This assignment is due **Friday, July 18 at 7 PM**. Late submissions will not be accepted. Please follow these guidelin

for each

Prepare your solution using LaTeX and submit a pdf file. Easiest will be to submit using the .tex file provided. For question

Enclose each paragraph of your solution with `\begin{soln}Your solution here...\end{soln}`. solnblueYour solution wi

question@14 question1@object14 question@24 question2@object14 question@35 question3@object15 question@

Submit the assignment via https://gradescope.ca/GradeScope. Your group must make a **single** submission via one group m

After uploading to Gradescope, link each question with the page of your pdf containing your solution.

Before we begin, a few notes on pseudocode throughout CPSC 320: Your pseudocode should communicate your algorit

Remember also to **justify/explain your answers**. We understand that gauging how much justification to provide ca

On the plus side, if you choose an incorrect answer when selecting an option but your reasoning shows partial understa

Ok, time to get started...

*Group Members
Please list the CWLs of all group members here (even if you are submitting by yourself). We will deduct a mark if this
Take Your Order
[10 points] Take the following functions and arrange them in ascending order of growth rate. That is, if $g(n)$ appears a
$f_1(n) = n$
$f_2(n) = 2^n$
$f_3(n) = \sqrt{n}\log n$
$f_4(n) = \frac{n\log n}{\log(\log n)}$
$f_5(n) = (n-2)!$
$f_6(n) = n^{\lg n}$ solnblue We first develop the following lemma: if $f \in O(g), g \in O(h)$ then $f \in O(h)$.
Suppose $f \in O(g), g \in O(h)$. Then there exists $c_1, c_2 \in R^+, n_1, n_2 \in N$ so that

Then we have that $f(n) \leq c_1c_2 \cdot h(n)$ whenever $n \geq \max(n_1, n_2)$. So, $f \in O(h)$.
Claim: The orderding is $(f_3, f_1, f_4, f_6, f_2, f_5)$.
It suffices to show for each neighbour pair (f_i, f_j) in the order that $f_i \in O(f_j)$ to get if g appears after f then $f \in O(g$
$f_3 \in O(f_1)$. We show that $\lim_{n\to\infty} \frac{f_1(n)}{f_3(n)}$ diverges.
So, $\frac{f_1(n)}{f_3(n)} = \frac{n}{\sqrt{n}\log(n)} = \frac{\sqrt{n}}{\log(n)}$.
Without loss of generaltity, assume the base is e, then using real valued limits and L'Hopital's rules we get:

And thus, we get that $f_3 \in O(f_1)$.
$f_1 \in O(f_4)$. We make use of the fact that $\log(n) \in O(n)$ and that $\log(n)$ is strictly increasing.
So then for some $c \in R^+$ and $n_1 \in N$, we have that $\log(n) \leq cn$ whenever $n \geq n_1$.
Applying the log function to both sides, we get $\log(\log(n)) \leq \log(c) + \log(n)$.
Then we see that $\log(c) + \log(n) \leq 2\log(n)$ whenever $n \geq \lceil c \rceil$.
Thus, we have that $\log\log(n) \leq 2\log(n)$ whenever $n \geq n_2$, where $n_2 = \max(n_1, \lceil c \rceil)$.
We know that $n \geq 1$, and hence we multiply both sides to get $n\log\log(n) \leq 2n\log(n)$.
Rearranging we get, $n \leq 2 \cdot \frac{n\log(n)}{\log\log(n)}$ whenever $n \geq n_2$, thus $f_1 \in O(f_4)$.
$f_4 \in O(f_6)$. We assume that the base is $b > 1$.
First, observe $\log(n) > b$ whenever $n \geq \lceil b^b \rceil + 1$. And so, $\log\log(n) > 1$ and thus $\frac{1}{\log\log(n)} < 1$.
Then we get that $\frac{n\log(n)}{\log\log(n)} \leq n\log(n)$. And so, it suffices to show $n\log(n) \in O(n^{\lg(n)})$.
Since the base does not matter, is also suffices to show that $\lg(n) \in O(n^{\lg(n)-1})$.
We will use a limit argument to show that it is the case. So then using real valued limits and L'Hopital's we get

Then $\lg(n) \in O(n^{\lg(n)-1})$. More formally, we can then write:

Then if we set $c_2 = c_1 \cdot \lg(b)$, we have

Now, if $n_2 = \max(\lceil b^b \rceil + 1, n_1)$, we can write

Hence, $f_4 \in O(f_6)$.
$f_6 \in O(f_2)$. $f_6 = n^{\lg(2)} = 2^{\lg(n^{\lg(2)})} = 2^{\lg^2(n)}$. $f_2 = 2^n$.
It suffices to show $\lg^2(n)$ is bounded by n since the exponential with base 2 is an increasing function.
We will again use a limit argument, so again we tak real valued limits,

Thus, $f_6 \in O(f_2)$.
$f_2 \in O(f_6)$. To show 2^n is bounded by $(n-2)!$, we show $2^n \leq (n-2)!$ for $n \geq 8$ using induction.
Base case: $n = 8$: $2^n = 2^8 = 256 \leq 720 = 6! = (8-2)! = (n-2)!$. Assume true for $k \in N$ with $k \geq 8$.
Notice, $k - 1 \geq 7 > 2$. Now, $2^{k+1} = 2 \cdot 2^k \leq 2 \cdot (k-2)! < 7 \cdot (k-2)! \leq (k-1)(k-2)! = (k-1)!$.
Hence, induction makes it true for $n \geq 8$. Thus, $2^n \in O((n-2)!)$.
All-Stars
Suppose you're organizing a tennis tournament between n players which we simply label as $1, 2, \ldots, n$. Each player com
You want to determine a way to rank these players, but a challenge here is that cycles **may** exist in your tournament –
A ranking for the tournament is an ordering $(r_1, r_2, \ldots, r_n)$ where node r_1 is established as the overall rank 1 player. D
[4] We decide that the top ranked player should always be some node whose out-degree in the directed graph is maximu
Hint: A proof by induction on n works well.
solnblue Let $G = (V, E)$ be a graph such that it is a tournament. Let $i \in V$ be the node such that it has maximum out