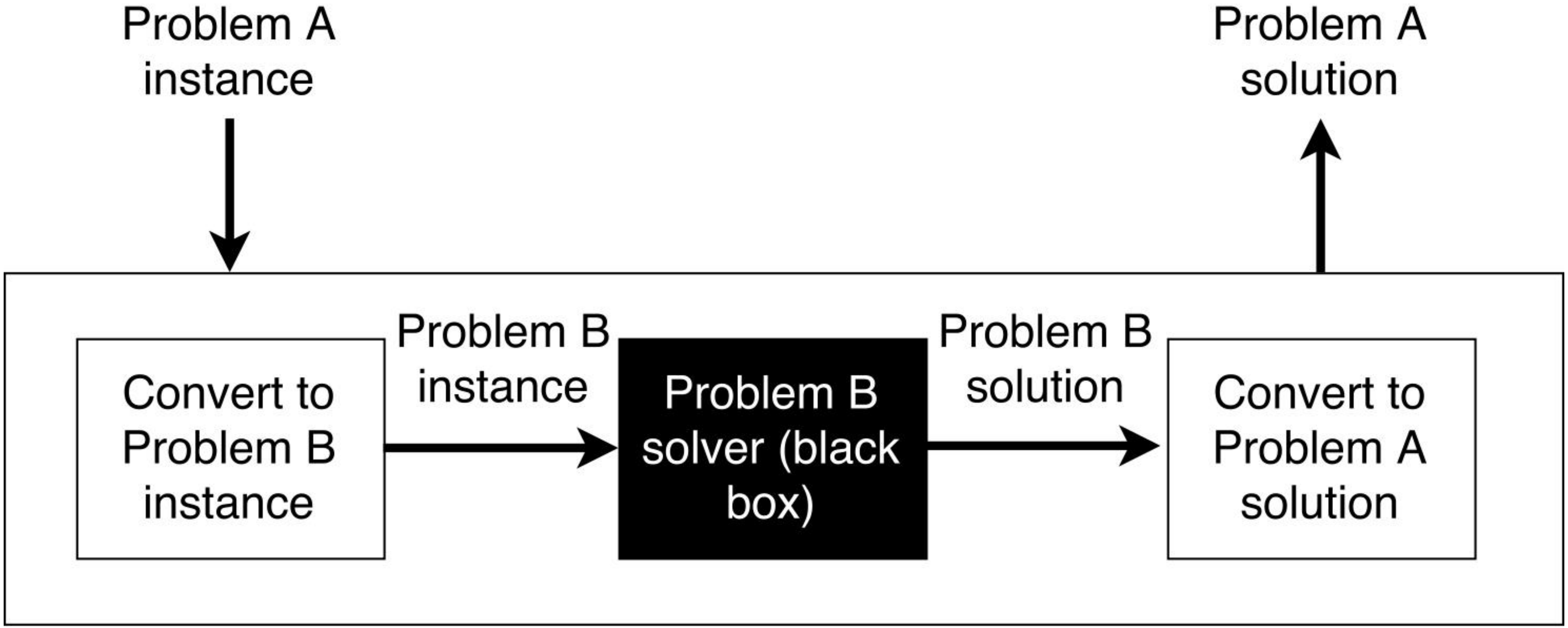
Reductions

CPSC 320 2024S

Context

- You encounter a new problem **A**
    - You don’t have an algorithm to solve it
    - But it seems similar to a problem **B** that you do know how to solve
    - … so, you might use your solver for **B** rather than writing an entirely new solver for **A**
- Definition: an *instance* of a problem is a valid input

How a reduction works


Problem A instance
Problem A solution
Convert to Problem B instance
Problem B instance
Problem B solver (black box)
Problem B solution
Convert to Problem A solution

How a reduction works (continued)

You need to:

- Show how to transform an arbitrary instance $\mathbf{I_A}$ of **A** into an instance $\mathbf{I_B}$ of **B**
- Show how to transform a solution $\mathbf{S_B}$ of $\mathbf{I_B}$ into a solution $\mathbf{S_A}$ of $\mathbf{I_A}$
- Prove that $\mathbf{S_A}$ is a correct solution to $\mathbf{I_A}$, assuming that $\mathbf{S_B}$ is a correct solution to $\mathbf{I_B}$

The total running time is:

- The sum of the two transformations
- Plus the time to solve $\mathbf{I_B}$

Reduction example

- **A:** Given a list of numbers **$[X_1, X_2,\ldots, X_n]$**, find the smallest gap between any two of them
- **B:** Sorting a list of values
- **Reduction:**
  - Given an instance **I_A** of **A**, let **$I_B = I_A$**
  - Sort the list **I_B** to get a list **$[Y_1, Y_2,\ldots, Y_n]$**
  - Return **$\min_{i=1,2,\ldots,n-1}\{Y_{i+1} - Y_i\}$**

Reduction example and clicker question

- **A:** Given a list of numbers **[X_1, X_2,..., X_n]**, determine if there are any duplicates in the list
- **B:** Given a list of numbers **[X_1, X_2,..., X_n]**, find the smallest gap between any two of them
- **Reduction:**
  - Given an instance **I_A** of **A**, let **I_B= I_A**
  - Solve **I_B** to get the value **smallestGap**
  - Return True to A if □; else return False

Fill the blank!

Reduction example and clicker question

- **A:** Given a list of numbers **[X_1, X_2,..., X_n]**, determine if there are any duplicates in the list
- **B:** Given a list of numbers **[X_1, X_2,..., X_n]**, find the smallest gap between any two of them
- **Reduction:**
  - Given an instance **I_A** of **A**, let **I_B= I_A**
  - Solve **I_B** to get the value **smallestGap**
  - Return True to A if **smallestGap = 0** ; else return False

Reduction example 3

- **A:** Given a SAT instance where all clauses have length 2, is there a way to assign truth values to variables such that all clauses evaluate to True? (AKA, 2-SAT)
  - E.g., $(x \vee \overline{y}) \wedge (y \vee z)$
- **B:** Given a SAT instance where all clauses have length 3, is there a way to assign truth values to variables such that all clauses evaluate to True? (3-SAT)
- **Reduction:**
  - Given an instance **I_A** of **A**, repeat the last literal in each clause and "OR" it to the previous literal to obtain **I_B**, e.g., $(x \vee \overline{y} \vee \overline{y}) \wedge (y \vee z \vee z)$
  - Solve **I_B** to obtain a truth assignment **S_B** (if it exists, or the value NO if **I_B** is not satisfiable
  - **$S_A = S_B$**