GENERAL ASSEMBLY

INTRO TO DATA SCIENCE

LECTURE 16: NETWORKS & GRAPHS

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LAST TIME:

- BIG DATA
- MAP REDUCE

QUESTIONS?

I. NETWORKS
II. NETWORK STATICS
III. NETWORK DYNAMICS

GUEST SPEAKER
SEAN TAYLOR – INFERENCE & CAUSALITY

I. NETWORKS

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The ubiquity of social networks gives rise to many interesting data-oriented questions that can be answered with analytical techniques.

Given a large set of social network data, what types of questions do you think would be interesting to ask?

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- What does the network look like?

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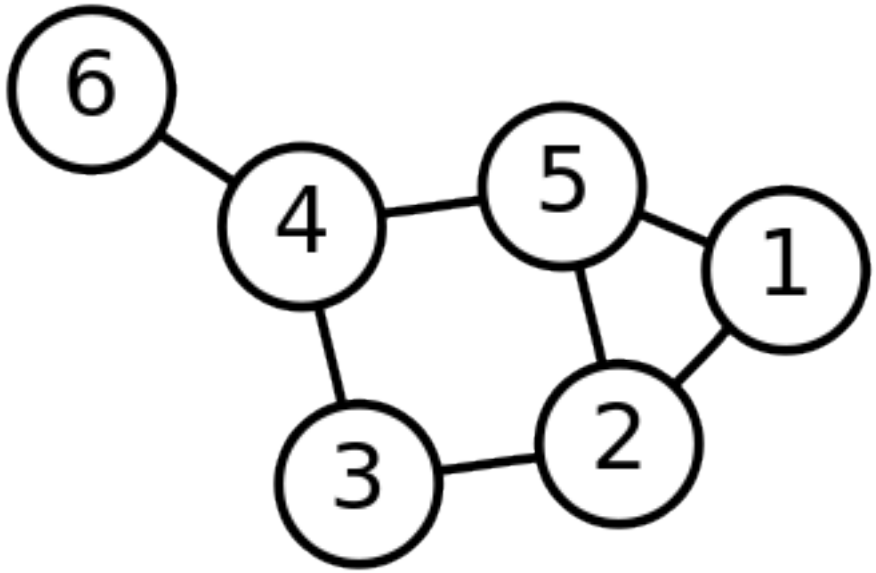
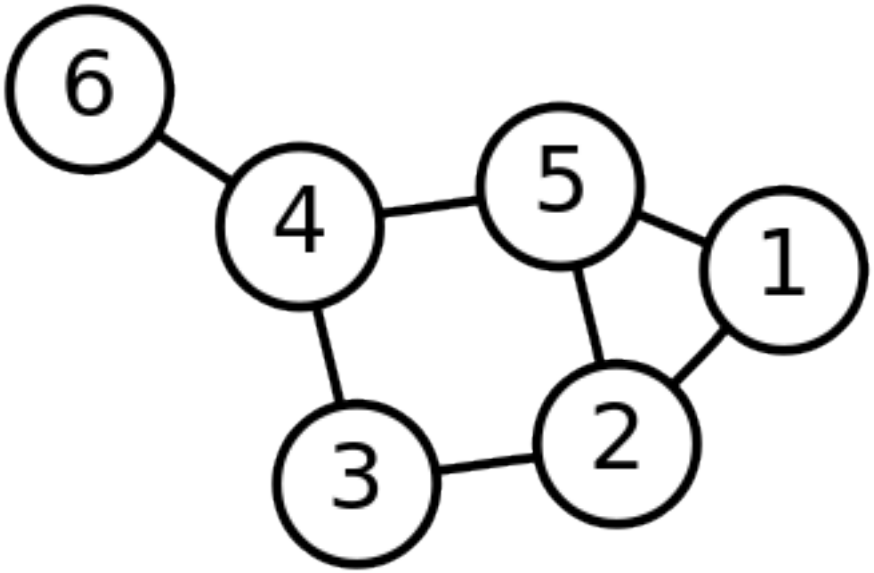
These are questions about network *behavior*.

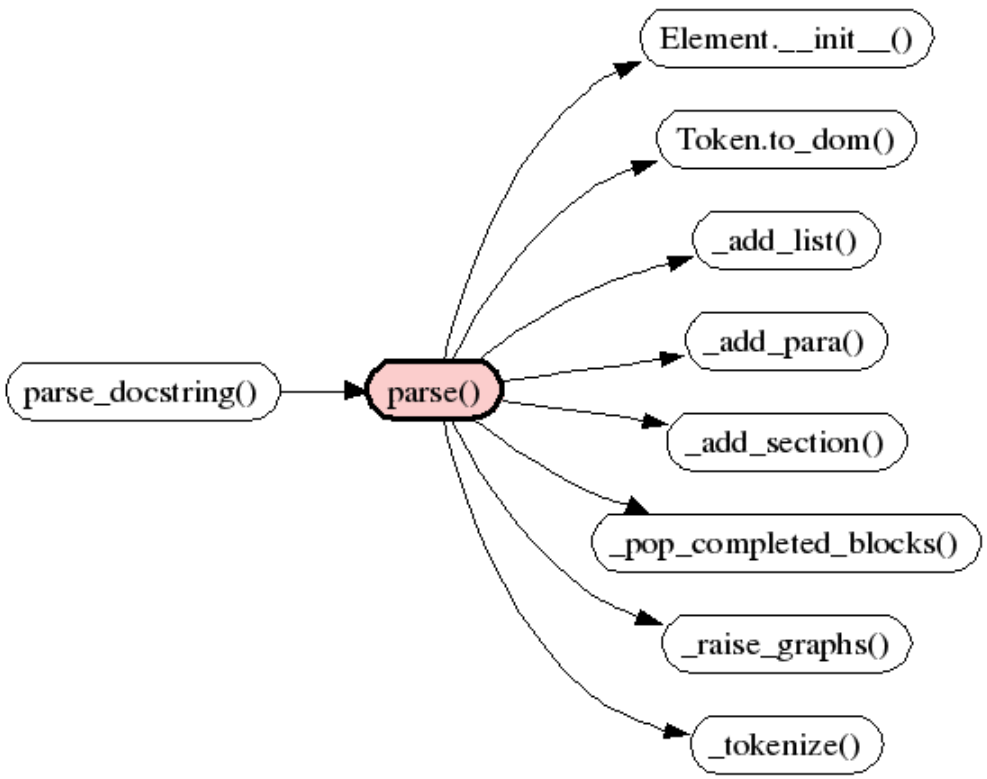
The mathematical representation of a network is an object called a **graph**, which is a configuration of *nodes* connected by *edges*.



Nodes represent *actors* in the graph, and edges represent the *relationships* between actors.



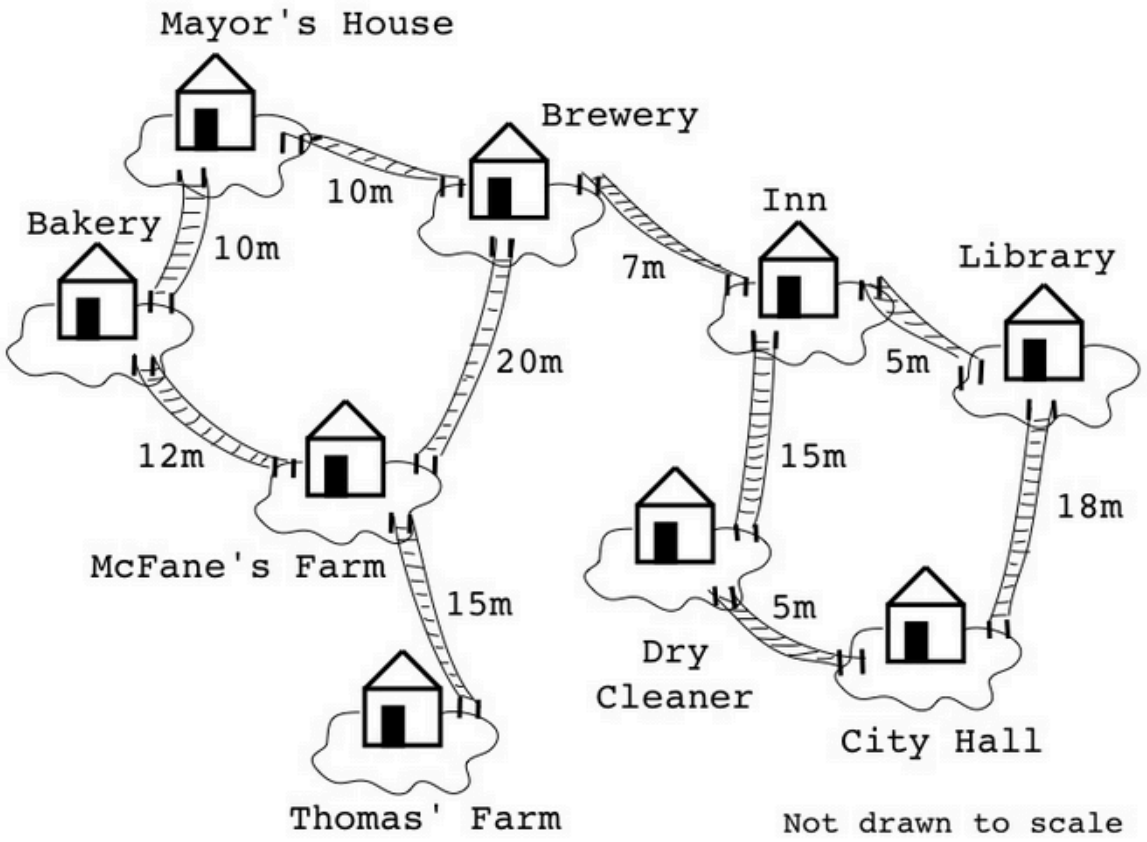
NETWORK REPRESENTATION



NOTE

A *directed graph* has edges that point from one node to another.

NETWORK REPRESENTATION



NOTE

A *weighted graph* contains edges associated with real-valued numbers, eg to measure distance or importance.

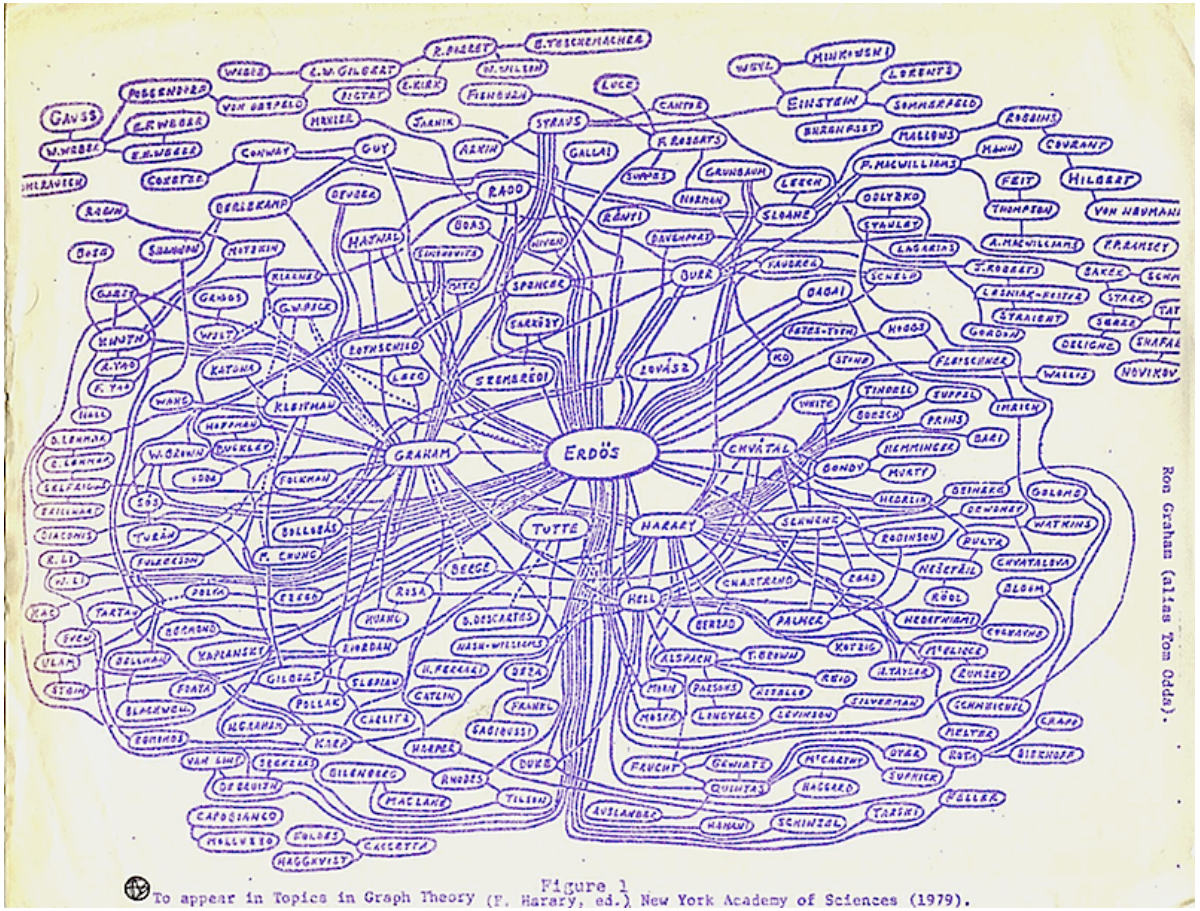
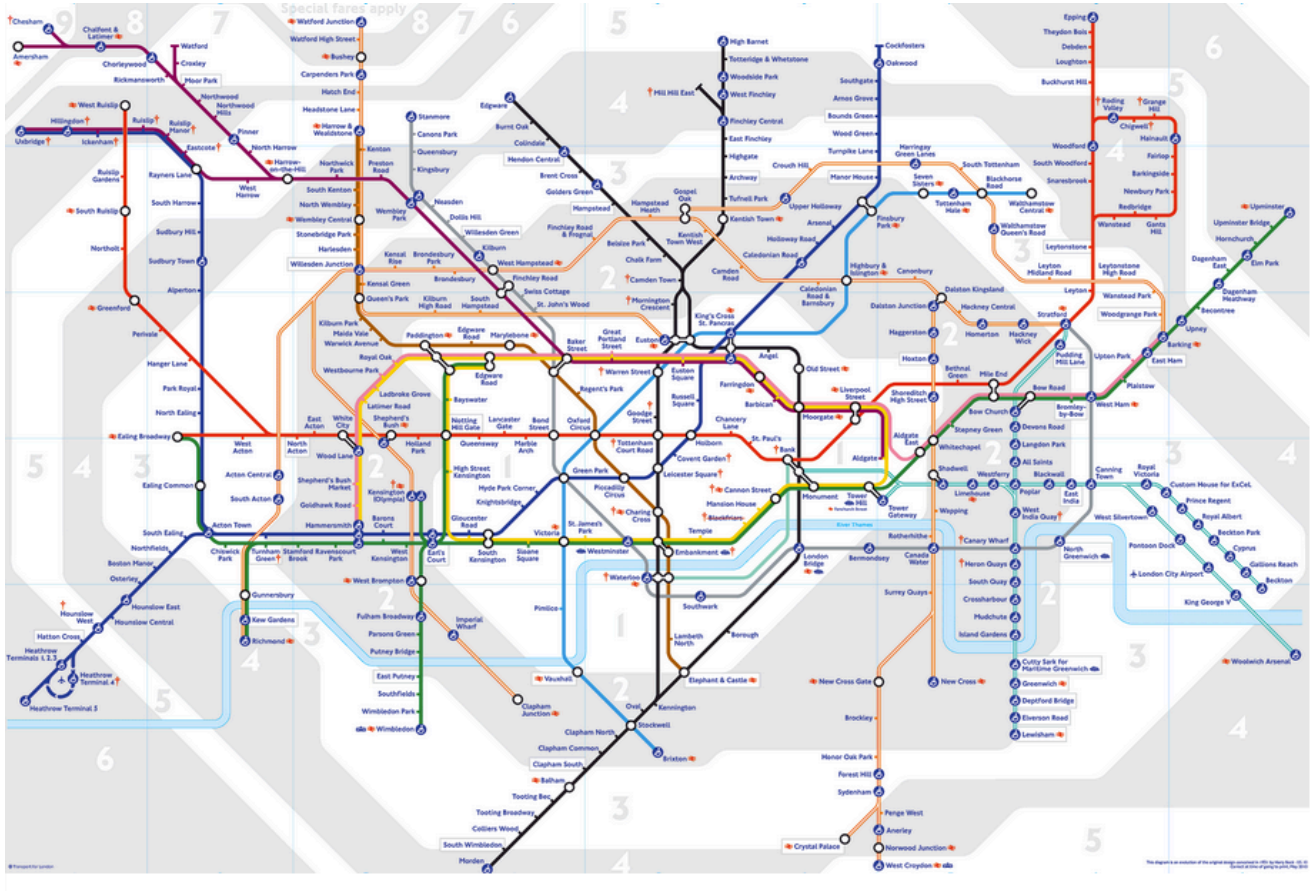


Figure 1

To appear in Topics in Graph Theory (F. Harary, ed.) New York Academy of Sciences (1979).

NETWORK REPRESENTATION



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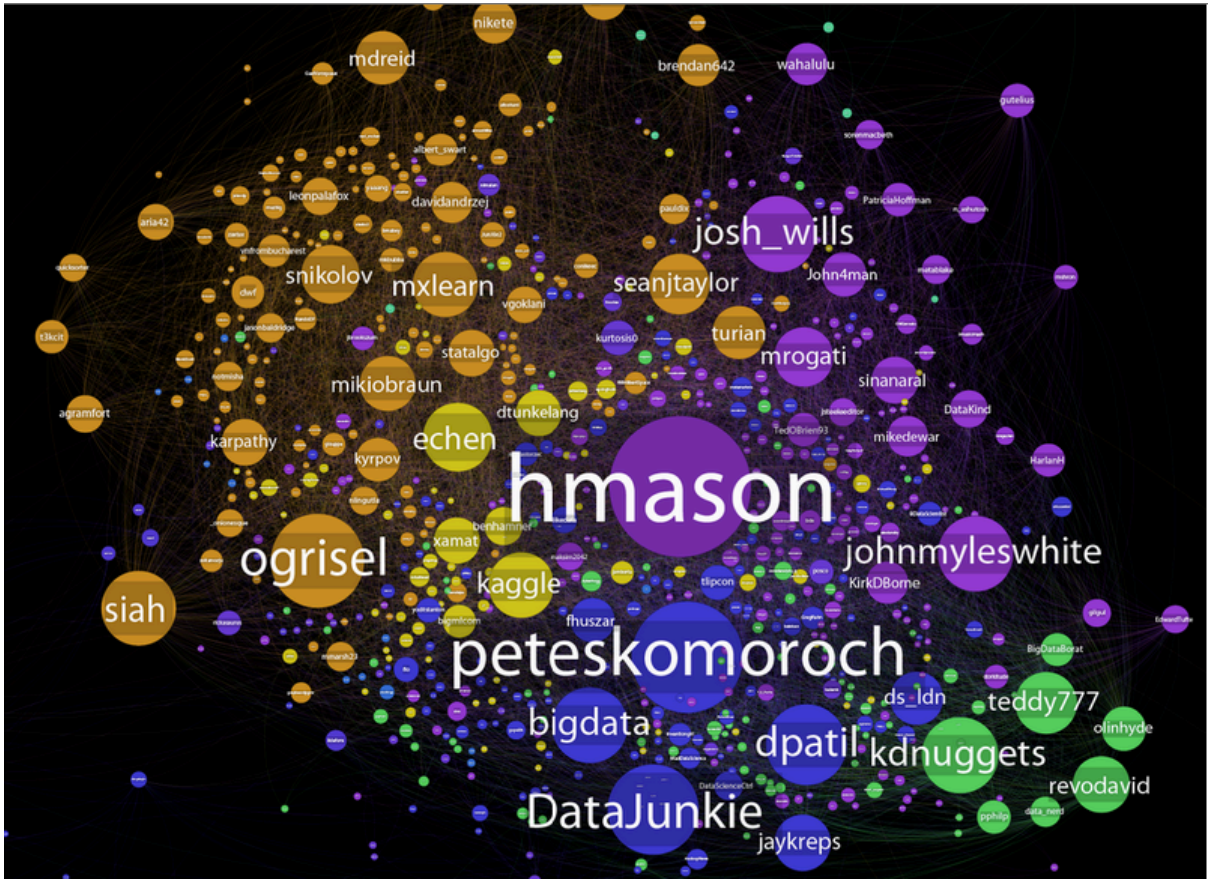

In practical terms, we need some data structures to represent and manipulate our network data.

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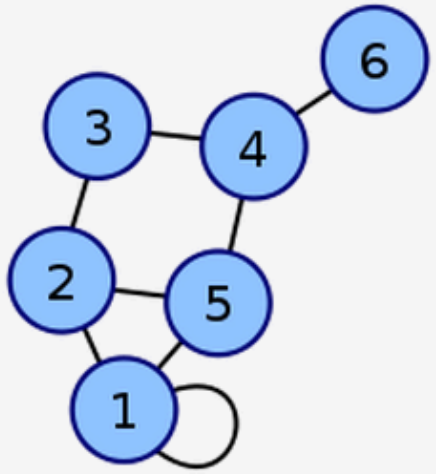
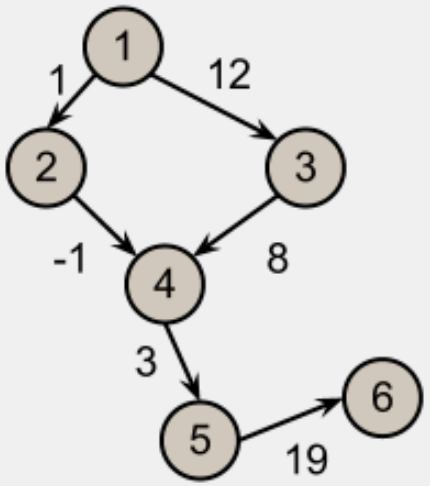
One common graph representation is the **adjacency matrix**. An n-node undirected graph can be represented by a symmetric n x n adjacency matrix A whose nonzero off-diagonal entries A_{ij} represent an edge between nodes i and j.

Labeled graph	Adjacency matrix
	$\begin{pmatrix} 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$ Coordinates are 1-6.

Weighted Directed Graph & Adjacency Matrix



Weighted Directed Graph

	1	2	3	4	5	6
1	0	1	12	0	0	0
2	-1	0	0	-1	0	0
3	-12	0	0	8	0	0
4	0	1	-8	0	3	0
5	0	0	0	-3	0	19
6	0	0	0	0	-19	0

Adjacency Matrix

NOTE

A directed graph has an *asymmetric* adjacency matrix.

Can you see why?

Another useful tool is the **adjacency list** (actually a dict!):

```
graph = {'A': ['B', 'C'],
         'B': ['C', 'D'],
         'C': ['D'],
         'D': ['C'],
         'E': ['F'],
         'F': ['C']}
```

Does this adjacency dict represent a directed or undirected graph?
How could you generalize this to represent a weighted graph?

II. NETWORK STATICS

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The simplest centrality measure is the **degree** of a node, which is simply the number of edges connected to it. Using the adjacency matrix notation for an undirected graph, we can express the degree k_i of node i as:

$$k_i = \sum_{j=1}^{n} A_{ij}.$$

A more sophisticated measure called **eigenvector centrality** allows important edges to give larger contributions to centrality:

$$x_i = \frac{1}{\lambda} \sum_{j=1}^{n} A_{ij} x_j,$$

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NOTE

To see where the name eigenvector centrality comes from, try to imagine this relation expressed in vector notation!

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Another useful centrality measure is based on the idea of shortest-distance (or *geodesic*) paths through the graph.

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If σ_{st} is the number of geodesic paths from node s to node t, and $\sigma_{st}(v)$ is the number of these paths that cross node v, then the **betweenness centrality** of node v is given by:

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NOTE

Betweenness centrality measures the proportion of geodesic paths passing through a node.

This gives an idea of the node's *influence* in the network.

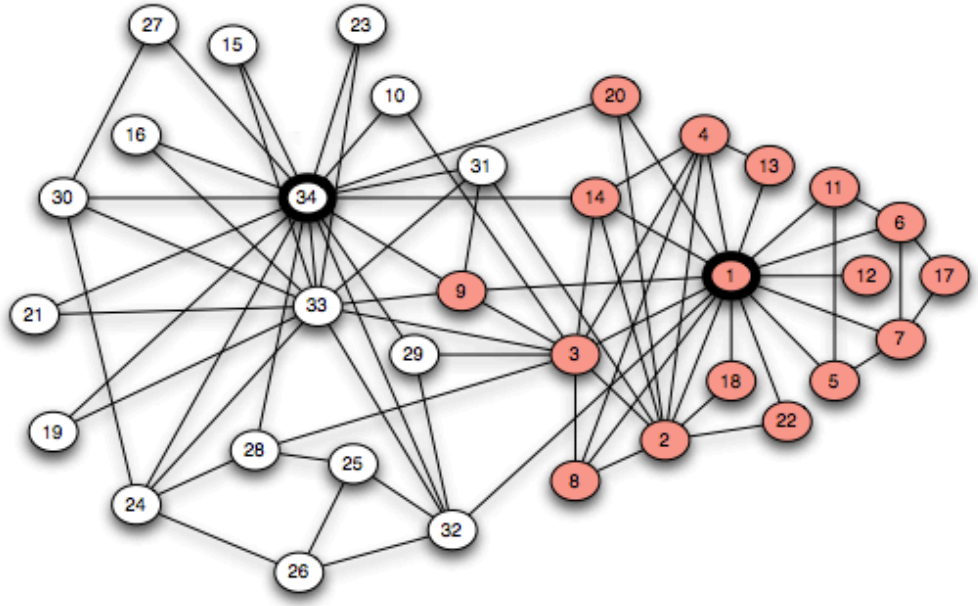

Figure 3.13: A karate club studied by Wayne Zachary [421] — a dispute during the course of the study caused it to split into two clubs. Could the boundaries of the two clubs be predicted from the network structure?

Geodesic paths form the basis of another well-known property of networks called the *small-world effect*.

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NOTE

This is where the phrase "six degrees of separation" comes from.

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INTRO TO DATA SCIENCE

II. NETWORK DYNAMICS

Suppose we're interested in the idea of how information (or behavior) spreads through a network:

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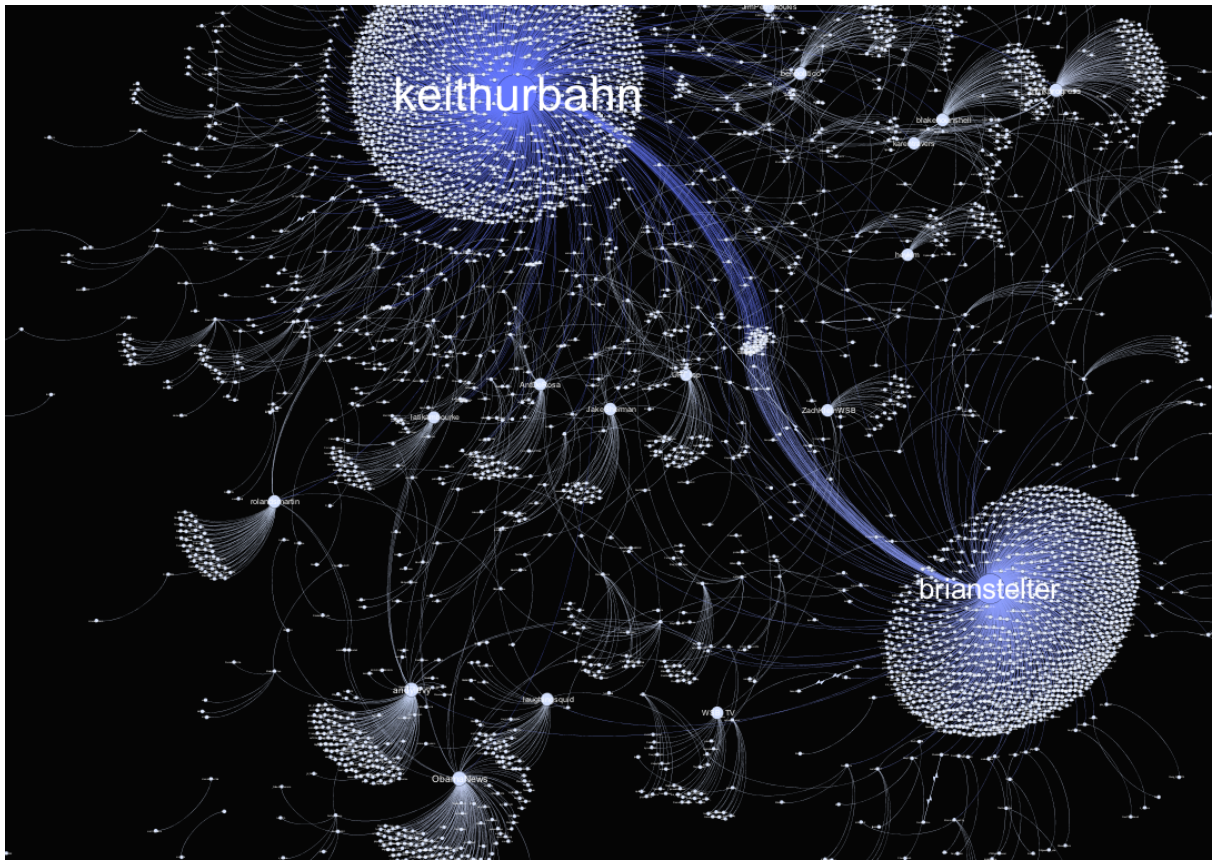
- How do members of a social network influence each other to adopt a new technology/product/behavior?
- How did information about the bin Laden raid spread over Twitter?
- What's the best way to use a social network to market your product?


kelthurbahn
brianstelter

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direct benefit effects – people may have incentives to use the same products/technology/etc as their network neighbors

Studies of informational effects have shown that while initial lack of information makes innovations risky to adopt, adopters ultimately benefit.

Furthermore, *early adopters* share certain common traits (eg higher socio-economic status, wider travel experience), and they influence their neighbors by providing indirect information about the innovation.

Consider the following model of information **diffusion** for two nodes v, w and two behaviors A, B (with payoffs a, b):

		w: A	w: B
v	A	a, a	$0, 0$
	B	$0, 0$	b, b

Figure 19.1: A-B Coordination Game

The question we'd like to answer is, how can v maximize its payoff given that some of its neighbors adopt A & some adopt B?

To start modeling this problem, suppose first that the proportion of v's neighbors selecting A is p, and the proportion selecting B is $(1\text{-}p)$.

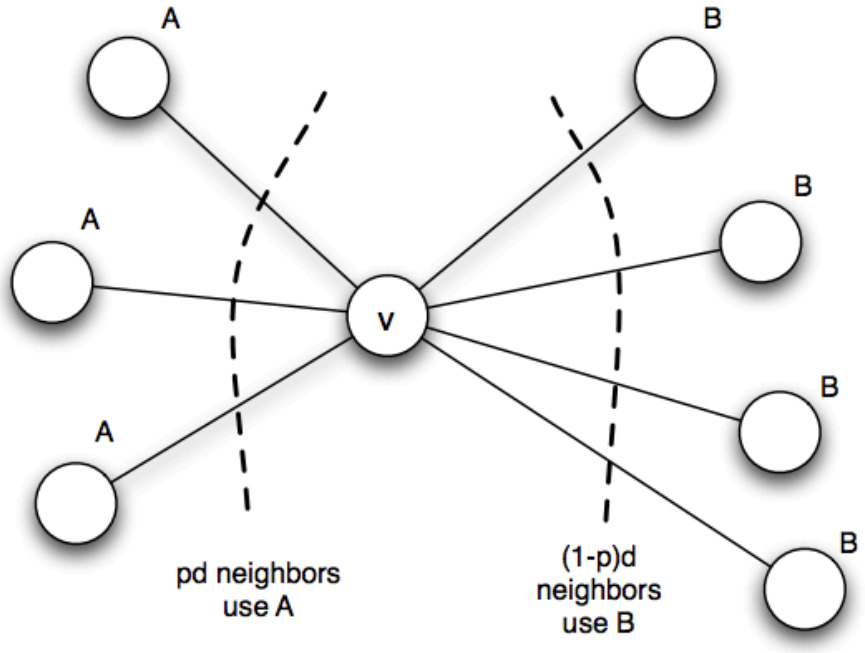

Figure 19.2: v must choose between behavior A and behavior B, based on what its neighbors are doing.

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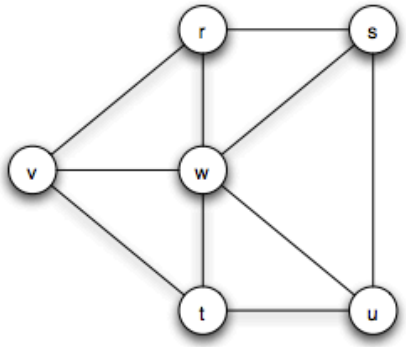
Therefore the payoff to v for choosing A is pda, and the payoff for choosing B is $(1\text{-}p)db$.

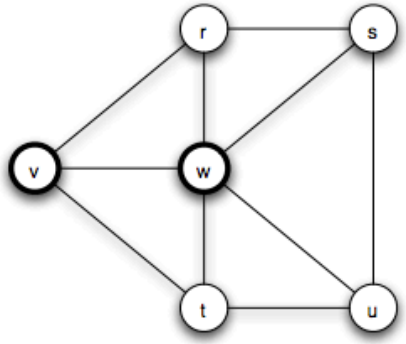
And thus v will adopt A if p (meets or) exceeds a *threshold* q:

$$pda \geq (1\text{-}p)db \quad \leftrightarrow \quad p \geq b/(a+b) = q$$

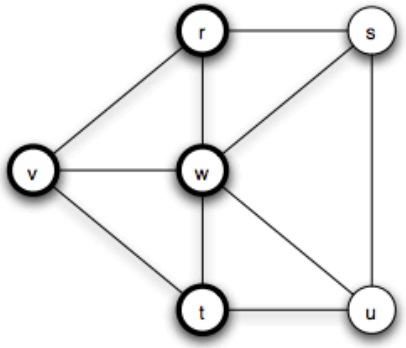
Adoption depends not only on the relative payoffs, but also on the structure of the network (eg, how many neighbors v has, and which particular nodes these neighbors are).



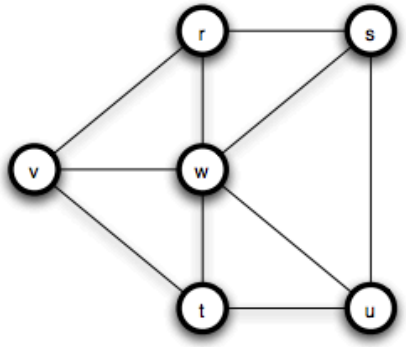
(a) *The underlying network*



(b) *Two nodes are the initial adopters*



(c) *After one step, two more nodes have adopted*

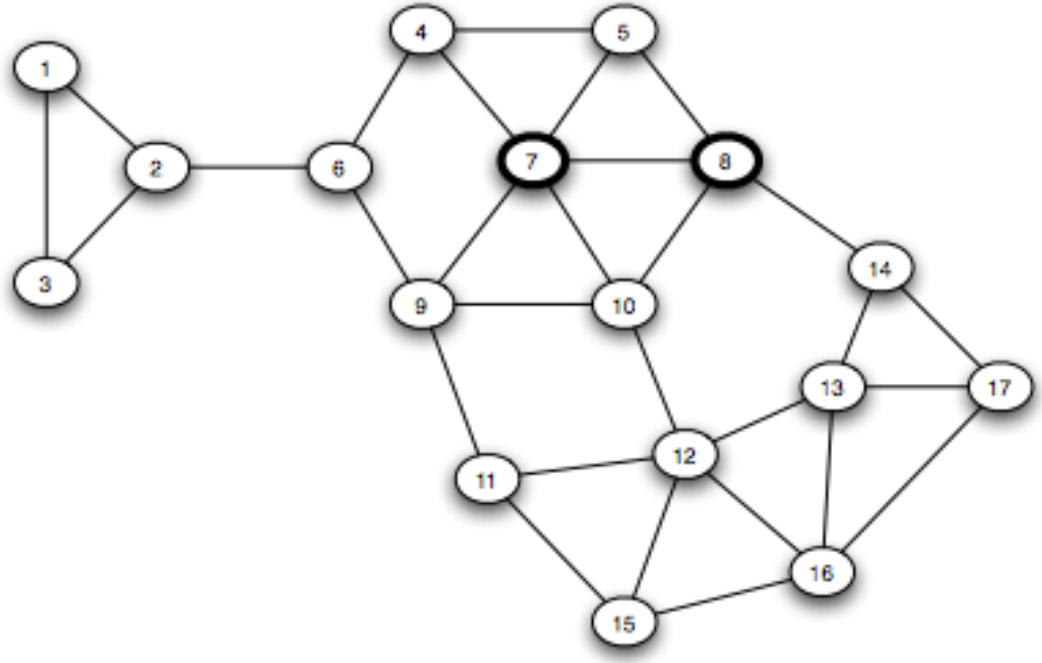


(d) *After a second step, everyone has adopted*

NOTE

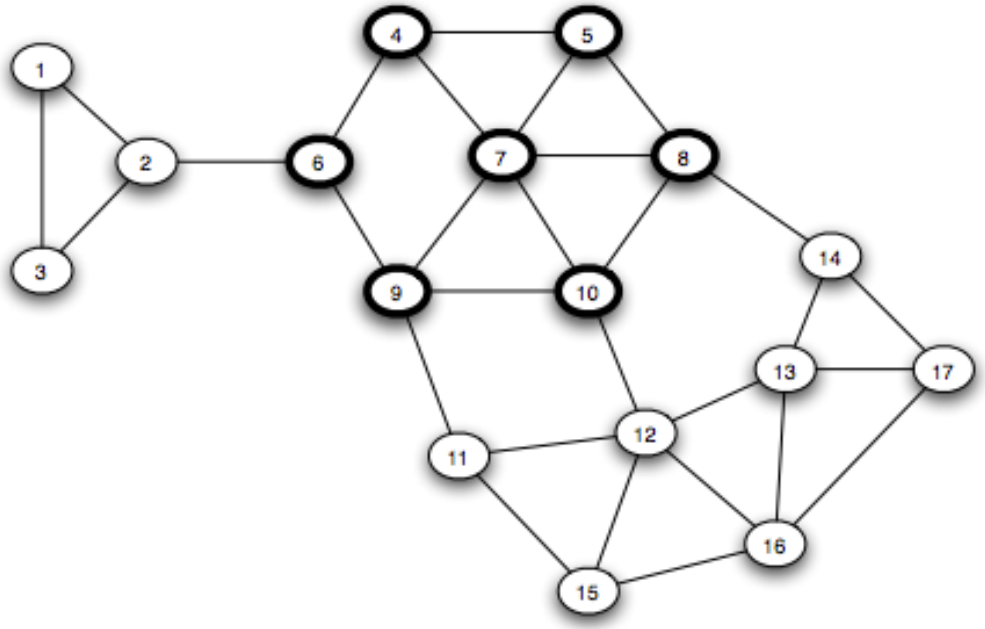
Since all nodes have adopted, this is called a *complete cascade* (at threshold q).

Consider the same diffusion now on another graph.



(a) *Two nodes are the initial adopters*

Since not all nodes adopt, this is called a *partial cascade*.



(b) *The process ends after three steps*

Here's an interesting question: how can you identify which (non-adopting) nodes are most important to allowing the cascade to continue?

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Answering this question effectively is the idea behind *viral marketing*.