Sums:

Let's say that you want to evaluate the following sum:

$$s = \sum_{k=1}^{100} k^{-1}$$

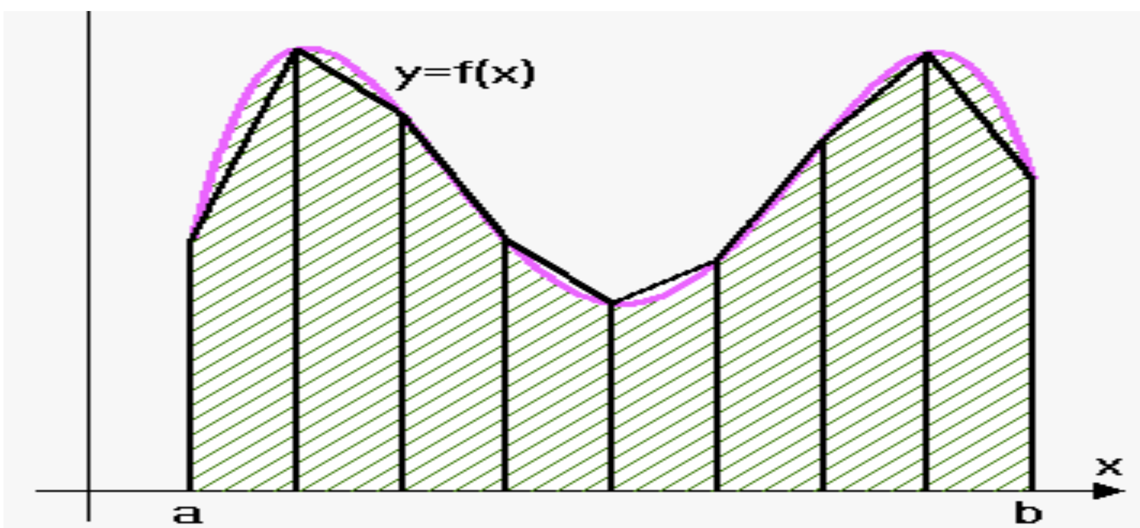
The python code will look like this:

```
s=0.
for k in range(1,101):
   s += 1/k
print(s)
```

Integration:

Now that you know how to sum, we can start integrating. Here we will explore several different methods.

Trapezoidal Rule



The procedure we must follow when using the trapezoidal rule is to divide each interval into trapezoids instead of rectangle

$$\int_a^b f(x)dx = \frac{b-a}{N}\left(\frac{1}{2}f(a) + \frac{1}{2}f(b) + \sum_{k=1}^{N-1} f(a + k\frac{b-a}{N})\right)$$

where N is the number of trapezoids and is the only free parameter in this method.

Use this method to integrate between 0 to 2, the function:

$$f(x) = x^4 - 2x + 1$$

Solve the integral with pen and paper before solving this problem numerically. How big must N be so that the analytical and numerical answers agree within 3 significant figures?

Simpson's Rule
Fitting quadratic functions instead of straight lines.

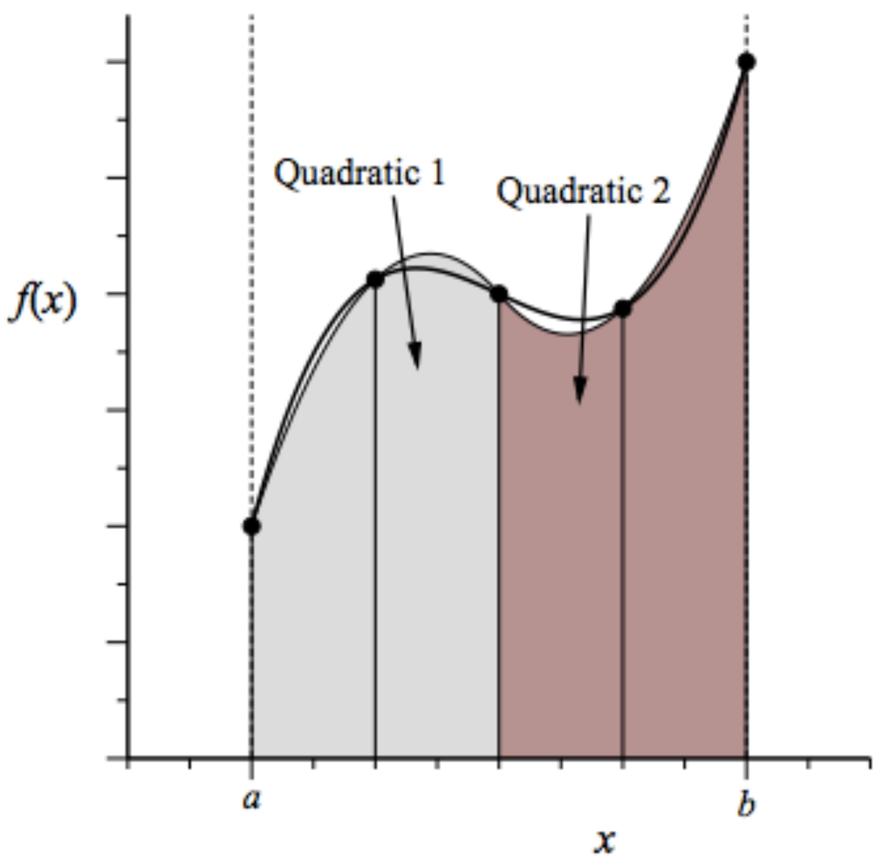


Figure 5.2 from Computational Physics by Mark Newman

Exercise 5.2:

a) Write a program to calculate an approximate value for the integral $\int_0^2 (x^4 - 2x + 1)\,dx$ from Example 5.1, but using Simpson's rule with 10 slices instead of the trapezoidal rule. You may wish to base your program on the trapezoidal rule program on page 142.

b) Run the program and compare your result to the known correct value of 4.4. What is the fractional error on your calculation?

c) Modify the program to use a hundred slices instead, then a thousand. Note the improvement in the result. How do the results compare with those from Example 5.1 for the trapezoidal rule with the same numbers of slices?

Exercise 5.3: Consider the integral

$$E(x) = \int_0^x e^{-t^2}\,dt.$$

a) Write a program to calculate $E(x)$ for values of x from 0 to 3 in steps of 0.1. Choose for yourself what method you will use for performing the integral and a suitable number of slices.