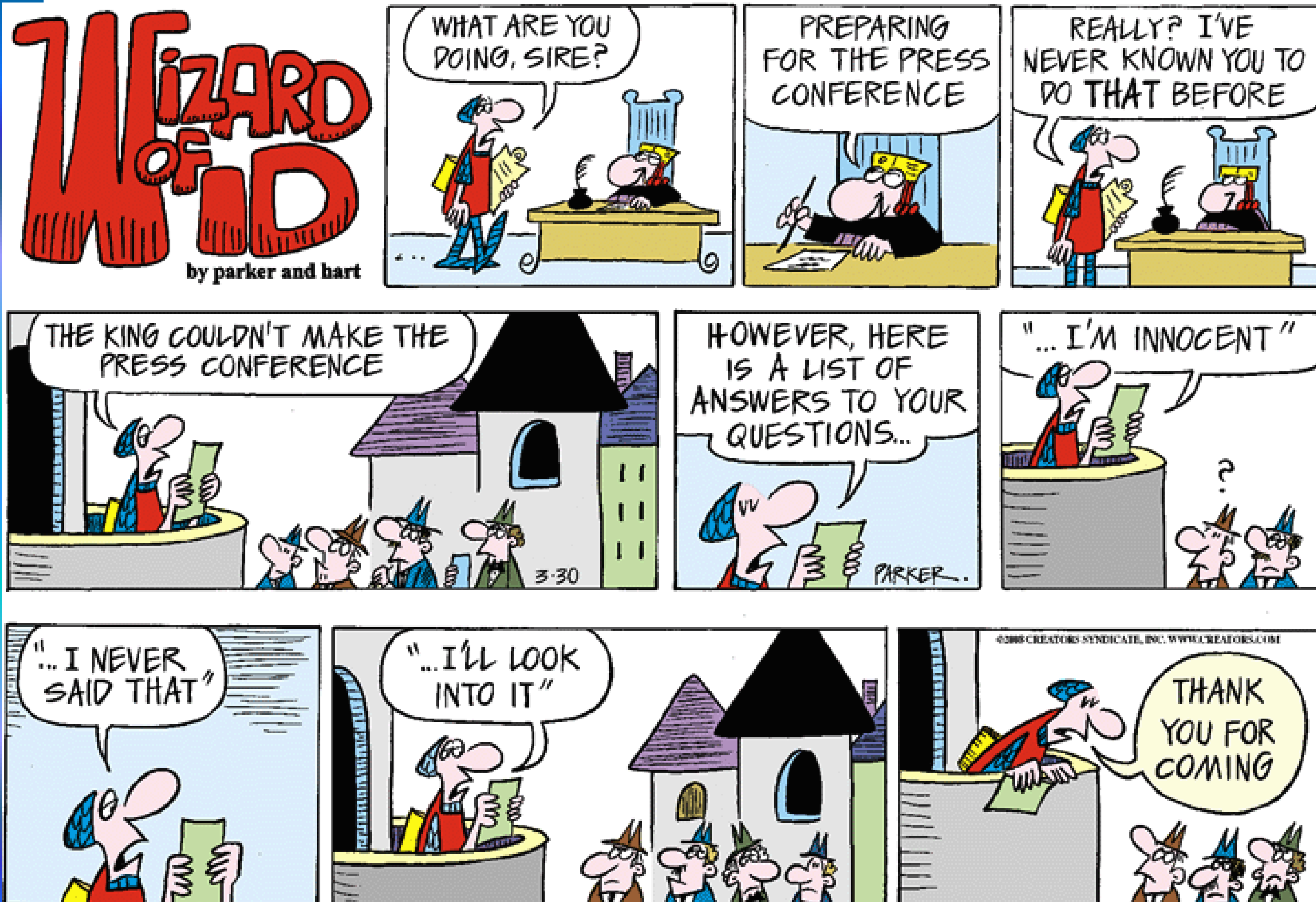
Introductory Statistics


Wizard of Id
by parker and hart
WHAT ARE YOU DOING, SIRE?
PREPARING FOR THE PRESS CONFERENCE
REALLY? I'VE NEVER KNOWN YOU TO DO THAT BEFORE
THE KING COULDN'T MAKE THE PRESS CONFERENCE
3-30
HOWEVER, HERE IS A LIST OF ANSWERS TO YOUR QUESTIONS...
PARKER
"... I'M INNOCENT"
?
"... I NEVER SAID THAT"
"... I'LL LOOK INTO IT"
THANK YOU FOR COMING

Chapter 9

Hypothesis Tests for One Population Mean Part Two

Section 9.5

Hypothesis Tests for One Population Mean When Sigma Is Unknown

Inference About mean μ (σ unknown)

- Inferences about μ are based on the sample mean $\bar{x}$
- If the sample size is large or the sample population is normal and σ is given: $z^* = (\bar{x} - \mu)/(\sigma/\sqrt{n})$ has a standard normal distribution
- If σ is unknown, use *s* as a point estimate for σ
- Estimated standard error of the mean: $s/\sqrt{n}$
- Test statistic: $t = (\bar{x} - \mu)/(s/\sqrt{n})$

Student's *t*-Statistic

1. When s is used as an estimate for σ, the test statistic has two sources of variation: $\bar{x}$ and s

2. The resulting test statistic:

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

Known as the **Student's *t*-statistic**

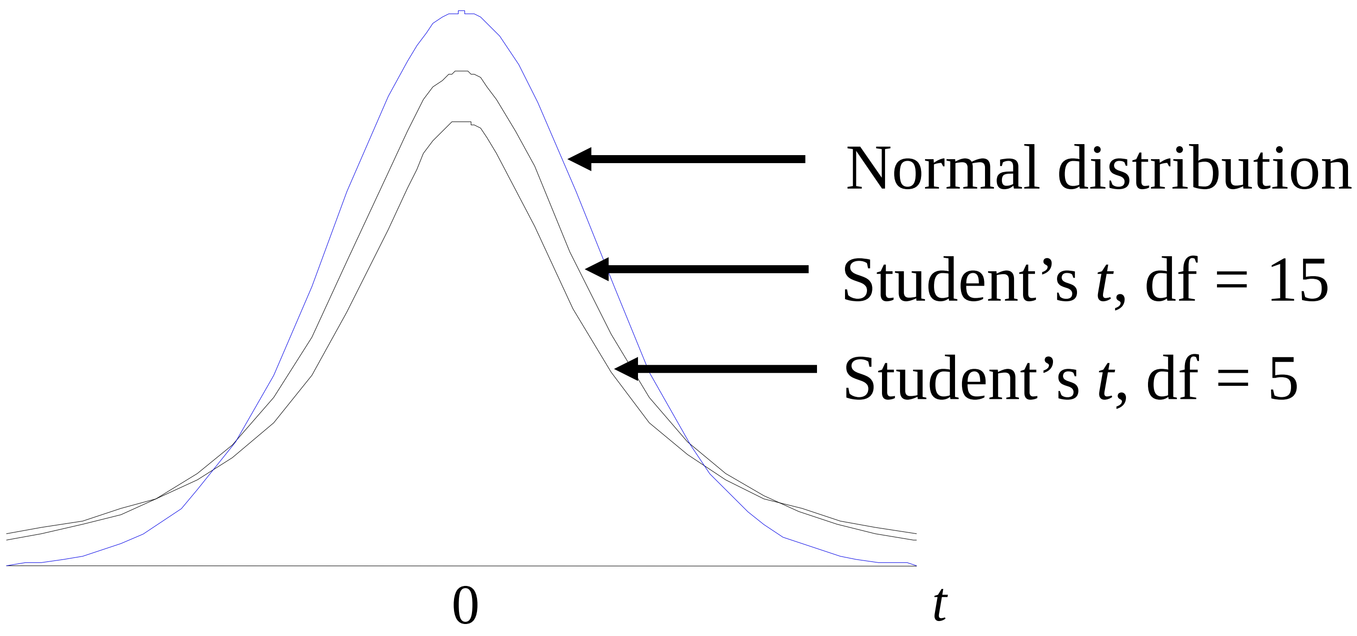
3. Assumption: samples are taken from normal populations

4. The population standard deviation, σ, is almost never known in real-world problems

   The standard error will almost always be estimated using $s/\sqrt{n}$

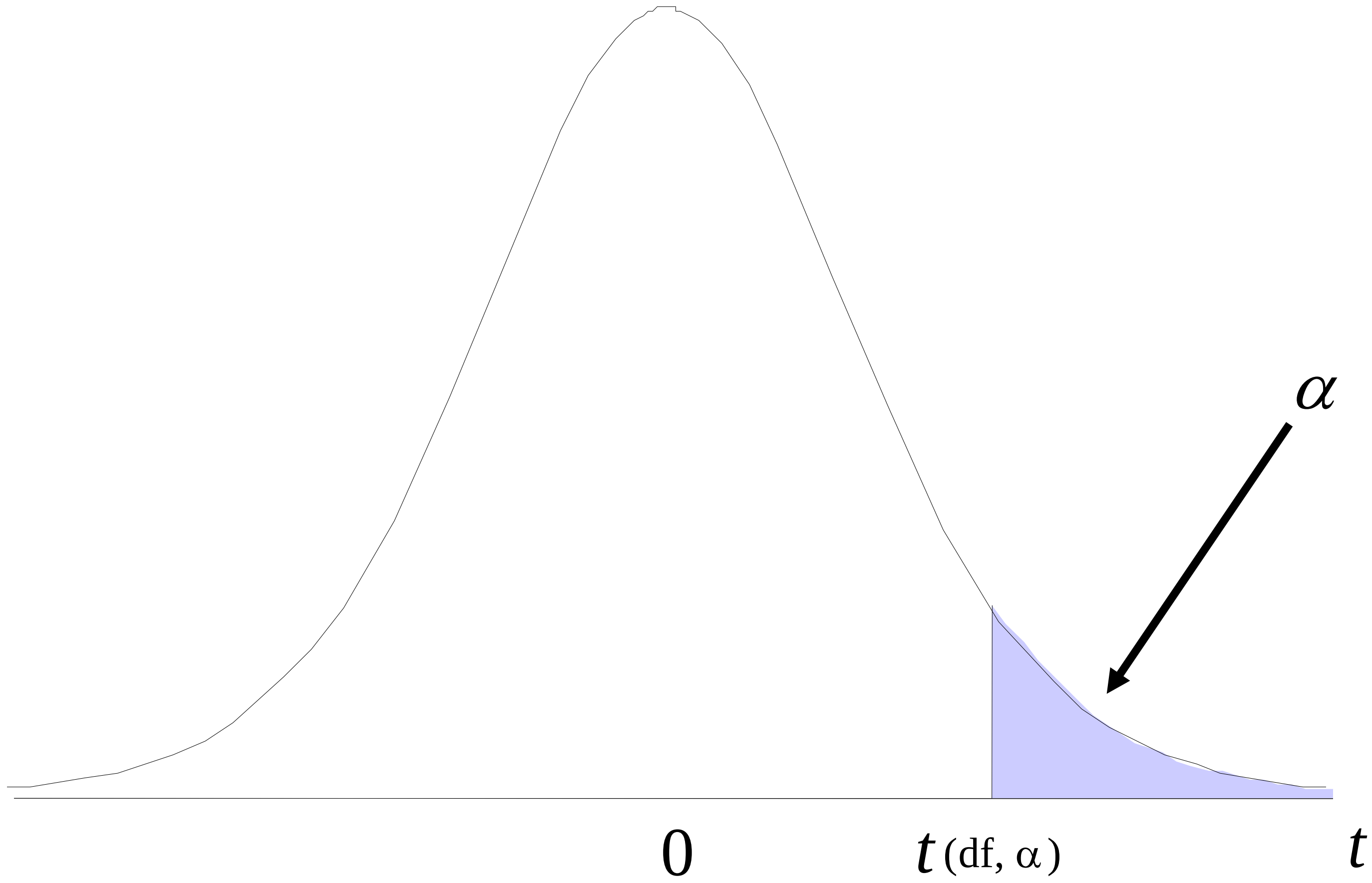
   Almost all real-world inference about the population mean will be completed using the Student's t-statistic

Student's *t*-Distributions



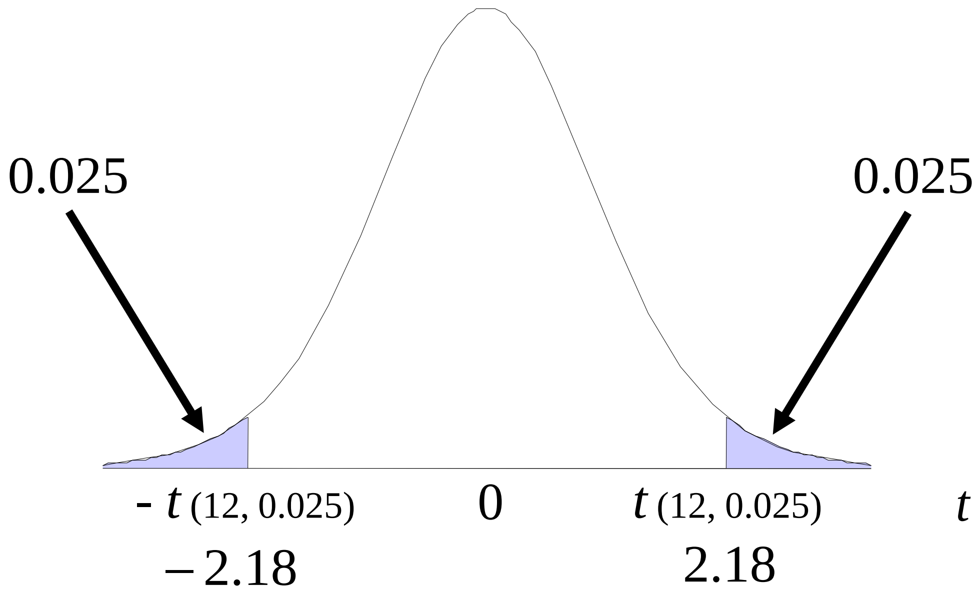
Degrees of Freedom, df: A parameter that identifies each different distribution of Student's *t*-distribution. For the methods presented in this chapter, the value of df will be the sample size minus 1, df = $n - 1$.

t-Distribution Showing *t*(df, α)



Example

Example: Find the value of $t_{(12, 0.025)}$



Portion of Table 6

df	Amount of α in one-tail		
	⋯	0.025	⋯
⋮			
12		2.18	

Notes

1. If the df is *not* listed in the left-hand column of the t-score table, use the next smaller value of df that is listed
2. Most computer software packages will calculate either the area related to a specified t-value or the t-value that bounds a specified area

Figure 9.17

Estimating the P-value of a left-tailed t-test with a sample size of 12 and test statistic t = −1.938

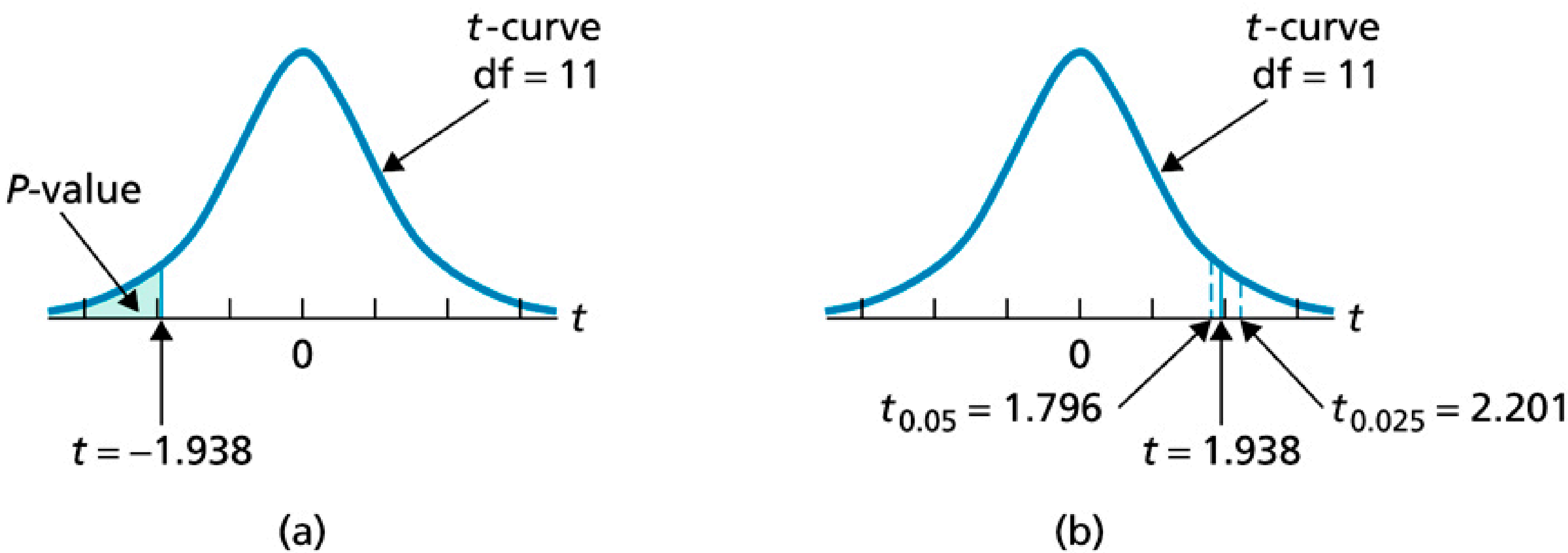
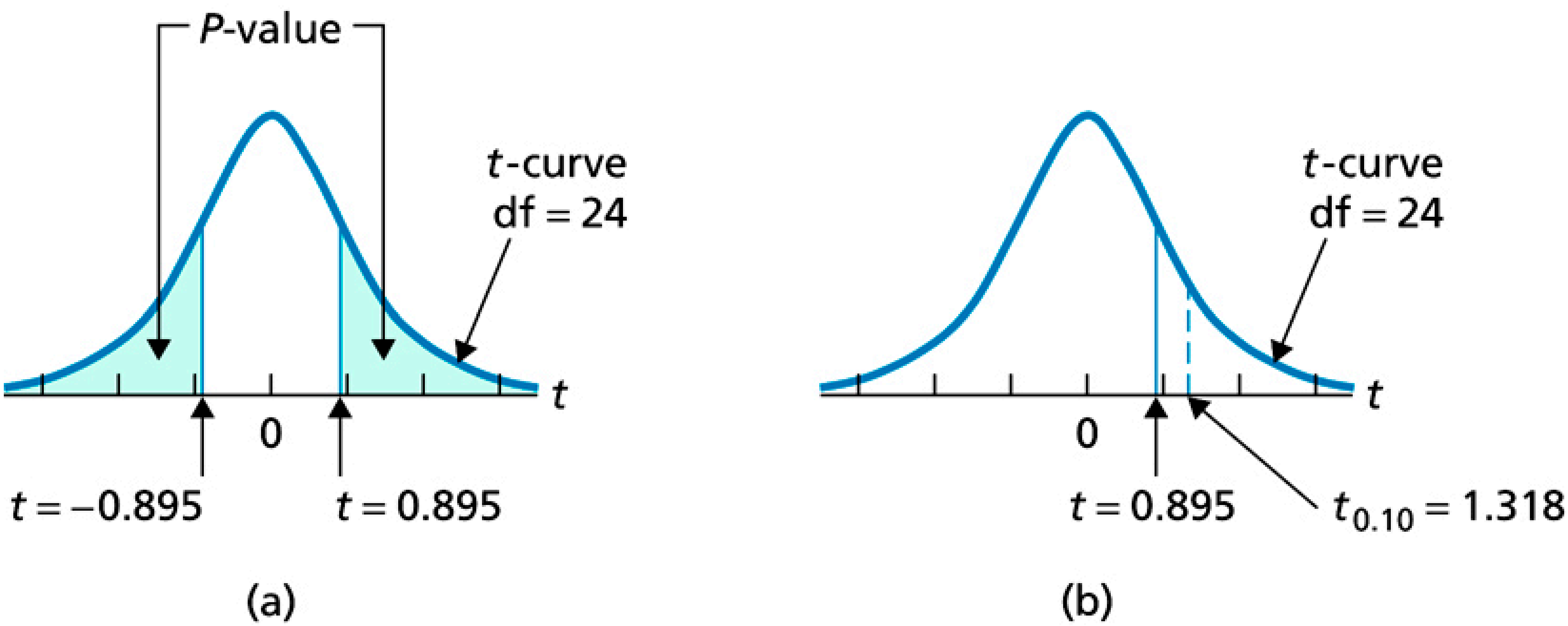


Figure 9.18

Estimating the P-value of a two-tailed t-test with a sample size of 25 and test statistic t = −0.895



Procedure 9.2

One-Mean *t*-Test

Purpose To perform a hypothesis test for a population mean, μ

Assumptions

1. Simple random sample
2. Normal population or large sample
3. σ unknown

Step 1 **The null hypothesis is H_0: $\mu = \mu_0$, and the alternative hypothesis is**

$$H_a\!: \mu \neq \mu_0 \text{ (Two tailed)} \quad \text{or} \quad H_a\!: \mu < \mu_0 \text{ (Left tailed)} \quad \text{or} \quad H_a\!: \mu > \mu_0 \text{ (Right tailed)}$$

Step 2 **Decide on the significance level, α.**

Step 3 **Compute the value of the test statistic**

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

and denote that value t_0.

Procedure 9.2 (cont.)

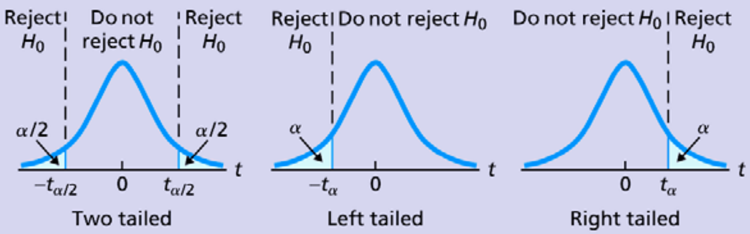
CRITICAL-VALUE APPROACH	OR	P-VALUE APPROACH

CRITICAL-VALUE APPROACH

Step 4 The critical value(s) are

$\pm t_{\alpha/2}$ (Two tailed) or $-t_{\alpha}$ (Left tailed) or t_{α} (Right tailed)

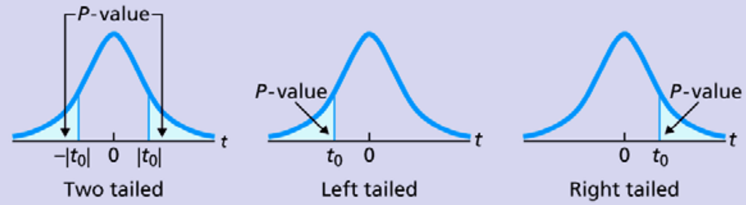
with df $= n - 1$. Use Table IV to find the critical value(s).



Step 5 If the value of the test statistic falls in the rejection region, reject H_0; otherwise, do not reject H_0.

P-VALUE APPROACH

Step 4 The t-statistic has df $= n - 1$. Use Table IV to estimate the P-value, or obtain it exactly by using technology.



Step 5 If $P \leq \alpha$, reject H_0; otherwise, do not reject H_0.

Step 6 Interpret the results of the hypothesis test.

Note: The hypothesis test is exact for normal populations and is approximately correct for large samples from nonnormal populations.

Example

✓ **Example:** A random sample of 25 students registering for classes showed the mean waiting time in the registration line was 22.6 minutes and the standard deviation was 8.0 minutes. Is there any evidence to support the student newspaper's claim that registration time takes longer than 20 minutes? Use $\alpha = 0.05$ and assume waiting time is approximately normal.

Solution:

1. *The Set-up*

   a. Population parameter of concern: the mean waiting time spent in the registration line

   b. State the null and alternative hypotheses:

   H_o: $\mu = 20$ ($\leq$) (no longer than)

   H_a: $\mu > 20$ (longer than)

Solution Continued

2. *The Hypothesis Test Criteria*

   a. Check the assumptions:
   The sampled population is approximately normal

   b. Test statistic: t^* with df = n - 1 = 24

   c. Level of significance: $\alpha = 0.05$

3. *The Sample Evidence*

   a. Sample information: $n = 25, \quad \bar{x} = 22.6, \quad \text{and} \quad s = 8$

   b. Calculate the value of the test statistic:

$$t^* = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{22.6 - 20}{8/\sqrt{25}} = \frac{2.6}{1.6} = 1.625$$

Solution Continued

Using the *p*-Value Procedure:

4. *The Probability Distribution*

   a. The p-value: $\mathbf{P} = P(t^* > 1.625, \text{ with df } = 24)$

Notes:

- If this hypothesis test is done with the aid of a computer, most likely the computer will compute the p-value for you
- Using Table 6: *place bounds* on the p-value
- Using Table 7: read the *p*-value directly from the table for many situations:

Using Table 6: $0.05 < \mathbf{P} < 0.10$

Using Table 7: $\mathbf{P} \approx 0.061$

b. The *p*-value is not smaller than the level of significance, α

Solution Continued

Using the Classical Procedure:

4. *The Probability Distribution*
   a. The critical value: $t_{(24,\ 0.05)} = 1.71$

   b. t^* is not in the critical region

5. *The Results*
   a. Decision: Fail to reject H_o
   b. Conclusion: There is insufficient evidence to show the mean waiting time is greater than 20 minutes at the 0.05 level of significance

Example

- **Example:** A new study indicates that higher than normal (220) cholesterol levels are a good indicator of possible heart attacks. A random sample of 27 heart attack victims showed a mean cholesterol level of 231 and a standard deviation of 20. Is there any evidence to suggest the mean cholesterol level is higher than normal for heart attack victims? Use $\alpha = 0.01$ and assume cholesterol levels is approximately normal.

Solution:

1. *The Set-up*

   a. Population parameter of concern: The mean cholesterol level of heart attack victims

   b. State the the null and alternative hypothesis:

   H_o: $\mu = 220$ ($\leq$) (mean is not greater than 220)
   H_a: $\mu > 220$ (mean is greater than 220)

Solution Continued

2. *The Hypothesis Test Criteria*

   a. Assumptions: We will assume cholesterol level is at least approximately normal.

   b. Test statistic: t^* (σ unknown), df = $n - 1 = 26$

   c. Level of significance: $\alpha = 0.01$ (given)

3. *The Sample Evidence*

   a. Sample information: $n = 27$, $\bar{x} = 231$, and $s = 20$

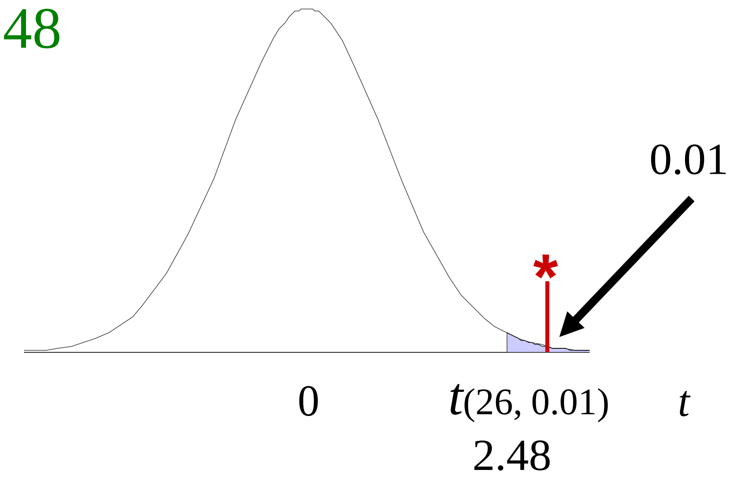
   b. Calculate the value of the test statistic:

$$t^* = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{231 - 220}{20/\sqrt{27}} = \frac{11}{3.849} = 2.858$$

Solution Continued

4. *The Probability Distribution*

   a. The critical value: $t_{(26,\ 0.01)} = 2.48$



   b. t^* falls in the critical region

5. *The Results*

   a. Decision: Reject H_0

   b. Conclusion: At the 0.01 level of significance, there is sufficient evidence to suggest the mean cholesterol level in heart attack victims is higher than normal

Section 9.6

The Wilcoxon Signed-Rank Test

Parametric vs. Nonparametric Statistics

- **Parametric Statistics:** are statistical techniques based on assumptions about the population from which the sample data are collected (i.e. Assume the population is at least approximately normal, or use the central limit theorem).
  - Assumption that data being analyzed are randomly selected from a normally distributed population.
  - Requires quantitative measurement that yield interval or ratio level data.

- **Nonparametric Statistics:** are based on fewer assumptions about the population and the parameters (i.e. Assume very little about the population, subject to less confining restrictions).
  - Sometimes called “distribution-free” statistics.
  - A variety of nonparametric statistics are available for use with nominal or ordinal data.

Advantages of Nonparametric Techniques

- Sometimes there is no parametric alternative to the use of nonparametric statistics (i.e. normality assumptions cannot be made).
- Certain nonparametric test can be used to analyze nominal data or ordinal data.
- The computations on nonparametric statistics are usually less complicated than those for parametric statistics, particularly for small samples (Generally easier to apply than their parametric counterparts).
- Probability statements obtained from most nonparametric tests are exact probabilities.
- Require few assumptions about the underlying population.
- Relatively easy to understand.

Disadvantages of Nonparametric Statistics

- Nonparametric tests can be wasteful of data if parametric tests are available for use with the data.
- Nonparametric tests are usually not as widely available and well know as parametric tests.
- For large samples, the calculations for many nonparametric statistics can be tedious.
- Generally only slightly less efficient than their parametric counterparts

Comparing Statistical Tests

- Various nonparametric tests presented in this chapter. There are many others.
- Many nonparametric tests may be used as well as certain parametric tests
- Which statistical test is appropriate: the parametric or nonparametric

RANKS

To rank observations, first arrange them in order from smallest to largest. The **rank** of each observation is its position in this ordered list, starting with rank 1 for the smallest observation.

The Rankings

Ranked			Ranked		
Data	Rank		Data	Rank	
61	1.5	1	69	11	11
61	1.5	2	69	11	12
63	4	3	70	13.5	13
63	4	4	70	13.5	14
63	4	5	71	15	15
65	6	6	75	16	16
66	7	7	76	17	17
68	8.5	8	77	18	18
68	8.5	9	78	19	19
69	11	10	83	20	20

Example 9.18, on P400, introducing the Wilcoxon Signed-Rank Test

Want to test the hypothesis that the mean expenses each family spend on food per week is \$157. A sample of 10 is recorded and summarized in the table in the next slide.

The null and the alternative hypothesis is the follow:

H_o: μ = \$157 (mean is not lesser than 157)

H_a: μ < \$157 (mean is less than 157)

Table 9.13

Sample of weekly food costs ($)

143	169	149	135	161
138	152	150	141	159

Table 9.14

Steps for ranking the data in Table 9.13 according to distance and direction from the null hypothesis mean

Cost ($) x	Difference $D = x - 157$	$\lvert D \rvert$	Rank of $\lvert D \rvert$	Signed rank R
143	−14	14	7	−7
138	−19	19	9	−9
169	12	12	6	6
152	−5	5	3	−3
149	−8	8	5	−5
150	−7	7	4	−4
135	−22	22	10	−10
141	−16	16	8	−8
161	4	4	2	2
159	2	2	1	1

Step 1 *Subtract μ_0 from x.*

Step 2 *Make each difference positive by taking absolute values.*

Step 3 *Rank the absolute differences in order from smallest (1) to largest (10).*

Step 4 *Give each rank the same sign as the sign in the Difference column.*

Procedure 9.3

Wilcoxon Signed-Rank Test

Purpose To perform a hypothesis test for a population mean, μ

Assumptions

1. Simple random sample
2. Symmetric population

Step 1 **The null hypothesis is H_0: $\mu = \mu_0$, and the alternative hypothesis is**

$$H_a: \mu \neq \mu_0 \text{ (Two tailed)} \quad \text{or} \quad H_a: \mu < \mu_0 \text{ (Left tailed)} \quad \text{or} \quad H_a: \mu > \mu_0 \text{ (Right tailed)}$$

Step 2 **Decide on the significance level, α.**

Step 3 **Compute the value of the test statistic**

$$W = \text{sum of the positive ranks}$$

and denote that value W_0. To do so, construct a work table of the following form.

Observation x	Difference $D = x - \mu_0$	$\lvert D \rvert$	Rank of $\lvert D \rvert$	Signed rank R
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.

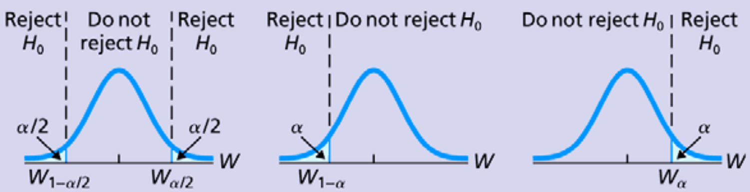
Procedure 9.3 (cont.)

CRITICAL-VALUE APPROACH	OR	P-VALUE APPROACH

Step 4 The critical value(s) are

$W_{1-\alpha/2}$ and $W_{\alpha/2}$ (Two tailed) or $W_{1-\alpha}$ (Left tailed) or W_{α} (Right tailed)

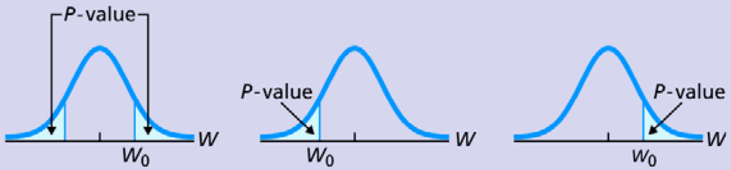
Use Table V to find the critical value(s). For a left-tailed or two-tailed test, you will also need the relation $W_{1-A} = n(n+1)/2 - W_A$.



Two tailed Left tailed Right tailed

Step 5 If the value of the test statistic falls in the rejection region, reject H_0; otherwise, do not reject H_0.

Step 4 Obtain the P-value by using technology.



Two tailed Left tailed Right tailed

Step 5 If $P \leq \alpha$, reject H_0; otherwise, do not reject H_0.

Step 6 Interpret the results of the hypothesis test.

Wilcoxon Signed-Rank Table (P464)

From Table V, we can find the value for W_α and $W_{\alpha/2}$ (right hand side of the value). However, we need the next two formulas for the value on the left, by using the fact of symmetric.

$$W_{1-\alpha} = \frac{n(n+1)}{2} - W_\alpha \qquad (9.1)$$

$$W_{1-\alpha/2} = \frac{n(n+1)}{2} - W_{\alpha/2} \qquad (9.2)$$

Example: According to an article, the quality of different vegetable, minestrone, and chicken noodle canned soups yielded the following data on the number of calories per serving. The serving size is typically 1 cup.

110	240	160	110	90	110	90
110	110	100	90	170	140	80

1. Obtain a normal-probability-plot for these data, what can you tell me?

2. Next conduct a hypothesis test at 5% significance, to check whether the mean number of calories per serving for all cans of noodle soups is greater than 120?

We have $n = 14$

A Normal-Probability-Plot

The Answer Is

Example Beverage Expenditures: A report publishes information on average annual expenditures by consumers. In 2002, the mean amount spent per consumer unit on beverages was $254. A random sample of 14 consumer unit yielded the following data on last year's expenditures on beverages.

395	210	218	254	293	299	315
274	307	293	283	228	254	292

Suppose that we assume the data is not normally distributed. At the 2% significance, do the data provide sufficient evidence to contradict the claim that the average annual expenditures by consumers remain unchanged?

Section 9.8

Which Procedure Should Be Used?

Figure 9.35

Flowchart for choosing the correct hypothesis testing procedure for a population mean