Problem 15

A system consists of a ball of mass M_2 and a uniform rod of mass M_1 and length d. The rod is attached to a horizontal frictionless table by a pivot at point P and initially rotates at an angular speed ω, as shown. The rotational inertia of the rod about point P is $\frac{1}{3}M_1d^2$. The rod strikes the ball, which is initially at rest. As a result of this collision, the rod is stopped and the ball moves in the direction shown. Express all answers in terms of M_1, M_2, ω, d, and fundamental constants.

a) Derive an expression for the angular momentum of the rod about point P before the collision.

b) Derive an expression for the speed v of the ball after the collision.

c) Assuming that this collision is elastic, calculate the numerical value of the ratio M_1/M_2.

d) A new ball with the same mass M_1 as the rod is now placed a distance x from the pivot, as shown. Again assuming the collision is elastic, for what value of x will the rod stop moving after hitting the ball?

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$$\left(\frac{1}{3}M_1 d^2\right)(\omega) = M_2 v d$$

$$\boxed{v = \frac{M_1 d \omega}{3M_2}}$$

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K_{\text{rod}} &= K_{\text{ball}} \\
\frac{1}{2} M_1 v_{\text{rod}}^2 &= \frac{1}{2} M_2 v_{\text{ball}}^2 \\
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$$\boxed{\frac{M_1}{M_2} = 3}$$

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$$\begin{aligned} E_{\mathsf{i}} &= E_{\mathsf{f}} \\ \frac{1}{2}I\omega^2 &= \frac{1}{2}Mv^2 \\ \left(\frac{1}{3}Md^2\right)\omega^2 &= Mv^2 \end{aligned}$$

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$$\boxed{x = \frac{d}{\sqrt{3}}}$$