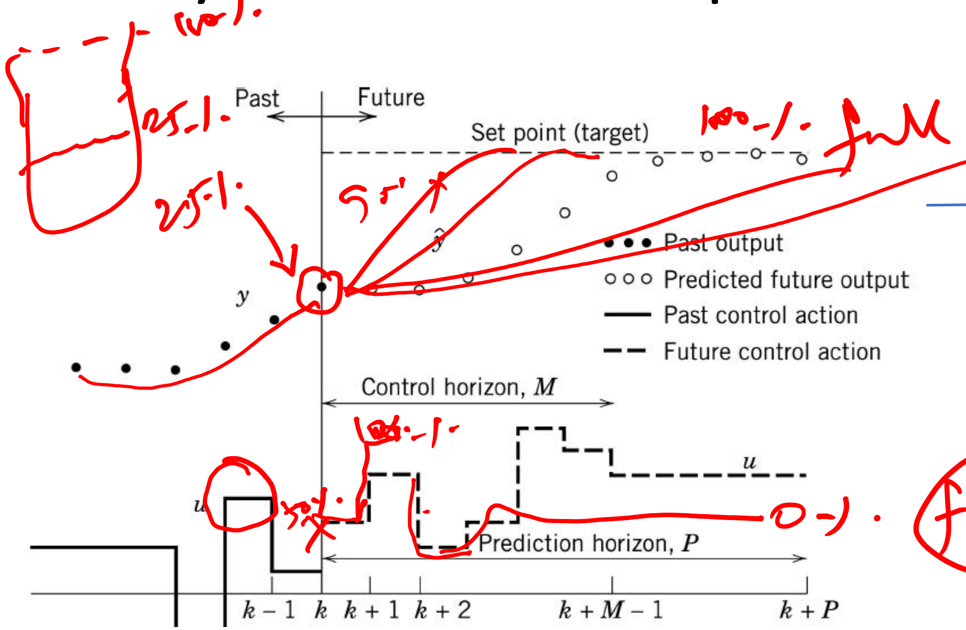
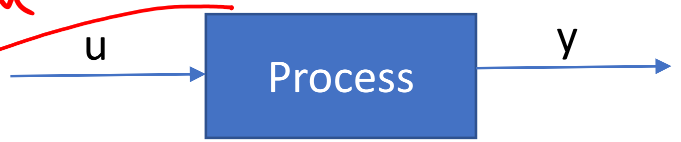
Modern Control Theory

Model Predict Control - MPC – Lecture 3

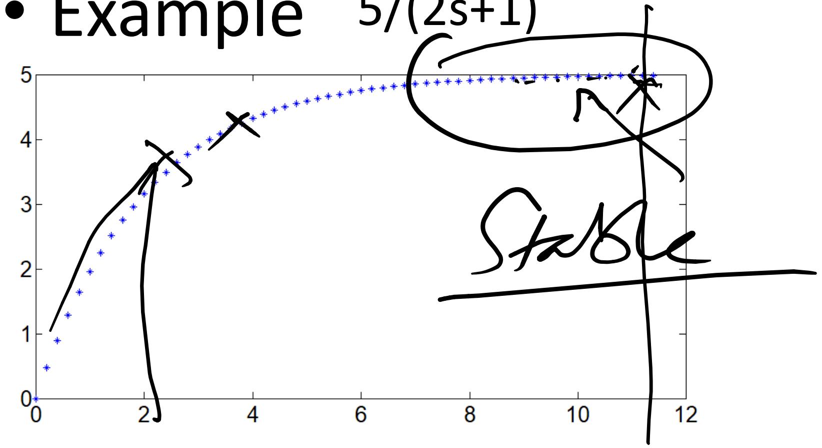
Dynamic Prediction Models

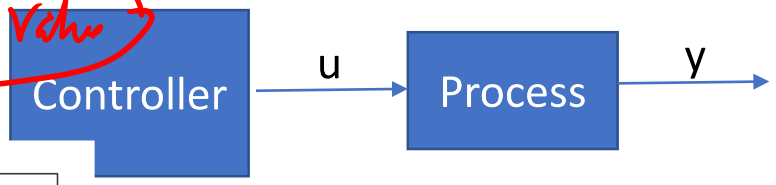
Why have multiple control moves?





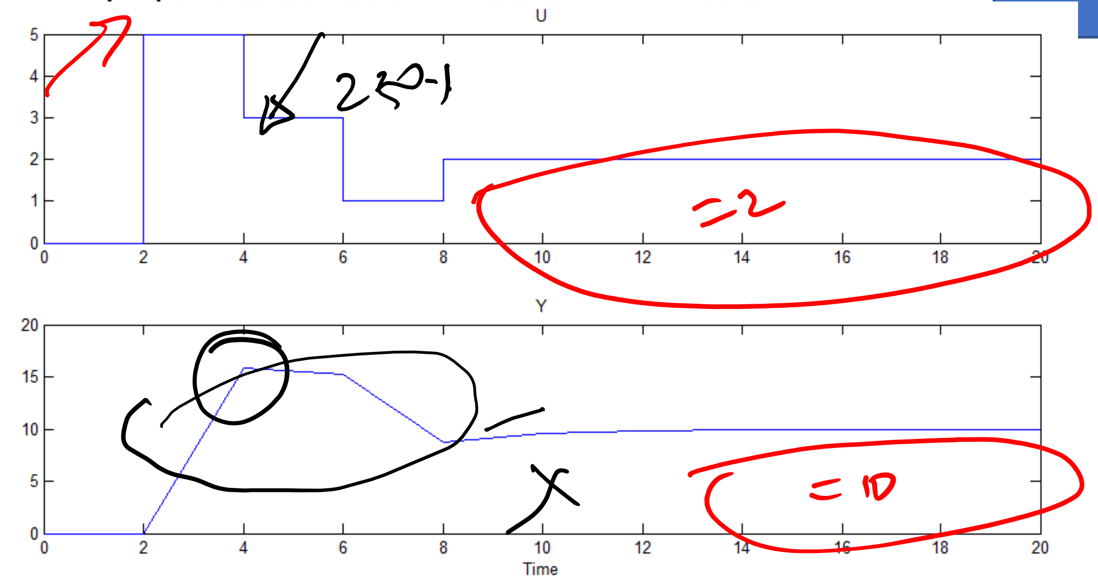
- Example $5/(2s+1)$

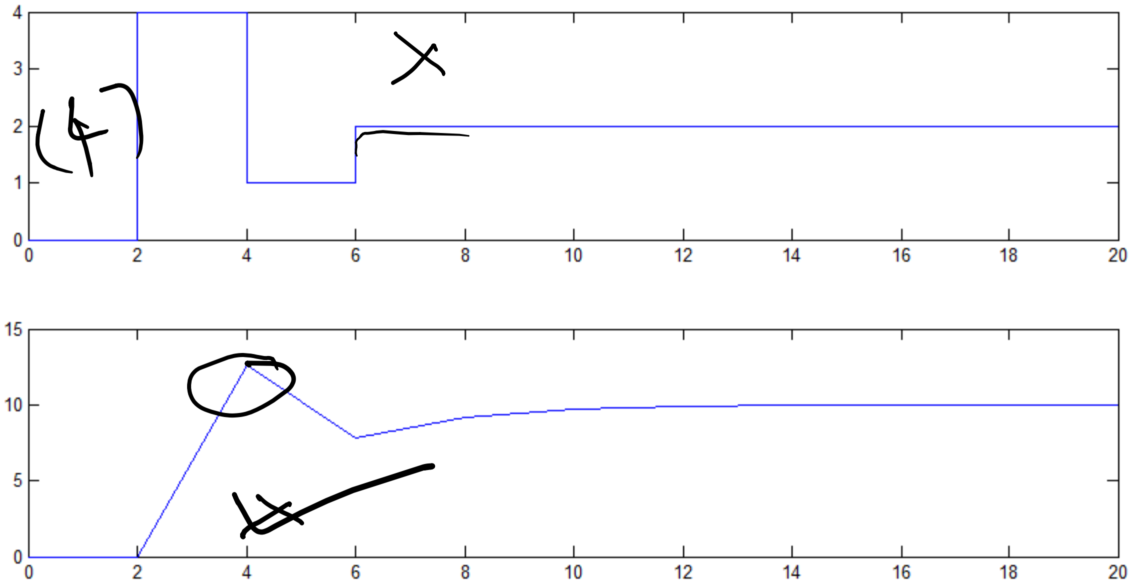




$y \rightarrow 10$, what should be u?

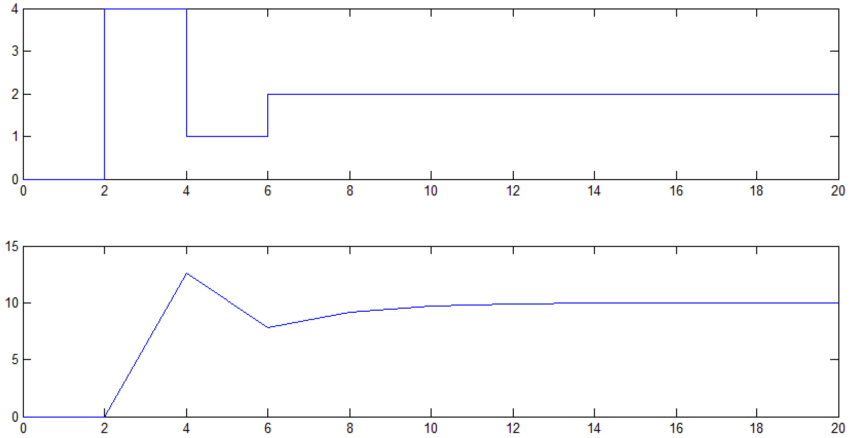
u = 2

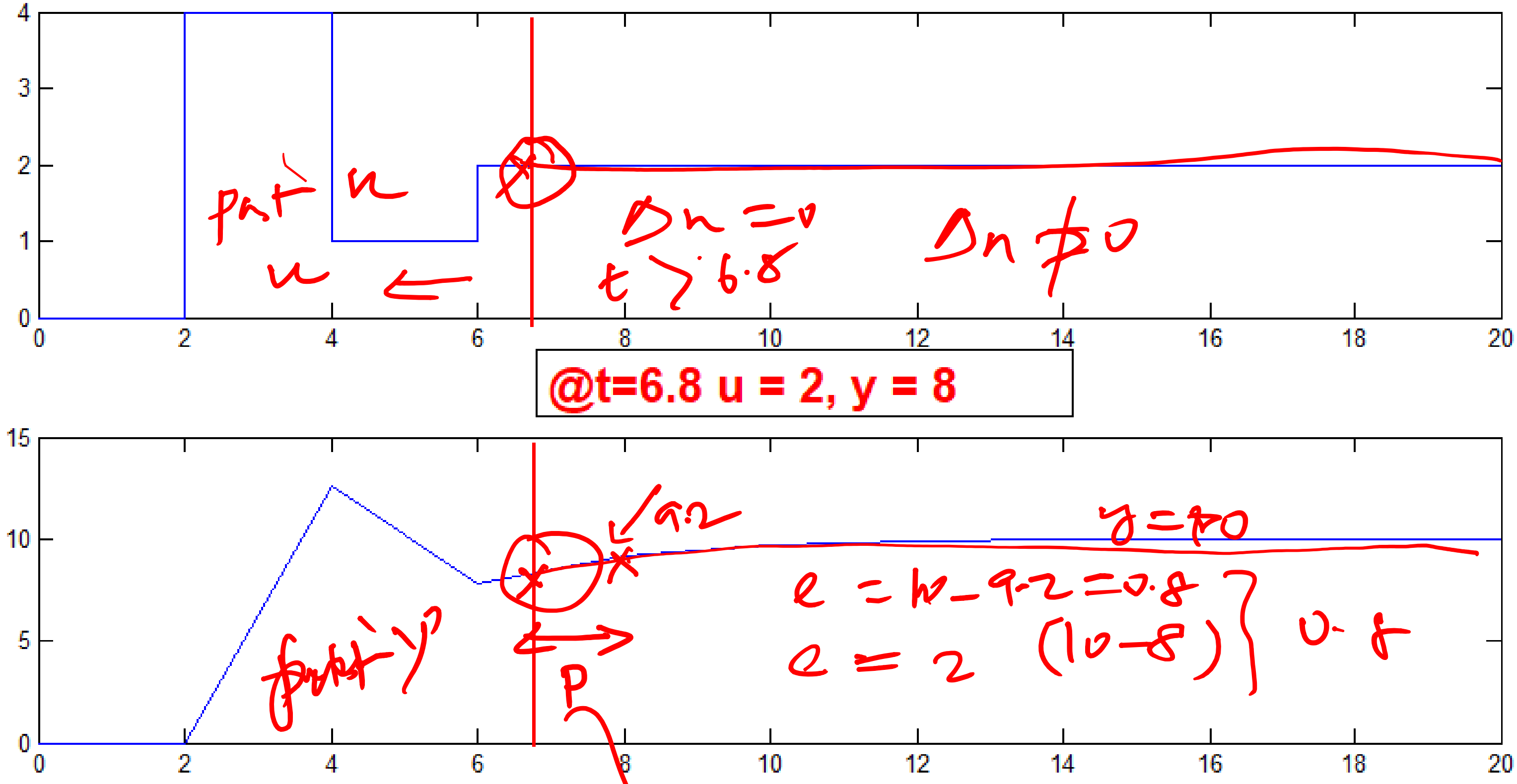


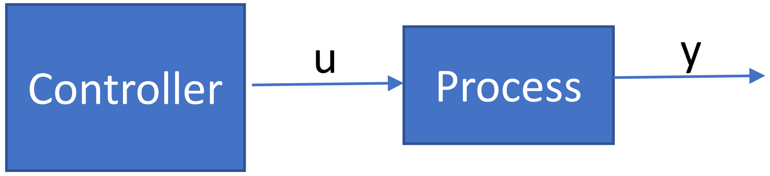


Does controller need to make control move all the time?

- Example 5/(2s+1)







y → 10

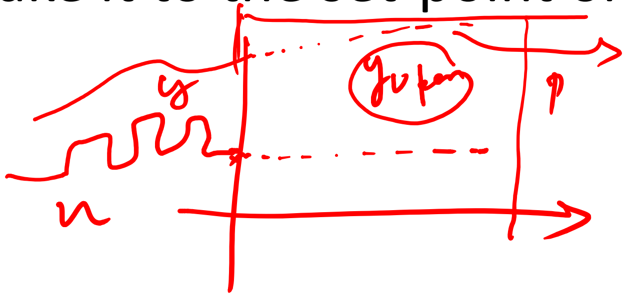
Prediction horizon?

~~What should controller~~ do?

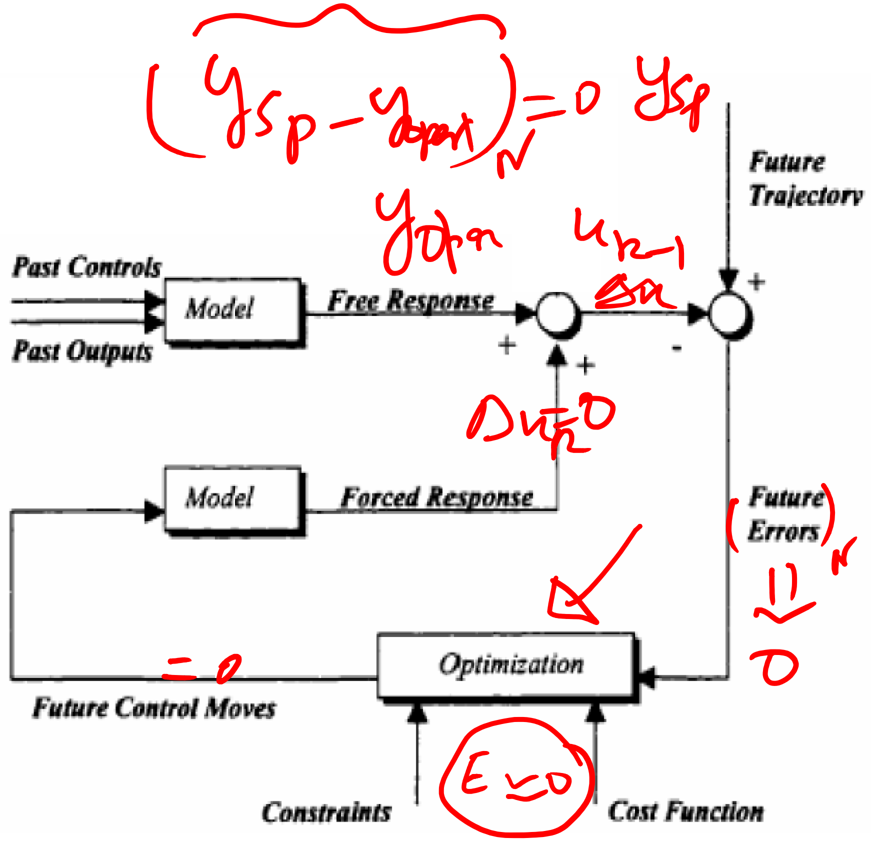
NO ACTION REQUIRED as it will reach to its Target

Free response and forced response

- Free response –
- Check if the system with current input can reach to the set-point or
- Find out how close it can take it to the set-point or objective



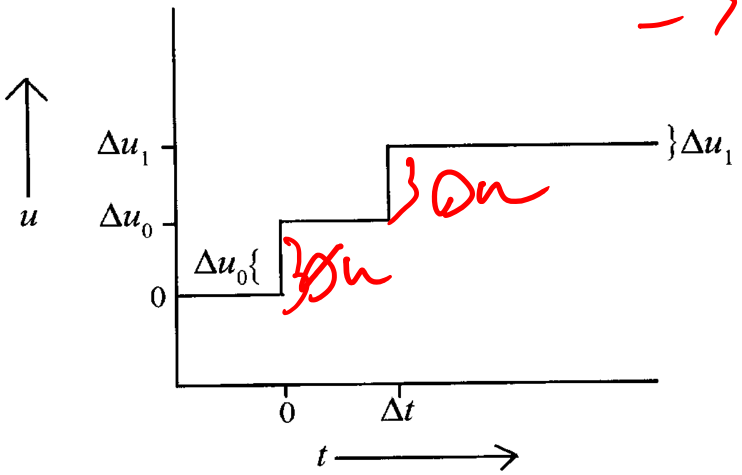
- Forced response –
- Don’t make any control action if the system can reach to the set-point or
- Compute control actions that can take it to the set-point or objective over and above the free response.



$\cancel{\Delta u_k = u_{k-1}}$

$u_k = u_{k-1} + \Delta u_k$

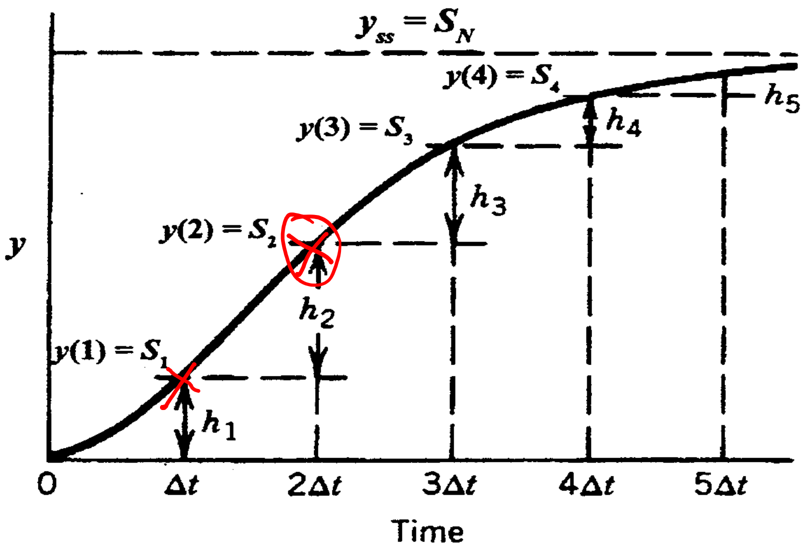
Multi move as breaking of multiple staggered step inputs

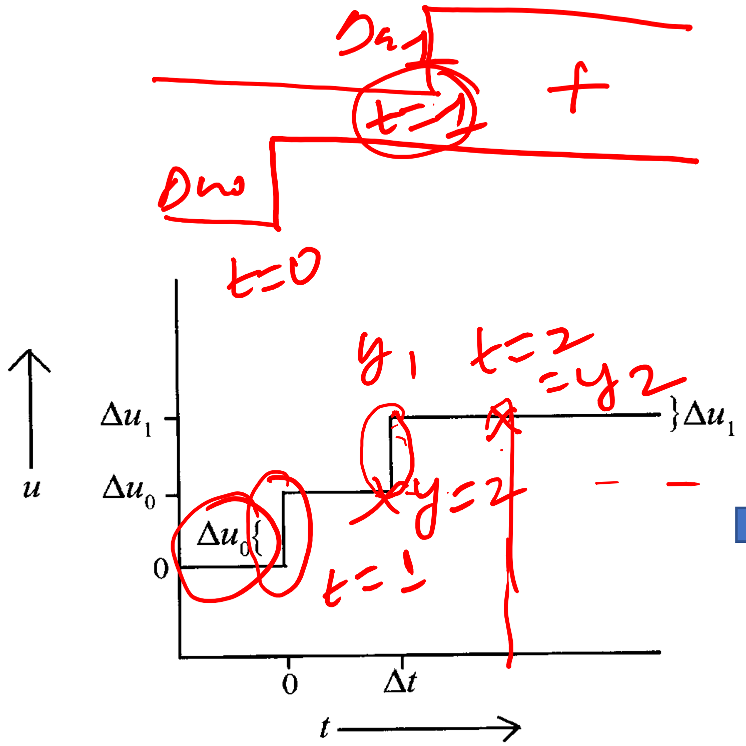


$\Rightarrow$

$$u_k = u_{k-1} + \Delta u$$

Step response predictions (1/2)





From the Principle of Superposition for linear systems:

$y_1 = y_0 + S_1\Delta u_0$

$y_2 = y_0 + \boxed{S_2\Delta u_0} + \boxed{S_1\Delta u_1}$

$y_3 = y_0 + S_3\Delta u_0 + S_2\Delta u_1$

. .

. .

$y_N = y_0 + S_N\Delta u_0 + S_{N-1}\Delta u_1$

$$y(k+1) = y_0 + \sum_{i=1}^{N-1} S_i \Delta u(k-i+1) + S_N\, u(k-N+1)$$

(u_{k-1})

Can extend also to MIMO Systems as simple Multi input Single output (MISO) systems

Step response prediction (2/2)

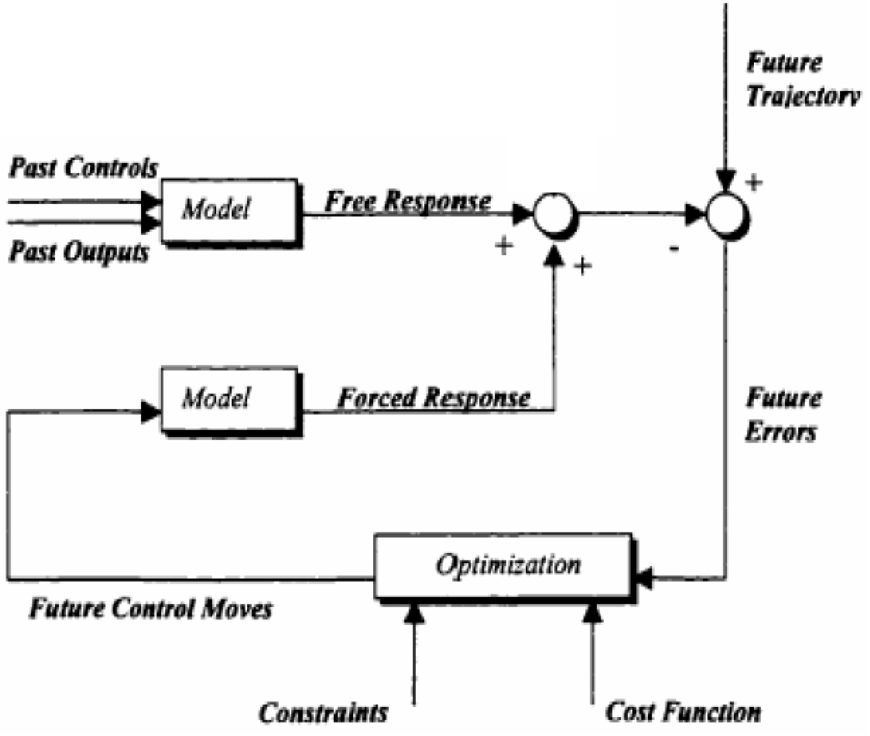
j^th^ step ahead prediction

$$\hat{y}(k+j) = \underbrace{\sum_{i=1}^{j} S_i \Delta u(k+j-i)}_{\text{Effects of current and future control actions}} + \underbrace{\sum_{i=j+1}^{N-1} S_i \Delta u(k+j-i) + S_N u(k+j-N)}_{\text{Effects of past control actions}}$$

What happens when u(k-i) = u(k-1) for all i > 0

Predicted FREE response

$$\hat{y}^o(k+j) = \sum_{i=j+1}^{N-1} S_i \Delta u(k+j-i) + S_N u(k+j-N)$$



Example – simple predictive receding horizon controller

- Find a predictive control law with single receding horizon move, such that $\hat{y}(k+J) = y_{sp}$

j^{th} step ahead prediction

$$\hat{y}(k+j) = \underbrace{\sum_{i=1}^{j} S_i \Delta u(k+j-i)}_{\text{Forced response}} + \underbrace{\hat{y}^o(k+j)}_{\text{Free response}}$$

Given

$j = J$, $\hat{y}(k+J) = y_{sp}$

$\Delta u(k+i) = 0$ for $i > 0$, i.e. $\Delta u(k+1) = \Delta u(k+2) \ldots = \Delta u(k+J-1) = 0$

$$\Downarrow$$

$$\hat{y}(k+J) = S_J \Delta u(k) + \hat{y}^o(k+J)$$

$$\Downarrow$$

$$y_{sp} = S_J \Delta u(k) + \hat{y}^o(k+J)$$

$$\Downarrow$$

$$\Delta u(k) = \frac{y_{sp} - \hat{y}^o(k+J)}{S_J}$$

SISO system with prediction-based control law using simple step response coefficients

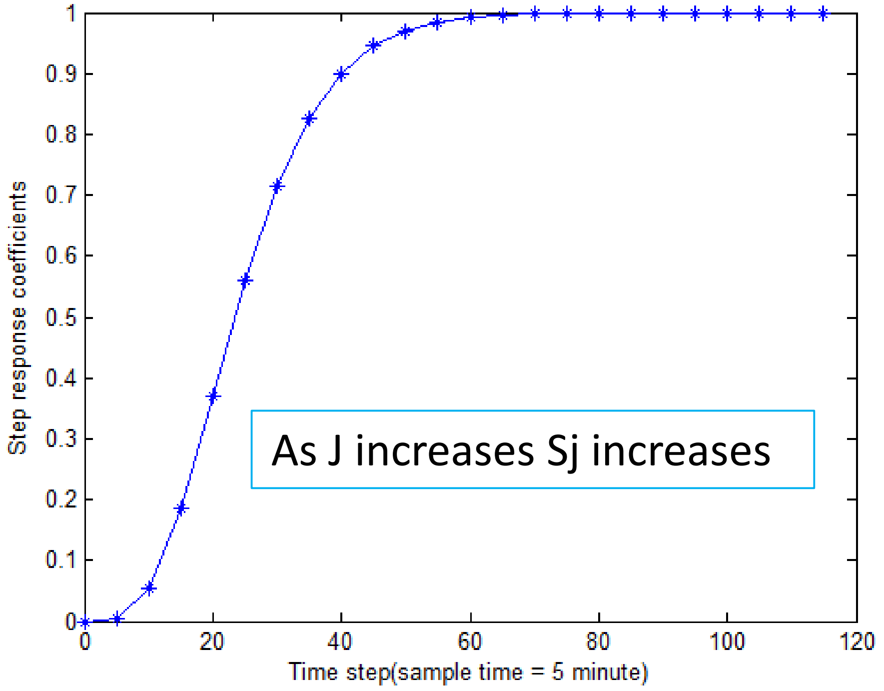
What are the pitfalls in this?

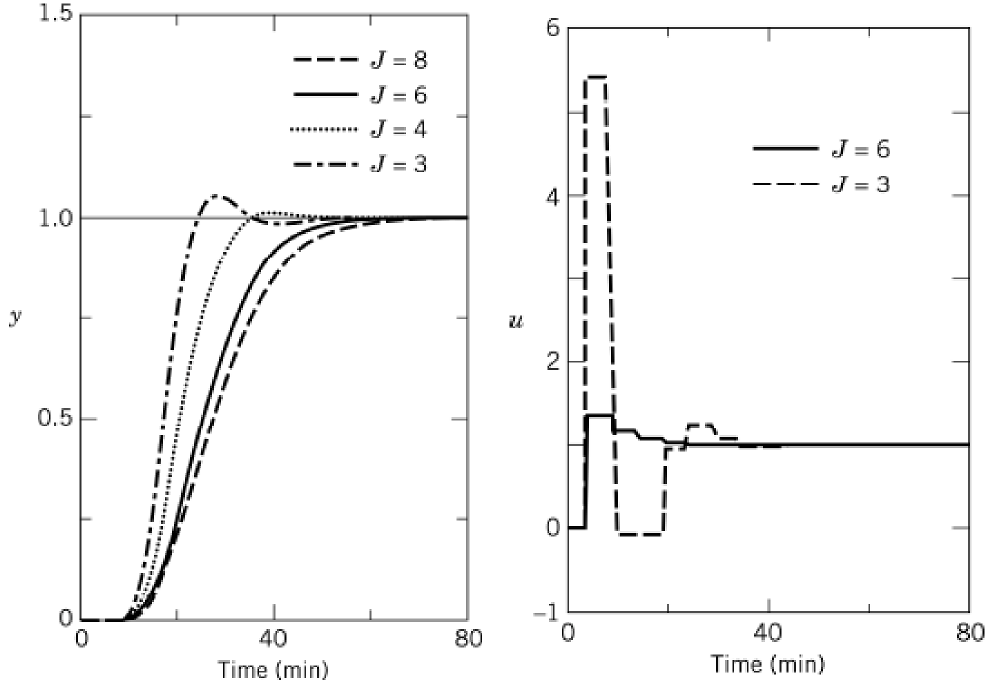
Example – simple predictive receding horizon controller

$$\frac{Y(s)}{U(s)} = \frac{1}{(5s + 1)^5}$$

J = 3, 4, 6, 8

Δt = 5 mins





$$\Delta u(k) = \frac{y_{sp} - \hat{y}^o(k + J)}{S_J}$$

The move required tend to be smaller as J increases

As J decreases, y becomes aggressive

Multi input Multi output prediction models - step response (1/2)

$$\hat{y}(k+1) = \underbrace{S_1 \Delta u(k)}_{\text{Effect of current}} + \underbrace{\sum_{i=2}^{N-1} S_i \Delta u(k-i+1) + S_N u(k-N+1)}_{}$$

$$\hat{y}(k+2) = \underbrace{S_1 \Delta u(k+1)}_{\substack{\text{Effect of future} \\ \text{control action}}} + \underbrace{S_2 \Delta u(k)}_{\substack{\text{Effect of current} \\ \text{control action}}} + \underbrace{\sum_{i=3}^{N-1} S_i \Delta u(k-i+2) + S_N u(k-N+2)}_{\text{Effect of past control actions}}$$

$$\hat{y}(k+j) = \underbrace{\sum_{i=1}^{j} S_i \Delta u(k+j-i)}_{\text{Forced response}} + \underbrace{\hat{y}^o(k+j)}_{\text{Free response}}$$

$$\begin{bmatrix} \hat{y}(k+1) \\ \hat{y}(k+2) \\ \hat{y}(k+j) \end{bmatrix} = \begin{bmatrix} S_1 & 0 & \cdots & 0 \\ S_2 & S_1 & 0 & \vdots \\ \vdots & \vdots & \ddots & 0 \\ S_j & S_{j-1} & & S_1 \end{bmatrix} \begin{bmatrix} \Delta u(k) \\ \Delta u(k+1) \\ \\ \Delta u(k+j-1) \end{bmatrix} + \begin{bmatrix} \hat{y}^o(k+1) \\ \hat{y}^o(k+2) \\ \hat{y}^o(k+j) \end{bmatrix}$$

$$\hat{\mathbf{Y}}(k+1) = S\Delta \mathbf{U}(k) + \hat{\mathbf{Y}}^o(k+1)$$

Multi input Multi output prediction models - step response (2/2)

Principle of superposition: Example 2 inputs and 2 outputs

$$\hat{y}_1(k+1) = \sum_{i=1}^{N-1} S_{11,i}\Delta u_1(k-i+1) + S_{11,N}\, u_1(k-N+1) + \sum_{i=1}^{N-1} S_{12,i}\Delta u_2(k-i+1) + S_{12,N}\, u_2(k-N+1)$$

$$\hat{y}_2(k+1) = \sum_{i=1}^{N-1} S_{21,i}\Delta u_1(k-i+1) + S_{21,N}\, u_1(k-N+1) + \sum_{i=1}^{N-1} S_{22,i}\Delta u_2(k-i+1) + S_{22,N}\, u_2(k-N+1)$$

P step ahead predictions for M future control moves

'r' inputs $\quad \dot{\mathbf{u}} = [u_1 \quad u_2 \quad \cdots \quad u_r]^T$

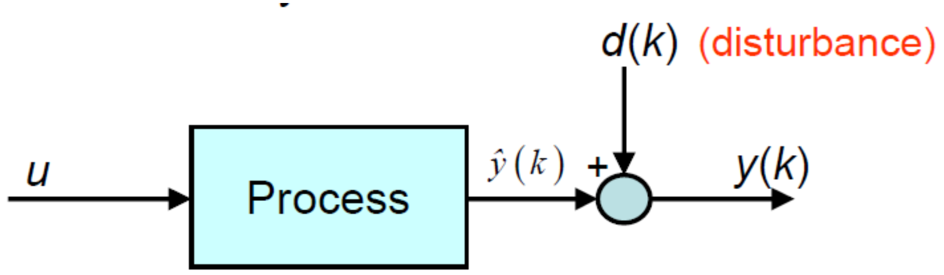
'm' outputs $\quad \mathbf{y} = [y_1 \quad y_2 \quad \cdots \quad y_m]^T$

$$\hat{Y}(k+1) \triangleq \begin{bmatrix} \hat{y}(k+1) \\ \hat{y}(k+2) \\ \vdots \\ \hat{y}(k+P) \end{bmatrix}_{mP\times 1} \quad \hat{Y}^o(k+1) \triangleq \begin{bmatrix} \hat{\mathbf{y}}^o(k+1) \\ \hat{\mathbf{y}}^o(k+2) \\ \vdots \\ \hat{\mathbf{y}}^o(k+P) \end{bmatrix}_{mP\times 1} \quad \Delta U(k) \triangleq \begin{bmatrix} \Delta\mathbf{u}(k) \\ \Delta\mathbf{u}(k+1) \\ \vdots \\ \Delta\mathbf{u}(k+M\text{-}1) \end{bmatrix}_{rM\times 1}$$

$$\boxed{\hat{\mathbf{Y}}(k+1) = \underbrace{S\Delta U(k)}_{\text{Forced response}} + \underbrace{\hat{\mathbf{Y}}^{o}(k+1)}_{\text{Free response}}}$$

$$\mathbf{S} \triangleq \begin{bmatrix} \mathbf{S}_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{S}_2 & \mathbf{S}_1 & \mathbf{0} & \vdots \\ \vdots & \vdots & \ddots & \mathbf{0} \\ \mathbf{S}_M & \mathbf{S}_{M-1} & \cdots & \mathbf{S}_1 \\ \mathbf{S}_{M+1} & \mathbf{S}_M & \cdots & \mathbf{S}_2 \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{S}_P & \mathbf{S}_{P-1} & \cdots & \mathbf{S}_{P-M+1} \end{bmatrix} \qquad \mathbf{S}_i \triangleq \begin{bmatrix} S_{11,i} & S_{12,i} & \cdots & S_{1r,i} \\ S_{21,i} & \cdots & \cdots & S_{2r,i} \\ \vdots & \vdots & \vdots & \vdots \\ S_{m1,i} & \cdots & \cdots & S_{mr,i} \end{bmatrix}$$

Correcting for model prediction errors – output feedback



When can predictions drift away actual?

- Inaccurate model
- Unmeasured disturbances

How do we correct the model predictions?

Output feedback based on the latest measurement

$$\tilde{y}(k+j) \triangleq \hat{y}(k+j)+\underbrace{\left[y(k)-\hat{y}(k)\right]}_{\text{Bias term or correction term or Estimated disturbance}}$$

MIMO Model prediction with bias correction

'r' inputs $\mathbf{u}=[u_1 \quad u_2 \quad \cdots \quad u_r]^T$

'm' outputs $\mathbf{y}=[y_1 \quad y_2 \quad \cdots \quad y_m]^T$

$$\tilde{Y}(k+1) = S\Delta U(k) + \hat{Y}^o(k+1)+\mathbf{\Phi}\left[\mathbf{y}(k)-\hat{\mathbf{y}}(k)\right]$$

$\mathbf{\Phi}$ is matrix of '1' with dimension mP xm

Nomenclature

- CVs, MVs, and DVs