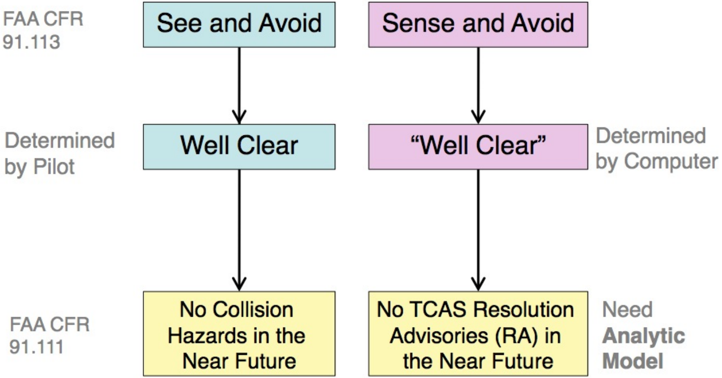
A Family of Well-Clear Boundary Models for the Integration of Unmanned Aircraft Systems in the National Airspace System

C. A. Muñoz   A. J. Narkawicz   M.C. Consiglio   J.M. Upchurch

NASA Langley Research Center
in Support of
the UAS in the NAS Project

March 24, 2015

See and Avoid vs. Sense and Avoid

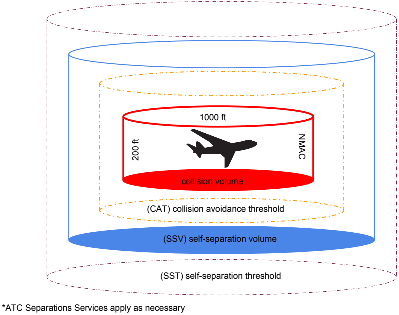


A Motivation for a Formal Definition of Well Clear

- The FAA SAA Workshop for UAS defines **sense and avoid** as: "the capability of a UAS to remain **well clear** from and avoid collisions with other airborne traffic."
- How will a UAS determine if it is **well clear** from other airborne traffic?
- In the absence of an on-board human pilot with the **experience** and **judgement** to determine well clear, a formal definition is needed to provide guidance to a ground pilot or possibly an automated algorithm.
- This definition should be more **conservative** than TCAS, a system intended to be the last resort in collision avoidance, so as to be compatible.
- NASA has examined and developed several formal definitions which considered to be a **family** of **well-clear boundary models**.

The Approach

A key characteristic of NASA's concept is that the self-separation threshold is a conservative extension of the collision avoidance threshold defined by TCAS.[1]



Volumes and thresholds are shown as cylinders for illustrative purposes only. In general, these shapes are irregular, with the exception of the collision volume.

[1] Consiglio, Chamberlain, Muñoz, and Hoffler, ICAS, 2012

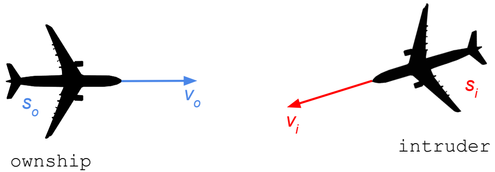
Interoperability with TCAS RA Logic

- TCAS is a family of airborne devices that are designed to reduce the risk of mid-air collisions between aircraft equipped with operating transponders. TCAS II, the current generation of TCAS devices, is mandated in the US for aircraft with greater than 30 seats or a maximum takeoff weight greater than 33,000 lbs,
- To ensure compatibility of NASA's self-separation concept and TCAS, the mathematical definition of the volume determined by the SST is considered to be a conservative extension of the core TCAS II Resolution Advisory logic which checks against independent horizontal and vertical time and distance threshold.[2]

[2] Muñoz, Narkawicz, and Chamberlain, GNC, 2013.

Assumptions

- Two aircraft, the *ownship* and *intruder*,
- Accurate aircraft state information is available for both, i.e.,
  - Horizontal positions $\mathbf{s}_o, \mathbf{s}_i$ and velocities $\mathbf{v}_o$, $\mathbf{v}_i$
  - Altitudes s_{oz}, s_{iz} and vertical speeds v_{oz}, v_{iz}
  - Relative position $\mathbf{s} = \mathbf{s}_o - \mathbf{s}_i$ and velocity $\mathbf{v} = \mathbf{v}_o - \mathbf{v}_i$
  - Relative altitude $s_z = s_{oz} - s_{iz}$ and vertical speed $v_z = v_{oz} - v_{iz}$
- Prediction at a particular time instant of a future well-clear violation is based on a straight-line trajectory from that time instant, i.e., constant velocity is assumed.



A Family of Well-Clear Boundary Models

Definition of the Well Clear Volume

$$WCV_{t_{\text{var}}}(\mathbf{s}, s_z, \mathbf{v}, v_z) \equiv \texttt{Horizontal\_WCV}_{t_{\text{var}}}(\mathbf{s}, \mathbf{v}) \text{ and } \texttt{Vertical\_WCV}(s_z, v_z), \tag{1}$$

Anywhere inside the volume determined by this function, the aircraft are **not well clear**.

$$\begin{aligned}
\texttt{Horizontal_WCV}_{t_{\text{var}}}(\mathbf{s}, \mathbf{v}) &\equiv \|\mathbf{s}\| \leq \texttt{DTHR} \text{ or} \\
&\quad (d_{\text{cpa}}(\mathbf{s}, \mathbf{v}) \leq \texttt{DTHR} \text{ and } 0 \leq t_{\text{var}}(\mathbf{s}, \mathbf{v}) \leq \texttt{TTHR}), \\
\texttt{Vertical_WCV}(s_z, v_z) &\equiv |s_z| \leq \texttt{ZTHR} \text{ or } 0 \leq t_{\text{coa}}(s_z, v_z) \leq \texttt{TCOA}.
\end{aligned}$$

$$\begin{aligned}
d_{\text{cpa}}(\mathbf{s}, \mathbf{v}) &\equiv r(t_{\text{cpa}}(\mathbf{s}, \mathbf{v})) = \|\mathbf{s} + t_{\text{cpa}}(\mathbf{s}, \mathbf{v})\mathbf{v}\|, \\
\|s\| &\equiv \sqrt{\mathbf{s}^2} = \sqrt{\mathbf{s} \cdot \mathbf{s}} \\
|s_z| &\equiv s_{oz} - s_{iz}
\end{aligned}$$

A Family of Well-Clear Boundary Models

Definition of the Well Clear Volume

$$WCV_{t_{\text{var}}}(\mathbf{s}, s_z, \mathbf{v}, v_z) \equiv \texttt{Horizontal\_WCV}_{t_{\text{var}}}(\mathbf{s}, \mathbf{v}) \text{ and } \texttt{Vertical\_WCV}(s_z, v_z), \tag{1}$$

Anywhere inside the volume determined by this function, the aircraft are **not well clear**.

The function $t_{\text{var}}(\mathbf{s}, \mathbf{v})$ is the only change between the models

$$\begin{aligned}
\texttt{Horizontal_WCV}_{t_{\text{var}}}(\mathbf{s}, \mathbf{v}) &\equiv \|\mathbf{s}\| \leq \texttt{DTHR} \text{ or} \\
&\quad (d_{\text{cpa}}(\mathbf{s}, \mathbf{v}) \leq \texttt{DTHR} \text{ and } 0 \leq t_{\text{var}}(\mathbf{s}, \mathbf{v}) \leq \texttt{TTHR}), \\
\texttt{Vertical_WCV}(s_z, v_z) &\equiv |s_z| \leq \texttt{ZTHR} \text{ or } 0 \leq t_{\text{coa}}(s_z, v_z) \leq \texttt{TCOA}.
\end{aligned}$$

$$\begin{aligned}
d_{\text{cpa}}(\mathbf{s}, \mathbf{v}) &\equiv r(t_{\text{cpa}}(\mathbf{s}, \mathbf{v})) = \|\mathbf{s} + t_{\text{cpa}}(\mathbf{s}, \mathbf{v})\mathbf{v}\|, \\
\|s\| &\equiv \sqrt{\mathbf{s}^2} = \sqrt{\mathbf{s} \cdot \mathbf{s}} \\
|s_z| &\equiv s_{oz} - s_{iz}
\end{aligned}$$

Parameter: Time Variables and Thresholds

Four choices for $t_{\text{var}}(\mathbf{s}, \mathbf{v})$:

$$\tau(\mathbf{s}, \mathbf{v}) \equiv \begin{cases} -\frac{\mathbf{s}^2}{\mathbf{s}\cdot\mathbf{v}} & \text{if } \mathbf{s}\cdot\mathbf{v} < 0, \\ -1 & \text{otherwise}, \end{cases} \tag{2}$$

$$t_{\text{cpa}}(\mathbf{s}, \mathbf{v}) \equiv \begin{cases} -\frac{\mathbf{s}\cdot\mathbf{v}}{\mathbf{v}^2} & \text{if } \mathbf{v} \neq \mathbf{0}, \\ 0 & \text{otherwise}, \end{cases} \tag{3}$$

$$\tau_{\text{mod}}(\mathbf{s}, \mathbf{v}) \equiv \begin{cases} \frac{\mathtt{DTHR}^2 - \mathbf{s}^2}{\mathbf{s}\cdot\mathbf{v}} & \text{if } \mathbf{s}\cdot\mathbf{v} < 0, \\ -1 & \text{otherwise}, \end{cases} \tag{4}$$

$$t_{\text{ep}}(\mathbf{s}, \mathbf{v}) \equiv \begin{cases} \Theta(\mathbf{s}, \mathbf{v}, \mathtt{DTHR}, -1) & \text{if } \mathbf{s}\cdot\mathbf{v} < 0 \text{ and } \Delta(\mathbf{s}, \mathbf{v}, \mathtt{DTHR}) \geq 0, \\ -1 & \text{otherwise}, \end{cases} \tag{5}$$

where

$$\Theta(\mathbf{s}, \mathbf{v}, D, \epsilon) \equiv \frac{-\mathbf{s}\cdot\mathbf{v} + \epsilon\sqrt{\Delta(\mathbf{s}, \mathbf{v}, D)}}{\mathbf{v}^2},$$

$$\Delta(\mathbf{s}, \mathbf{v}, D) \equiv D^2\mathbf{v}^2 - (\mathbf{s}\cdot\mathbf{v}^{\perp})^2.$$

All four models use the same vertical time variable to compare to TCOA:

$$t_{\text{coa}}(s_z, v_z) \equiv \begin{cases} -\frac{s_z}{v_z} & \text{if } s_z v_z < 0, \\ -1 & \text{otherwise}. \end{cases} \tag{6}$$

Parameter: Time Variables and Thresholds

Four choices for $t_{\text{var}}(\mathbf{s}, \mathbf{v})$:

$$\tau(\mathbf{s}, \mathbf{v}) \equiv \begin{cases} -\frac{\mathbf{s}^2}{\mathbf{s}\cdot\mathbf{v}} & \text{if } \mathbf{s}\cdot\mathbf{v} < 0, \\ -1 & \text{otherwise,} \end{cases} \tag{2}$$

$$t_{\text{cpa}}(\mathbf{s}, \mathbf{v}) \equiv \begin{cases} -\frac{\mathbf{s}\cdot\mathbf{v}}{\mathbf{v}^2} & \text{if } \mathbf{v} \neq \mathbf{0}, \\ 0 & \text{otherwise,} \end{cases} \tag{3}$$

$$\tau_{\text{mod}}(\mathbf{s}, \mathbf{v}) \equiv \begin{cases} \frac{\mathtt{DTHR}^2 - \mathbf{s}^2}{\mathbf{s}\cdot\mathbf{v}} & \text{if } \mathbf{s}\cdot\mathbf{v} < 0, \\ -1 & \text{otherwise,} \end{cases} \tag{4}$$

$$t_{\text{ep}}(\mathbf{s}, \mathbf{v}) \equiv \begin{cases} \Theta(\mathbf{s}, \mathbf{v}, \mathtt{DTHR}, -1) & \text{if } \mathbf{s}\cdot\mathbf{v} < 0 \text{ and } \Delta(\mathbf{s}, \mathbf{v}, \mathtt{DTHR}) \geq 0, \\ -1 & \text{otherwise,} \end{cases} \tag{5}$$

where

$$\Theta(\mathbf{s}, \mathbf{v}, D, \epsilon) \equiv \frac{-\mathbf{s}\cdot\mathbf{v} + \epsilon\sqrt{\Delta(\mathbf{s}, \mathbf{v}, D)}}{\mathbf{v}^2},$$

$$\Delta(\mathbf{s}, \mathbf{v}, D) \equiv D^2\mathbf{v}^2 - (\mathbf{s}\cdot\mathbf{v}^{\perp})^2.$$

The four well clear volumes are in order of increasing containment

All four models use the same vertical time variable to compare to `TCOA`:

$$t_{\text{coa}}(s_z, v_z) \equiv \begin{cases} -\frac{s_z}{v_z} & \text{if } s_z v_z < 0, \\ -1 & \text{otherwise.} \end{cases} \tag{6}$$

Parameter: Time Variables and Thresholds, continued

$$\mathtt{Horizontal\_WCV}_{t_{\mathsf{var}}}(\mathbf{s},\mathbf{v}) \equiv \|\mathbf{s}\| \leq \mathtt{DTHR} \text{ or } (d_{\mathsf{cpa}}(\mathbf{s},\mathbf{v}) \leq \mathtt{DTHR} \text{ and } 0 \leq t_{\mathsf{var}}(\mathbf{s},\mathbf{v}) \leq \mathtt{TTHR})$$

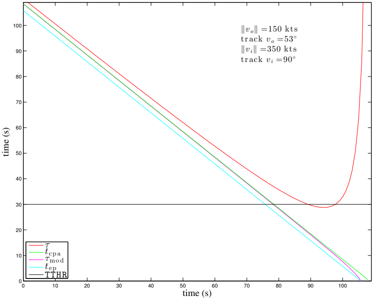


Figure : The 4 well clear volumes are in order of increasing containment

Conceptualizing the Well-Clear Boundary

- Sweep the ownship trajectory around 360° while holding v_{oz} constant,
- a boundary in three dimensions is determined by calling $WCV_{t_{\mathrm{var}}}$ along each trajectory,
- project the resulting surface into the horizontal plane containing $\mathbf{s}_o$.

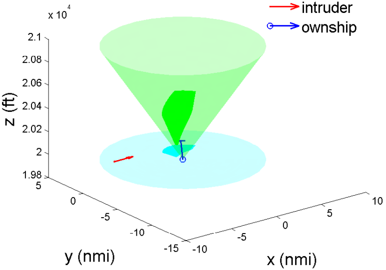
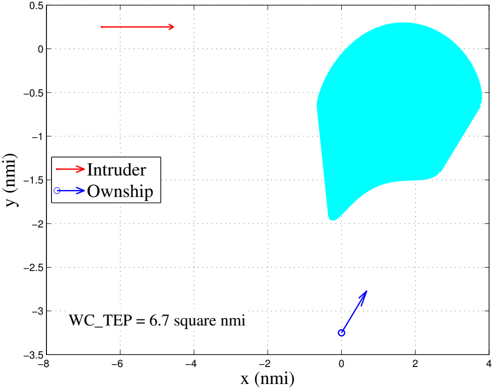


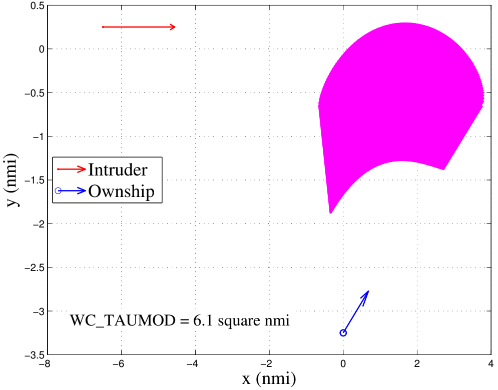
Figure : Illustration of a 3-dimensional encounter projected into 2 dimensions

WC_TEP



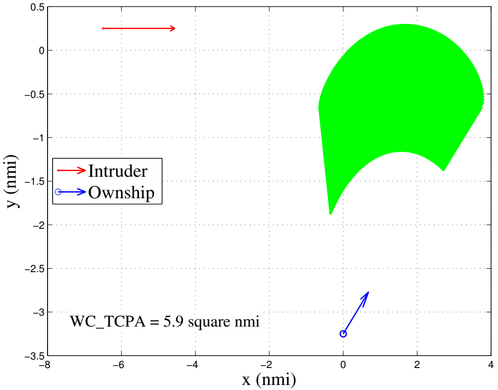
$$WCV_{t_{ep}}(\mathbf{s}, s_z, \mathbf{v}, v_z) \equiv \texttt{Horizontal\_WCV}_{t_{ep}}(\mathbf{s}, \mathbf{v}) \text{ and } \texttt{Vertical\_WCV}(s_z, v_z)$$

WC_TAUMOD



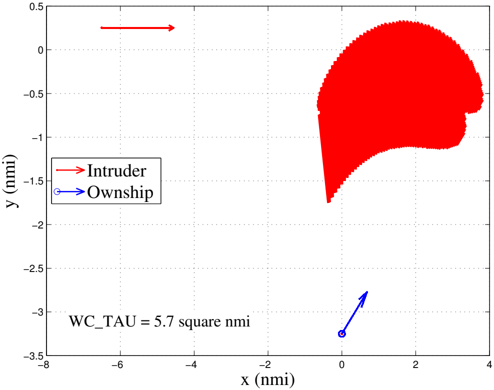
$$WCV_{\tau_{\text{mod}}}(\mathbf{s}, s_z, \mathbf{v}, v_z) \equiv \texttt{Horizontal\_WCV}_{\tau_{\text{mod}}}(\mathbf{s}, \mathbf{v}) \text{ and } \texttt{Vertical\_WCV}(s_z, v_z)$$

WC_TCPA



$$WCV_{t_{\text{cpa}}}(\mathbf{s}, s_z, \mathbf{v}, v_z) \equiv \texttt{Horizontal\_WCV}_{t_{\text{cpa}}}(\mathbf{s}, \mathbf{v}) \text{ and } \texttt{Vertical\_WCV}(s_z, v_z)$$

WC_TAU



$$WCV_\tau(\mathbf{s}, s_z, \mathbf{v}, v_z) \equiv \texttt{Horizontal\_WCV}_\tau(\mathbf{s}, \mathbf{v}) \texttt{ and } \texttt{Vertical\_WCV}(s_z, v_z)$$

Properties of Interest: Symmetry

Definition (Symmetry)

A well-clear boundary model specified by $WCV_{t_{var}}$, for a given time variable t_{var}, is symmetric *if and only if*

$$WCV_{t_{var}}(\mathbf{s}, s_z, \mathbf{v}, v_z) = WCV_{t_{var}}(-\mathbf{s}, -s_z, -\mathbf{v}, -v_z).$$

The ownship and intruder agree on whether they are well clear.

Theorem (Symmetry)

The well-clear boundary models WC_TAU, WC_TAUMOD, WC_TCPA, and WC_TEP are symmetric for any choice of threshold values `DTHR`, `TTHR`, `ZTHR`, and `TCOA`.

Properties of Interest: Inclusion

Theorem (Inclusion)

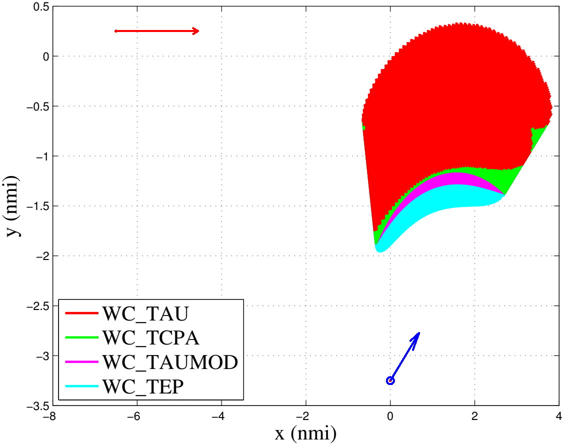
For all $\mathbf{s}, s_z, \mathbf{v}, v_z$ *and choice of threshold values* $\mathtt{DTHR}$, $\mathtt{TTHR}$, $\mathtt{ZTHR}$, *and* $\mathtt{TCOA}$, *the following implications hold*

(i) $WCV_{\tau}(\mathbf{s}, s_z, \mathbf{v}, v_z) \implies WCV_{t_{cpa}}(\mathbf{s}, s_z, \mathbf{v}, v_z)$,

(ii) $WCV_{t_{cpa}}(\mathbf{s}, s_z, \mathbf{v}, v_z) \implies WCV_{\tau_{mod}}(\mathbf{s}, s_z, \mathbf{v}, v_z)$, *and*

(iii) $WCV_{\tau_{mod}}(\mathbf{s}, s_z, \mathbf{v}, v_z) \implies WCV_{t_{ep}}(\mathbf{s}, s_z, \mathbf{v}, v_z)$.

Properties of Interest: Inclusion, continued



$$WCV_{t_{\mathrm{var}}}(\mathbf{s}, s_z, \mathbf{v}, v_z) \equiv \texttt{Horizontal\_WCV}_{t_{\mathrm{var}}}(\mathbf{s}, \mathbf{v}) \text{ and } \texttt{Vertical\_WCV}(s_z, v_z)$$

Properties of Interest: Local Convexity

A well-clear boundary model specified by $WCV_{t_{\text{var}}}$, for a given time variable t_{var}, is *locally convex* if and only if there are no times $0 \leq t_1 \leq t_2 \leq t_3 \leq T$ such that

1. the aircraft are not well clear at time t_1, i.e., $WCV_{t_{\text{var}}}(\mathbf{s} + t_1\mathbf{v}, s_z + t_1 v_z, \mathbf{v}, v_z)$,
2. the aircraft are well clear at time t_2, i.e., $\neg WCV_{t_{\text{var}}}(\mathbf{s} + t_2\mathbf{v}, s_z + t_2 v_z, \mathbf{v}, v_z)$, and
3. the aircraft not well clear at time t_3, i.e., $WCV_{t_{\text{var}}}(\mathbf{s} + t_3\mathbf{v}, s_z + t_3 v_z, \mathbf{v}, v_z)$.

Local Convexity: Along a linear trajectory, the aicraft does not lose well clear, gain it back, and lose it again.

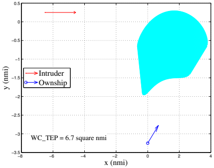


Figure : WC_TEP

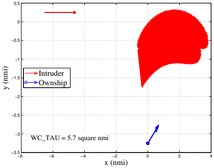


Figure : WC_TAU

Properties of Interest: Local Convexity, continued

Theorem

For any choice of threshold values, the well-clear boundary models WC_TCPA, WC_TAUMOD, and WC_TEP are locally convex.

Theorem

For some choices of threshold values, the well-clear boundary model WC_TAU is not locally convex.

Conclusion

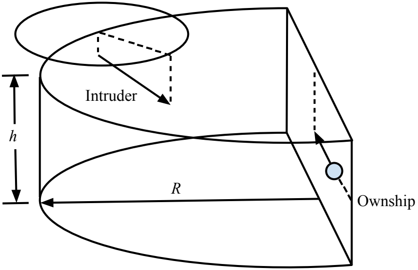
- A formal definition of *well clear* is motivated by the need for UAS to operate safely in the presence of other aircraft in the airspace
- A family of well-clear boundary models is introduced which are extensions of the TCAS II RA logic
- Characterizing concepts for these models are:
  - Symmetry
  - Inclusion
  - Local convexity
- $\mathrm{WC\_TAU}$ has instances of non-local convexity and is the least conservative model
- $\mathrm{WC\_TEP}$ is the most conservative model

References

The End

Questions?

Encounter Space for Randomly-Generated Trajectories



- Ownship position, and horizontal direction fixed,
- Ownship and intruder horizontal velocity randomly chosen 849 velocities,
- Intruder horizontal position chosen from $\mathcal{U}[\pi, 2\pi]$,
- Intruder vertical position chosen from $\mathcal{N}(s_{oz}, h/6)$,
- Intruder horizontal velocity direction chosen from $\mathcal{U}[0, 2\pi]$,
- Intruder vertical velocity chosen from $\mathcal{N}(0, v_{iz,\max})$.

Example Encounters of Interest

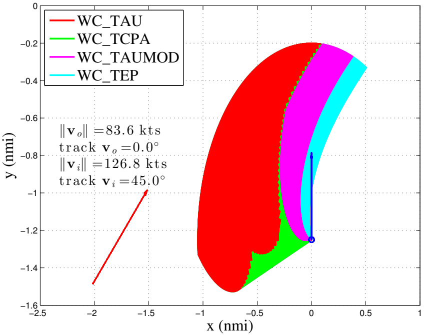


Figure : Large difference in t_{in}

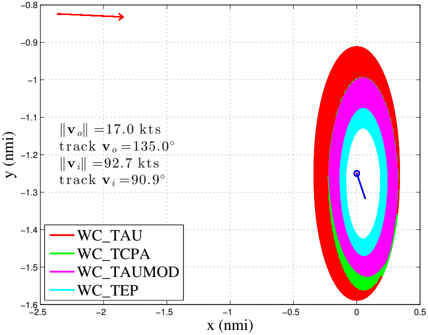


Figure : Disagreement in $WCV_{t_{\mathrm{var}}}$