EXPONENTIAL STABILITY OF LARGE-SCALE NONLINEAR SYSTEMS WITH MULTIPLE TIME DELAYS

S.P.Banks D.McCaffrey

University of Sheffield, Dept. Automatic Control & Systems Engineering, Mappin St., Sheffield S1 3JD, England.

Abstract: The exponential stability of large-scale nonlinear parabolic systems with time-varying delays is studied in this paper using a new generalization of Gronwall's inequality and Lyapunov theory. The systems are interconnected by bounded operators, although this can be generalized. All the time-delays will be assumed to have bounded time derivatives for simplicity here.

Keywords: Exponential Stability, Nonlinear Delay Systems

1. INTRODUCTION

In this paper we shall study the exponential stability of large scale nonlinear distributed systems with delay of the form

$$\begin{aligned}\dot{x}(t) = &-A(x(t),t)x(t) + B(x(t),t)u(t) + \\ &\sum_{k=1}^{L} C^k(x(t),t)x(t-\tau_k(t)) + \\ &f(t,x(t),x(t-\tau_k(t))) \qquad (1)\\ x_i(t) = &\phi_i(t)\ ,\ -\infty < t \le t_0,\ i = 1,\cdots,m\end{aligned}$$

Here, $x(t)$ is of the form $x(t) = (x_1(t),\cdots,x_m(t))^T$, where $x_i = (x_1^i,\cdots,x_{n_i}^i)^T$ is the state of the i^{th} isolated subsystem given by

$$\dot{x}_i(t) = -A_{ii}(x_i(t),t)x_i(t) + B_{ii}(x_i(t),t)u_i(t)$$

where $u_i = (u_1^i,\cdots,u_{r_i}^i)^T$. Stabilization will be achieved by finding a feedback control of the form

$$u_i(t) = K_{ii}(x_i(t),t)x_i(t)$$

so that the system

$$\begin{aligned}\dot{x}_i(t) = &-A_{ii}(x_i(t),t)x_i(t) + \\ &B_{ii}(x_i(t),t)K_{ii}(x_i(t),t)x_i(t) \qquad (2)\end{aligned}$$

is stable. The resulting operator $-A_{ii}(x_i(t),t) + B_{ii}(x_i(t),t)K_{ii}(x_i(t),t)$ generates a nonlinear evolution operator which can be approximated arbitrarily closely by the systems

$$\begin{aligned}\dot{x}_i^{[k]}(t) = &-A_{ii}(x_i^{[k-1]}(t),t)x_i^{[k]}(t) + \\ &B_{ii}(x_i^{[k-1]}(t),t)K_{ii}(x_i^{[k-1]}(t),t)x_i^{[k]}(t)\end{aligned}$$

Details of the convergence of the sequence $x_i^{[k]}(t)$ over k are given in (Banks *et al.*, 1995; Banks and McCaffrey, 1997). This means that we can work with a linear evolution operator when considering the perturbed system (1) and obtain an exponential stability result.

We shall assume that the operator A_{ii} satisfies the following conditions:

- For each ϕ, $\{A_{ii}(t,\phi) : t \ge 0\}$ is a family of closed, densely defined linear operators in H with domain $D(A_{ii}(t,\phi))$ independent of t and $\phi \in H$.
- The resolvent $R(\lambda, A_{ii}(t,\phi))$ exists for all λ with Re$\lambda \le 0$ and

$$\|R(\lambda, A_{ii}(t,\phi))\| \le C(1+|\lambda|)^{-1}$$

for some constant C. (Thus, $(A_{ii}(t,\phi))^{-1}$ exists in $L(H)$ and we put $A_{ii} = A_{ii}(0,0)$.)

- $A_{ii}(\cdot,\phi) : [0,\infty) \to L(D(A_{ii}), H)$ is Holder continuous.
- Let $A_{ii}^{\alpha}(t,\phi)$ denote the usual fractional power, $0 < \alpha \le 1$. Then we also assume $\|A_{ii}^{-\alpha}(A_{ii}(t_1,\phi) - A_{ii}(t_2,\psi))\| \le C_1(|t_1 - t_2|^{\mu} + \|\phi - \psi\|_{\alpha})$ for all $\phi, \psi \in D(A_{ii}^{\alpha})$ for some constant C_1 and $\mu > 0$, where $\|x\|_{\alpha} = \|A_{ii}^{\alpha}x\|$. (We shall assume a similar bound on B_{ii}.)

The following are well-known (Amann, 1978; Henry, 1981):

(1) $D(A_{ii}^{\alpha}(t,\phi)) \subseteq D(A_{ii}^{\beta}(t,\phi))$, if $\alpha \ge \beta > 0$. Put $H_{\alpha} = (D(A_{ii}^{\alpha}), \|\cdot\|_{\alpha})$. Then H_{α} is a Hilbert space and $H_{\beta} \subseteq H_{\alpha}$, for $0 \le \alpha \le \beta \le 1$.

(2) Let $U_{ii}(t,\tau;\phi) \in L(H)$ denote the evolution operator generated by $A_{ii}(t,\phi)$. Then
  (a) $U_{ii}(t,t;\phi) = I$
  (b) $U_{ii}(t,s) \cdot U_{ii}(s,\tau) = U_{ii}(t,\tau)$, $0 \le \tau \le s \le t$
  (c) $\|U_{ii}(t,\tau;\phi)\|_{\alpha,\beta} \le \frac{\text{const}}{(t-\tau)^{\gamma}}$, if $0 \le \alpha \le \beta \le 1$ and $\beta - \alpha < \gamma < 1$
  (d) $\|U_{ii}(t,\tau;\phi)\|_{\alpha,\beta} \le$ const if $0 \le \beta < \alpha < 1$
  (e) $\|U_{ii}(t,\tau;\phi) - U_{ii}(s,\tau;\phi)\|_{\beta,\alpha} \le$ const $\cdot |t-s|^{\gamma}$ for $0 \le \gamma < \beta - \alpha$

  Here, $\|\cdot\|_{\alpha,\beta}$ denotes the norm of an operator from H_{α} to H_{β}.

Moreover, we shall assume for simplicity that the interconnection operators are bounded. (By a suitable extension we can consider unbounded operators here.) Thus we assume

$$\|A_{ij}(x(t),t)\|_{\alpha} \le a_{ij} \ ,$$
$$\|B_{ij}(x(t),t)\|_{\alpha} \le b_{ij}$$
$$\left\|C_{ij}^{k}(x(t),t)\right\|_{\alpha} \le c_{ij}^{k}$$
$$\|f_i(t,x(t),x(t-\tau_k(t)))\| \le \sum_{j=1}^{m} e_{ij}\,\|x_j(t)\|_{\alpha}$$
$$+\sum_{k=1}^{K}\sum_{j=1}^{m} d_{ij}^{k}\,\|x_j(t-\tau_k(t))\|_{\alpha}$$

for all $(x(t),t)$. (Again, with a more careful analysis, these conditions can be replaced by local versions, e.g. $\|A_{ij}(x(t),t)\|_{\alpha} \le a_{ij}(x(t))$, etc.)

To prove uniform stabilizability of the isolated subsystems we shall require the following generalization of the extended Gronwall inequality for parabolic PDEs, see (Henry, 1981).

Lemma 1 *Let $a,b,c \ge 0$, $0 < \alpha < 1$, $\alpha+\beta-1 > 0$, $\gamma > 0$, $\alpha+\beta+\gamma > 2$, $[c\Gamma(\alpha)]^{1/\alpha} < \varepsilon/2$ and suppose that $t^{\gamma-1}x(t)$ is locally integrable and*

$$x(t) \le ae^{-\varepsilon t} + b\int_0^t e^{-\varepsilon(t-s)}(t-s)^{\beta-1}s^{\gamma-1} \times$$
$$x(s)ds + c\int_0^t e^{-\varepsilon(t-s)}(t-s)^{\alpha-1}x(s)ds$$

a.e. on $[0,\infty)$. Then

$$x(t) \le ae^{-\varepsilon t}\left(1+\frac{t^{\alpha}}{\alpha}\right) \times$$

$$E_{\alpha+\beta-1,\gamma}\left(\left(L(1+t^{1-\alpha})\Gamma(\alpha+\beta-1)\right)^{1/\nu}t\right)$$

where $\nu = \alpha+\beta+\gamma-2$, $E_{\alpha+\beta-1,\gamma}(s) = \sum_{m=0}^{\infty} c_m s^{m\nu}$ with $c_0 = 1, c_{m+1}/c_m = \Gamma(m\nu+\gamma)/\Gamma(m\nu+\alpha+\beta+\gamma)$ and $L = b\cdot max(1, beta(\alpha,\beta))$.

Proof Put $y(t) = e^{\varepsilon t}x(t)$. Then

$$y(t) \le a + b\int_0^t (t-s)^{\beta-1}s^{\gamma-1}y(s)ds$$
$$+c\int_0^t (t-s)^{\alpha-1}y(s)ds \ .$$

Now define

$$p(t) = a + b\int_0^t (t-s)^{\beta-1}s^{\gamma-1}y(s)ds \ .$$

Then we have

$$y(t) \le p(t) + c\int_0^t (t-s)^{\alpha-1}y(s)ds \ .$$

Hence, by (Henry, 1981) lemma 7.1.1, we have

$$y(t) \le p(t) + \theta\int_0^t E'_{\alpha}(\theta(t-s))p(s)ds$$

where $\theta = [c\Gamma(\alpha)]^{1/\alpha}$, $E_{\alpha}(z) = \sum_{n=0}^{\infty} z^{n\alpha}/\Gamma(n\alpha+1)$, $E'_{\alpha} = \frac{d}{dz}E_{\alpha}(z)$. Since $E'_{\alpha}(z) \sim z^{\alpha-1}/\Gamma(\alpha)$ as $z \to 0+$ and $E'_{\alpha}(z) \sim \frac{1}{\alpha}e^{z}$ as $z \to \infty$, we have

$$E'_{\alpha}(\theta(t-s)) \le C_1 e^{\theta(t-s)}(t-s)^{\alpha-1}$$

for some constant C_1 depending on c, and so

$$y(t) \le p(t) + \theta\int_0^t C_1 e^{\theta(t-s)}(t-s)^{\alpha-1}p(s)ds \ .$$

Now,

$$\int_0^t e^{\theta(t-s)}(t-s)^{\alpha-1}p(s)ds =$$
$$a\int_0^t e^{\theta(t-s)}(t-s)^{\alpha-1}ds$$
$$+b\int_0^t e^{\theta(t-s)}(t-s)^{\alpha-1}ds$$
$$\times\int_0^s (s-\tau)^{\beta-1}\tau^{\gamma-1}y(\tau)d\tau$$
$$= a\int_0^t e^{\theta(t-s)}(t-s)^{\alpha-1}ds +$$
$$b\int_0^t \tau^{\gamma-1}y(\tau)d\tau$$
$$\times\int_\tau^t e^{\theta(t-s)}(t-s)^{\alpha-1}(s-\tau)^{\beta-1}ds$$
$$\leq a\int_0^t e^{\theta(t-s)}(t-s)^{\alpha-1}ds +$$
$$b\int_0^t e^{\theta(t-\tau)}(t-\tau)^{\alpha+\beta-2}$$
$$\times\tau^{\gamma-1}y(\tau)d\tau\int_0^1 \eta^{\beta-1}(1-\eta)^{\alpha-1}d\eta$$
$$\leq a\int_0^t e^{\theta(t-s)}(t-s)^{\alpha-1}ds +$$
$$C_3\int_0^t e^{\theta(t-\tau)}(t-\tau)^{\alpha+\beta-2}\tau^{\gamma-1}y(\tau)d\tau$$

Hence,

$$\begin{aligned} y(t) \leq a + b\int_0^t (t-s)^{\beta-1}s^{\gamma-1}y(s)ds &+ \\ aC_1\theta\int_0^t e^{\theta(t-s)}(t-s)^{\alpha-1}ds &+ \\ C_3\int_0^t e^{\theta(t-s)}(t-s)^{\alpha+\beta-2}s^{\gamma-1}y(s)ds& \end{aligned}$$

and so

$$\begin{aligned} q(t) \leq a + b\int_0^t (t-s)^{\beta-1}s^{\gamma-1}q(s)ds &+ \\ aC_1\theta\int_0^t (t-s)^{\alpha-1}ds &+ \\ C_3\int_0^t (t-s)^{\alpha+\beta-2}s^{\gamma-1}q(s)ds& \end{aligned}$$

where $q(t) = e^{-\epsilon/2t}y(t)$. Hence

$$q(t) \leq a\left(1 + C_4\frac{t^\alpha}{\alpha}\right) + C_5(1+t^{1-\alpha})\int_0^t (t-s)^{\alpha+\beta-2}s^{\gamma-1}q(s)ds$$

and the result follows from (Henry, 1981), lemma 7.1.2. □

2. EXPONENTIAL STABILITY OF THE ISOLATED SUBSYSTEMS

In this section we shall consider the i^{th} isolated subsystem given by

$$\dot{x}_i(t) = -A_{ii}(x_i(t),t)x_i(t) + B_{ii}(x_i(t),t)u_i(t)$$

For simplicity of exposition, we shall assume that $B_{ii}(x(t),t)$ is a bounded operator for all $x(t),t$. However, the theory is easily extended to the case of unbounded B (i.e. boundary control) in the usual way, see (Curtain and Pritchard, 1978). We shall assume that the system

$$\dot{x}_i(t) = -A_{ii}(0,0)x_i(t) + B_{ii}(0,0)u_i(t) \qquad (3)$$

is stabilizable with (linear) stabilizing feedback

$$u_i(t) = K_{ii}(0,0)x_i(t)$$

so that $K_{ii}(0,0) \in L(L^2(\Omega))$ and the operator $-A_{ii}(0,0) + B_{ii}(0,0)K_{ii}(0,0)$ generates an analytic semigroup with

$$\|T_{ii}(t)\| \leq Me^{a_{ii}t}$$
$$\|A_{ii}(0,0)T_{ii}(t)\| \leq Me^{a_{ii}t}/t\ ,\ t>0$$

where $a_{ii} < 0$.

We have

$$\begin{aligned} \dot{x}_i(t) = &-A_{ii}(0,0)x_i(t) + B_{ii}(0,0)u_i(t) \\ &+(-A_{ii}(x_i(t),t) + A_{ii}(0,0))x_i(t) \\ &+(B_{ii}(x_i(t),t) - B_{ii}(0,0))u_i(t) \end{aligned}$$

so that

$$\begin{aligned} x_i(t) = &T_{ii}(t)x_{i0} + \int_0^t T_{ii}(t-s) \\ &\times(-A_{ii}(x_i(s),s) + A_{ii}(0,0))\,x_i(s)ds \\ &+\int_0^t T_{ii}(t-s)(B_{ii}(x_i(s),s) - \\ &B_{ii}(0,0))K_{ii}(0,0)x_i(s)ds \end{aligned}$$

whence

$$\|x_i(t)\|_\alpha \le Me^{a_{ii}t}\|x_{i0}\|_\alpha + \int_0^t \frac{Me^{a_{ii}(t-s)}}{(t-s)^\alpha}$$
$$(C_1 s^\mu + C_2\|x_i(s)\|_\alpha)\,\Delta\,\|x_i(s)\|_\alpha\,ds$$
$$+\int_0^t \frac{Me^{a_{ii}(t-s)}}{(t-s)^\alpha}(C_4 s^\mu + C_5\|x_i(s)\|_\alpha)$$
$$\Delta\,\|K_{ii}(0,0)\|\,\|x_i(s)\|_\alpha\,ds$$
$$= Me^{a_{ii}t}\|x_{i0}\|_\alpha + M\Delta(C_1 +$$
$$C_4\|K_{ii}(0,0)\|)\int_0^t \frac{e^{a_{ii}(t-s)}}{(t-s)^\alpha}s^\mu\|x_i(s)\|_\alpha\,ds$$
$$+M\Delta(C_{21} + C_5\|K_{ii}(0,0)\|)$$
$$\times\int_0^t \frac{e^{a_{ii}(t-s)}}{(t-s)^\alpha}\|x_i(s)\|_\alpha\|x_i(s)\|_\alpha\,ds$$

for some new constants C_i, Δ. Suppose that $\|x_{i0}\|_\alpha < \lambda < 1$ and that $\|x_i(t)\|_\alpha < 1$ for $t \in [0,T)$ for some λ to be specified. Then, by lemma 1, for $t \in [0,T)$, we have

$$\|x_i(t)\|_\alpha \le Me^{a_{ii}t}\|x_{i0}\|_\alpha\left(1 + C\frac{t^\alpha}{\alpha}\right)\times$$
$$E_{\alpha+\beta-1,\gamma}\left((L(1+t^{1-\alpha})\Gamma(\alpha+\beta-1))^{1/\nu}t\right)$$
$$= M\|x_{i0}\|_\alpha v(t)$$

say, where

$$L = M\Delta(C_1 + C_4)\|K_{ii}(0,0)\|$$
$$\times\max(1, \mathrm{beta}(\alpha,\beta))$$

and

$$v(t) = \left(1 + C\frac{t^\alpha}{\alpha}\right)\times$$
$$E_{\alpha+\beta-1,\gamma}\left((L(1+t^{1-\alpha})\Gamma(\alpha+\beta-1))^{1/\nu}t\right).$$

Now $E_{\alpha+\beta-1,\gamma}(z) \sim z^{1/2(\nu/\beta-\gamma)}\exp\left(\frac{\beta}{\nu}z^{\nu/\beta}\right)$, and since $\nu/\beta = (\alpha+\beta+\gamma-2)/\beta$ we see that if $-a_{ii} > (\alpha+\gamma-2)/\beta + 1$ then

$$\zeta \doteq \max_{t\in[0,\infty)} v(t)$$

exists. Hence, if we take $\lambda = \frac{1}{M\zeta}$ then $\|x_i(t)\|_\alpha \le 1$ for $t = T$ and we can extend the set of t for which $\|x_i(t)\|_\alpha \le 1$ to $[0,\infty)$. Hence $\|x_i(t)\|_\alpha \to 0$ as $t \to \infty$ and the system is stabilized. Summarizing, we have

Theorem 1 *If $-a_{ii} > (\alpha + \gamma - 2)/\beta + 1$ the system (2.1) can be stabilized for small enough initial conditions. Moreover, a similar proof shows that the nonlinear evolution operator $\tilde{U}_{ii}(t,s)$ generated by $-A_{ii}(x_i(t),t)x_i(t) + B_{ii}(x_i(t),t)K_{ii}(0,0)$ can be approximated arbitrarily closely by a linear evolution operator $U_{ii}(t,s)$ given by the approximate system*

$$\dot{x}_i^{[k]}(t) = -A_{ii}(x_i^{[k-1]}(t),t)x_i^{[k]}(t)$$
$$+B_{ii}(x_i^{[k-1]}(t),t)K_{ii}(0,0)x_i^{[k]}(t)$$

(for details see (Banks et al.*, 1995; Banks and McCaffrey, 1997)). We also have*

$$\|U_{ii}(t,s)\|_\alpha \le M_{ii}e^{a_{ii}(t-s)}.\square$$

Remark A second much simpler condition for the stabilization of (2) can be given at the expense of a much stronger condition on A_{ii} and B_{ii} namely, if there exists a uniformly bounded operator $K_{ii}(x_i(t),t)$ for each fixed $(x_i(t),t)$ such that

$$\langle x, (A_{ii}(y,\tau) + B_{ii}(y,\tau)K_{ii}(y,\tau))x\rangle \le -\varepsilon\|x\|^2$$

for each y,τ then the system (2) is clearly stabilized by K_{ii}.

3. STABILITY OF THE COMPLETE SYSTEM

The main result of the paper is

Theorem 2 *If $K_{ii}(0,0) \in L(L^2(\Omega)) \cap L(D(A^\alpha))$ exists which stabilizes the system (1) and the conditions of lemma 1 and theorem 1 are satisfied and, moreover, we have*

(i) $\lambda_k = \sup_{t\ge t_0}\{\dot{\tau}_k(t)\} < 1$
(ii) $a_{ii} + b_{ii} < 0, i = 1,\cdots,m$
(iii) $M_{ii}[(a_{ij} + e_{ij} + \alpha_i b_{ij}) + \sum_{k=1}^K v_k(c_{ij}^k + d_{ij}^k)] < 0, i,j = 1,\cdots m$

where $\alpha_i = \|K_{ii}(0,0)\|$ and $v_k = (1-\lambda_k)^{-1} > 0, k = 1,\cdots,K$, then the zero solution of (1) is exponentially stable.

Proof From the above remarks we may assume that the stabilized isolated system generates a linear evolution operator $U_{ii}(t,s)$. Then we have

$$x_i(t) = U_{ii}(t,t_0)\phi_i(t) + \int_{t_0}^t U_{ii}(t,s)$$
$$[\sum_{j\ne i}^m A_{ij}(x(s),s)x_j(s) + \sum_{j\ne i}^m B_{ij}(x(s),s)u_j(s)$$
$$+\sum_{k=1}^K\sum_{j=1}^m C_{ij}^k(x(s),s)x_j(s-\tau_k(s))$$
$$+f_i(s,x(s),x(s-\tau_k(s)))]ds$$

Hence,

$$
\begin{aligned}
&\|x_i(t)\|_\alpha \le M_{ii}e^{a_{ii}(t-t_0)}\,\|\phi_i(t)\|_\alpha + \\
&\int_{t_0}^{t} M_{ii}e^{a_{ii}(t-s)}\sum_{j\ne i}^{m}\|A_{ij}(x(s),s)\|_\alpha\,\|x_j(s)\|_\alpha \\
&+\sum_{j\ne i}^{m}\|B_{ij}(x(s),s)\|_\alpha\,\|u_j(s)\|_\alpha + \\
&\sum_{k=1}^{K}\sum_{j=1}^{m}\|C_{ij}(x(s),s)\|_\alpha\,\|x_j(s-\tau_k(s))\|_\alpha \\
&+\|f_i(s,x(s),x(s-\tau_k(s)))\|_\alpha]ds \\
\le\ & M_{ii}e^{a_{ii}(t-t_0)}\,\|\phi_i(t)\|_\alpha + \\
&\int_{t_0}^{t} M_{ii}e^{a_{ii}(t-s)}[\sum_{j\ne i}^{m}(a_{ij}+e_{ij}+\alpha_i b_{ij})\times \\
&\|x_j(s)\|_\alpha + e_{ii}\,\|x_i(s)\|_\alpha + \\
&\sum_{k=1}^{K}\sum_{j=1}^{m}(c_{ij}^k+d_{ij}^k)\times \\
&\|x_j(s-\tau_k(s))\|_\alpha]ds
\end{aligned}
$$

Now let $q_i(t)$ be defined by

$$
\begin{aligned}
&q_i(t) = \|\phi_i(t)\|_\alpha\ ,\ -\infty < t \le t_0 \\
&q_i(t) = M_{ii}e^{a_{ii}(t-t_0)}\,\|\phi_i(t)\|_\alpha + \\
&\int_{t_0}^{t} M_{ii}e^{a_{ii}(t-s)}[\sum_{j\ne i}^{m}(a_{ij}+e_{ij}+\alpha_i b_{ij})\times \\
&\|x_j(s)\|_\alpha + e_{ii}\,\|x_i(s)\|_\alpha + \\
&\sum_{k=1}^{K}\sum_{j=1}^{m}(c_{ij}^k+d_{ij}^k)\times \\
&\|x_j(s-\tau_k(s))\|_\alpha]ds\ ,\ t \ge t_0
\end{aligned}
$$

Then we have

$$
\|x_i(t)\|_\alpha \le q_i(t)
$$

From the definition of $q_i(t)$ we have

$$
\begin{aligned}
&\dot{q}_i(t) = a_{ii}q_i(t) + M_{ii}e^{a_{ii}(t-t_0)} \\
&[\sum_{j\ne i}^{m}(a_{ij}+e_{ij}+\alpha_i b_{ij})\,\|x_j(s)\|_\alpha \\
&+e_{ii}\,\|x_i(s)\|_\alpha + \sum_{k=1}^{K}\sum_{j=1}^{m}(c_{ij}^k+d_{ij}^k) \\
&\times\|x_j(s-\tau_k(s))\|_\alpha]ds
\end{aligned}
$$

so that

$$
\begin{aligned}
\dot{q}_i(t) \le M_{ii}&\sum_{j=i}^{m}(a_{ij}+e_{ij}+\alpha_i b_{ij}) - \\
&\alpha_i b_{ij}q_i(t) + \\
&\sum_{k=1}^{K}\sum_{j=1}^{m}(c_{ij}^k+d_{ij}^k)q_i(t-\tau_k(t))
\end{aligned}
$$

Define a Lyapunov (-like) function

$$
\begin{aligned}
&V_i(t,q_i(t)) = q_i(t) + \\
&\sum_{k=1}^{K}v_k\int_{t-\tau_k(t)}^{t}\sum_{j=1}^{m}(c_{ij}^k+d_{ij}^k)q_j(s)ds
\end{aligned}
$$

and

$$
V(t,q(t)) = \sum_{i=1}^{m}\mu_i V_i(t,q_i(t))
$$

where $\mu_i > 0$. Elementary computation shows that

$$
\dot{V}(t,q(t)) \le -\beta\sum_{j=1}^{m}q_j(t)
$$

where

$$
\begin{aligned}
\beta = &-\max_{1\le j\le m}\sum_{i=1}^{m}\mu_i[M_{ii}\{(a_{ij}+e_{ij}+\alpha_i b_{ij}) \\
&+\sum_{k=1}^{K}v_k(c_{ij}^k+d_{ij}^k)\}] \\
&> 0
\end{aligned}
$$

The result now follows from Krasovskii's theorem, see (Hale, 1977).□

4. CONCLUSIONS

In this paper we have studied the exponential stability of nonlinear large-scale parabolic systems by stabilizing the isolated subsystems. These subsystems are assumed to dominate the interconnection terms in some sense and a fairly standard application of Lyapunov theory proves stability of the entire system. The results may be generalized in various ways. First, local Lipschitz bounds on the operators can be assumed with a little more effort. Secondly, boundary control can be treated in a similar way.

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