Statistics (I) Quiz 4-Date: December-25, 2017

姓名：

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1. Assume that the mean systolic blood pressure of normal adults is 120 millimeters of mercury (mm Hg) and the standard deviation is 6. Assume the variable is normal distributed. Find the following probability.
   a. If an individual is selected, that the individual's pressure will be between 116.1 and 121.8 mm Hg.
   b. If a sample of 25 adults is randomly selected, the sample mean will be between 116.1 and 121.8 mm Hg.
   c. Why is the answer to part *a* so much smaller than the answer to part *b*?

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$\mu=120$, $\sigma=6$

a. $P(116.1 < X < 121.8)$ 7

$= P\left(\frac{116.1-120}{6} < \frac{X-\mu}{\sigma/\sqrt{n}} < \frac{121.8-120}{6}\right)$

$= P(-0.65 < Z < 0.3)$

$= 0.3601$

b. $P(116.1 < \bar{X} < 121.8)$ 7

$= P\left(\frac{116.1-120}{6/\sqrt{25}} < \frac{\bar{X}-\mu}{\sigma/\sqrt{n}} < \frac{121.8-120}{6/\sqrt{25}}\right)$

$= P(-3.25 < Z < 1.5) = 0.9326$

3 c. sample mean ~~are~~ is less variable than individual data.

2. The average cholesterol content of a certain brand of eggs is 215 milligrams, and the standard deviation is 15 milligrams. Assume the variable is normally distributed.
   a. If a single egg is selected, find the probability that the cholesterol content will be greater than 220 milligrams.
   b. If a sample of 36 eggs is selected, find the probability that the mean of the sample will be larger than 220 milligrams.

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$\mu=215$, $\sigma=15$

a. $P(X > 220)$ 5

$= P\left(\frac{X-\mu}{\sigma} > \frac{220-215}{15}\right)$

$= P(Z > 0.33)$

$= 0.3707$

b. $P(\bar{X} > 220)$ 5

$= P\left(\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} > \frac{220-215}{15/\sqrt{36}}\right)$

$= P(Z > 2)$

$= 0.0228$

3. The standard deviation of variable is 15. If a sample of 100 individuals is selected, compute the standard error of the mean. (a) What size sample is necessary to double the standard error of the mean?
   (b)What size sample is needed to cut the standard error of the mean in half?

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$\sigma=15$, $n=100$

a $\sigma_{\bar{x}} = \sigma/\sqrt{n}$ 2

$= 15/\sqrt{100}$

$= 1.5$

a. 4

$2(1.5) = \frac{15}{\sqrt{n}}$

$3\cdot\sqrt{n} = 15$

$\sqrt{n} = 5$

$\therefore n = 25$

b. 4

$\frac{1.5}{2} = \frac{15}{\sqrt{n}}$

$0.75\cdot\sqrt{n} = 15$

$\sqrt{n} = 20$

$\therefore n = 400$

4. A mail order company has an 8% success rate. If it mails advertisements to 600 people,
   (a) find the probability of getting fewer than 40 sales. (b) find the probability of getting between 45 and 52 sales.

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$\mu = np = 48$ 2

$\sigma = \sqrt{npq} = 6.65$ 2

a. $P(X < 40)$ 6

$= P(X \le 39.5)$

$= P\left(Z < \frac{39.5-48}{6.65}\right)$

$= P(Z < -1.28) = 0.1003$

b. $P(45 \le X \le 52)$ 8

$= P(44.5 < X < 52.5)$

$= P\left(\frac{44.5-48}{6.65} < Z < \frac{52.5-48}{6.65}\right)$

$= P(-0.53 < Z < 0.68)$

$= 0.7517 - 0.2981 = 0.4536$

5. How large a sample is needed to estimate the population mean for the amount of money a person spends on Christmas presents within 500 and be 95% confident? The standard deviation of the population is \$1500.

5

$e \le 500$

95% C.I.

$\sigma = 1500$

$e = Z\cdot\sigma/\sqrt{n} \Rightarrow n = \left(\frac{Z\cdot\sigma}{e}\right)^2$ 2

1

$= 35$ 2

6. A survey show that 85% of households own smartphones. If a random sample of 200 households is selected, what is the probability that (a) more than 160 but fewer than 180 have smartphones? (b) at least 155 have smartphones?

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$p = 85\%$

$n = 200$

$\mu = np = 170$. 2

$\sigma^2 = npq = 25.5$ 2

$\sigma = \sqrt{npq} = 5.05$

a. $P(160 < X < 180)$

8

$= P(160.5 < X < 179.5)$

$= P\left(\frac{160.5-170}{5.05} < Z < \frac{179.5-170}{5.05}\right)$

$= P(-1.88 < Z < 1.88)$

$= 0.9398$

b. $P(X \geq 155)$

6

$= P(X > 154.5)$

$= P\left(Z > \frac{154.5-170}{5.05}\right)$

$= P(Z > -3.07)$

$= 0.9989$

7. A sociologist found that in a random sample of 64 retired men, the average number of jobs they had during their lifetimes was 7.2. The population standard deviation is 2.1.
   a. Find the best point estimate of the population mean.
   b. Find the 95% confidence interval of the mean number of jobs.
   c. Find the 99% confidence interval of the mean number of jobs.
   d. Which is smaller? Explain why.

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$n = 64$

$\bar{x} = 7.2$

$\sigma = 2.1$

2 a. Sample mean 7.2

b. 95% C.I. $\Rightarrow Z = 1.96$ 1

$\bar{x} \pm Z \cdot \sigma/\sqrt{n}$

$\Rightarrow 7.2 \pm 1.96 \times 2.1/\sqrt{64}$ 2

$\Rightarrow (6.6855, 7.7145)$ 2

c. $99\% \Rightarrow Z = 2.33$ 1

$\bar{x} \pm Z \cdot \sigma/\sqrt{n}$ 2

$= (6.523, 7.877)$

2

d. 2 b. 小 ∵ $\overset{b}{95\%} < \overset{c}{99\%}$

8. A researcher wishes to estimate the average number of minutes per day a person spends on the Internet. Assume the population standard deviation is 42 minutes.
   (a) How large a sample must she select is she wishes to be 90% confident that the population mean is within 10 minutes of the sample mean?
   (b) How large a sample must she select is she wishes to be 98% confident that the population mean is within 10 minutes of the sample mean?

$\sigma = 42$.

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a. $e = 10$. 90% C.I. 1

$Z = 1.65$

$e = Z \cdot \frac{\sigma}{\sqrt{n}}$

$\Rightarrow n = \left(\frac{Z \cdot \sigma}{e}\right)^2$ 2

$= 49$ 2

b. $e = 10$, 98% C.I.

$\Rightarrow Z = 2.33$ 1

$n = \left(\frac{Z \cdot \sigma}{e}\right)^2$ 2

$= 96$ 2