gFHR Load Follows SYSTEM Surrogate Model

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1 Nomenclature

K_p, K_s	=	Primary, Secondary pump degradation
λ_D	=	Fractional head loss when pump is fully degraded [%]
$\dot{m}_p, \dot{m}_s$	=	Primary, Secondary pump flow rate [kg/s]
n_p, n_s	=	Primary, Secondary pump rotational speed [RPM]
N_I	=	Global Iodine-135 concentration [Δk/k]
N_{Xe}	=	Global Xenon-135 concentration [Δk/k]
$P_{c,in}$	=	Core inlet pressure [Pa]
$P_{ihx,p,in}$	=	IHX primary inlet pressure [Pa]
$P_{ihx,p,out}$	=	IHX primary outlet pressure [Pa]
$P_{ihx,s,in}$	=	IHX secondary inlet pressure [Pa]
$P_{ihx,s,out}$	=	IHX secondary outlet pressure [Pa]
$\Delta P_p, \Delta P_s$	=	Primary, Secondary pump head [Pa]
Q_p, Q_s	=	Primary, Secondary pump required electrical power [W]
$Q_{D,p}, Q_{D,s}$	=	Primary, Secondary pump required electrical power with degradation [W]
$\dot{Q}_{RX}$	=	Core power [W]
$\dot{Q}_{RX,T}$	=	Target core power [W]
σ_1	=	Pump degradation uncertainty std. dev.
σ_2	=	Pump degradation trajectory uncertainty std. dev.
t	=	Time [s]
Δt	=	Time step [s]
Δt_D	=	Time when pump is fully degraded [s]
$T_{ihx,s,out}$	=	IHX secondary outlet temperature [K]
z_{ext}	=	Control rod position [m]

2 General Fluoride-Salt-Cooled High-Temperature Reactor (gFHR)

The System Analysis Modelule (SAM) [2] is a system analysis tool that provides fast, intermediate-fidelity simulations, with whole-plant transient capabilities for quick design analysis of advanced reactor concepts. One advanced reactor is the fluoride-salt-cooled high-temperature reactor (FHR) which contains a solid fuel and liquid fluoride salt coolant in the reactor core. The generic FHR (gFHR) is a publicly accessible benchmark

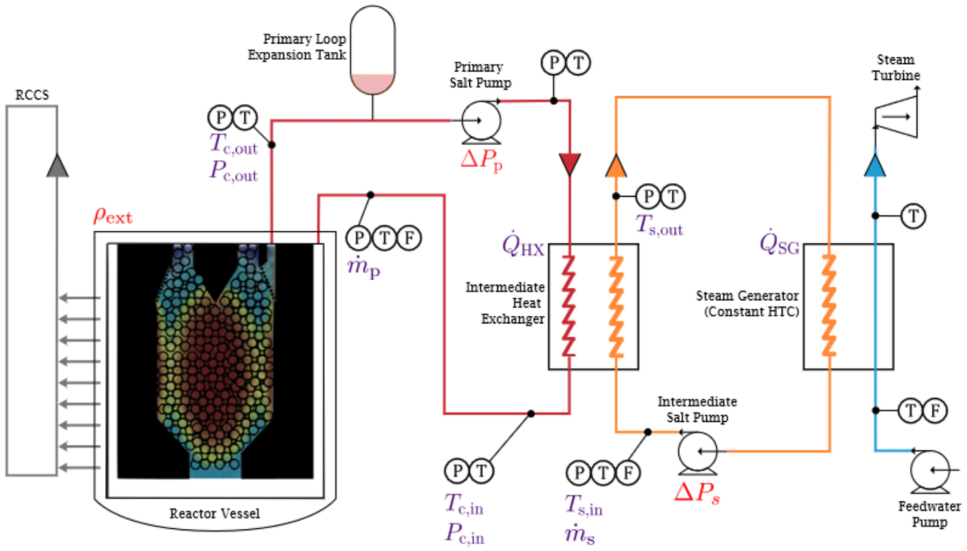

Figure 1: gFHR system: reactor configuration

model created by Kairos Power [4] that retains the main neutroincs and thermal-hydraulics information of their proprietary Karios Power FHR (KP-FHR) model.

The gFHR is modeled in SAM with the gFHR core modeled using a point kinetics model (PKE) with xenon reactivity effects. The reactor configuration, shown in Figure 1, contains the gFHR core, a primary and secondary loop with PID controllers for the primary pump, secondary pump and the reactor power. The inputs to the SAM gFHR model are the time-dependent target electric power $\left(\mathbf{t}, \dot{\mathbf{Q}}_{HX,T}\right)$ and the primary and secondary pump degradation coefficients K_p, K_s.

3 System Surrogate Models

The system surrogate model provides predictions to both physical and computational assets of the gFHR system. Physical assets include measurable quantities (e.g. temperature, pressure, mass flow rate) of the reactor's physical entities (e.g. reactor core, heat exchangers, pumps). The computational assets are all other state values such as controllers (e.g. control rod position) and un-observable variables (e.g. product concentrations, degradation). The system surrogate is to provide a system state prediction given an inputted target reactor power trajectory.

The system surrogate model is hybrid, using both physics-based and data-driven models to form its state prediction.

3.1 gFHR System Surrogate

With the System Surrogate model, a prediction of the system states is computed using the Iodine & Xenon concentration physical model, two VARMAX models and the homologous pump model. The System Surrogate model framework is shown in Figure 2.

The inputs to the System Surrogate model are the state vector values at the previous time step $\mathbf{x}^{(k-1)}$ and the

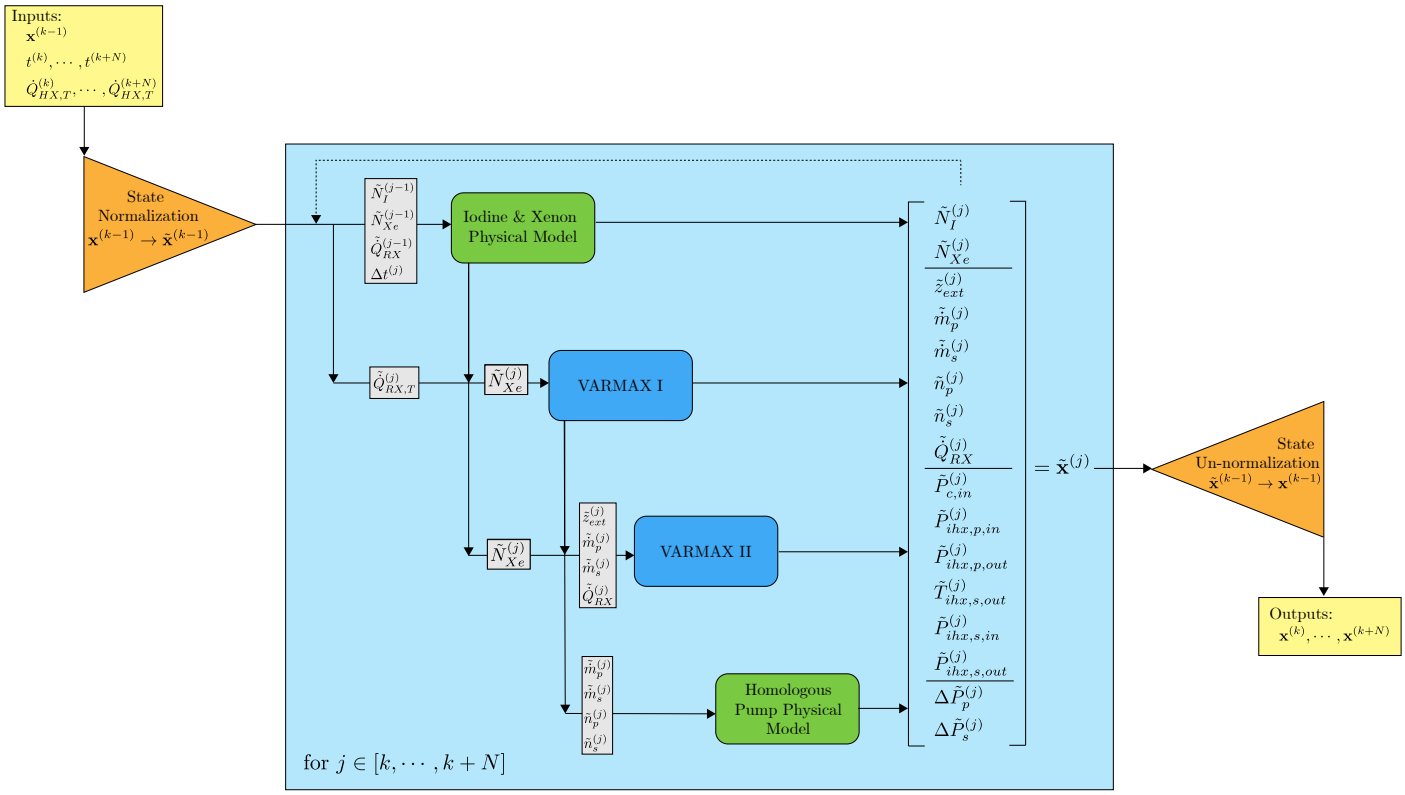

Figure 2: System surrogate framework

target electric power $\left\{\dot{Q}^{(j)}_{HX,T}\right\}_{j=k}^{k+N}$ for the next N time-steps for the time domain $\left\{t^{(j)}\right\}_{j=1}^{k+N}$. Normalization of the states $\mathbf{x}_{(k-1)}$ is done by dividing each quantities by their total values. The total values are define to be the state values when the reactor is at steady-state full-power. Once the full state vector is computed for the prediction time horizon, the state vector is un-normalized by multiplication with the total values.

3.2 gFHR System Surrogate with Pump Power Model

The inputs to the System Surrogate with Pump Power model include the inputs of the System Surrogate model which are the state vector values at the previous time step $\mathbf{x}^{(k-1)}$ and the target electric power $\left\{\dot{Q}^{(j)}_{HX,T}\right\}_{j=k}^{k+N}$ for the next N time-steps for the time domain $\left\{t^{(j)}\right\}_{j=1}^{k+N}$. Additionally, the parameters for pump degradation are included for the pump model. In this model, it is assumed that the primary and secondary pumps degrade at the same rate and have the same uncertainties. λ_D is the fractional head loss when the pump is fully degraded, Δt_D is the time when the pump is fully degraded, σ_1 is the uncertainty of the degradation rate and σ_2 determines the spread of the degradation rate over time.

The System Surrogate with Pump Power model framework is shown in Figure 3.

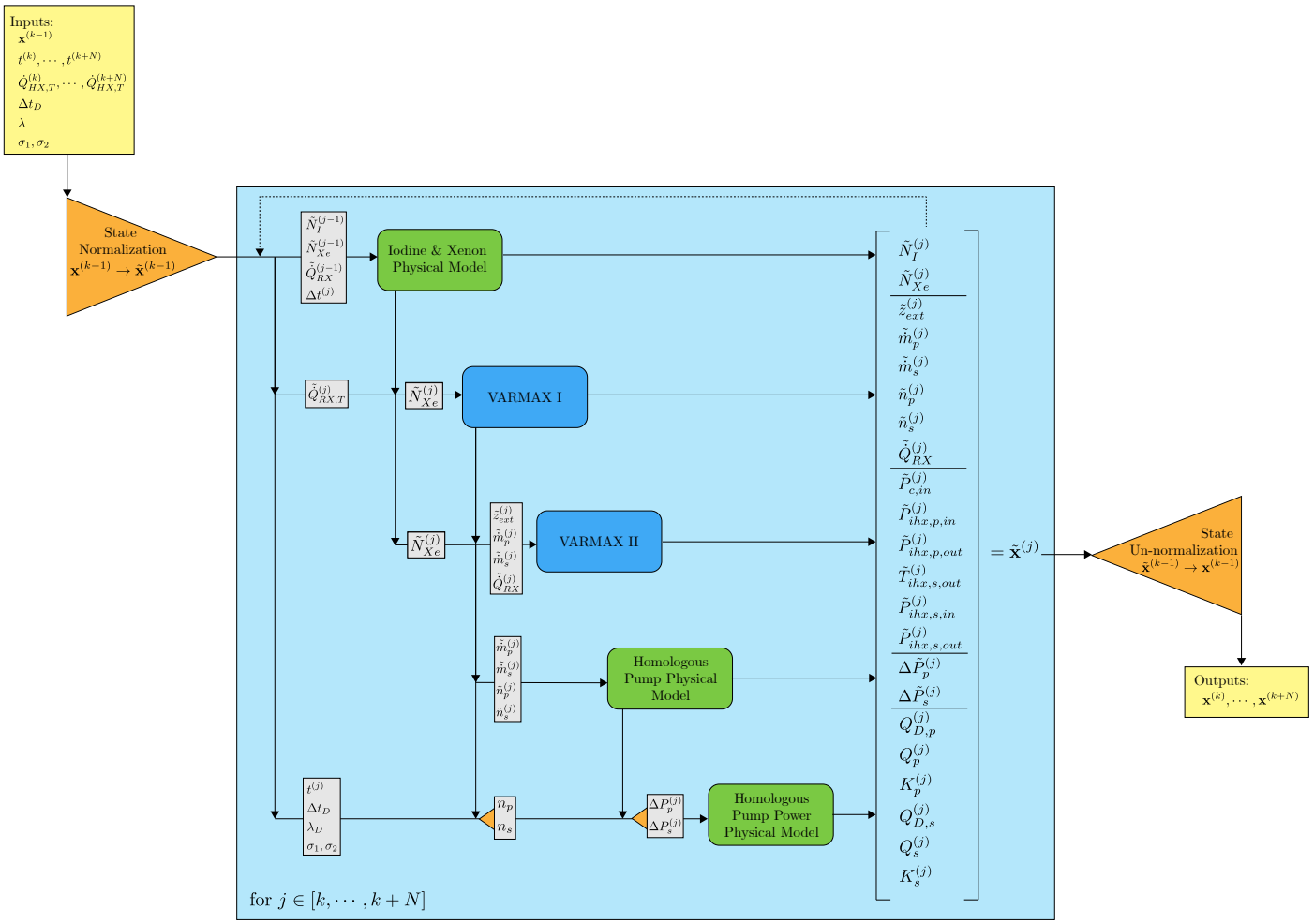

Figure 3: System surrogate with pump power framework

4 Physical Models

4.1 Xenon Reactivity Feedback

The xenon reactivity model computes the reactivity feedback in response to transient changes in the global Xenon-135 concentration. The following system computes the relative Iodine-135 and Xenon-135 relative concentrations; this is modeled is used by SAM in the PKE model [CITE]:

$$\frac{d\tilde{N}_I}{dt} = \lambda_I \left(\tilde{\dot{Q}}_{RX}(t) - \tilde{N}_I \right) \tag{1}$$

$$\frac{d\tilde{N}_{Xe}}{dt} = \lambda_{Xe} \left(\tilde{\dot{Q}}_{RX}(t) - \tilde{N}_{Xe} \right) + \sum_g \sigma_{a,g}^{Xe} \phi_{g,0} \tilde{\dot{Q}}_{RX}(t) \left(1 - \tilde{N}_{Xe} \right) + \frac{\sum_g \gamma_{I,g} \Sigma_{f,g} \phi_{g,0}}{\sum_g \left(\gamma_{I,g} + \gamma_{Xe,g} \right) \Sigma_{f,g} \phi_{g,0}} \left(\lambda_{Xe} + \sum_g \sigma_{a,g}^{Xe} \phi_{g,0} \right) \left(\tilde{N}_I - \tilde{\dot{Q}}_{RX}(t) \right) \tag{2}$$

where g is the energy group; $\gamma_{I,g}, \gamma_{Xe,g}$ are the fission product yield fractions; $\Sigma_{f,g}$ is the macroscopic fission cross section; $\phi_{g,0}$ is the initial neutron flux; $\sigma_{a,g}^{Xe}$ is the microscopic neutron absorption cross section of xenon; and λ_I, λ_{Xe} are the decay constants. $\tilde{N}$ is the normalized value of concentration; and $\tilde{\dot{Q}}_{RX}(t)$ is normalized power of the reactor. The Iodine and Xenon values are specified with the I and Xe subscripts.

In the SAM gFHR model, only one energy group is considered with the following constant values: $\lambda_I = 2.9306 \times 10^{-5}$, $\lambda_{Xe} = 2.1066 \times 10^{-5}$, $\sigma_a^{Xe} = 1.5883 \times 10^{-19}$, $\phi_0 = 7.3663 \times 10^{14}$, $\gamma_I = 6.3858 \times 10^{-2}$, $\gamma_{Xe} = 4.4411 \times 10^{-3}$, $\Sigma_f = 8.2980 \times 10^{-4}$. Using these constants, an Explicit Euler time integration scheme is implemented. With the time-step Δt and the previous concentrations and power, the concentrations at the next time step is:

$$\begin{bmatrix} \tilde{N}_I^{(k)} \\ \tilde{N}_{Xe}^{(k)} \end{bmatrix} = \begin{bmatrix} \tilde{N}_I^{(k-1)} \\ \tilde{N}_{Xe}^{(k-1)} \end{bmatrix} + \Delta t \begin{bmatrix} \frac{d\tilde{N}_I^{(k-1)}}{dt} \\ \frac{d\tilde{N}_{Xe}^{(k-1)}}{dt} \end{bmatrix}. \tag{3}$$

4.2 Pump

See Pump Documentation in Git Repo

5 Data-Driven Models

5.1 Vector Autoregressive Moving Average with Exogenous Term

The Vector Autoregressive Moving Average with Exogenous term (VARMAX) is the vectorized form of the Autoregressive Moving Average with Exogenous Input [6]. VARMAX is a statistical model used to describe time-series data with the assumption that future state values are dependent on previous state values. The state value at the next time step $\mathbf{s}^{(k)} \in \mathbb{R}^{n_s}$ is determined by the linear combination of p of its own lagged variables $\mathbf{s}^{k-1}, \mathbf{s}^{(k-2)}, \ldots, \mathbf{s}^{(k-p)}$; the exogenous terms $\mathbf{u}^{(k)} \in \mathbb{R}^{n_u}$; and a moving average model of order q. The trainable parameters are: the autoregressive (AR) coefficients $\mathbf{a} \in \mathbb{R}^{n_s}, \mathbf{A}_i \in \mathbb{R}^{(n_s \times n_s)}$ for $i \in [1, \ldots, p]$; the input coefficients $\mathbf{B} \in \mathbb{R}^{n_s \times n_u}$; and the moving average (MA) coefficients $\mathbf{M}_j \in \mathbb{R}^{n_s \times n_s}$ for $j = [1, \ldots, q]$. The

VARMAX model is written as:

$$\mathbf{s}^{(k)} = V_{p,q}\left(\mathbf{s},\mathbf{u}\right) = \mathbf{a} + \mathbf{A}_1\mathbf{s}^{(k-1)} + \ldots + \mathbf{A}_p\mathbf{s}^{(k-p)} + \mathbf{B}\mathbf{u}^{(k)} + \mathbf{M}_1\boldsymbol{\varepsilon}^{(k-1)} + \ldots + \mathbf{M}_q\boldsymbol{\varepsilon}^{(k-q)},$$
$$\boldsymbol{\varepsilon}^{(k)} \sim \mathcal{N}(0,\boldsymbol{\Omega}), \tag{4}$$

where $\boldsymbol{\varepsilon}^{(k)} \in \mathbb{R}^{n_s}$ are independently identically distributed (i.i.d.) white noise error terms from a normal distribution with zero bias and $\boldsymbol{\Omega}$ covariance. The estimation of the VARMAX model is determined by using the Box-Jenkins method [1]. First the (p,q) parameters are determined by examining partial autocorrelations and the autocorrelations respectively; then coefficients are determined by performing least squares regression to minimize the model error.

The python package *statsmodel* [5] implementation is utilized for the VARMAX modeling and analysis of the generated data. The AR component of this model assumes that the process $\mathbf{s}^{(k)}$ is a stationary, meaning the first and second moments are time-invariant. In order to enforce stationary, the *statsmodel* optimizer considers an unconstrained parameter space for the autoregressive terms, then transforms them to a stationary space (Monahan, 1984).

References

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