Time Response Specifications:

The time domain specifications are usually restricted to the step response of the underdamped system for ease in analysis. The time domain specifications usually refer to the performance indices of the step response of system. These indices are specified as part of the design requirements of the control systems. The time domain specifications usually answer following questions:

How fast the system moves to follow the input?

How Oscillatory is the response (indicative of damping) ?

How long does it take to practically reach the final value?

Delay Time t_d :

Delay time t_d is the time required for the response to reach 50% of the final value at first instance.

$$t_d = \frac{1 + 0.7\zeta}{w_n}$$

Rise Time t_r :

Time required for the response to rise from 10% to 90% of the final value for overdamped systems and 0 to 100% of the final value for underdamped systems at first instance.

We have step response of the under damped second order system as:

$$\therefore\ y(t) = 1 - \frac{e^{-\zeta w_n t}}{\sqrt{1 - \zeta^2}} \sin(w_d t + \theta)$$

Where, $\theta = \cos^{-1}\zeta = \sin^{-1}(\sqrt{1 - \zeta^2})$

To find expression for t_r :

$$\therefore\ y(t_r) = 1 - \frac{e^{-\zeta w_n t_r}}{\sqrt{1 - \zeta^2}} \sin(w_d t_r + \theta) = 1$$

$$\rightarrow \sin(w_d t_r + \theta) = 0$$

$$\therefore\ t_r = \frac{\pi - \theta}{w_d} = \frac{\pi - \cos^{-1}\zeta}{w_n\sqrt{1 - \zeta^2}}$$

Peak Time t_p :

Time required for the response to reach the peak value of the time response.

Peak Overshoot M_p :

It is normalized difference between the peak value of the time response and the steady state value

$$M_p = \frac{y(t_p) - y(\infty)}{y(\infty)} x\, 100\%$$

The amount of maximum percent overshoot directly indicates the relative stability of the system.

Peak Time and Peak overshoot are not defined for overdamped and critically damped systems.

At peak value of the response curve the first derivative with respect to time is zero. So we can obtain peak time by differentiating response $y(t)$ and equating with zero.

$$y(t_p) = 1 - \frac{e^{-\zeta w_n t_p}}{\sqrt{1 - \zeta^2}} \sin(w_d t_p + \theta)$$

$$\frac{dy(t_p)}{dt} = \frac{e^{-\zeta w_n t_p}}{\sqrt{1-\zeta^2}}\left[\zeta w_n \sin(w_d t_p+\theta) - w_d \cos(w_d t_p+\theta)\right]$$

$$\text{or, } 0 = \frac{w_n e^{-\zeta w_n t_p}}{\sqrt{1-\zeta^2}}\left[\cos(\theta)\sin(w_d t_p+\theta) - \sin(\theta)\cos(w_d t_p+\theta)\right]$$

$$\text{or, } \sin(w_d t_p) = 0$$

$$\therefore\ w_d t_p = 0, \pi, 2\pi, 3\pi, \ldots$$

Since the peak time corresponds to the first peak overshoot $w_d t_p = \pi$

Hence $t_p = \frac{\pi}{w_d}$.

Settling Time t_s :

Time required for the response to reach and stay within a specified tolerance band of its final value or steady state value. Usually the tolerance band is 2% or 5%.

Except for certain applications where oscillations can not be tolerated, it is desirable that the transient response be sufficiently fast and be sufficiently damped. Thus, for a desirable transient response of a second order system, the damping ratio must be between 0.4 and 0.8. Small value of ζ that is $\zeta<0.4$ yield excessive overshoot in the transient response and a large value of ζ that is $\zeta>0.8$ responds sluggishly. Maximum overshoot and the rise time conflict with each other. In other words both the maximum overshoot and the rise time can not be made smaller simultaneously. If one is reduced other becomes larger.