CH5120: Modern Control Theory Assignment 2

September 2022

1. A device is assumed to be running at constant wattage at all times. Assuming a uniform voltage supply, we know that the noisy reading caused the measured current values to deviate from actual value. The table shows the measured current at 10 time instances. Assuming R=0.1, initial state, $x_0 = 0$, $P_0 = 1$. Find the estimate of x at the 10th time step (k=10)

$$x_k = x_{k-1} + w_k; \quad z_k = x_k + v_k$$

k	1	2	3	4	5	6	7	8	9	10
z_k	0.5	0.52	0.49	0.53	0.45	0.54	0.48	0.47	0.46	0.53

2. Consider the following scalar system

$$x_k = x_{k-1} + w_{k-1}; \quad y_k = x_k + v_k$$

where w_k and v_k are Gaussian distributions with zero mean and variances Q = 0.001 and R = 0.1 respectively. Using Kalman filter, with the initial state and state uncertainty values as $x_{0/0} = 0$ and $P_0 = 1000$, find the estimates of x up to four-time steps i.e., $x_{1/1}$, $x_{2/2}$, $x_{3/3}$ and $x_{4/4}$ if the measured values for the four-time steps are [y1, y2, y3, y4] = [0.9, 0.8, 1.1, 1]. Round up the values to 3 decimal places.

3. .

Consider a state space model given by

$$X_{k+1} = AX_k + BU_k + \omega_k \sim N(0, Q)$$
$$Y_k = C\,X_k + v_k \sim N(0, R)$$

Where $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, B = [-0.5 \;\; 1\,]^T, C = [\,1 \quad 0]$ with $U_k = 1 \quad \forall\, k$

The measurement of Y at instances k, k+1, and k+2 are

$$\mathrm{Y(k)} = 100\,; Y(k+1) = 97.9 \; and \; y(k+2) = 94.4.$$

Starting with the estimate of $\hat{X}_{k|k} = [95 \;\; 1]^T$, with $P_{k|k} = \begin{bmatrix} 10 & 0 \\ 0 & 1 \end{bmatrix}, Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, R = 1,$

implement Kalman filter for two steps, k+1 and k+2. Calculate the Kalman gain at the K+2.

4. Find the gain sequence for the (i) first three steps using hand (ii) first ten steps using MATLAB of the Kalman filter for state estimation for the systems

(a)

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} & 1 \\ \frac{1}{2} & 0 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(k)$$

$$\mathrm{y(k)} = \begin{bmatrix} 2 & 1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$$

Q = 1; R = 5

(b)

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ x_3(k+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ \frac{1}{4} & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{bmatrix} + \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} u(k)$$

$$\begin{bmatrix} y_1(k) \\ y_2(k) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \mathrm{u(k)}$$

$$\mathrm{Q} = 5;\ \mathrm{R} = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$$