ASTR 260 Fall 2014 Takamiya
December 15, 2014

ASTR 260 Final Exam
Physics and Astronomy Department
University of Hawaii Hilo

All the work must be your own. No collaboration is allowed.
You may use your notes, books and internet.

Provide the 1) source codes and 2) answer to the question at the end of the problem. Leave the code in your computer station.

The student honor's code states that students should not plagiarize or cheat. If your work resembles somebody else's both students will receive a zero mark for the exam.

PROBLEM: The radiation of a black body is given by the Planck function:

$$B(\nu) = \frac{2h\nu^3}{c^2} \frac{1}{e^{\frac{h\nu}{kT}} - 1}$$

Let F be the flux radiated by the body. F is related to the Planck function in the following way:

$$F = \pi \int_0^\infty B(\nu) d\nu = \pi \int_0^\infty \frac{2h\nu^3}{c^2} \frac{1}{e^{\frac{h\nu}{kT}} - 1} d\nu$$

Using the change of variable $x = \frac{h\nu}{kT}$

show that the integral can be expressed as (5 points):

$$F = \frac{2\pi h}{c^2} \left(\frac{kT}{h}\right)^4 \int_0^\infty \frac{x^3}{e^x - 1} dx$$

Plot the integrant (7 pts). Compute the integral in F numerically. Use the Trapezoid (7 pt), Simpson rule (7 pt), and Monte Carlo (7 pt) integration methods to compute the value of the Stefan-Boltzmann constant σ given by:

$$F = \sigma T^4$$

Knowing the value of $\sigma = 5.670400 \times 10^{-8}\ \mathrm{J s^{-1} m^{-2} K^{-4}}$, which method provides the most accurate value for the integral?