The state-dependent nonlinear regulator with state constraints

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Juan Cockburn
Rochester Institute of Technology

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James R. Cloutier
U.S. Air Force Research Laboratory
Munitions Directorate
101 W. Eglin Blvd. Ste 339
Eglin, FL 32542-6810
cloutierj@eglin.af.mil

Juan C. Cockburn
Department of Electrical Engineering
FAMU–FSU College of Engineering
2525 Pottsdamer Street
Tallahassee, FL 32310-6046
cockburn@eng.fsu.edu

Abstract

In this paper an extension of the state–dependent Riccati equation method to regulation of systems with state constraints is considered. First, a sufficient condition is introduced to characterize state constraints and it is shown how to design state-dependent regulators to satisfy this condition. Then the more general nonlinear regulation problem with state constraints is solved using the state–dependent Riccati equation approach.

1 Introduction

Most practical control problems involve in one way or another hard constraints on states and inputs. Since there are very few design approaches that can handle these constraints *a priori*, the designer has to tweak the controller using ad-hoc "anti-windup" schemes *a posteriori*. In the last decade there has been significant advances both from the theoretical and practical aspects of this problem. As a result, several approaches that explicitly handle state and input constraints are available. These include command governors [GT91, GKT95, Bem97, GK99, ACM99], nonlinear controllers with special structure [Tee96, KTD98], set theoretic methods [ST97, Bla99] and receding horizon techniques [MM93, RM93, LK99]. In this work we will concentrate on the extension of the state–dependent Riccati equation method to problems with state constraints. Input constraints have already been considered in [CDM96, MC98].

A promising technique for designing nonlinear controllers for nonlinear systems is the state-dependent Riccati equation (SDRE) approach [CDM96, C97]. In its classical formulation, this extended linearization method uses a state-dependent coefficient (SDC) parameterization of the nonlinear system together with a nonlinear performance index with quadratic-like structure to reduce the control problem to the solution of a state-dependent Riccati equation. The objective of this paper is to study possible extensions of the SDRE approach for systems with state constraints. First let us look at a general nonlinear regulation problem with state constraints.

Consider an input-affine nonlinear system described by the state-space equations

$$\dot{x} = f(x) + g(x)u, \quad x(0) = x_o \in \mathcal{X}, \tag{1}$$

where $x \in \mathbb{R}^n$ are the states, $u \in \mathbb{R}^m$ are the inputs and

$$\mathcal{X} = \{x : h(x) \leq 0, h(x) \in \mathbb{R}^p, h(\cdot) \in \mathcal{C}^1\}, \tag{2}$$

is a set of allowable states. Assume that $f(x)$ and $g(x)$ are smooth with $f(0) = 0$ and $g(x) \neq 0, \forall x \in \mathcal{X}$. The objective is to design a state feedback controller of the form $u = k(x)$, with $k(0) = 0$, and $k(x)$ smooth, such that the closed-loop system is stable and $x \in \mathcal{X}$ for all $t > 0$. Any feasible trajectory of the closed-loop system must not cross $\partial\mathcal{X}$, the boundary of $\mathcal{X}$ defined as

$$\partial\mathcal{X} = \{x : h(x) = 0, h(x) \in \mathbb{R}^p, h(\cdot) \in \mathcal{C}^1\}. \tag{3}$$

A sufficient condition for x to remain in $\mathcal{X}$ is that $\nabla h(x)\dot{x} = 0$, or equivalently

$$\nabla h(x)\,[f(x) + g(x)u] = 0, \tag{4}$$

A controller that satisfies (4) forces the closed–loop trajectories to follow level sets of $\mathcal{X}$. This condition will be exploited in the design of state–dependent nonlinear regulators that render $\mathcal{X}$ invariant.

2 The State-Dependent Nonlinear Regulator

In [CDM96] it has been shown that it is always possible to find a non-unique SDC representation of (1) as

$$\dot{x} = A(x)x + B(x)u, \tag{5}$$

with $\text{col}\{A(\cdot)\} \in \mathcal{C}^1$. For this representation the following system theoretic concepts can be defined pointwise in x.

Definition 1 *The SDC representation* (5) *is stabilizable in a region* $\mathcal{X}$ *if the pair* $\{A(x), B(x)\}$ *is pointwise stabilizable in the linear sense* $\forall x \in \mathcal{X}$.

Definition 2 *The SDC representation* (5) *is detectable in a region* $\mathcal{X}$ *if the pair* $\{A(x), C(x)\}$ *is pointwise detectable in the linear sense* $\forall x \in \mathcal{X}$.

Definition 3 *The SDC representation* (5) *is pointwise Hurwitz in a region $\mathcal{X}$ if the eigenvalues of $A(x)$ have negative real part $\forall x \in \mathcal{X}$.*

The SDC representation (5) is suitable for designing nonlinear controllers using the state–dependent Riccati equation (SDRE) approach. This method treats at each instant $A(x)$ and $B(x)$ as being constant and computes a control action solving a linear quadratic optimal control problem.

In its most general form the state–dependent nonlinear regulator (SDNR) problem seeks to find a controller

$$u = K(x)x, \tag{6}$$

such that the closed–loop system is at least locally asymptotically stable which approximately minimizes

$$J(x,u) = \frac{1}{2}\int_0^\infty \big(x^T Q(x) x + 2x^T S(x) u + u^T R(x) u\big)\, dt, \quad x(0) = x_o, \tag{7}$$

subject to (5), where $Q(x) \geq 0$, $S(x) = S(x)^T$, $R(x) > 0$, $\mathrm{col}\{Q(\cdot)\} \in \mathcal{C}^1$, $\mathrm{col}\{S(\cdot)\} \in \mathcal{C}^1$ and $\mathrm{col}\{R(\cdot)\} \in \mathcal{C}^1$. The state feedback gain that minimizes (7) is

$$K(x) = R(x)^{-1}\left[B(x)^T P(x) + S(x)^T\right], \tag{8}$$

where $P(x) \geq 0$ is a solution to the state-dependent Riccati equation

$$P(x)\tilde{A}(x) + \tilde{A}(x)^T P(x) + P(x)B(x)R(x)^{-1}B(x)^T P(x) + \tilde{Q}(x) = 0, \tag{9}$$

with

$$\begin{aligned} \tilde{A}(x) &= A(x) - B(x)R(x)^{-1}S(x)^T, && (10)\\ \tilde{Q}(x) &= Q(x) - S(x)R(x)^{-1}S(x)^T. && (11) \end{aligned}$$

It is well known [AM92] that a positive semi–definite solution to (9) exists provided that
A1. $\tilde{Q}(x) > 0$, ,
A2. $\{\tilde{A}(x), B(x)\}$ stabilizable,
A3. $\{\tilde{A}(x), \tilde{Q}(x)^{1/2}\}$ detectable.

For more details on optimaliy and stability properties of the state–dependent nonlinear regulator see [CDM96].

In the next section the nonlinear regulator problem with state constraints of the form (2) will be studied using the state-dependent Riccati equation approach.

3 The State–Dependent Regulator with State Constraints

Our strategy to design state–dependent nonlinear regulators with state constraints is based on the enforcement of the sufficient condition (4) when the states are close to *the boundary*, $\partial\mathcal{X}$, and total relaxation of condition (4) when the states are far from $\partial\mathcal{X}$. A SDC representation of the left hand side of (4) is

$$\begin{aligned} z &= \nabla h(x)\left[A(x)x + B(x)u\right], && (12)\\ &= C(x)x + D(x)u, && (13) \end{aligned}$$

where $z \in \mathbb{R}^p$ is a fictitious output. Assume now that x is close to $\partial\mathcal{X}$. Then, the state-feedback law that satisfies the algebraic equation $z = 0$ is

$$u(x) = -D(x)^\dagger C(x)x \tag{14}$$

where

$$D(x)^\dagger = D(x)^T(D(x)D(x)^T)^{-1} \tag{15}$$

is the right inverse of $D(x)$, e.g. $D(x)D(x)^\dagger = I$. This right inverse exists provided that $D(x)$ has full row rank for all x.

Remark 1 *For $D(x)$ to be full row rank the number of inputs of the original control system, m, must be as large as the number of constraints, p. For a single input system with a single constraint this is trivially satisfied.*

The control law (14) can be asymptotically recovered by solving a state-dependent nonlinear regulator problem that minimizes $J_{\mathcal{X}}(x,u) = \frac{1}{2}z^T W(x) z$, $W(x) > 0$, subject to (5). That is,

$$J_{\mathcal{X}}(x,u) = \frac{1}{2}\int_0^\infty \big(x^T Q_{\mathcal{X}}(x) x + 2x^T S_{\mathcal{X}}(x) u + u^T R_{\mathcal{X}}(x) u\big)\, dt, \tag{16}$$

where

$$\begin{aligned} Q_{\mathcal{X}}(x) &= C(x)^T W(x) C(x),\\ R_{\mathcal{X}}(x) &= D(x)^T W(x) D(x),\\ S_{\mathcal{X}}(x) &= C(x)^T W(x) D(x). \end{aligned} \tag{17}$$

and $W(x)$ is a $p \times p$ diagonal weighting matrix such that its i^{th} element is large when x is close to the boundary of the i^{th} constraint and small otherwise.

The state feedback gain that minimizes (16) is

$$K(x) = R_{\mathcal{X}}(x)^{-1}\left[B(x)P(x) + S_{\mathcal{X}}(x)^T\right] \tag{18}$$

where $P(x) \geq 0$ is obtained by solving the state-dependent Riccati equation (9) with coefficients $Q(x) = Q_{\mathcal{X}}(x)$, $R(x) = R_{\mathcal{X}}(x)$ and $S(x) = S_{\mathcal{X}}(x)$.

Remark 2 *In general $R_{\mathcal{X}}(x)$ is invertible only if $W(x)$ is invertible and $D(x)$ is full column rank. Assume that $R_{\mathcal{X}}(x)$ is invertible. Then it is easy to check that $Q_{\mathcal{X}}(x) - S_{\mathcal{X}}(x)R_{\mathcal{X}}(x)^{-1}S_{\mathcal{X}}(x)^T = 0$. Since this violates the detectability assumption A3, a positive semi–definite solution $P(x)$ to (9) that makes $\tilde{A}(x)$ pointwise Hurwitz does not exits.*

In general, the minimization of (16) leads to singular regulators (e.g. $R_{\mathcal{X}}(x)$ not invertible) and makes the level sets of $\mathcal{X}$ positively invariant. However our regulation objective is to drive the states to a desired equilibrium while remaining in the set $\mathcal{X}$. This can be achieved by minimizing the augmented cost functional

$$J(x,u) = J_o(x,u) + J_{\mathcal{X}}(x,u) \tag{19}$$

where

$$J_o(x,u) = \frac{1}{2}\int_0^\infty \Big(\rho_x x^T Q_o(x)\, x \ + \rho_u u^T R_o(x)\, u\Big) dt, \tag{20}$$

and $\rho_x > 0$, $\rho_u > 0$ are state–dependent weights. Without loss of generality let $W(x) = I$ (e.g. W can be absorbed in C and D). Dropping the dependence on x the coefficients of the state-dependent Riccati equation become

$$\begin{aligned} R &= \rho_u R_o + D^T D, \\ Q &= \rho_x Q_o + C^T \Big[I - \\ &\qquad D\left(\rho_u R_o + D^T D\right)^{-1} D^T \Big] C, \end{aligned}$$

and the feedback gain

$$u = K(x)x = \left[K_o(x) + K_{\mathcal{X}}(x)\right] x, \tag{21}$$

where

$$\begin{aligned} K_o(x) &= [\rho_u R_o + D(x)^T D(x)]^{-1} B(x)^T P(x), \\ K_{\mathcal{X}}(x) &= [\rho_u R_o + \\ &\qquad D(x)^T D(x)\big]^{-1} D(x)^T C(x), \end{aligned}$$

and $P(x)$ satisfies the state–dependent Riccati equation (9) with coefficients as above.

One interpretation of (21) is that $K_o(x)$ is for stabilization/performance and $K_{\mathcal{X}}(x)$ to satisfy (4).

It is interesting to see under which conditions, if any, the control law given by (21) reduces to (14). This is summarized in the following lemma.

Lemma 1 *Let $\rho_x = 0$ and $W(x) = I$. If $D(x)$ invertible and $A(x) - B(x)D(x)^\dagger C(x)$ pointwise Hurwitz $\forall x \in \mathcal{X}$, then, as $\rho_u \to 0$, $K_{\mathcal{X}}(x) \to D^{-1}(x)C(x)$ and $K_o(x) \to 0$.*

Proof 1 *As $\rho_u \to 0$, $K_{\mathcal{X}}(x) \to D(x)^\dagger C(x)$ and $K_o(x) \to (D(x)^T D(x))^{-1} B(x)^T P(x)$. Consider the limiting case when $\rho_u = 0$. Assuming that $D(x)$ is invertible, then $K_o(x) = 0 \Leftarrow B(x)^T P(x) = 0 \Leftrightarrow \mathcal{R}\{B(x)\} \subset \mathcal{N}\{P(x)\}$ $\forall x \in \mathcal{X}$, where $\mathcal{R}\{\cdot\}$ and $\mathcal{N}\{\cdot\}$ denote the range and kernel of a matrix, respectively. This can occur when $Q(x) = 0$. As $\rho_u \to 0$, $Q(x) \to \rho_x Q_o$. If $\rho_x = 0$, then $Q(x) = 0$ and the maximal solution $P_+(x)$ of (9) is such that $\mathcal{N}\{P_+\} = \mathcal{A}^-$, where $\mathcal{A}^-$ denotes the invariant subspace spanned by the eigenvectors of $\tilde{A}$ associated with its eigenvalues in the closed left half plane [Ni91]. Therefore $\mathcal{R}\{B\} \subset \mathcal{N}\{P\} \Leftrightarrow \mathcal{R}\{B\} \subset \mathcal{A}^-$ and P_+ is pointwise stabilizing provided that $\tilde{A}(x)$ is pointwise Hurwitz. This implies that $\mathcal{A}^- = \mathbb{R}^n$. Therefore $P = P_+ = 0$ and $K_o = 0$.*

Remark 3 *When both $\nabla h(x)$ and $B(x)$ are invertible $\forall x \in \mathcal{X}$, then $D = B^{-1}(\nabla h)^{-1}$ and $A - BD^\dagger C = 0$ which renders the associated Hamiltonian singular. Therefore, a positive semi–definite solution to the Riccati equation does not exist. This is easy to verify in the scalar case where the solutions to (9) are $p_1 = 0$ and $p_2 = -2\tilde{d}(-a\tilde{d} + \tilde{c}b)/b^2 = 0$. In this case $a - b\tilde{c}/\tilde{d} = a - a = 0$ is not pointwise Hurwitz.*

If $\nabla h(x)$ is orthogonal to $B(x)$, $D(x) = 0$ and $R_{\mathcal{X}}(x) = 0$. In this degenerate case $R(x) = \rho_u R_o$, $\tilde{Q}(x) = \rho_x Q_o + C(x)^T C(x)$ and $\tilde{A}(x) = A(x)$. The the controller gains become $K_{\mathcal{X}}(x) = 0$ and $K_o(x) = \rho_u^{-1} R_o^{-1} B(x)^T P(x)$ where $P(x) \geq 0$ provided that A3 is satisfied.

4 Weights for SDNR with State Constraints

The control law that results from solving the state-dependent regulator problem with state constraints exhibits an additive multi–objective structure of the state–dependent gain, where K_o is designed for performance and $K_{\mathcal{X}}$ for satisfaction of the state constraints. In this section suitable weighting strategies for regulation with state constraints will be introduced.

Consider the following cost functional

$$J(x,u) = J_o(x,u) + J_{\mathcal{X}}(x,u), \tag{22}$$

where $J_o(x,u)$ is given by (20), $J_{\mathcal{X}}(x,u)$ is given by (16). The objective is to choose $W(x) > 0$ such that (4) is enforced as x approaches $\partial\mathcal{X}$ and $x \to 0$ asymptotically.

4.1 Weighting based on Distance to Boundary

A simple choice for $W(x)$ is based on the distance of x to $\partial\mathcal{X}$. For each fixed x, let

$$\psi_i(x) = \left\| h_i(x)/h_i(0) \right\|_2, i = 1, \ldots, p \tag{23}$$

$$\phi_i(x) = \frac{1}{(\psi_i(x) + \epsilon_i)^{2N_i}}, \ i = 1, \ldots, p \tag{24}$$

with $N_i \in \mathbb{Z}$, $N_i > 1$ and $0 < \epsilon_i < 1$.

Define

$$W(x) = \mathrm{diag}(\phi_1(x), \ldots, \phi_p(x)) \tag{25}$$

Then, as $x \to \partial\mathcal{X}$, $\phi_i(x) \to \dfrac{1}{\epsilon_i^{2N_i}}$ and as $x \to 0$, $\phi_i(x) \to \dfrac{1}{(1+\epsilon)^{2N_i}}$. The tuning parameters, N_i and ϵ_i can be selected to make ϕ_i as large as necessary as x approaches $\partial\mathcal{X}$.

4.2 Weighting based on State Penalty

Another weighting strategy for SDNR of systems with state constraints was considered in [Fri98]. This strategy can be formulated in the framework introduced in the last section. Let $\mathcal{I}$ denote the set of all the states to be penalized and define $z = Cx$ where

$$C = \mathrm{diag}(\chi_1, \ldots, \chi_n)$$

and $\chi_i = 1\, \forall i \in \mathcal{I}$ and 0 otherwise. For the particular case of symmetric state constraints $h_i(x) = |x_i| - B_i, i \in \mathcal{I}$ and

$$\phi_i(x) = \left(\frac{x_i}{\nu_i B_i}\right)^{2N_i} \tag{26}$$

where $0 < \nu_i \leq 1$. In this case, the weight $W(x)$ is also given by (25) with $p = n$. Clearly, as $x \to \partial\mathcal{X}$, $\phi_i(x) \to \dfrac{1}{\nu^{2N_i}}$ for some i, and as $x \to 0$, $\phi_i(x) \to 0$. The satisfaction of the state constraints can be guaranteed by a suitable choice of N_i and ν_i.

5 Example

This example is based on [Fri98]. Consider the state-space dynamics of an inverted pendulum given by

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ c_1 \sin x_1 \end{bmatrix} + \begin{bmatrix} 0 \\ c_2 \end{bmatrix} \tau,$$

where x_1 is the angular position of the pendulum, in radians, τ is the control torque and $c_1 = g/l$, $c_2 = 1/(ml^2)$ are parameters of the system. For this example $c_1 = 0.6333$ and $c_2 = 1$. The objective is to control the pendulum to its inverted equilibrium position so that $|x_1| < 1$ (e.g. 58°). The pendulum starts at the inverted equilibrium position with an initial velocity of 5 rad/sec. It is assumed that both states are available for measurement, otherwise, a nonlinear observer can be employed.

5.1 LQR design

A linear quadratic regulator based on Jacobian linearization about the equilibrium $x_1 = 0, x_2 = 0$, is designed as the baseline controller for performance. The linearized model of the plant is

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ c_1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ c_2 \end{bmatrix} u$$

and the quadratic performance index $J_o(x, u)$ has coefficients $Q_o = \mathrm{diag}(10, 0)$ and $R_o = 1$. Solving the corresponding ARE gives the state feedback gain

$$K_{lqr} = \begin{bmatrix} 3.858 & 2.778 \end{bmatrix}$$

5.2 SDNR design I

To design a state–dependent nonlinear regulator we first find a state–dependent coefficient parameterization of the system. One such parameterization is

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ c_1 \mathrm{sinc}(x_1) & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ c_2 \end{bmatrix} u.$$

were $\mathrm{sinc}(x_1) = \sin(x_1)/x_1$. The state constraint set is

$$\mathcal{X} = \{x \in \mathbb{R}^2 : |x_1| - 1 \leq 0\},$$

Then $h(x) = |x_1| - 1$ and $\nabla h(x) = \begin{bmatrix} \mathrm{sgn}(x_1) & 0 \end{bmatrix}$.

Following the procedure developed in this paper, the coefficient matrices associated to the fictitious output z representing the state constraints are

$$C = \begin{bmatrix} 0 & \mathrm{sgn}(x_1) \end{bmatrix}, \quad D = 0.$$

Since $D(x) = 0\ \forall x \in \mathcal{X}$, $R_{\mathcal{X}}(x) = 0$, $S_{\mathcal{X}}(x) = 0$ and u in (12) does not affect z directly. This implies that $C(x) \to 0$ asymptotically, indicating the need to make x_2 small.

Using the weights suggested in section 4.1 we obtain

$$Q_{\mathcal{X}}(x) = \mathrm{diag}\Big(0, \frac{1}{(\sqrt{h(x)} + \epsilon)^{2N}}\Big).$$

Thus,

$$\begin{aligned} \text{as } x \to \partial\mathcal{X}, \quad Q_{\mathcal{X}}(x) &\to \mathrm{diag}(0, 1/\epsilon^{2N}) \\ \text{as } x \to 0, \quad Q_{\mathcal{X}}(x) &\to \mathrm{diag}(0, 1/(1+\epsilon)^{2N}) \end{aligned}$$

This weighting strategy imposes a severe penalty on x_2, the velocity of the pendulum, as x_1 approaches its allowable bounds. The objective is to satisfy the state constraints with the same control authority as the LQ regulator. The performance weights Q_o and R_o were not changed. For this controller $N = 2$ and $\epsilon = 0.25$ are satisfactory. In the simulations, at each integration step, a state–dependent Riccati equation is solved to obtain the state feedback control.

5.3 SDNR design II

Another controller based on the alternative approach of [Fri98] with the modifications introduced in section 4.2 was designed for comparison. In this case $C(x) = \mathrm{diag}(1, 0)$, $D(x) = 0$ and

$$Q_{\mathcal{X}}(x) = \mathrm{diag}((x_1/\nu_1)^{2N_1}, 0).$$

Thus,

$$\begin{aligned} \text{as } x \to \partial\mathcal{X}, \quad Q_{\mathcal{X}}(x) &\to \mathrm{diag}(1/\nu_1^{2N_1}, 0) \\ \text{as } x \to 0, \quad Q_{\mathcal{X}}(x) &\to \mathrm{diag}(0, 0) \end{aligned}$$

This weighting strategy imposes a severe penalty on x_1, the position of the pendulum, as x_1 approaches its allowable bounds. The performance weights Q_o and R_o were not changed. For this controller $N_1 = 2$ and $\nu = 0.32$. In the simulations, at each integration step, a state–dependent Riccati equation is solved to obtain the state feedback control.

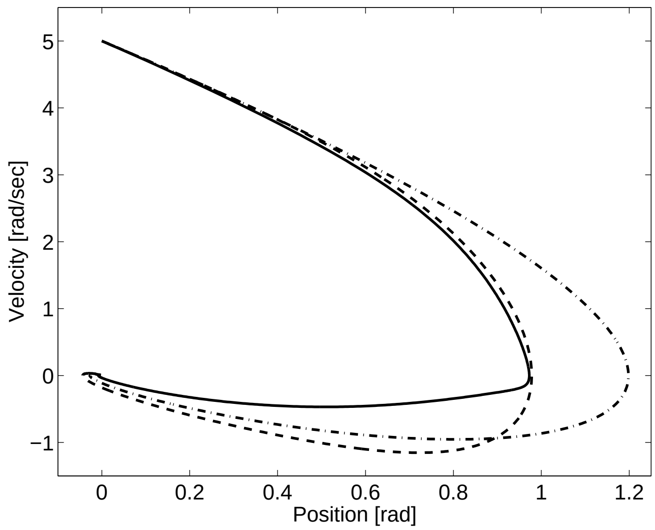

Figure 1: LQR (−·) SDRE I (—) SDRE II (−−)

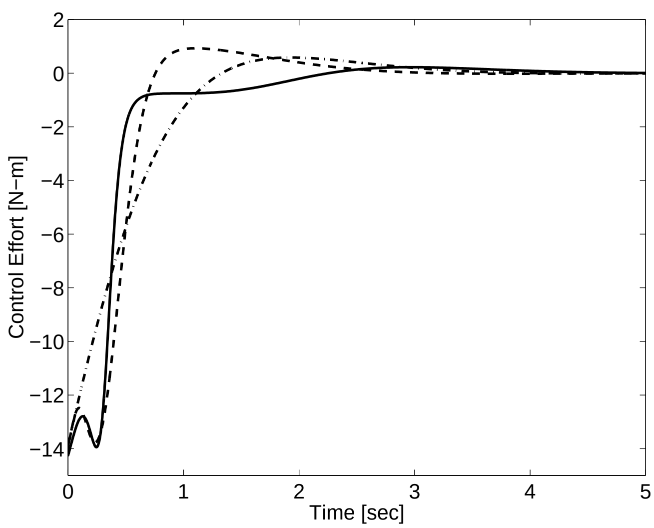

Figure 2: LQR (−·) SDRE I (—) SDRE II (−−)

5.4 Discussion

The results of the nonlinear simulations with each of these controllers are shown in the Figures 1 to 3. The LQ controller fails to satisfy the state constraints as shown in the phase–plane plot of Fig. 1 and the position plot of Fig. 3. This is not unexpected since this controller was not designed to take state constraints into account. Both SDRE controllers satisfy the state constraints. As mentioned before, all of the controllers were designed to use the same control authority as shown in Fig. 2. Both SDRE controllers satisfy the constraints and stabilize the pendulum around $x_1 = 0, x_2 = 0$ as desired. The main difference between these nonlinear controllers is in their velocity profiles shown in Fig. 4.

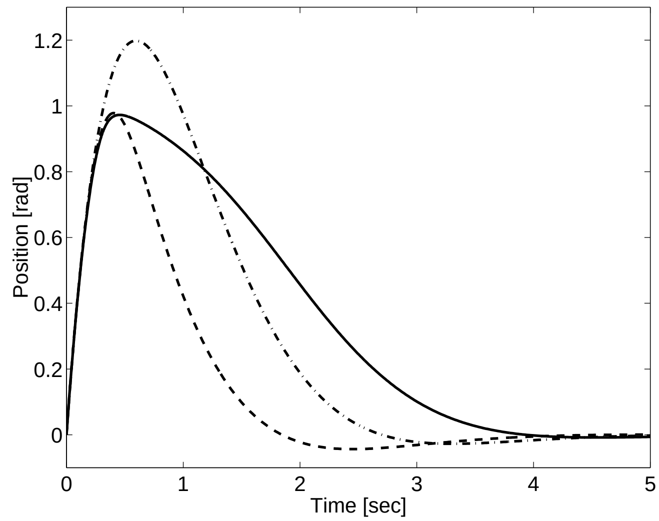

Figure 3: LQR (−·) SDRE I (—) SDRE II (−−)

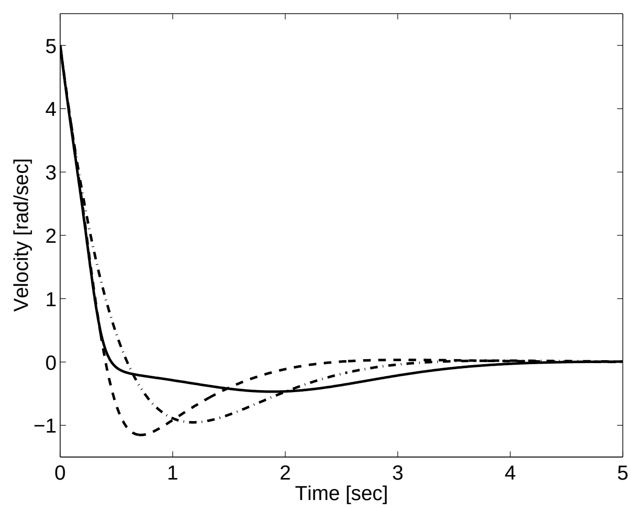

Figure 4: LQR (−·) SDRE I (—) SDRE II (−−)

6 Conclusions

In this paper an extension of the state–dependent Riccati equation approach with state constraints was investigated. To achieve regulation with state constraints a sufficient condition that forces the state trajectories of the closed-loop system to be parallel to the constraint levels sets is used in the controller design. The design procedure involves two steps. First a base-line controller is designed without taking into account the state constraints. Then a SDNR regulator is designed augmenting the cost functional to take into account the state constraints and the base-line performance. This step involves several iterations where performance and state constraints weights are tuned to achieve satisfactory state-constrained controller performance.

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