If $L[x(t)] = X(s)$, then the inverse transform is defined as

$$x(t) = L^{-1}[X(s)] = \frac{1}{2\pi j}\int e^{st} X(s) ds$$

Every s-domain expression does not have equivalent time domain representation. This means that the inverse Laplace transform need not exist for all X(s).

Properties of Inverse Laplace Transform

Linearity

if $L^{-1}[X_1(s)] = x_1(t)$ and
$L^{-1}[X_2(s)] = x_2(t)$ then,
$L^{-1}[aX_1(s) + bX_2(s)] = ax_1(t) + bx_2(t)$

Time Shifting

$L^{-1}[X(s)] = x(t)$ then,
$L^{-1}[X(s - s_o)] = e^{s_o t} x(t)$

Example

$$L^{-1}\left[\frac{a}{s^2 + a^2}\right] = \sin(at)$$
$$L^{-1}\left[\frac{a}{(s-b)^2 + a^2}\right] = e^{bt} \sin(at)$$

Time Scaling

$L^{-1}[X(s)] = x(t)$ then,
$$L^{-1}[X(as)] = \frac{1}{|a|} x\left(\frac{t}{a}\right)$$

Time Reversal Property:

$$L^{-1}[X(s)] = x(t) \ ; \ L^{-1}[X(-s)] = x(-t)$$

Multiplication by s:

$L^{-1}[X(s)] = x(t)$ then,
$L^{-1}[sX(s)] = \frac{d\ x(t)}{dt}$ assuming x(0) = 0.

Division by s:

$$L^{-1}[X(s)] = x(t) \text{ then,}$$
$$L^{-1}\left[\frac{X(s)}{s}\right] = \int x(\tau)d\tau$$

Frequency Differentiation

$$L^{-1}[X(s)] = x(t) \text{ then,}$$
$$L^{-1}\left[\frac{dX(s)}{ds}\right] = -t\,x(t)dt$$
$$L^{-1}\left[\frac{d^n X(s)}{ds^n}\right] = (-1)^n t^n x(t)dt$$

Frequency Integration

$$L^{-1}[X(s)] = x(t) \text{ then,}$$
$$L^{-1}\left[\int_s^{\infty} X(u)du\right] = \frac{x(t)}{t}$$

Convolution:

Convolution is a mathematical way of combining two signals to get a third. Convolution is integral that measures the area overlap of one signal x(t) as it is time shifted over the other signal y(t).

$$x(t) * y(t) = \int_{\infty^-}^{\infty} x(\tau)y(t-\tau)\,d\tau = \int_{\infty^-}^{\infty} y(\tau)x(t-\tau)\,d\tau = y(t) * x(t)$$

Convolution is important tool in study of the response of a system to given input signal.

If y(t) describes the dynamics of system in time domain and if the system is subjected to a time varying input signal x(t) then the response of the system to input signal can't be directly know by algebraic multiplication of x(t) and y(t). This is because the input signal is time varying and the system dynamics is also dependent on the present and past condition. The present and past condition of the system should be considered at every instant. So integration becomes useful here. Convolution is just integration.

Convolution in s-domain:

In time domain, determining the convolution of two signals becomes very complicated depending on the signals. When the signals are transformed to s-domain, convolution becomes very easy.
In s-domain the convolution becomes simple multiplication. By convolution theorem:

$$L[x(t) * y(t)] = X(s)Y(s)$$
$$L^{-1}[X(s)Y(s)] = x(t) * y(t)$$