Name：

Statistics (2) Quiz-1 Date: Mar. 27, 2018

ID：

1. A recent survey of 9 randomly selected social networking sites has a mean of 13.1 million visitors for a specific month. The standard deviation is 4.2 million. Find the 90% confidence interval of true mean.

90% C.I. for μ $\Rightarrow \bar{X} \pm t \cdot S/\sqrt{n}$

σ未

$n=9$ σ未知 ⇒ t

(10.496 , 15.704)

5

$13.1 \pm 1.86 \times \frac{4.2}{\sqrt{9}} \Rightarrow$

$df=8 \Rightarrow t=1.86$ ①

2. A survey of 120 Americans showed that 90 said they find it hard to buy holiday gifts that convey their true feelings. Find the 95% confidence interval of the population proportion.

7

$\hat{p}=\frac{90}{120}=0.75 \Rightarrow$ C.I. : $\hat{p} \pm z \cdot \sqrt{\hat{p}\hat{q}/n}$

$np>5$ & $nq>5$ $\Rightarrow 0.75 \pm 1.96 \times \sqrt{\frac{0.75 \times 0.25}{120}} \Rightarrow$ (0.67 , 0.8274)

3. A researcher wishes to estimate the average number of minutes per day a person spends on the Internet. How large a sample must she select if she wishes to be 98% confident that the population mean is within 10 minutes of the sample mean? Assume the population standard deviation is 42 minutes.

$e=10.$

$\sigma=42$

$e=z \cdot \frac{\sigma}{\sqrt{n}}$

⇒ 取 $n=96.$

5

$\Rightarrow n=\left(\frac{z \cdot \sigma}{e}\right)^2=\left(\frac{2.33 \times 42}{10}\right)^2=95.8$

4. A researcher wishes to be 95% confident that her estimate of the true proportion of individuals who travel overseas is within 4% of the true proportion. Find the sample necessary if, in a prior study, a sample of 200 people showed that 40 traveled overseas last year. If no estimate of the sample proportion is available, how large should the sample be?

10.

$\hat{p}=20\%.$ $e=4\%$

$e=z \cdot \sqrt{\hat{p}\hat{q}/n}$

$n=\hat{p}\hat{q} \cdot \left(\frac{z}{e}\right)^2=0.2 \times 0.8 \times \left(\frac{1.96}{4\%}\right)^2$

$=384.16$ 取 $n=385$ ①

② $\hat{p}$ 未知 ⇒ $\hat{p}=50\%.$

2

$n=0.5 \times 0.5 \times \left(\frac{1.96}{0.04}\right)^2$

$=600.25$

取 $n=601$

5. Obesity is defined as a body mass index (BMI) of 30kg/m^2 or more. A 95% confidence interval for the percentage of U.S. adults aged 20 years and over who were obese was found to be 21.2 to 22.4%. What was the sample size?

7

$\Rightarrow e=\frac{22.4\%-21.2\%}{2}$

$=0.6\%$

$n=\hat{p} \cdot \hat{q} \cdot \left(\frac{z}{e}\right)^2$

$=0.5 \times 0.5 \times \left(\frac{1.96}{0.6\%}\right)^2$

$=26677.8$

取 $n=26678.$ 1

6. Find the 90% confidence interval for the variance and standard deviation for the lifetimes of inexpensive wristwatches if a random sample of 24 watches has a standard deviation of 4.8 months. Assume the variable is normally distributed.

10

90% C.I. for σ^2 $df=23$

$\frac{(n-1)S^2}{\chi^2_R} < \sigma^2 < \frac{(n-1)S^2}{\chi^2_L}$

$\chi^2_{Right}=35.172$

$\chi^2_{Left}=13.091$

$\frac{(24-1) \times 4.8^2}{35.172} < \sigma^2 < \frac{(24-1) \times 4.8^2}{13.091}$

$15.07 < \sigma^2 < 40.48$

$3.88 < \sigma < 6.36$

7. A researcher claims that the yearly consumption of soft drinks per person is 200 liters. In a sample of 49 randomly selected people, the mean of the yearly consumption was 216 liters. The standard deviation of the population is 36.4 liters. (a) Use the sample data to construct the 95% confidence interval for the true mean. (b) Is the researcher's claim valid (use *P*-value)?

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$\bar{x}=216,\ \sigma^2=36.4 \Rightarrow$ Z

95% C.I. for μ

$\bar{x} \pm Z \cdot \frac{\sigma}{\sqrt{n}}$ 5

$\Rightarrow 216 \pm 1.96 \times 36.4/\sqrt{49}$

$(205.8,\ 226.2)$

① $H_0: \mu = 200$ (claim)
$H_1: \mu \neq 200$

② $\alpha = 0.05$

③ test value

$z = \frac{216-200}{36.4/\sqrt{49}} = 3.08$

p-value $= 2 \times p(z > 3.08) = 0.002$

④ Decision

$\because$ p-value $< \alpha = 0.005$

$\Rightarrow$ reject H_0

⑤ summary

有足夠的証据証明宣称有誤。

8. Suppose a statistician chose to test a hypothesis at α=0.01. The critical value of a right-tailed test is +2.33. If the test value were 1.97, what would the decision be? What would happen if, after seeing the test value, she decided to choose α=0.05? What would the decision be? Explain the contradiction, if there is one.

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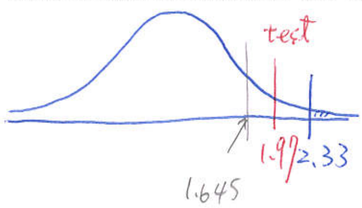

$\alpha = 0.05$

① 1.97 ∉ critical region

~~do not rej~~ do not reject H_0

② $\alpha = 0.05 \Rightarrow$ 1.97 ∈ critical region

$\Rightarrow$ reject H_0.

9. (a) State the alternative hypothesis (μ_0 as the hypothesized value) and draw a picture to show the critical region(s).
(b) Find the *P*-value interval for the test value and determine whether the null hypothesis should be rejected.

(1) t = 2.321, n=15, right-tailed **(2) t = 1.945, n=28, two-tailed**

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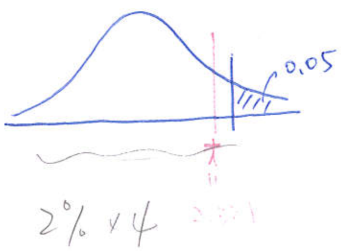
$H_1: \mu > \mu_0$



2% × 4

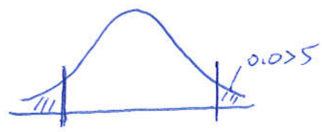
$\because t = 2.321$

$\Rightarrow 0.01 <$ p-value < 0.025

$\because$ p-value $< \alpha$

$\Rightarrow$ reject H_0.

$H_1: \mu \neq \mu_0$



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$0.05 <$ p-value < 0.1

$\Rightarrow$ p-value $> \alpha \Rightarrow$ do not rej. H_0

10. An obstetrician read that a newborn baby loses on average 215 grams in the first two days of his or her life. He feels that in the hospital where he works, the average weight loss of a newborn baby is less than 215 grams. A random sample of 36 newborn babies has a mean weight loss of 195 grams and the standard deviation is 54 grams. Is there enough evidence at α = 0.03 to support his claim (use P-value method)?

$n=36$
↓
Z

10.

① $H_0: \mu \geq 215$
$H_1: \mu < 215$ (claim)

② $\alpha = 0.05$

③ test value

$z = \frac{195-215}{54/\sqrt{36}}$

$= -2.22$

$\Rightarrow$ p-value $= P(z < -2.22) = 0.0132$

④ p-value $< \alpha$

$\Rightarrow$ ~~do not~~ rej. H_0.

⑤ ~~無~~有足夠証据支持宣称.

11. A survey of 15 large U.S. cities finds that the average commute time one way is 25.4 minutes. A chamber of commerce executive feels that the commute is his city is less and wants to publicize this. He randomly selects 25 commuters and finds the average is 22.1 minutes with a standard deviation of 5.5 minutes. At α=0.10, is he correct?

$n=25$
σ未知
10 ⇓
t

① $H_0: \mu \geq 25.4$
$H_1: \mu < 25.4$ (claim)

② $\alpha = 0.1$

CV: $t = 1.711$

$df = 24$

③ test value

$t = \frac{\bar{x}-\mu}{s/\sqrt{n}} = \frac{22.1-25.4}{5.5/\sqrt{25}} = -3$

1.5

④ $\Rightarrow$ reject H_0. $\because$ test value ∈ critical region

⑤ 有足夠証据支持宣称.