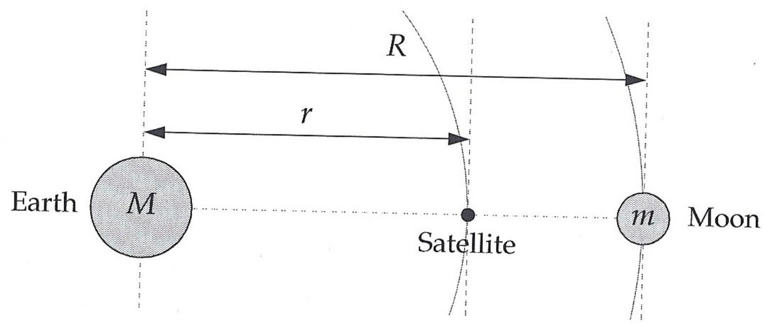
Assignment #5 due October 22 at 12pm:

1) There is magical point between the Earth and the Moon, called the L_1 Lagrange point, at which a satellite will orbit the Earth in perfect synchrony with the Moon staying always in between the two. This works because the inward pull of the Earth and the outward pull of the Moon combine to create exactly the needed centripetal acceleration that keeps the satellite in its orbit. Here is the setup:



a) Assuming circular orbits, and assuming that the Earth is much more massive than either the Moon or the satellite, show that the distance r from the center of the Earth to the L_1 point satisfies

$$\frac{GM}{r^2} - \frac{Gm}{(R-r)^2} = \omega^2 r$$

where M and m are the Earth and Moon masses, G is Newton's gravitational constant, and ω is the angular velocity of both the Moon and the satellite.

b) The equation above is a fifth-order polynomial equation in r. Such equations cannot be solved exactly in closed form, but it's straightforward to solve them numerically. Write a program that uses the Newton-Raphson method to solve for the distance r from the Earth to the L_1 point. Compute a solution accurate to at least four significant figures. You will need to choose a suitable starting value for r. Use:

$$G = 6.674 \times 10^{-11}\ m^3 kg^{-1} s^{-2}$$

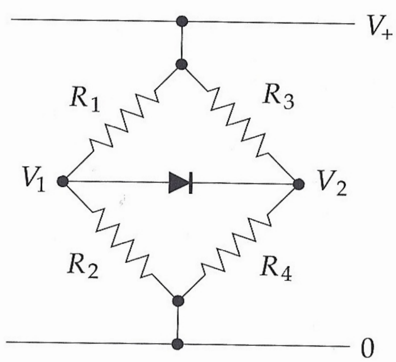
$$M = 5.974 \times 10^{24}\ kg$$

$$m = 7.348 \times 10^{22}\ kg$$

$$R = 3.844 \times 10^{8} m$$

$$\omega = 2.662 \times 10^{-6}\ s^{-1}$$

2) Consider the following simple circuit, a variation on the classic Wheatstone bridge:



The resistors obey the normal Ohm law, but the diode obeys the diode equation:

$$I = I_0(e^{V/V_T} - 1)$$

where V is the voltage across the diode and I0 and VT are constants.

a) The Kirchhoff current law says that the total net current flowing into or out of every point in a circuit must be zero. Applying the law to voltages V_1 in the circuit above we get:

$$\frac{V_1 - V_+}{R_1} + \frac{V_1}{R_2} + I_0(e^{(V_1 - V_2)/V_T} - 1) = 0$$

Derive the corresponding equation for voltage V_2.

b) Solve the two nonlinear equations for the voltages V1 and V2 with the conditions:

$$V_+ = 5V$$
$$R_1 = 1k\Omega \quad R_2 = 4k\Omega \quad R_3 = 3k\Omega \quad R_4 = 2k\Omega$$
$$I_0 = 3nA \quad V_T = 0.05V$$

Note that there are two nonlinear equations that need to be solved simultaneously.

c) The electronic engineer's rule of thumb for diodes is that the voltage across a (forward biased) diode is always about 0.6 V. Confirm that your results agree with this rule.