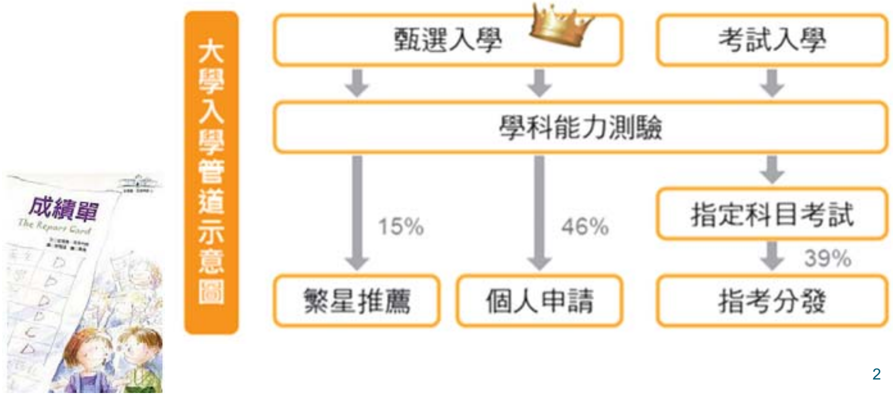
Chapter 12

ANalysis Of VAriance (ANOVA)

變異數分析

- ◆檢定三個以上的獨立<u>母體之平均值是否相等</u>時，可採用變異數分析(analysis of variance; ANOVA)
- ◆例：不同入學管道學生之課業表現是否具有差異？

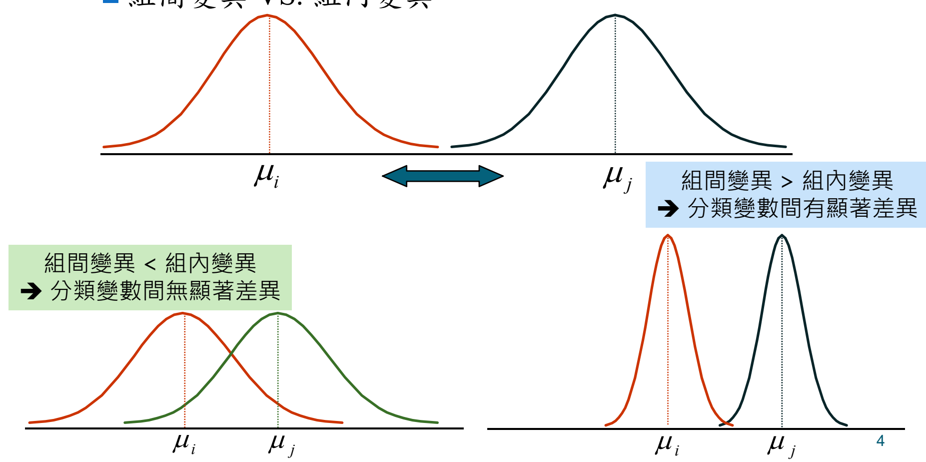


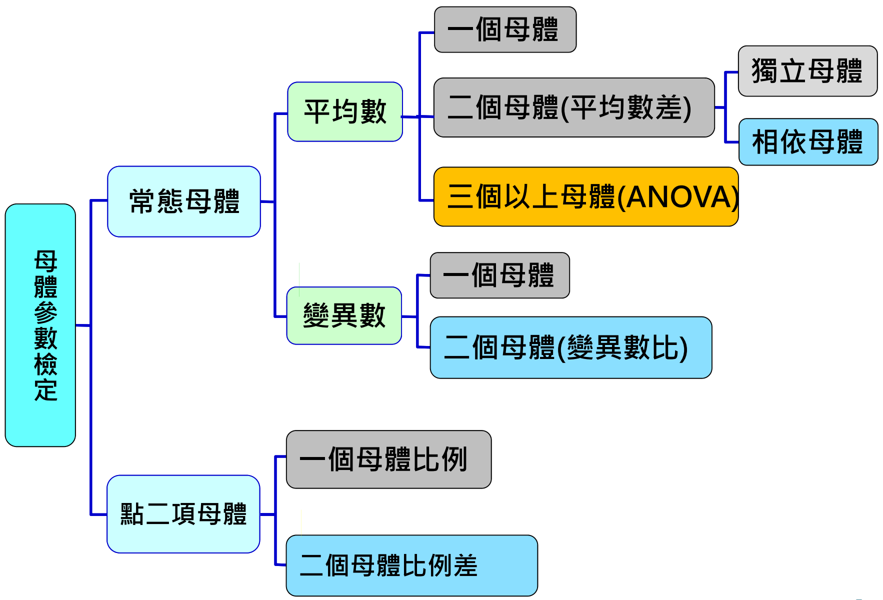
促銷活動

那些<u>促銷方式</u>
可有效提升<u>營業額</u>?

ANOVA基本概念

- ◆檢定三個以上的獨立<u>母體之平均值是否相等</u>
- ◆各組之間的平均數差異是否具顯著性
  - ■組間變異 VS. 組內變異





變異數分析的基本觀念

◆在比較多組母體的平均值時，我們通常不採用兩兩比較的方式，原因：這種做法太浪費時間，因為比較多個母體可能產生很多的比較組

- 例如比較五個母體的平均值差異，如果以兩兩比較的方式，我們必須進行C^5_2=10次的比較。
- 採用變異數分析

◆所謂變異數分析是指，檢定三個或三個以上的**母體平均數是否相等**的檢定方法，或檢定因子對依變數是否有影響的統計方法

變異數分析類別

◆檢定三個以上的獨立母體之平均值是否相等時

◆變異數分析種類繁多，如下表：

依變數個數	自變數個數	名稱
1 (單變數變異數分析)	1	單因子變異數分析
	2 (以上)	多因子變異數分析
2 (以上) (多變數變異數分析)	1	單因子多變量分析
	2 (以上)	多因子多變量分析

ANOVA分類(依因子數目)

◆單因子變異數分析

- 係指探討**單一分類性**的解釋變數對依變數之間的關係
  - 大學中各年級的同學智商是否有別？
  - 三種不同的教學方法對於學生的成績是否有影響？

◆多因子變異數分析

- 係指探討兩個以上分類性的解釋變數對依變數之間的關係
  - 探討教室氣氛和教學方法對學生學習成就的影響
  - 分析性別與不同廠牌機器對產量的影響

單因子變異數分析

- ◆單因子變異數分析係指探討單一分類性的解釋變數對依變數之間的關係
- ◆資料格式

觀測值	母體 1	母體 2	…	母體 j	…	母體 k	總平均
樣本 1	x_{11}	x_{12}	…	x_{1j}	…	x_{1k}	
樣本 2	x_{21}	x_{22}	…	x_{2j}	…	x_{2k}	
⋮	⋮	⋮	⋮	⋮	⋮	⋮	
樣本 i	x_{i1}	x_{i2}	…	x_{ij}	…	x_{ik}	
⋮	⋮	⋮	⋮	⋮	…	⋮	
樣本 n	$x_{n_1 1}$	$x_{n_2 2}$	…	$x_{n_j j}$	…	$x_{n_k k}$	
平均數	$\bar{x}_1$	$\bar{x}_2$	…	$\bar{x}_j$	…	$\bar{x}_k$	
變異數	s_1^2	s_2^2	…	s_j^2	…	s_k^2	$\bar{\bar{x}}$

範例: 單因子變異數分析-1

- ◆問題：三種入學管道，哪一類學生成績較優？
- ◆方法：記錄<u>三種入學管道</u>各五位學生第一學期末之學業總平均分數

	A	B	C
樣本資料	82.8	88.9	88.2
	79.4	76.5	91.3
	76.8	85.0	86.4
	73.4	82.7	82.5
	74.5	79.5	81.6

範例：單因子變異數分析-2

- ◆ 問題：某知名飲料公司欲了解其所生產的<u>**飲料容量是否因機器的種類而不同**</u>
- ◆方法：分別在<u>4台機器</u>獨立隨機抽取5罐飲料測量其容量

	機器一	機器二	機器三	機器四
樣本資料	7	5	3	5
	6	8	4	3
	4	7	6	4
	4	6	3	7
	6	4	6	5

多因子變異數分析

- ◆係指探討兩個以上分類性的解釋變數對依變數之間的關係，最常見者為二因子變異數分析
- ◆二因子變異數分析係探討兩個分類性的解釋變數對依變數之間的關係
- ◆資料格式(二因子)

Factor A

Factor B

因子	A_1	A_2		A_c	平均
B_1	x_{11}	x_{12}	…	x_{1c}	$\bar{B}_1$
B_2	x_{21}	x_{22}	…	x_{2c}	$\bar{B}_2$
⋮	⋮	⋮	…	⋮	⋮
B_r	x_{r1}	x_{r2}	…	x_{rc}	$\bar{B}_r$
平均	$\bar{A}_1$	$\bar{A}_2$	…	$\bar{A}_c$	$\bar{\bar{x}}$

範例：二因子變異數分析

◆探討性別與A, B, C三種不同機器對產量是否有影響?

Factor A: 機 器 類 型

Factor B: 性別

		X Type	Y Type	Z Type
性別	男性	4, 9, 8, 9, 6, 8	1, 3, 4 5, 3, 3	3, 9, 6 5, 9, 8
	女性	3, 8, 5 6, 3, 7	7, 3, 4 2, 5, 3	11, 8, 10 12, 9, 13

自變數與依變數

◆自變數或因子 (Factor)

- 自變數指引起資料發生差異的原因，故可稱為因子，亦稱獨立變數、自變數、實驗變數等
- 因子水準：每因子內之處理方式稱為「水準」(levels)

◆依變數 (dependent variable)

- 研究者欲觀察的反應變數

自變數與依變數

◆自變數或因子 (Factor)

- 自變數指引起資料發生差異的原因，故可稱為因子，亦稱獨立變數、自變數、實驗變數等
- 因子水準：每因子內之處理方式稱為「水準」(levels)
  - 範例一，入學方式因子，ABC三種入學管道為三個衡量水準(level)
  - 範例二，生產機器為因子，衡量水準為4(考慮四種不同機器)
  - 範例三，考慮兩種不同的因子：性別與機器

◆依變數 (dependent variable)

- 研究者欲觀察的反應變數
  - 範例一，汽油每公升可以行駛的公里數
  - 範例二，飲料容量
  - 範例三，生產量

範例: 單因子變異數分析-1

◆問題：三種入學管道，哪一類學生成績較優？

◆方法：記錄三種入學管道各五位學生第一學期末之學業總平均分數

	A	B	C
樣本資料	18.2	19.8	21.2
	19.4	21.0	21.8
	19.6	20.0	22.4
	19.0	20.8	22.0
	18.8	20.4	21.6

◆**自變數**：入學管道➔一個自變數、三個衡量水準

◆**依變數**：第一學期末之學業總平均分數➔一個依變數

範例：單因子變異數分析-2

◆ 問題：某知名飲料公司欲了解其所生產的**飲料容量是否因機器的種類而不同**

◆方法：分別在4台機器獨立隨機抽取5罐飲料測量其容量

	機器一	機器二	機器三	機器四
樣本資料	7	5	3	5
	6	8	4	3
	4	7	6	4
	4	6	3	7
	6	4	6	5

◆**自變數**：**機器別 ➔ 一個一變數、四個衡量水準**

◆**依變數**：**飲料容量➔一個依變數**

範例: 二因子變異數分析

◆探討性別與A, B, C三種不同機器對產量是否有影響?

Factor A: 機 器 類 型

Factor B: 性別	X Type	Y Type	Z Type
男性	4, 9, 8, 9, 6, 8	1, 3, 4 5, 3, 3	3, 9, 6 5, 9, 8
女性	3, 8, 5 6, 3, 7	7, 3, 4 2, 5, 3	11, 8, 10 12, 9, 13

◆**自變數**：**➔ 兩個自變數➔二因子變異數分析**

- **性別 (2 levels) & 機器別 (3 levels)**

◆**依變數**：**產量 ➔ 一個依變數**

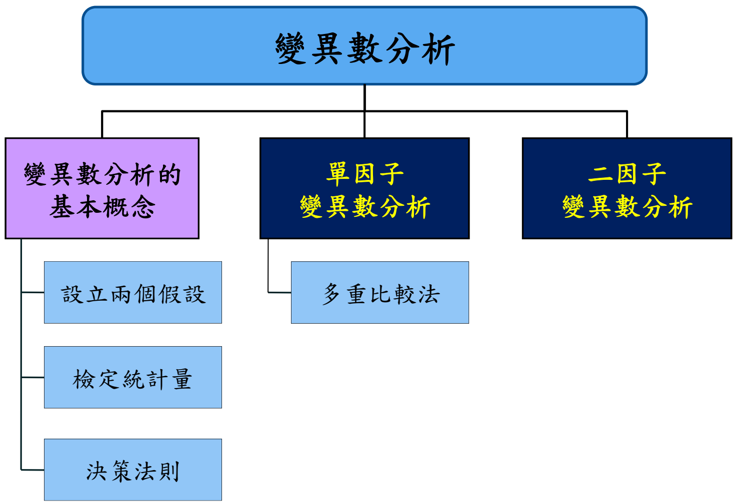
Chapter 12 Overview

Introduction

◆12-1 One-Way Analysis of Variance

◆12-2 The Scheffé Test and the Tukey Test

◆12-3 Two-Way Analysis of Variance



Introduction

- The *F* test, used to compare two variances, can also be used to compare three or more means.
- This technique is called **ANalysis Of VAriance** or **ANOVA**.
- For three groups, the *F* test can only show whether or not a difference exists among the three means, not where the difference lies.
- Other statistical tests, **Scheffé test** and the **Tukey test**, are used to find where the difference exists.

Section 12-1

One-Way Analysis of Variance

12-1 One-Way Analysis of Variance

- When an *F* test is used to test a hypothesis concerning the means of three or more populations, the technique is called **analysis of variance** (commonly abbreviated as **ANOVA**).
- Although the *t* test is commonly used to compare two means, it should not be used to compare three or more.

Assumptions for the *F* Test

The following assumptions apply when using the *F* test to compare three or more means.

1. The populations from which the samples were obtained must be normally or approximately normally distributed.
2. The samples must be independent of each other.
3. The variances of the populations must be equal.

變異數分析介紹

H_0: $\mu_1 = \mu_2 = \mu_3 = \cdots = \mu_k$

H_a: 所有母體平均**不全相等**

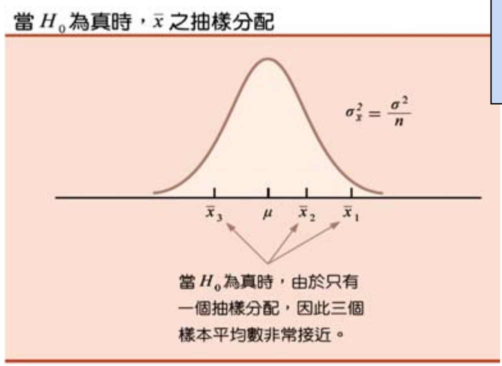
- 如果拒絕H_0，我們**不能**下結論說所有的母體平均數**都不相等**。
- 拒絕H_0意指至少有兩個母體平均數不相等。

One way ANOVA (單因子ANOVA)

H_0: $\mu_1 = \mu_2 = \mu_3 = \Lambda = \mu_k$

H_1 : Not all μ_j are the same

All means are the same:
The null hypothesis is true
(no treatment effect)

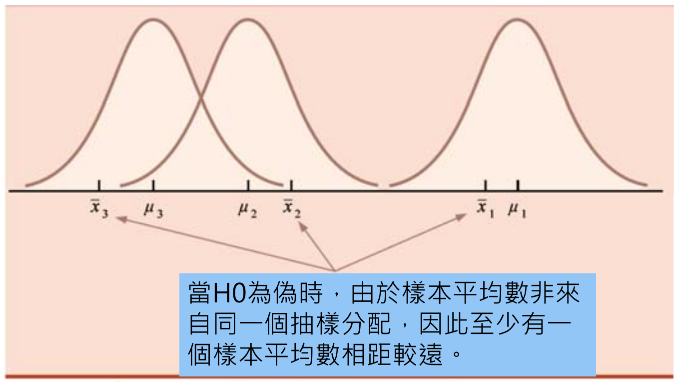


One way ANOVA (單因子ANOVA)

H_0: $\mu_1 = \mu_2 = \mu_3 = \ldots = \mu_k$

H_1 : Not all μ_j are the same

At least one mean is different:
The null hypothesis is NOT true
(treatment effect is present)

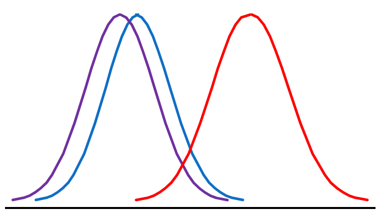


One way ANOVA (單因子ANOVA)

H_0: $\mu_1 = \mu_2 = \mu_3 = \ldots = \mu_k$

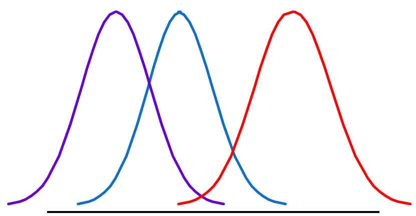
H_1 : Not all μ_j are the same

At least one mean is different:
The null hypothesis is NOT true
(treatment effect is present)



$\mu_1 = \mu_2 \neq \mu_3$

or



$\mu_1 \neq \mu_2 \neq \mu_3$

分割總變異數

◆總變異(Total variation)可分割成二部份:

$$\boxed{SST = SSB\ (SSTr) + SSW\ (SSE)}$$

SST = Total Sum of Squares
(Total variation)

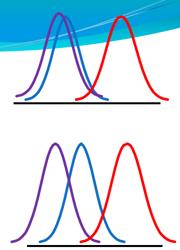
SSB (SSTr) = Sum of Squares Between Groups (Treatments)
(Between-group variation)

SSW (SSE) = Sum of Squares Within Groups
(Within-group variation)

The *F* Test

- In the *F* test, two different estimates of the population variance are made.
- The first estimate is called the **between-group variance**, and it involves finding the variance of the means.
- The second estimate, the **within-group variance**, is made by computing the variance using all the data and is not affected by differences in the means.

Total Variation



總變異數 (SST)

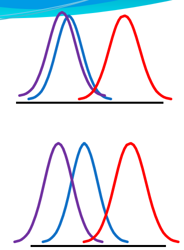
d.f. = n – 1

= 由於因子所形成平方和 (SSB) + 由於隨機抽樣所形成平方和 (SSW)

d.f. = $k-1$ d.f. = $N-k$

Commonly referred to as:	Commonly referred to as:
Sum of Squares Between Groups	Sum of Squares Within Groups
Sum of Squares Among Groups	Sum of Squares Error
Sum of Squares Explained	Sum of Squares Unexplained
Among Groups Variation	Within-Group Variation

Total Variation



$\boxed{SST}$ = SSB + SSW

$$SST = \sum_{j=1}^{k}\sum_{i=1}^{n_j}(X_{ij} - \overline{\overline{X}})^2$$

Where:

$$\overline{\overline{X}} = \overline{X}_{GM} = \frac{\sum\sum X_{ij}}{\sum n_i}$$

SST : Total sum of squares

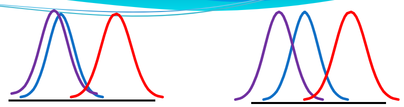
k : number of groups (levels or treatments)

n_j : number of observations in the j^{th} group

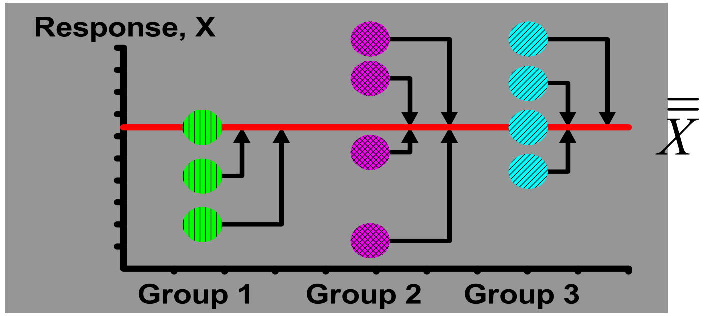
X_{ij} = the i^{th} observation in the j^{th} group

$\overline{\overline{X}}$ = the aggregate mean (the grand mean, $\overline{X}_{GM}$)

Total Variation
(continued)



$$SST = (X_{11} - \overline{\overline{X}})^2 + (X_{12} - \overline{\overline{X}})^2 + \ldots + (X_{k\,n_k} - \overline{\overline{X}})^2$$



Sum of Squares Between Groups

$$SST = \boxed{SSB} + SSW$$

$$SSB = \sum_{j=1}^{k} n_j (\overline{X}_j - \overline{\overline{X}})^2$$

Where:

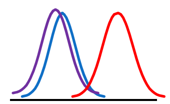
SSB : Sum of Squares Between Groups

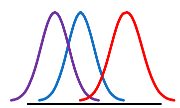
k : number of groups (levels or treatments)

n_j : number of observations in the j^{th} group

$\overline{X}_j$: Mean value of the j^{th} treatment

$\overline{\overline{X}}$: Aggregate mean



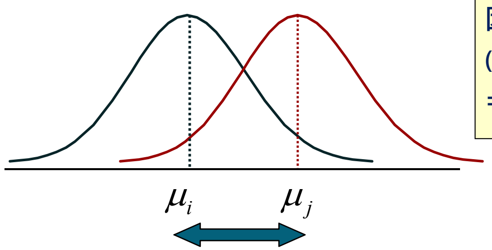


Sum of Squares Between Groups

(continued)

$$SSB = \sum_{j=1}^{k} n_j (\overline{X}_j - \overline{\overline{X}})^2$$

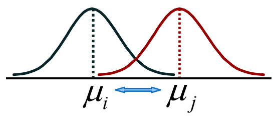
Variation Due to
Differences Between Groups



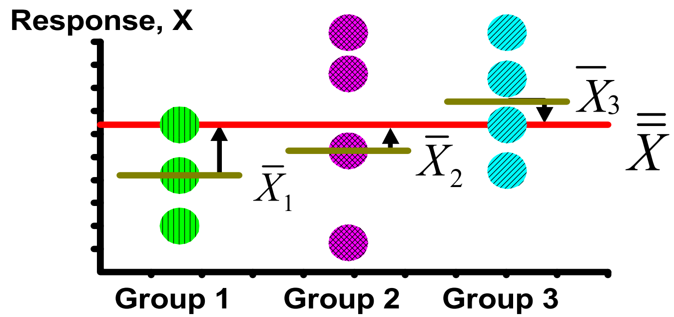
$$MSB = \frac{SSB}{k-1}$$

因子間之平均變異
(Mean Square Between Groups)
=因子間之總變異數 / 自由度

Sum of Squares Between Groups *(continued)*



$$SSB = n_1(\bar{x}_1 - \bar{\bar{x}})^2 + n_2(\bar{x}_2 - \bar{\bar{x}})^2 + \ldots + n_k(\bar{x}_k - \bar{\bar{x}})^2$$



Sum of Squares Within Groups

$$SST = SSB + \boxed{SSW}$$

$$SSW = \sum_{j=1}^{k} \sum_{i=1}^{n_j} (X_{ij} - \overline{X}_j)^2$$

Where:

SSW : Sum of Squares Within Groups

k : number of groups (levels or treatments)

n_j : number of observations in the j^{th} group

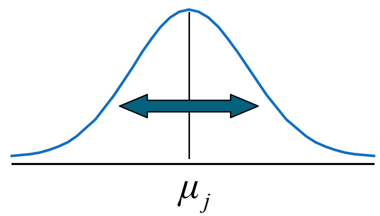
$\overline{X}_j$: Mean value of the j^{th} treatment

X_{ij} : the i^{th} observation in the j^{th} group

Sum of Squares Within Groups *(continued)*

$$SSW = \sum_{j=1}^{k} \sum_{i=1}^{n_j} (X_{ij} - \overline{X}_j)^2$$

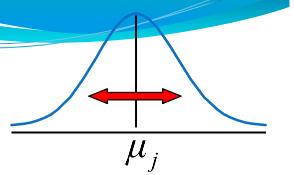
Summing the variation within each group and then adding over all groups



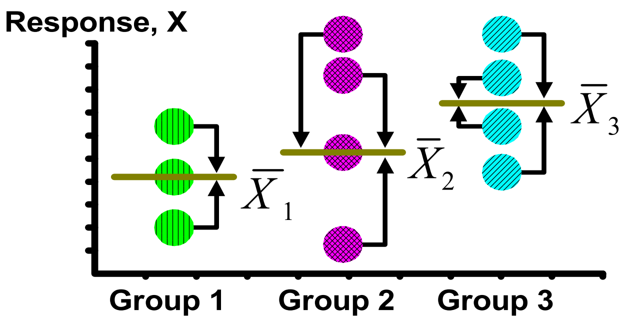
$$MSW = \frac{SSW}{N-k}$$

因子內之平均變異數
(Mean Square Within)
=因子內之變異數 /自由度

Sum of Squares Within Groups *(continued)*

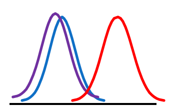


$$SSW = (x_{11} - \overline{X}_1)^2 + (X_{12} - \overline{X}_2)^2 + \ldots + (X_{k\,n_k} - \overline{X}_k)^2$$

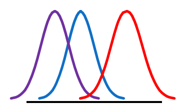


平均變異數

$$MSB = \frac{SSB}{k-1}$$



$$MSW = \frac{SSW}{N-k}$$



$$\text{Test value:} \quad F = \frac{MSB}{MSW}$$

One-Way ANOVA Table (p.650)

Source of Variation	SS	df	MS (Variance)	F ratio
Among Groups	SSB	k - 1	MSB = $\frac{\text{SSB}}{\text{k - 1}}$	F = $\frac{\text{MSB}}{\text{MSW}}$
Within Groups	SSW	N - k	MSW = $\frac{\text{SSW}}{\text{N - k}}$	
Total	SST = SSB+SSW	N - 1		

c = number of groups
n = sum of the sample sizes from all groups
df = degrees of freedom

F Test for One-Way ANOVA

H_0: $\mu_1 = \mu_2 = \ldots = \mu_c$

H_1: At least one mean is different from the others

- test value

$$F = \frac{MSB}{MSW}$$

MSB is the **Mean Square Between Groups**
MSW is the **Mean Square Within Groups**

- Degrees of freedom
  - $df_1 = k - 1$ (k = number of groups)
  - $df_2 = N - k$ (n = sum of sample sizes from all populations)

The F test to compare the means is always right-tailed.

The procedure of ANOVA

- **Step 1 State the hypotheses and identify the claim.**

H_0: $\mu_1 = \mu_2 = \ldots = \mu_k$

H_1: At least one mean is different from the others

- **Step 2 Find the critical value.**
  - **F-test,**
    - **df.N = *k*-1, df.D = *N* - *k***
- **Step 3 Compute the test value**
- **Step 4 Make the decision.**
- **Step 5 Summarize the results.**

The procedure of ANOVA

- **Step 3 Compute the test value**
  - Find the mean and variance of each sample
  - Find the **Mean Square Between Groups** or **between-group variance**, MSB or s_B^2
    - Sum of Squares Between Groups (SSB) ➔ $MSB = S_B^2 = \frac{SSB}{k\text{-}1}$
  - Find the **Mean Square Within Groups** or **within-group variance**, MSW or s_W^2
    - Sum of Squares Within Groups (SSW) ➔ $MSW = S_W^2 = \frac{SSW}{N - k}$
  - **Test value:** $F = \frac{MSB}{MSW} = \frac{S_B^2}{S_W^2}$
- **Step 4 Make the decision.**
- **Step 5 Summarize the results.**

The F test to compare the means is always right-tailed.

The procedure of ANOVA

- **Step 1 State the hypotheses and identify the claim.**
- **Step 2 Find the critical value.**
- **Step 3 Compute the test value**
- **Step 4 Make the decision.**
- **Step 5 Summarize the results.**

The *F* Test

- If there is no difference in the means, the between-group variance will be approximately equal to the within-group variance, and the *F* test value will be close to 1—*do not reject null hypothesis*.
- However, when the means differ significantly, the between-group variance will be much larger than the within-group variance; the *F* test will be significantly greater than 1—*reject null hypothesis*. (組間變異大、組內變異小)

Example 12-1: Miles per Gallon

A researcher wishes to see if there is a difference in the fuel economy for city driving for three different types of automobiles: small automobiles, sedans, and luxury automobiles. He randomly samples four small automobiles, five sedans, and three luxury automobiles. The miles per gallon for each is shown. At α = 0.05, test the claim that there is no difference among the means.

Small	Sedans	Luxury
36	43	29
44	35	25
34	30	24
35	29	
	40	

Example 12-1: Miles per Gallon

Small	Sedans	Luxury
36	43	29
44	35	25
34	30	24
35	29	
	40	

Step 1: State the hypotheses and identify the claim.

H_0: $\mu_1 = \mu_2 = \mu_3$ (claim)

H_1: At least one mean is different from the others.

Step 2: Find the critical value.

Since k = 3, n = 12, α = 0.05

d.f.N. = k – 1 = 3 – 1 = 2

d.f.D. = N – k = 12 – 3 = 9

From Table H ➔ The critical value, $F_{(3,12,\ 0.05)} = 4.26$

Example 12-1: Miles per Gallon

Step 3: Compute the test value. (F = MSB / MSW)

a. Find the mean and variance of each sample.

b. Find the **grand mean.**

c. Find the **between-group variance (SSB → MSB).**

d. Find the **within-group variance (SSW → MSW).**

e. Find the test value, **F = MSB / MSW** .

Example 12-1: Miles per Gallon

Step 3: Compute the test value. (F = MSB / MSW)

a. Find the mean and variance of each sample.

	Small	Sedans	Luxury
	36	43	29
	44	35	25
	34	30	24
	35	29	
		40	
Mean	37.25	35.4	26
Variance	20.92	37.3	7

b. Find the **grand mean.**

$$\overline{X}_{GM} = \frac{\sum X}{N} = \frac{36+44+...+25+44}{12} = 33.67$$

Example 12-1: Miles per Gallon

Step 3: Compute the test value. (continued)

c. Find the **SSB** and **MSB**.

$$SSB = \sum n_i(\overline{X}_i - \overline{X}_{GM})^2$$
$$= 4(37.25-33.67)^2 + 5(35.4-33.67)^2 + 3(26-22.67)^2$$
$$= 242.72$$

$$MSB = S_B^2 = \frac{SSB}{k-1} = \frac{242.72}{3-1} = 121.36$$

d. Find the **SSW** and **MSW**.

$$SSW = \sum(n_i - 1)S_i^2 = (4-1)20.92 + (5-1)37.3 + (3-1)7$$
$$= 225.95$$

$$MSW = S_W^2 = \frac{225.95}{9} = 25.11$$

Example 12-1: Miles per Gallon

Step 3: Compute the test value. (continued)

e. Compute the F value.

$$F = \frac{\text{variance between groups}}{\text{variance within groups}}$$

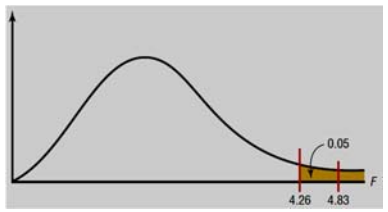
$$= \frac{MSB}{MSW} = \frac{S_B^2}{S_W^2} = \frac{121.36}{25.11} = 4.83$$

Step 4: Make the decision.

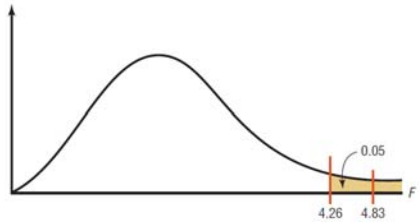
Reject the null hypothesis,

since test-value > C.V.

(4.83 > 4.26)



Example 12-1: Miles per Gallon



Step 5: Summarize the results.

There is enough evidence to reject the claim and conclude that at least one mean is different from the others.

ANOVA Summary Table for Example 12-1

Source	Sum of Squares	d.f.	Mean Squares	*F*
Between				
Within (error)				
Total				

Example: Lowering Blood Pressure

A researcher wishes to try three different techniques to lower the blood pressure of individuals diagnosed with high blood pressure. The subjects are randomly assigned to three groups; the first group takes medication, the second group exercises, and the third group follows a special diet. After four weeks, the reduction in each person's blood pressure is recorded. At α = 0.05, test the claim that there is no difference among the means.

Medication	Exercise	Diet
10	6	5
12	8	9
9	3	12
15	0	8
13	2	4
$\bar{X}_1 = 11.8$	$\bar{X}_2 = 3.8$	$\bar{X}_3 = 7.6$
$s_1^2 = 5.7$	$s_2^2 = 10.2$	$s_3^2 = 10.3$

Example: Lowering Blood Pressure

Medication	Exercise	Diet
10	6	5
12	8	9
9	3	12
15	0	8
13	2	4
$\bar{X}_1 = 11.8$	$\bar{X}_2 = 3.8$	$\bar{X}_3 = 7.6$
$s_1^2 = 5.7$	$s_2^2 = 10.2$	$s_3^2 = 10.3$

Step 1: State the hypotheses and identify the claim.

H_0: $\mu_1 = \mu_2 = \mu_3$ (claim)

H_1: At least one mean is different from the others.

Example: Lowering Blood Pressure

Medication	Exercise	Diet
10	6	5
12	8	9
9	3	12
15	0	8
13	2	4
$\overline{X}_1 = 11.8$	$\overline{X}_2 = 3.8$	$\overline{X}_3 = 7.6$
$s_1^2 = 5.7$	$s_2^2 = 10.2$	$s_3^2 = 10.3$

Step 2: Find the critical value.

Since $k = 3$, $n = 15$, and $\alpha = 0.05$,

d.f.N. = $k - 1 = 3 - 1 = 2$

d.f.D. = $N - k = 15 - 3 = 12$

➔ The critical value, $F_{(2,12,\ 0.05)} = 3.89$.

(obtained from Table H)

Step 3: Compute the test value. **(F = MSB / MSW)**

a. Find the mean and variance of each sample. (given)

b. Find the **grand mean.**

$$\overline{X}_{GM} = \frac{\sum X}{N} = \frac{10+12+9+\cdots+4}{15} = \frac{116}{15} = 7.73$$

b. Find the **between-group variance (SSB → MSB)**.

d. Find the **within-group variance (SSW → MSW)**.

e. Find the test value, **F = MSB / MSW** .

Example: Lowering Blood Pressure

Step 3: Compute the test value.

c. Find the **between-group variance**, MSB or s_B^2 . **(SSB→MSB)**

$$SSB = \sum n_i(\overline{X}_i - \overline{X}_{GM})^2$$

$$MSB = S_B^2 = \frac{SSB}{k-1}$$

$$s_B^2 = \frac{5(11.8-7.73)^2 + 5(3.8-7.73)^2 + 5(7.6-7.73)^2}{3-1}$$

$$= \frac{160.13}{2} = 80.07$$

Example: Lowering Blood Pressure

Step 3: Compute the test value. (continued)

d. Find the **within-group variance**, MSW, s_W^2.

$$SSW = \sum(n_i - 1)S_i^2$$

$$MSW = S_W^2 = \frac{SSW}{\sum(n_i - 1)}$$

$$s_W^2 = \frac{\sum(n_i - 1)s_i^2}{\sum(n_i - 1)}$$

$$= \frac{4(5.7)+4(10.2)+4(10.3)}{4+4+4} = \frac{104.80}{12} = 8.73$$

Example: Lowering Blood Pressure

Step 3: Compute the test value. (continued)

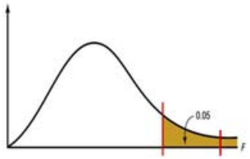
e. Compute the F value.

$$F = \frac{\text{variance between groups}}{\text{variance within groups}}$$

$$= \frac{MSB}{MSW} = \frac{S_B^2}{S_W^2} = \frac{80.07}{8.73} = 9.17$$

Step 4: Make the decision.

Reject the null hypothesis, since 9.17 > 3.89.

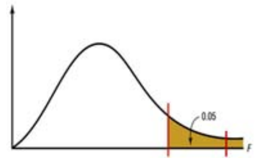


Step 5: Summarize the results.

There is enough evidence to reject the claim and conclude that at least one mean is different from the others.

ANOVA Table
Example of Lowering Blood Pressure

Source	Sum of Squares	d.f.	Mean Squares	*F*
Between				
Within (error)				
Total				



Report from MINITAB

One-Way ANOVA: Medication, Exercise, Diet

```
Source    DF      SS      MS      F      P
Factor     2  160.13   80.07   9.17  0.004
Error     12  104.80    8.73
Total     14  264.93

                                Individual 95% CIs For Mean
                                Based on Pooled StDev
Level        N    Mean   StDev  -------+---------+---------+---------+--
Medication   5  11.800   2.387                       (--------*-------)
Exercise     5   3.800   3.194  (-------*-------)
Diet         5   7.600   3.209           (-------*-------)
                                -------+---------+---------+---------+--
                                     3.5       7.0      10.5      14.0
Pooled StDev = 2.955
```

Reject the null hypothesis. There is enough evidence to conclude that there is a difference between the treatments.

ANOVA

- ◆The between-group variance is sometimes called the **mean square between groups, *MSB***.
- ◆The numerator of the formula to compute ***MSB*** is called the **sum of squares between groups, *SSB***.
- ◆The within-group variance is sometimes called the **mean square within group, *MSW***.
- ◆The numerator of the formula to compute ***MSW*** is called the **sum of squares within groups, *SSW***.

變異數分析的基本概念

- 總差異

  總差異 ＝ 因子引起的差異 ＋ 隨機差異

  $SST = SSB + SSW$

- 因子引起的差異(組間差異）SSB

$$SSB = \sum_{i=1}^{k}\sum_{j=1}^{n_i}(\overline{X}_i - \overline{X}_{GM})^2$$

$$= \sum_{i=1}^{k} n_i(\overline{X}_i - \overline{X}_{GM})^2$$

- 隨機差異(組內差異） SSE or SSW

$$SSW = \sum_{i=1}^{k}\sum_{j=1}^{n_i}(X_{ij} - \overline{X}_i)^2$$

$$= \sum_{i=1}^{k}(n_i - 1)S_i^2$$

變異數分析的基本概念

- F檢定統計量

$$F = \frac{MSB}{MSW}$$

- 決策法則

  ①若 $F > F_{k-1,N-k,\alpha}$ ，則拒絕 H_0 。

  ②若 $F \leq F_{k-1,N-k,\alpha}$ ，則接受 H_0 。

ANOVA Summary Table

Source	Sum of Squares	d.f.	Mean Squares	F
Between	***SSB***	$k-1$	$MSB = \frac{SSB}{k-1}$	$\frac{MSB}{MSW}$
Within (error)	***SSW***	$N-k$	$MSW = \frac{SSW}{N-k}$	
Total	SST	$N-1$		

Exercise

Method I	Method II	Method III
48	55	84
73	86	65
52	70	95
65	69	74
87	90	67
$\Sigma X_1 = 325$	$\Sigma X_2 = 370$	$\Sigma X_3 = 385$
$n_1 = 5$	$n_2 = 5$	$n_3 = 5$