Modern Control Theory

Model Predictive Control - MPC – Lecture 2

Dynamic Prediction Models

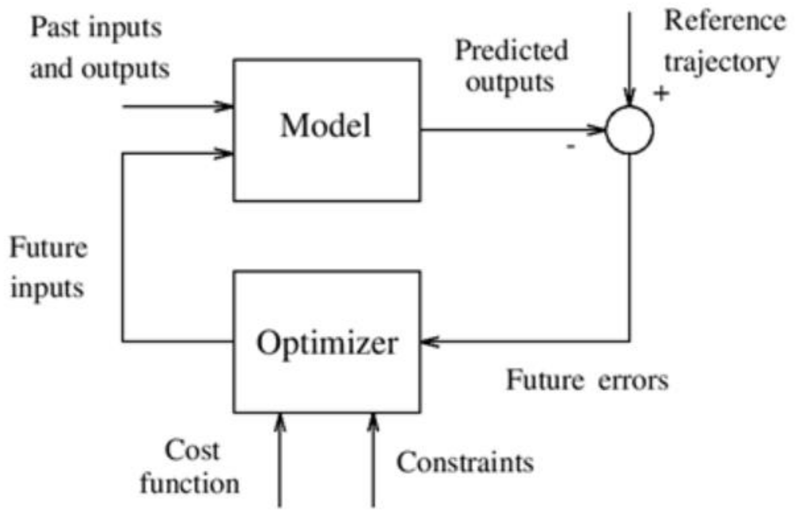
RECAP

Second definition of MPC:

MPC is a controller which uses prediction that includes the effect of past and future control actions such that to satisfy the desired objectives without violation of constraints but implements only the first time-step action at **EVERY TIME STEP**.

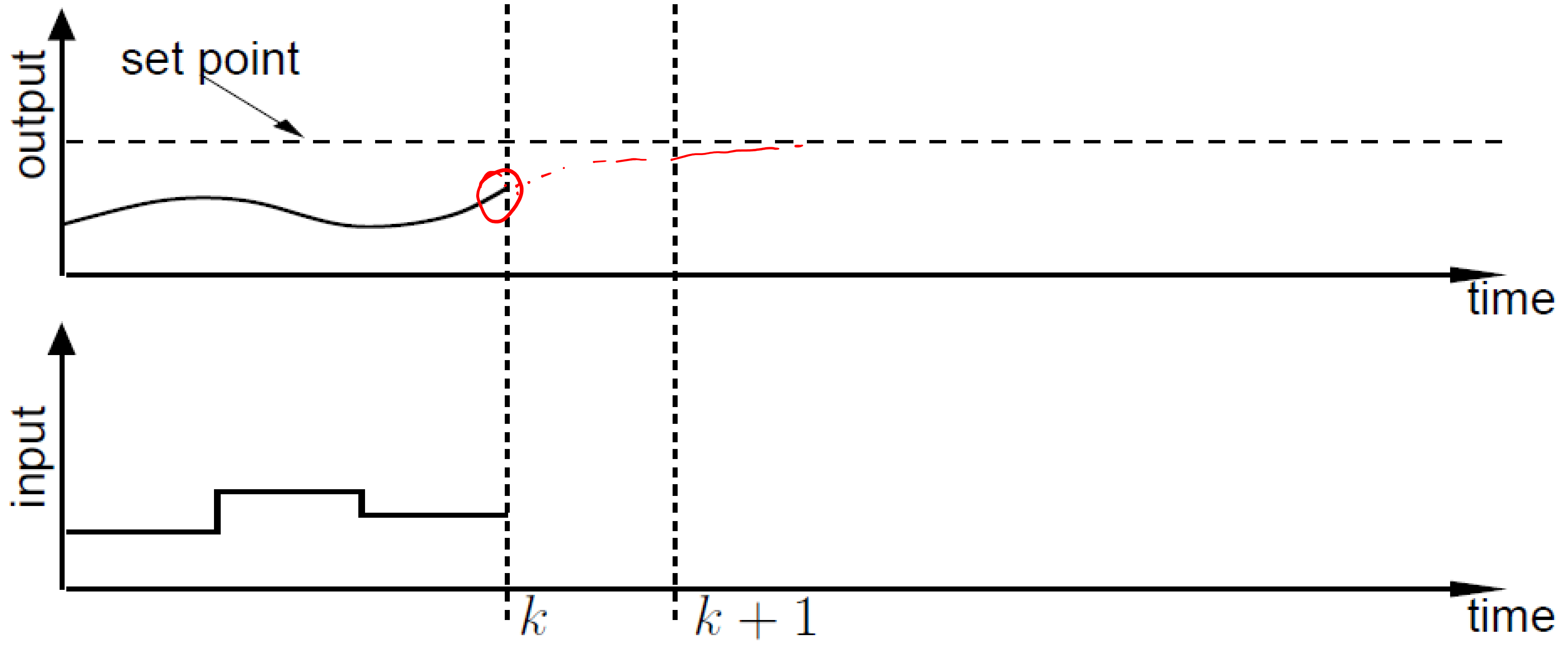
KEY ELEMENTS OF MPC

- Prediction Model
  - Effect of past actions onto the future **(implemented)**
  - Effect of current and future actions (**to be implemented**)
- **Objectives** with Constraints to be met on Input action, Output deviations
- Optimizer to obtain the control actions

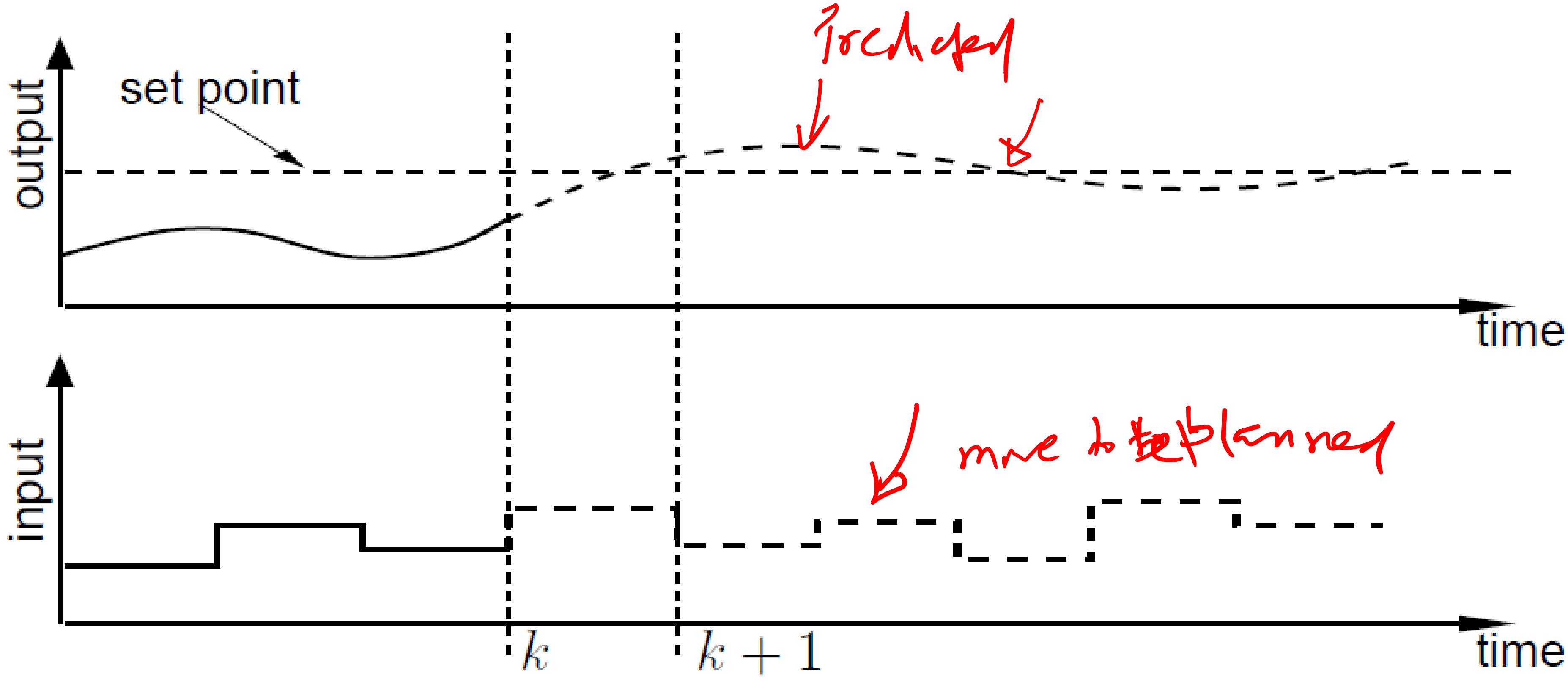


Implementing the first step and re-computing the control trajectory at each time step is called Receding Horizon control

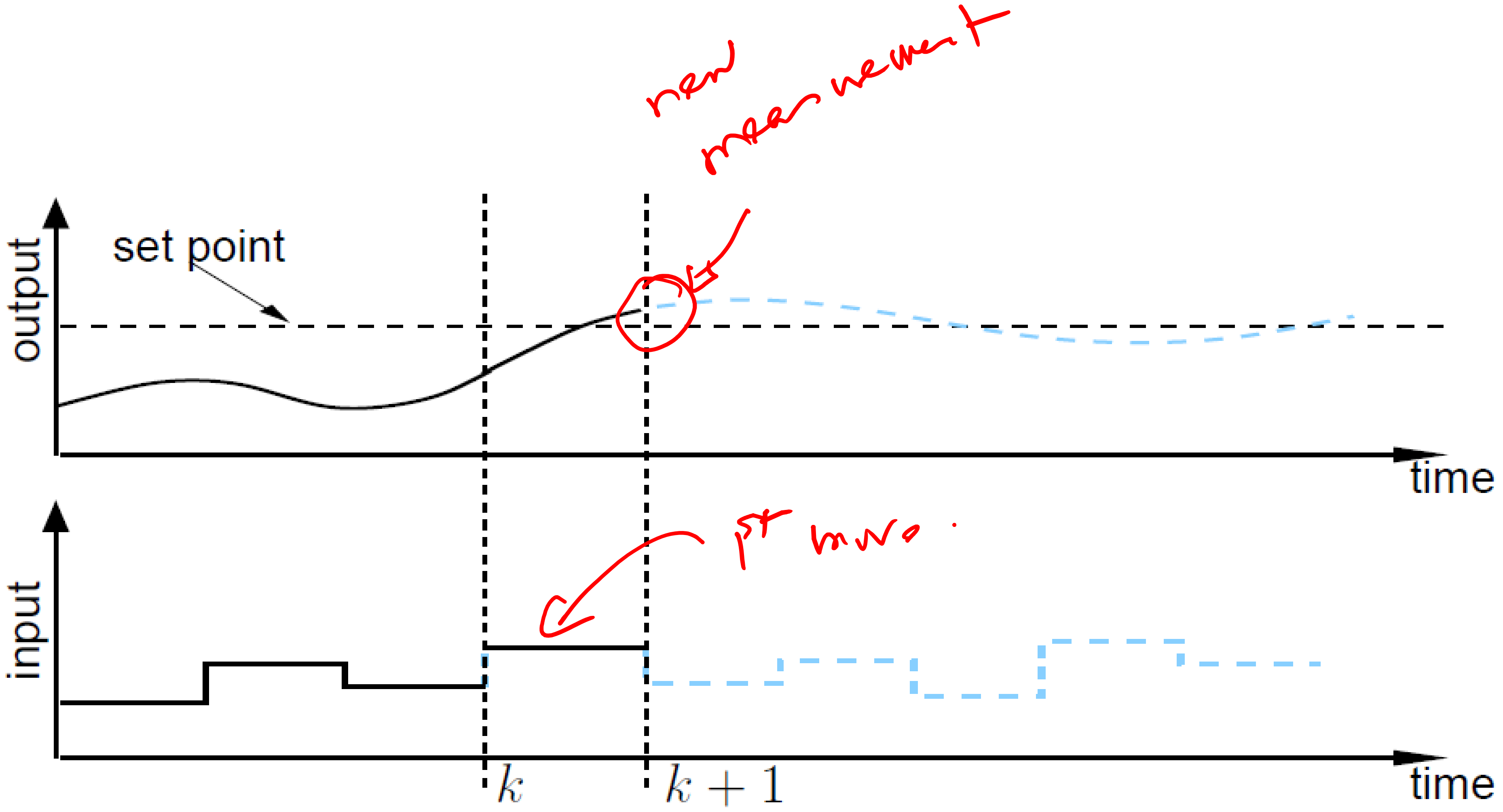
Receding Horizon Control



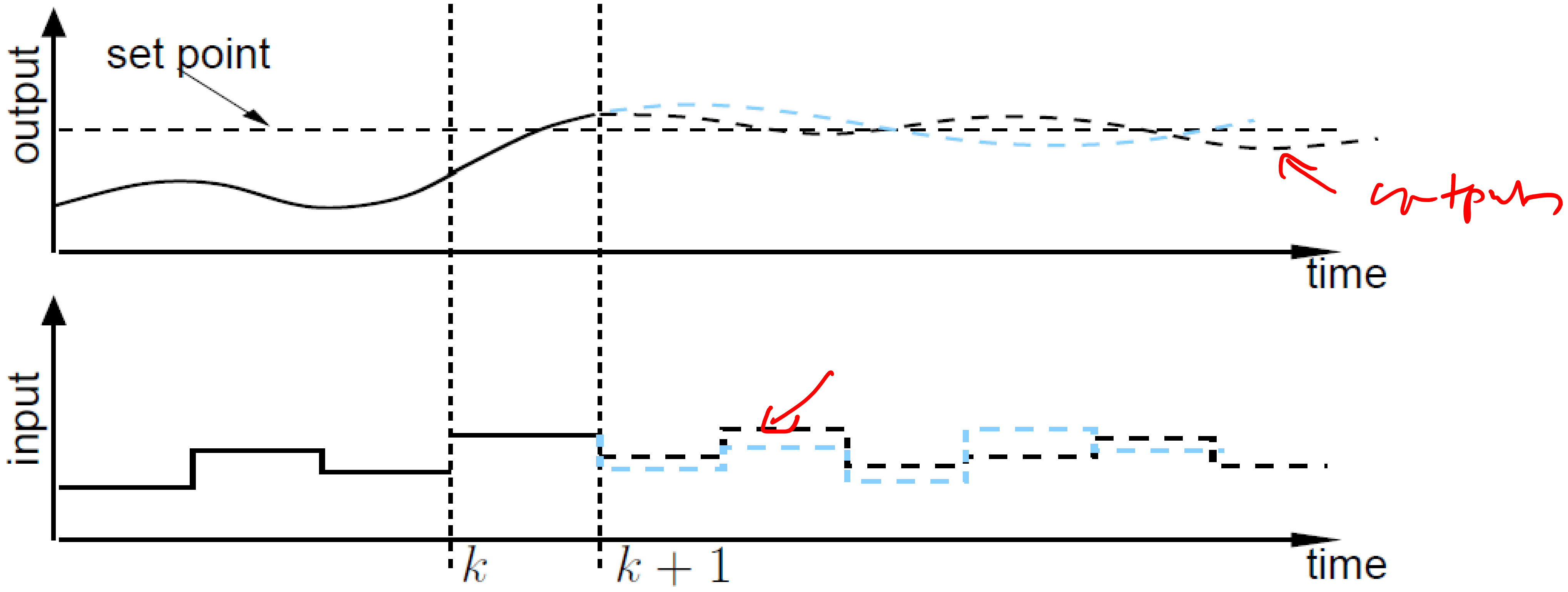
Receding Horizon Control



Receding Horizon Control



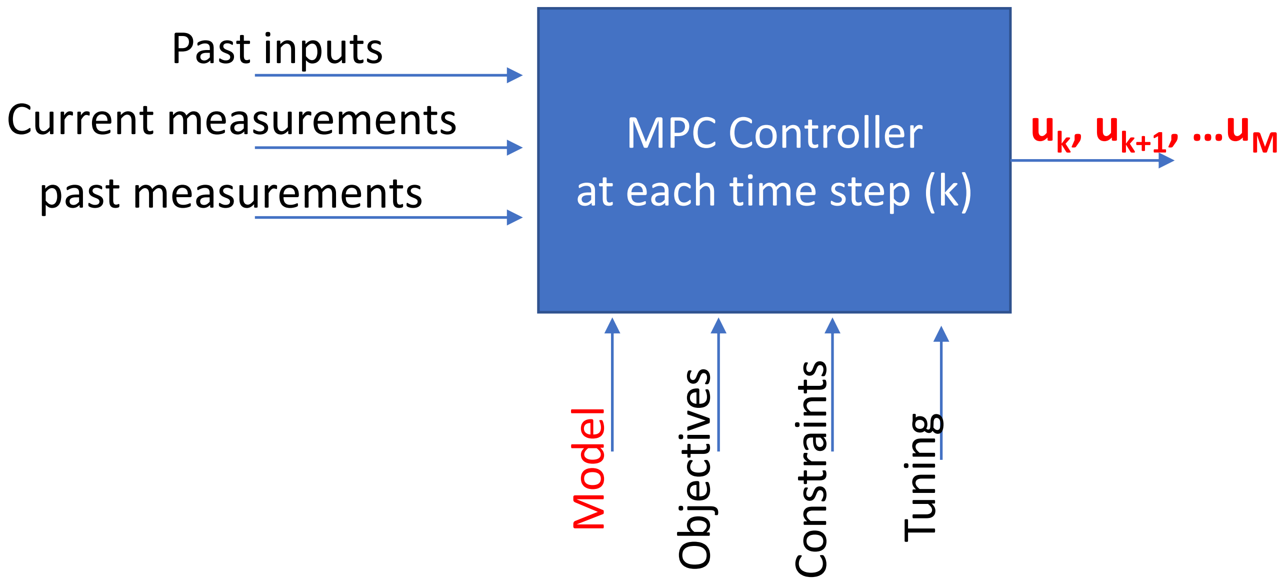
Receding Horizon Control



Receding Horizon Control/ MPC

- Implementation steps ONLINE

1. Obtain the current measurements of states
2. Compute the optimal finite horizon control sequence (u_k, u_{k+1}, …u_M)
3. Implement first move u_k
4. Repeat from Step 1



What are offline activities for a MPC controller?

- Identifying a model – system identification – 60-70% of effort
- Identifying the objectives and cost function
- Tuning

Prediction models in MPC

Dynamic prediction models

Model types

- Physics/ first principles based
- Data-based (or empirical) models

Input/ Output Relationships

- Linear (L) or
- non-linear (NL)

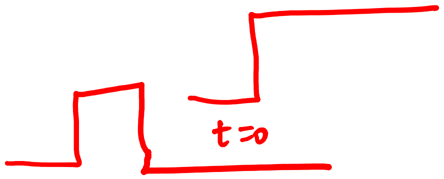
Linearizing

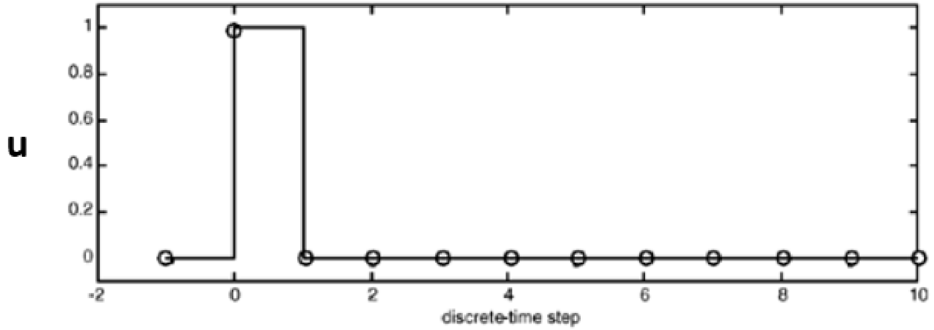
Types of LINEAR models

- Impulse response coefficients (Linear, Stable, data based)
- Step response coefficients – Large industry adoption (Linear, Stable, data based)
- State-space – Recent applications such autonomous vehicles, robots, satellite systems (Linear, Non-linear, Stable/ Unstable, physics and data based)

Discrete impulse response models

Consider a single input, single output process:





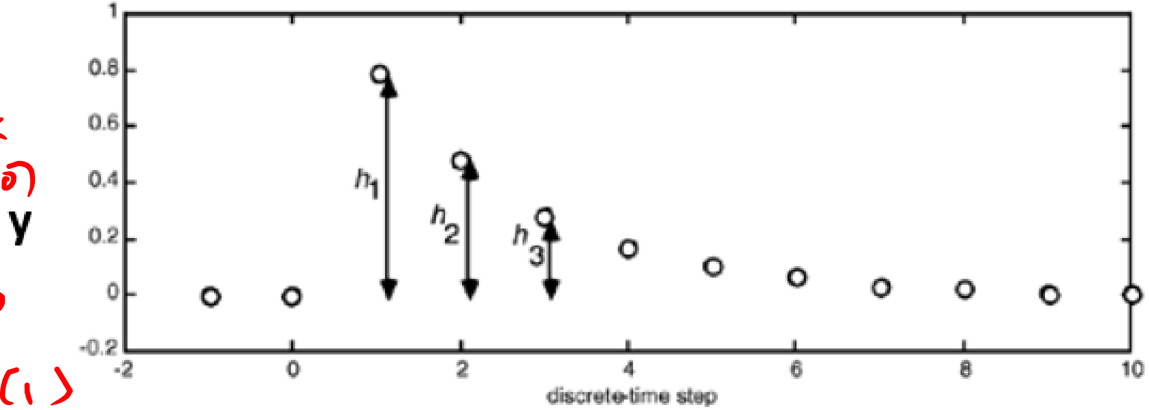
- **Definition**: impulse response is the response of a relaxed process to a unit pulse (impulse) excitation at $t = 0$

y $y(1) = u(0)$

$h(1)$

$\neq y(1)$

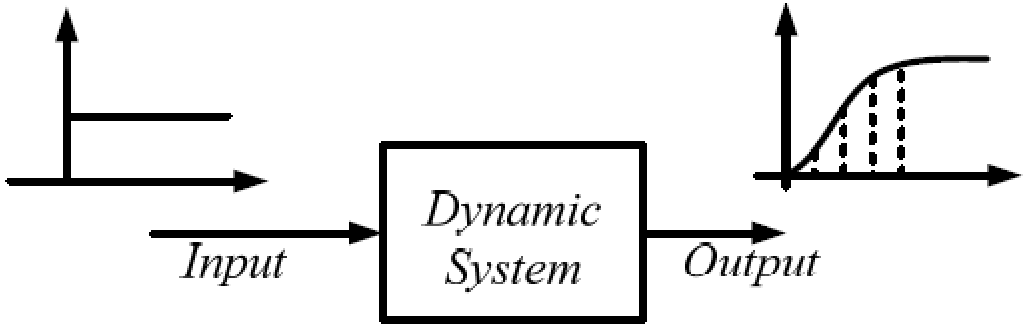
$$\{h_i\} \quad i = 0,\ 1, 2, 3, \cdots$$



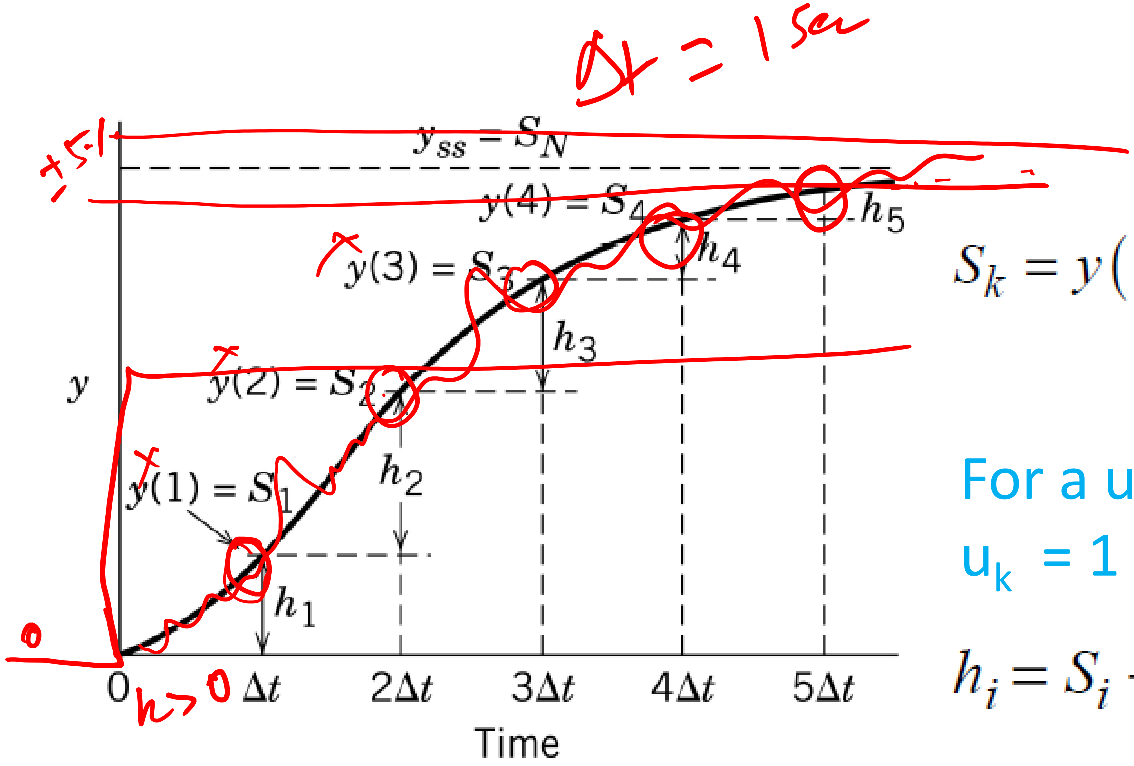
Process input-output relationship

$$y(k) = \sum_{i=0}^{\infty} h_i u(k-i) \qquad \text{(convolution summation)}$$

Step Response models (1/4)



Why are step responses popular?



$$S_k = y(k) = \sum_{i=0}^{N} h_i\, u(k-i)$$

For a unit step

u_k = 1 for all k

$$S_k = \sum_{i=0}^{k} h_i$$

$$h_i = S_i - S_{i-1}$$

$\Delta u_k = u_k - u_{k-1}$

$u_k = u_{k-1} + \Delta u_k$

$u_{k-1} = 0$ $t<0$

$u_k = 1$ $t \geq 0$

$\boxed{\Delta u_0 = u_0 - u_{-1}}$

$y_0 = 0$

$y_1 = y_0 + S_1 \Delta u_0$

$y_2 = y_0 + S_2 \Delta u_0$

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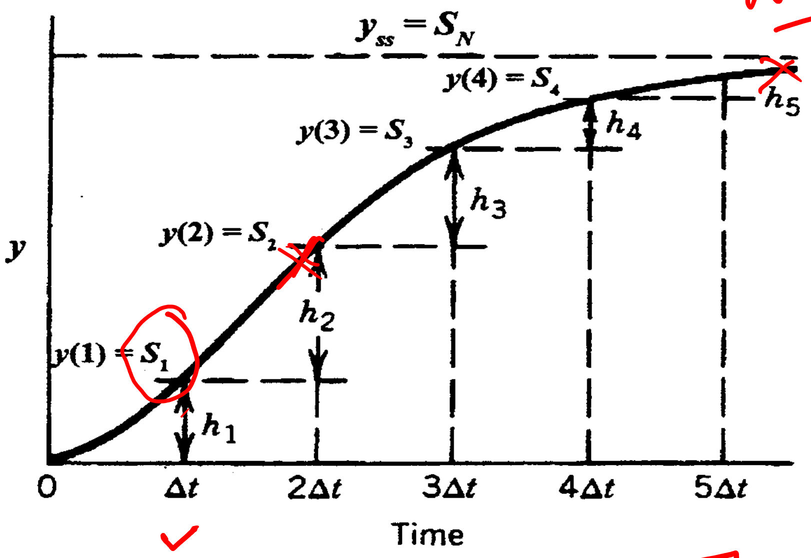
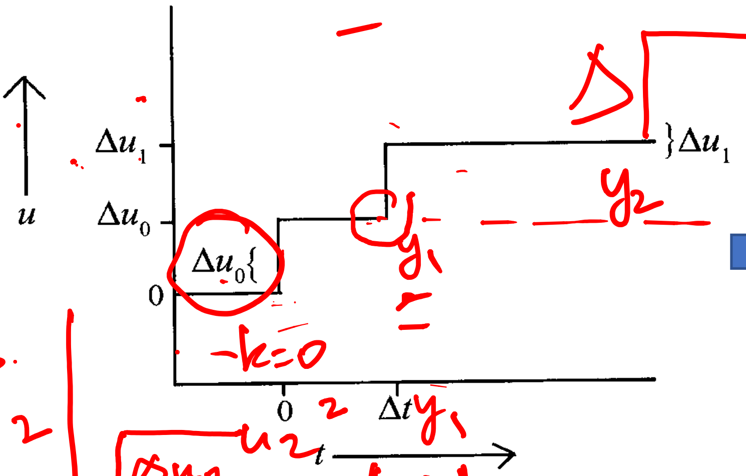
$y_n = y_0 + S_n \Delta u_0$

$\Delta u_0 = 1$

Step response predictions (2/4)





From the Principle of Superposition for linear systems:

$y_1 = y_0 + S_1\Delta u_0$

$y_2 = y_0 + S_2\Delta u_0 + S_1\Delta u_1$

$y_3 = y_0 + S_3\Delta u_0 + S_2\Delta u_1$

. .

. .

$y_N = y_0 + S_N\Delta u_0 + S_{N-1}\Delta u_1$

$$y(k+1) = y_0 + \sum_{i=1}^{N-1} S_i \Delta u(k-i+1) + S_N\, u(k-N+1)$$

Can extend also to MIMO Systems as simple Multi Input Single Output (MISO) systems

$\Delta u_1 = u_2 - u_1$

$\Delta u_2 = u_3 - u_2$

Step response predictions (3/4)

S^i = i^{th} step response coefficient

$\Delta uk = u(k) - u(k-1)$

N = prediction window or horizon

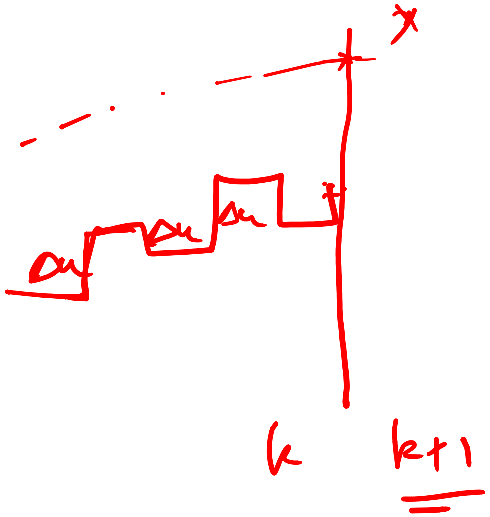
One step ahead prediction

$$y(k+1) = \sum_{i=1}^{N} h_i \, u(k-i+1) = \sum_{i=1}^{N} (S_i - S_{i-1}) \, u(k-i+1)$$

$$\Rightarrow \boxed{y(k+1) = \sum_{i=1}^{N-1} S_i \, \Delta u(k-i+1) \; + \; S_N \, u(k-N+1)}$$

$\Delta u(k)$

$$\hat{y}(k+1) = \underbrace{S_1 \Delta u(k)}_{\text{Effect of current control action}} + \underbrace{\sum_{i=2}^{N-1} S_i \Delta u(k-i+1) \; + \; S_N \, u(k-N+1)}_{\text{Effect of past control actions}}$$



Two step ahead prediction

$S_2 \Delta u(-1)$

$$\hat{y}(k+2) = \underbrace{S_1 \Delta u(k+1)}_{\text{Effect of future control action}} + \underbrace{S_2 \Delta u(k)}_{\text{Effect of current control action}} + \underbrace{\sum_{i=3}^{N-1} S_i \Delta u(k-i+2) \; + \; S_N \, u(k-N+2)}_{\text{Effect of past control actions}}$$

Step response prediction (4/4)

j^{th} step ahead prediction

future past

$$\hat{y}(k+j) = \underbrace{\sum_{i=1}^{j} S_i \Delta u(k+j-i)}_{\text{Effects of current and future control actions}} + \underbrace{\sum_{i=j+1}^{N-1} S_i \Delta u(k+j-i) + S_N u(k+j-N)}_{\text{Effects of past control actions}}$$

$\Delta u = 0$

$y_{sp} - y_{past}$

$\Downarrow$ future move

$\Downarrow$

y_{sp} Target

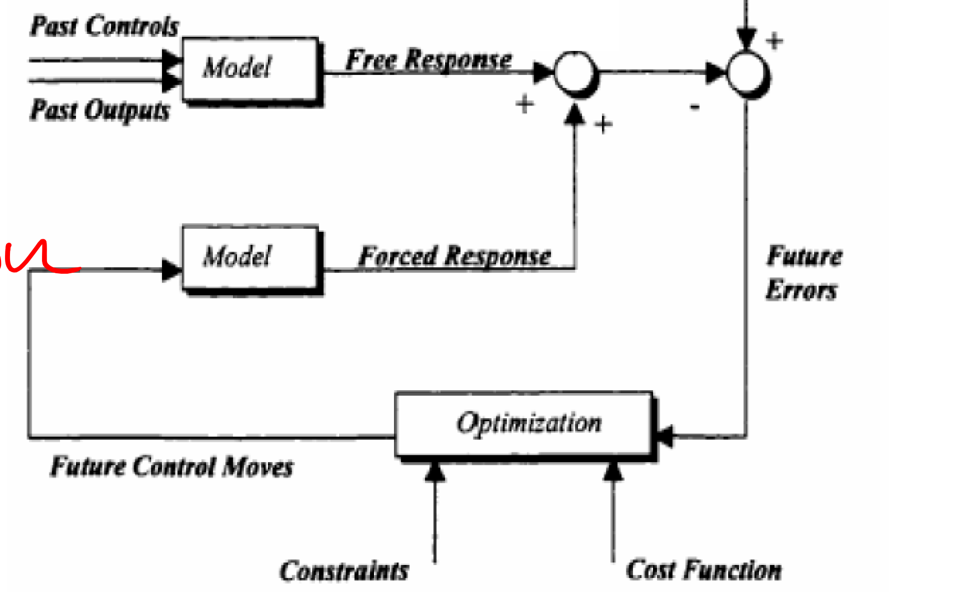
What happens when $u(k-i) = u(k-1)$ for all $i > 0$

jth step in the future

Predicted FREE response $\Rightarrow$ $\Delta u = 0$ future

$$\hat{y}^o(k+j) = \sum_{i=j+1}^{N-1} S_i \Delta u(k+j-i) + S_N u(k+j-N)$$

$$\hat{y}(k+j) = \underbrace{\sum_{i=1}^{j} S_i \Delta u(k+j-i)}_{\text{Forced response}} + \underbrace{\hat{y}^o(k+j)}_{\text{Free response}}$$



Example – simple predictive receding horizon controller

- Find a predictive control law with single receding horizon move, such that $\hat{y}(k+J) = y_{sp}$

j^{th} step ahead prediction

$$\hat{y}(k+j) = \underbrace{\sum_{i=1}^{j} S_i \Delta u(k+j-i)}_{\text{Forced response}} + \underbrace{\hat{y}^{o}(k+j)}_{\text{Free response}}$$

Given

$j = J$, $\hat{y}(k+J) = y_{sp}$

$\Delta u(k+i) = 0$ for $i > 0$, i.e. $\Delta u(k+1) = \Delta u(k+2) \ldots = \Delta u(k+J-1) = 0$

$$\Downarrow$$

$$\hat{y}(k+J) = S_J \Delta u(k) + \hat{y}^{o}(k+J)$$

$$\Downarrow$$

$$y_{sp} = S_J \Delta u(k) + \hat{y}^{o}(k+J)$$

$$\Downarrow$$

$$\Delta u(k) = \frac{y_{sp} - \hat{y}^{o}(k+J)}{S_J}$$

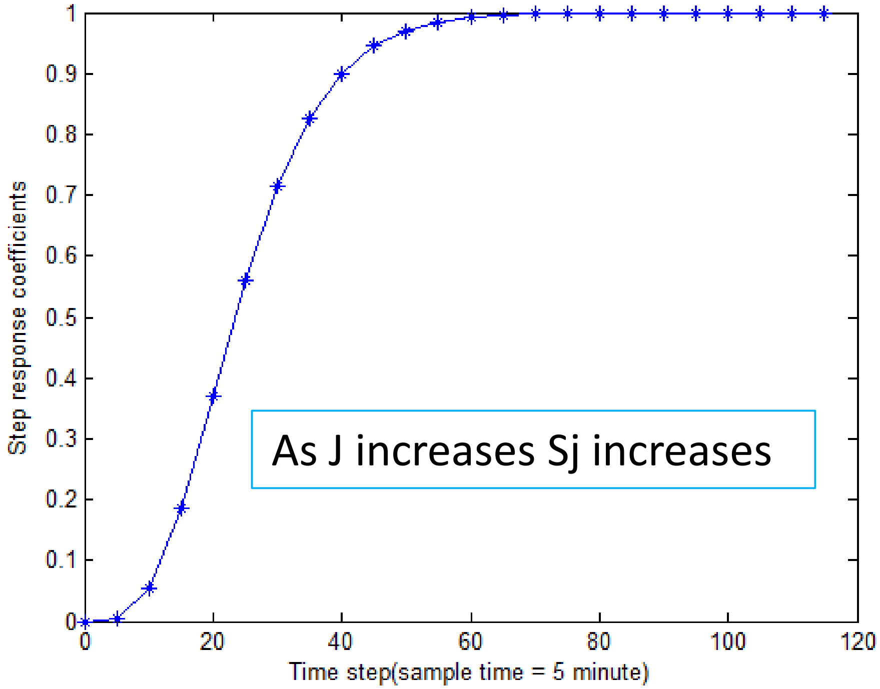
SISO system with prediction-based control law using simple step response coefficients

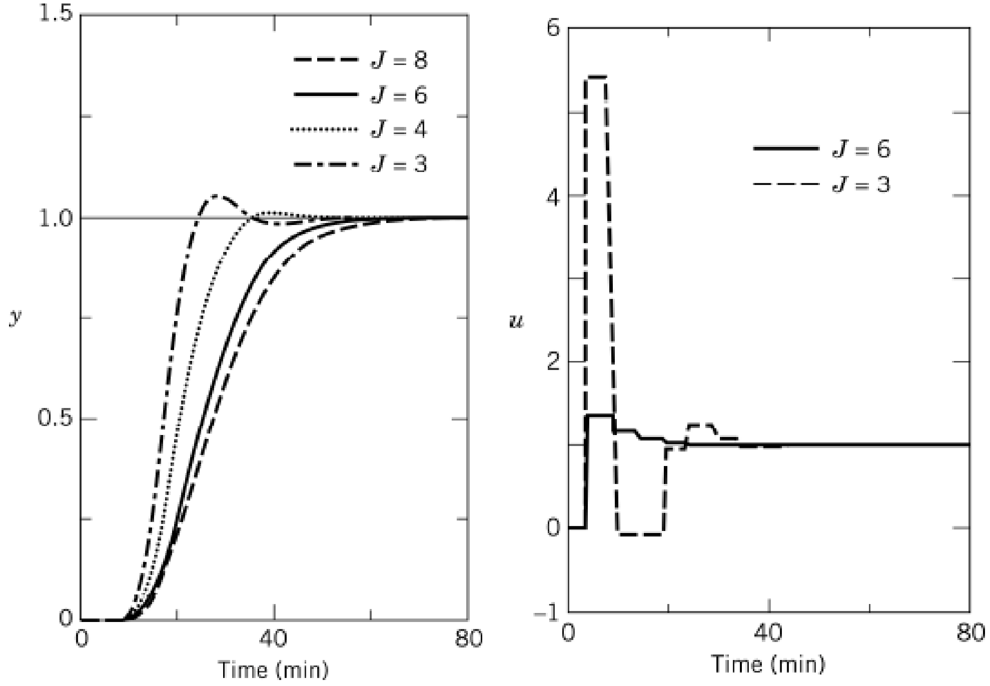
What are the pitfalls in this?

Example – simple predictive receding horizon controller

$$\frac{Y(s)}{U(s)} = \frac{1}{(5s+1)^5}$$

J = 3, 4, 6, 8
Δt = 5 mins





$$\Delta u(k) = \frac{y_{sp} - \hat{y}^o(k+J)}{S_J}$$

The move required tend to be smaller as J increases

As J decreases, y becomes aggressive

Multi input Multi output prediction models - step response (1/2)

$$\hat{y}(k+1) = \underbrace{S_1 \Delta u(k)}_{\text{Effect of current}} + \underbrace{\sum_{i=2}^{N-1} S_i \Delta u(k-i+1) + S_N\, u(k-N+1)}$$

$$\hat{y}(k+2) = \underbrace{S_1 \Delta u(k+1)}_{\substack{\text{Effect of future}\\ \text{control action}}} + \underbrace{S_2 \Delta u(k)}_{\substack{\text{Effect of current}\\ \text{control action}}} + \underbrace{\sum_{i=3}^{N-1} S_i \Delta u(k-i+2) + S_N\, u(k-N+2)}_{\text{Effect of past control actions}}$$

$$\hat{y}(k+j) = \underbrace{\sum_{i=1}^{j} S_i \Delta u(k+j-i)}_{\text{Forced response}} + \underbrace{\hat{y}^o(k+j)}_{\text{Free response}}$$

$$\begin{Bmatrix} \hat{y}(k+1) \\ \hat{y}(k+2) \\ \hat{y}(k+j) \end{Bmatrix} = \begin{Bmatrix} S_1 & 0 & \cdots & 0 \\ S_2 & S_1 & 0 & \vdots \\ \vdots & \vdots & \ddots & 0 \\ S_j & S_{j-1} & & S_1 \end{Bmatrix} \begin{Bmatrix} \Delta u(k) \\ \Delta u(k+1) \\ \\ \Delta u(k+j-1) \end{Bmatrix} + \begin{Bmatrix} \hat{y}^o(k+1) \\ \hat{y}^o(k+2) \\ \hat{y}^o(k+j) \end{Bmatrix}$$

$$\hat{\mathbf{Y}}(k+1) = S\Delta \mathbf{U}(k) + \hat{\mathbf{Y}}^{\mathbf{o}}(k+1)$$