Modern Control Theory

Model Predict Control - MPC – Lecture 6

Dynamic Prediction Models

Prediction model - Recap

Recap 1: Free response and forced response

- Free response –
- Check if the system with current input can reach to the set-point or
- Find out how close it can take it to the set-point or objective

- Forced response –
- Don't make any control action if the system can reach to the set-point or
- Compute control actions that can take it to the set-point or objective over and above the free response.

force $\qquad\qquad$ free $\qquad\qquad$ M control

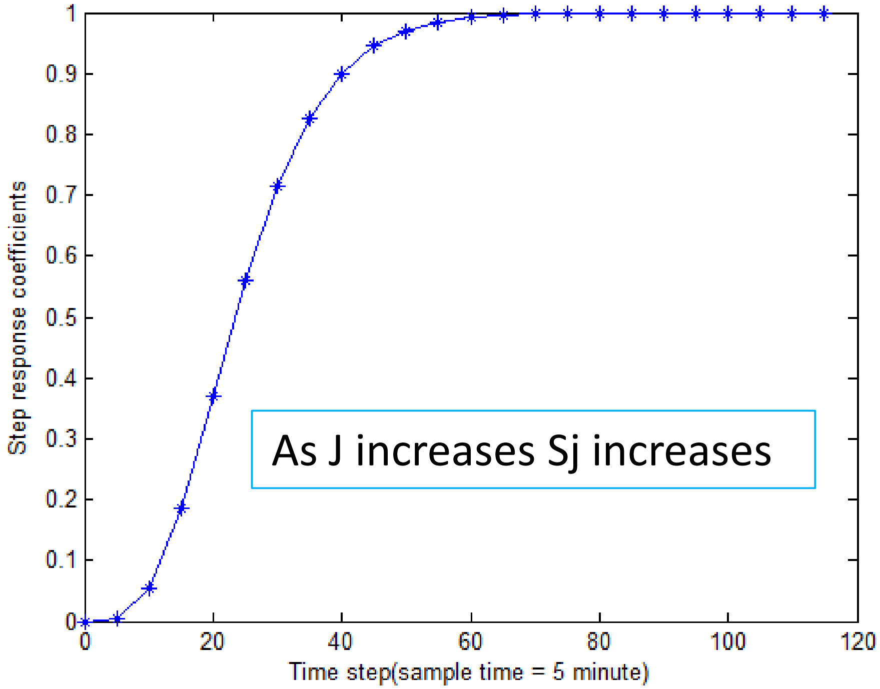
$$\hat{\mathbf{Y}}(k+1) = S\Delta\boldsymbol{U}(k) + \underbrace{\hat{\mathbf{Y}}^{o}(k+1)}_{\text{free}}$$

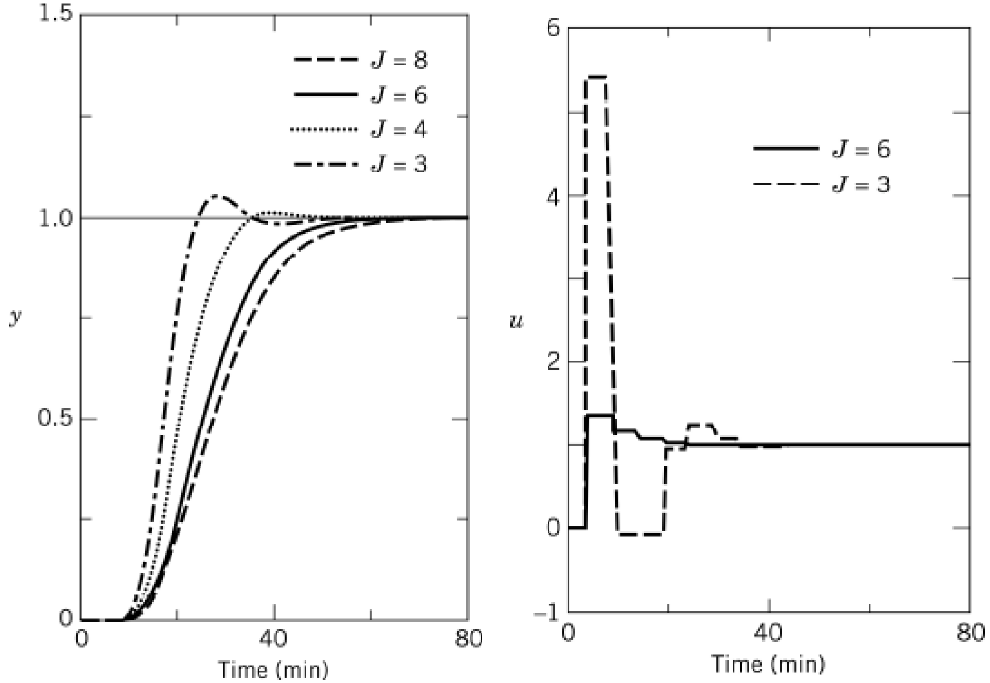
$u(k) \; \ldots \; u(k+M)$

Recap 2: Minimal look ahead or prediction required

$$\frac{Y(s)}{U(s)} = \frac{1}{(5s + 1)^5}$$

J = 3, 4, 6, 8
Δt = 5 mins





$$\boxed{\Delta u(k) = \frac{y_{sp} - \hat{y}^{o}(k + J)}{S_J}}$$

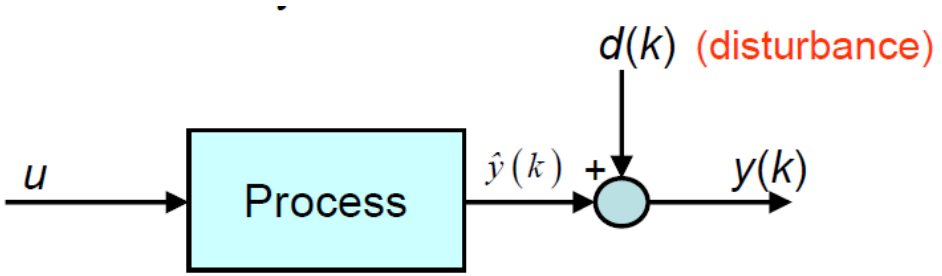
The move required tend to be smaller as J increases

As J decreases, y becomes aggressive

Recap 3: Correcting for model prediction errors

When can predictions drift away actual?

- Inaccurate model
- Unmeasured disturbances



How do we correct the model predictions?

Output feedback based on the latest measurement

$$\tilde{y}(k+j) \triangleq \hat{y}(k+j) + \underbrace{\left[y(k) - \hat{y}(k) \right]}_{\text{Bias term or correction term or Estimated disturbance}}$$

MIMO Model prediction with bias correction

'r' inputs $\mathbf{u} = [u_1 \quad u_2 \quad \cdots \quad u_r]^T$

'm' outputs $\mathbf{y} = [y_1 \quad y_2 \quad \cdots \quad y_m]^T$

$$\tilde{Y}(k+1) = S\Delta U(k) + \hat{Y}^o(k+1) + \Phi\left[\mathbf{y}(k) - \hat{\mathbf{y}}(k)\right]$$

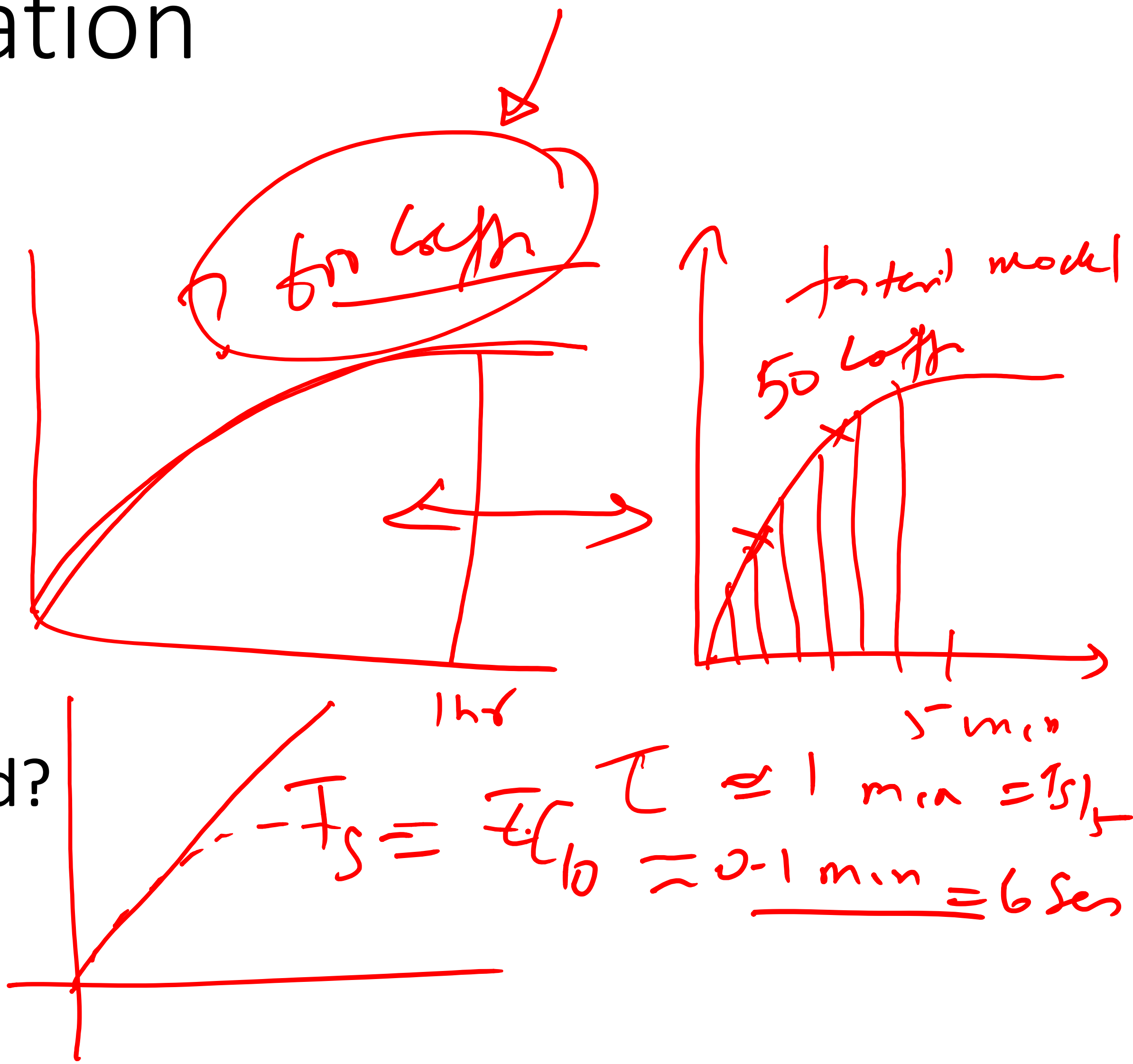
Φ is matrix of '1' with dimension mP xm

Dynamic prediction models

- Model types
  - Physics based or data-based (or empirical) models
  - Linear or non-linear relationships
- Types of Linear models
  - Impulse response coefficients
  - Step response coefficients – Most process industry implementations
  - Transfer function
  - State-space – Recent applications such autonomous vehicles, robots, satellite systems, etc

State space models - Motivation

- 5 CVs * 5 MVs
- Each model has 30 coefficients
- Total - ?
- Mixed time scale – what is this?
- Unstable system?
- Can I represent as step coefficients?
- Type of disturbances that can be modeled?



Prediction using State space models (1/6)

$$x(k+1) = Ax(k) + Bu(k)$$

$y(k) = Cx(k)$ State measurements

$z(k) = Hx(k)$ Output measurements H = C and we can omit *z(k)*

(A, B) are controllable

(C, A) are observable

Prediction using state space models (2/6)

$y(k) = u(k-1)$

Free resp

Forced

1st input

2nd input

$$x(k+1) = Ax(k) + Bu(k)$$

$$x_1 = Ax_0 + Bu_0 \qquad x_1 = Ax_0 + Bu_0$$

$$x_2 = Ax_1 + Bu_1 \qquad x_2 = A(Ax_0 + Bu_0) + Bu_1 = A^2x_0 + ABu_0 + Bu_1$$

$$x_3 = Ax_2 + Bu_2 \qquad x_3 = A^3x_0 + A^2Bu_0 + ABu_1 + Bu_2$$

.

.

.

$$x_N = Ax_{N-1} + Bu_{N-1} \qquad x_N = A^N x_0 + A^{N-1}Bu_0 + \cdots + ABu_{N-2} + Bu_{N-1}$$

Prediction using State space models (3/6)

Re-arranging the terms in matrix/ vector form

$$\begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{pmatrix} := \begin{pmatrix} A \\ A^2 \\ \vdots \\ A^N \end{pmatrix} x_0 + \begin{pmatrix} B & 0 & \cdots & 0 \\ AB & B & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ A^{N-1}B & A^{N-2}B & \cdots & B \end{pmatrix} \begin{pmatrix} u_0 \\ u_1 \\ \vdots \\ u_{N-1} \end{pmatrix}$$

$$X := \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_N \end{pmatrix} \qquad U := \begin{pmatrix} u_0 \\ u_1 \\ u_2 \\ \vdots \\ u_{N-1} \end{pmatrix} \qquad \Phi := \begin{pmatrix} A \\ A^2 \\ \vdots \\ A^N \end{pmatrix} \qquad \Gamma := \begin{pmatrix} B & 0 & \cdots & 0 \\ AB & B & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ A^{N-1}B & A^{N-2}B & \cdots & B \end{pmatrix}$$

F R

$$\boxed{X = \Phi x_0 + T U}$$

Forced

response

Free response

Forced response

The measurement model y

Prediction using State space models (4/6)

$$X = \Phi x_0 + TU$$

absolute 'U'

Free response

Forced response

Not in incremental of control moves

System representation

$$\boldsymbol{x_m}(k+1) = A_m \boldsymbol{x_m}(k) + B_m \boldsymbol{u}(k)$$
$$y(k) = C_m \boldsymbol{x_m}(k)$$

Difference

$$\boldsymbol{x_m}(k+1) - \boldsymbol{x_m}(k) = A_m(\boldsymbol{x_m}(k) - \boldsymbol{x_m}(k-1)) + B_m(u(k) - u(k-1))$$

Define

$$\Delta \boldsymbol{x_m}(k+1) = \boldsymbol{x_m}(k+1) - \boldsymbol{x_m}(k)$$
$$\Delta \boldsymbol{x_m}(k) = \boldsymbol{x_m}(k) - \boldsymbol{x_m}(k-1)$$

$$\Delta u(k) = u(k) - u(k-1)$$

System in incremental form

$$\boxed{\Delta \boldsymbol{x_m}(k+1) = A_m \Delta \boldsymbol{x_m}(k) + B_m \Delta u(k)}$$

Prediction using State space models (5/6)

System in incremental form

$$\Delta \boldsymbol{x}_m(k+1) = A_m \Delta \boldsymbol{x}_m(k) + B_m \Delta u(k)$$

Difference in outputs

$$y(k+1) - y(k) = C_m(\boldsymbol{x}_m(k+1) - \boldsymbol{x}_m(k)) = C_m \Delta \boldsymbol{x}_m(k+1)$$

$$\Delta y(k) = C_m A_m \Delta \boldsymbol{x}_m(k) + C_m B_m \Delta u(k)$$

$$y(k+1) = C_m A_m \Delta x_m(k) + y(k) + C_m B_m \Delta u(k)$$

Define

$$\boldsymbol{x}(k) = \begin{bmatrix} \Delta \boldsymbol{x}_m(k) \\ y(k) \end{bmatrix}$$

We get

$$\underbrace{\begin{bmatrix} \Delta \boldsymbol{x}_m(k+1) \\ y(k+1) \end{bmatrix}}_{x(k+1)} = \begin{bmatrix} A_m & 0_m^T \\ C_m A_m & 1 \end{bmatrix} \overbrace{\begin{bmatrix} \Delta \boldsymbol{x}_m(k) \\ y(k) \end{bmatrix}}^{x(k)} + \begin{bmatrix} B_m \\ C_m B_m \end{bmatrix} \Delta u(k)$$

$$\boxed{\boldsymbol{x}(k+1) = A\boldsymbol{x}(k) + B\Delta u(k)}$$

$$y(k) = \begin{bmatrix} 0_m & 1 \end{bmatrix} \begin{bmatrix} \Delta x_m(k) \\ y(k) \end{bmatrix}$$

$$\boxed{y(k+1) = C\,x(k)}$$

Prediction using State space models (6/6)

System representation

$$\boldsymbol{x}_m(k+1) = A_m \boldsymbol{x}_m(k) + B_m \boldsymbol{u}(k)$$
$$y(k) = C_m \boldsymbol{x}_m(k)$$

System in incremental form

$$\boxed{\boldsymbol{x}(k+1) = A\boldsymbol{x}(k) + B\,\Delta u(k)}$$
$$\boxed{y(k+1) = C\,x(k)}$$

State Prediction equation

$$\boxed{X(k) = \Phi\, x_0 + T\Delta U(k)}$$

$$X := \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_N \end{pmatrix} \qquad U := \begin{pmatrix} u_0 \\ u_1 \\ u_2 \\ \vdots \\ u_{N-1} \end{pmatrix}$$

$u(k)$
$-u(k-1)$
$\Delta u =$

$$\Phi := \begin{pmatrix} A \\ A^2 \\ \vdots \\ A^N \end{pmatrix} \qquad \Gamma := \begin{pmatrix} B & 0 & \cdots & 0 \\ AB & B & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ A^{N-1}B & A^{N-2}B & \cdots & B \end{pmatrix}$$

$$A = \begin{bmatrix} A_m & 0_m^T \\ C_m A_m & 1 \end{bmatrix} \qquad B = \begin{bmatrix} B_m \\ C_m B_m \end{bmatrix} \qquad C = \begin{bmatrix} 0_m & 1 \end{bmatrix}$$

Output Prediction equation

$$Y(k+1) = C\,X(k)$$

Prediction equation comparison

$h \geq 0$ future

$x_0 = 0$

$\Delta u(h) = 0$

State space prediction equation

$$X = \Phi x_0 + \cancel{T\Delta U(k)}$$

all future inputs are zero

State space Output Prediction equation

$$Y(k+1) = CX(k) = C\phi x_0 \quad \underline{\text{zero}}$$

$f(\Delta u(k)) \quad k < 0$

Step response Prediction equation

$$\hat{\mathbf{Y}}(k+1) = S\Delta\boldsymbol{U}(k) + \hat{\mathbf{Y}}^{o}(k+1) = $$

$x_1 = Ax_0 + Bu_0$

$x_2 = Ax_1 + Bu_1$

Free response = $C\,\Phi x_0 = \hat{\mathbf{Y}}^{o}(k+1)$

$$\boxed{\text{Forced response} = C \times T\Delta U(k) = S\Delta\boldsymbol{U}(k)}$$