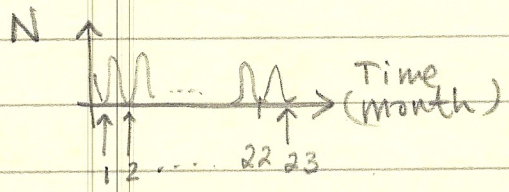
Due

Homework 6 ASTR 260 Nov 7, '14

① Plotting code is part of TakamiyaM_HW6_p1.py

a) There are 23-1 periods in the plot:



The 22 cycles span a range in time of:

$$\Delta t = 3128 - 81 = 3047 \text{ months}$$

The values in Δt, i.e. 3128 and 81 months, are read from the plot produced by python in the native interactive interface of the show() function.

Since there are 22 cycles within 3047 months, each cycle has a duration of:

$$T = 3047/22 = 138.5 \text{ months}$$

The period is thus

$$T = 138.5 \text{ months}$$

OR

$$T = \frac{138.5 \text{ months}}{12 \text{ month/yr}} = \underline{11.5 \text{ yr}}$$

The figure generated by this code is in

TakamiyaM_HW6_fig.png

1 b) The code is also in TakamiyaM_HW6_p1.py identified as 1b in the comments.
The plot is in TakamiyaM_HW6_f1b.png and in log scale (y-axis) in TakamiyaM_HW6_f1blog.png.
The first non-zero peak in the power spectrum is at $k = 24$.

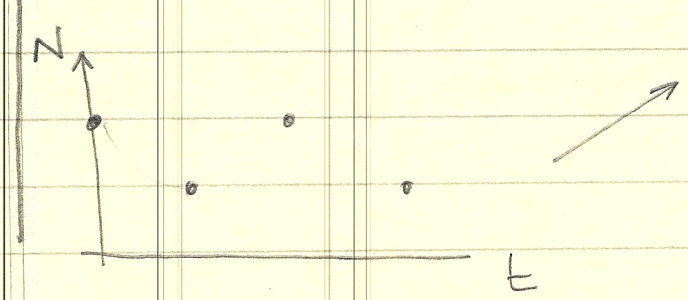
c) We find that C_{24} is the largest non-zero value in the power spectrum of the sunspots.
To obtain the corresponding frequency and thus periodicity of the data we must first relate k to f (frequency); $f = \frac{1}{T}$ where T is the period.

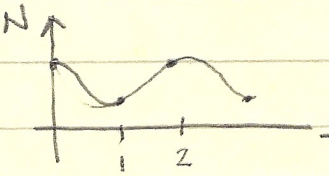
The sunspots are sampled every month for 3143 months.
The shortest frequency we can extract from the data is

$$f_{min} = \frac{1}{3143} \text{ month}^{-1} = 0.00032 \text{ month}^{-1}$$

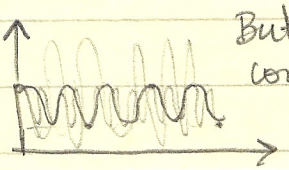
The largest frequency is twice the sampling time intervals, where the sampling interval is 1 month.

If the data looked like this





Fig①



But higher frequencies could also be a possibility.

Fig②

Unfortunately, we can't constrain frequencies higher than what is shown in Fig ①.

What is the period in Fig ①? → 2 months

" " " frequency " " ? → $f = \frac{1}{T} = \frac{1}{2}$ month^{-1}

Thus, the largest frequency is then $f_{max} = 0.5$ month^{-1}

The power spectrum plot goes from $k=0$ to $k = \frac{N}{2} = \frac{3143}{2} = 1571$, there are 1572 C_k values.

The largest k value corresponds to the largest frequency:

$$k_{max} = 1571 \iff f_{max} = 0,5 \text{ month}^{-1}$$

The smallest k value corresponds to the smallest frequency:

$$k_{min} = 0 \iff f_{min} = 0.00032 \text{ month}^{-1}$$

To know the value of the frequency at $k=24$ we divide the k range $(0-1571)$ into the frequency interval $(0.00032 - 0.5 \text{ month}^{-1})$; to determine the value α:

$$(k_{max} - k_{min})\,\alpha = (0.5 - 0.00032) \text{ month}^{-1}$$

$$\alpha =$$

And so

$$\left.\begin{aligned} f_k &= f_{min} + \alpha k \\ &= 0.00032 + 0.00032 \times k \end{aligned}\right\} f_{24} = 0.008 \text{ month}^{-1}$$

And the corresponding Period for $k=24$ is

$$T_{24} = \frac{1}{f_{24}} = 126 \text{ month} = 10.5 \text{ yr.}$$

2. Lotka-Volterra Equations

x is rabbit population in thousands
y is fox population "
Allowed values of x & y:
$x = N_x\ 0.001$ $\quad y = N_y\ 0.001$

$$\frac{dx}{dt} = \alpha x - \beta xy \qquad \frac{dy}{dt} = \gamma xy - \delta y$$

$\alpha, \beta, \gamma, \delta$ constants

a) The Runge-Kutta order 4 method rk4.py in Laulima is modified to solve both equations simultaneously. $\alpha = 1$, $\beta = \gamma = 0.5$, $\delta = 2$.
see code Takamiya M_HW6_p2.py.

b) The number of rabbits & foxes increase & decrease periodically.
First the # rabbits increases while the # of foxes decreases. The # foxes increases while the # of rabbits decrease. The two populations have the same periodicity but offset by 1 unit of time.
First the number of foxes decreases from 2000 to 700 while the " " rabbits idoubles. The # of rabbits increases even further to about 7000 after which it decreases to its original 2000.
The # of foxes doesn't go above ~4200.

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3) This program is a modification of Takamiya_HW6_p2.py to include a third dimension to $\vec{r}$.