Assignment #3 due October 1 at 12pm:

1) Stefan-Boltzmann constant

The Planck theory of thermal radiation tells us that in the (angular) frequency interval ω to $\omega + d\omega$, a black body of unit area radiates electromagnetically an amount of thermal energy per second equal to $I(\omega)\,d\omega$, where

$$I(\omega) = \frac{\hbar}{4\pi^2 c^2} \frac{\omega^3}{(e^{\hbar\omega/k_B T} - 1)}.$$

Here $\hbar$ is Planck's constant over 2π, c is the speed of light, and k_B is Boltzmann's constant.

a) Show that the total energy per unit area radiated by a black body is

$$W = \frac{k_B^4 T^4}{4\pi^2 c^2 \hbar^3} \int_0^\infty \frac{x^3}{e^x - 1}\,dx.$$

b) Write a program to evaluate the integral in this expression. Explain what method you used, and how accurate you think your answer is.

c) Even before Planck gave his theory of thermal radiation around the turn of the 20th century, it was known that the total energy W given off by a black body per unit area per second followed Stefan's law: $W = \sigma T^4$, where σ is the Stefan–Boltzmann constant. Use your value for the integral above to compute a value for the Stefan–Boltzmann constant (in SI units) to three significant figures. Check your result against the known value, which you can find in books or on-line. You should get good agreement.

What to turn in:

- ☐ For part a) Work out the problem starting with the energy per second function I to show the expression for W. Turn this in in class on a separate sheet of paper.
- ☐ For part b) Upload your code into laulima with the convention: LastnameFI_HW3_p1.py, where Lastname is your lastname with the first letter capitalized, FI is your first initial also capitalized and the rest should be exactly the same. E.g. for John Doe this would be: DoeJ_HW3_p1.py. Upload a README file (for John Doe this would be: DoeJ_HW3_README.txt) in ASCII (do not use any word program) with any special instructions needed to run the code. In addition, your explanation and the accuracy you achieved must be turned in on paper.
- ☐ For Part c) On paper, compare the value you obtained with your code and the accepted value of the Stefan-Boltzman constant.