CH5120: Assignment 3 – Controllability, Observability and Pole placement

Note

- Submit the assignment on or before **September 22nd 2022**
- Submission link for assignment 3 will be open in Moodle.
- Ensure the filename is in the format **<Rollno.pdf>**
- Attach the codes and results for the respective questions, if MATLAB or any software is used.

Questions

1. Consider the state-space representation of a system described by x' = Ax + Bu and y = Cx where x is the state vector, u is the input and y is the output vector. Is the system controllable and observable?
   (i) A = [1 0 1; 1 1 1; -1 -1 0]; B = [0 ; 1 ; 0];C = [0 0 1];D = [0]
   (ii) A = [-1, 0, -1; 1, 1, 0 ; 0, -1, -1]; B = [0; 4; 0] ;C = [1, 1, 1]
   Note: Use the inbuilt MATLAB function *ctrb* and *obsv* only for verification.

2. Consider the following statements related to controllability and observability for a state-space system representation x' = Ax + Bu and y = Cx + Du and choose the correct option(s):

(i) Output plays an important role in controllability.
(ii) Observability and estimating state-space matrices A, B, C, and D are the same.
(iii) Observability deals with whether or not the initial state can be observed from the output.
(iv) Controllability studies whether or not the state of a state-space equation can be controlled from both the input and output.

Note: For questions 3&4
Continuous-time State equation : d(x(t))/dt = Ac x(t) + Bc u(t)
Continuous-time output equation : y(t) = Cc * x(t)

3. Find the observability matrix for the given state space model.
   Ac = [-5 -4 -1 ; 5 -9 -4 ; -4 -2 -4] Bc = [-5 ; 8 ; 7] Cc = [-2 0 1]

4. Find the controllability matrix for the given state space model.
   Ac = [8 1 -6 ; -8 5 -3 ; -9 -4 -6] Bc = [0 ; 8 ; 2] Cc = [-8 -2 -8]

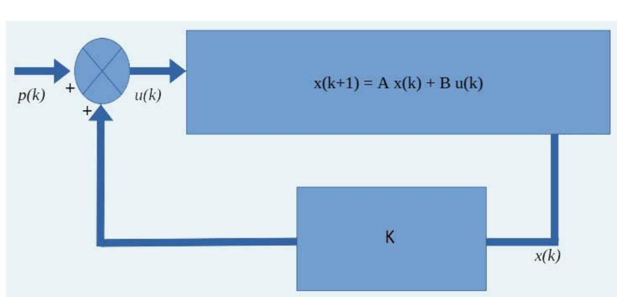
5. Is the system represented by the transfer function H(s) = (5*s + 5)/(s^2 + 2*s + 1), observable and controllable?

6. Obtain the gain matrix for the given system such that the closed loop poles are placed at -5 and -4
   A = [1 2; 1 1]; B = [1; 0]; C = [1 1]; D = [0]

7. Consider the state space model of the single input system given below:

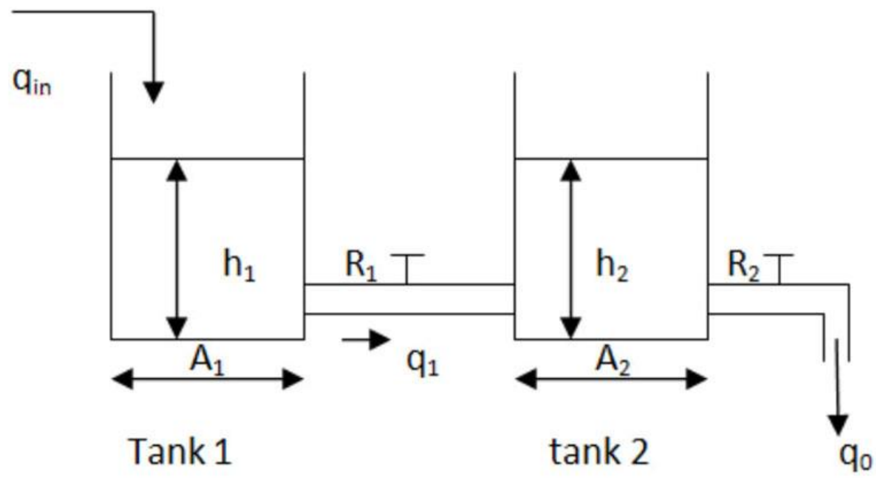
$$\begin{bmatrix} x1(k+1) \\ x2(k+1) \\ x3(k+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -4 & -2 & -1 \end{bmatrix} \begin{bmatrix} x1(k) \\ x2(k) \\ x3(k) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u(k)$$

Derive the gain matrix of the state feedback system such that the system is supposed to have the following eigenvalues. $\lambda 1 = 0$; $\lambda 2 = -0.5 - j\,0.5$; $\lambda 3 = -0.5 + j\,0.5$



8. The figure shows the interaction of two tanks. Assuming the rate of flow from each tank, q(t) = driving force (h(t))/resistance (R), find the coefficient array of the denominator of the transfer function h2(s)/qin(s) along with the controllability and observability matrix.

   Given: A1 = 1; A2 = 1; R1 = 2; R2 = 2



9. Is the system (MIMO) represented by the state space matrices controllable and observable?

   A = [0.9984, 0, 0.0042, 0; 0, 0.9989, 0, 0.0033; 0, 0, 0.9958, 0; 0, 0, 0, 0.9967]

   B = [0.0083, 0; 0, 0.0063; 0, 0.0048; 0.0031, 0]

   C = [0.5, 0, 0, 0; 0, 0.5, 0, 0]