ROLL NO: ____________________

CH5120: MODERN CONTROL THEORY
MID-SEMESTER EXAM

60 minutes **20 marks**

INSTRUCTIONS

- *Fill in the blanks and return the question paper.*
- *Answers in the question paper alone will be graded.*
- *MATLAB or any software can be used for computation.*
- *Make reasonable assumptions wherever necessary and mention it.*

Question 1: Identify whether the given matrix is positive or negative, definite or semi-definite or indefinite provided a, b and c are non-zero real values. *(1 mark)*

$$\begin{bmatrix} a^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{bmatrix}$$

Type of matrix (with reason):____

Question 2: Select the correct statement(s)
Given the state equation : x(k) = A x(k-1) + B u(k-1)+ w(k-1)
Measurement equation : y(k) = H(k) x(k)+v(k) with Q and R being the process and measurement covariance matrices. *(1 mark)*

(a) B and H can be non-square matrices but A is always a square matrix.

(b) 'A' matrix is responsible for the dilation and rotation of covariance in Kalman updation step.

(c) Kalman gain initially gives low weightage to the measured values (y measured).

(d) Larger Q than R means that the filter corrects less with measurement update

Question 3: Given the state space model:

$$x' = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -3 & -1 & 2 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 0 \\ 4 \end{bmatrix} u(t)$$

$y=\begin{bmatrix}1 & 0 & 0\end{bmatrix}x(t) + \begin{bmatrix}1\end{bmatrix}u(t)$

The equivalent transfer function: ____

The coefficient of the numerator: ____ (2 marks)

Question 4: Identify the transfer function model of the Mass-Spring-Damper model shown in the figure 1.

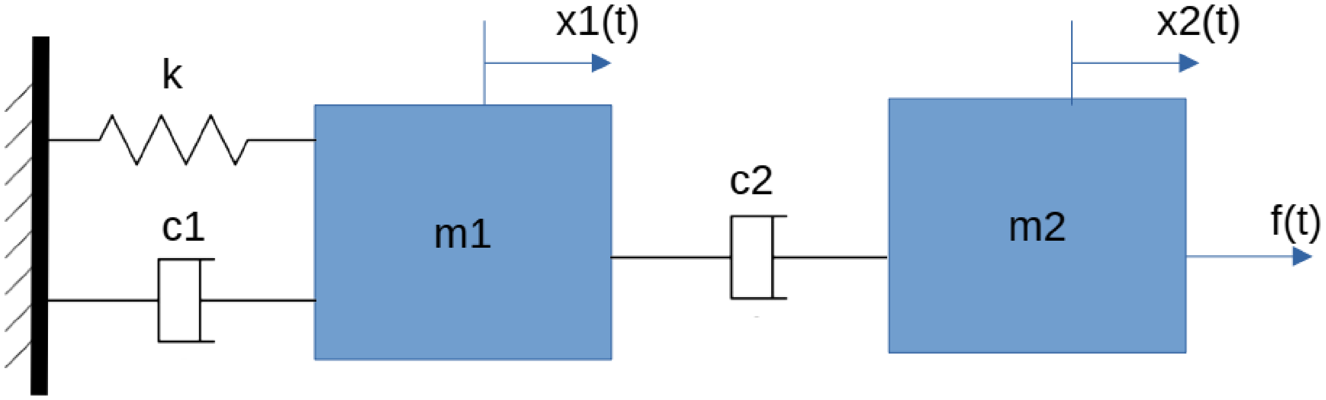

Figure 1: Question 4

Note that the input to the system is the force f(t) and the output is displacement x2(t). Consider $fk = kx$, $fc = c \cdot dx/dt$ and $fm = m \cdot d^2x/dt^2$. The force balance equation for masses would be:

Mass 1 : ____

Mass 2 : ____

Given: m1 = 9; m2 = 10; c1 = 0.5; c2 = 0.4; k = 1. Assume zero initial conditions, then the coefficient matrix of numerator in the form $as^2 + bs + c$ is:

______________________. *(4 marks)*

Question 5:

State equation: $d(x(t))/dt=\begin{bmatrix}-6 & 0 \\ -1 & -7\end{bmatrix}x(t)+\begin{bmatrix}1 \\ 0\end{bmatrix}u(t)$

Measurement equation: $y(t)=\begin{bmatrix}2 & -5\end{bmatrix}x(t)$

If the closed loop poles of the system are -3 and -5, then the gain matrix is ____________ *(1 mark)*

Question 6: The non-interacting system is given with input u(t) = q(t) and the flow rate relation as h/R, provided R, h, A, q indicates the resistance of the valve, liquid level in the tank, area of the tank and flow rate respectively. Given the parameters are A1 = 7; R1 = 0.4; A2 = 8; R2 = 0.2. Analyze the system and compute the following.

(3 marks)

(a) System matrix A = ____________________

(b) Input matrix B = ____________________

(c) Controllability matrix = ____________________

(d) Is the system controllable and why?

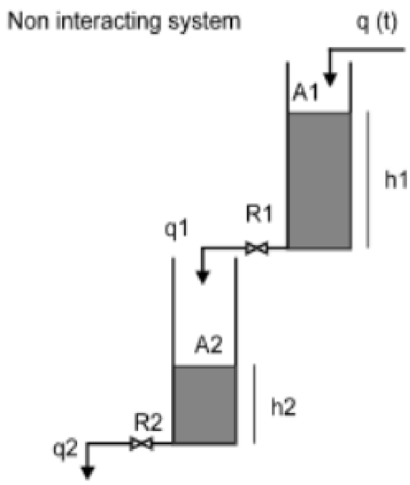

Figure 2: Question 6

Question 7: Analyze the given electrical circuit with Vi as the input voltage and the current passing through the resistor as the output. Provided the values are R = 2, C = 0.3F, L=1 H

(4 marks)
(a) State variables = ____________________

(b) System matrix A = ____________________

(c) Input matrix B = ____________________

(d) Output matrix C = ____________________

(e) Controllability matrix = ____________________

(f) Is the system controllable and why?

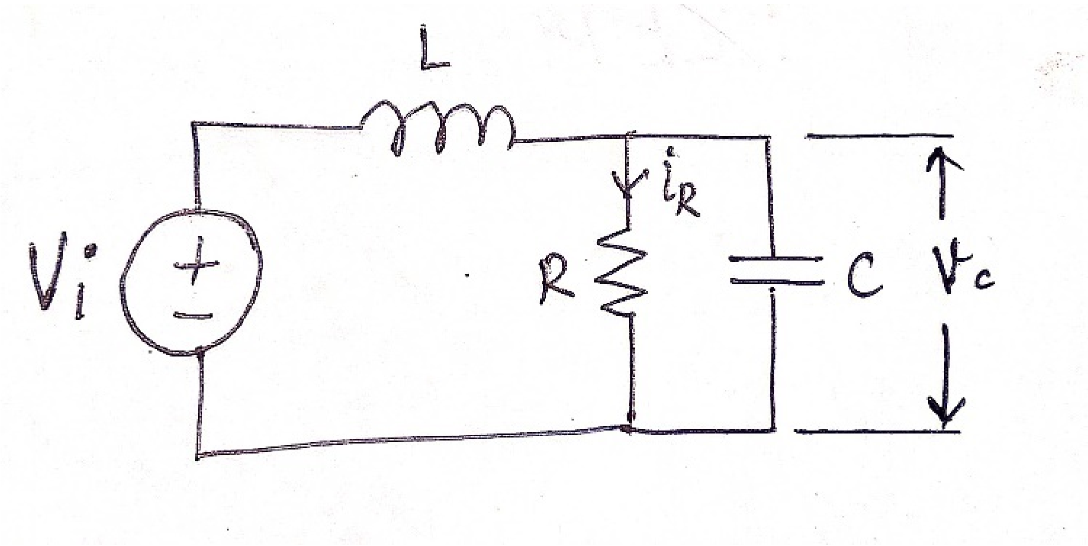

Figure 3: Question 7

Question 8: A device is assumed to be running at constant wattage at all times. Assuming a uniform voltage supply, we know that the noisy readings caused the measured current values to deviate from the actual value. System is represented as xk =xk-1 +wk and yk = xk + vk where wk and vk indicate process and measurement noise respectively. Given the values of R = 0.1, Q = 0.01, initial state, xo = 0, Po = 1 and actual measurements of response [y1 y2 y3 y4] = [0.5 0.5085 0.5123 0.5234]. Then the updated or estimates of x would be:

x(2/2) = ______________________________;

x(4/4) = ______________________________ *(2 marks)*

Question 9: There is a certain parameter that you would like to measure which is known to be constant throughout the experiments. It is measured directly but the measurements are noisy. The model that describes the process is given by: x(k+1) = x(k) + w(k); and y(k+1) = x(k+1) + v(k+1) where w(k) and v(k+1) are Gaussian distributions with zero mean and variances Q = 0.001 and R = 0.01. The initial state and state uncertainty values are x(0/0) = 0 and P = 1000. If the measured values for the three-time steps are [y(1), y(2), y(3)] = [0.99, 0.94, 0.96], the estimates of x up to the first three steps (Round off to 3 decimal places) would be:

x(1/1) = ______________________________

x(3/3) = ______________________________ *(2 marks)*