Log. Reg. Cost fxn

$$\hat{y} = \sigma(w^T x + b) \quad \text{where } \sigma(z) = \frac{1}{1+e^{-z}}.$$

likelihood fxn:

$$\left.\begin{array}{ll} \text{if } y = 1: & p(y|x) = \hat{y} \\ \text{if } y = 0: & p(y|x) = 1 - \hat{y} \end{array}\right\} p(y|x)$$

$$p(y|x) = \hat{y}^{\,y} (1-\hat{y})^{(1-y)}$$

$$\text{if } y = 1: \quad \hat{y}^{\,1} \underbrace{(1-\hat{y})^{(1-1)}}_{1} = \hat{y}$$

$$\text{if } y = 0: \quad \underbrace{\hat{y}^{\,0}}_{1} (1-\hat{y})^{(1-0)} = 1 - \hat{y}$$

Take the log of the above fxn. Log likelihood

$$\log(p(y|x)) = y \log \hat{y} + (1-y) \log(1-\hat{y})$$

$$= -\underbrace{\mathcal{L}(\hat{y}, y)}_{\text{MINIMIZE this = maximize log likelihood}}$$

$p(\text{labels in training set}) = \prod_{i=1}^{m} p(y_i | x_i)$ assuming i.i.d

Maximizing this probability → (use the log)

$$= \sum_{i=1}^{m} \underbrace{\log p(y^{(i)} | x^{(i)})}_{-\mathcal{L}(\hat{y}^{(i)}, y^{(i)})}$$

MLE

Maximum Likelihood

Estimate

$$= -\sum_{i=1}^{m} \mathcal{L}(\hat{y}^{(i)}, y^{(i)})$$

Cost : $J(w,b) = \frac{1}{m} \sum_{i=1}^{m} \mathcal{L}(\hat{y}^{(i)}, y^{(i)})$

Minimizing

$\equiv$ MLE problem