COMP0205 Mechatronics and Making

Mechanical Components – Shafts

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Today’s Objectives

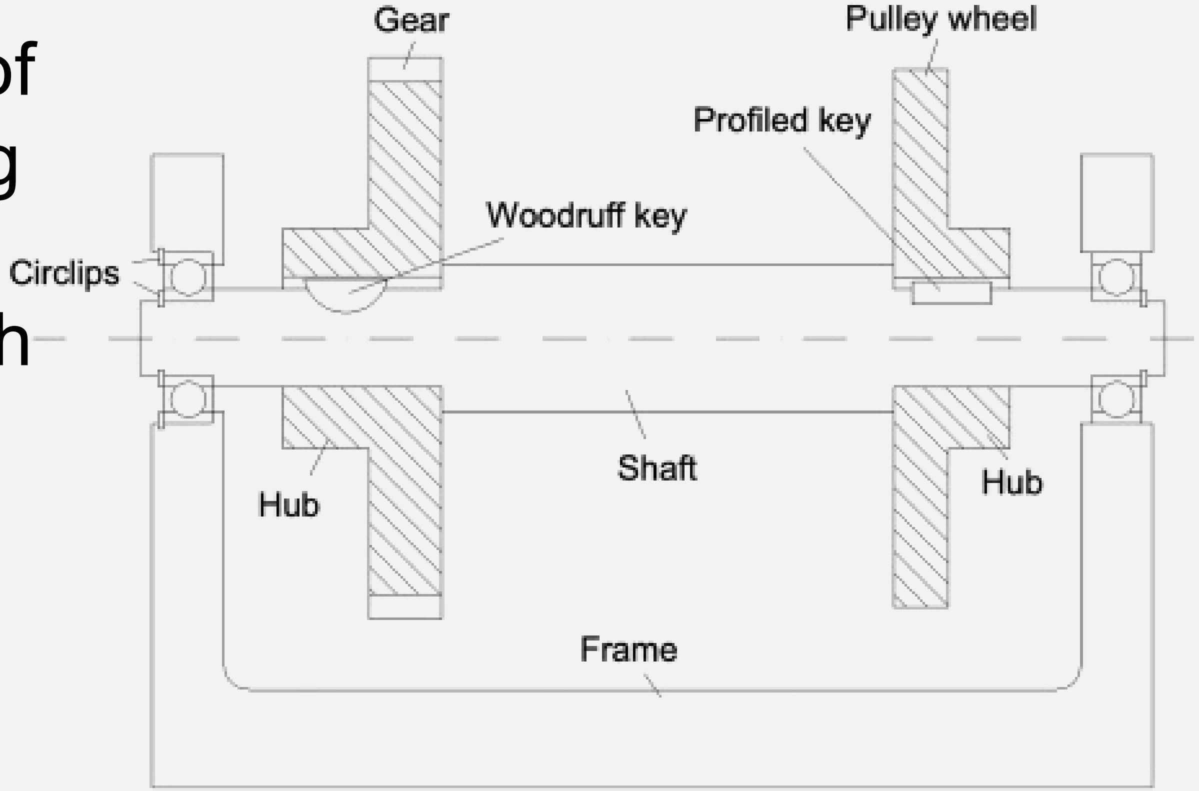
- To understand the fundamental concepts of shafts.
- To analyse shaft loading and deflection.
- To evaluate shaft-hub and shaft-shaft connections.
- To be aware of design principles of shafts.

What is a shaft

- Shaft refers to a component of circular cross section that rotates and transmits power from a driving device, such as a motor or engine, through a machine.
- Shafts can carry gears, pulleys, and sprockets to transmit rotary motion and power via mating gears, belts, and chains. Alternatively, a shaft may simply connect to another via a mechanical coupling.

Shaft arrangement

- Shafts typically consist of a series of **stepped** diameters accommodating bearing mounts and providing **shoulders** for locating devices such as gears, sprockets, and pulleys to butt up against, and keys are often used to prevent rotation, relative to the shaft, of these "added" components.

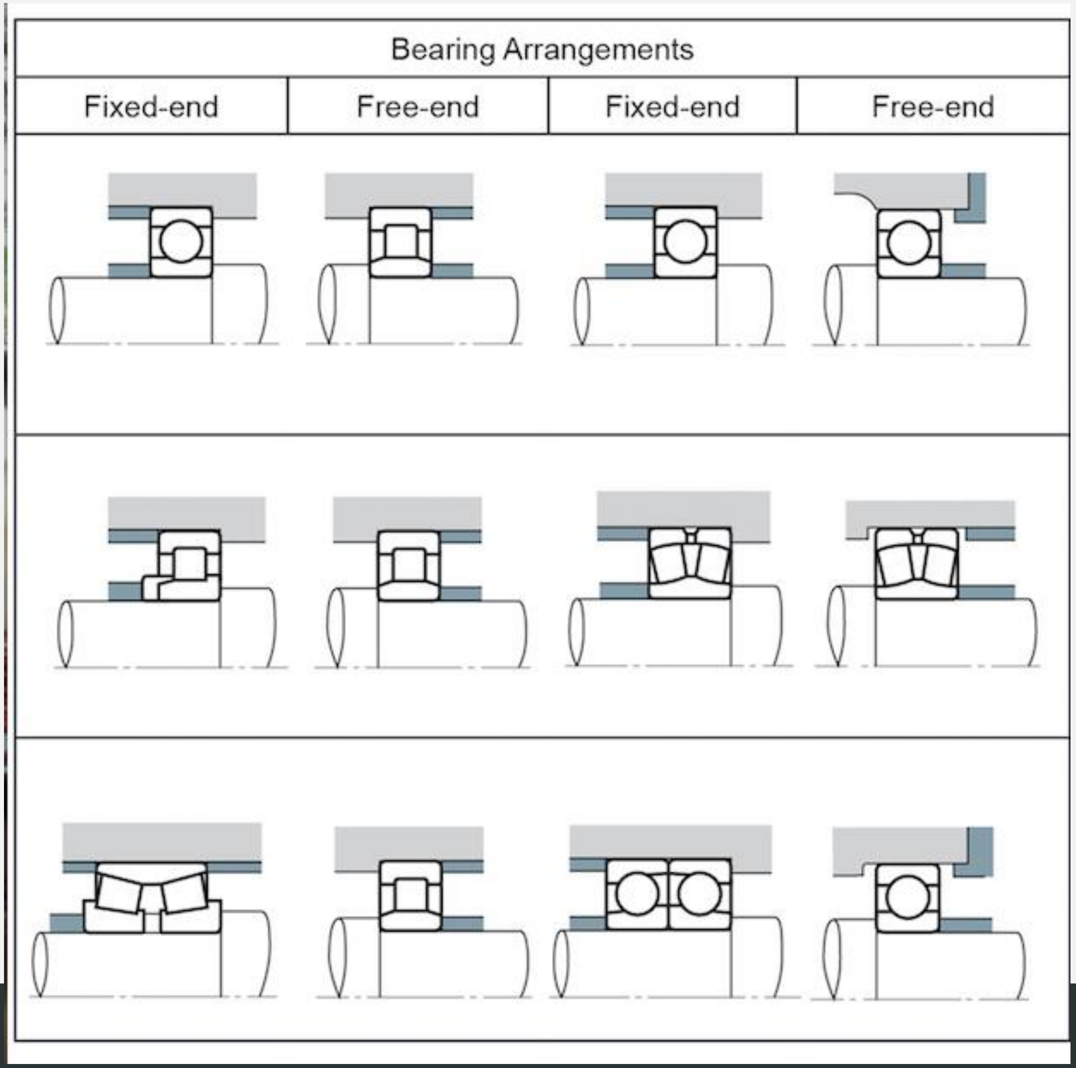


Bearing arrangements on a shaft

- Rotating shafts must generally be supported by bearings. Normally, **two** or more bearings are used on one shaft. Why?
- Provision for thrust load capability and axial location of the shaft is normally supplied by just one thrust bearing taking thrust in each direction.
- It is important that the structural members supporting the shaft bearings are sufficiently strong and rigid.
- **Locating** (fixed-end) bearing and **non-locating** (free) bearing. Why?

Most common bearing arrangements

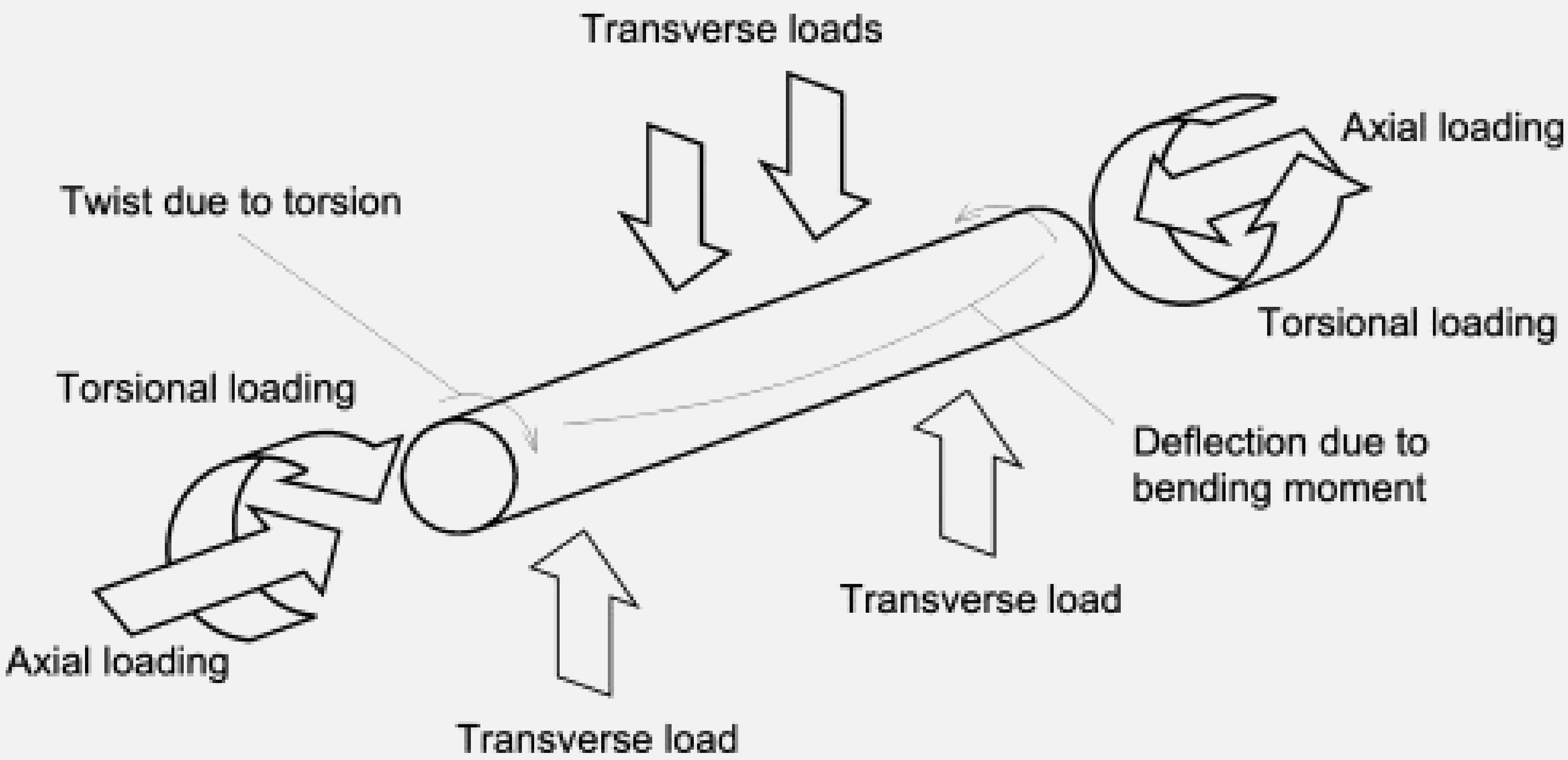
Axial movement of two rings relative to each other



Displacement between outer ring and housing

Shaft loading and deflection

Shaft loading and deflection



Shaft deflections

- Shafts must be designed so that **deflections are** within **acceptable** levels. Too much deflection can, for example, degrade gear performance and cause noise and vibration.
- The **maximum allowable deflection** of a shaft is usually determined by limitations set on the critical speed, minimum deflections required for gear operation and bearing requirements.
- In general, deflections should not cause mating-gear teeth to separate more than about 0.13 mm, and the slope of the gear axes should not exceed about 0.03 degrees.

Deflection calculation

- Deflection, δ, as a function of bending moment, M.

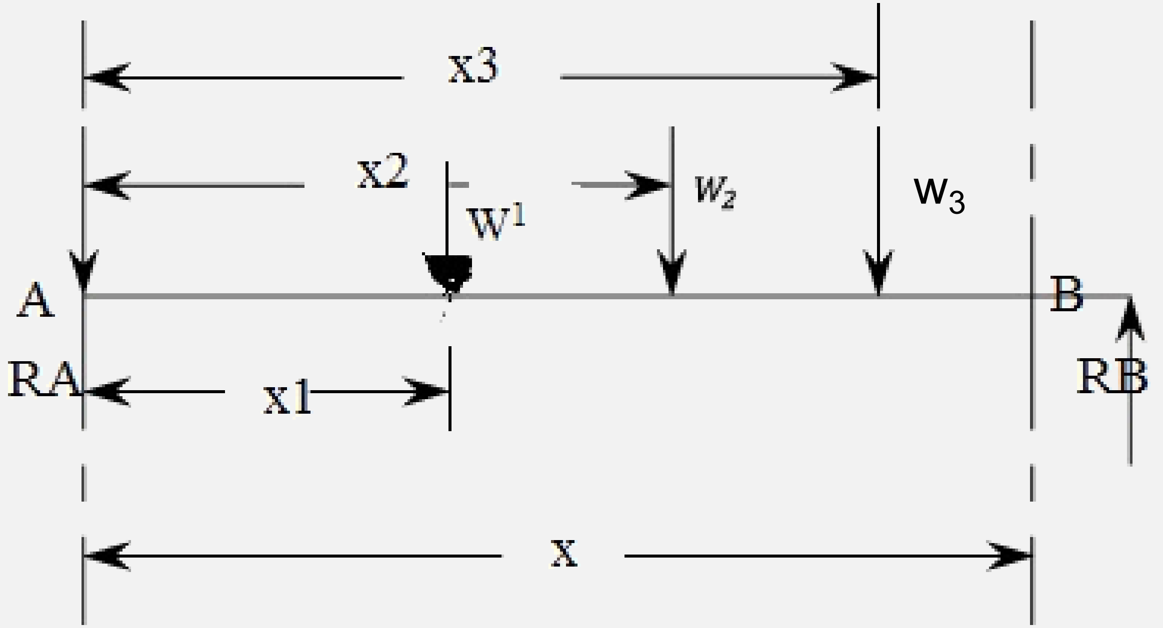
$$\frac{d^2\delta}{dx^2} = \frac{M}{EI}$$

- Integrate twice to obtain δ.

$$\frac{d\delta}{dx} = \int \frac{M}{EI} dx, \qquad \delta = \iint \frac{M}{EI} dx dx$$

For a shaft of given length and loading, the bending deflection is inversely proportional to the product EI. Therefore, the effective way to increase the rigidity of a shaft is to increase the diameter of shaft.

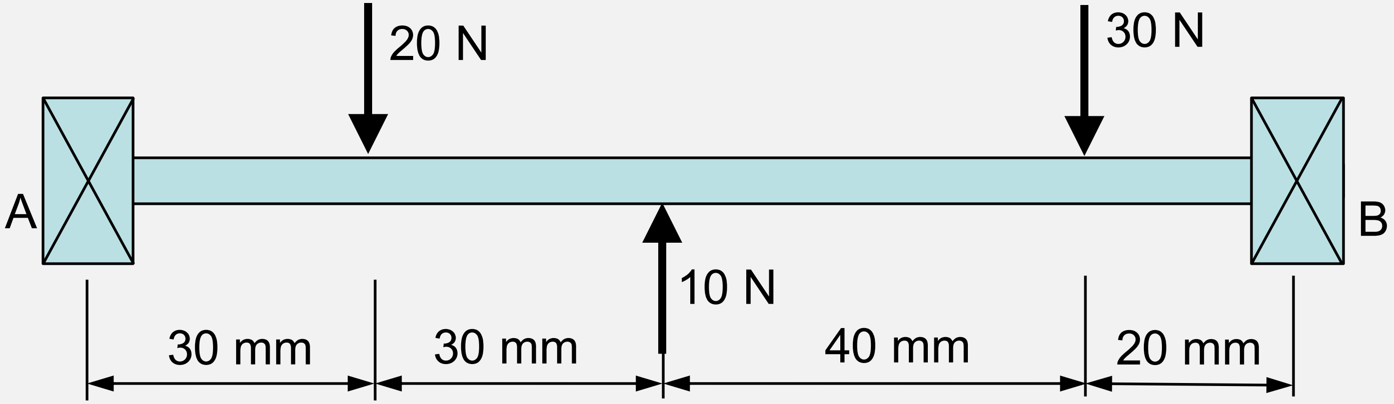
Macaulay’s Method



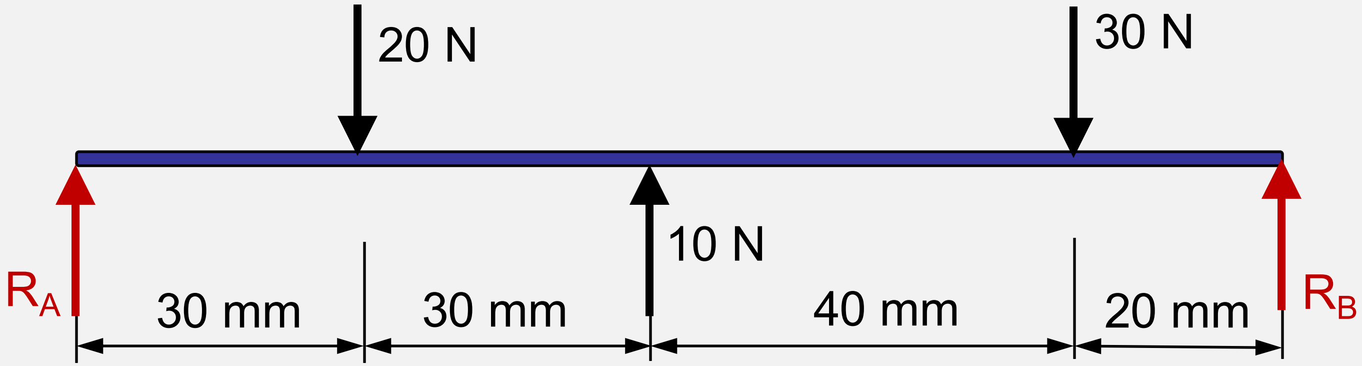
$$EI\,\frac{d^2\delta(x)}{dx^2} = M(x) = R_A x + w_1 < x - x_1 > + w_2 < x - x_2 > + w_3 < x - x_3 >$$

Macaulay bracket: $< x - a >^n = \begin{cases} 0 & for\ x < a \\ (x-a)^n & for\ x \geq a \end{cases}$

Example - Macaulay’s Method

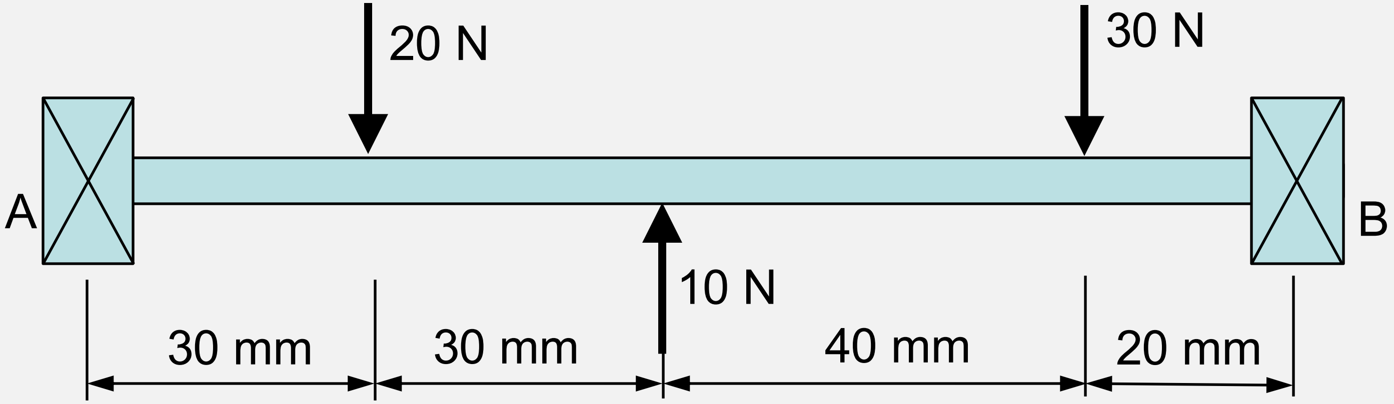


FBD



What is the deflection in the **center**?

Example - Macaulay's Method



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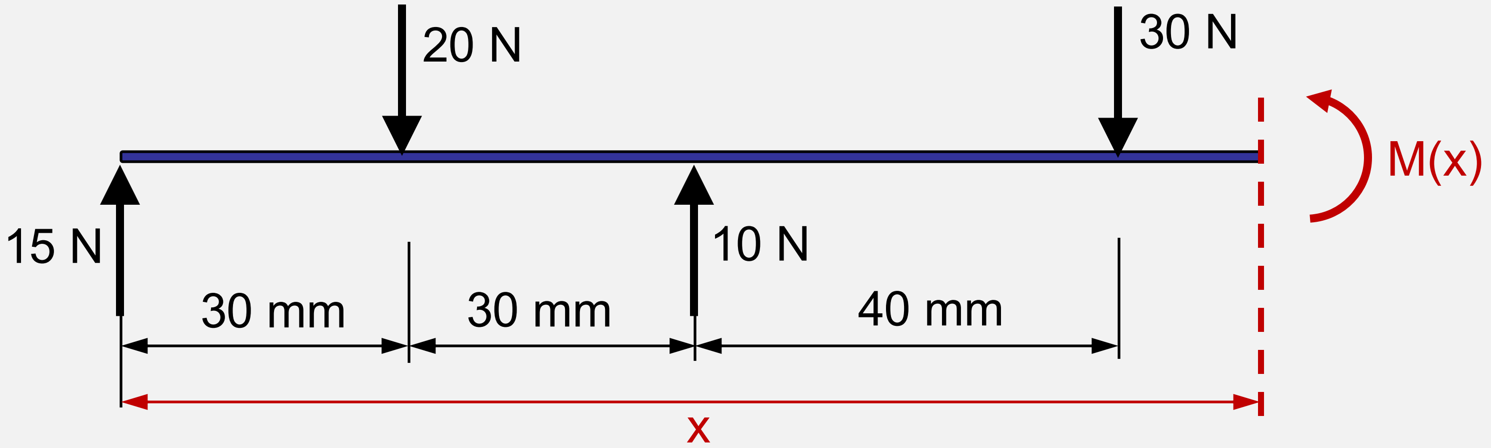
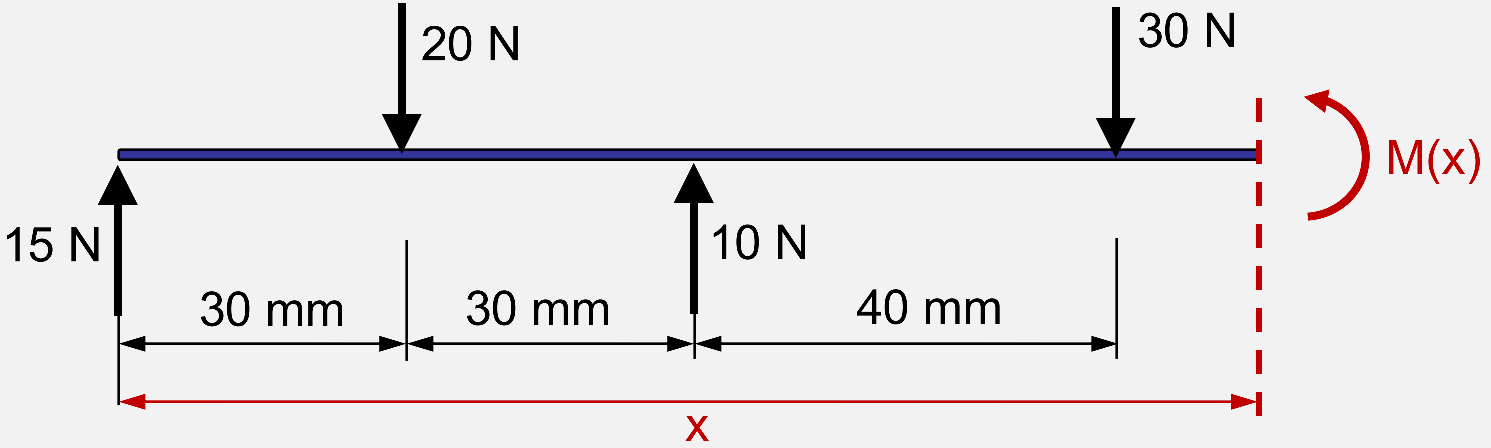


Example - Macaulay's Method

FBD

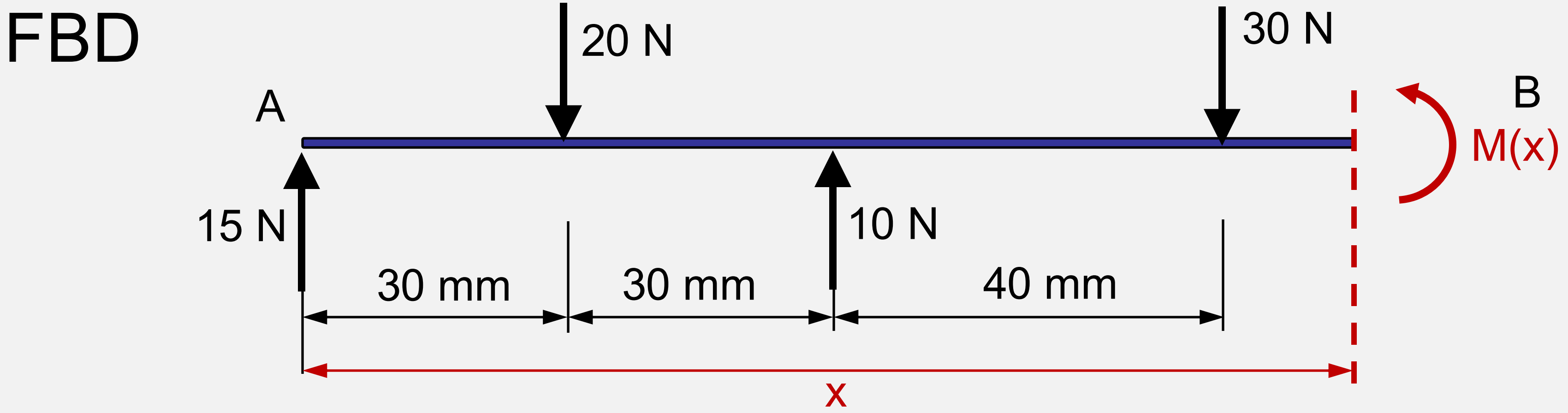


$$EI\frac{d^2\delta(x)}{dx^2} = M(x) = 15x - 20 < x - 30 > +10 < x - 60 > -30 < x - 100 >$$

$$EI\frac{d\delta(x)}{dx} = 15\frac{x^2}{2} - 20\frac{< x - 30 >^2}{2} + 10\frac{< x - 60 >^2}{2} - 30\frac{< x - 100 >^2}{2} + C_1$$

$$EI\delta(x) = 15\frac{x^3}{6} - 20\frac{< x - 30 >^3}{6} + 10\frac{< x - 60 >^3}{6} - 30\frac{< x - 100 >^3}{6} + C_1 x + C_2$$

Example - Macaulay's Method

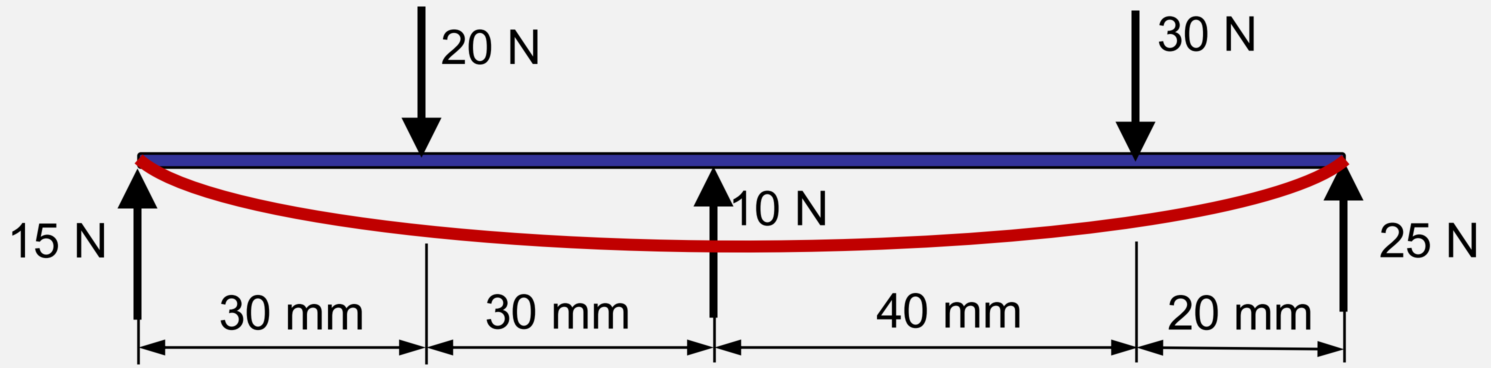


$$EI\delta(x) = 15\frac{x^3}{6} - 20\frac{< x - 30 >^3}{6} + 10\frac{< x - 60 >^3}{6} - 30\frac{< x - 100 >^3}{6} + C_1 x + C_2$$

- Using the boundary conditions to determine the integration constants
  - at point A, $x = 0, \delta(0) = 0 \;\; \rightarrow \;\; C_2 = 0$
  - at point B, $x = 120, \delta(120) = 0 \rightarrow 0 = 15\frac{120^3}{6} - 20\frac{90^3}{6} + 10\frac{60^3}{6} - 30\frac{20^3}{6} + 120C_1$

$$\rightarrow C_1 = -18416.7$$

Example - Macaulay's Method



$$EI\delta(x) = 15\frac{x^3}{6} - 20\frac{< x - 30 >^3}{6} + 10\frac{< x - 60 >^3}{6} - 30\frac{< x - 100 >^3}{6} - 18416.7x$$

- Calculate deflection at mid-span
  - $x = 60 \quad \rightarrow EI\delta(60) = 15\frac{60^3}{6} - 20\frac{30^3}{6} + 0 - 0 - 18416.7 \times 60 = -655002 \text{ Nmm}^3$
  - $\delta(60) = -\frac{655002\text{Nmm}^3}{\text{EI}}$
- Question: how to find the maximum deflection location?

E & I

- As a rough guideline, the allowable bending deflections for a transmission shaft is (0.0003–0.0005)**L**, where **L** is span between bearings.
- When a shaft supports gears, the allowable bending deflections is selected as (0.01–0.03)**m**, where **m** is the normal module of mounted gears.

- Approximate Young's modulus (**E**) for various materials
  - https://en.wikipedia.org/wiki/Young%27s_modulus#Examples

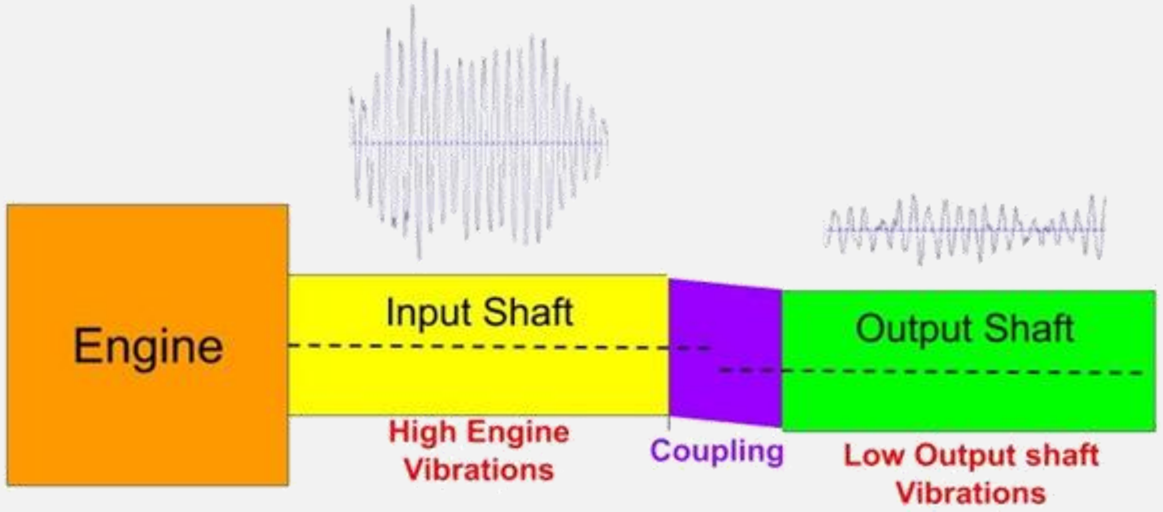
- List of second moments of area (**I**) of various shapes
  - https://en.wikipedia.org/wiki/List_of_second_moments_of_area

Shaft–Shaft connection

Why are mechanical couplings used?

- In order to transmit power from one shaft to another, a **coupling** or clutch can be used.
- But can't we just weld two shafts or use a gearbox instead?

- Transmit power
- Protection against overloading
- Absorbs misalignment
- Absorb shock and vibrations



Shaft–Shaft Connection: Couplings

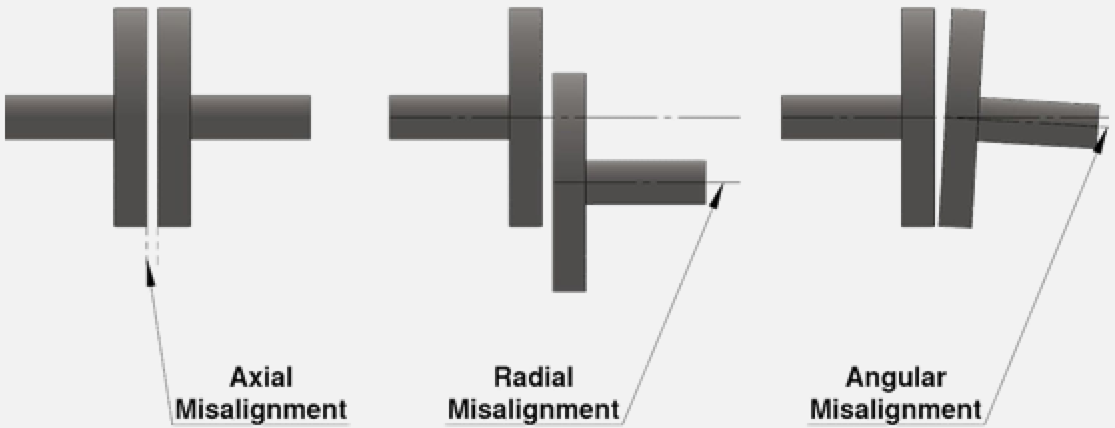
- In order to transmit power from one shaft to another, a **coupling** or clutch can be used.
- There are two general types of coupling: **rigid** and **flexible**.
- **Rigid couplings** are suitable when precise alignment of two shafts is required.
- **Flexible couplings** are designed to transmit torque while permitting some axial, radial, and angular misalignment.
- Each coupling is designed to transmit a given limiting torque.

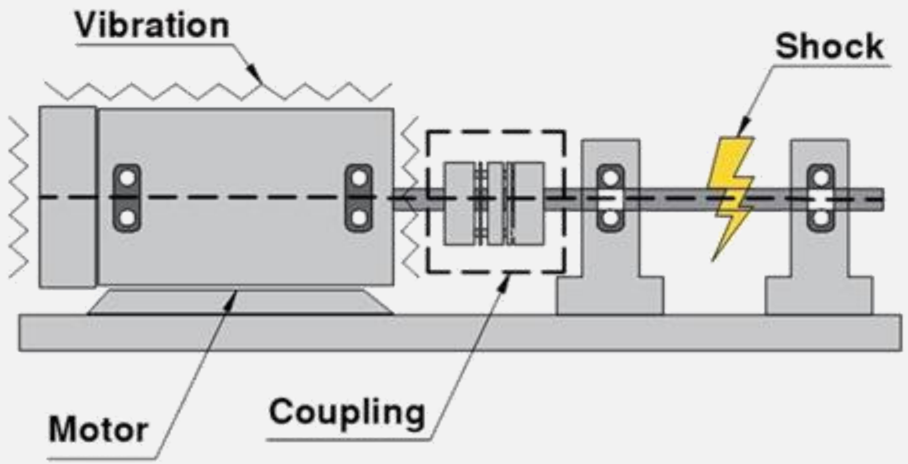
Rigid couplings

- Rigid couplings are available in **straight-through bores** (both shafts are the same size) and **stepped bores** (shafts are different sizes).
- Common compatibility problems, such as variations in shaft length or size, can be resolved using **shaft adapters** to resize shafts.
- Measurement system compatibility issues can be resolved with **inch-metric couplings** when connecting two incompatible shafts.
- Note that using the wrong rigid coupling can result in vibration dampening and poor shock absorption.


Flexible couplings

- Flexible shaft couplings are mechanical devices used to connect two shafts to transmit power while **accommodating misalignments** and **reducing** transmission of **vibrations** or shock loads.
- Selecting the wrong flexible coupling can lead to inefficient torque transmission and potential wear over time





Universal joint

- Generally, flexible couplings are able to tolerate up to ±3 degrees of angular misalignment and up to 0.75 mm parallel misalignment depending on their design.
- If more misalignment is required, a universal joint can be used

Rigid Coupling

Sleeve Coupling

Flange Coupling

Split Muff Coupling

Gear Coupling

Fluid Coupling

Universal Coupling

Oldham Coupling

Jaw Coupling

Bellow Coupling

Flexible Coupling

Constant Speed Coupling

Diaphragm Coupling

FLUDEX Coupling

Bushed Pin Type Coupling

Shaft–Hub connection

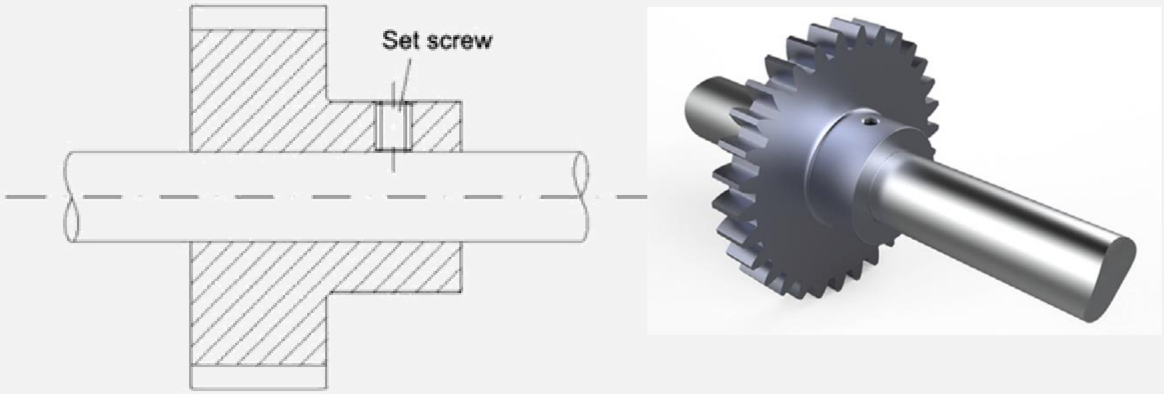
Shaft–Hub connection

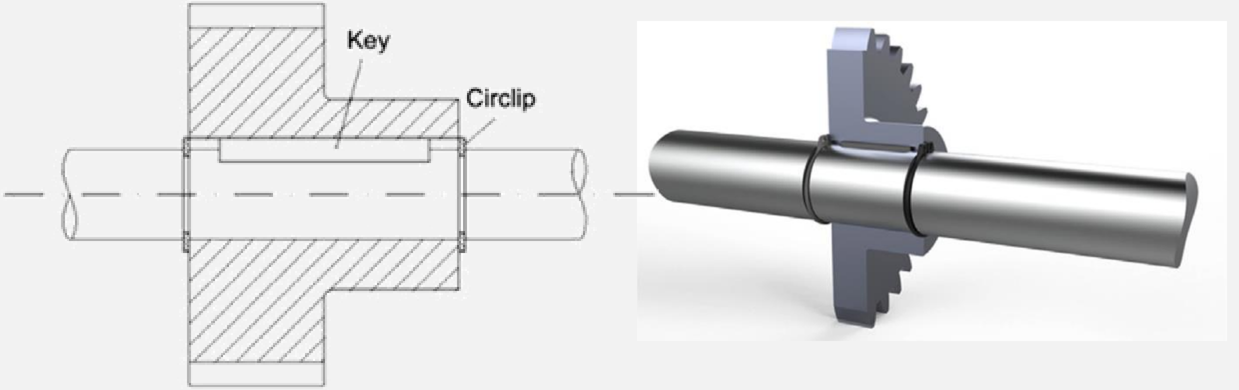
- Power transmitting components such as gears, pulleys, and sprockets need to be mounted on shafts securely and located axially with respect to mating components.
- In addition, a method of transmitting torque between the shaft and the component must be supplied.
- The portion of the component in contact with the shaft is called the **hub** and can be attached to, or driven by, the shaft by keys, pins, setscrews, press and shrink fits, splines, and taper bushes.

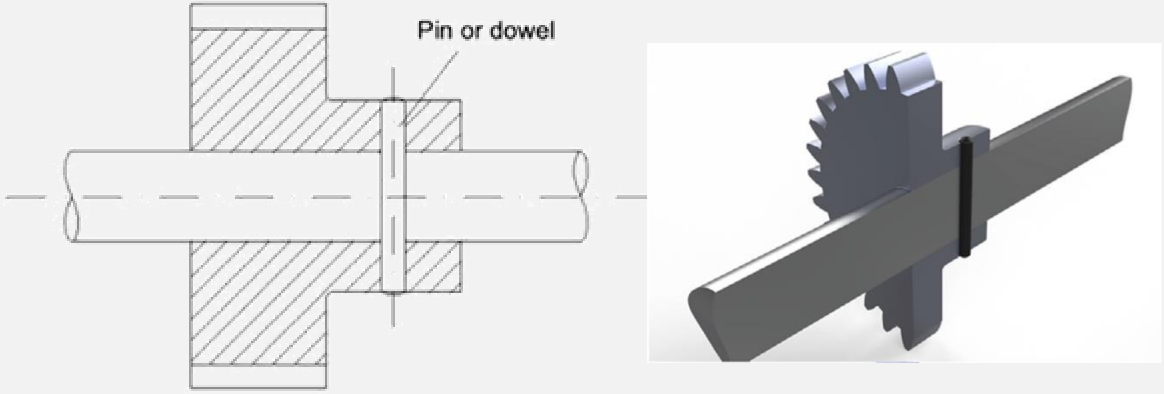
Interference fit

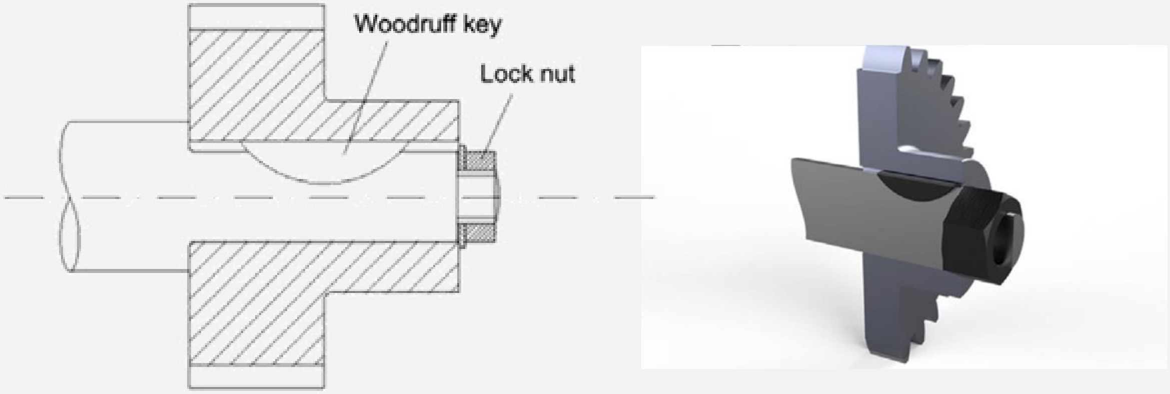
- One of the simplest shaft-hub attachments is to use an **interference fit,** where the hub bore is slightly **smaller** than the shaft diameter.
- Assembly is achieved by press fitting, or thermal expansion of the outer ring by heating and thermal contraction of the inner by use of liquid nitrogen.

Shaft–hub connection methods



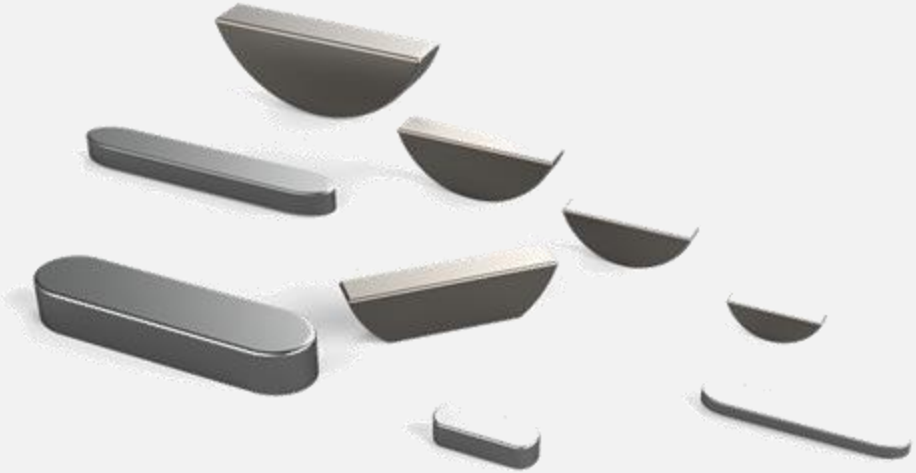


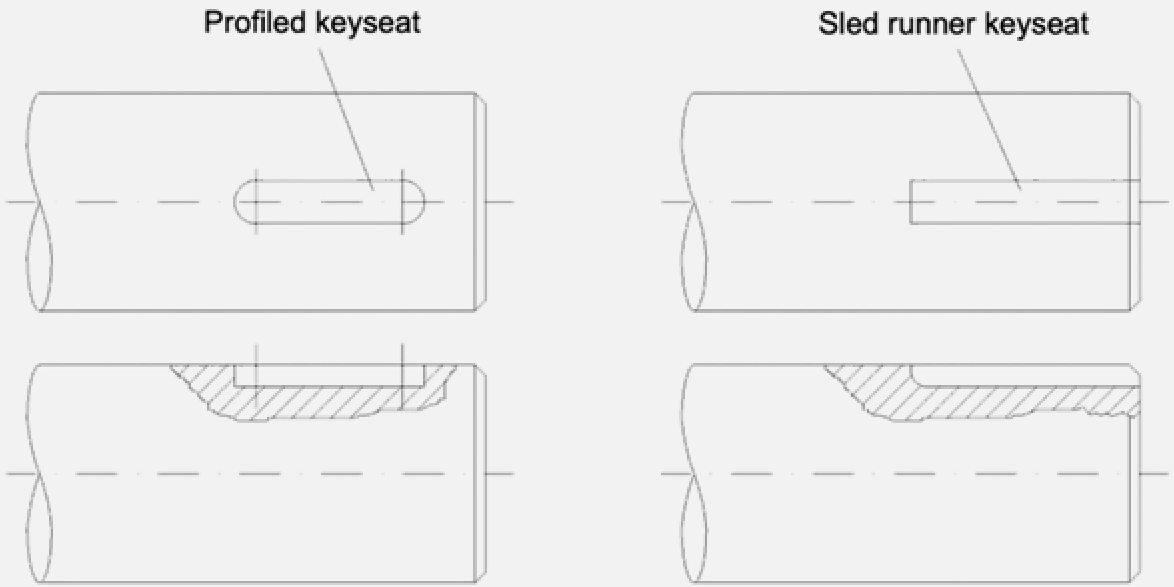


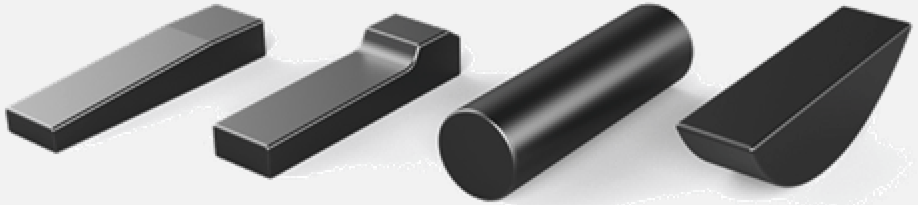


Keyways and keys

- The grooves in the shaft and hub into which the key fits are called **keyways** or keyseats.

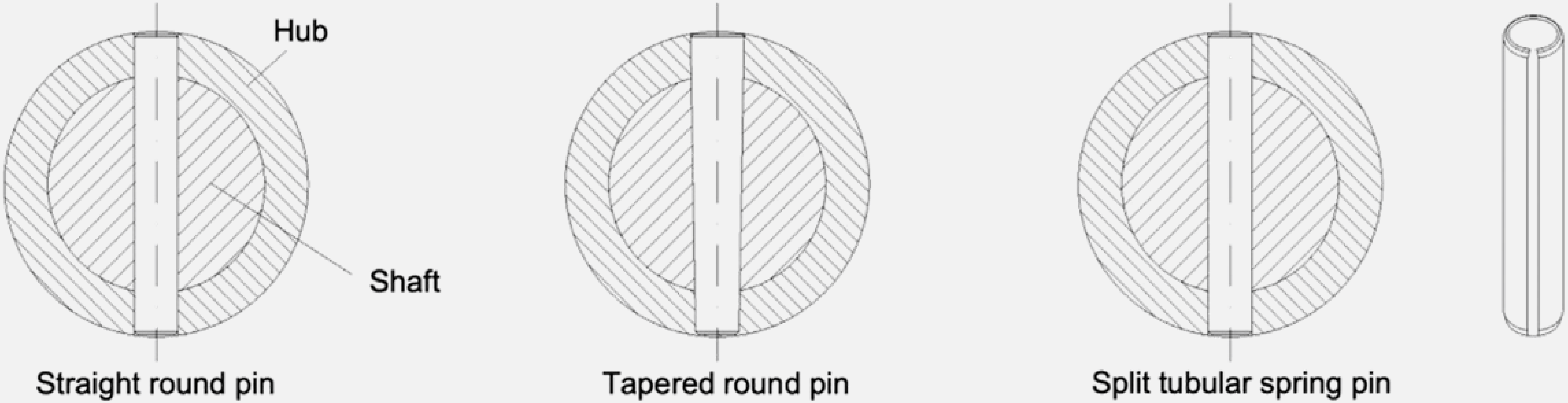






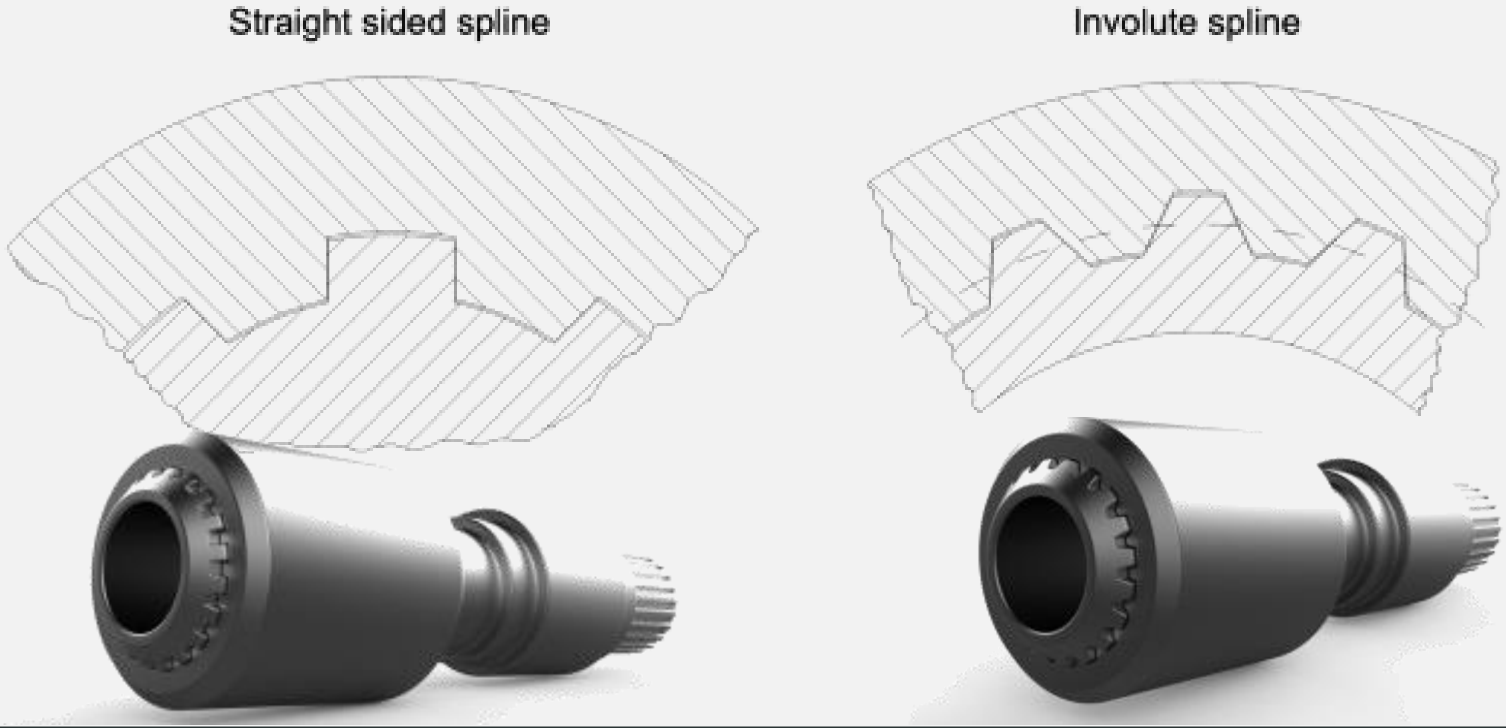
Pins

- A simpler and less expensive method for transmitting light loads is to use **pins**.



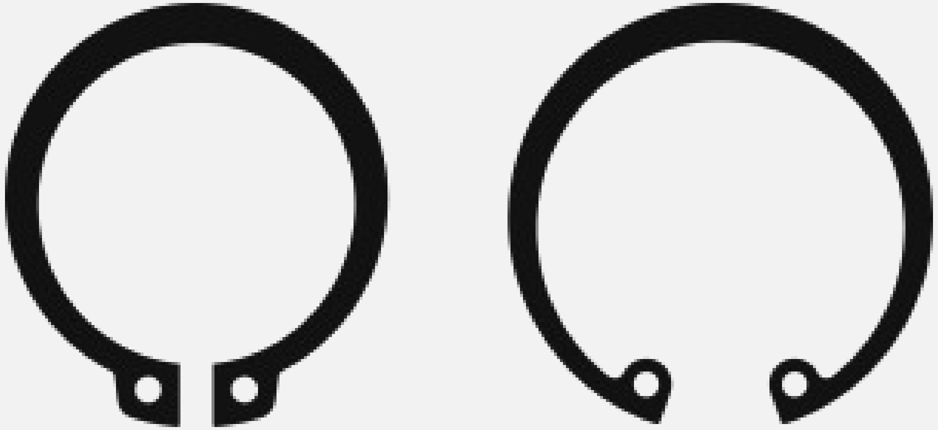
Splines

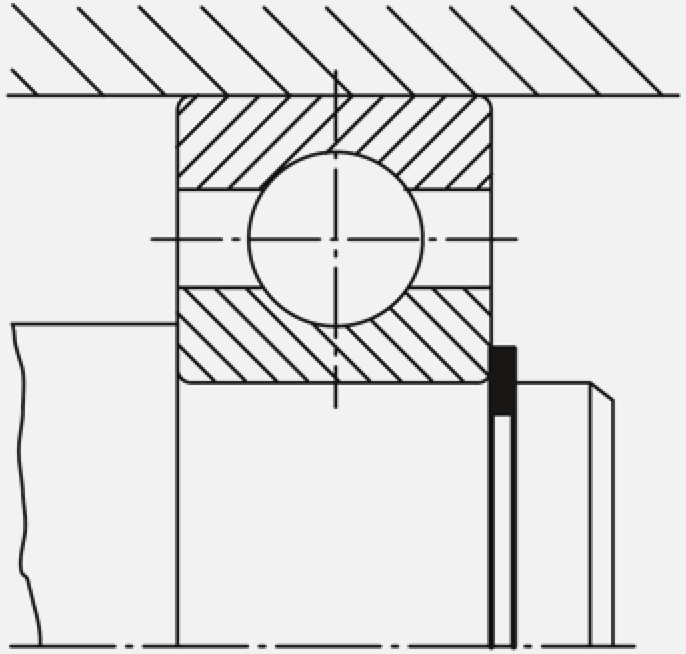
- Mating splines comprise teeth cut into both the shaft and the hub and provide one of the strongest methods of transmitting torque.

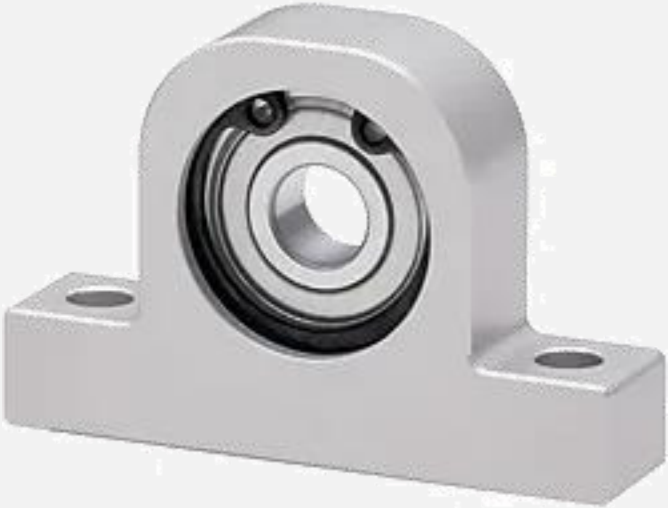


Snap rings or circlips

- An inexpensive method of providing **axial location** of hubs and bearings on shafts is to use circlips







Merits of various shaft-hub connections

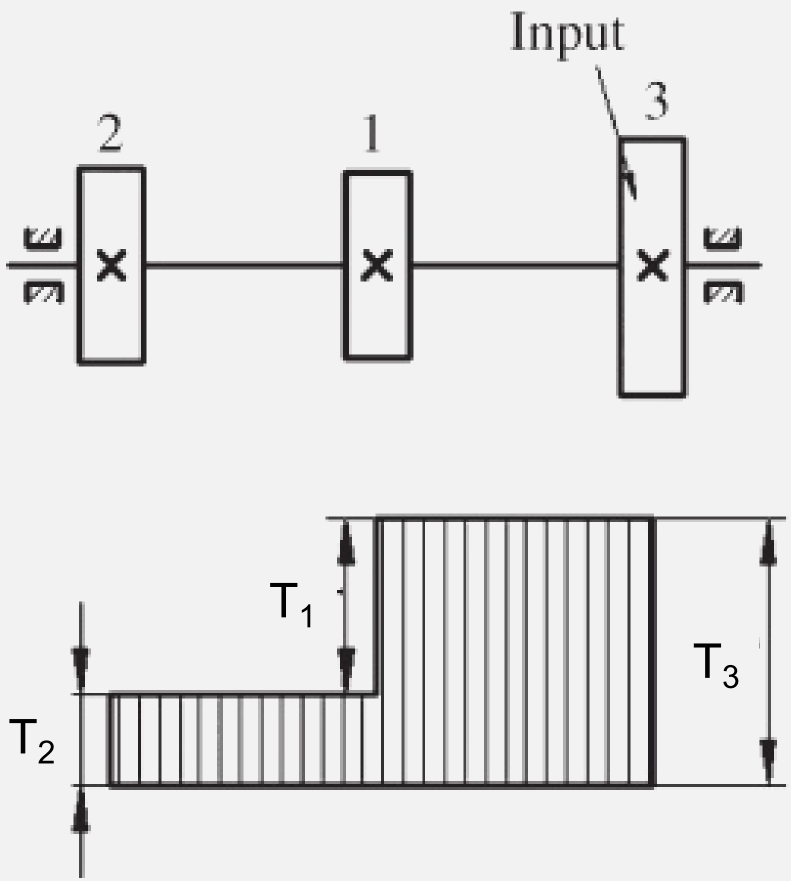
	Pin	Grub screw	Press fit	Shrink fit	Split clamp	Splines	Key	Taper bush	Cone clamping elements	Shrink disk	Star disk
High torque capacity	×	×	×	✓	✓	✓	✓	✓	✓	✓	✓
High axial loads	✓	×	×	✓	✓	×	×	✓	✓	✓	✓
Axially compact	×	×	✓	✓	×	✓	✓	✓	✓	✓	✓
Axial location provision	✓	✓	✓	✓	✓	×	×	✓	✓	✓	✓
Easy hub replacement	×	✓	×	×	✓	✓	✓	✓	✓	✓	✓
Fatigue (cyclic)	×	×	✓	✓	✓	×	×	✓	✓	✓	✓
Accurate angular positioning	✓	×	×	×	×	✓	✓	(✓)	(✓)	(✓)	(✓)
Easy position adjustment	×	✓	×	×	✓	×	×	✓	✓	✓	✓
Low cost	✓	✓	✓	×	✓	×	×	×	×	×	×

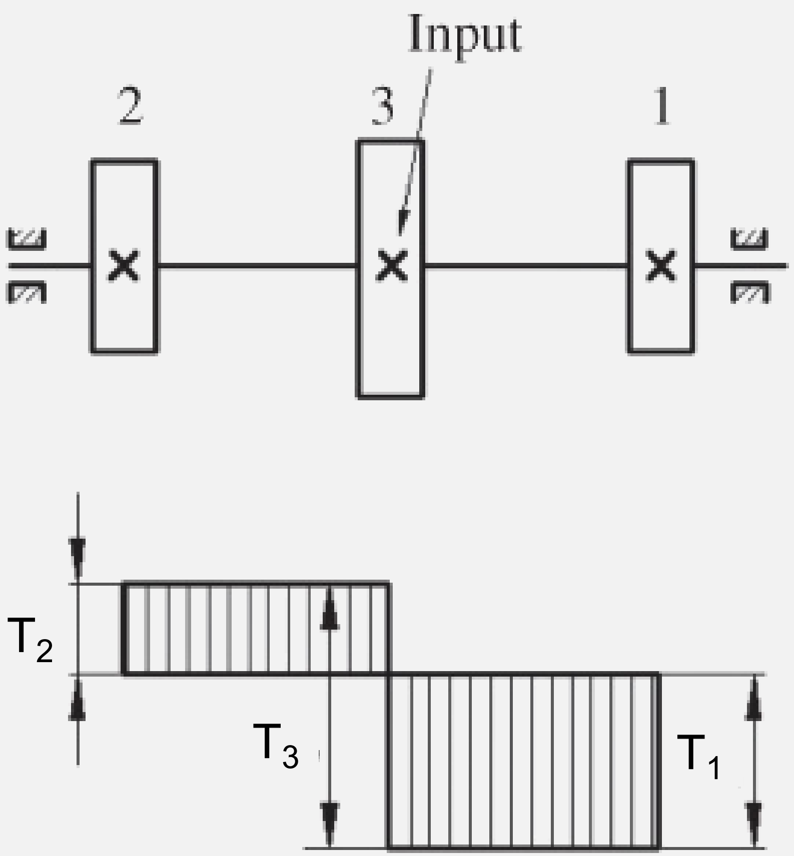
Design of shafts

General principles in shaft design

- Keep shafts **as short as possible** with the bearings close to applied loads. This will reduce shaft deflection and bending moments and increase critical speeds.
- If possible, locate stress raisers, such as sharp radii, slots, and changes in diameter, away from highly stressed regions of the shaft.
- Use generous fillet radii and smooth surface finishes and consider using **local surface strengthening** processes such as shot peening and cold rolling.
- If weight is critical, use **hollow** shafts.

Example - Layout of shaft mating elements



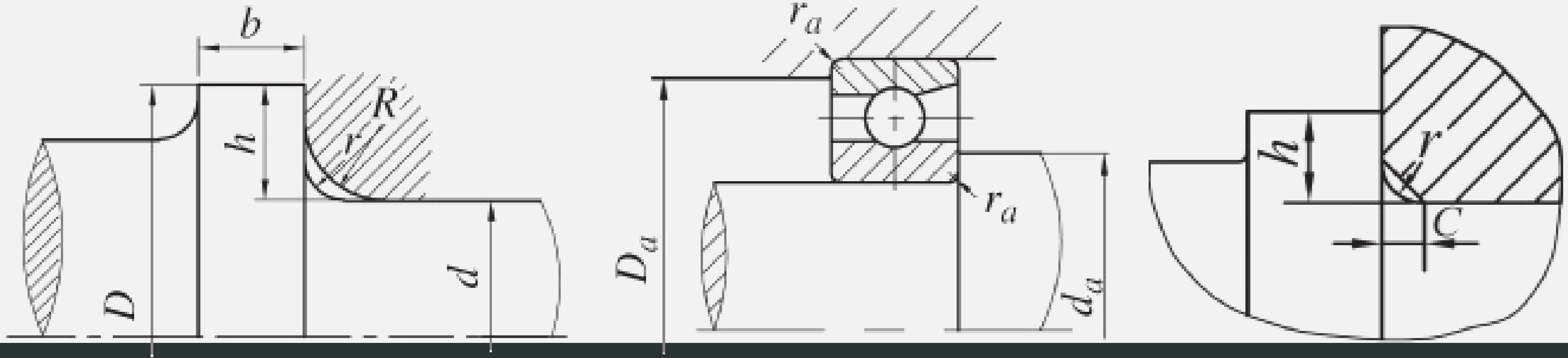


Locating and fastening elements on a shaft

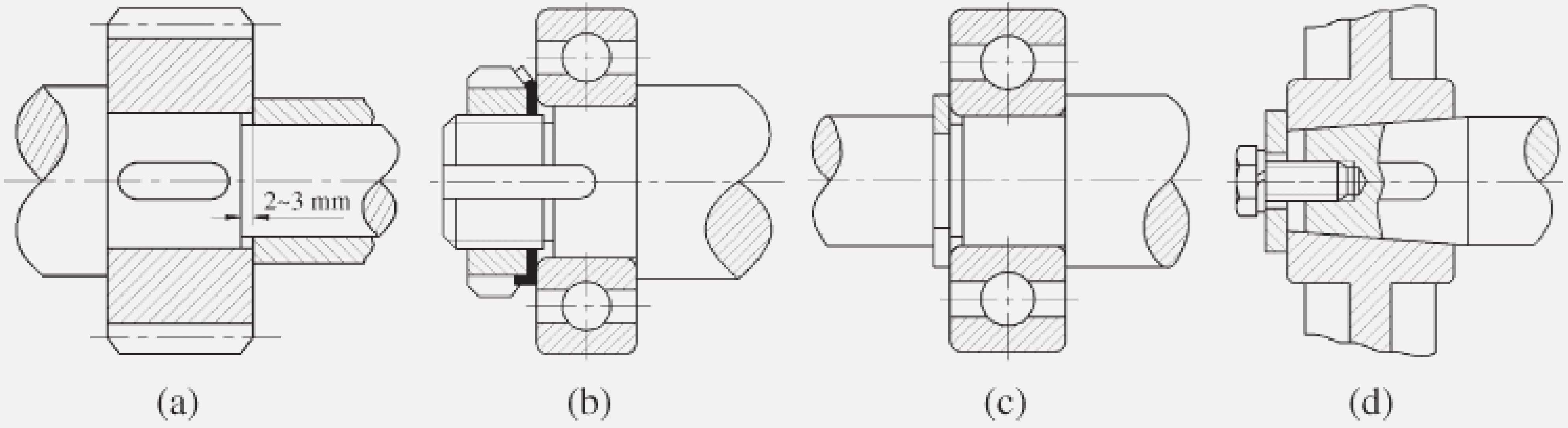
- Elements must be held in position along shafts, especially for thrust loads producing elements, such as helical or bevel gears, or tapered roller bearings.
- Accurate **axial positioning** of gears or bearings requires a shoulder, spacer, locknut and lockwasher, retaining ring, collar and screw, tapered surface and so on.
- Keys, splines, setscrews, pins, interference fit and tapered fit, can be used to **circumferentially locating** and fastening elements on a shaft.

Axially locating and fastening elements on a shaft

- A shoulder is a change in the shaft diameter against which to locate an element carrying large thrust loads.
- For a locating shoulder, the diameter of shaft shoulder must be sufficient to ensure solid seating, with a height of h = (0.07–0.1)d.
- Shoulder rings have the same function as shoulders, width b ≥ 1.4h.
- Shoulder fillet radius is the maximum permissible radius on the shaft, smaller than that on the mating elements, that is, r < R in left figure, or r < C in right figure.



Axially locating and fastening elements on a shaft



Spacer

Locknut & lockwasher

Retaining ring

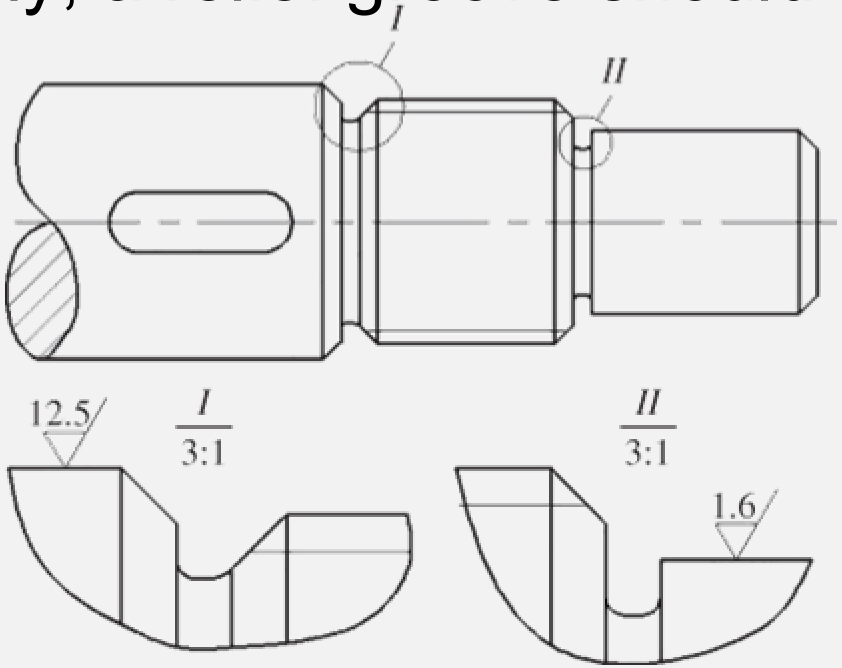
Tapered surface & collar and screw

Machinability and Assemblability of Shafts

- The structure of shaft should **facilitate manufacture, assembly**, measurement and maintenance.
- The dimensions of fillet radius, chamfers, keyways, grooves and suchlike on different shaft segments with similar diameters has to be **consistent**.
- The keyways on different segments of a shaft should be on the **same generatrix**.

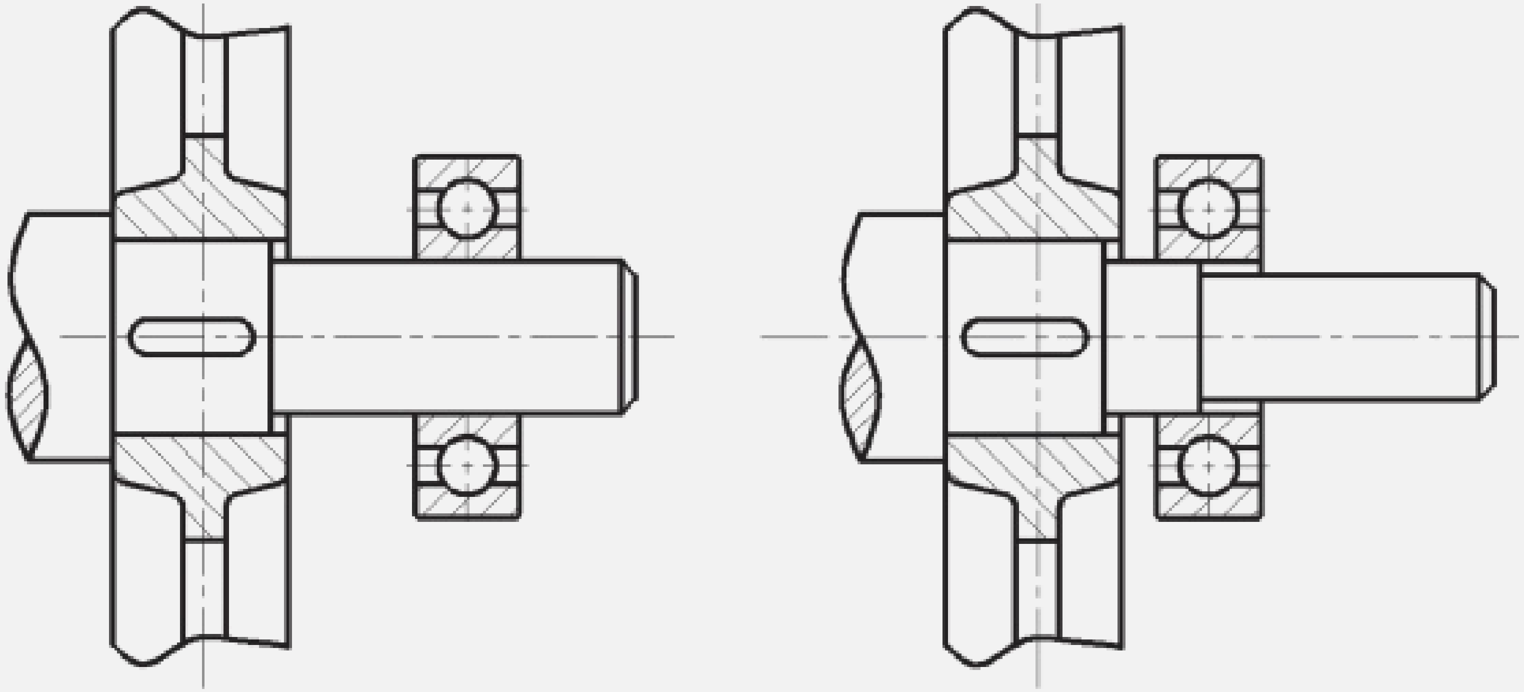
Relief grooves

- A grinding-relief groove should be provided for the grinded shaft segment to prevent the grinding operation from going all the way to the shoulder. Similarly, a relief groove should be designed for screw fabrication,



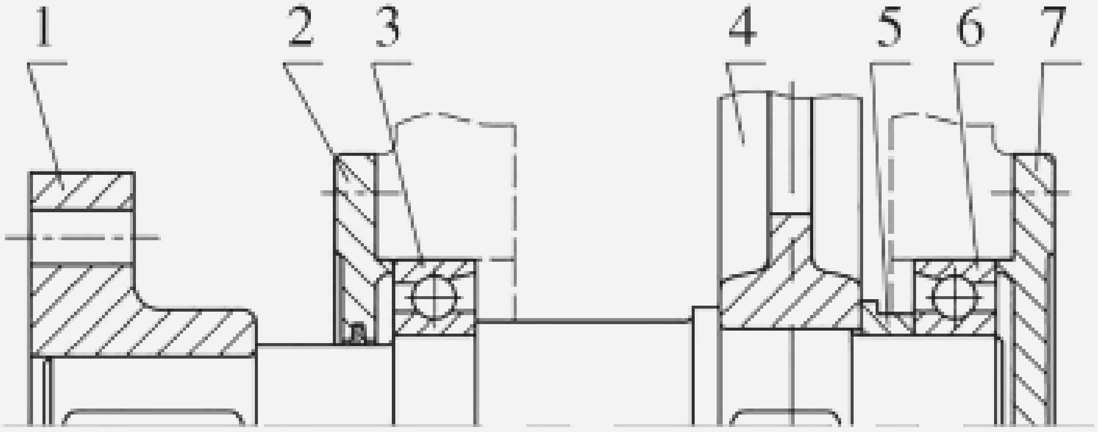
Relief groove I and grinding-relief groove II on a shaft.

Example - Mating shaft segment design



The length of mating shaft should be shortened and the diameter of the rest shaft segment be slightly reduced to allow the bearing to slide easily over the shaft up to its final position.

Assembly sequence



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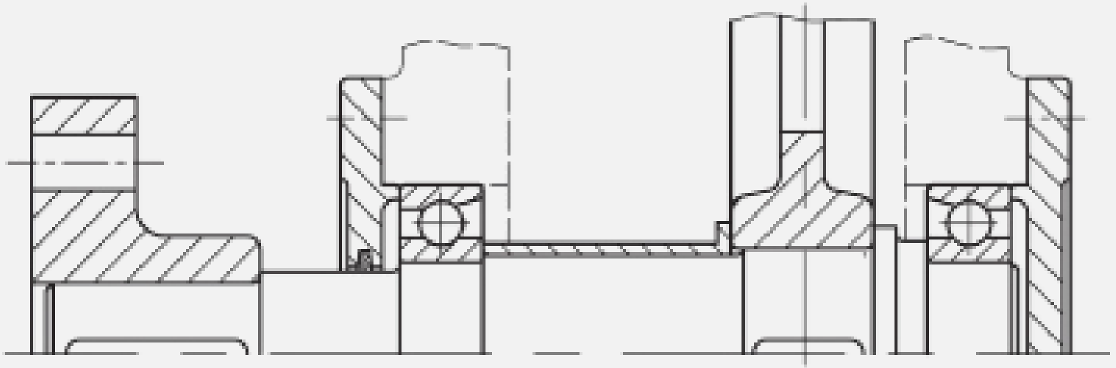

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