ASSIGNMENT-1

Note : 5 set of question papers were circulated. This is the answer key for one of them. I/P numericals may be different.

1. $$M = \begin{bmatrix} -2 & -7 & -2 & 8 \\ -8 & -6 & -9 & 0 \\ -5 & -5 & 8 & 0 \end{bmatrix}_{3 \times 4}$$

Rank $(r) = 3$ (Dimension)

Row space $R(M) \Rightarrow 4$ (No. of columns)

Column space $C(M) \Rightarrow 3$ (No. of rows).

Null space $N(M) \Rightarrow R(M) - r = 1$

Left null space $L(M) \Rightarrow C(M) - r = 0$.

Note : Or you can explain in detail too.

2. (i) T (explanation should have: <u>Linearly independent eig vectors</u>)

(ii) F

(iii) T

3. $A = \begin{bmatrix} 1 & 1 & 1 \\ -1 & -3 & -3 \\ 2 & 4 & 4 \end{bmatrix}$

<u>Step 1</u> : eig (A) $\Rightarrow$ 2, 0, 0.

(Any method of solving except direct expm(At) using MATLAB, is evaluated accordingly)

<u>step 2</u> : <u>Let</u> $\exp(At) = \beta_0 + \beta_1 A + \beta_2 A^2$ — ①

<u>Cayley - Hamilton theorem,</u>

$$\exp(\lambda t) = \beta_0 + \beta_1 . \lambda + \beta_2 \lambda^2$$

$$\exp(2t) = \beta_0 + 2\beta_1 + 4\beta_2$$

$$\exp(0t) = \beta_0 \Rightarrow \boxed{\beta_0 = 1}$$

<u>First derivative</u> $t \exp(\lambda t) = \beta_1 + 2\beta_2 \lambda$.

$$\lambda = 0 \Rightarrow t.(1) = \beta_1 \Rightarrow \boxed{\beta_1 = t}$$

$$\therefore \beta_2 = \left(\frac{e^{2t} - 1 - 2t}{4} \right)$$

Sub all in ①,

$$e^{At} = \begin{bmatrix} \frac{e^{2t}+1}{2} & \frac{e^{2t}-1}{2} & \frac{e^{2t}-1}{2} \\ 1+t-e^{2t} & \boxed{2-t-e^{2t}} & 1-t-e^{2t} \\ \frac{3e^{2t}-3-t}{2} & \frac{+3e^{t}-3}{2}+t & +3\frac{e^{2t}-1}{2}+t \end{bmatrix}$$

<u>β_2 Answer</u>

4. Sum of eigen values

= Trace of Matrix

= $1 + 2 + 2 = 5$

5. Prd of eigen values

= Det of Matrix = -4

6. Left eigen vectors

$\Rightarrow$ $V . A = V . \lambda$ (Solve)

Or use $[V, D, W] = eig(M)$

$\downarrow$ Left E.V

$$W = \begin{bmatrix} -0.7863 & -0.6077 & 0.1108 \\ 0.4796 & -0.4876 & 0.7296 \\ -0.3894 & 0.6268 & 0.6749 \end{bmatrix}$$

7. SV Dec

$[U, S, V] = svd(M)$

$S \Rightarrow$ Singular value matrix $\Rightarrow 16.2, 10.6, 7.4$

8. Rotation $\boxed{\text{Do: } R \times M}$

Since rotation abt origin or with respect to any fixed point is not mentioned in question, based on assumption, evaluated.

$\theta = -90^\circ$ (~~anti~~ clockwise)

R rot. matx $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

9. $\exp(At) = b_0 + b_1 A + b_2 A^2$

Similar to Problem 3.

Solution :

$$\beta_0 = 6e^{2t} + 3e^{4t} - 8e^{3t}$$

$$\beta_1 = 6e^{3t} - \frac{7}{2}e^{2t} - \frac{5}{2}e^{4t}$$

$$\beta_2 = \frac{1}{2}e^{2t} - e^{3t} + \frac{e^{4t}}{2}$$

10. Given $M = \begin{bmatrix} 55 & -1 & 7 \\ -1 & 12 & 13 \\ 7 & 13 & 38 \end{bmatrix}$

eig vectors $(V) = \begin{bmatrix} 0.0754 & -0.3857 & -0.9195 \\ 0.9164 & 0.3902 & -0.0886 \\ -0.3930 & 0.8360 & -0.3829 \end{bmatrix}$

eig. values = $[6.3429 \quad 40.8387 \quad 57.8184]$

Note : $V^{-1} = V^T$ (orthogonal) $\rightarrow$ expected in ans

$\therefore$ Trace (V^{-1}) = Trace (V^T) = Trace (V).

$\therefore$ Trace = 0.0827

ASSIGNMENT-2

1. (i) False.

   (ii) True : Poles of TF = Eigenvalues of SS (A)

2. Use MATLAB command,

$$[n,d] = ss2tf(A, B, C, D)$$

(or) $TF = C(sI - A)^{-1} \cdot B + D$

Soln : $n = [1 \; -3 \; -4 \; 0]$

$d = [1 \; -3 \; -4 \; -1]$

3. Similar to 2 sum

Soln : $n = [0 \; 0 \; -1 \; -2]$

$d = [1 \; -1 \; -1 \; -2]$

4. $$4\frac{d^3y}{dt^3} - 6\frac{d^2y}{dt^2} + \frac{dy}{dt} + 7y = 2u(t)$$

Note : 3rd order

$$4y''' - 6y'' + y' + 7y = 2u(t)$$

Taking LT, $(4s^3 - 6s^2 + s + 7)\, y(s) = 2u(s)$

$$\frac{y(s)}{u(s)} = \frac{2}{4s^3 - 6s^2 + s + 7} \quad \text{(or)} \quad \frac{1/2}{s^3 - \frac{3}{2}s^2 + \frac{1}{4}s + \frac{7}{4}}$$

Corresponding SS, (Any methods)

① Can use MATLAB tf2ss

② Write directly from above tf.

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -7/4 & -1/4 & 3/2 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1/2 \end{bmatrix} u \qquad y = [1 \; 0 \; 0]\, x$$

5. <u>Discretize</u> (Any method).

① $sys = ss(A, B, C, D)$

$tsample = 0.1$

$sys\_disc = c2d(sys, tsample)$

② $A_d = \exp.(A_c \cdot t_s) \Rightarrow$ We know

<u>Use</u> : $expm(A_c \times 0.1)$

③ Or can find eigen values of A, express $A = f(t)$ and sub $\underline{t = 0.1\,s}$ as in ASSIGNMENT I

Soln : $A = \begin{bmatrix} 0.682 & -0.667 \\ 0.222 & 1.572 \end{bmatrix}$

6. <u>Balance,</u>

$u_1 = iR + u_2 \Rightarrow i = \dfrac{u_1 - u_2}{R}$

$u_2 = V_c$

$$\frac{dV_c}{dt} = \frac{i}{c}$$

Sub i, $\dfrac{dV_c}{dt} = \dfrac{u_1 - u_2}{RC}$

$$\dot{V_c} = \frac{u_1}{RC} - \frac{V_c}{RC}.$$

$$\therefore \dot{V_c} = \left[\frac{-1}{RC}\right] V_c + \left[\frac{1}{RC}\right] u_1 .$$

$$\therefore A = -1/RC = \frac{-1}{130 \times 2 \times 10^{-6}} = -3846.2$$

7. $$\frac{Y(s)}{U(s)} = \frac{G_1 . G_2}{1 + G_1 G_2 G_3} = \frac{\frac{7}{s+9} \cdot \frac{9}{s+1}}{1 + \frac{7}{s+9} \cdot \frac{9}{s+1} \cdot \frac{3}{s+1}}$$

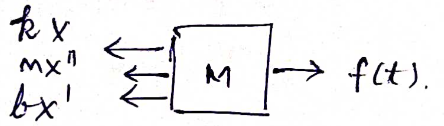
$$\frac{Y(s)}{U(s)} = \frac{63\,(s+1)}{(s+1)^2 (s+9) + 63 \times 3} = \frac{63s + 63}{s^3 + 11s^2 + 19s + 198}$$

Can use tf2ss in MATLAB or straightly write,

$$A = \begin{bmatrix} -11 & -19 & -198 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 63 & 63 \end{bmatrix} \qquad D = [0]$$

8.



$\therefore$ $f(t) = kx + bx' + mx''$

input $\longmapsto$ forces $\longleftarrow$

$\therefore$ $X_1 = x'$

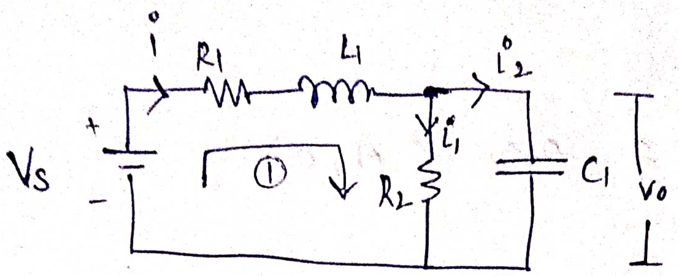
$X_1' = x'' \Rightarrow \frac{f}{m} - \frac{k}{m} x - \frac{b}{m} X_1$

$$\therefore \begin{bmatrix} X' \\ X_1' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{b}{m} \end{bmatrix} \begin{bmatrix} X \\ X_1 \end{bmatrix} + \begin{bmatrix} 0 \\ 1/M \end{bmatrix} f$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} X \\ X_1 \end{bmatrix}$$

Sub, $$A = \begin{bmatrix} 0 & 1 \\ -0.75 & -0.05 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 0.25 \end{bmatrix}$$

9. Note: Can be represented by more than one state space model. Answers are evaluated based on approach.



$$-V_s + iR_1 + L\cdot\frac{di}{dt} + \underbrace{i_1 \cdot R_2}_{V_o} = 0 \qquad \text{Loop 1}$$

$$i = i_1 + i_2 \Rightarrow i = \frac{V_o}{R_2} + C\cdot\frac{dV_o}{dt}$$

Take, i & V_o as variables.

$$\frac{di}{dt} = \frac{V_s}{L} - \frac{iR_1}{L} - \frac{V_o}{L}$$

$$\frac{dV_o}{dt} = \frac{i}{C} - \frac{V_o}{CR_2}$$

$$\begin{bmatrix} i' \\ V_o' \end{bmatrix} = \begin{bmatrix} -R_1/L & -1/L \\ 1/C & -1/CR_2 \end{bmatrix} \begin{bmatrix} i \\ V_o \end{bmatrix} + \begin{bmatrix} 1/L \\ 0 \end{bmatrix} V_s$$

Sub values $\Rightarrow$

$$A = \begin{bmatrix} -2 & -1 \\ 10/3 & -10/21 \end{bmatrix} \qquad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

ASSIGNMENT - 3

1. Controll-matrix $= [B \quad AB \quad A^2B]$

Observ- matrix $= [C \quad CA \quad CA^2]^T$

Can check

ctrb (A, B) & obsv (A, C)

Soln :

(a)

$$C = \begin{bmatrix} 0 & 0 & -1 \\ 1 & 1 & 0 \\ 0 & -1 & -1 \end{bmatrix}, \quad O = \begin{bmatrix} 0 & 0 & 1 \\ -1 & -1 & 0 \\ -2 & -1 & -2 \end{bmatrix}$$

$\boxed{R=3}$ $\boxed{R=3}$

(b)

$$C = \begin{bmatrix} 0 & 0 & 4 \\ 4 & 4 & 4 \\ 0 & -4 & 0 \end{bmatrix} \quad O = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & -2 \\ 0 & 2 & 2 \end{bmatrix}$$

$\boxed{R=3}$ $\boxed{R=3}$

Matrix has full rank $\Rightarrow$ Controllable ✓ & Observable ✓

2. Correct statement $\Rightarrow$ (iii)

3. Soln :

$$O = \begin{bmatrix} -2 & 0 & 1 \\ 6 & 6 & -2 \\ 8 & -74 & -22 \end{bmatrix}$$

4. Soln

$$C = \begin{bmatrix} 0 & -4 & 266 \\ 8 & 34 & 334 \\ 2 & -44 & 164 \end{bmatrix}$$

5. State space obtain $n = [5 \quad 5]$, $d = [1 \quad 2 \quad 1]$

$[A, B, C, D] = \text{tf2ss}(n, d)$

Find $\quad$ Contro = $[B \quad AB]$ $\quad$ Obsv = $[C \quad CA]^T$

$$\text{Control} = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix} \qquad \text{Obsv} = \begin{bmatrix} 5 & 5 \\ -5 & -5 \end{bmatrix}_{2\times 2}$$

$\boxed{R = 2}$ $\qquad$ $\boxed{R = 1}$

Full rank matrix $\qquad$ Rank = 1 < 2

$\therefore$ Not full rank

System is controllable but not observable.

6. Pole placement

General : $(A - BK) \Rightarrow$ closed loop poles.

① Can do

Poles = $[-5 \quad -4]$

k = place (A, B, poles) $\Rightarrow$ in MATLAB

② Manual,

poles @ $-5 \; -4 \Rightarrow \quad (s+5)(s+4) = 0$

(i.e) $\begin{Bmatrix} s = -5 \\ s = -4 \end{Bmatrix}$ $\qquad s^2 + 9s + 20 = 0 \longrightarrow$ ①

$$A_{cl} = (A - BK) = \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \end{bmatrix}_{2\times 1} \begin{bmatrix} k_1 & k_2 \end{bmatrix}_{1\times 2} = \begin{bmatrix} 1-k_1 & 2-k_2 \\ 1 & 1 \end{bmatrix}$$

$$\text{eig}(A_{cl}) \Rightarrow |A_{cl} - sI| = \begin{vmatrix} 1-k_1-s & 2-k_2 \\ 1 & 1-s \end{vmatrix} = 0.$$

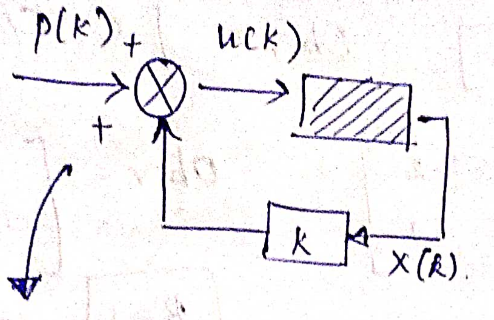
$\Rightarrow (1-k_1-s)(1-s) - (2-k_2)$

$\Rightarrow 1 - s - k_1 + k_1 s - s + s^2 - 2 + k_2$

$\Rightarrow s^2 + s(k_1 - 2) + (k_2 - k_1 - 1) = 0. \longrightarrow$ ②

Comparing ① & ②, $\quad k_1 - 2 = 9 \Rightarrow \boxed{k_1 = 11 \;\&\; k_2 = 32}$

7.



$$P(k) + X.k = u(k)$$

$$\therefore \dot{X} = AX + Bu = AX + B\,(p(k) + \cancel{X}\cdot k)$$

$$= AX + Bp(k) + Bk.\cancel{X}$$

$$= \underbrace{(A+Bk)}_{A_{closed}}X + Bp(k)$$

$\therefore$ $(A+Bk) \rightarrow$ eigen values to be found.

<u>Similar to p6 except</u>

① place (A, −B, poles)

② eig values of $(A+Bk)$ & compare with $(s+\lambda_1)(s+\lambda_2)(s+\lambda_3)=0$

<u>Soln</u> : $K = [4 \;\; 1.5 \;\; 0]$

8. <u>Interacting tank,</u>

$$q = \frac{h}{R} \qquad q_{in} - q_1 = A_1 \cdot \frac{dh_1}{dt}$$

① —— $$q_{in} - \frac{(h_1 - h_2)}{R_1} = A_1 \cdot \frac{dh_1}{dt}.$$

② —— $$\frac{h_1 - h_2}{R_1} - \frac{h_2}{R_2} = A_2 \frac{dh_2}{dt}.$$

$$\begin{bmatrix} dh_1/dt \\ dh_2/dt \end{bmatrix} = \begin{bmatrix} -1/A_1R_1 & 1/A_1R_1 \\ 1/R_1A_2 & -1/A_2\left(1/R_2 + 1/R_1\right) \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} + \begin{bmatrix} 1/A \\ 0 \end{bmatrix} q_{in}$$

Output : $h_2 = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix}$

Soln : (sub values) $A = \begin{bmatrix} -1/2 & 1/2 \\ 1/2 & -1 \end{bmatrix}$ $B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $C = \begin{bmatrix} 0 & 1 \end{bmatrix}$

Can use ss2tf $\Rightarrow$ $tf = \dfrac{0.5}{1s^2 + 1.5s + 0.25}$ (or equivalent)

ctrb (A, B) $= \begin{bmatrix} 1 & -0.5 \\ 0 & 0.5 \end{bmatrix}$, $R = 2$, controllable ✓

obsv (A, C) $= \begin{bmatrix} 0 & 1 \\ 0.5 & -1 \end{bmatrix}$, $R = 2$, Observable ✓

Note : Can represent by more than one state space model, every form is evaluated.

9. Use,

→ ctrb (A, B) & obsv (A, C)

→ Full rank matrix. (Controllable & observable) ✓

Note : As it was not mentioned explicitly the interactions or effects of each input with output, individual effects of inputs were analyzed. And so both the cases were considered this time.

ASSIGNMENT-4

Codes are same as explained in class. Final answers :

1. 10th step : 0.4921

2. $x_1 = 0.9$, $x_2 = 0.85$, $x_3 = 0.934$, $x_4 = 0.951$

3. k required $= [0.8 \quad 0.4]$

4. Note : This problem doesnot require y actual. Even when used, the output for k would be same. As k is unaffected by it.

Given, $A, B, C, D \rightarrow$ SS., $Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $R = 5$.

Assume : $P_0 \Rightarrow \begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix}$, provided a is very high value so that KF converges.

→ Taking $a = 0$ or very low value is not a good assumption.

Step 1 : $P_k^- = A P_{k-1} \cdot A^T + Q$

Step 2 : $K_k = P_k^- \cdot C^T (C . P_k^- C^T + R)^{-1}$

Step 3 : $P_k^+ = (I - K_k \cdot C) P_k^-$

→ Do the above steps to get K_k & repeat it for n. specified times.

(i) $\underline{\underline{K_K}}$

$$K_1 = \begin{bmatrix} 0.5286 \\ -0.0585 \end{bmatrix}$$

$$K_2 = \begin{bmatrix} 0.5528 \\ -0.1097 \end{bmatrix}$$

$$K_3 = \begin{bmatrix} 0.3827 \\ -0.0289 \end{bmatrix}$$

$$K_4 = \begin{bmatrix} 0.3636 \\ -0.0081 \end{bmatrix}$$

$$K_5 = \begin{bmatrix} 0.3534 \\ -0.0002 \end{bmatrix}$$

$$K_6 = \begin{bmatrix} 0.3506 \\ 0.0024 \end{bmatrix}$$

$$K_7 = \begin{bmatrix} 0.3496 \\ 0.0032 \end{bmatrix}$$

$$K_8 = \begin{bmatrix} 0.3492 \\ 0.0036 \end{bmatrix}$$

$$K_9 = \begin{bmatrix} 0.3491 \\ 0.0037 \end{bmatrix}$$

$$K_{10} = \begin{bmatrix} 0.3491 \\ 0.0037 \end{bmatrix}$$

(ii) Assume

$$P_0 = \begin{bmatrix} 1000 & 0 & 0 \\ 0 & 1000 & 0 \\ 0 & 0 & 1000 \end{bmatrix}$$

$$K_1 = \begin{bmatrix} 0.9962 & 0.0019 \\ 0.9888 & -0.9907 \\ -0.0046 & 0.0046 \end{bmatrix}$$

$$K_2 = \begin{bmatrix} 0.8445 & 0.0665 \\ 0.5783 & -0.4577 \\ -0.1663 & 0.2974 \end{bmatrix}$$

$$K_3 = \begin{bmatrix} 0.7858 & 0.0735 \\ 0.4919 & -0.4469 \\ -0.1837 & 0.2998 \end{bmatrix}$$

$$K_4 = \begin{bmatrix} 0.7820 & 0.0734 \\ 0.4883 & -0.4469 \\ -0.1836 & 0.2998 \end{bmatrix}$$

$$K_5 = \begin{bmatrix} 0.7819 & 0.0734 \\ 0.4882 & -0.4469 \\ -0.1836 & 0.2998 \end{bmatrix}$$

Same for K_6 to K_{10} as K_5

Note: Depending on P_0 the initial values of K might slightly vary, but converges to K_{10} finally.

MID. SEM

1. Solution

$|A - \lambda I| = 0$

$\lambda = 0, 0, a^2 + b^2 + c^2$

$\therefore$ Positive semi definite.

2. (a) True

~~false~~.

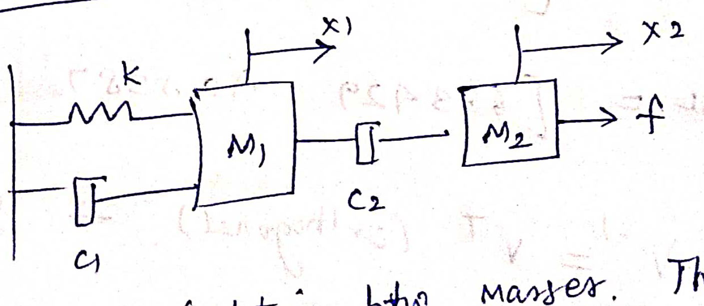
3. Can use ss2tf or

$TF = C(sI - A)^{-1}B + D$

Soln : $n = [1 \ -2 \ , \ 7]$

$d = [1 \ -2 \ 1 \ 3]$

4. Force balance.



C_2 acts as isolation btw masses. Therefore, effect of k on m_2 is negligible.

Mass1 : $m_1 x_1'' + c_1 . x_1' + c_2 (x_1' - x_2') + kx_1 = 0$

Mass 2 : $m_2 x_2'' + c_2 (x_2' - x_1') = f(t)$.

L.T. $\Rightarrow$ $m_1 s^2 X_1 = c_2 s (X_2 - X_1) - kX_1 - c_1 s X_1$

$m_2 s^2 X_2 = f(s) - c_2 s (X_2 - X_1)$

$$\Rightarrow \quad m_1 s^2 X_1 = C_2 s X_2 - C_2 s X_1 - k X_1 - C_1 s X_1$$

$$m_2 s^2 X_2 = f(s) - C_2 X_2 s + C_2 X_1 s$$

$$\Rightarrow \quad C_2 s X_2 = (m_1 s^2 + C_2 s + C_1 s + k) X_1$$

$$(m_2 s^2 + C_2 s) X_2 = f(s) + C_2 X_1 s$$

$$\Rightarrow \quad (m_2 s^2 + C_2 s) X_2 = f(s) + \frac{C_2 s \cdot (C_2 s X_2)}{m_1 s^2 + (C_1 + C_2) s + k}$$

$$\Rightarrow \quad \left(m_2 s^2 + C_2 s - \frac{C_2^2 s^2}{m_1 s^2 + C_2 s + C_1 s + k} \right) X_2 = u(s)$$

$$\Rightarrow \quad \frac{X_2}{u(s)} = \frac{m_1 s^2 + (C_2 + C_1) s + k}{(m_2 s^2 + C_2 s)(m_1 s^2 + C_2 s + C_1 s + k) - C_2^2 s^2}$$

Numerator: $m_1 s^2 + (C_2 + C_1) s + k$

$9s^2 + 0.98 + 1.$

Note: Writing equivalent forms of tf will also be considered.

5. Closed loop $(A - BK)$

$$= \begin{bmatrix} -6 & 0 \\ -1 & -7 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} k_1 & k_2 \end{bmatrix} = \begin{bmatrix} -6-k_1 & -k_2 \\ -1 & -7 \end{bmatrix}$$

Poles $\quad |sI - A_{cl}| = \left| \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -6-k_1 & -k_2 \\ -1 & -7 \end{bmatrix} \right| = 0$

① $\Rightarrow \quad s^2 + (6 + k_1 + 7) s + (42 + 7k_1 - k_2) = 0.$

② $\Rightarrow$ poles @ $-3, -5 \Rightarrow \quad (s+3)(s+3) = 0.$

$s^2 + 8s + 15 = 0 \quad \Rightarrow$ Comparing: $\boxed{k_1 = -5 \\ k_2 = -8}$

6. Non. interacting tank & $q = h/R$.

Tank 1 : $f_1 - \frac{h_1}{R_1} = A_1 \frac{dh_1}{dt}$.

Tank 2 : $\frac{h_1}{R_1} - \frac{h_2}{R_2} = A_2 \frac{dh_2}{dt}$.

$$\begin{bmatrix} h_1' \\ h_2' \end{bmatrix} = \begin{bmatrix} -1/R_1A_1 & 0 \\ 1/R_2A_2 & -1/R_2A_2 \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} + \begin{bmatrix} 1/A_1 \\ 0 \end{bmatrix} f_i.$$

Sub values : $A = \begin{bmatrix} -0.357 & 0 \\ 0.3125 & -0.625 \end{bmatrix}$ $B = \begin{bmatrix} 1/7 \\ 0 \end{bmatrix}$

$$[B, AB] = \begin{bmatrix} 0.1429 & -0.0510 \\ 0 & 0.0446 \end{bmatrix}$$

Rank = 2 ; controllable ✓

Note : Any other representation of state space Model is also considered.

8. Kalman filter

$x(1/1) = 0.4550$

$x(2/2) = 0.4819$

$x(3/3) = 0.4933$

$x(4/4) = 0.5030$

9. Kalman filter

$x(1/1) = 0.99$

$x(2/2) = 0.964$

$x(3/3) = 0.962$

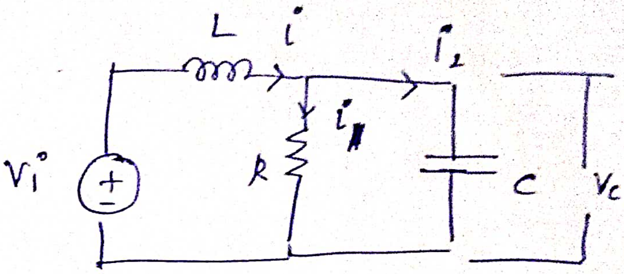
7. Electrical circuit problem

Output : $i_{Resistor}$ ⇒ given (i_1)

⇒ State variables : Depending on the variables taken, evaluation is done.

One of the solution,

(a) Variables : i & V_C.



$\therefore -V_i + V_L + V_R = 0.$

$$V_L + V_R = V_i$$

$$L.\frac{di}{dt} + i_1.R_1 = V_i \Rightarrow \boxed{\frac{di}{dt} = \frac{V_i}{L} - \frac{V_C}{L}} \rightarrow (1)$$

(Since $V_C = V_R = i_1 R_1$)

(across capacitor)

WK : $C.\frac{dV_C}{dt} = i_2$

$$i = i_1 + i_2$$

$$i = \frac{V_C}{R_1} + C.\frac{dV_C}{dt}.$$

$$\Rightarrow \boxed{\frac{dV_C}{dt} = \frac{i}{C} - \frac{V_C}{CR_1}}$$

$$\therefore \begin{bmatrix} di/dt \\ dV_C/dt \end{bmatrix} = \begin{bmatrix} -1/L & 0 \\ 1/C & -1/CR_1 \end{bmatrix} \begin{bmatrix} i \\ V_C \end{bmatrix} + \begin{bmatrix} 1/L \\ 0 \end{bmatrix} V_i.$$

(b) $\therefore$ $A = \begin{bmatrix} -1 & 0 \\ 10/3 & -10/6 \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ (c)

$$i_R = y = \begin{bmatrix} 0 & 1/R_1 \end{bmatrix} \begin{bmatrix} i \\ V_C \end{bmatrix} \Rightarrow C = \begin{bmatrix} 0 & 1/2 \end{bmatrix}$$

(d)

Controllability : [B AB]

$$\Rightarrow \begin{bmatrix} 0 & -1 \\ 1 & 10/3 \end{bmatrix} \quad [\text{Det} \neq 0]$$

Rank : 2.

$\therefore$ Controllable ✓