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Sliding mode control for a class of non-affine nonlinear systems

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ABSTRACT

The objective of this paper is to present a sliding mode control (SMC) technique for a class of nonlinear and control non-affine systems. A rotor supported by a pair of magnetic bearings is introduced as a nonlinear system which is designed as a sequence of linear time-varying (LTV) systems and iterated for each time. The sliding surface is designed for each iterated LTV system by using the time response of the previous iteration. The surface parameters are selected so that the poles of reduced order LTV system remain on the left-hand side of the complex plane. It is shown that the response of LTV approximations converges to the nonlinear system's response which is validated by using the non-affine nonlinear dynamical equations of rotor-active magnetic bearing system.

1. Introduction

Active magnetic bearings (AMBs) are receiving increasing attention due to their unique characteristics. They have some significant advantages compared to conventional bearings. The AMB systems do not require any sort of lubrication between its rotating parts and rarely need maintenance due to their contactless behaviour. The rotor is levitated in the air electromagnetically so there will not be any contact between the rotating part and the bearing. The AMBs provide variable stiffness and damping and hence the vibration in the rotor can be suppressed to a lower level.

High nonlinearity in systems and the complexity of control algorithms are some of the disadvantages of AMBs. The main sources of nonlinearity in an AMB system are the forces generated in the electromagnetic actuator [1]. In addition to these nonlinearities there exist some uncertainties and external disturbances in the nonlinear model of rotor-AMB system. Thus it is required to design a robust control to overcome these ambiguities.

Sliding mode control (SMC) is a well-known robust control technique against external disturbances, parameter uncertainties and unmodeled dynamics. The dynamics of the system is altered with a designed hyperplane hence the SMC can conquer these uncertainties. The SMC design for linear and nonlinear systems have been studied by various authors [10]. Linear time-varying surfaces for a class of nonlinear systems are designed by [2,3]. Hyperplane design using stable manifolds was studied by [4]. For active magnetic bearings sliding mode control is used in [5,6].

In this paper a new SMC design method is suggested for non-affine nonlinear equations of the rotor-AMB system. The iteration scheme used in [7,2] is extended to the SMC of non-affine nonlinear systems in the presence of synchronous disturbances. The control is generated from the control of a sequence of LTV iterations. Instead of designing the hyperplane for nonlinear systems it is designed for an LTV system and it is shown that after a few iterations response of time-varying systems converges to the nonlinear dynamics of the system.

The results show that the technique can successfully be applied to the rotor-AMB systems and this can be extended to the other non-affine nonlinear systems.

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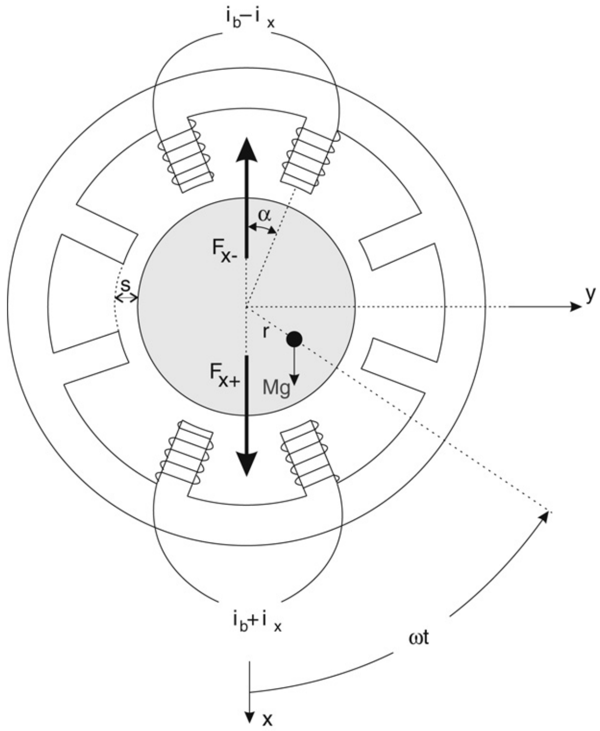

Fig. 1. Differential driving mode for an 8-pole AMB.

2. Mathematical model of rotor-AMB system

2.1. Nonlinear electromagnetic force

Assuming there is no eddy current losses, magnetic flux leakage, hysteresis, saturation and also relative permeability of the material $\mu_r \gg 1$, electromagnetic force is given as [12],

$$f = \frac{1}{4}\mu_0 N^2 A \frac{i^2}{s^2}\cos\alpha \tag{1}$$

where μ_0 is the permeability of vacuum, A is the cross sectional area of the electromagnet, N is the number of windings, s is the nominal air gap between the rotor and the stator and α is the angle between stator pole and its effecting axis. This equation is derived only for 2-pole magnetic bearing element but the AMB used in this paper is an 8-pole magnetic bearing system which works in differential driving mode. This working principle is depicted in Fig. 1.

Here the current i_b is called bias current. Bias current is supplied constantly to each 2-pole element. In the differential driving mode, control current is added to the bias current while in the opposite direction it is subtracted from it. In the positive directions the gap between the rotor and the AMB is assumed to be decreasing while the resultant current is the summation of bias current and regulating current. Ignoring the geometric coupling between horizontal and vertical axes net forces acting on the rotor in positive and negative directions are

$$F_{x+} = \frac{1}{4}\mu_0 N^2 A\cos\alpha \frac{(i_b+i_x)^2}{(s-x)^2} \tag{2}$$

$$F_{x-} = \frac{1}{4}\mu_0 N^2 A\cos\alpha \frac{(i_b-i_x)^2}{(s+x)^2}. \tag{3}$$

The net resulting force is calculated from the difference of these two forces:

$$F_x = \kappa\left[\frac{(i_b+i_x)^2}{(s-x)^2} - \frac{(i_b-i_x)^2}{(s+x)^2}\right], \tag{4}$$

where $\kappa = \frac{1}{4}\mu_0 N^2 A\cos\alpha$. For simplicity only the force acting on vertical direction is given. Horizontal force can easily be derived with the same procedure as described above.

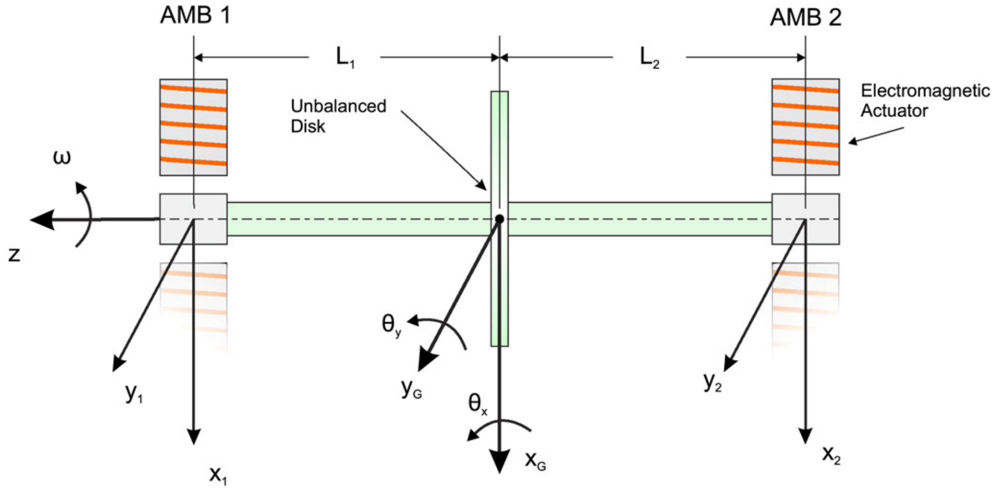

Fig. 2. Schematic view of rotor-AMB system with coordinates that describes its motion.

2.2. Rigid rotor dynamics

In this section dynamical equations of a rigid rotor supported by two AMBs are derived. Magnetic bearings levitate the rotor with variable stiffness and damping in each axis separately. When the rotor approaches the bearing stator, AMB produces a net force in the opposite direction to move the rotor to the bearing rotation centre. The rotor-AMB system is depicted in Fig. 2 where this rigid rotor with a mass unbalance rotates about z-axis with an angular velocity of ω.

Similar to the mathematical model given in [11], equations of motion of the rigid rotor with an unbalanced disk can be derived by using Newton–Euler formulae where $\sum F = ma$ and $\sum M = I\alpha$.

$$M\ddot{x}_G = F_{x1} + F_{x2} + F_{xu} + Mg \tag{5}$$

$$I_x\ddot{\theta}_y = \omega I_z\dot{\theta}_x + F_{x1}L_1 - F_{x2}L_2 \tag{6}$$

$$M\ddot{y}_G = F_{y1} + F_{y2} + F_{yu} \tag{7}$$

$$I_y\ddot{\theta}_x = -\omega I_z\dot{\theta}_y - F_{y1}L_1 + F_{y2}L_2, \tag{8}$$

where I_x and I_y are transverse mass moment of inertia about x- and y-axes respectively, I_z is polar moment of inertia about z-axis, M denotes mass of rotor, L_1 and L_2 are the distances between the centre of mass and magnetic bearings on the left-hand side and on the right-hand side respectively. F_{xu} and F_{yu} are the components of force created due to mass unbalance in horizontal and vertical axis respectively. Mass unbalance is shown in Fig. 1. The following equations are the components of unbalance force:

$$F_{ux} = M\omega^2 r\cos(\omega t) \tag{9}$$

$$F_{uy} = M\omega^2 r\sin(\omega t). \tag{10}$$

The dynamical equations of the rigid rotor are derived with respect to the geometric centre. Since the rotor is supported by its two ends and it is assumed that the sensors are located in the middle of magnetic bearings, it is necessary to re-write the equations in terms of bearing coordinates. Using trigonometric relations and assuming small oscillations of θ (i.e. $sin(\theta) \cong \theta$), the following equations are obtained:

$$x_1 = x_G + L_1\theta_y \qquad y_1 = y_G + L_1\theta_x \tag{11a}$$

$$x_2 = x_G - L_2\theta_y \qquad y_2 = y_G - L_2\theta_x. \tag{11b}$$

Taking the second derivatives of (11a) and (11b) and substituting the equations into (5) to (8), then the acceleration components for each end of the rotor are obtained as follows:

$$\ddot{x}_1 = \frac{1}{M}[F_{x1} + F_{x2}] + L_1\left[\frac{\omega I_z\dot{\theta}_x}{I_x} + \frac{F_{x1}L_1}{I_x} - \frac{F_{x2}L_2}{I_x}\right] + \omega^2 r\cos\omega t + g \tag{12}$$

$$\ddot{y}_1 = \frac{1}{M}[F_{y1} + F_{y2}] + L_1\left[-\frac{\omega I_z\dot{\theta}_y}{I_y} - \frac{F_{y1}L_1}{I_y} + \frac{F_{y2}L_2}{I_y}\right] + \omega^2 r\cos\omega t \tag{13}$$

$$\ddot{x}_2 = \frac{1}{M}[F_{x1} + F_{x2}] - L_2\left[\frac{\omega I_z\dot{\theta}_x}{I_x} + \frac{F_{x1}L_1}{I_x} - \frac{F_{x2}L_2}{I_x}\right] + \omega^2 r\cos\omega t + g \tag{14}$$

$$\ddot{y}_2 = \frac{1}{M}[F_{y1} + F_{y2}] - L_2\left[-\frac{\omega I_z \dot{\theta}_y}{I_y} - \frac{F_{y1}L_1}{I_y} + \frac{F_{y2}L_2}{I_y}\right] + \omega^2 r\cos\omega t, \tag{15}$$

where $\dot{\theta}_x$ and $\dot{\theta}_y$ are angular velocities around x_G and y_G respectively and they are derived from the derivatives of Eqs. (11a) and (11b) as follows:

$$\dot{\theta}_x = \frac{\dot{y}_1 - \dot{y}_2}{L_1 + L_2} \qquad \dot{\theta}_y = \frac{\dot{x}_1 - \dot{x}_2}{L_1 + L_2}. \tag{16}$$

The dynamic equations of AMB which are given in Eqs. (12)–(15) are used in the following section for their control system design.

3. Robust control design

In this section, the SMC of nonlinear and control non-affine system with disturbances of the form $\dot{x} = f(x, u, t)$ is considered. The nonlinear dynamics of the system are assumed to be represented in the state space form as:

$$\dot{x} = A(x(t))x(t) + B(x(t), u(t))u(t) + F(t) \tag{17}$$

where $A \in \mathfrak{R}^{n\times n}$ is a nonlinear function of states x and $B \in \mathfrak{R}^{n\times m}$ is a nonlinear function of both x and the control input u. The SMC for control affine nonlinear systems is studied by [2,8]. In these studies, it has been shown that the nonlinear system's response is reached by its successive LTV approximations which give a great opportunity to design the control for the successive LTV systems. Nevertheless, the approach has not been extended to the SMC of control non-affine nonlinear systems. It is well known that for nonlinear systems the SMC cannot be designed in a straightforward way, since the design of sliding surface needs careful attention. This is because a nonlinear sliding surface (hyperplane) is to be designed to overcome the system's nonlinearity. In general, this nonlinearity is linked with the states (i.e. the surface is nonlinear in terms of states) for control affine nonlinear systems. The problem of sliding surface design for non-affine nonlinear systems becomes much more sophisticated since the surface is now to be defined in terms of control as well. This makes the problem almost unsolvable even numerically.

In this paper, we show that – at least by doing simulations – a time-varying sliding surface may be used for the SMC of a class of non-affine nonlinear systems. We have shown that the response of non-affine nonlinear system is reached by its successive LTV approximations which gives us a chance to design an SMC for each LTV system. Therefore we will be able to design the sliding surface for the LTV system instead of designing it for the non-affine nonlinear control problem.

In the following subsection, we shall overview the iteration scheme for the sake of completeness.

3.1. The iteration scheme

This section extends a previously studied technique for nonlinear dynamical systems in the form of

$$\dot{x} = A(x)x + B(x)u$$

to Eq. (17). It has been shown [7] that equations of nonlinear dynamical systems can be replaced by a sequence of LTV equations as the responses of recursive LTV approximations converge to the response of the nonlinear equations.

Consider now the nonlinear, control non-affine system with disturbances in the form of (17). This equation is approximated by a sequence of LTV equations as follows:

$$\begin{aligned}
&\dot{x}^{[1]} = A\left(x_0\right)x^{[1]}(t) + B\left(x_0, u_0\right)u^{[1]}(t) + F\left(t_0\right)\\
&\dot{x}^{[2]} = A\left(x^{[1]}(t)\right)x^{[2]}(t) + B\left(x^{[1]}(t), u^{[1]}(t)\right)u^{[2]}(t) + F\left(t\right)\\
&\vdots\\
&\dot{x}^{[i]} = A\left(x^{[i-1]}(t)\right)x^{[i]}(t) + B\left(x^{[i-1]}(t), u^{[i-1]}(t)\right)u^{[i]}(t) + F\left(t\right)\\
&\left.\begin{array}{l} x^{[i]}(t_0) = x_0 \in \mathfrak{R}^n \\ u^{[i]}(t_0) = u_0 \in \mathfrak{R}^m \end{array}\right\} \quad \text{for } i \geq 1.
\end{aligned}$$

The convergence of the approximations is proved in the case of smooth functions A and B [9]. In the sense of sliding control, however, we have a discontinuous function *sgn* in the control. The proof of convergence in this case is equivalent to proving the convergence of a sequence of functions $\xi^{[i]}(t)$ which satisfy equations of the form

$$\dot{\xi}^{[i]}(t) = A\left(\xi[i-1](t)\right)\xi^{[i]}(t) + f\left(\xi^{[i-1]}(t)\right) \tag{18}$$

where f is discontinuous and A is locally Lipschitz. Let F any $\epsilon > 0, f_\epsilon$ be a locally Lipschitz function such that

$$\int_0^t g(s)\left(f\left(\xi(s)\right) - f_\epsilon\left(\xi(s)\right)\right)\,\mathrm{d}s \leq \epsilon \tag{19}$$

for any bounded functions $g(s)$ and $\xi(s)$. This is clearly possible (for example, if f contains the *sgn* function we can use an appropriately scaled *arctan* function for f_ϵ).

Next consider the systems

$$\dot{\eta}^{[i]}(t) = A\left(\eta^{[i-1]}(t)\right)\eta^{[i]}(t) + f_\epsilon\left(\eta[i-1](t)\right), \tag{20}$$

for any fixed $\epsilon > 0$. These systems converge uniformly or any compact time interval by the general theory and by Eq. (19) it is now easy to show that

$$\left\|\xi^{[i]}(t) - \eta^{[i]}(t)\right\| \longrightarrow 0 \tag{21}$$

as $\epsilon \longrightarrow 0$ so the solutions $\xi^{[i]}(t)$ also converge on any compact time interval.

Lemma 1. *Supposing the nonlinear dynamical equation (17) has a unique solution on the interval* $[0, \tau]$*, sequence of LTV system equations converges uniformly to the nonlinear solution of* $x(t)$ *after a series of iterations.*

3.2. Sliding mode controller

Consider the n-dimensional non-affine nonlinear system given in Eq. (17). Assume $[A(x), B(x, u)]$ is a controllable pair. In order to design the SMC, the system is defined in the so-called regular form in which $B = [0\ B_2]^{\mathrm{T}}$. If the system is not in the regular form then it is necessary to transform the system to the canonical form to obtain two subsystems, one has control input and one does not have. A sliding hyperplane will be designed by using the subsystem without control which will alter the system's behaviour with desired dynamics. Suppose that x is in the current coordinates and ξ is in the transformed coordinates. The following transformation is applied to the above equation:

$$\xi(t) = T(t)\cdot x(t) \tag{22}$$

$$x(t) = T^{-1}(t)\cdot \xi(t) \tag{23}$$

where $T \in \mathfrak{R}^{n\times n}$ is nonsingular.

After a proper coordinate transformation and taking the iteration scheme into account, Eq. (17) becomes:

$$\dot{\xi}^{[i]}(t) = \widetilde{A}\left(\xi^{[i-1]}(t)\right)\xi^{[i]}(t) + \widetilde{B}\left(\xi^{[i-1]}(t), u^{[i-1]}(t)\right)u^{[i]}(t) + \widetilde{F}(t) \tag{24}$$

where $\widetilde{A}\left(\xi^{[i-1]}(t)\right)$, $\widetilde{B}\left(\xi^{[i-1]}(t), u^{[i-1]}(t)\right)$ and $\widetilde{F}(t)$ are respectively,

$$\widetilde{A} = TAT^{-1} + \dot{T}T^{-1} = \begin{bmatrix} \widetilde{A}_{11} & \widetilde{A}_{12} \\ \widetilde{A}_{21} & \widetilde{A}_{22} \end{bmatrix}$$

$$\widetilde{B} = TB = \begin{bmatrix} 0 \\ \widetilde{B}_2 \end{bmatrix}$$

$$\widetilde{F} = TF = \begin{bmatrix} \widetilde{F}_1 \\ \widetilde{F}_2 \end{bmatrix}.$$

The transformation matrix T can be found by performing QR decomposition on B matrix in order to divide control matrix into two parts as shown. Then the following equations can be written easily from the above matrices:

$$\dot{\xi}_1^{[i]} = \widetilde{A}_{11}^{[i-1]}\xi_1^{[i]} + \widetilde{A}_{12}^{[i-1]}\xi_2^{[i]} + \widetilde{F}_1 \tag{25}$$

$$\dot{\xi}_2^{[i]} = \widetilde{A}_{21}^{[i-1]}\xi_1^{[i]} + \widetilde{A}_{22}^{[i-1]}\xi_2^{[i]} + \widetilde{B}_2^{[i-1]}u^{[i]} + \widetilde{F}_2. \tag{26}$$

We now define the sliding surface for each transformed LTV equations:

$$\sigma^{[i]}(t) = C^{[i]}\xi_1^{[i]} + \xi_2^{[i]} = 0 \tag{27}$$

from which the derivative of $\sigma^{[i]}(t)$ is

$$\dot{\sigma}^{[i]}(t) = \dot{C}^{[i]}\xi_1^{[i]} + C^{[i]}\dot{\xi}_1^{[i]} + \dot{\xi}_2^{[i]} = -\delta\,\mathrm{sign}\left(\sigma^{[i]}(t)\right) \tag{28}$$

so that the sliding surface is reached in a finite time. In Eq. (28), δ is a positive real number and $C \in \mathfrak{R}^{m\times m}$ is found by the pole placement method to stabilise the reduced order system. Using the uncontrolled part of the transformed system and omitting $\widetilde{F}_1(t)$ for simplicity the reduced order model can be obtained by substituting (27) into (25):

$$\dot{\xi}_1^{[i]} = \underbrace{\left[\widetilde{A}_{11}^{[i-1]} - \widetilde{A}_{12}^{[i-1]}C^{[i]}\right]}_{M}\xi_1^{[i]}, \tag{29}$$

where C is chosen to stabilise M by placing the poles on the left-hand side plane. Conditions for stability of linear time-varying systems are given in [8]. Due to its dependency on $A_{11}(\xi(t))$ and $A_{12}(\xi(t))$, C varies with time. By substituting Eqs. (25) and (26) into Eq. (28),

$$\begin{aligned}\dot{\sigma}^{[i]}(t) &= \dot{C}^{[i]}\xi_1^{[i]} + C^{[i]}\left[\widetilde{A}_{11}(\xi^{[i-1]})\xi_1^{[i]} + \widetilde{A}_{11}(\xi^{[i-1]})\xi_2^{[i]}\right] + \widetilde{A}_{21}(\xi^{[i-1]})\xi_1^{[i]} \\ &\quad + \widetilde{A}_{22}(\xi^{[i-1]})\xi_2^{[i]} + \widetilde{B}_2(\xi^{[i-1]}, u^{[i-1]}) = \delta \cdot \mathrm{sign}(\sigma^{[i]}(t)).\end{aligned} \tag{30}$$

Therefore a sequence of sliding mode controls $u^{[i]}(t)$ is obtained:

$$u^{[i]} = \frac{-1}{\widetilde{B}_2(\xi^{[i-1]}, u^{[i-1]})}\left\{\begin{array}{l}\dot{C}^{[i]}\xi_1^{[i]} + C^{[i]}\left[\widetilde{A}_{11}(\xi^{[i-1]})\xi_1^{[i]} + \widetilde{A}_{11}(\xi^{[i-1]})\xi_2^{[i]}\right] \\ \quad + \widetilde{A}_{21}(\xi^{[i-1]})\xi_1^{[i]} + \widetilde{A}_{22}(\xi^{[i-1]})\xi_2^{[i]} \\ \\ \quad + \widetilde{B}_2(\xi^{[i-1]}, u^{[i-1]})u^{[i]} + \delta \cdot \mathrm{sign}(\sigma^{[i]})\end{array}\right\}. \tag{31}$$

For simplicity, the time dependency of $\xi(t)$, $\sigma(t)$, $C(t)$ and submatrices of $\widetilde{A}(t)$ is not shown.

Note that the sliding surface design for the non-affine nonlinear system is simplified with its LTV approximations which introduce a time-varying sliding surface for the nonlinear system. In the following section the methodology is applied to the nonlinear model of the AMB system.

4. Numerical application

In order to apply the proposed method to the equations of motion for the AMB derived in Section 2, we express them in state space form. For this purpose the states of the system are selected as follows:

$$\begin{array}{ll}\xi_1 = x_1 & \xi_5 = x_2 \\ \xi_2 = \dot{x}_1 & \xi_6 = \dot{x}_2 \\ \xi_3 = y_1 & \xi_7 = y_2 \\ \xi_4 = \dot{y}_1 & \xi_8 = \dot{y}_2.\end{array} \tag{32}$$

Control currents are renamed as $u_1 = i_{x1}$, $u_2 = i_{y1}$, $u_3 = i_{x2}$ and $u_1 = i_{y2}$. After renaming the variables, state space form is obtained as in Eq. (17). The state matrix is only a function of x while control matrix is function of both x and u. $F(t)$ is time-varying matrix including unbalance force and gravity.

$$A(\xi) = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ a_{21}(\xi_1) & 0 & 0 & a_{24}(\omega) & a_{25}(\xi_5) & 0 & 0 & a_{28}(\omega) \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & a_{42}(\omega) & a_{43}(\xi_3) & 0 & 0 & a_{46}(\omega) & a_{47}(\xi_7) & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ a_{61}(\xi_1) & 0 & 0 & a_{64}(\omega) & a_{65}(\xi_5) & 0 & 0 & a_{68}(\omega) \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & a_{82}(\omega) & a_{83}(\xi_3) & 0 & 0 & a_{86}(\omega) & a_{87}(\xi_7) & 0 \end{bmatrix} \tag{33}$$

$$B(\xi, u) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ b_{21}(u_1) & 0 & b_{23}(u_3) & 0 \\ 0 & 0 & 0 & 0 \\ 0 & b_{42}(u_2) & 0 & b_{44}(u_4) \\ 0 & 0 & 0 & 0 \\ b_{61}(u_1) & 0 & b_{64}(u_3) & 0 \\ 0 & 0 & 0 & 0 \\ 0 & b_{82}(u_2) & 0 & b_{84}(u_4) \end{bmatrix} \tag{34}$$

where parameters a_{ij} and b_{ij} are given in Appendix. The open loop poles of the system at the initial time $t = 0$ consist of four negative and four positive poles, which make the system unstable. By selecting the poles of reduced order system in the left-hand plane and designing a sliding surface, the system can be stabilised under the conditions given in [8]. For this purpose the poles of the reduced order system are selected as $[-152.69, -116.3, -135.9, -137.3]$. Model parameters of the rotor and electromagnetic actuators are given in Table 1.

Numerical simulations are carried out by using *Matlab* ® software. For solving the differential equations, first order Euler equation is used with a fixed time increment. Using a rotational speed of 250 rad/s and initial positions of

$$[x_1, y_1; x_2; y_2] = [0.35, -0.35; -0.35; 0.35]\ \text{mm},$$

the response of system in some iterations and nonlinear response is depicted in Fig. 3 for x_1 and $\dot{x}_1$. It can be seen from the figure that the states are converging to the nonlinear system's response after a few iterations. The error graphs show us how

Table 1
Model parameters.

Parameter	Value	Parameter	Value
M	1 kg	L_1	0.15 m
I_x, I_y	0.029 kg/m^2	L_2	0.19 m
I_z	0.00222 kg/m^2	s	1 mm
μ_0	$4\pi\,10^{-7}$ N/A^2	N	500
i_b	2 A	r	0.005 m

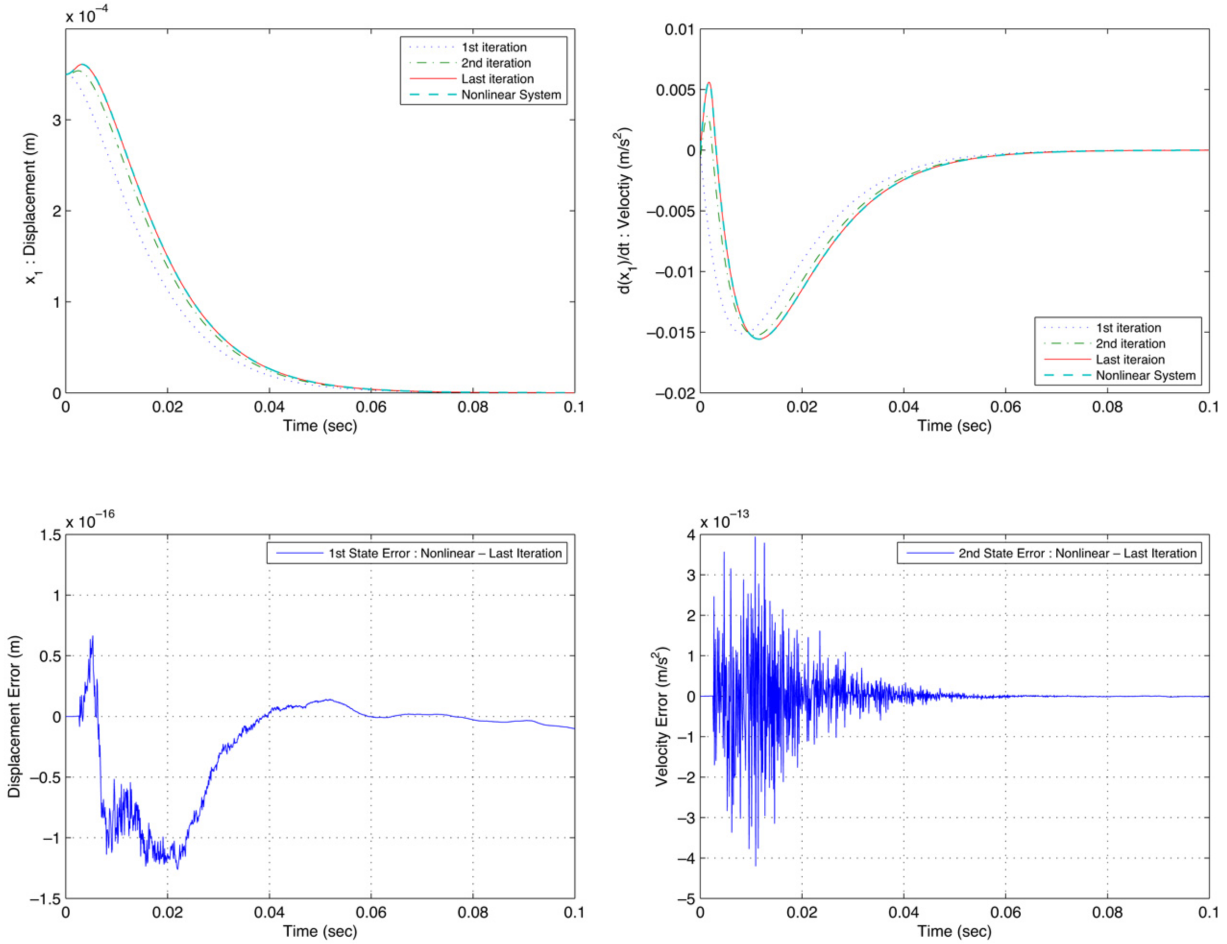

Fig. 3. 1st and 2nd states in some iterations and error graphs.

close the last iterations results to the nonlinear system's response are. In Fig. 4 the control currents for one magnetic bearing is given. Control currents oscillate in order to compensate the unbalance force and keep the rotor at the bearing centre. To eliminate the chatter phenomenon in the control signal a tanh(.) function is used instead of the signum function. So we obtain a smooth switching function which forces the system to slide on the hyperplane.

5. Conclusion

In this study, we have designed a sliding mode controller for a class of nonlinear non-affine systems in the presence of disturbances. The SMC design for the nonlinear system is simplified by using the successive LTV approximations of the nonlinear system and hence the difficulties in the design of hyperplane for the non-affine nonlinear system are eliminated. The proposed control strategy is then applied to the nonlinear equations of rotor-AMB system. The simulation results show the success of the method even for the disturbances. It should be noted that the proposed method can be applied to non-affine nonlinear systems when the dynamics of the system changes slowly. In our further works, we shall explore possible implementations of the proposed control technique.

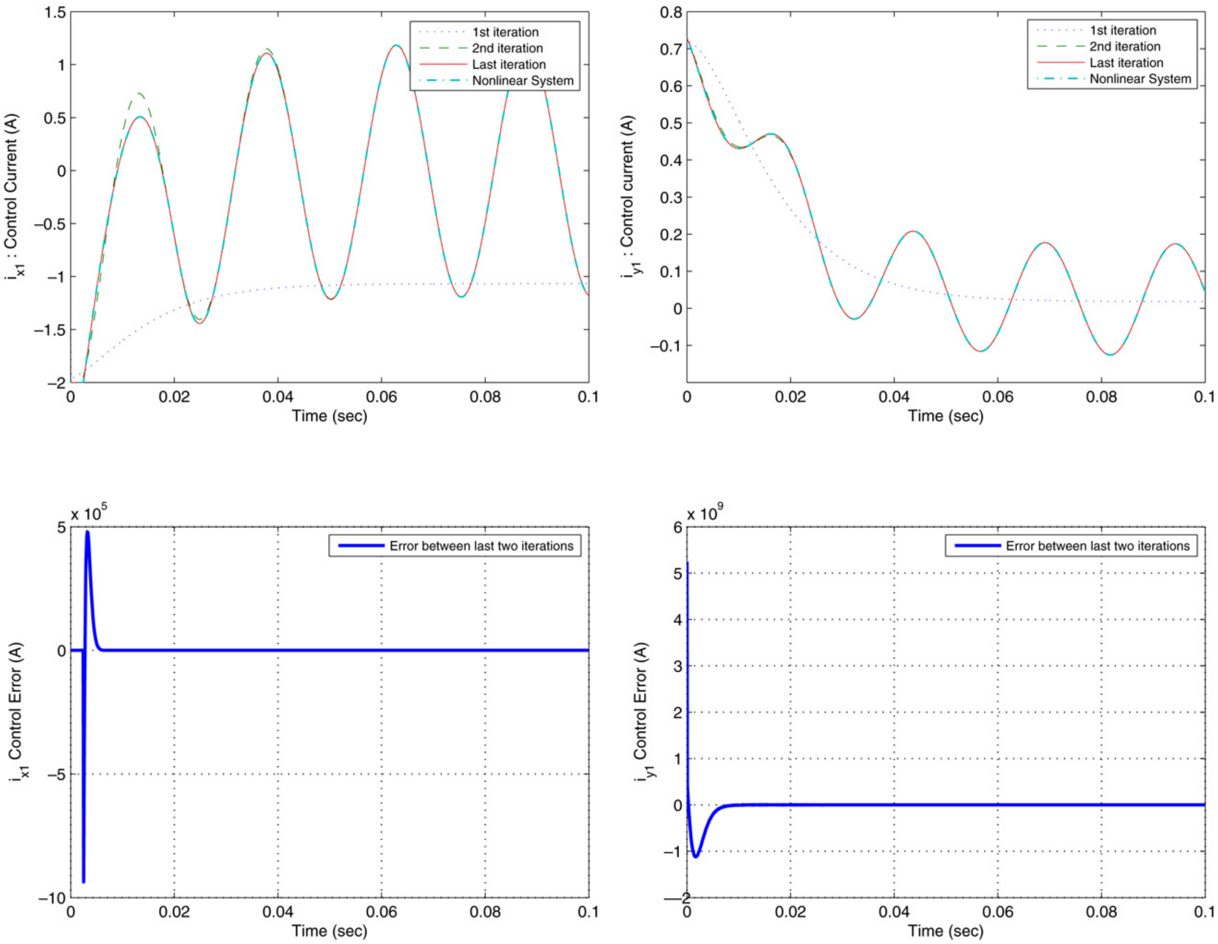

Fig. 4. i_{x1} and i_{y1} in some iterations and error graphs.

Appendix

Parameters of A and B matrices are:

$$a_{24} = a_{46} = a_{68} = a_{82} = -\frac{wJ_zL_1}{J_x(L_1+L_2)} \qquad a_{28} = a_{42} = a_{64} = a_{86} = \frac{wJ_zL_1}{J_x(L_1+L_2)}$$

$$a_{21} = \left(\frac{1}{M} + \frac{L_1^2}{J_x}\right)\frac{\mu_0 N^2 S \cos\alpha \; s \; i_b^2}{(s-\xi_1)^2(s+\xi_1)^2} \qquad a_{25} = \left(\frac{1}{M} - \frac{L_1L_2}{J_x}\right)\frac{\mu_0 N^2 S \cos\alpha \; s \; i_b^2}{(s-\xi_5)^2(s+\xi_5)^2}$$

$$a_{43} = \left(\frac{1}{M} + \frac{L_1^2}{J_y}\right)\frac{\mu_0 N^2 S \cos\alpha \; s \; i_b^2}{(s-\xi_3)^2(s+\xi_3)^2} \qquad a_{47} = \left(\frac{1}{M} - \frac{L_1L_2}{J_y}\right)\frac{\mu_0 N^2 S \cos\alpha \; s \; i_b^2}{(s-\xi_7)^2(s+\xi_7)^2}$$

$$a_{61} = \left(\frac{1}{M} - \frac{L_1L_2}{J_x}\right)\frac{\mu_0 N^2 S \cos\alpha \; s \; i_b^2}{(s-\xi_1)^2(s+\xi_1)^2} \qquad a_{65} = \left(\frac{1}{M} + \frac{L_2^2}{J_x}\right)\frac{\mu_0 N^2 S \cos\alpha \; s \; i_b^2}{(s-\xi_5)^2(s+\xi_5)^2}$$

$$a_{83} = \left(\frac{1}{M} - \frac{L_1L_2}{J_y}\right)\frac{\mu_0 N^2 S \cos\alpha \; s \; i_b^2}{(s-\xi_3)^2(s+\xi_3)^2} \qquad a_{87} = \left(\frac{1}{M} + \frac{L_2^2}{J_y}\right)\frac{\mu_0 N^2 S \cos\alpha \; s \; i_b^2}{(s-\xi_7)^2(s+\xi_7)^2}$$

$$b_{21} = \frac{1}{4}\left(\frac{1}{M} + \frac{L_1^2}{J_x}\right)\frac{\mu_0 N^2 S \cos\alpha \left(4u_1 s\xi_1 + 4i_b(s^2+\xi_1^2)\right)}{(s-\xi_1)^2(s+\xi_1)^2}$$

$$b_{23} = \frac{1}{4}\left(\frac{1}{M} - \frac{L_1L_2}{J_x}\right)\frac{\mu_0 N^2 S \cos\alpha \left(4u_3 s\xi_5 + 4i_b(s^2+\xi_5^2)\right)}{(s-\xi_5)^2(s+\xi_5)^2}$$

$$b_{42} = \frac{1}{4}\left(\frac{1}{M} + \frac{L_1^2}{J_y}\right)\frac{\mu_0 N^2 S \cos\alpha \left(4u_2 s\xi_3 + 4i_b(s^2+\xi_3^2)\right)}{(s-\xi_3)^2(s+\xi_3)^2}$$

$$b_{44} = \frac{1}{4}\left(\frac{1}{M} - \frac{L_1 L_2}{J_y}\right)\frac{\mu_0 N^2 S \cos\alpha \left(4u_4 s\xi_7 + 4i_b(s^2 + \xi_7^2)\right)}{(s-\xi_7)^2(s+\xi_7)^2}$$
$$b_{61} = \frac{1}{4}\left(\frac{1}{M} - \frac{L_1 L_2}{J_x}\right)\frac{\mu_0 N^2 S \cos\alpha \left(4u_1 s\xi_1 + 4i_b(s^2 + \xi_1^2)\right)}{(s-\xi_1)^2(s+\xi_1)^2}$$
$$b_{63} = \frac{1}{4}\left(\frac{1}{M} + \frac{L_2^2}{J_x}\right)\frac{\mu_0 N^2 S \cos\alpha \left(4u_3 s\xi_5 + 4i_b(s^2 + \xi_5^2)\right)}{(s-\xi_5)^2(s+\xi_5)^2}$$
$$b_{82} = \frac{1}{4}\left(\frac{1}{M} - \frac{L_1 L_2}{J_y}\right)\frac{\mu_0 N^2 S \cos\alpha \left(4u_2 s\xi_3 + 4i_b(s^2 + \xi_3^2)\right)}{(s-\xi_3)^2(s+\xi_3)^2}$$
$$b_{84} = \frac{1}{4}\left(\frac{1}{M} + \frac{L_2^2}{J_y}\right)\frac{\mu_0 N^2 S \cos\alpha \left(4u_4 s\xi_7 + 4i_b(s^2 + \xi_7^2)\right)}{(s-\xi_7)^2(s+\xi_7)^2}.$$

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