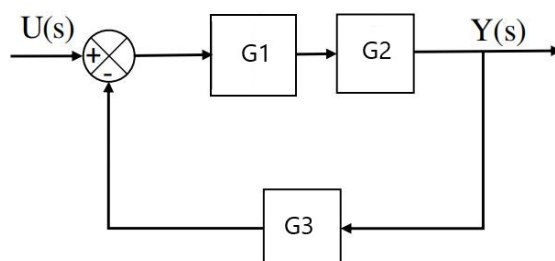
$$\text{Discrete-time State Equation} : x(k+1) = A x(k) + B_c u(k)$$

$$\text{Discrete-time Output Equation} : y(k) = C x(k)$$

6. Identify the state space representation (in the form $\dot{x}'(t) = A x(t) + Bu(t)$ and $y(t) = C x(t) + D u(t)$) for the transient response of the circuit shown in the figure and choose the right option for the A matrix. Such that the input to the system is the voltage $u_1(t)$ and the output is the voltage $u_2(t)$. Given: $R = 130\Omega$, $C = 2.000000e-06F$

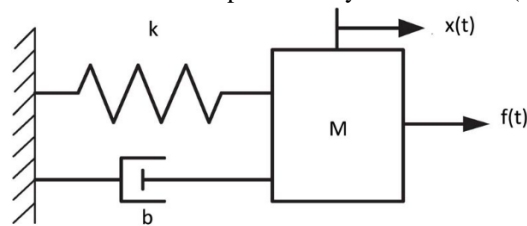


7. If the state-space model is represented as $\dot{x}' = Ax + Bu$ and $y = Cx + Du$ where x , u , and y are the state variable vector, input vector, and measurement vector respectively. What is the A matrix for the following closed loop system?



$$\text{Given: } G1 = 7/(s+9), \quad G2 = 9/(s+1), \quad G3 = 3/(s+1)$$

8. Identify the state space representation (in the form $\dot{x} = Ax + Bu$ and $y = Cx + Du$) of the Mass-Spring-Damper model shown in the figure and find the matrix A. Note that the input to the system is force $f(t)$ and output is displacement $x(t)$.



Given: $M = 4$; $k = 3$; $b = 2.000000e-01$

9. Identify the state space representation (in the form $\dot{x} = Ax + Bu$ and $y = Cx + Du$) of the circuit shown in the figure and find the matrix A. Given: $R1 = 2$; $R2 = 7$; $L1 = 1$; $C1 = 3.000000e-01$. Follow SI units for R, L, and C. Assume Zero initial conditions. Also, consider the circuit to be at time $t=0$ just after it is switched on. (Hint: Refer Nodal Analysis Technique for finding V_o/V_s)

