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Control of nonlinear functional differential equations

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ABSTRACT

In this paper, we extend a recently developed approximation method to nonlinear systems given by functional differential equations. The nonlinear system is approached by a sequence of linear time-varying systems, which allows many linear control techniques to be applied. Here we design a sliding mode controller for a hydraulic press system to demonstrate the effectiveness of this method.

1. Introduction

The control of finite-dimensional nonlinear system of the form

$$\dot{x} = A(x)x + B(x)u \tag{1}$$

has recently been studied via a sequence of time-varying approximations of the form

$$\dot{x}^{[i]}(t) = A(x^{[i-1]}(t))x^{[i]}(t) + B(x^{[i-1]}(t))u^{[i]}(t). \tag{2}$$

In particular, it has been shown to be effective in optimal control [1], sliding control [2], frequency-domain theory and control of chaos for communications [3]. The basic theory and convergence are presented in [4]. Many real systems are, however, distributed and are given, in particular, by functional differential equations. Here we shall extend the above theory to systems of the form

$$\dot{x} = A(x, N(x, \theta))x + B(x, N(x, \theta))u \tag{3}$$

where $N(x, \theta)$ is some nonlinear function defined over the interval $[t - \theta, t]$; for example

$$N(x, \theta) = \int_{(t-\theta)}^{t} \sin(x(t))\,dt.$$

2. An example: Velocity tracking problem of the hydraulic press

Hydraulic presses are widely used in metal forging process. In some new technologies, such as isothermal forging process, a specific forging trajectory, pressure or velocity is required.

Trajectory and pressure control of hydraulic systems have been approached by different kinds of control techniques [5,6]. In this paper, we consider the velocity tracking problem when the hydraulic press is under a certain reaction force, the model of which is given by nonlinear functional differential equations.

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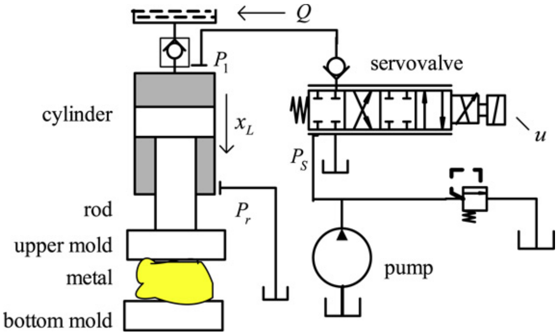

Fig. 1. The hydraulic press system.

2.1. The principles of hydraulic press systems

Most hydraulic presses can be simplified to single-rod hydraulic systems, as is shown in Fig. 1. In the forging process, the rod and the upper mold go downward. The dynamical model of them can be described by

$$m\ddot{x}_L = b\dot{x}_L - P_1A - f \tag{4}$$

where x_L is the displacement of the rod; m is the mass of the rod and the upper mold; P_1 is the pressure of the chamber; b is the combined coefficient of the friction forces of the rod and metal; f is the reaction force and other hard-to-model terms. The dynamical model of the upper cylinder can be described by

$$\frac{V}{\beta_e}\dot{P}_1 = -A\dot{x}_L - (C_{em} + C_{tm})(P_1 - P_r) + Q \tag{5}$$

where $V = V_0 + Ax_L$ is the total volume of the chamber; V_0 is the initial volume of the chamber; β_e is the effective bulk modulus; C_{em} is the external coefficient of the chamber; C_{tm} is the internal coefficient of the cylinder; P_r is the reference pressure, Q is the flow rate of the upper chamber, which is regulated by the servovalve during the forging process

$$Q = k_q x_v \sqrt{P_s - P_1} \tag{6}$$

where k_q is the flow gain of the servovalve; P_s is the supply pressure of the valve; x_v is the servovalve displacement related to the control input given by

$$\tau_v \dot{x_v} = -x_v + K_v u \tag{7}$$

where τ_v and K_v are the time constant and gain of the valve dynamics, respectively.

2.2. The model of the velocity tracking problem

In the velocity tracking problem, define the state variable $x = [x_1, x_2, x_3]^{\mathrm{T}} \triangleq [\dot{x}_L - v_d, P_1 - P_{1d}, x_v - x_{vd}]^{\mathrm{T}}$, where v_d is the desired velocity; P_{1d} and x_{vd} are the equivalent values of P_1 and x_v respectively, when $\dot{x}_L = v_d$. Define $v = u - u_d$, where u_d is the equivalent value of control input. Then the system model (4)–(7) can be written as

$$\begin{aligned}
\dot{x}_1 &= \frac{1}{m}[-b(x_1 + v_d) + A(x_2 + P_{1d}) - f] \\
\dot{x}_2 &= \frac{\beta_e}{V_0 + \int_{t_0}^{t}(x_1 + v_d)\,\mathrm{d}t}[-A(x_1 + v_d) + k_q(x_3 + x_{vd})\sqrt{P_s - x_2 - P_{1d}} - (C_{em} + C_{tm})(x_2 + P_{1d} - P_r)] \\
\dot{x}_3 &= \frac{1}{\tau_v}[-(x_3 + x_{vd}) + K_v(v + u_d)].
\end{aligned} \tag{8}$$

From (4)–(7), we can easily get the equivalent values by setting $\dot{x}_L = v_d$, $\ddot{x}_L = 0$, $\dot{P}_1 = 0$, and $\dot{x_v} = 0$

$$\begin{aligned}
P_{1d} &= \frac{bv_d + f}{A} \\
x_{vd} &= \frac{Av_d + (C_{em} + C_{tm})(P_{1d} - P_r)}{k_q\sqrt{P_s - P_{1d}}} \\
u_d &= \frac{x_{vd}}{k_v}.
\end{aligned} \tag{9}$$

To simplify the model, we expand $\sqrt{P_s - x_2 - P_{1d}}$ in the second equation of (8) as

$$\sqrt{P_s - x_2 - P_{1d}} = \sqrt{P_s - P_{1d}}\left(1 - \frac{x_2}{2(P_s - P_{1d})} - \frac{x_2^2}{8(P_s - P_{1d})^2}\right). \tag{10}$$

Substituting (9) and (10) into (8), we get the error model

$$\begin{aligned}
\dot{x}_1 &= \frac{1}{m}(-bx_1 + Ax_2)\\
\dot{x}_2 &= \frac{\beta_e}{V_0 + \int_{t_0}^{t}(x_1 + v_d)\,\mathrm{d}t}(-Ax_1 + F_1(x_2)x_2 + F_2(x_2)x_3)\\
\dot{x}_3 &= \frac{1}{\tau_v}(-x_3 + K_v v)
\end{aligned} \tag{11}$$

where

$$\begin{aligned}
F_1(x_2) &= k_q x_{vd}\sqrt{P_s - P_{1d}}\left(-\frac{1}{2(P_s - P_{1d})} - \frac{x_2}{8(P_s - P_{1d})^2}\right) - (C_{em} + C_{tm})x_2\\
F_2(x_2) &= k_q x_{vd}\sqrt{P_s - P_{1d}}\left(1 - \frac{x_2}{2(P_s - P_{1d})} - \frac{x_2^2}{8(P_s - P_{1d})^2}\right).
\end{aligned} \tag{12}$$

3. Linear, time-varying approximation

The iteration technique for finite-dimensional nonlinear systems of the form (1) is proved to be globally convergent under a mild Lipschitz condition [4]. In this section we extend the technique to nonlinear functional differential equations given by

$$\dot{x} = A(x, N(x, \theta))x, \quad x(0) = x_0 \in R^n \tag{13}$$

where $N(x, \theta)$ is some nonlinear function defined over the interval $[t - \theta, t]$. We introduce the following sequence of linear, time-varying (LTV) approximations

$$\dot{x}^{[0]}(t) = A(x_0, N(x_0, \theta))x^{[0]}(t), \quad x^{[0]}(0) = x_0 \tag{14}$$

$$\dot{x}^{[i]}(t) = A(x^{[i-1]}(t), N(x^{[i-1]}(t), \theta))x^{[i]}(t), \quad x^{[i]}(0) = x_0 \tag{15}$$

for $i \geq 1$. In the following part of this section we will prove the sequences given by (14) and (15) are globally convergent to the solution of (13) under certain conditions.

Lemma 1. *Suppose the following condition holds:*

$$\|A(x_1, N(x_1, \theta)) - A(x_2, N(x_2, \theta))\| \leq \alpha(K)\|x_1 - x_2\|, \quad \text{for } x_1, x_2 \in B(K, x_0) \tag{16}$$

where $B(K, x_0) \in R^N$ is a ball of radius K centered at x_0, α is a constant related to K. Then the sequence of $x^{[i]}(t)$ defined in (14) and (15) converges uniformly on $[0, T]$, in the space $C([0, T]; R^n)$.

Proof. Let $\phi^{[i-1]}(t, t_0)$ denote the transition matrix of $A(x^{[i-1]}(t), N(x^{[i-1]}(t)))$. That is

$$\phi^{[i-1]}(t, t_0) = \exp[A(x^{[i-1]}(t), N(x^{[i-1]}(t)))(t, t_0)].$$

So we have [7]

$$\|\phi^{[i-1]}(t, t_0)\| \leq \exp\left[\int_{t_0}^{t} \mu(A(x^{[i-1]}(\tau), N(x^{[i-1]}(\tau), \theta)))\,\mathrm{d}\tau\right] \tag{17}$$

where $\mu(A)$ is the logarithmic norm of matrix A:

$$\mu(A) = \lim_{h \to 0+}(\|1 + hA\| - 1)/h.$$

If (16) holds, then

$$\|A(x^{[i-1]}(t), N(x^{[i-1]}(t), \theta))\| \leq \alpha(K)\|x^{[i-1]}(t) - x_0\| + \|A(x_0, N(x_0, \theta))\|.$$

And since

$$\mu(A) = \frac{1}{2}\max[\sigma(A + A^{\mathrm{T}})]$$

in the logarithmic matrix norm (where $\sigma(A + A^{\mathrm{T}})$ denotes the eigenvalues of matrix $A + A^{\mathrm{T}}$), for some constant μ_0 we have

$$\mu(A(x^{[i-1]}(t), N(x^{[i-1]}(t), \theta))) \leq \mu_0, \quad \text{for all } t \in [0, T], \text{ and } i \geq 1. \tag{18}$$

From (15), we get

$$x^{[i]}(t) - x_0 = \phi^{[0]}(t,0)x_0 - x_0 + \int_0^t \phi^{[0]}(t,s)[A(x^{[i-1]}, N(x^{[i-1]},\theta)) - A(x_0, N(x_0,\theta))]x^{[i-1]}(s)\,\mathrm{d}s.$$

Assuming (16) holds, from (17) and (18) we have, for any $T > 0$,

$$\|x^{[i]}(t) - x_0\| \le \sup_{t\in[0,T]} \|\phi^{[0]}(t,t_0) - I\| \cdot \|x_0\| + T\alpha \exp(\mu_0 t)\|x_0\| \sup_{t\in[0,T]} \|x^{[i-1]}(t) - x_0\|.$$

Therefore, for $t \in [0,T]$, if $x^{[i-1]}(t) \in B(K,x_0)$, then $x^{[i]}(t) \in B(K,x_0)$ if T is small enough. Since $x^{[0]}(t) \in B(K,x_0)$, then $x^{[i]}(t) \in B(K,x_0)$. Also from (15) we have

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}[x^{[i]}(t) - x^{[i-1]}(t)] &= A(x^{[i-1]}(t), N(x^{[i-1]}(t),\theta))[x^{[i]}(t) - x^{[i-1]}(t)] \\ &\quad + [A(x^{[i-1]}(t), N(x^{[i-1]}(t),\theta)) - A(x^{[i-2]}(t), N(x^{[i-2]}(t),\theta))]x^{[i-1]}(t)\end{aligned}$$

then we get

$$x^{[i]}(t) - x^{[i-1]}(t) = \int_0^t \phi^{[i-1]}(t,s)[A(x^{[i-1]}, N(x^{[i-1]},\theta)) - A(x_0, N(x_0,\theta))]x^{[i-1]}(s)\,\mathrm{d}s.$$

Thus

$$\|x^{[i]}(t) - x^{[i-1]}(t)\| \le \int_0^t \|\phi^{[i-1]}(t,s)\|\alpha\|x^{[i-1]}(t) - x^{[i-2]}(t)\| \cdot \|x^{[i-1]}(s)\|\,\mathrm{d}s.$$

Suppose that

$$\xi^{[i]}(t) \triangleq \sup_{s\in[0,t]} \|x^{[i]}(s) - x^{[i-1]}(s)\|.$$

Since $x^{[i]}(t) \in B(K,x_0)$, then

$$\xi^{[i]}(t) \le \int_0^t \exp[\mu_0(t-s)]K\alpha\xi^{[i-1]}(s)\,\mathrm{d}s$$

so

$$\begin{aligned}\xi^{[i]}(T) &\le \sup_{s\in[0,t]} \exp[\mu_0(t-s)]TK\alpha\xi^{[i-1]}(T)\,\mathrm{d}s \\ &\le \lambda\xi^{[i-1]}(T)\end{aligned}$$

where $\lambda = \sup_{s\in[0,t]} \exp[\mu(t-s)]TK\alpha$. If T is small enough, then $\lambda < 1$. For any $i \ge j$

$$\|x^{[i]}(s) - x^{[j]}(s)\| \le \|x^{[i]}(s) - x^{[i-1]}(s)\| + \|x^{[i-1]}(s) - x^{[i-2]}(s)\| + \cdots + \|x^{[j+1]}(s) - x^{[j]}(s)\|$$

so

$$\sup_{s\in[0,t]} \|x^{[i]}(s) - x^{[j]}(s)\| \le \lambda^{i-j}\xi^{[j]}(T) + \lambda^{i-j-1}\xi^{[j]}(T) + \cdots + \lambda^{j}\xi^{[j]}(T) = \lambda\left(\frac{1-\lambda^{i-j}}{1-\lambda}\right)\xi^{[j]}(T).$$

Hence, if N is a fixed positive integer and $i \ge j > N$, then

$$\sup_{s\in[0,t]} \|x^{[i]}(s) - x^{[j]}(s)\| \le \lambda^{j-N+1}\left(\frac{1-\lambda^{i-j}}{1-\lambda}\right)\xi^{[N]}(T).$$

Since $\xi^{[N]}(T)$ is bounded, the right-hand side is arbitrarily small if j is large and so $x^{[i]}(t)$ is a Cauchy sequence in $C([0,T];R^n)$.

Thus we have proven the local convergence of the sequences $x^{[i]}(t)$ in $C([0,T];R^n)$. Based on it, the global convergence can be easily proved, using the same method as in [1,4].

Assume that the nonlinear functional system has bounded solutions (bounded by K_{B_ρ}) for any time, for all initial states in a ball $B_\rho \subseteq R^n$ of radius ρ, and for all initial functions with values in B_ρ.

Suppose the sequence of solutions $x^{[i]}(t;x_0,x_I(\cdot))$ (where $x_I(\cdot)$ is the initial function) of the LTV approximations to the nonlinear system do not converge uniformly on every compact time interval, and we derive a contradiction. Since the sequence converges on some interval by the local convergence proof, there must be a minimum time, say T, at which the sequence does not converge.

Suppose, by local convergence, that the sequence converges uniformly on the time interval $[0,\varepsilon]$ for all initial states in the ball $B_{2\rho}$ and all initial functions with values in $B_{2\rho}$. (That this is possible follows from the local convergence proof.) Now consider the sequence of approximations on the interval $[0, T-\varepsilon/2]$. By definition of T, the sequence $x^{[i]}(t;x_0,x_I(\cdot))$ converges uniformly on $[0, T-\varepsilon/2]$ as so, in particular, the sequence of points

$$x_\varepsilon^{[i]} \triangleq x^{[i]}(T-\varepsilon/2; x_0, x_I(\cdot))$$

converges (to the solution $x(T-\varepsilon/2; x_0, x_I(\cdot))$ of the nonlinear system through x_0 and $x_I(\cdot)$). Moreover, we have

$$x_\varepsilon^{[i]} \in B_{2\rho}$$

and

$$\lim_{i\to\infty} x_\varepsilon^{[i]} = x(T-\varepsilon/2; x_0, x_I(\cdot)) \in B_\rho$$

by assumption. Hence $x_\varepsilon^{[i]} \in B_{\rho+\delta}$ for arbitrarily small $\delta > 0$ and large enough i.

Now consider, for each i, the approximating sequences through $x_\varepsilon^{[i]}$ at time $T-\varepsilon/2$:

$$x^{[i,j]}(t; x_\varepsilon^{[i]}, x^{[i]}(T-\varepsilon/2-\theta, T-\varepsilon/2))$$

where $x^{[i]}(T-\varepsilon/2-\theta, \varepsilon/2)$ is the solution $x^{[i]}$ on the interval θ prior to $T-\varepsilon/2$ (i.e. the initial function for $x^{[i,j]}$, where θ is the length of the interval on which the initial functions are defined). By the local convergence result, these will converge uniformly, for each i, on the interval $[T-\varepsilon/2, T+\varepsilon/2]$. A simple diagonal argument now gives a contradiction. Thus we could give the following theorem.

Theorem 2. *Suppose that the nonlinear functional differential equation (13) has a unique solution on the interval* $[0,\tau]$ *and assume that the following condition holds*

$$\|A(x_1, N(x_1,\theta)) - A(x_2, N(x_2,\theta))\| \le \alpha(K)\|x_1 - x_2\|, \quad \text{for } x_1, x_2 \in B(K, x_0).$$

Then the sequence of $x^{[i]}(t)$ *defined in (14) and (15) converges uniformly on* $[0,\tau]$.

4. Sliding mode control for the hydraulic press

The linear approximation method stated in Section 3 makes it convenient to control nonlinear system in the form of (3) via linear feedback control technique. In this section, we apply sliding mode control to the velocity tracking problem stated in Section 2. Firstly, we recall the principles of sliding mode control for LTV systems and a recently introduced sufficient condition for sliding surface design. Secondly, the controller design for the velocity tracking problem is illustrated.

4.1. Sliding Mode control of LTV systems

Consider an n dimension LTV system with a scalar input in the form

$$\begin{pmatrix}\dot\xi\\ \dot\eta\end{pmatrix} = \begin{pmatrix}A_{11}(t) & A_{12}(t)\\ A_{21}(t) & A_{22}(t)\end{pmatrix}\begin{pmatrix}\xi\\ \eta\end{pmatrix} + \begin{pmatrix}o\\ 1\end{pmatrix}u(t) \tag{19}$$

where $\xi = (x_1, x_2, \ldots, x_{n-1})^{\mathrm{T}}$, $\eta = x_n$. Define a sliding surface

$$\sigma = c_1(t)x_1 + c_2(t)x_2 + \cdots + c_{n-1}(t)x_{n-1} + x_n = 0. \tag{20}$$

Solve (20), we can get

$$x_n = -c_1(t)x_1 - c_2(t)x_2 - \cdots - c_{n-1}(t)x_{n-1}. \tag{21}$$

Substituting (21) into (19), we can get the reduced order system

$$\dot\xi = \tilde A\xi$$

where

$$\tilde A = \begin{pmatrix} a_{11} - a_{1n}c_1 & a_{12} - a_{1n}c_2 & \cdots & a_{1(n-1)} - a_{1n}c_{n-1}\\ a_{21} - a_{2n}c_1 & a_{22} - a_{2n}c_2 & \cdots & a_{2(n-1)} - a_{2n}c_{n-1}\\ \cdots & \cdots & \ddots & \cdots\\ a_{(n-1)1} - a_{(n-1)n}c_1 & a_{(n-1)2} - a_{(n-1)n}c_2 & \cdots & a_{(n-1)(n-1)} - a_{(n-1)n}c_{n-1}\end{pmatrix}.$$

The first step of sliding mode controller design is to choose a proper sliding surface to make the reduced order system stable. In this paper, we use a recently introduced sufficient condition [8] for the stability of LTV system to design the sliding surface

$$\begin{aligned} &\tilde a_{ii}(t) < 0\\ &\tilde a_{ij}(t) = -\tilde a_{ji}(t)\\ &i,j = 1,2,\ldots,n-1, \quad i \ne j. \end{aligned} \tag{22}$$

The second step is to choose control input u to make the system converge to the sliding surface. Recall the sliding equation (20)

$$\sigma = c_1(t)x_1 + c_2(t)x_2 + \cdots + c_{n-1}(t)x_{n-1} + x_n = 0.$$

To obtain a suitable input, assume that system states are not on the sliding surface σ. Therefore σ is positive or negative. To bring the states to this surface where σ is zero and the motion is asymptotically stable, derivative of σ is set equal to minus sign of σ.

$$\begin{aligned}\dot{\sigma} &= \dot{c}_1(t)x_1 + \dot{c}_2(t)x_2 + \cdots + \dot{c}_{n-1}(t)x_{n-1} + c_1(t)\dot{x}_1 + c_2(t)\dot{x}_2 + \cdots + c_{n-1}(t)\dot{x}_{n-1} + \dot{x}_n \\ &= -\mathrm{sign}(\sigma).\end{aligned} \tag{23}$$

Substituting $\dot{x}_n$ from (19) to (23), we obtain

$$\begin{aligned}\dot{\sigma} &= \sum_{i=1}^{n-1}(\dot{c}_i(t)x_i + c_i(t)\dot{x}_i) + a_{n1}(t)x_1 + a_{n2}(t)x_2 + \cdots + a_{nn}(t)x_n + u \\ &= -\mathrm{sign}(\sigma).\end{aligned}$$

Solving for control input we get

$$u(t) = -\mathrm{sign}(\sigma) - \sum_{i=1}^{n-1}(\dot{c}_i(t)x_i + c_i(t)\dot{x}_i) + a_{n1}(t)x_1 + a_{n2}(t)x_2 + \cdots + a_{nn}(t)x_n.$$

When nonlinear functional differential equations are approximated by a sequence of LTV systems, we can design a sliding mode controller $u^{[i]}(t)$ for every LTV system. The controllers converge as $i \longrightarrow \infty$, and stabilize the original system [2].

4.2. Sliding Mode control for the hydraulic press

From (11), we can get the linear approximation of the model

$$\begin{aligned}\dot{x}_1^{[i]}(t) &= \frac{1}{m}(-bx_1^{[i]}(t) + Ax_2^{[i]}(t)) \\ \dot{x}_2^{[i]}(t) &= \frac{\beta_e}{V_0 + \int_{t_0}^{t}(x_1^{[i-1]}(t) + v_d)\,\mathrm{d}t}(-Ax_1^{[i]}(t) + F_1(x_2^{[i-1]}(t))x_2^{[i]}(t) + F_2(x_2^{[i-1]}(t))x_3^{[i]}(t)) \\ \dot{x}_3^{[i]}(t) &= \frac{1}{\tau_v}(-x_3^{[i]}(t) + K_v v^{[i]}(t))\end{aligned} \tag{24}$$

where the initial value $x_1^{[i]}(0) = -v_d$, $x_2^{[i]}(0) = P_r - P_d$, $x_3^{[i]}(0) = -x_d$, for all $i \geq 0$. The sliding equation is

$$\sigma = c_1^{[i]}(t)x_1^{[i]}(t) + c_2^{[i]}(t)x_2^{[i]}(t) + x_3^{[i]}(t) = 0$$

then

$$x_3^{[i]}(t) = c_1^{[i]}(t)x_1^{[i]}(t) + c_2^{[i]}(t)x_2^{[i]}(t). \tag{25}$$

Substituting (25) into (24), we get the reduced system

$$\begin{pmatrix}\dot{x}_1^{[i]}(t) \\ \dot{x}_2^{[i]}(t)\end{pmatrix} = \begin{pmatrix}\dfrac{-b}{m} & \dfrac{A}{m} \\ \dfrac{-\beta_e(A + c_1^{[i]}(t)F_2(x_2^{[i-1]}(t)))}{V_0 + \int_{t_0}^{t}(x_1^{[i-1]}(t) + v_d)\,\mathrm{d}t} & \dfrac{\beta_e(F_1(x_2^{[i-1]}(t)) - c_2^{[i]}(t)F_2(x_2^{[i-1]}(t)))}{V_0 + \int_{t_0}^{t}(x_1^{[i-1]}(t) + v_d)\,\mathrm{d}t}\end{pmatrix}\begin{pmatrix}x_1^{[i]}(t) \\ x_2^{[i]}(t)\end{pmatrix}.$$

For stability of the reduced system according to condition (22)

$$\begin{aligned}&\frac{-b}{m} < 0 \\ &\frac{\beta_e(F_1(x_2^{[i-1]}(t)) - c_2^{[i]}(t)F_2(x_2^{[i-1]}(t)))}{V_0 + \int_{t_0}^{t}(x_1^{[i-1]}(t) + v_d)\,\mathrm{d}t} < 0 \\ &\frac{-\beta_e(A + c_1^{[i]}(t)F_2(x_2^{[i-1]}(t)))}{V_0 + \int_{t_0}^{t}(x_1^{[i-1]}(t) + v_d)\,\mathrm{d}t} = -\frac{A}{m}.\end{aligned}$$

Notice that $\frac{-b}{m} < 0$, $F_1(x_2^{[i-1]}(t)) < 0$, $F_2(x_2^{[i-1]}(t)) > 0$ always hold, we can choose

$$c_1^{[i]}(t) = \frac{1}{F_2(x_2^{[i-1]}(t))}\left[\frac{A(V_0 + \int_{t_0}^{t}(x_1^{[i-1]}(t) + v_d)\,\mathrm{d}t)}{m\beta_e} - A\right] \tag{26}$$

$$c_2^{[i]}(t) = 0 \tag{27}$$

then the stability of reduced system is guaranteed. Since

$$\dot{\sigma} = \dot{c}_1^{[i]}(t)x_1^{[i]}(t) + c_1^{[i]}(t)\dot{x}_1^{[i]}(t) = -\mathrm{sign}(\sigma)$$

then

$$\dot{c}_1^{[i]}(t)x_1^{[i]}(t) + c_1^{[i]}(t)\left[\frac{1}{m}(-bx_1^{[i]}(t) + Ax_3^{[i]}(t))\right] + \frac{1}{\tau_v}(-x_3^{[i]}(t) + K_v v^{[i]}(t)) = -\mathrm{sign}(\sigma).$$

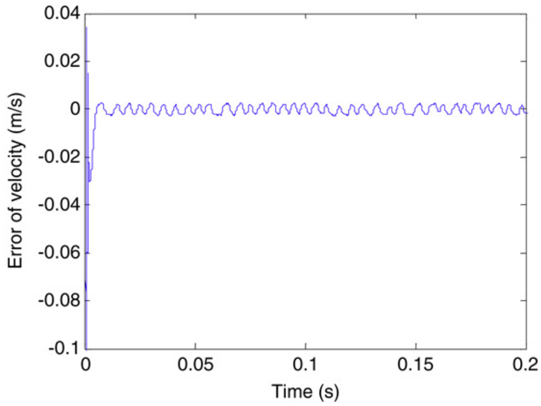

Fig. 2. Velocity tracking error.

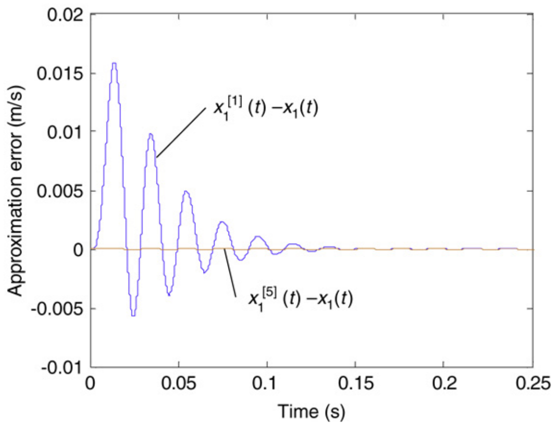

Fig. 3. Approximation error.

We get the control input

$$u^{[i]}(t) = \frac{1}{K_v}\left[\tau_v(-\mathrm{sign}(\sigma) - \dot{c}_1^{[i]}(t)x_1^{[i]}(t) - \frac{c_1^{[i]}(t)}{m}(-bx_1^{[i]}(t) + Ax_3^{[i]}(t))) + x_3^{[i]}(t)\right] + u_d.$$

5. Result

The simulation of the sliding mode control for the hydraulic system is performed in Matlab by using Euler numerical integration technique. The parameters are: $m = 1000$ kg; $b = 2 \times 10^4$ (Ns/m); $f = 3 \times 10^5$ N; $V_0 = 4.9 \times 10^{-1}$ m^3; $A = 2.46 \times 10^{-1}$ m^2; $\beta_e = 5 \times 10^8$ Pa; $P_r = 1 \times 10^5$ Pa; $Ps = 31.5 \times 10^6$ Pa; $C_{tm} = 1.5 \times 10^{-14}$ (m^3/sPa); $C_{em} = 2.1 \times 10^{-14}$ (m^3/sPa); $k_q = 1 \times 10^{-4}$ (m$^3/\sqrt{Pa}$vS); $\tau_v = 0.01$ s; $Kv = 1 \times 10^{-3}$ (m$^3/\sqrt{\mathrm{Pa}}$ vS); $vd = 0.5$ m/s

Fig. 2 shows the tracking error of the velocity when $u^{[5]}(t)$ is applied to (11). Fig. 3 shows the approximation error of $x_1^{[1]}(t)$ and $x_1^{[5]}(t)$.

6. Conclusions

In this paper, we extend a recently introduced approximation method for nonlinear functional differential equations. The nonlinear system is replaced by a sequence of linear time-varying systems which are proved to be globally convergent under certain conditions. This allows many results of linear control theory to be applied to nonlinear systems. Based on this method, we have designed a sliding mode controller for a hydraulic press system. And the control result shows the effectiveness of this method.

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