CH5120: Assignment 1 – Linear Algebra

Note

- Submit the assignment on or before **September 10th 2022**
- Submission link for assignment will be open in Moodle.
- Ensure the filename is in the format **<Rollno.pdf>**
- Attach the codes and results for the respective questions, if MATLAB or any software is used.

1. What is the correct vector space and corresponding dimensions of the four fundamental subspaces of the matrix M (C(M), R(M), N(M), L(M))? (Note: C(M) is the column space of M, N(M) is the null space of M, R(M) is the row space of M, and L(M) is the left null space of M. Dimension of a subspace is the number of basis vectors spanning the subspace)

$$M = \begin{bmatrix} -3 & -4 & 7 & -4 & 9 \\ -8 & -9 & 3 & 8 & -2 \\ -4 & -6 & 7 & -6 & -5 \end{bmatrix}$$

2. State true/false for the following statements with reasoning.
   (i) If P is real symmetric matrix, then any two linearly independent eigenvectors of P are perpendicular
   (ii) If all entries of A are positive, then A is positive-definite matrix
   (iii) If A is positive-definite matrix, then Inverse(A) is also a positive-definite matrix.

3. For the given matrix (A), calculate exp(At) such that f(x)=exp(xt) is a characteristic polynomial of A. What is the expression for b3 if exp(At) is expressed as [a1 b1 c1; a2 b2 c2; a3 b3 c3] ?

$$A = \begin{bmatrix} 2.5 & 0.5 & 0.5 \\ -1 & 4 & 0.5 \\ 1 & -1 & 5 \end{bmatrix}$$

4. Find the sum of the eigen values of the 3-dimensional matrix

$$A = \begin{bmatrix} 0 & 3 & 2.5 \\ 3 & 1 & 0.5 \\ 2.5 & 0.5 & 7 \end{bmatrix}$$

5. Find the product of eigen values of the given 3-dimensional matrix

$$A = \begin{bmatrix} -2 & -4 & -6.5 \\ -4 & -4 & -4.5 \\ -6.5 & -4.5 & -2 \end{bmatrix}$$

6. Find the left eigen vector of the given matrix.

$$M = \begin{bmatrix} 0 & -3 & 3 \\ 3 & 5 & 5 \\ -6 & 8 & 2 \end{bmatrix}$$

7. Compute the singular values of the given matrix.

$$M = \begin{bmatrix} -8 & -5 & 4 & 2 \\ -5 & -5 & 6 & 8 \\ -4 & 3 & 4 & -8 \end{bmatrix}$$

8. Compute the matrix when M is rotated clockwise by 90°.

$$M = \begin{bmatrix} 13 & -7 \\ 9 & 1 \end{bmatrix}$$

9. For the given matrix (A), calculate exp(At) such that f(x)=exp(xt) is a characteristic polynomial of A. What is the expression for b_2 if $\exp(At) = b_0+b_1A+b_2A^2$?

$$A= \begin{bmatrix} 2.5 & 1 & -0.5 \\ 0.5 & 3 & -0.5 \\ 0.5 & -1 & 3.5 \end{bmatrix}$$

10. M is a square matrix of dimension 3. Perform the eigen value decomposition of M and calculate the trace of the inverse of the eigen vectors of matrix M.

$$M = \begin{bmatrix} 128 & 32 & 120 \\ 32 & 187 & 47 \\ 120 & 47 & 129 \end{bmatrix}$$