Qno. 1

$$A=\begin{bmatrix} -3 & -4 & 7 & -4 & 9 \\ -8 & -9 & 3 & 8 & -2 \\ -4 & -6 & 7 & -6 & -5 \end{bmatrix}$$

- Column space $\Rightarrow$ Span(col of A) = Image(A)
- Null space $\Rightarrow \{\vec{x} : A\vec{x} = 0\}$
- Row space $\rightarrow$ span (rows of A) = Image(A^T)
- Left null space $\Rightarrow$ kernel (A^T) $\{\vec{y} : A^T\vec{y} = 0\}$

$$= \begin{bmatrix} 1 & 2 & 0 & 2 & 14 \\ 0 & 7 & 3 & 24 & 110 \\ 0 & 2 & 7 & 2 & 51 \end{bmatrix} \quad \begin{matrix} R_1 \to R_1 - R_3 \\ R_2 \to R_2 + 8R_1 \\ R_3 \to R_3 + 4R_1 \end{matrix}$$

$$= \begin{bmatrix} 1 & 2 & 0 & 2 & 14 \\ 0 & 1 & -18 & 18 & -43 \\ 0 & 0 & 43 & -34 & 137 \end{bmatrix} \quad \begin{matrix} R_2 \to R_2 - 3R_3 \\ R_3 \to R_3 - 2R_2 \end{matrix}$$

$$\begin{bmatrix} 1 & 2 & 0 & 2 & 14 \\ 0 & 1 & -18 & 18 & -43 \\ 0 & 0 & \frac{43}{43} & \frac{-34}{43} & \frac{137}{43} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 36 & -34 & 100 \\ 0 & 1 & 0 & \frac{162}{43} & \frac{617}{43} \\ 0 & 0 & 1 & \frac{-34}{43} & \frac{137}{43} \end{bmatrix} \quad \begin{matrix} R_1 \to R_1 - 2R_2 \\ R_2 \to R_2 + 18R_3 \end{matrix}$$

$18 \times \frac{34}{43}$

$$\begin{bmatrix} 1 & 0 & 0 & \frac{-238}{43} & \frac{-632}{43} \\ 0 & 1 & 0 & \frac{162}{43} & \frac{617}{43} \\ 0 & 0 & 1 & \frac{-34}{43} & \frac{137}{43} \end{bmatrix} \quad R_1 \to R_1 - 36R_3$$

The rank of matrix is 3 so, Column space of A is span of pivot columns

Basis of Column space is $\left\{ \begin{bmatrix} -3 \\ 8 \\ -4 \end{bmatrix}, \begin{bmatrix} -4 \\ -9 \\ -6 \end{bmatrix}, \begin{bmatrix} 7 \\ 3 \\ 7 \end{bmatrix} \right\}$

Dimension of col space = Rank of A = 3

$Ax = 0$

$$\begin{bmatrix} 1 & 0 & 0 & \frac{-238}{43} & \frac{-632}{43} \\ 0 & 1 & 0 & \frac{163}{43} & \frac{617}{43} \\ 0 & 0 & 1 & \frac{-34}{43} & \frac{137}{43} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x_4 = s_1$ and $x_5 = s_2$

$$x_1 = +\frac{238}{43} s_1 + \frac{632}{43} s_2$$

$$x_2 = -\frac{162}{43} s_1 - \frac{617}{43} s_2$$

$$x_3 = +\frac{34}{43} s_1 - \frac{137}{43} s_2$$

$$\vec{x} = \begin{bmatrix} +\frac{238}{43} s_1 + \frac{632}{43} s_2 \\ \frac{162}{43} s_1 - \frac{617}{43} s_2 \\ +\frac{34}{43} s_1 - \frac{137}{43} s_2 \\ s_1 \\ s_2 \end{bmatrix} = \begin{bmatrix} +\frac{238}{43} \\ -\frac{162}{43} \\ +\frac{34}{43} \\ 1 \\ 0 \end{bmatrix} s_1 + \begin{bmatrix} +\frac{632}{43} \\ -\frac{617}{43} \\ -\frac{137}{43} \\ 0 \\ 1 \end{bmatrix} s_2$$

are the null spaces of A.

Basis for null space is $\left\{ \begin{bmatrix} +238/43 \\ 162/43 \\ +34/43 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} +632/43 \\ -617/43 \\ -137/43 \\ 0 \\ 1 \end{bmatrix} \right\}$

Nullity of matrix is dimension of basis for null space. i.e 2

$\boxed{\text{Nullity} = 2}$

The row space of matrix is the span of the vectors that create the rows. The basis for rowspace of A is

$$\boxed{\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ \frac{-243}{43} \\ \frac{-632}{43} \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ \frac{163}{43} \\ \frac{617}{43} \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ \frac{-34}{43} \\ \frac{137}{43} \end{bmatrix} \right\} \qquad \text{Dimension of Rowspace} = \text{Rank of A} = 3}$$

The left null space is the null space of A^T.

$$A^T = \begin{bmatrix} -3 & -8 & -4 \\ -4 & -9 & -6 \\ 7 & 3 & 7 \\ -4 & 8 & -6 \\ 9 & -2 & -5 \end{bmatrix} \begin{matrix} R_1 \to R_1 - R_2 \\ R_2 \to R_2 + 4R_1 \\ R_3 \to R_3 - 7R_1 \\ R_4 \to R_4 + 4R_1 \\ R_5 \to R_5 - 9R_1 \end{matrix} = \begin{bmatrix} 1 & 1 & 2 \\ 0 & -5 & 2 \\ 0 & -4 & -7 \\ 0 & 12 & 2 \\ 0 & -11 & -23 \end{bmatrix} \xrightarrow{R_2 \to R_2 + R_3 \pm R_1} = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & -9 \\ 0 & -4 & -7 \\ 0 & 12 & 2 \\ 0 & -11 & -23 \end{bmatrix}$$

$$\xrightarrow{R_3 + 4R_2} = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & -9 \\ 0 & 0 & -43 \\ 0 & 12 & 2 \\ 0 & -11 & -23 \end{bmatrix}$$

We can see that the rank of A^T is 3. Since rank + nullity = no of variables
rank = 3, no of variables then nullity = 0

$$\boxed{\text{The null space is } \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \text{ and nullity} = 0}$$

2. State True/False for following statements with reasoning

(ii) If all entries of A are positive, then A is positive-definite matrix.

⇒ False

Say, $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ where all elements are positive

Determinant of all possible kxk upper sub matrices.

$|1| = 1$ (+ve) $\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = 4-6 = -2$ (-ve)

The determinant is negative so, it is not positive definite.

(iii) If A is positive-definite matrix, then Inverse (A) is also positive definite matrix.

⇒ True

If A is a positive definite matrix then, all its eigenvalue are positive.

The eigenvalues of A^{-1} are inverse of eigenvalues of A i.e

$\lambda(A^{-1}) = \dfrac{1}{\lambda(A)}$ which implies it is positive definite matrix.

(i) If P is real symmetric matrix, then any two linearly independent eigenvectors of P are perpendicular.

⇒ True

Qno 4: Sum of eigenvalues of 3d matrix.

```
>> M_4 = [0 3 2.5; 3 1 0.5; 2.5 0.5 7]

M_4 =

         0    3.0000    2.5000
    3.0000    1.0000    0.5000
    2.5000    0.5000    7.0000

>> [P,D] = eig(M_4);
>> sum(D)

ans =

   -2.8146    2.7055    8.1092

>> sum(eig(M_4))

ans =

    8.0000
```

The sum of eigenvalues is 8

Qno 5: Product of eigenvalues of 3d matrix.
The product of eigenvalues is the determinant of matrix

```
M_5 =

   -2.0000   -4.0000   -6.5000
   -4.0000   -4.0000   -4.5000
   -6.5000   -4.5000   -2.0000

>> det(sym(M_5))

ans =

-17/2
```

The product of the eigenvalues is -8.5

Qno 6: Left eigen vector of 3d matrix
[V,D,W] = eig(A,B) returns full matrix W.
Columns of W are the corresponding left eigenvectors, so that W'*A = D*W'*B.

```
>> M_6 = [0 -3 3; 3 5 5; -6 8 2]

M_6 =

     0    -3     3
     3     5     5
    -6     8     2

>>
>> [V,D,W]=eig(M_6);
>> W

W =

  -0.7746 + 0.0000i  -0.7746 + 0.0000i  -0.0524 + 0.0000i
   0.1291 + 0.4282i   0.1291 - 0.4282i   0.8555 + 0.0000i
  -0.1291 - 0.4282i  -0.1291 + 0.4282i   0.5151 + 0.0000i
```

W is the left eigenvector of the matrix.

Qno 7. Singular values of matrix

Singular values of A are the square roots of the nonzero eigenvalues of ATA or AAT.

```
>> M_7 = [-8 -5 4 2; -5 -5 6 8; -4 3 4 -8]

M_7 =

    -8    -5     4     2
    -5    -5     6     8
    -4     3     4    -8

>> sqrt(eig(M_7*transpose(M_7)))

ans =

    3.0471
   10.7785
   15.4447
```

which are the singular values of the matrix.

The non zero values obtained from A^TA or the values obtained from AA^T are the singular values of matrix A.

Qno 8. Clockwise rotation of matrix

```
>> M_8 = [13 -7; 9 1]

M_8 =

    13    -7
     9     1

>> transpose(rot90(transpose(M_8)))

ans =

     9    13
     1    -7
```

which is the clockwise rotation of matrix by 90^{O}

Qno 10. Eigenvalue decomposition of M and
We calculate the trace of the inverse of the eigenvectors of matrix M.

```
>> M_10 = [128 32 120; 32 187 47; 120 47 129]

M_10 =

   128    32   120
    32   187    47
   120    47   129

>> [V,D,W] = eig(M_10);
>> V

V =

   -0.6968    0.4036    0.5929
   -0.0630   -0.8579    0.5100
    0.7145    0.3180    0.6232

>> trace(inv(V))

ans =

   -0.9315
```

Eigen value decomposition is diagonalization
i.e. inv(Eigenvector)*Matrix*Eigenvector

which is the trace of the inverse of the eigenvectors of matrix M.