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A Mapping Technique from $\mathbb{R}^n$ to $\mathbb{I}$ and Its Application to Nonlinear Sliding Control Systems

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Abstract: In this paper, we introduce a bijective mapping $\mathbf{v}_\mathbf{n}''$ which maps an n-dimensional unit space $\mathbb{I}^\mathbf{n}$ to a subset of the one-dimensional interval $\mathbb{I}$, by using a symbolic, 2-adic representation of each point of the space. We also demonstrate an application of this method to obtaining sliding manifolds of certain classes of sliding mode control systems.

Keywords: bijective mapping, nonlinear systems, sliding mode control, sliding surfaces, polynomial functions.

1. INTRODUCTION

Variable structure control theory has been introduced by Utkin [Utkin (1977)] and has been used widely afterwards [Utkin (1992), Young (1993), Yan et al. (2004)] since this control method is robust against external disturbances, parameter changes and unmodelled dynamics. Sliding mode control is a specific type of variable structure control problem which drives the system from an initial condition to a stable sliding manifold and then keeps it on.

Recent research in the field of sliding mode control has been mainly focusing on specific implementations of this control method, sometimes in hybrid form with other control methods to achieve preferential results. Examples of this include the wavelet-neural-network sliding mode control of an induction servo motor [Wai, et al. (2003)], the sliding mode and fuzzy controller for vehicle suspension control [Huang, et al. (2003)], the sliding control of automatic car steering [Guldner, et. al (2005)], etc. There have also been reports on reducing the chattering phenomenon of sliding mode control systems (e.g. [Young, et al. (1992)]).

Although this control method has many advantages, there exists a challenging problem of determining the sliding manifolds for the sliding mode systems in general, particularly in the case of high-dimensional, nonlinear sliding systems.

Gelbaum and Olmsted presented a many-to-one mapping of the interval [0,1] onto the square [0,1]×[0,1] (see [Gelbaum, et al.(1964)]). Since then, there appears to be few applications of this method. In Section 2, we shall extend this mapping technique to the case of mapping the n-dimensional space $\mathbb{R}^\mathrm{n}$ to a subset of the one-dimensional interval $\mathbb{I}$ by using a symbolic and 2-adic representation of each space element. In addition, we shall refine the mapping into a bijective transformation. In Section 3, we will demonstrate an application of the mapping technique in finding the sliding surfaces of a group of high-dimensional nonlinear sliding mode control systems. This is an extension of the idea firstly presented in [Xu, (2009)] with more illustrations of application examples and of the type of control systems this method could be applied to. The main advantage of the proposed method is that it saves online computing time during the sliding mode control because information of the corresponding sliding manifolds of the nonlinear control systems is computed off-line and is then stored separately to be used.

2. THE BASIC IDEA OF THE MAPPING TECHNIQUE

Firstly, the n-dimensional space $\mathbb{R}^\mathrm{n}$ can be easily normalised onto $\mathbb{I}^\mathrm{n}$ (for example, by choosing an 'arctan' function), where $\mathbb{I}$=[0,1] and $\mathbb{I}^\mathrm{n}=\mathbb{I}\times\cdots\times\mathbb{I}$.

Next, consider the map

$$(x_1,\cdots,x_n)\in\mathbb{I}^n \xrightarrow{\nu_n} \widetilde{x_1\cdots x_n}\in\mathbb{I},$$

where $\widetilde{x_1\cdots x_n}$ is given as follows: Write $x_1, x_2\cdots,x_n$ in terms of their binary representations:

$$x_1 = 0.a_1^1 a_2^1 a_3^1 \cdots\cdots$$
$$x_2 = 0.a_1^2 a_2^2 a_3^2 \cdots\cdots$$
$$\vdots$$
$$x_n = 0.a_1^n a_2^n a_3^n \cdots\cdots$$

where each $a_i^j \in \{0,1\}$. Then define

$$\widetilde{x_1 \cdots x_n} = 0.a_1^1 a_1^2 \cdots a_1^n a_2^1 a_2^2 \cdots a_2^n a_3^1 a_3^2 \cdots a_3^n \cdots\cdots$$

Consequently, we obtain a transformation $\nu_n: \mathbb{I}^n \to \mathbb{I}$, i.e. $\nu_n(x_1, \cdots, x_n) = \widetilde{x_1 \cdots x_n}$. For simplicity of exposition, we shall only consider mapping the two-dimensional interval space $\mathbb{I}^2$ onto a one-dimensional line $\mathbb{I}$ under the transformation ν_2 in the following part of the section. However, the idea applies for general cases of mappings $\nu_n: \mathbb{I}^n \to \mathbb{I}$, where $n > 2$.

Obviously, a binary string ending in an infinite sequence of 1s may represent the same number as a corresponding string ending in an infinite number of 0s. For example, $0.0011111111\cdots$ and $0.0100000000\cdots$ both represent the number 0.25. We define points in the form of the above binary strings **double-valued points**. These double-valued points, in fact, belong to a subset (say D) of the set of rational numbers. Therefore, D is a countable set. Thus, points of which either coordinate is double-valued in the two-dimensional plane have more than one correspondences on the one-dimensional interval according to the mapping ν_2. Therefore, ν_2 is not one-to-one. In order to refine the mapping, we remove all points ending with an infinite number of 1s (apart from $0.1111111111\cdots$, which is number 1 that corresponds to infinity of one of $\mathbb{R}^2$'s coordinates) for both x_1 and x_2 coordinates, i.e. remove points in the form of

$$0.\sim 1111111111\cdots\cdots$$

on either coordinate, where '$\sim$' denotes any binary string and the remainder represents an infinite string of 1s. After removing all such points, remaining elements of $\mathbb{I}^2$ have unique correspondences in $\mathbb{I}$. Let D' be the union of all removed binary points, where $D' \subset D$ which is also countable, we obtain a one-to-one mapping ν_2':

$$\mathbb{I}^2 \setminus \mathrm{D}' \xrightarrow{\nu_2'} \mathbb{I}.$$

Lemma 2.1 *The mapping* ν_2' has range in $\mathbb{I}$ *consisting of all points of* $\mathbb{I}$ *apart from points of the form*

$$0.\sim 1 * 1 * 1 * 1 * 1 \cdots\cdots$$

where '$\sim$' represents any binary string but all 1s, '$*$' denotes either 0 or 1, and '$\cdots\cdots$' means an infinite recursion of the binary pattern '$*\,1$'.

Consider elements in the one-dimensional space; it is obvious that binary points in the form of $0.\sim 1 * 1 * 1 * 1 * 1 \cdots\cdots$ have no corresponding elements on $\mathbb{I}^2 \setminus \mathrm{D}'$ according to the new function ν_2' since points in $\mathbb{I}^2 \setminus \mathrm{D}'$ which have coordinates like $0.\sim 1111111111\cdots\cdots$ have been removed in the earlier process. Therefore, ν_2' is not onto. Similarly as before, if we exclude the union of points which are in the form of $0.\sim 1 * 1 * 1 * 1 * 1 \cdots\cdots$ (say U) from $\mathbb{I}$, then $\mathbb{I}\setminus\mathrm{U}$ is the set which contains all elements with unique correspondences on $\mathbb{I}^2 \setminus \mathrm{D}'$. Clearly, most elements of U are not double-valued and the set U is uncountable.

Therefore, from the above, we have the following result:

Theorem 2.2 *There exists a countable subset $D' \subseteq \mathbb{I}^2$ and an uncountable subset $U \subseteq \mathbb{I}$ such that the induced mapping ν_2'':*

$$\mathbb{I}^2 \setminus D' \xrightarrow{\nu_2''} \mathbb{I}\setminus U.$$

is both one-to-one and onto.

Remark 1 (i). It is worth noticing that not every point on the one-dimensional interval $\mathbb{I}$ has a representation on the two-dimensional plane $\mathbb{I}^2$ according to the refined map ν_2'' since U contains an uncountable number of non-double-valued points, i.e. all points on $\mathbb{I}^2$ can be mapped onto a sub-interval of $\mathbb{I}$, but not vice versa.

(ii). Of course, the Euclidean Spaces $\mathbb{I}^2$ and $\mathbb{I}$ have different topologies. The map ν_2'' is discontinuous with respect to the ordinary topologies of Euclidean metric spaces. Two points which are arbitrarily close in $\mathbb{I}^2 \setminus \mathrm{D}'$ can be far away in $\mathbb{I}\setminus\mathrm{U}$. For the convenience of denotation, we refer to $\mathbb{I}\setminus\mathrm{U}$ as $\tilde{\mathbb{I}}$ in the following part of the paper.

3. APPLICATION TO SLIDING MODE CONTROL PROBLEMS

A general n-dimensional dynamical nonlinear sliding mode control system is given in the following form

$$\dot{\xi} = f_1(\xi, \eta), \tag{1}$$

$$\dot{\eta} = f_2(\xi, \eta) + f_3(\xi, \eta)u, \tag{2}$$

where $\xi \in \mathbb{R}^{n-m}$ with $n, m \in \mathbb{Z}$, $\eta, u \in \mathbb{R}^m$ for any $n \geq m > 0$; $f_3 \neq 0$ for all points apart from equilibriums of the system; and u represents the sliding mode control. We know that u is designed to drive the system to the m-dimensional sliding manifold $\sigma(\xi, \eta)$, which is defined as:

$$\sigma(\xi, \eta) = \Phi(\xi) - \eta = 0, \tag{3}$$

and then force the system to slide to the equilibrium. Therefore, when the system lies on the sliding hyperplane, the system behaviour is governed by the following expression:

$$\eta = \Phi(\xi). \tag{4}$$

If we substitute (4) into (1), we obtain

$$\dot{\xi} = f_1(\xi, \Phi(\xi)). \tag{5}$$

Equations (5) and (2) together now describe the system dynamics on the sliding surface $\sigma(\xi, \eta)$. The sliding surface also needs to satisfy the following:

$$\dot{\sigma} = -\mathrm{sgn}(\sigma(\xi, \eta)). \tag{6}$$

The appropriate control u can be obtained as follows:

$$u = \frac{\mathrm{sgn}(\Phi(\xi,\eta)-\eta) + \frac{\partial\Phi}{\partial\xi} f_1(\xi,\eta) - f_2(\xi,\eta)}{f_3(\xi,\eta)} \tag{7}$$

to make the system stay on the sliding manifold.

Now, if we assume that

$$f(\xi,\eta)=\sum_{i,j=0}^{K_1}\alpha_{ij}\,\xi^i\eta^j \tag{8}$$

is a polynomial, vector-valued function, where $K_1\in\overline{\mathbb{Z}^-}$, and α_{ij} s are some given coefficients. Then, the subvector functions f_1, f_2, and f_3 are defined by the following:

$$f_d(\xi,\eta)=\sum_{i,j=0}^{K_1}\alpha_{ij}^{(d)}\,\xi^i\eta^j, \tag{9}$$

where $d=1,2,3\cdots\cdots$. Thus, systems defined by (1) and (2) are specified as n-dimensional systems given by polynomial vector fields containing some parameters. Note that, we only study the problem of finding sliding manifolds for this particular type of systems; it does not include all n-dimensional systems, of course, however, it represents a great amount of systems with very different dynamical behaviours (In fact, it contains all linear systems and most of the nonlinear systems, or at least it provides a good approximation of some complicated nonlinear systems.).

Next, suppose that a sliding surface is a polynomial function of the following form:

$$\eta=\Phi(\xi,\beta_k)=\sum_{k=0}^{K_2}\beta_k\xi^k, \tag{10}$$

where $K_2\in\overline{\mathbb{Z}^-}$, and the β_ks are time-invariant parameters to be determined. From (5) and (9), we have the following

$$\begin{aligned}\dot{\xi}&=f_1\left(\xi,\sum_{k=0}^{K_2}\beta_k\xi^k\right)=\sum_{i,j=0}^{K_1}\alpha_{ij}^{(1)}\,\xi^i\left(\sum_{k=0}^{K_2}\beta_k\xi^k\right)^j\\&\triangleq g(\alpha,\beta,\xi),\end{aligned} \tag{11}$$

which describes the system behaviour on the sliding manifold given by the polynomial function (10), where g is a function of parameters α,β , and of time-dependent variable ξ. We know that there is an equilibrium point of the system (11) at the origin, which means that:

$$g(\alpha,\beta,\xi)=0. \tag{12}$$

Let

$$q(\alpha,\beta)=\frac{\partial g}{\partial\xi}(\alpha,\beta,\xi)\Big|_{\xi=0}, \tag{13}$$

then by Taylor's theorem, we obtain the following:

$$\dot{\xi}\cong q(\alpha,\beta)\xi \tag{14}$$

which is the linearised (n-m)-dimensional equation of (11) at the origin. Then let $S_\alpha\colon\beta\to\wp(\mathbb{R}^{n-m})$ be the mapping which for a given vector containing a series of α parameters, takes the parameter vector β to the subset of $\mathbb{R}^{n-m}$ including the origin on which (14) is stable. Of course, for some uncontrollable systems generated by (1) and (2), or in the case of the systems only having non-polynomial sliding surfaces, the right-hand side of the mapping S_α is empty.

It becomes straightforward at this stage that for a system governed by (1) and (2) with given parameters α_{ij}s, sliding manifolds in the form of a polynomial function (10) with desired parameters β_ks are to be determined to make the linearised system (14) stable at the equilibrium point, hence when the system trajectory has been pushed onto a sliding manifold by control variable u , system dynamics slides towards the origin and eventually reaches the stable equilibrium point.

Now, recall the dimension reduction method we presented in Section 2, two maps $v''_{N_1}\colon\mathbb{I}^{N_1}\to\tilde{\mathbb{I}}$ and $v''_{N_2}\colon\mathbb{I}^{N_2}\to\tilde{\mathbb{I}}$ are introduced here, where $N_1=\dim(\alpha)$ and $N_2=\dim(\beta)$. Then, we can plot a graph showing all suitable values of $\tilde{\beta}$ for corresponding values of $\tilde{\alpha}$. The sliding surface for each system can then be defined as the form of (10). The graph can be used as a 'look-up' table; i.e. for a given system specified by the αs, we can find a stable sliding surface (if one exists) from the βs by splitting the 2-adic representation of the corresponding $\tilde{\beta}$ we find from the graph into N_2-dimensional β s. For some system, there might exist several sliding manifolds which drive the system to the stable equilibrium point; in this case, several points suggesting several $\tilde{\beta}$s will be found corresponding to that system in the graph. This technique enables us to store the parameters of sliding surfaces which are in the form of polynomial functions for a whole class of systems. Next, we show two simple examples for finding stable sliding surfaces for two classes of sliding mode control systems.

Example 3.1 Consider systems in the following form:

$$\begin{aligned}\dot{x}_1&=\alpha_1x_1+\alpha_2x_1x_2\\\dot{x}_2&=x_1^2+x_2^2+u\end{aligned} \tag{15}$$

with two α coefficients: α_1 and α_2. We can map them onto $\mathbb{I}$ first and then code them on $\tilde{\mathbb{I}}$ that $\tilde{\alpha}=\widetilde{\alpha_1\alpha_2}$. For simplicity, let

$$\sigma(x_1,x_2)=x_2-\beta_1x_1-\beta_2x_1^2 \tag{16}$$

be the equation describing the sliding surface. Similarly, we can map β_1 and β_2 onto $\tilde{\mathbb{I}}$: $\tilde{\beta}=\widetilde{\beta_1\beta_2}$. We know that the sliding surface is given by $\sigma(x_1,x_2)=0$, i.e.

$$x_2=\beta_1x_1+\beta_2x_1^2. \tag{17}$$

Substituting (17) into (15) gives the following:

$$\begin{aligned}\dot{x}_1&=\alpha_1x_1+\alpha_2x_1(\beta_1x_1+\beta_2x_1^2)\\&=x_1(\alpha_1+\alpha_2\beta_1x_1+\alpha_2\beta_2x_1^2)\end{aligned} \tag{18}$$

Hence, in order to make the sliding surface stable, the quadratic equation

$$s(\alpha,\beta,x_1)=\alpha_1+\alpha_2\beta_1x_1+\alpha_2\beta_2x_1^2 \tag{19}$$

needs to be negative. The problem now comes to the range of x_1. To get local stability at the origin, $\alpha_1<0$ is a necessary condition which needs to be satisfied; furthermore, the range of the stable sliding curve depends on the real roots of x_1 when $s=0$, and the direction of hyperbola s. But the sliding surface is guaranteed to be globally stable if $\alpha_1<0$ and $(\alpha_2\beta_1)^2-4\alpha_1\alpha_2\beta_2<0$ (i.e. the quadratic function $s=0$ has two imaginary roots of x_1). In this example, it is possible to obtain some choices of βs for given αs to get a globally stable surface for the system (15). Fig. 1 is a plot of system parameter $\tilde{\alpha}$s against sliding manifold parameter $\tilde{\beta}$s.

We can see from the plot that some systems defined by (15) have a number of globally stable sliding manifolds described by (16) with various β parameters while some other systems do not have any. Note that we only stored stable sliding manifolds in the form of (16) for a class of systems defined by (15) in this plot. Systems which do not appear to have solutions from the plot may have sliding manifolds described by equations other than (16).

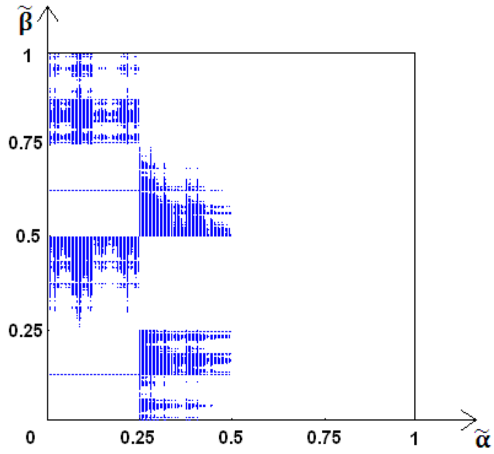

Fig. 1. A plot of system parameter $\tilde{\alpha}$ of (15) against sliding manifold parameter $\tilde{\beta}$.

Example 3.2 A group of two-dimensional dynamical systems with three α coefficients are defined as

$$\begin{aligned} \dot{x}_1 &= \alpha_1 x_2 + \alpha_2 x_1^2 + \alpha_3 x_1 x_2 \\ \dot{x}_2 &= x_1 x_2 + u \end{aligned} \tag{20}$$

Again, we consider the sliding manifold in the form of (16) for simplicity. Then similar procedures are carried out as in Example 3.1 that α s and β s are mapped onto $\mathbb{I}$ by $\tilde{\alpha} = \widetilde{\alpha_1\alpha_2\alpha_3}$ and $\tilde{\beta} = \widetilde{\beta_1\beta_2}$. We can get the quadratic equation:

$$s(\alpha, \beta, x_1) = \alpha_1\beta_1 + (\alpha_1\beta_2 + \alpha_2 + \alpha_3\beta_1)x_1 + \alpha_3\beta_2 x_1^2 \tag{21}$$

for this group of systems. In order to get global stability, two conditions $\alpha_1\beta_1 < 0$ and $(\alpha_1\beta_2 + \alpha_2 + \alpha_3\beta_1)^2 - 4\alpha_1\beta_1\alpha_3\beta_2 < 0$ need to be satisfied. Hence, according to the conditions, a plot is obtained as below:

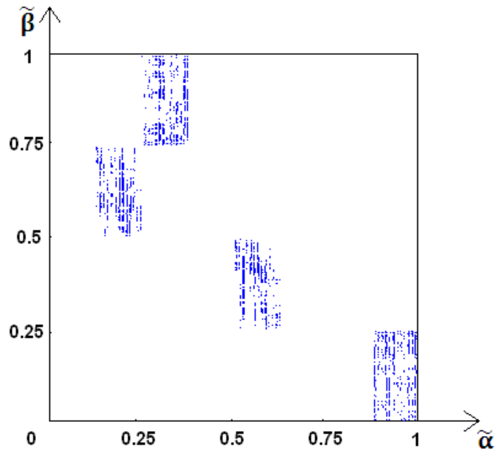

Fig. 2. A plot of system parameter $\tilde{\alpha}$ of (20) against sliding manifold parameter $\tilde{\beta}$.

We see from Fig. 2 that globally stable solutions of sliding manifold appear as patterns (less dense than in the previous example) in some areas but other areas are blank. For the systems which do not have any corresponding points on the plot, different sliding functions should be tested.

For the systems which have plotted points on the graph, knowing the system parameter α s, we can find the corresponding stable sliding manifold from the graph according to the β parameters. Then control u of the system can be gained accordingly. As an example, $(\tilde{\alpha}, \tilde{\beta})$:

$(0.200000000000045, 0.600000000000364)$ is a point found on the plot. Split $\tilde{\alpha}$ into three α values and then map them back to $\mathbb{R}$, we have $\alpha_1 = -3.24844838582168$, $\alpha_2 = -13.7628826848268$, $\alpha_3 = -3.2501376980962$. Also similarly for β parameters, we get $\beta_1 = 5.773516007 70939$, $\beta_2 = -5.7735559552409$. Checking the conditions mentioned earlier gives $\alpha_1\beta_1 \approx -18.75 < 0$ and $(\alpha_1\beta_2 + \alpha_2 + \alpha_3\beta_1)^2 - 4\alpha_1\beta_1\alpha_3\beta_2 \approx -843.34 < 0$. So the two conditions are satisfied. Therefore,

$$\begin{aligned} &\sigma(x_1, x_2) \\ &= x_2 - 5.77351600770939x_1 + 5.7735559552409x_1^2 \\ &= 0 \end{aligned} \tag{22}$$

is a globally stable sliding surface for system defined by (20). The corresponding control u is given by

$$\begin{aligned} u &= -x_1x_2 + (\beta_1 + 2\beta_2x_1)(\alpha_1x_2 + \alpha_2x_1^2 + \alpha_3x_1x_2) \\ &\quad -\mathrm{sng}(x_2 - \beta_1x_1 - \beta_2x_1^2) \end{aligned}$$

$$\begin{aligned} &= -x_1x_2 \\ &+ (5.77351600770939 - 11.5471119104818x_1) \\ &\times (-3.24844838582168x_2 - 13.7628826848268x_1^2 + \\ &3.25013769809629x_1x_2) - \\ &\mathrm{sgn}(x_2 - 5.77351600770939x_1 + 5.7735559552409x_1^2). \end{aligned}$$

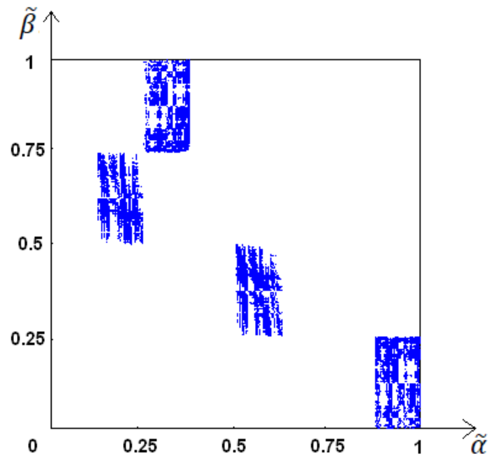

Fig. 3. A plot of system parameter $\tilde{\alpha}$ of (20) against sliding manifold parameter $\tilde{\beta}$ (1000×1000 points).

If the plotted points appear to be sparse, there are two ways in general, of potentially showing more solutions for the sliding manifolds. The first one is to increase the number of points selected and computed during the process. For example, in Fig. 2, 250×250 points are used for checking possible system parameter $\tilde{\alpha}$ of (20) against sliding parameter manifold parameters $\tilde{\beta}$ of (16). More points can be adopted for the checking to show a denser set of solutions (see Fig. 3 where 1000×1000 points are used for parameters $\tilde{\alpha}$ and $\tilde{\beta}$). The second approach is to enlarge a certain region in order to achieve clearer and more accurate plot of solutions within the area. For example, if we are interested in the region $\tilde{\alpha}:(0.5\sim0.75)$, $\tilde{\beta}:(0.25\sim0.5)$, we can zoom in this region onto a separate plot (see Fig. 4).

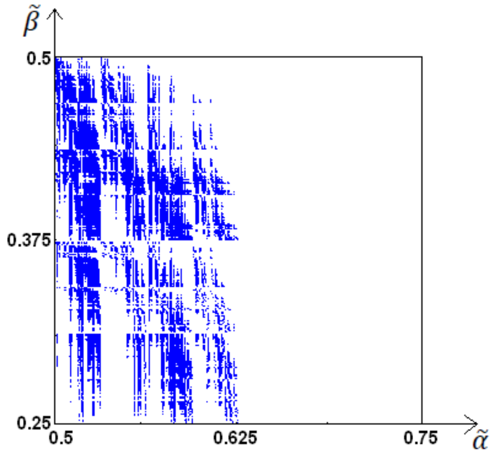

Fig. 4. A zoomed-in plot of the system parameter $\tilde{\alpha}$ of (20) against sliding manifold parameter $\tilde{\beta}$.

It is also worth mentioning that for a system which has a known or a small $\tilde{\alpha}$ range**,** attention should be given on this particular area to reduce computing time and to increase the accuracy of the sliding mode manifold parameter solutions.
In this section, we have applied the dimension reduction method in considering the general problem of sliding control for a class of systems defined by polynomial vector fields containing a number of parameters. We have shown how to associate these parameters with a scalar variable and also a similar scalar variable to the parameters of a polynomial sliding surface. These scalar variables $\tilde{\alpha}$ and $\tilde{\beta}$ can be put into a two-dimensional graph which stores the possible stable sliding surfaces for each set of systems with certain $\tilde{\alpha}$ parameters.

The main advantage of using the dimension reduction method in sliding control is that the sliding manifold is coded into the parameter vector which is mapped into the unit interval, so that finding and visualising the n-dimensional sliding manifold are simple – each of them corresponds to a point in the two-dimensional plot. On the other hand, there exist some drawbacks of this application. A theoretical problem is that only sliding manifolds in the form of polynomial functions can be obtained by using this method. Note, however, Stone-Weierstrass theorem states that any compact manifold can be represented arbitrarily closely by a polynomial function [Stone (1948)]. It is also worth mentioning that we only showed two simple examples with two-dimensional systems which have globally stable sliding manifolds; a great amount of systems might not have globally stable sliding manifolds. For systems which only have locally stable sliding surfaces, more than one plot should be provided to show the possible sliding surfaces having local stability for different ranges of x_1.

4. CONCLUSIONS

In this paper, we presented a method which provides a bijective transformation v_n'' which maps the n-dimensional unit space $\mathbb{I}^n$ into a one-dimensional set $\tilde{\mathbb{I}}$. Since the contraction and expansion between $\mathbb{R}^n$ and $\mathbb{I}^n$ can be easily achieved, the technique can be essentially regarded as giving a unique representation of each point of $\mathbb{R}^n$ on $\tilde{\mathbb{I}}$.

We also showed an application of the dimension reduction method in storing parameters information of sliding surfaces of a class of sliding mode control systems. It therefore provides a simple way of finding sliding manifolds and sliding controllers for such high-dimensional nonlinear systems. Note that although the computing time required for obtaining sliding manifolds for high-dimensional nonlinear control systems can be very long (depending on the speed and the resources of the computer), the main advantage of the presented technique is that the computing can be implemented off-line to find and to store the relevant parameters for the suitable sliding mode manifolds; therefore once the satisfactory sliding manifolds are achieved, the sliding mode controllers in real-time are straightforward and do not need extra on-line computing.

Future work can be carried out on the potential usage of the mapping technique in finding crucial dynamics of complex nonlinear dynamical systems, and in helping implement other types of control systems (for example, in finding switching surfaces of time-optimal control systems).

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