Towards the near-infrared detection of brown dwarf companions: Exploring methods to detect low mass stellar companions from blended spectra ⋆

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ABSTRACT
In this paper we attempt to directly detect the near-infrared spectrum of candidate brown dwarf (BD) companions around FGK stars to assert or discard their stellar nature. We explore two different methods to probe the faint spectra of BD companions. The first technique involves the direct subtraction of two observations shifted to mutually cancel out the spectra of the host star. The second technique applies a χ^2 fit to the individual observations of a synthetic binary model comprised of two PHOENIX-ACES spectra. The observed spectra are wavelength calibrated and corrected for the atmospheric absorption with the aid of synthetic telluric models. The direct subtraction method failed to detect the signature of the companions due to poor observational constraints (the observations were insufficiently separated in radial velocity). The χ^2 technique developed here is able to fit the component for the host star but unable to successfully detect the faint spectra of even the largest companion in our sample. We explore how the companion recovery fitting performs on simulated observations and discuss reasons for the non-detection observed. From the injection-recovery analysis, this technique in its current form, is insufficient to recover a companion below 3800 which corresponds to an upper mass limit for the companions around 600 M_{Jup}. This work highlights the challenges in the spectral detection of faint companions. We explore the limitations of the direct subtraction method in the case of small RV separation, and companion detection with synthetic models at low companion/host flux ratios below 1%, made difficult by the discrepancy between observed and synthetic spectra around 2100.

Key words: brown dwarfs – binaries: spectroscopic – infrared: stars – techniques: spectroscopic – methods: miscellaneous

1 INTRODUCTION

Brown dwarfs (BDs) are sub-stellar objects unable to achieve hydrogen fusion, with masses around 13 – 80 M_{jup} (**?**), bridging the gap between low-mass stars and giant planets. Without sustained fusion, brown-dwarfs cool down over time with an age-dependent cooling rate. Therefore, there is an inherent degeneracy between the mass, age and luminosity of a given BD(**?**). This degeneracy may be broken by observation of several parameters, for instance when a BD is in a binary system with a main sequence host star, using both the host stars age and the masses derived from the dynamical motion.

A paucity of BD companions exists in short period orbits around Sun-like stars ($\lesssim$ 5 AU), compared to stellar or planetary companions, termed the *brown dwarf desert* (**???**). As the number of known BDs orbiting solar type stars is low, the characterization of benchmark BDs in the brown dwarf desert (e.g. **?**) is beneficial in understanding this sub-stellar population and to help constrain formation and evolution theories (**?**). The BD desert also provides a greater challenge as it reduces the amount of good BD candidates to study.

BDs in binary systems, unlike free-floating BDs, allow for the determination of their masses, when complemented with radial velocity (RV) and astrometry measurements. The RV technique provides the mass lower-limit ($M_2 \sin i$) of binary and planetary companions, while complementary astrometry measurements can often provide mass upper-limits (e.g. **?**). Measuring or tightening the constraints of BD masses improves the understanding of mass

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dependence on BD formation processes. For instance, there is growing evidence that the larger giant planets and BD companions do not follow the well known metallicity-giant planet correlation seen in main-sequence stars with planets (e.g. **???**). Photometry along with stellar evolution models (e.g. **??**) can also be used to estimate the mass of BD companions (e.g. **?**) if there is sufficient orbital separation, and a precise determination of the age (**?**).

Recently, there has been a renewed interest in BD candidates triggered by exoplanetary searches. While several works found similar properties on the two populations, like a similar density (**?**), others found intriguing differences. One of the most recent is the different host metallicity of the Brown Dwarf and giant planet populations (**??**), a very strong hint of different formation mechanisms.

Spectral observations of binary systems contain the spectra of both bodies, in proportion to their flux ratio, and Doppler shifted relative to each other due to their orbital motion. One technique to recover the spectra of the companion is secondary reconstruction through a differential spectrum (**?**). Spectra from different phases are shifted to the host stars rest frame and subtracted to mutually cancel out the spectrum of the host star allowing the faint companion spectra to become visible. Advances in high-resolution and near-infrared (nIR) capabilities should enable this technique to be applied to BDs and planet companions, in which smaller RV shifts can be resolved and the contrast ratio of the smaller companion is improved.

Observing in the nIR is specifically desirable for the cooler sub-stellar and giant planet companions as their thermal emission is stronger in the infrared compared to the optical. This improves the contrast ratio between the host star and companion, providing favourable conditions for their detection and spectral separation. CRIRES, a high resolution nIR spectrograph, has made many prominent advances in recent years with the detection of atmospheric constituents, such as CO and H_2O, atmospheric winds and thermal profiles, rotation and orbital motion, for both transiting and non-transiting planets (e.g. **?????????**).

The higher temperature and relatively larger size of BDs compared to giant-planets makes the development of spectral recovery techniques for BD companions a logical step towards the spectroscopic detection of planetary atmospheres. There has been the recent installation and continued development of many new high-resolution nIR spectrographs, such as, CARMENES (**?**), NIRPS (**?**) or SPIRou (**?**), as well as, the CRIRES+ (**?**) upgrade. These new instruments motivate the study of nIR-oriented methodologies for spectral recovery, and are of high importance due to the larger planet-to-star flux ratio provided by near-IR compared to the visible.

The search and detection of faint secondary spectra is not only relevant to planetary atmospheres. **?** developed a method to detect the presence of optical secondary spectra down to a flux ratio of 1% in the hosts of *Kepler* transit candidates. The presence of which can cause ambiguities in the system configuration, and increase the uncertainty of the measured planet radius. The characterization of the false positive probability rate for Kepler has been found to be as high as ∼35% **?**.

In this paper we apply two different techniques on FGK stars with BD companions with the aim to spectroscopically detect their companions. In Sect. 3 we present the observations and reduction process as well as the spectral models used in this work. In Sect. 4 we explain the differential spectral technique and its applicability to these observations while in Sect. **??** we apply companion recovery using a χ^2 approach. In Sect. 5 we discuss our results and in Sect. **??** we present our conclusions.

2 MOTIVATION AND TARGET SELECTION

The work of **?** identified several candidate brown dwarf companions of FGK stars with $M_2 \sin i$ values >10 M_{Jup}. Seven candidates from **?**, which were visible in Period 89, were selected for further observation in order to identify their stellar nature. The target host stars are presented in Table **??** with their stellar parameters, while the companion orbital parameters are provided in Table **??**.

We note that some of these orbital parameters have been refined in the literature since observations took place. For example three candidates have had their masses refined in recent works. The companion to HD 211847 was determined to be a low mass star with an $M_2 = 155\ M_{Jup}$ (**?**), while the companion to HD 4747 was found to have a mass of $M_2 = 60\ M_{Jup}$ (**?**). The two companions of HD 202206 (B and c) were found to have masses of $M_B = 93.6\ M_{Jup}$ and $M_c = 17.9\ M_{Jup}$, respectively, classifying HD 202206c as a “circumbinary brown dwarf” (**?**). These targets with recently refined masses create good benchmarks for us to compare any results of the techniques developed in this paper, and show that these masses do span the BD – low mass star range. All target companions except HD 162020 (P=8.4 days) are in (very) long period orbits (P=0.7–38 years) with masses (or $M \sin i$) greater than 10 M_{Jup}.

K-band spectra were obtained with the goal to achieve a direct detection of the companion spectra through the application of a spectral-differential approach. Doing so would enable a further constraint to be placed on their masses. The *K*-band is used to achieve a high contrast relative to the host star, detected in the extreme V-K colour indexes (>7.8).

3 DATA AND DATA REDUCTION

In this section we explain the observations and the data reduction process used. We also discuss the models used to correct for atmospheric absorption.

3.1 CRIRES data

Observations were performed with the CRIRES instrument (**?**) configured to observe a narrow wavelength domain of the *K*-band between 2120–2160 using the Ks and the Hx5e-2 filters. The slit width of 0.4 sec resulted in an instrumental resolving power of R = 50 000, with no adaptive optics to ensure that the slit was entirely covered by each target. This prevents strong slit illumination variations that could change the shape of spectral lines.

The observations were performed in service mode during period 89 with run ID. 089.C-0977(A) between April and August 2012. An observation is composed of 8 individual spectra each with an integration time of 180 seconds, observed in a ABBAABBA nod cycle pattern to obtain a high (>180) signal-to-noise when combined.

The list of observations obtained with CRIRES are provided in Table **??**. The SNR is calculated with the formula SNR $= \frac{\mu}{\sigma}$ where μ and σ are the mean and standard deviation in the continuum of detector 2 between 2130 and 2134 (see Fig. A1). The estimated RV values for the host and companion at the time of each observation are calculated using Equation B1 using the best known orbital parameters and the companion masses from Tables **??** and **??**. For hosts with multiple companions the RV value is for the largest companion only, i.e. HD 202206B and HD 168443c. The RV difference between the host and the companion $rv_2 = (RV_2 - RV_1)$ is parameter used in the binary model from Sect. **??**.

Figure 1. Example of an artefact in the optimally extracted spectra. The top panel contains the 8 normalized nod spectra after optimal extraction, while the middle panel shows the rectangular extraction for the exact same spectra. A vertical offset is included between each spectra. A single large spike in the seventh nod (pink) near pixel 230 creates a wide and noticeable artefact in the optimal extraction. The bottom panel shows the difference between a combined spectrum using the optimal nods only and a combined spectrum in which the seventh nod is replaced with its rectangular counterpart. The nod spectra are in observation order from top to bottom.

3.1.1 *Data reduction*

The observations were reduced using a custom reduction pipeline (**?**). Written in IRAF's CL[1] (**?**) it provides for automated dark and non-linearity corrections (using the non-linearity coefficients provided by ESO), as well as the flagging and replacement of bad pixels. The images were corrected from sensitivity variations by dividing by a flat-field corrected from the blaze function effect. The nodding pairs were mutually subtracted and the order tracing was fitted by linear or cubic splines selected for each detector.

There are two types of extraction commonly used. The *rectangular* extraction performs a rectangular aperture sum in the spatial direction while the *optimal* extraction (**?**) also includes variance weighting to reduce the impact of the noise and deviant pixels on the total flux measurement. The extracted nod-cycle spectra are normalized by dividing each by a polynomial fitted to the continuum.

At this point one would normally combine all the optimally extracted nod spectra together to improve the signal-to.noise. However, we discovered extended artefacts in the optimally reduced nod spectra. We were not able to identify why these artefacts are large and extended but they occur near the presence of a large cosmic rays or bad pixel spike seen in the rectangular extraction. These artefacts were not observed in previous works using this pipeline in the *H*-band, so there may be a wavelength dependent affect. The variance weighting procedure during the optimal extraction is somehow being heavily affected by these spikes. An example of this can be seen in Fig. 1 where a spike in the rectangular extraction corresponds to the large extended features in the optimal extraction. The amplitude of these artefacts created flux deviations in the combined optimally extracted spectra up to ~0.5%. Therefore, we took measures to remove these artefacts before combining the nod spectra as we are trying to recover companion spectra with expected flux ratios $F_2/F_1 < 1\%$. Parameters of the pipeline were adjusted to try and remove these artefacts, such as the σ rejection limits and the aperture width with limited success. In a small number of instances allowing the aperture width to be automatically adjusted removed the artefacts.

All nod spectra for each observation were visually inspected together, as shown in Fig. 1, and any spectra containing artefacts were marked. The optimally extracted nods (top panel) that were identified were replaced with their rectangularly extracted counterparts (middle panel). An iterative 4-σ rejection algorithm[2] was applied to the replacement rectangular extractions to remove the erroneous pixels that created the artefacts. The σ value for each pixel was calculated as the standard deviation of the nearest 2 pixels on either side of all 8 nod spectra. The rejected pixels were replaced using linear interpolation along the dispersion axis. Out of the 544 nod spectra from individual detectors, 79 (14%) contained artefacts and were replaced using this technique.

For the remainder of the paper we use combined spectra constructed by averaging the 8 nod-cycle spectra together, where some of the optimally extracted spectra have been replaced using the above method. The last panel of Fig. 1 shows that the difference between the combined optimally extracted spectra and the *combined extraction with replacements*.

The continuum normalization and nod combining steps were also performed using IRAF while the following post reduction procedures and analysis all utilize *Python*. This pipeline was chosen over the ESO CRIRES pipeline because it seemed relatively simple to use, being semi-automated, and appeared to have less bad pixel/cosmic ray artefacts in the extracted spectra. In hindsight this was not the case, with the removal of the extended artefacts that appeared.

3.1.2 *Telluric correction*

Ground based observations require the removal of the absorption lines introduced by Earth's atmosphere. These observations were first taken in an atmospheric window of the *K*-band in order to reduce the absorption introduced by the atmosphere (**?**). To correct for the remaining telluric line contamination the spectra were divided by the TAPAS(**?**) atmospheric transmission models for each observation. Synthetic telluric models were used to avoid the observing overhead necessary to perform telluric standard star exposures (**?**), and they have been demonstrated to be superior in the quality of the correction relative to the telluric standard approach (e.g. **?**).

Before the correction, the depth of the telluric lines were rescaled to match the airmass of the observation using the relation $T = T^{\beta}$, where T is the telluric spectrum and β is the airmass ratio between the observation and model. This changed the depth of most absorption lines to match the observations, but does not correctly scale the deeper H_2O lines. The scaled telluric model is interpolated to the wavelengths of the observed spectrum and then used to correct the observed spectra through division, leaving behind a telluric corrected spectra. An example of a telluric corrected spectra is shown in the middle panel of Fig. A1, with the light blue shading indicating where the deeper telluric lines were.

We attempted the technique suggested by **?** to address the poor H_2O airmass scaling, to fit a scaling factor to the H_2O absorption lines before convolution to the instrument resolution. This was achieved by first dividing the spectrum by a telluric model with only non-H_2O constituents, convolved to the observed resolution, and scaled by the airmass to remove the non-H_2O lines. Then a model with only H_2O lines at full resolution was scaled by a factor T^x, convolved to R = 50 000 and compared to the observed spectra. The factor x was fitted to find the best scaling factor for the H_2O lines.

We found that for a few spectra in our sample this method corrected the deeper telluric lines well, but in many cases we found that the fitted scaling factor was affected by the presence of blended stellar lines (attempting to fit those also). It was also strongly influenced by the deepest *H_2O* telluric lines present. We find that the telluric correction of the deep H_2O lines could be improved with this technique, but, at the cost of worsening the correction of the many smaller H_2O lines. Since the smaller H_2O lines covered more of the spectrum in this region than the larger lines we chose not to

[1] IRAF is distributed by the National Optical Astronomy Observatories, which are operated by the Association of Universities for Research in Astronomy, Inc., under cooperative agreement with the National Science Foundation.

[2] Found at `https://github.com/jason-neal/nod_combination`

continue with this separate H_2O scaling. One possible solution for this would be to perform a piece-wise telluric correction, performing this step only for the deeper H_2O lines, or by using one of the other tools that fits the telluric model to the observations. This technique could also benefit from a larger wavelength span that would enable blended lines to be ignored while having sufficient deep H_2O lines to fit the scaling factor correctly. This small experiment shows that a simple scaling is not enough to correct for the absorption in an effective way, for this case.

After the telluric correction is performed, the spectra are corrected for Earth's barycentric RV using the *helcor* PyAstronomy[3] function ported from the REDUCE IDL package (See **?**).

3.2 Tapas models

We used telluric line models to wavelength calibrate the reduced spectra in Sect. **??** and to correct for the atmospheric absorption in Sect. 3.1.2. We utilized the TAPAS (Transmissions of the AtmosPhere for AStronomical data) web-service[4] (**?**) to obtain atmospheric transmission models for each observation. TAPAS uses the standard line-by-line radiation transfer model code LBLRTM (**?**) along with the 2008 HITRAN spectroscopic database (**?**) and Arletty atmospheric profiles derived using meteorological measurements from the ETHER data center[5], which has a 6 hour resolution in atmospheric profiles. We use the mid-observation time to retrieve transmission models for each observation, with the Arletty atmospheric profiles[6] and vacuum wavelengths. The telluric models were retrieved without any barycentric correction to keep the telluric lines at a RV of zero with respect to the instrument. We obtained one model with all provided species present, convolved to a resolution of R = 50 000, and another two models without an instrumental profile convolution applied. For these two extra models, one contained only the transmission spectra of H_2O while the other was for all other constituents except H_2O. This was to explore a known issue (**?**) with the depth of H_2O absorption lines in Sect. 3.1.2.

3.3 Wavelength masking

We apply several wavelength masks to remove wavelength regions from which we cannot extract information. Firstly, regions near the edges of each detector where the wavelength solution is extrapolated outside of the calibrating telluric lines are removed, reducing the effective size of each detector by about 10% or ∼100 pixels.

Secondly, we mask out any remaining artefacts present in the spectra and the centers of deep telluric lines where telluric correction was not corrected properly, sometimes resulting in "emission-like" peaks in the corrected spectrum. These factors combined result in masking out around a further 10% of the observed spectra.

In Sect. **??** we also apply a further wavelength restriction to mask out regions where there is a large mismatch between the observed spectrum and the closest synthetic spectra to the host. This significantly restricts the wavelength span utilized for that purpose to around only 43% and the masked regions are visible in Fig. **??**.

[3] https://pyastronomy.readthedocs.io
[4] `http://www.pole-ether.fr/tapas/`
[5] `http://www.pole-ether.fr`
[6] Nearest of the 6 hourly profiles

4 DIFFERENTIAL SUBTRACTION

The observations were gathered having in mind the application of a differential subtraction method (e.g. **??**) to detect the spectra of the faint BD companions. In short, we Doppler shift each observation to the rest frame of the host star and then mutually subtract the spectra from pairs of observations to cancel out the spectra of the brighter host star. The residuals from this method should contain two copies of the faint companion, subtracted from each other with a radial velocity offset ΔRV between them. This RV offset, if detectable, would allow the mass ratio to be determined and hence the companion mass.

Due to the poorly separated observation times relative to the long orbital periods, this method was revealed to be inappropriate for these observations as the RV separation of the companion spectra between observations ($\leq$ 2.3 kms^{-1}) is significantly smaller than the FWHM (full width half maximum) of individual spectral lines (∼6 kms^{-1} at R = 50 000). The small separation of the companion causes the lines of the companion to also mutually cancel, severely reducing the residual signal to well below the available noise level. The requirement of well separated RVs for the companion spectra was clearly stated in the original proposal but was not satisfied when the observations were obtained[7]. The very large orbital periods of some of the targets would not produce a sufficient RV signal during one semester. This was a possible oversight during the proposal stage. The largest estimated companion ΔRV separation between the observations of each target is provided in the seventh column of Table **??**. Radial velocity constraints are also valid for other studies such as the detection of reflected light from exoplanets **?**.

Despite these problems, we still tried to apply the method to our data. Given the negative result, however, we decided to move this section to Appendix A, where we describe the method and some exploratory results that may be useful for future studies.

5 DISCUSSION

The spectral differential and the synthetic recovery methods attempted here were both unsuccessful in a detection of the BD companion spectra. The upper mass limits of 600^{+20}_{-40} we set for these companions is very high, roughly six times higher then the BD mass limit ∼80–90 M_{Jup}. We discuss potential reasons and solutions for these poor results below, list the lessons learned in this exploratory study of this dataset, and provide some guidance for any future attempts with these methods.

5.1 Synthetic recovery limitations

In this section we discuss some of the limitations from this synthetic binary method and some options to overcome some of these.

5.1.1 Mismatch in synthetic models

We believe the spectral mismatch between the observation and synthetic spectra is the main cause of the unsuccessful companion detection with the χ^2 method with several strong lines in the model not observed in the spectra. This impacts the recovery in two ways; the spectral mismatch causes the χ^2 values to be large in general, but also causes the companion temperature to be pushed to higher temperatures, up to the constraints allowed by the grid.

[7] see Sect. 5.2.2 for more details

In our examples the logg and metallicity of the synthetic models are held fixed, leaving only temperature to vary. The temperature impacts the synthetic spectral models in two main ways: the flux level of the continuum; and the number and strength of the absorption lines. In the binary model the contributions from the individual components is scaled by the flux ratio. If the temperature of the companion increases then the flux and radius of the companion increases. The contribution of the companion to the binary model increases and the flux ratio F_1/F_2 decreases. This effectively makes the lines of the spectrum of the host component relatively smaller in the normalized binary model spectrum. Due to the large initial mismatch of synthetic spectral lines of the host, a decrease in relative strength of the host lines decreases the χ^2 value, and is a better "match" to the observation. This causes the recovered temperature of the companion to be much higher than expected. If the companion temperature grid is allowed to extend it will recover a companion with a temperature >2000 above the expected temperature. The χ^2 approach is dominated by reducing the mismatch in the spectrum of the host rather than actually detecting the spectra of the companion. When preforming the simulations in Sect. **??** there is no spectral mismatch between the simulation and the models, hence they do recover the correct host spectra and get closer values for the companion.

To specifically check that the mismatch was not due to errors in our reduction we obtained the final spectral atlas of 10 LEO from the CRIRES-POP archive, fully reduced and telluric corrected (**?**). Comparing the spectrum of 10 LEO to a PHOENIX-ACES model with the corresponding parameters T_{eff_1} = 4800, logg=3.0, =0.0 and convolved to R=90 000, similar line mismatches are observed. For 10 LEO, there is more spectral lines that agree with the model, but in the region of our observations between 2110–2160 there are deep lines in the model that are not observed, lines that have significantly different strengths, lines observed that are not modelled whilst other lines 'appear' to be offset in wavelength, at the same positions observed in the work presented here. This indicates that the mismatch is not specific to our reduction only and that there is still room for improvement in synthetic models around 2100.

Other works have also indicated regions or specific lines in which synthetic models did not reproduce all of the spectral features seen in stellar spectra (e.g. **????**).

5.1.2 Line contribution of faint companions

We calculate the line depths of the synthetic companion spectra to determine the SNR levels required to detect the lines of the binary companions. One thing easy to overlook when attempting to detect the binary companion at low flux ratios is the actual contribution of the spectral lines of the companion. The flux ratio of the continuum for our most promising target is F_2/F_1~1% with the other targets having an expected flux ratio 0.5%, and some well below. The spectral lines of the individual components which are the features we are trying to detect with the binary model, have depths on average around 10–20% of their respective continua, at-least between 2110–2160. In effect, the companion line features have a depth $\ll$ 1% relative to the continuum of the combined spectra.

In Table **??** we calculate some properties of the spectral lines in the PHOENIX-ACES library between 2110–2160. We count the number of spectral lines (*no. lines*) deeper than 5%, and take the average depth (*avg. depth*) of these lines. The contribution *cont. depth* of the companion lines to a combined spectrum accounts for the flux ratio between the two components. Here we use a Sun-like host with T_{eff_1} = 5800. This simplified combination neglects the continuum shapes of both spectral components and uses the average flux ratios for this wavelength range. The PHOENIX-ACES spectra in the temperature range of 2500–5000 shown in Fig. 2 can be used to get a visual indication of the line density and depth measured here.

There are more lines >5% deep for the lower temperature spectra, with 360–460 lines in this wavelength range, to be compared with the 31 deep absorption lines found in a Sun-like spectrum. The average line depth of these lines is also larger than the Sun-like spectrum, around twice as deep. However, when combined, the contribution of the companion lines is 1–2 orders of magnitude smaller than the hosts lines due to the low continuum flux ratios.

For example, with the synthetic model for the companion of HD 211847, the average contributions of lines >5% become only 0.3% deep in a binary with the Sun-like spectrum. For a companion with a temperature of 2300 (the lower PHOENIX-ACES temperature limit) the deepest lines contribute lines around 0.1%.

The SNR of the observed spectra is between 150–350, which is below the SNR of 323 needed for the detection of the low-mass star companion of HD 211847 with temperature 3200 and logg 5.0. For our other targets with BD companions at and below the PHOENIX-ACES temperature range, we would need observed SNR >800 to detect the individual spectral lines of the companion. With the SNR increasing with $\sqrt{N}$ this would require the observational time for each target to be increased by a factor of ~10–64. This is in line with the recent detection of the spectrum of a non-transiting giant planet by **?** which utilized nIR spectra with SNR > 2000, from 1–3 hours of observation each.

Our non-detection of binary companions with low flux ratios is consistent with results from other works. For example **?** performed extensive spectral analysis of a quadruple-star system ξ Tauri using 227 spectra in 3 different wavelength bands. Of the four stars in the system they were unable to detect the spectral component of the one which had a luminosity ratio below 1%. The secondary detection in optical spectra using spectral matching of KOI was also only able to reach flux ratios of 1% **?**.

5.1.3 χ^2 asymmetry

In Fig. **??** we showed that the shape of the recovered χ^2 becomes asymmetric when dealing with companion temperatures below around 3800. A visual inspection of the spectra reveals the likely cause. In Fig. 2 we show the corresponding spectra between 2111–2165. As the temperature decreases the strongest lines become less prominent, disappearing progressively among the other many small lines that appear at lower temperatures. Hence there are no strong companion lines to easily distinguish one temperature from another. In the flatter part of the χ^2 curves several low temperature companions are equally well fitted to the simulation/observation.

Figures **??** and **??** show different recovered temperatures but both agree above 3800. A higher companion temperature is recovered between 2800 and 3800, where as in Fig. **??** a lower temperature is recovered. This is probably due to a combination of the noise added, and the asymmetries of the χ^2 lines. Figure **??** uses the noise level from the observed spectrum while Fig. **??** has a SNR of 300. This large asymmetry can also explain the jump observed in the synthetic recovery temperature around 2700 in Fig. **??**.

The asymmetry also causes an asymmetry in the χ^2 error bars which can be seen in the bottom panel of Fig. **??**. For instance the recovered value and 1-σ error bars on the 3000 injected companion is 2800^{+20}_{-100}, with an asymmetric error bar skewed towards lower temperatures.

Figure 2. PHOENIX-ACES spectra for temperatures between 2500 and 5000, corresponding to to the same lines in Fig. **??**. The flux units are the native units of the PHOENIX-ACES spectrum, (erg s^{-1} cm^2 cm^{-1}), and have not been scaled by the stellar radii. All spectra have a logg = 5.0 and = 0.0. The vertical dotted lines indicate the edges of the CRIRES detectors.

Figure 3. Detector 1 spectrum for HD 211847 (blue) alongside the PHOENIX-ACES (orange dash-dot) and BT-Settl (green dashed) synthetic spectra for the host star only, with parameters $T_{\rm eff}$ = 5700, logg = 4.5 and = 0.0. Both synthetic models have been normalized and convolved to R = 50 000. There is a 0.05 off-set between each spectrum

The bump observed at 5100 in the χ^2 curves is due to a discontinuity in the PHOENIX-ACES modelling. The "reference wavelength defining the mean optical depth grid" is changed at 5000 (**?**, Sect. 2.3). Care needs to be taken if trying to detect a companion near this temperature.

5.1.4 Component RV separation

Another factor which could contribute to an unsuccessful detection is the RV separation between the host and companion, rv_2. Estimates for our observations are given in the last column of Table **??**. If rv_2 is small compared to the line width, then all the same lines of both components will be blended. This is indeed the case for HD 4747, HD 211847, and HD 202206 with expected $|rv_2| < 2$ kms^{-1}, due to poor observational planning. This may have contributed to the lack of recovery with both components of the binary model attempting to fit the same features. This may even cause correlation between the parameters of the two components. The RV separation of the two components changes with orbital phase. Having multiple spectra of the same target distributed in phase may allow the RV of the spectral components to be better recovered (e.g. **??**). Similarly **?** were unable to detect companion stars within 10 kms^{-1} of the host using optical spectra.

5.1.5 Wavelength range

The wavelength range for these observations was chosen specifically due to the location of the *K*-band telluric absorption window. This was to reduce telluric contamination present in the spectra intended for the spectral differential technique. The wavelength range is also very narrow (~50) and was set by the CRIRES instrument. The small number and inconsistent distribution telluric lines made the wavelength calibration method using the telluric lines difficult in some regions (specifically detectors 2 and 3). For the χ^2 fitting of faint companions this narrow wavelength region is likely not the best choice given the small number of stellar lines and unique spectral features of the companion. For example **?** use 4 different wavelength regions with lines from different species to fit PHOENIX-ACES models to M-dwarf stars while other studies aiming to detect planetary companions choose wavelength regions which contain strong planetary absorption features such as the absorption of CO and H_2O near 2.3 μm (e.g. **??**). Applying the binary fitting to a different wavelength region with lines more sensitive to stellar parameters for both stars and BDs, as well as using a larger wavelength range (i.e. provided by the cross-dispersion on CRIRES+ (**?**)), should improve the recovery results of the technique presented here. We note that if the wavelength range is increased by taking separate observations at different wavelengths, not covered by a single exposure, then changes in the RV of both components between the different wavelength observations will need to be accounted for.

5.1.6 The BT-Settl models

We note that the PHOENIX-ACES models are not the only spectral libraries available with the other notable library considered for this work is the BT-Settl models (**???**). The included modelling of dust and cloud formation, as well as hydro-dynamical modelling atmospheric mixing/settling for atmospheres with $T_{\rm eff}$ below ~2600, make the BT-Settl models valid across the regime from stars to BDs as cool as 400. As the BT-Settl models are suitable to model the atmospheres of the brown dwarfs they would be useful for the companion recovery technique developed here. However, as shown in Sect. **??** and **??**, we were unable to successfully recover the 155 M_J ($T_{\rm eff}$ ~3200) low mass star companion of HD 211847 and derived a temperature upper limit for our methodology of around 3800. These are both well above the 2300 cut-off of the PHOENIX-ACES models and for the onset of dust- and cloud-formation phenomena, at 2600 K..

Fig. 3 shows again the minimum χ^2 solution for detector 1 of the second HD 211847 observation, this time including the BT-Settl solution with the same parameters. Although the PHOENIX-ACES and BT-Settl models differ slightly they both have a large spectral mismatch to the observations. As such, we did not use the BT-Settl models for the simulation and results above as we did not see any special advantage in using them.

The ease of access to find, download, and use PHOENIX-ACES spectral library, available in the fits file format, compared to older BT-Settl libraries is another reason for the current favoured use of the PHOENIX-ACES library.

Although the newer generations synthetic spectral models are improving and match the overall spectral energy distribution reasonably well there are still regions in the H and *K*-band where there is room for improvement **?**. The spectral mismatch in the region studied here is still too large for spectral recovery of companion brown dwarfs. In the nIR we have compounding problem: the model input physics of sub-stellar temperatures and chemistry combined with the general difficulty of the nIR.

5.1.7 Impact of logg

Logg, a measure of surface gravity, is related to evolutionary state and the size of the star with smaller logg values usually indicating larger radii stars. This parameter has a large impact on the radius and flux ratio of the binary models. In the PHOENIX-ACES models a decrease in logg from 5.0 to 4.5 increases the models effective radius by ~1.75 in the temperature range investigated here. This change in radius alone roughly triples (1.75^2) the absolute flux of the synthetic spectrum, neglecting any changes to the shape of the actual spectrum. Therefore, there are large jumps in the model flux ratios if the logg is allowed to vary, with lower logg values for the companion being favoured as the increased flux ratio decreases the mismatch of the host component to the observations. This large impact of logg on the spectral library absolute flux is one reason for keeping the logg of each component fixed in the χ^2 results presented in Sect. **??**.

5.1.8 Interpolation

It is common to interpolate between the synthetic spectral grids to fit and derive parameters in between the grid points (e.g. **??**).

Instead of interpolation **?** use a spectral emulator to use Principal Component Analysis to create eigenspectra for the synthetic library and Gaussian processes to derive a probability distribution function of possible interpolation spectra to account for uncertainties in the interpolation required for high signal-to-noise spectra.

However, we did not incorporate any interpolation into the companion recovery at this stage. This could be something to be added in the future to refine the recovered parameters, and to help the transition between the grid logg values. Codes are readily available to perform spectral interpolation which could be utilized for this, two of them are *pyterpol*[8] **?** and *Starfish*[9] **?**.

5.2 Future implementation

5.2.1 High resolution instrumentation

The future of high resolution near- and mid- IR spectrographs is looking bright, with many new ground- and space-based instruments currently being developed. Notable examples include CARMENES (550–1710, R=82 000) which is now operational (**?**), while SPIRou (980–2350, R=73 500) **?** and NIRPS (970–1810, R=100 000) **?** are still being assembled and installed. The eagerly awaited JWST **cite** will also be launched soon[10] providing observations in both the near-IR (600–5300, R=2700) and mid-IR (4900–28 800, R∼1550–3250) regions without contamination from our atmosphere.

The upgrade of CRIRES to CRIRES+ (**?**) will increase the wavelength coverage of a single shot capture by at least a factor of 3–5. This larger wavelength span would be extremely beneficial for the χ^2 performance of the spectral recovery method, increasing the number of useful lines and spectral features to be fitted with the models.

On the modelling side, there are continual improvements in atmospheric modelling and their associated synthetic spectral libraries: as seen with the evolution of the BT-Settl models **?**. With additional physics and improved line lists and solar abundances (e.g. **??**), the synthetic libraries are reaching a better agreement with nIR observations. An improved agreement between the nIR observations and synthetic spectra will be crucial to improve the performance of the spectral recovery technique presented here.

Although not successful with the CRIRES data used here, the instrumental stage is set to attempt these techniques presented here using the next-generation of high resolution spectrographs. The lessons learned in this analysis need to be taken into account in order to achieve the best chance of a successful detection.

5.2.2 Differential scheduling challenges

This work has revealed that more care needs to be taken in planning the observations for the spectral differential analysis of faint companions in the future. Paying attention in particular to the FWHM of the lines in the region (governed by resolution and wavelength); the estimated companion ΔRV; the previous observations from different observing periods; and keeping consistent detector settings.

The original goal for the observations was to obtain two different and "clearly separated radial-velocities" for the secondary companion. However, the program was assigned a low-priority (C, in ESO grading) and, possibly due to operational reasons, the original time requirements necessary to secure well separated RVs for the companion spectra could not be met. This meant that all observations were insufficiently separated to extract a differential spectra for the companion.

The long orbital periods of these targets is also a contributing factor to the insufficient separations. Most of the targets observed here have orbital periods much longer than an observing semester (183 days). An optimal pair of observations (achieved at the extrema) would need to have been obtained from separate observing periods (between 2 months and 19 years apart). In some cases, even observations taken at the beginning and end of the semester would not be sufficient to achieve companion separation (depending on the phase). Requiring separate observing periods to even achieve the minimum ΔRV larger than the line FHWM.At the time it was impossible to ask for time over several semesters in a regular proposal.

Our study demonstrates the importance of proposals for projects that need to be extended over several semesters or years. In the ESO context, this corresponds to "Monitoring proposals" (e.g. **?**, pg. 18). Observations of the targets explored here, with long orbital periods in particular, would benefit from the ability for multi-period proposals and newer scheduling systems which allow for tighter scheduling constraints, such as a companion RV separation.

For future observations we suggest that the known orbital solution of the companion be used to estimate the companions' RV curve during the observing period, with the companion $M_2 \sin i$ providing an RV upper-limit. Knowing the instrumental wavelength and resolution, a constraint can then be set to avoid taking observations when the companion spectra are insufficiently separated, or $\Delta RV <$ FWHM.This constraint can be set using the absolute and relative *time-link* constraints available in ESO's Phase 2 Proposal Preparation (P2PP) tool. Additionally, analysing the known orbital solution before-hand, to determine RV constraints will also help identify the best time to observe, if observations from separate periods will be required or, if an optimally separated companion differential is even feasible.

5.3 Other techniques

We note that there are many other disentangling techniques to separate mixed spectra of binary systems, (e.g. **?**). These require more than two observations, with $n + 1$ observations used to set up a system of linear equations to solve for n spectral components (e.g. **???**). These methods are ideal for many well spaced observations. For example the ideal situation for the SVD method of **?** is homogeneous samples of at least half the period, to identify the moving spectral components. Recently the cross-correlation and maximum-likelihood techniques (e.g. **??**) have been successful in detecting the faint companion spectra of giant planets using several spectra with very-high SNR (>2000) obtained with longer observational time (1–3 hours) each taken across the full orbital range. The few, short and insufficiently separated observations we analyse here are not suitable to apply these advanced techniques and are beyond what we have attempted here.

The recent work of **?** use a PCA technique to correct for the telluric spectra by applying it to several (number not specified) AB nodding pairs over their 60–180 minute integration time. We are uncertain if this technique would work as effectively on our observations due to our shorter integration time (24 minutes) would have less telluric variation present across the 8 nod spectra. A recent comparison of three telluric correction methods, *Molecfit* and TelFit

[8] https://github.com/chrysante87/pyterpol
[9] https://github.com/iancze/Starfish
[10] Recently pushed to around May 2020

and TAPAS to the standard star method by (**?**, in prep.) found that all three synthetic models preform better at correcting for atmospheric H_2O compared to the standard star method with *Molecfit*, being a more complete tool, preforming slightly better over TAPAS.

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REFERENCES

APPENDIX A: DIRECT SUBTRACTION METHOD

Here we outline the direct subtraction method used, which is similar to previous works (**??**). Assuming that the instrumental profile and atmospheric absorption are dealt with appropriately, the spectrum received from the host-companion pair is given by the superposition of two spectral components (J_1, J_2), where $\lambda - v$ represents the Doppler shift $\lambda(1 - v/c)$ by velocity v.

$$\mathrm{I}(\lambda) = \mathrm{J}_1(\lambda - v_1) + \mathrm{J}_2(\lambda - v_2) \tag{A1}$$

or shifted to the host stars rest frame,

$$\mathrm{I}(\lambda + v_1) = \mathrm{J}_1(\lambda) + \mathrm{J}_2(\lambda - v_2 + v_1) \tag{A2}$$

Here, the subscripts 1 and 2 indicate the spectrum of the host and companion respectively, and λ represents the wavelength of the spectra. The spectra of the host star (J_1) is removed though the mutual subtraction of the host spectrum from two separate observations, denoted with subscripts a and b, correcting for RV motion of the host star.

$$\begin{aligned} S(\lambda) &= \mathrm{I}_a(\lambda + v_{1a}) - \mathrm{I}_b(\lambda + v_{1b}) \\ &= \mathrm{J}_2(\lambda - v_{2a} + v_{1a}) - \mathrm{J}_2(\lambda - v_{2b} + v_{1b}) \\ S(\lambda + v_{2a}) &= \mathrm{J}_2(\lambda) - \mathrm{J}_2(\lambda + \Delta RV) \end{aligned} \tag{A3}$$

where,

$$\Delta RV = v_{2a} - v_{2b} - v_{1a} + v_{1b} \tag{A4}$$

is the actual RV difference (ΔRV) between the two companion spectra.

The resulting differential spectra, dubbed *s-profile* by **?**, is composed of just the companion spectra, shifted and subtracted from itself.

From binary dynamics (e.g. **?**) the RV amplitudes of the host and companion are related through the mass ratio, q, while having an opposite sign.

$$v_2 = -\mathrm{q} * v_1 \tag{A5}$$

This equation was used to calculate the expected companion RV for each observation in Table **??**.

We can simplify Eq. A4 by expressing it in terms of the host RV and mass ratio,

$$\begin{aligned} k &= -qv_{1a} + qv_{1b} + v_{1a} - v_{1b} \\ &= (1 - \mathrm{q})(v_{1a} - v_{1b}). \end{aligned} \tag{A6}$$

If we are able to determine the ΔRV between the the companion spectra, k, from the s-profile (see **?**) then we can determine the mass ratio of the system, q, thereby constraining the mass of the companion. The values v_{1a} and v_{1b} are calculated from the orbital parameters of the system and are the same values already used to shift and mutually cancel the host spectrum.

This method is very similar to **?** except that they focus on M-dwarfs host stars with the observations taken at the extrema, in which the companion lines are well separated.

A1 Estimating parameters of observations

To estimate the differential amplitude signal we estimated the expected parameters of the impact of the

A2 Results of spectral differential analysis

We applied the spectral differential procedure outlined above on the wavelength-calibrated and telluric-corrected CRIRES observations. The spectra were corrected for Earth's barycentric RV using the *helcor* PyAstronomy[12] function ported from the REDUCE IDL package (See **?**) and Doppler shifted to the rest frame velocity of the system. The spectra were then subtracted from each other and analysed as described above.

It is necessary to have a consistent instrumental setup **?**, to avoid introducing extra instrumental effects (e.g. slit-width and/or filters) into the spectral differentials and to always observe the same wavelength range and maximize the information to be extracted. In our case, the second observation of HD 202206 and fourth of HD 30501 were taken with different filters compared to the other observations. Therefore, these two observations could not be used for this differential analysis. As noted in (**?**), any spectral differences in the filters would add extra unknown signal/noise making it harder to disentangle the faint spectral differences.

Figure A1. (Top) A reduced CRIRES observation of HD 30501 (blue) for detector 1 between 2112–2124 along with the tapas telluric absorption model (orange dashed) used for the wavelength calibration and telluric correction. (Middle) The telluric corrected spectra. (Bottom) (blue) Differential spectra for HD 30501 between observations 1 and 3. (orange dashed) Simulated "perfect" differential using PHOENIX-ACES spectra with parameters T_{eff} = 2500, logg = 5.0, and = 0.0, with the same ΔRV as the observations. The shaded regions indicate where the telluric green and host star blue spectra are > 4% deep.

[11] PyAstronomy can be found at https://pyastronomy.readthedocs.io

[12] https://pyastronomy.readthedocs.io

We performed the differential analysis for all targets but only show our most favourable case here, HD 30501, because it is the second largest companion in our sample at 90 M_{Jup} and also has the second largest RV separation between observations. The differential spectra recovered for HD 30501 is shown at the bottom panel of Fig. A1. The presence of deep (> 4%) stellar and telluric lines in the original spectrum is shaded by the blue and green regions respectively. This indicates that the features of the differential spectrum near these shaded regions are likely due to imperfect telluric correction and host mutual cancellation. The mutual cancellation of the stellar host works well for the ∼40% deep line near 2117, being completely removed, but it does not do so well for the smaller ∼10% deep line around 2121.5. The residual for the large ∼40% deep telluric line near 2118.5 is quite prominent. There is also a wider residual due to three neighbouring lines ∼10% deep around 2120 which cause features in the differential spectrum. One possible explanation is that the continuum normalization near 2120 was influenced by this grouping of lines.

To understand the observed differential signal we simulated a differential spectrum of HD 30501 using a synthetic PHOENIX-ACES spectra with parameters $T_{\rm eff}$ = 2500, logg = 5.0, and = 0.0, with a RV offset estimated from the observation times. These parameters represent an estimated companion $T_{\rm eff}$ with the metallicity and logg similar to the host (closest grid model). The model spectra were convolved to R = 50 000, continuum normalized and scaled by the estimated flux ratio of the companion. We do not include any synthetic host or telluric spectra and as such simulate the differential result of a "perfect" host cancellation with no telluric contamination present. This is the ideal-case scenario, and we stress that it is impossible to simulate the effect of improper telluric correction in a meaningful way. When comparing the simulated and observed differential in the bottom panel of Fig. A1, there is a striking amplitude difference. The orange-dashed line of the simulated differential spectrum amplitude is of a much smaller scale than the observed differential. This demonstrates that the amplitude of the differential signal we are trying to detect is much smaller than the residuals created by this differential technique.

The amplitude of the differential signal is lower than we expected due to the very low ΔRV between the observation pairs. The maximum ΔRV between observation pairs, for the observations investigated in this work, are provided in Table **??**. There is no ΔRV for HD 4747 as there was only a single observation. We also provide the phase coverage for our targets. We calculate this as the ratio of time between the observed pairs and the orbital period, and show that the fraction of the orbit covered is very small, all except one covering less than 1 percent of the orbit.

In our best case, HD 30501, the ΔRV of the companion between observations is 1.346 km s^{-1}. For comparison, a single Gaussian absorption line, to be shifted by $\Delta\lambda$ = FWHM would need a ΔRV of $v_{\rm FWHM} = c/R =$ ∼6 km s^{-1}. Since the ΔRV are shifted by a smaller value than the FWHM, the spectral lines of the reconstruction mutually cancel themselves, diminishing the amplitude of the differential signal significantly. As the companion spectra are already faint (with a flux ratio at the percent level) the differential signal is not detectable within these observations and noise level. When the ΔRV of the companion is smaller than the FWHM of a line there is a mutual subtraction of the companion spectra, diminishing the detected amplitude of the differential signal, and removing the ability to detect the companions using this method. Observations need to be spaced further apart in time/phase to achieve a larger ΔRV separation and increase the amplitude of the differential. Of course once there is a separation there will be complex interactions between neighbouring lines that need to be accounted for.

Figure A2. Simulated relative amplitude of differential spectra at different RV separations revealing the diminished amplitude at small orbital separations. The solid blue line shows the maximum relative amplitude of the differential signal (from a shifted copy of itself) of a PHOENIX-ACES spectrum with $T_{\rm eff}$ = 2500, logg = 5.0, = 0.0, in the wavelength region 2110–2123. The maximum difference is normalized by the average amplitude between 7–10 km s^{-1}, representing a complete line separation. The orange (dashed) and green (dot-dashed) lines represent the relative amplitude of a differential spectrum for a single Gaussian and single Lorentzian absorption line respectively, each with a unitary amplitude and a FWHM = λ/R. Beyond the FWHM lines the difference of the synthetic spectrum becomes more complicated due to the interaction of neighbouring lines. The solid vertical lines indicate the estimated companion ΔRV in these observations while the dashed vertical lines indicate the RV corresponding to the FWHM at this wavelength and resolution. Beyond the FWHM RV the synthetic spectrum becomes complicated due to the interaction of neighbouring lines.

A3 Relative differential amplitude

To probe this issue further we investigated how the amplitude of the differential signal changed with ΔRV. Taking the same PHOENIX-ACES spectra ($T_{\rm eff}$ = 2500, logg = 5.0, = 0.0), we computed differential spectra for a range of ΔRVs between ±10 km s^{-1}. Figure A2 shows how the relative amplitude of the differential spectrum in the wavelength range 2110–2123 (detector 1 of our CRIRES observations) changed as the spectra are offset. At each RV step we take the maximum absolute differential value found. These are then normalized by the median value outside of the line FWHM (dashed vertical lines), between ±(7 − 10) km s^{-1}, to give a relative differential amplitude, independent from the depth of a specific line. For comparison we also show the relative amplitude of the differential spectrum for a single Gaussian and Lorentzian line when Doppler shifted by the same ΔRVs. The shape of these results is also consistent with the analytical form of the differential spectra **?**, eqn. A.1.

At a ΔRV difference of zero, spectral lines of the companion completely cancel each other out, resulting in zero amplitude. As the RV separation increases, the individual lines stop cancelling themselves until a maximum differential amplitude is achieved when the lines are fully separated from themselves (equal to the line depth).

The two solid vertical lines in Fig. A2 indicate the estimated ΔRV=1.346 km s^{-1} separation for our best target, HD 30501 from Table **??**, given known orbital parameters and the observation times. This shows that our differentials have severely reduced amplitude, < 20% relative to well separated individual lines. As the companion spectra are faint and in combination with a host star at 1% flux ratio the >80% extra reduction in signal amplitude makes this detection impossible with these observations.

In the synthetic spectrum (and of course real spectra) neighbouring spectral lines begin to interfere, leading to an impact on the measured relative amplitude. We suspect that the interaction of neighbouring lines is one possible cause for the difference between the theoretical and simulated shape between 2 and 6 km s^{-1}. Beyond the RV range present the amplitude becomes complicated due to neighbouring line interaction, but as the ΔRV for all our spectra fall well short of this region we did not investigate this further.

APPENDIX B: ESTIMATING COMPANION PARAMETERS

In this appendix we detail how we calculate RV of the components and determine the flux ratio of our targets given their literature masses M_2 or $M_2 \sin i$. We refer to these calculations as estimates as in some cases we only have the companions minimum mass.

B1 RV equation

We use the Keplerian orbit RV equation to estimate the RV of the host and companions at the time of each observation, t:

$$RV = K[\cos(\nu(t) + \omega) + e\cos(\omega)] \tag{B1}$$

Here, K is the *semi-major amplitude*, ν is the *true anomaly*, e is the orbital *eccentricity*, and *omega* is the *argument of periastron*. The true anomaly is not only a function time, t, but also the orbital period P, the *time of periastron passage*, T_0, and eccentricity. The literature parameters for each target are provided in Table **??**. To determine the RV of the companion we transformed the RV semi-amplitude of the host K_1 into the semi-amplitude for the companion K_2 using the mass ratio,

$$q = \mathrm{M_2/M_1} = \mathrm{K_1/K_2} \tag{B2}$$

We note that for the targets in which only the minimum mass ($\mathrm{M}\sin i$) is known, this equation will indicate the maximum K_2 value for the companion. The estimated K_2 for each companion is provided in Table **??** while the RV for both components at the time of each observation is provided in Table **??**.

The error on estimated RV values, shown in Fig. **??** is calculated by applying the general error propagation formula (**?**) and using the errors on the orbital parameters. For a function, f, with errors on the inputs δx, δy etc., it follows:

$$f = f(x, y, z, \ldots) \tag{B3}$$

$$\delta f = \sqrt{\left(\frac{\partial f}{\partial x}\delta x\right)^2 + \left(\frac{\partial f}{\partial y}\delta y\right)^2 + \left(\frac{\partial f}{\partial z}\delta z\right)^2 + \ldots} \tag{B4}$$

B2 Estimating Companion-host Flux ratio

The companion-host flux or contrast ratio of the systems are calculated using $\frac{F_2}{F_1} \approx 2.512^{m_1 - m_2}$, where m_1 and m_2 are the magnitude of the host and companion respectively. The noise ratio between the host and companion is also calculated using $N_2/N_1 = \sqrt{2}\times\sqrt{F_1/F_2}$. For this work we specifically use the magnitudes in K-band. The magnitudes of the hosts, m_1, are obtained from SIMBAD (**?**) while the magnitudes for the companions, m_2, are estimated from stellar evolution models **??**. The stellar evolution models are interpolated to the companion mass (or $M_2 \sin i$) and system age, then the magnitude in the K-band magnitudes extracted. The K-band magnitudes, used for the host and companions along with the calculated flux ratio for each target is given in Table **??**. For the companions in which only the minimum mass is known then the flux-ratio will be the lower limit, or worst case scenario.

These evolution models also provide a first estimate of the companions other properties such as $T_{\rm eff}$, logg, $R/R_\odot$ and the magnitude in many other wavelength bands. The companion $T_{\rm eff}$ and logg values specifically were utilized to influence the selection of synthetic model grids to perform the χ^2 analysis over in Section **??**.

A simple tool[13] was created to calculate the flux ratio using the (**??**) evolution tables. Given the host star name, the companion mass and a stellar age it interpolates the available Baraffe tables to the companion mass and age specified. The host's name is used to query the SIMBAD database to obtain stellar properties, specifically magnitudes, to calculate the flux ratios. It can also work in reverse to estimate a companion mass when provided with a flux ratio.

[13] Available at https://github.com/jason-neal/baraffe_tables

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