Hypothesis Testing

For one population parameter

Introduction

- 統計學鼻祖—英國學者 R.A. Fisher
  - 某女士提出：奶茶調製順序對風味有很大影響
    - 方法一：把茶加進牛奶裡
    - 方法二：把牛奶加進茶裡
  - 真的嗎？如何確認？

例:奶茶調製

是否能正確指出先放茶或牛奶

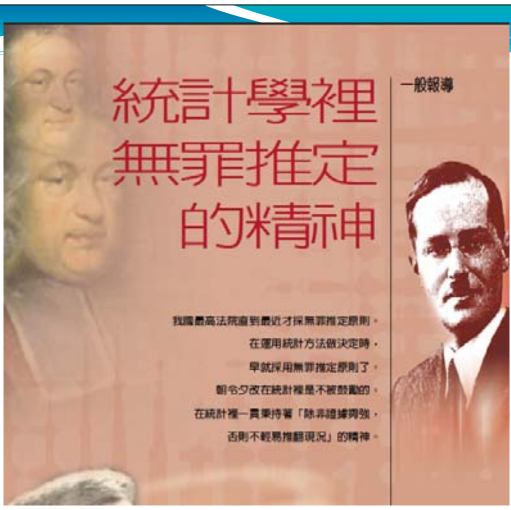
- 連續 1 次猜中的機率為 $(0.5)^1 = 0.5$
- 連續 2 次猜中的機率為 $(0.5)^2 = 0.25$
- 連續 3 次猜中的機率為 $(0.5)^3 = 0.125$
- 連續 4 次猜中的機率為 $(0.5)^4 = 0.0625$
- 連續 5 次猜中的機率為 $(0.5)^5 = 0.03125$
- 連續 6 次猜中的機率為 $(0.5)^6 = 0.015625$

- 連續 10 次猜中的機率為 $(0.5)^{10} = 0.00009765$

真的
有差嗎??

假設檢定的意義

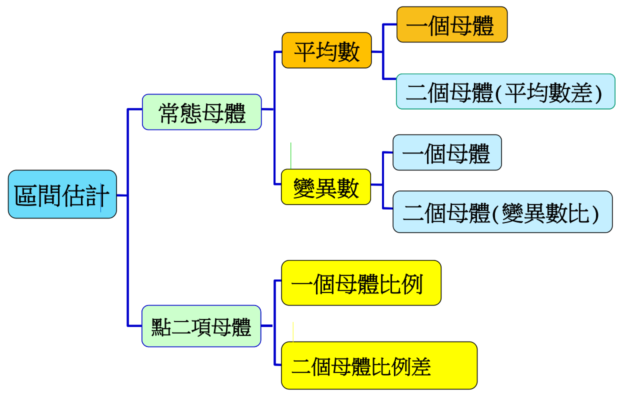
- 假設(Hypothesis)
  - 是一種說法，或是一種傳說，它是對或是錯，需要經過驗證，統計上稱此驗證過程為檢定(Testing)。
- 對有關母體參數的假設，利用樣本的訊息，決定接受(不拒絕)該假設或拒絕該假設的統計方法。
- 例如：「奶茶的調製順序會影響風味」，這樣的說法是真是假？ 有待證明。

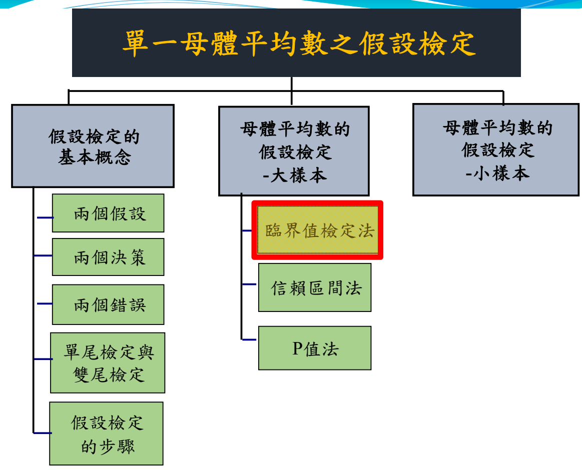


Source: http://ejournal.stpi.narl.org.tw/NSC_INDEX/Journal/EJ0001/9311/9311-10.pdf

法官判案與假設檢定

- 專制時代，寧可錯殺一百，不能錯放一個
  - 有罪推定原則
    - 舉證推翻“有罪”的假設
    - 必須找證據證明嫌疑犯沒有做、或「除非法官確信你無罪，才會判你無罪」
- 民主時代：被告未經審判證明有罪確定前，推定其為無罪
  - 無罪推定原則
    - 舉證推翻“無罪”的假設
    - 找出犯罪事實
    - 我國司法於2003年推動無罪推定原則



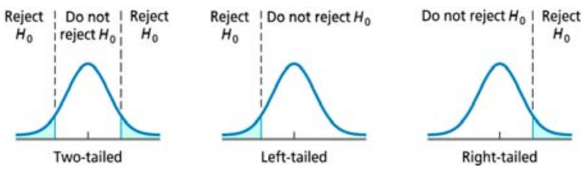


Hypothesis Testing

Researchers are interested in answering many types of questions. For example,

- Is the earth warming up?
- Does a new medication lower blood pressure?
- Does the public prefer a certain color in a new fashion line?
- Is a new teaching technique better than a traditional one?
- Do seat belts reduce the severity of injuries?

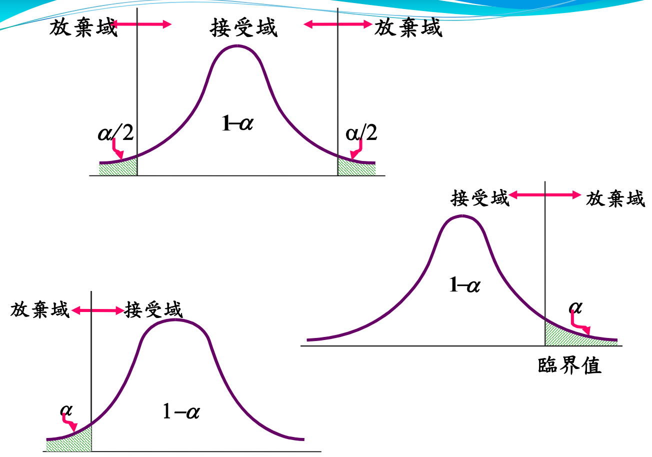
These types of questions can be addressed through statistical hypothesis testing, which is a decision-making process for evaluating claims about a population.



	Two-Tailed Test	Left-Tailed Test	Right-Tailed Test
Sign in H_0	=	≥	≤
Sign in H_1	≠	<	>
Rejection region	In both tails	In the left tail	In the right tail

Table 8–1 Hypothesis-Testing Common Phrases

>	<
Is greater than	Is less than
Is above	Is below
Is higher than	Is lower than
Is longer than	Is shorter than
Is bigger than	Is smaller than
Is increased	Is decreased or reduced from
≥	**≤**
Is greater than or equal to	Is less than or equal to
Is at least	Is at most
Is not less than	Is not more than
=	**≠**
Is equal to	Is not equal to
Is exactly the same as	Is different from
Has not changed from	Has changed from
Is the same as	Is not the same as



Hypothesis Testing

Three methods used to test hypotheses:

1. The critical value method
2. The *P*-value method
3. The confidence interval method

Procedure Table

Solving Hypothesis-Testing Problems
(critical value Method)

Step 1 State the hypotheses and identify the claim.

Step 2 Find the critical value(s) from the appropriate table in Appendix.

Step 3 Compute the test value.

Step 4 Make the decision to reject or not reject the null hypothesis.

Step 5 Summarize the results.

z Test for a Mean

The **z test** is a statistical test for the **mean of a population**. It can be used when **$n \geq 30$, or when the population is normally distributed and σ is known**.

The formula for the *z test is*

$$z = \frac{\overline{X} - \mu}{\sigma / \sqrt{n}}$$

where

$\overline{X}$ = sample mean

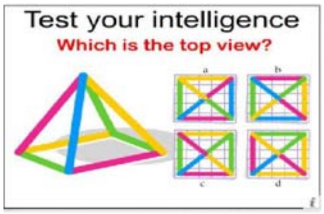
μ = hypothesized population mean

σ = population standard deviation

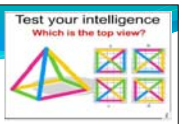
n = sample size

Example: Intelligence Tests

In a survey, the average IQ score of high school students is 101.5. The variable is normally distributed, and the population standard deviation is 15. A school superintendent claims that the students in her school district have an IQ higher than the average of 101.5. She selects a random sample of 30 students and finds the mean of the test scores is 106.4. Test the claim at α = 0.05.



Example: Intelligence Tests



Step 1 State the hypotheses and identify the claim.

H_0: $\mu = 101.5$ and H_1: $\mu > 101.5$ (claim)

Step 2 Find the critical value.

Since $\alpha = 0.05$ and the test is a right-tailed test, the critical value is $z = +1.65$.

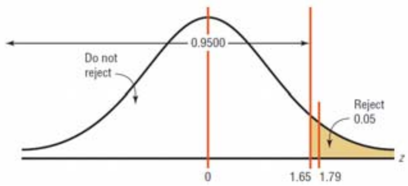
Step 3 Compute the test value.

$$z = \frac{\overline{X} - \mu}{\sigma / \sqrt{n}} = \frac{106.4 - 101.5}{15 / \sqrt{30}} = 1.79$$

Example: Intelligence Tests

Step 4 Make the decision.

Since the test value, 1.79, is greater than the critical value, 1.65, the decision is to reject the null hypothesis.



Step 5 Summarize the results.

There is enough evidence to support the claim that the IQ of the students is higher than the state average IQ.

Important Comments

In the example of "Intelligence Test", the difference is said to be statistically significant. However, when the null hypothesis is rejected, there is always a chance of a type I error. In this case, the probability of a type I error is at most 0.05, or 5%.

◆ In the hypothesis-testing situation, there are four possible outcomes.

流感篩檢為例：
H_0: A君未患流感
H_1: A君已患流感

	H_0 true	H_0 false
Reject H_0	**Error** Type I	Correct decision
Do not reject H_0	Correct decision	**Error** Type II

Hypothesis Testing

- In reality, the null hypothesis may or may not be true, and a decision is made to reject or not to reject it on the basis of the data obtained from a sample.

Two types of errors

Type I and Type II Errors

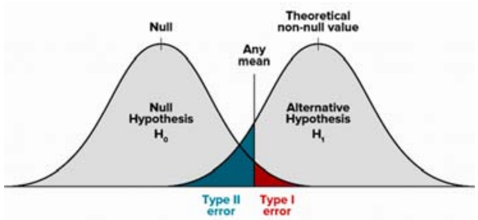
Type I error: Rejecting the null hypothesis when it is in fact true.
Type II error: Not rejecting the null hypothesis when it is in fact false.

Hypothesis Testing

- **Level of significance**
  - The maximum probability of committing a type I error.
  - This probability is symbolized by α.

  α = P (type I error) = P (H_o is rejected | H_o is true).

  β = P (type II error) = P (H_o is not rejected | H_o is false).



Example

- 某製造商成立一條自動化罐裝生產線，每罐平均裝量為200cc，標準差為10cc, 假設每罐裝量呈常態分配。今工廠隨機抽取25罐以檢定是否合乎標準，若未符標準將重新調整操作過程，試問：

1.在5%之顯著水準下，最佳規則為何？

2.若每罐平均量為195cc, 則發生型II誤差之機率為何？

Procedure

One-Mean z-Test (Critical-Value Approach)

Purpose To perform a hypothesis test for a population mean, μ

Assumptions
1. Simple random sample
2. Normal population or large sample
3. σ known

STEP 1 The null hypothesis is H_0: $\mu = \mu_0$, and the alternative hypothesis is

H_a: $\mu \neq \mu_0$ (Two tailed) or H_a: $\mu < \mu_0$ (Left tailed) or H_a: $\mu > \mu_0$ (Right tailed)

STEP 2 Decide on the significance level, α.

STEP 3 Compute the value of the test statistic

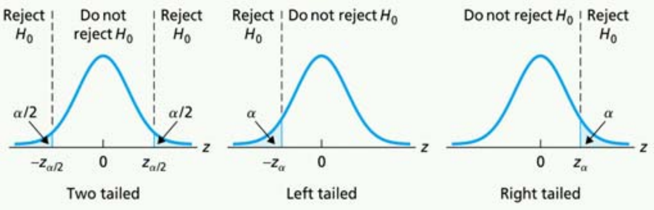
$$z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}.$$

STEP 4 The critical value(s) are

$\pm z_{\alpha/2}$ (Two tailed) or $-z_{\alpha}$ (Left tailed) or z_{α} (Right tailed)

Use Table II to find the critical value(s).

Procedure (cont.)



STEP 5 If the value of the test statistic falls in the rejection region, reject H_0; otherwise, do not reject H_0.

STEP 6 Interpret the results of the hypothesis test.

The hypothesis test is exact for normal populations and is approximately correct for large samples from nonnormal populations.

大樣本單一母體之母體平均數的檢定 (Z-檢定)

統計假設的配置法	棄卻域	檢定統計量
$H_0: \mu = \mu_0$，$H_1: \mu \neq \mu_0$ (雙尾檢定)	$C = \{\lvert Z\rvert > z_{\frac{\alpha}{2}}\}$	$Z = \dfrac{\overline{X} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$
$H_0: \mu \geq \mu_0$，$H_1: \mu < \mu_0$ (左尾檢定)	$C = \{Z < -z_\alpha\}$	$Z = \dfrac{\overline{X} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$
$H_0: \mu \leq \mu_0$，$H_1: \mu > \mu_0$ (右尾檢定)	$C = \{Z > z_\alpha\}$	$Z = \dfrac{\overline{X} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$

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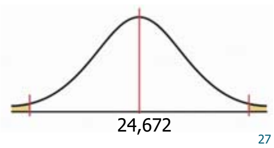
Example 8-5: Cost of Rehabilitation

The Medical Rehabilitation Education Foundation reports that the average cost of rehabilitation for stroke victims is $24,672. To see if the average cost of rehabilitation **is different** at a particular hospital, a researcher selects a random sample of 35 stroke victims at the hospital and finds that the average cost of their rehabilitation is $25,226. The standard deviation of the population is $3251. At α = 0.01, can it be concluded that the average cost of stroke rehabilitation at a particular hospital is different from $24,672?

Step 1: State the hypotheses and identify the claim.

H_0: μ = $24,672
H_1: $\mu \neq$ $24,672 (claim)



Example 8-5: Cost of Rehabilitation

Step 1: State the hypotheses and identify the claim.

H_0: μ = $24,672 and H_1: $\mu \neq$ $24,672 (claim)

Step 2: Find the critical value.

Since α = 0.01 and a two-tailed test

➔the critical values are z = ±2.58

Step 3: Find the test value.

$$z = \frac{\overline{X} - \mu}{\sigma/\sqrt{n}} = \frac{25,226 - 24,672}{3251/\sqrt{35}} = 1.01$$

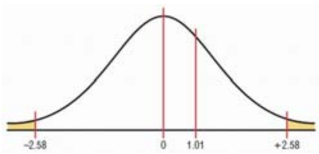
- a random sample of 35 stroke victims
- the average cost of their rehabilitation is $25,226
- the standard deviation of the population is $3251

Example 8-5: Cost of Rehabilitation

Step 4: Make the decision.

Since the test value falls in the noncritical region.

➔**Do not reject the null hypothesis**



H_0: μ = $24,672
H_1: $\mu \neq$ $24,672 (claim)

Step 5: Summarize the results.

There is **not enough evidence to support the claim** that the average cost of rehabilitation at the particular hospital is different from $24,672.

Confidence Interval method

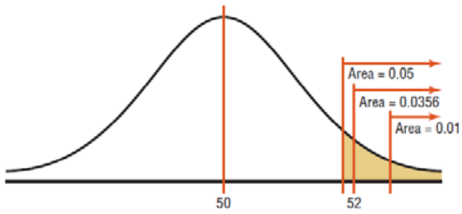
- **C.I. for Example 8-5**
- **C.I. for "Intelligence Tests"**
- $n = 35,\ \overline{X} = 25226,\ \sigma = 3251$

Hypothesis Testing

***P*-value Method**

Hypothesis Testing

The ***P*-value** (or probability value) is the probability of getting a sample statistic (such as the mean) or a more extreme sample statistic in the direction of the alternative hypothesis when the null hypothesis is true.



Hypothesis Testing

Decision rule when using a P-value

If P-value $\le \alpha$, reject the null hypothesis

If P-value $> \alpha$, do not reject the null hypothesis

Example 7: lose weights

The management of Priority Health Club claims that its members lose an average of 10 pounds or more within the first month after joining the club. A consumer agency that wanted to check this claim took a random sample of 36 members of this health club and found that they lost an average of 9.2 pounds within the first month of membership with a standard deviation of 2.4 pounds. Find the p – value for this test.

What will you decision be if α= .01? What if α= .05?

Solution : ($n > 30$)

Step 1. H_0: μ ≥ 10 vs. H_1: μ < 10

Step 2. $\alpha = .05$

Example 7: lose weights

Step 3. Statistic value (Z) (大樣本，σ未知，以s取代σ)

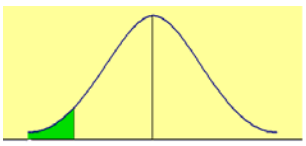
$$s_{\bar{x}} = \frac{s}{\sqrt{n}} = \frac{2.4}{\sqrt{36}} = .40$$

$$z = \frac{\bar{x} - \mu}{s_{\bar{x}}} = \frac{9.2 - 10}{.40} = -2.00$$

Step 4. P-value

$$P - \text{value} = P(z < -2.00) = 0.0228$$

Step 5. Decision

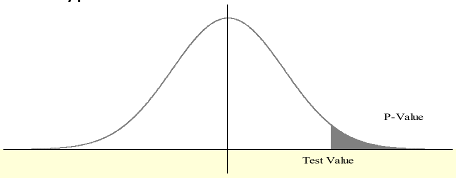


Solution—P-Value test

- The p – value is .0228 (p-value=P(Z<-2.00))
- If α = .01
  - p – value > α
  - Therefore, we do not reject the null hypothesis
- If α = .05
  - p – value < α
  - Therefore, we reject the null hypothesis.

Hypothesis Testing—P-value

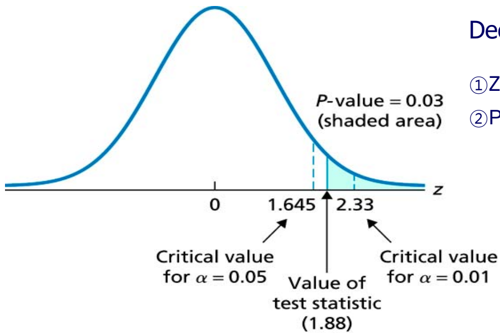
The ***P*-value** (or probability value) is the probability of getting a sample statistic (such as the mean) or a more extreme sample statistic in the direction of the alternative hypothesis when the null hypothesis is true.



***P*-Value**

To obtain the ***P*-value** of a hypothesis test, we assume that the null hypothesis is true and compute the probability of observing a value of the test statistic as extreme as or more extreme than that observed. By *extreme* we mean "far from what we would expect to observe if the null hypothesis is true." We use the letter P to denote the P-value.

Figure for right-tailed test

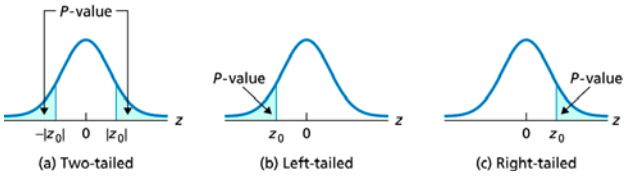


Decision: reject H_0

① $Z=1.88>Z_{\alpha}=1.645$

② P-value < $\alpha=0.05$

Figures for P-value



Hypothesis Testing

- The traditional method for solving hypothesis-testing problems compares **z-values**:
  - critical value
  - test value
- The *P*-value method for solving hypothesis-testing problems compares **areas**:
  - alpha
  - *P*-value

Procedure Table

Solving Hypothesis-Testing Problems
(*P*-Value Method)

Step 1 State the hypotheses and identify the claim.

Step 2 Compute the test value.

Step 3 Find the *P*-value.

Step 4 Make the decision.

Step 5 Summarize the results.

Example 8-6: Cost of College Tuition

A researcher wishes to test the claim that the average cost of tuition and fees at a four-year public college is greater than $5700. She selects a random sample of 36 four-year public colleges and finds the mean to be $5950. The population standard deviation is $659. Is there evidence to support the claim at a 0.05? Use the *P*-value method.

Step 1: State the hypotheses and identify the claim.

H_0: $\mu \leqq \$5700$ and H_1: $\mu > \$5700$ (claim)

Step 2: Compute the test value.

$$z = \frac{\overline{X} - \mu}{\sigma/\sqrt{n}} = \frac{5950 - 5700}{659/\sqrt{36}} = 2.28$$

Example 8-6: Cost of College Tuition

Step 3: Find the *P*-value.

Using Table II, find the area for z = 2.28.

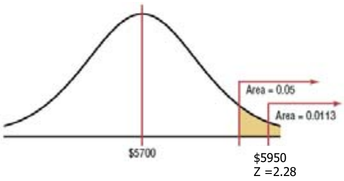
The area is 0.9887.

Subtract from 1.0000 to find the area of the tail.

Hence, the *P*-value is 1.0000 – 0.9887 = 0.0113.

Step 4: Make the decision.

Since the *P*-value is less than 0.05, the decision is to reject the null hypothesis.

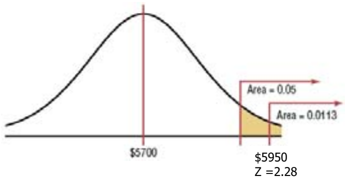


Example 8-6: Cost of College Tuition

Step 5: Summarize the results.

There is enough evidence to support the claim that the tuition and fees at four-year public colleges are greater than $5700.

Note: If α = 0.01, the null hypothesis would not be rejected.



Example 8-7: Wind Speed

A researcher claims that the average wind speed in a certain city is 8 miles per hour. A sample of 32 days has an average wind speed of 8.2 miles per hour. The standard deviation of the population is 0.6 mile per hour. At α = 0.05, is there enough evidence to reject the claim? Use the *P*-value method.

Step 1: State the hypotheses and identify the claim.

H_0: $\mu = 8$ (claim) and H_1: $\mu \neq 8$

Step 2: Compute the test value.

$$z = \frac{\overline{X} - \mu}{\sigma / \sqrt{n}} = \frac{8.2 - 8}{0.6 / \sqrt{32}} = 1.89$$

Step 3: Find the *P*-value.

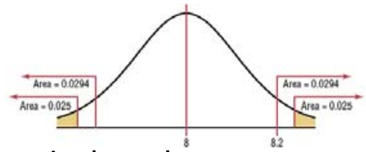
P-value $= 2 \times (1 - P(z < 1.89)) = 2 \times 0.0294 = 0.0588$.

Since this is a two-tailed test

Example 8-7: Wind Speed

Step 4: Make the decision.

The decision is to not reject the null hypothesis, since the *P*-value is greater than 0.05.



Step 5: Summarize the results.

There is not enough evidence to reject the claim that the average wind speed is 8 miles per hour.

One-Mean *z*-Test (*P*-Value Approach)

Purpose To perform a hypothesis test for a population mean, μ

Assumptions

1. Simple random sample
2. Normal population or large sample
3. σ known

STEP 1 The null hypothesis is H_0: $\mu = \mu_0$, and the alternative hypothesis is

H_a: $\mu \neq \mu_0$ (Two tailed) or H_a: $\mu < \mu_0$ (Left tailed) or H_a: $\mu > \mu_0$ (Right tailed)

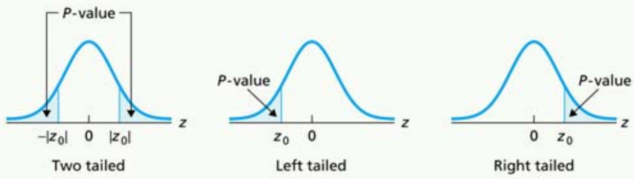
STEP 2 Decide on the significance level, α.

STEP 3 Compute the value of the test statistic

$$z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$$

and denote that value z_0.

STEP 4 Use Table II to obtain the *P*-value.



STEP 5 $\boxed{\text{If } P \leq \alpha, \text{ reject } H_0; \text{ otherwise, do not reject } H_0.}$

STEP 6 Interpret the results of the hypothesis test.

The hypothesis test is exact for normal populations and is approximately correct for large samples from nonnormal populations.

CRITICAL-VALUE APPROACH	P-VALUE APPROACH
STEP 1 State the null and alternative hypotheses.	**STEP 1** State the null and alternative hypotheses.
STEP 2 Decide on the significance level, α.	**STEP 2** Decide on the significance level, α.
STEP 3 Compute the value of the test statistic.	**STEP 3** Compute the value of the test statistic.
STEP 4 Determine the critical value(s).	**STEP 4** Determine the *P*-value.
STEP 5 If the value of the test statistic falls in the rejection region, reject H_0; otherwise, do not reject H_0.	**STEP 5** If $P \le \alpha$, reject H_0; otherwise, do not reject H_0.
STEP 6 Interpret the result of the hypothesis test.	**STEP 6** Interpret the result of the hypothesis test.

Guidelines for *P*-Values With No α

- If *P*-value ≤ 0.01, reject the null hypothesis.
  - The difference is highly significant.
- If P-value > 0.01 but P-value ≤ 0.05, reject the null hypothesis.
  - The difference is significant.
- If P-value > 0.05 but P-value ≤ 0.10, consider the consequences of type I error before rejecting the null hypothesis.
- If P-value > 0.10, do not reject the null hypothesis.
  - The difference is not significant.

P-value	Evidence against H_0
$P > 0.10$	Weak or none
$0.05 < P \le 0.10$	Moderate
$0.01 < P \le 0.05$	Strong
$P \le 0.01$	Very strong

Significance

- The researcher should distinguish between **statistical significance** and **practical significance**.
- When the null hypothesis is rejected at a specific significance level, it can be concluded that the difference is probably not due to chance and thus is statistically significant. However, the results may not have any practical significance.
- It is up to the researcher to use common sense when interpreting the results of a statistical test.

t Test for a Mean

t Test for a Mean

The *t* test is a statistical test for the mean of a population and is used when the population is normally or approximately normally distributed, σ is unknown.

The formula for the *t test is*

$$t = \frac{\overline{X} - \mu}{s/\sqrt{n}}$$

The degrees of freedom are d.f. = *n* – 1.

Example 8-12: Hospital Infections

A medical investigation claims that the average number of infections per week at a hospital in southwestern Pennsylvania **is** 16.3. A random sample of 10 weeks had a mean number of 17.7 infections. The sample standard deviation is 1.8. Is there enough evidence to reject the investigator's claim at *α* = 0.05?

Step 1: State the hypotheses and identify the claim.

H_0: $\mu = 16.3$ (claim) and H_1: $\mu \ne 16.3$

Step 2: Find the critical value.

The critical values are 2.262 and -2.262 for α = 0.05 and d.f. = 9

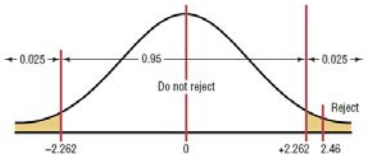
Step 3: Find the test value.

$$z = \frac{\overline{X} - \mu}{s/\sqrt{n}} = \frac{17.7 - 16.3}{1.8/\sqrt{10}} = 2.46$$

Example 8-12: Hospital Infections

Step 4: Make the decision.

Reject the null hypothesis since 2.46 > 2.262.



Step 5: Summarize the results.

There is enough evidence to reject the claim that the average number of infections is 16.3.

Example: find the P-value

- Find the following P-value for each
  - The t test value is 2.056 with sample size is 11 (right-tailed test)
  - The t test value is 2.983 with sample size is 6 (right-tailed test)

FIGURE 8–22
Finding the *P*-Value for Example 8–14

d.f.	Confidence intervals	80%	90%	95%	98%	99%
	One tail, α	0.10	0.05	0.025	0.01	0.005
	Two tails, α	0.20	0.10	0.05	0.02	0.01
1		3.078	6.314	12.706	31.821	63.657
2		1.886	2.920	4.303	6.965	9.925
3		1.638	2.353	3.182	4.541	5.841
4		1.533	2.132	2.776	3.747	4.604
5		1.476	2.015	2.571	3.365	4.032
6		1.440	1.943	2.447	3.143	3.707
7		1.415	1.895	2.365	2.998	3.499
8		1.397	1.860	2.306	2.896	3.355
9		1.383	1.833	2.262	2.821	3.250
10		1.372	1.812 *	2.228	2.764	3.169
11		1.363	1.796	2.201	2.718	3.106
12		1.356	1.782	2.179	2.681	3.055
13		1.350	1.771	2.160	2.650	3.012
14		1.345	1.761	2.145	2.624	2.977
15		1.341	1.753	2.131	2.602	2.947
⋮		⋮	⋮	⋮	⋮	⋮
(z) ∞		1.282	1.645	1.960	2.326	2.576

*2.056 falls between 1.812 and 2.228.

Hence, the *P*-value would be contained in the interval 0.025 < *P*-value < 0.05. This means that the *P*-value is between 0.025 and 0.05. If α were 0.05, you would reject the null hypothesis since the *P*-value is less than 0.05. But if α were 0.01, you would not reject the null hypothesis since the *P*-value is greater than 0.01. (Actually, it is greater than 0.025.)

Example 8-16: Jogger's Oxygen Intake

A physician claims that joggers' maximal volume oxygen uptake is greater than the average of all adults. A sample of 15 joggers has a mean of 40.6 milliliters per kilogram (ml/kg) and a standard deviation of 6 ml/kg. If the average of all adults is 36.7 ml/kg, is there enough evidence to support the physician's claim at α = 0.05? (use P-value method)

Step 1: State the hypotheses and identify the claim.

H_0: $\mu \leqq 36.7$ and H_1: $\mu > 36.7$ (claim)

Step 2: Compute the test value.

$$t = \frac{\overline{X} - \mu}{s/\sqrt{n}} = \frac{40.6 - 36.7}{6/\sqrt{15}} = 2.517$$

Step 3: Find the *P*-value. (d.f. = 14, one-tailed test.)

2.517 falls between 2.145 and 2.624 ➔ P = 0.01~0.025.

Thus, the *P*-value is somewhere between 0.01 and 0.025

Example 8-16: Jogger's Oxygen Intake

Step 4: Make the decision.

The decision is to reject the null hypothesis, since the *P*-value < 0.05.

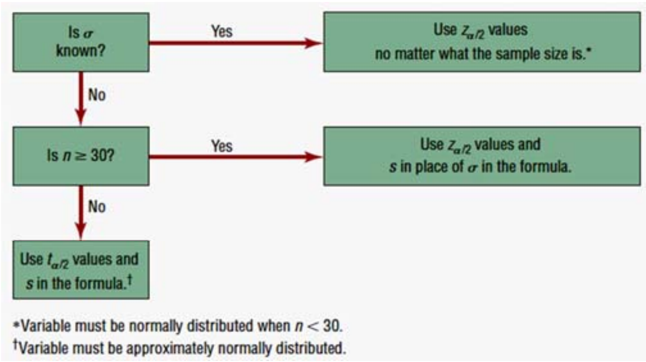
Step 5: Summarize the results.

There is enough evidence to support the claim that the joggers' maximal volume oxygen uptake is greater than 36.7 ml/kg.

小樣本下單一母體之母體平均數的檢定 (*t* 檢定)

統計假設的配置法	棄卻域	檢定統計量
$H_0: \mu = \mu_0$，$H_1: \mu \neq \mu_0$ (雙尾檢定)	$C = \{\lvert T\rvert > t_{\frac{\alpha}{2}}(n-1)\}$	$T = \frac{\overline{X} - \mu_0}{S/\sqrt{n}}$
$H_0: \mu \geq \mu_0$，$H_1: \mu < \mu_0$ (左尾檢定)	$C = \{T < -t_\alpha(n-1)\}$	$T = \frac{\overline{X} - \mu_0}{S/\sqrt{n}}$
$H_0: \mu \leq \mu_0$，$H_1: \mu > \mu_0$ (右尾檢定)	$C = \{T > t_\alpha(n-1)\}$	$T = \frac{\overline{X} - \mu_0}{S/\sqrt{n}}$

Whether to use *z* or *t*



Confidence intervals and hypothesis testing.

Confidence intervals and hypothesis testing

- There is a relationship between confidence intervals and hypothesis testing.
- When the null hypothesis is rejected in a hypothesis-testing situation, the confidence interval for the mean using the same level of significance *will not* contain the hypothesized mean.
- Likewise, when the null hypothesis is not rejected, the confidence interval computed using the same level of significance *will* contain the hypothesized mean.

Exercise (p.435 #4)

- **Moviegoers**
- The average "moviegoer" sees 8.5 movies a year. A ***moviegoer*** is defined as a person who sees at least one movie in a theater in a 12-month period. A random sample of 36 moviegoers from a large university revealed that the average number of movies seen per person was 9.6. the population standard deviation is 3.6 movies.
- Find the 95% confidence interval of the mean. Is μ contained in the interval?
- At α = 0.05, does the evidence refute the sociologist's assertion?

Example 13: Sugar Production

Sugar is packed in 5-pound bags. An inspector suspects the bags may not contain 5 pounds. A sample of 50 bags produces a mean of 4.6 pounds and a standard deviation of 0.7 pound. Is there enough evidence to conclude that the bags do not contain 5 pounds as stated at $\alpha = 0.05$? Also, find the 95% confidence interval of the true mean.

Step 1: State the hypotheses and identify the claim.

H_0: $\mu = 5$ and H_1: $\mu \neq 5$ (claim)

Step 2: Find the critical value. (α/2=0.025)

The critical values are $z = \pm 1.96$.

Example 13: Sugar Production

Step 3: Compute the test value.

$$z = \frac{X - \mu}{\sigma / \sqrt{n}} = \frac{4.6 - 5.0}{0.7/\sqrt{50}} = -4.04$$

Step 4: Make the decision.

Reject the null hypothesis. (∵$|z|$ > critical value=1.96)

Step 5: Summarize the results.

There is enough evidence to support the claim that the bags do not weigh 5 pounds.

Example 13: Sugar Production

The 95% confidence interval for the mean is

$$\bar{X} - z_{\alpha/2}\frac{s}{\sqrt{n}} < \mu < \bar{X} + z_{\alpha/2}\frac{s}{\sqrt{n}}$$

$$4.6-(1.96)\left(\frac{0.7}{\sqrt{50}}\right) < \mu < 4.6+(1.96)\left(\frac{0.7}{\sqrt{50}}\right)$$

$$4.4 < \mu < 4.8$$

Notice that the 95% confidence interval of m does *not* contain the hypothesized value $\mu = 5$.

Hence, there is agreement between the hypothesis test and the confidence interval.

信賴區間檢定 1/4

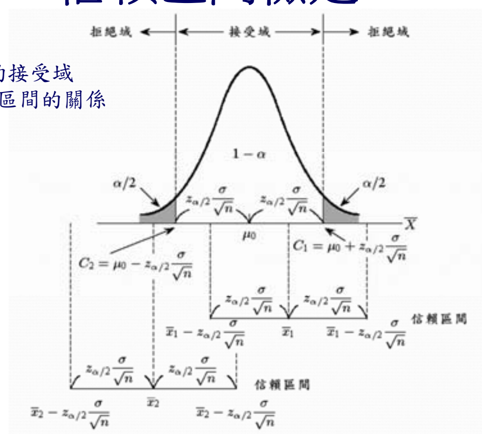
- 在雙尾檢定中，H_0:μ=μ$_0$；H_1:μ≠μ$_0$，如果母體平均數μ的(1−α)100%之信賴區間包含μ$_0$，則樣本平均數 $\bar{X}$ 的觀察值會落於接受域，此時將做出接受H_0的結論。

信賴區間檢定 2/4

- 臨界值檢定
  計算臨界值，求出接受域(或拒絕域)，若檢定統計量的觀察值落於接受域，則接受H_0；反之，則拒絕H_0。
- 信賴區間檢定
  計算母體參數的信賴區間，若此區間包含H_0成立時的假想值(如μ$_0$)，則接受H_0；反之，則拒絕H_0。

信賴區間檢定 3/4

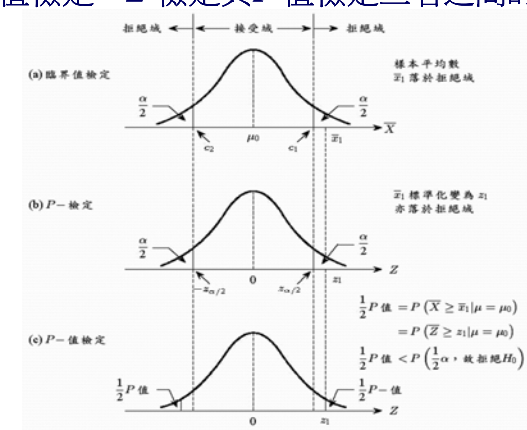
雙尾檢定的接受域
與μ之信賴區間的關係



信賴區間檢定 4/4

	$\bar{X}$ 抽樣分配屬於常態(若 σ 未知，可以 s 取代)	$\bar{X}$ 抽樣分配屬於 t 分配
(1)雙尾檢定 H_0:μ=μ$_0$ H_1:μ≠μ$_0$	$\left(\bar{x}-z_{\alpha/2}\cdot\frac{\sigma}{\sqrt{n}}, \bar{x}+z_{\alpha/2}\cdot\frac{\sigma}{\sqrt{n}}\right)$	$\left(\bar{x}-t_{\alpha/2}\cdot\frac{s}{\sqrt{n}}, \bar{x}+t_{\alpha/2}\cdot\frac{s}{\sqrt{n}}\right)$
(2)左尾檢定 H_0:μ≥μ$_0$ H_1:μ<μ$_0$	$\left(-\infty, \bar{x}+z_{\alpha}\cdot\frac{\sigma}{\sqrt{n}}\right)$	$\left(-\infty, \bar{x}+t_{\alpha}\cdot\frac{s}{\sqrt{n}}\right)$
(2)右尾檢定 H_0:μ≤μ$_0$ H_1:μ>μ$_0$	$\left(\bar{x}-z_{\alpha}\cdot\frac{\sigma}{\sqrt{n}}, \infty\right)$	$\left(\bar{x}-t_{\alpha}\cdot\frac{s}{\sqrt{n}}, \infty\right)$

臨界值檢定、Z-檢定與P-值檢定三者之間的關係



臨界值檢定與Z檢定(或t檢定)之比較--1/2

(a)Z—檢定的情況	臨界值檢定 檢定統計量：$\bar{X}$	Z—檢定 檢定統計量：$z=\frac{\bar{X}-\mu_0}{\sigma/\sqrt{n}}$ (大樣本，σ 未知，以S取代)
(1)雙尾檢定：$\begin{cases} H_0:\mu=\mu_0 \\ H_1:\mu\neq\mu_0 \end{cases}$	拒絕域：$\bar{X}\ge c_1$或$\bar{X}\le c_2$	拒絕域：$z\ge z_{\alpha/2}$或 $z\le -z_{\alpha/2}$
(2)左尾檢定：$\begin{cases} H_0:\mu\ge\mu_0 \\ H_1:\mu<\mu_0 \end{cases}$	拒絕域：$\bar{X}\le c_1$ $c=\mu_0-z_\alpha\frac{\sigma}{\sqrt{n}}$	拒絕域：$z\le -z_\alpha$
(3)右尾檢定：$\begin{cases} H_0:\mu\le\mu_0 \\ H_1:\mu>\mu_0 \end{cases}$	拒絕域：$\bar{X}>c$ $c=\mu_0+z_\alpha\frac{\sigma}{\sqrt{n}}$	拒絕域：$z\ge z_\alpha$

Source: 方世榮，基礎統計學，華泰書局， 臨界值檢定與Z–檢定之比較的圖例

臨界值檢定與Z檢定(或t檢定)之比較(續)--2/2

(b)t—檢定的情況	臨界值檢定 檢定統計量：$\bar{X}$	t—檢定 檢定統計量：$t=\frac{\bar{X}-\mu_0}{S/\sqrt{n}}$
(4)雙尾檢定：$\begin{cases} H_0:\mu=\mu_0 \\ H_1:\mu\neq\mu_0 \end{cases}$	拒絕域：$\bar{X}\ge c_1$或$\bar{X}\le c_2$	拒絕域：$t\ge t_{\alpha/2}(n-1)$或 $t\le -t_{\alpha/2}(n-1)$
(5)左尾檢定：$\begin{cases} H_0:\mu\ge\mu_0 \\ H_1:\mu<\mu_0 \end{cases}$	拒絕域：$\bar{X}\le c_1$ $c=\mu_0-t_\alpha(n-1)\frac{S}{\sqrt{n}}$	拒絕域：$t\le -t_\alpha(n-1)$
(6)右尾檢定：$\begin{cases} H_0:\mu\le\mu_0 \\ H_1:\mu>\mu_0 \end{cases}$	拒絕域：$\bar{X}>c$ $c=\mu_0+t_\alpha(n-1)\frac{S}{\sqrt{n}}$	拒絕域：$t\ge t_\alpha(n-1)$

Source: 方世榮，基礎統計學，華泰書局， 臨界值檢定與Z–檢定之比較的圖例

8.6 Additional Topics Regarding Hypothesis Testing

- There is a relationship between confidence intervals and hypothesis testing.
- When the null hypothesis is rejected in a hypothesis-testing situation, the confidence interval for the mean using the same level of significance *will not* contain the hypothesized mean.
- Likewise, when the null hypothesis is not rejected, the confidence interval computed using the same level of significance *will* contain the hypothesized mean.

Example 8-30: Sugar Production

Sugar is packed in 5-pound bags. An inspector suspects the bags may not contain 5 pounds. A sample of 50 bags produces a mean of 4.6 pounds and a standard deviation of 0.7 pound. Is there enough evidence to conclude that the bags do not contain 5 pounds as stated at $\alpha = 0.05$? Also, find the 95% confidence interval of the true mean.

Step 1: State the hypotheses and identify the claim.

H_0: $\mu = 5$ and H_1: $\mu \neq 5$ (claim)

Step 2: Find the critical value.

The critical values are $z = \pm1.96$.

Example 8-30: Sugar Production

Sugar is packed in 5-pound bags. An inspector suspects the bags may not contain 5 pounds. A sample of 50 bags produces a mean of 4.6 pounds and a standard deviation of 0.7 pound. Is there enough evidence to conclude that the bags do not contain 5 pounds as stated at $\alpha = 0.05$? Also, find the 95% confidence interval of the true mean.

Step 3: Compute the test value.

$$z = \frac{X-\mu}{\sigma/\sqrt{n}} = \frac{4.6-5.0}{0.7/\sqrt{50}} = -4.04$$

Example 8-30: Sugar Production

Step 4: Make the decision.

Reject the null hypothesis.

Step 5: Summarize the results.

There is enough evidence to support the claim that the bags do not weigh 5 pounds.

Example 8-30: Sugar Production

The 95% confidence interval for the mean is

$$\bar{X} - z_{\alpha/2}\frac{s}{\sqrt{n}} < \mu < \bar{X} + z_{\alpha/2}\frac{s}{\sqrt{n}}$$

$$4.6-(1.96)\left(\frac{0.7}{\sqrt{50}}\right) < \mu < 4.6+(1.96)\left(\frac{0.7}{\sqrt{50}}\right)$$

$$4.4 < \mu < 4.8$$

Notice that the 95% confidence interval of m does *not* contain the hypothesized value $\mu = 5$.

Hence, there is agreement between the hypothesis test and the confidence interval.

Power of a Statistical Test

The **power of a test** measures the sensitivity of the test to detect a real difference in parameters if one actually exists. The higher the power, the more sensitive the test. The power is $1-\beta$.

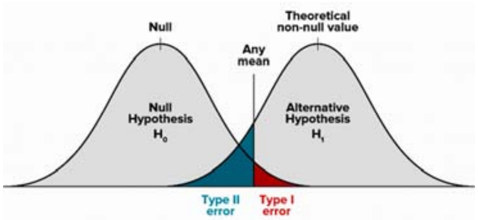
	H_0 true	H_0 false
Reject H_0	**Type I error** α	Correct decision $1-\beta$
Do not reject H_0	Correct decision $1-\alpha$	**Type II error** β

Hypothesis Testing

- **Level of significance**
  - The maximum probability of committing a type I error.
  - This probability is symbolized by α.

  α = P (type I error) = P (H_o is rejected | H_o is true).

  β = P (type II error) =P (H_o is not rejected | H_o is false).



Example I

- 某製造商成立一條自動化罐裝生產線，每罐平均裝量為200cc，標準差為10cc, 假設每罐裝量呈常態分配。今工廠隨機抽取25罐以檢定是否合乎標準，若未符標準將重新調整操作過程，試問：

1.在5%之顯著水準下，最佳規則為何？

2.若每罐平均量為195cc, 則發生型II誤差之機率為何？

資料來源：程大器，統計學新增600題, p.378

Example II

- A產品平均壽命μ=1100小時，金製造商宣稱新型產品壽命已提升，自改善後產品中隨機抽取一組n=36之樣本進行測試，得知平均壽命為1125小時，標準差為300小時，則：

  (1)在α=0.05下，檢定改善後之產品壽命是否已提升？

  (2)求(1)中之p-value

  (3)求在μ=1225小時之type II error機率

  (4)求(1)之檢定力函數，

Example III

- X~N(50, 6^2), 欲檢定 H0: μ ≦50, H1: μ>50
- 取樣n=16, 且取棄卻域 CR={X-bar|X-bar>53}, 則:

  (1)求此檢定的顯著水準α

  (2)求當μ=55時，犯型II誤差的機率

  (3)求當X-bar=54.5時之p-value，並利用此值檢定此假設

Section 8-4

z Test for a Proportion

z Test for a Proportion

Since a normal distribution can be used to approximate the binomial distribution when $np \geq 5$ and $nq \geq 5$, the standard normal distribution can be used to test hypotheses for proportions.

The formula for the z test for a proportion is

$$z = \frac{\hat{p} - p}{\sqrt{pq/n}} \text{, where } \hat{p} = \frac{X}{n} \text{ (sample proportion)}$$

$p =$ population proportion

$n =$ sample size

C.I. for a true proportion

$$\hat{p} - Z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}} < p < \hat{p} + Z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

Example: Avoiding Trans Fats

A dietitian claims that 60% of people are trying to avoid trans fats in their diets. She randomly selected 200 people and found that 128 people stated that they were trying to avoid trans fats in their diets. At $\alpha = 0.05$, is there enough evidence to reject the dietitian's claim?

Step 1: State the hypotheses and identify the claim.

H_0: $p = 0.60$ (claim) and H_1: $p \neq 0.60$

Step 2: Find the critical value.

Since $\alpha = 0.05$ and the test is a two-tailed test, the critical value is $z = \pm 1.96$.

Example: Avoiding Trans Fats

A dietitian claims that 60% of people are trying to avoid trans fats in their diets. She randomly selected 200 people and found that 128 people stated that they were trying to avoid trans fats in their diets. At $\alpha = 0.05$, is there enough evidence to reject the dietitian's claim?

Step 3: Compute the test value.

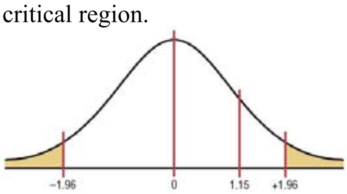
$$\hat{p} = \frac{X}{n} = \frac{128}{200} = 0.64$$

$$z = \frac{\hat{p} - p}{\sqrt{pq/n}} = \frac{0.64 - 0.60}{\sqrt{(0.60)(0.40)/200}} = 1.15$$

Example: Avoiding Trans Fats

Step 4: Make the decision.

Do not reject the null hypothesis since the test value falls outside the critical region.



Step 5: Summarize the results.

There is not enough evidence to reject the claim that 60% of people are trying to avoid trans fats in their diets.

Example: Call-Waiting Service

A telephone company representative estimates that less than 23% of its customers have call-waiting service. To test this hypothesis, she selected a sample of 100 customers and found that 11% had call waiting. At $\alpha = 0.01$, is there enough evidence to reject the claim?

Step 1: State the hypotheses and identify the claim.

H_0: $p \geqq 0.23$

H_1: $p < 0.23$ (claim)

Step 2: Find the critical value.

Since $\alpha = 0.01$ and the test is a two-tailed test, the critical value is $z = -2.33$.

Example: Call-Waiting Service

A telephone company representative estimates that less than 23% of its customers have call-waiting service. To test this hypothesis, she selected a sample of 100 customers and found that 11% had call waiting. At $\alpha = 0.01$, is there enough evidence to reject the claim?

Step 3: Compute the test value.

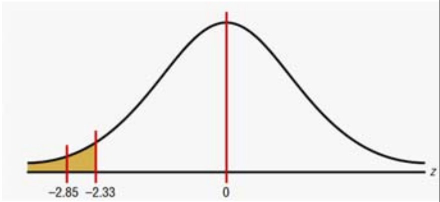
$$p = 0.23,\ q = 1 - 0.23 = 0.77,\ \hat{p} = 0.11$$

$$z = \frac{\hat{p} - p}{\sqrt{pq/n}} = \frac{0.11 - 0.23}{\sqrt{0.23 \times 0.77/100}} = -2.85$$

Example: Call-Waiting Service

Step 4: Make the decision

Reject the null hypothesis since the test value falls in the critical region.



Step 5: Summarize the results.

There is enough evidence to support the claim that less than 23% of the telephone company's customers have call waiting.

Example: Call-Waiting Service

A telephone company representative estimates that less than 23% of its customers have call-waiting service. To test this hypothesis, she selected a sample of 100 customers and found that 11% had call waiting. At $\alpha = 0.01$, is there enough evidence to reject the claim? **Use p-value method**.

One-Proportion z-Interval Procedure

Purpose To find a confidence interval for a population proportion, p

Assumptions

1. Simple random sample
2. The number of successes, x, and the number of failures, $n - x$, are both 5 or greater.

STEP 1 For a confidence level of $1 - \alpha$, use Table II to find $z_{\alpha/2}$.

STEP 2 The confidence interval for p is from

$$\hat{p} - z_{\alpha/2} \cdot \sqrt{\hat{p}(1-\hat{p})/n} \quad \text{to} \quad \hat{p} + z_{\alpha/2} \cdot \sqrt{\hat{p}(1-\hat{p})/n}$$

where $z_{\alpha/2}$ is found in Step 1, n is the sample size, and $\hat{p} = x/n$ is the sample proportion.

STEP 3 Interpret the confidence interval.

Procedure

One-Proportion z-Test

Purpose To perform a hypothesis test for a population proportion, p

Assumptions

1. Simple random sample
2. Both np_0 and $n(1 - p_0)$ are 5 or greater

STEP 1 The null hypothesis is H_0: $p = p_0$, and the alternative hypothesis is

H_a: $p \neq p_0$ (Two tailed) or H_a: $p < p_0$ (Left tailed) or H_a: $p > p_0$ (Right tailed)

STEP 2 Decide on the significance level, α.

STEP 3 Compute the value of the test statistic

$$z = \frac{\hat{p} - p_0}{\sqrt{p_0(1 - p_0)/n}}$$

and denote that value z_0.

Procedure (cont.)

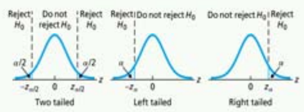
CRITICAL-VALUE APPROACH or *P*-VALUE APPROACH

CRITICAL-VALUE APPROACH

STEP 4 The critical value(s) are

$\pm z_{\alpha/2}$ (Two tailed) or $-z_\alpha$ (Left tailed) or z_α (Right tailed)

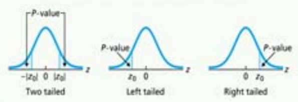
Use Table II to find the critical value(s).



STEP 5 If the value of the test statistic falls in the rejection region, reject H_0; otherwise, do not reject H_0.

***P*-VALUE APPROACH**

STEP 4 Use Table II to obtain the P-value.



STEP 5 If $P \le \alpha$, reject H_0; otherwise, do not reject H_0.

STEP 6 Interpret the results of the hypothesis test.

母體比例的檢定-- 臨界值檢定法

	臨界值	決策法則
(1)雙尾檢定：$\begin{cases} H_0: P = P_0 \\ H_1: P \neq P_0 \end{cases}$	$c_1 = P_0 + z_{\alpha/2} \cdot \sigma_{\bar{P}}$ $c_1 = P_0 - z_{\alpha/2} \cdot \sigma_{\bar{P}}$	若$\bar{P} \geq c_1$或$\bar{P} \leq c_2$，則拒絕H_0; 若$c_2 < \bar{P} < c_2$，則接受H_0。
(2)左尾檢定：$\begin{cases} H_0: P \geq P_0 \\ H_1: P < P_0 \end{cases}$	$c = P_0 - z_{\alpha} \cdot \sigma_{\bar{P}}$	若$\bar{P} \leq c$，則拒絕H_0; 若$\bar{P} > c$，則接受H_0。
(3)右尾檢定：$\begin{cases} H_0: P \leq P_0 \\ H_1: P > P_0 \end{cases}$	$c = P_0 + z_{\alpha} \cdot \sigma_{\bar{P}}$	若$\bar{P} \geq c$，則拒絕H_0; 若$\bar{P} < c$，則接受H_0。
註：$\sigma_{\bar{P}} = \sqrt{\dfrac{P_0(1-P_0)}{n}}$		

母體比例的檢定- Z值檢定法

	檢定統計量	決策法則
(1)雙尾檢定 H_0:P=P_0 H_1:P≠P_0	$Z = \dfrac{\bar{P} - P_0}{\sqrt{\dfrac{P_0(1-P_0)}{n}}}$	若$z \geq z_{\alpha/2}$或$z \leq -z_{\alpha/2}$，則拒絕H_0; 若$-z_{\alpha/2} < z < z_{\alpha/2}$，則接受$H_0$。
(2)左尾檢定 H_0:P≥P_0 H_1:P<P_0	同上	若$z \leq -z_{\alpha}$，則拒絕H_0; 若$z > -z_{\alpha}$，則接受H_0。
(3)右尾檢定 H_0:P≤P_0 H_1:P>P_0	同上	若$z \geq z_{\alpha}$，則拒絕H_0; 若$z < z_{\alpha}$，則接受H_0。

母體比例的檢定–信賴區間檢定法

	母體比例之信賴區間	決策法則
(1)雙尾檢定 H_0:P=P_0 H_1:P≠P_0	$\left(\bar{P} - z_{\alpha/2} \cdot \sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}, \bar{P} + z_{\alpha/2} \cdot \sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}\right)$	若區間包含P_0，則接受H_0，反之，則拒絕H_0。
(2)左尾檢定 H_0:P≥P_0 H_1:P<P_0	$\left(-\infty, \bar{P} + z_{\alpha} \cdot \sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}\right)$	同上
(3)右尾檢定 H_0:P≤P_0 H_1:P>P_0	$\left(\bar{P} - z \cdot \sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}, \infty\right)$	同上

Section 8-5

Test for a Variance or Standard Deviation

Introduction

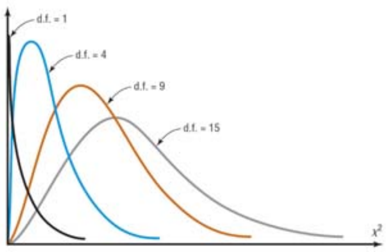


Chi-Square Distributions

- The chi-square distribution must be used to calculate confidence intervals for one population variance and standard deviation.
- The chi-square variable is similar to the *t* variable in that its distribution is a family of curves based on the number of degrees of freedom.
- The symbol for chi-square is χ^2 (Greek letter chi, pronounced "ki").
- A chi-square variable cannot be negative, and the distributions are skewed to the right.

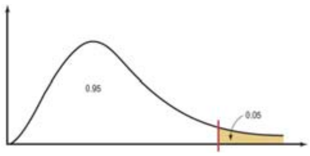
Chi-Square Distributions

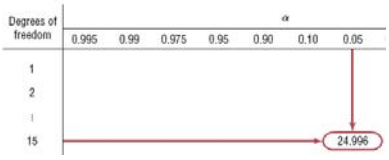
- At about 100 degrees of freedom, the chi-square distribution becomes somewhat symmetric.
- The area under each chi-square distribution is equal to 1.00, or 100%.



Example 8-21: Table G

Find the critical chi-square value for 15 degrees of freedom when $\alpha = 0.05$ and the test is right-tailed.

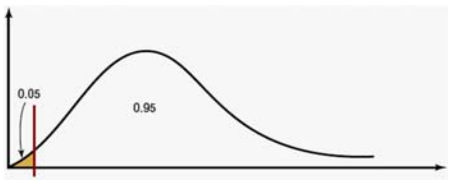




$$\chi^2 = 24.996$$

Example 8-22: Table G

Find the critical chi-square value for 10 degrees of freedom when $\alpha = 0.05$ and the test is left-tailed.

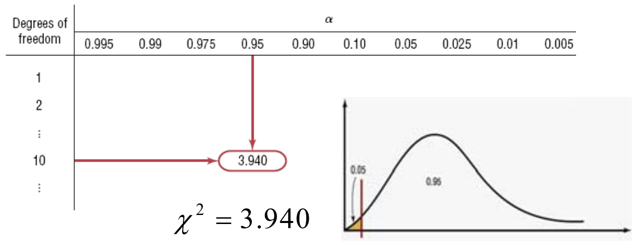


When the test is left-tailed, the α value must be subtracted from 1, that is, 1 – 0.05 = 0.95. The left side of the table is used, because the chi-square table gives the area to the right of the critical value, and the chi-square statistic cannot be negative.

Example 8-22: Table G

Find the critical chi-square value for 10 degrees of freedom when $\alpha = 0.05$ and the test is left-tailed.

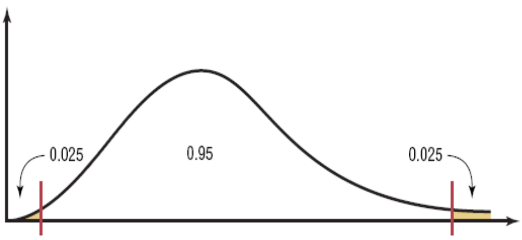
Use Table G, looking in row 10 and column 0.95.



$$\chi^2 = 3.940$$

Example 8-23: Table G

Find the critical chi-square value for 22 degrees of freedom when $\alpha = 0.05$ and a two-tailed test is conducted.

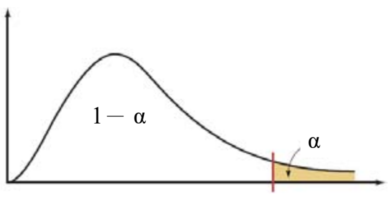


With 22 degrees of freedom, areas 0.025 and 0.975 correspond to chi-square values of 36.781 and 10.982.

Right-tailed test

- If a researcher believes the variance of a population to be greater than some specific value, say, 225, then the researcher states the hypotheses as

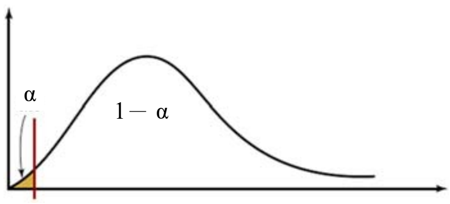
$$H_0: \sigma^2 \leq 225 \quad \text{and} \quad H_1: \sigma^2 > 225$$



Left-tailed test

- If the researcher believes the variance of a population to be less than 225, then the researcher states the hypotheses as

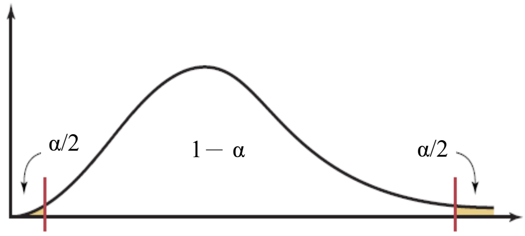
H_0: $\sigma^2 \geq 225$ and H_1: $\sigma^2 < 225$



Two-tailed test

- If a researcher does not wish to specify a direction, he or she states the hypotheses as

H_0: $\sigma^2 = 225$ and H_1: $\sigma^2 \neq 225$



8-5 χ^2 Test for σ² or σ

The chi-square distribution is also used to test a claim about a single variance or standard deviation.

The formula for the chi-square test for a variance is

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2}$$

with degrees of freedom d.f. = $n - 1$ and

n = sample size

s^2 = sample variance

σ^2 = population variance

The Assumptions

The Assumptions for the χ^2 test for σ^2 or σ

1. The sample must be randomly selected from the population.
2. The population must be normally distributed for the variable under study.
3. The observations must be independent of one another.

Example: 螺絲釘標準差

- 螺絲釘製品，根據以往經驗，半徑長度的變異數少於8.5cm^2。今改善其作方法後，以隨機方法抽得20個樣本，並求出其變異數為7.2cm^2，試用α = 5%，檢定螺絲釘製品有無改進？

Example: 機器螺絲釘

- A工廠製造某種機器螺絲釘，規定其長度之標準差不得超過0.10cm。今於其產品中抽查10支，求得其標準差為0.15cm。若以α = 0.025，請問該廠產品是否符合規定？

Example: Math SAT Test

The standard deviation for the Math SAT test is 100. The variance is 10,000. An instructor wishes to see if the variance of the 23 randomly selected students in her school is less than 10,000. The variance for the 23 test scores is 7225. Is there enough evidence to support the claim that the variance of the students in her school is less than 10,000 at α = 0.05? Assume that the scores are normally distributed.

Step 1: State the hypotheses and identify the claim.

Ste $H_0: \sigma^2 = 10{,}000$ and $H_1: \sigma^2 < 10{,}000$ (claim)

The critical value is 12.338.

Example: Math SAT Test

An instructor wishes to see whether the variation in scores of the 23 students in her class is less than the variance of the population. The variance of the class is 198. Is there enough evidence to support the claim that the variation of the students is less than the population variance (σ^2 =225) at α = 0.05? Assume that the scores are normally distributed.

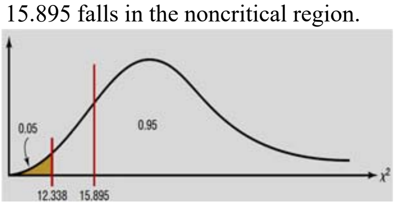
Step 3: Compute the test value.

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(23-1)7225}{10{,}000} = 15.895$$

Example: Math SAT Test

Step 4: Make the decision.

Do not reject the null hypothesis since the test value 15.895 falls in the noncritical region.



Step 5: Summarize the results.

There is not enough evidence to support the claim that the variation of the students' test scores is less than the population variance.

Example 8-26: Nicotine Content

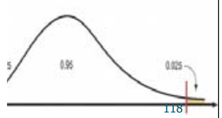
A cigarette manufacturer wishes to test the claim that the variance of the nicotine content of its cigarettes is 0.644. Nicotine content is measured in milligrams, and assume that it is normally distributed. A sample of 20 cigarettes has a standard deviation of 1.00 milligram. At α = 0.05, is there enough evidence to reject the manufacturer's claim?

Step 1: Sta **e claim.**

H_0: c

Step 2: Fir

The critical



Example 8-26: Nicotine Content

A cigarette manufacturer wishes to test the claim that the variance of the nicotine content of its cigarettes is 0.644. Nicotine content is measured in milligrams, and assume that it is normally distributed. A sample of 20 cigarettes has a standard deviation of 1.00 milligram. At α = 0.05, is there enough evidence to reject the manufacturer's claim?

Step 3: Compute the test value.

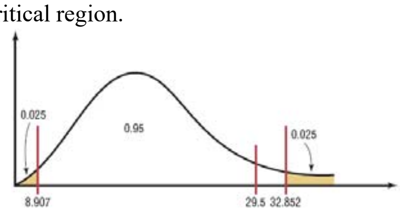
The standard deviation s must be squared in the formula.

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(19)(1.00)^2}{0.644} = 29.5$$

Example 8-26: Nicotine Content

Step 4: Make the decision.

Do not reject the null hypothesis, since the test value falls in the noncritical region.



Step 5: Summarize the results.

There is not enough evidence to reject the manufacturer's claim that the variance of the nicotine content of the cigarettes is 0.644.

Example 8-27: P-value

EXAMPLE 8–27

Find the P-value when $\chi^2 = 19.274$, $n = 8$, and the test is right-tailed.

Degrees of freedom	α									
	0.995	0.99	0.975	0.95	0.90	0.10	0.05	0.025	0.01	0.005
1	—	—	0.001	0.004	0.016	2.706	3.841	5.024	6.635	7.879
2	0.010	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210	10.597
3	0.072	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.345	12.838
4	0.207	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277	14.860
5	0.412	0.554	0.831	1.145	1.610	9.236	11.071	12.833	15.086	16.750
6	0.676	0.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812	18.548
7	0.989	1.239	1.690	2.167	2.833	12.017	14.067	16.013	18.475	20.278
8	1.344	1.646	2.180	2.733	3.490	13.362	15.507	17.535	20.090	21.955
9	1.735	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666	23.589
10	2.156	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209	25.188
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
100	67.328	70.065	74.222	77.929	82.358	118.498	124.342	129.561	135.807	140.169

*19.274 falls between 18.475 and 20.278

See Figure 8–37. Hence, the P-value is contained in the interval $0.005 < P\text{-value} < 0.01$. (The P-value obtained from a calculator is 0.007.)

EXAMPLE 8–28

Find the P-value when $\chi^2 = 3.823$, $n = 13$, and the test is left-tailed.

Hence, the P-value falls in the interval

$$0.01 < P\text{-value} < 0.025$$

(The P-value obtained from a calculator is 0.014.)

When the χ^2 test is two-tailed, both interval values must be doubled. If a two-tailed test were being used in Example 8–28, then the interval would be $2(0.01) < P\text{-value} < 2(0.025)$, or $0.02 < P\text{-value} < 0.05$.

The P-value method for hypothesis testing for a variance or standard deviation follows the same steps shown in the preceding sections.

Step 1 State the hypotheses and identify the claim.

Step 2 Compute the test value.

Step 3 Find the P-value.

Step 4 Make the decision.

Step 5 Summarize the results.

Example 8–29 shows the P-value method for variances or standard deviations.

EXAMPLE 8–29 Car Inspection Times

A researcher knows from past studies that the standard deviation of the time it takes to inspect a car is 16.8 minutes. A random sample of 24 cars is selected and inspected. The standard deviation is 12.5 minutes. At $\alpha = 0.05$, can it be concluded that the standard deviation has changed? Use the P-value method. Assume the variable is normally distributed.

SOLUTION

Step 1 State the hypotheses and identify the claim.

$$H_0: \sigma = 16.8 \quad \text{and} \quad H_1: \sigma \neq 16.8 \text{ (claim)}$$

Step 2 Compute the test value.

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(24-1)(12.5)^2}{(16.8)^2} = 12.733$$

Step 3 Find the P-value. Using Table G with d.f. = 23,

$0.05 < P\text{-value} < 0.10$

EXAMPLE 8–29 Car Inspection Times

A researcher knows from past studies that the standard deviation of the time it takes to inspect a car is 16.8 minutes. A random sample of 24 cars is selected and inspected. The standard deviation is 12.5 minutes. At $\alpha = 0.05$, can it be concluded that the standard deviation has changed? Use the P-value method. Assume the variable is normally distributed.

$\because 0.05 < P\text{-value} < 0.10$

Step 4 Make the decision. Since $\alpha = 0.05$ and the P-value is between 0.05 and 0.10, the decision is to not reject the null hypothesis since $P\text{-value} > \alpha$.

Step 5 Summarize the results. There is not enough evidence to support the claim that the standard deviation of the time it takes to inspect a car has changed.