Statistics (2) Midterm

Date: Apr. 24, 2018

Name :

ID : total 117

9 9 $n = 81 \Rightarrow z$

1\. A study of 81 apple trees showed that the average number of apples per tree was 825 and the standard deviation is 135.
(a) What is the 95% confidence interval for the mean number of apples per tree for all trees?
(b) State the hypothesis and use confidence interval method to test whether the mean number of the apples per tree is 800 ($\alpha = 0.05$)?

(a) 95% C.I. for μ

$\bar{x} \pm z \, s/\sqrt{n}$ (z = 1.96)

② $\Rightarrow (795.6, 854.4)$

(b) ① $\begin{cases} H_0: \mu = 800 \text{ (claim)} \\ H_1: \mu \neq 800 \end{cases}$

② $\because$ 800 $\in$ 95% C.I.

$\Rightarrow$ do not rej. H_0.

③ 亦即.無足夠証據宣稱有誤.

$\mu_1 - \mu_2$ n大 $\Rightarrow z$

2\. A sociologist expects the life expectancy of people in Africa is ~~different~~ than the life expectancy of people in Asia. The data obtained is shown in the table below.
(a) Determine the 95% confidence interval for the difference in the population means.
(b) Can it conclude that the life expectancy of people in Africa is ~~different~~ less than the life expectancy of people in Asia? ($\alpha = 0.10$)

Africa	Asia
$\bar{x}_1=55.3$ $\sigma_1=8.1$ $n_1=53$	$\bar{x}_2=65.2$ $\sigma_2=9.3$ $n_2=42$

10

(a) 95% C.I. for $(\mu_1 - \mu_2)$

$(\bar{x}_1 - \bar{x}_2) \pm z \cdot \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$ (① 1.96)

② $\Rightarrow (-13.46, -6.34)$

(b) ① $\begin{cases} H_0: \mu_1 \geq \mu_2 \\ H_1: \mu_1 < \mu_2 \text{ (claim)} \end{cases}$

② $\alpha = 0.1 \Rightarrow$ CV. $z_c = -1.28$

③ $z = \dfrac{(\bar{x}_1 - \bar{x}_2) - 0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = -5.45$

④ Decision:

$\because$ test value < CV

$\Rightarrow$ rej. H_0

⑤ 非洲人の壽命有顯著低於亞洲人.

非く亞

3. How large a sample is needed to estimate the population mean for the amount of money a person spends on Christmas presents within \$100 and be 95% confident? The standard deviation of the population is \$600.

3 3

1

$$n = \left(\frac{z \cdot s}{e}\right)^2 \quad ① \; 1.96.$$

$$= ① \; 138.4$$

① 取 $n = 139$

10 10

4. A reporter bought hamburgers at randomly selected stores of two different restaurant chains, and had the number of Calories in each hamburger measured.

1+1 (a) Construct the 95% confidence interval of the different number of Calories for the two restaurant chains. 3

(b) Can the reporter conclude, at $\alpha = 0.05$, that the hamburgers from the two chains have a different number of Calories?

	Chain A	Chain B
Sample size	6	8
Sample mean	230 Cal	285 Cal
Sample standard deviation	23 Cal	25 Cal

n↓1 σ未知 ⇓ t $df = 5$.

(a) 95% CI for $(\mu_1 - \mu_2)$

$$(\bar{x}_1 - \bar{x}_2) \pm t \cdot \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \quad ① \; 2.571$$

$$\Rightarrow ② \; (-88.15, -21.85)$$

(b) 2 ①
$$\begin{cases} H_0: \mu_1 = \mu_2 \\ H_1: \mu_1 \neq \mu_2 \text{ (claim)} \end{cases}$$

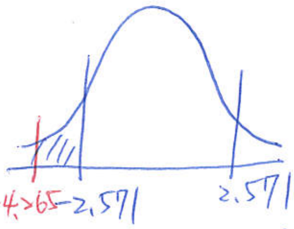
$\alpha = 0.05$

1 ② C.V. $t_c = \pm 2.571$

2 ③ test value

$$t = (\bar{x}_1 - \bar{x}_2) \Big/ \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

$$= -4.265.$$



1 ④ rej. H_0. < test

1 有顯著差異

5. A researcher claims that students in a private school have exam scores that are at most 5 points higher than those of students in public schools. Random samples of 60 students from each type of school are selected and given an exam. The results are shown. At $\alpha = 0.05$, test the claim.

Private School	Public School
$\bar{x}_1=112$	$\bar{x}_2=104$
$\sigma_1=15$	$\sigma_2=15$
$n_1=60$ 50	$n_2=60$ 50

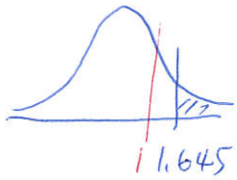
8

① $\begin{cases} H_0: \mu_1 - \mu_2 \le 5 \text{ (claim)} \\ H_1: \mu_1 - \mu_2 > 5 \end{cases}$

2

② $\alpha = 0.05$, CV: $Z = 1.645$

1



③ $Z = \dfrac{(\bar{X}_1 - \bar{X}_2) - 5}{\sqrt{\dfrac{\sigma_1^2}{50} + \dfrac{\sigma_2^2}{50}}} = 1.$

3

④ Do not rej. Ho.

1

私

1 私校成績高於公校不超过5分.

6. A researcher claim that the number of monthly visitors of a social networking site is more than 13.5 million. A recent survey of 9 randomly selected social networking sites has a mean of 15.6 million visitors for a specific month. The sample standard deviation is 4.2 million.
(a) Use the sample data to find the 90% confidence interval of true mean.
(b) At $\alpha = 0.05$, use **p-value method** to test the researcher's claim.

$\bar{x} = 15.6$, $s = 4.2$ $n = 9$, σ未知 ⇒ t.

$df = 8$

11

(a) 90% C.I. for μ

① 1.86

$\bar{x} \pm t \cdot \sqrt{s^2/n}$

⇒② (12.996, 18.204)

(b) ② ① $\begin{cases} H_0: \mu \le 13.5 \\ H_1: \mu > 13.5 \text{ (claim)} \end{cases}$

② $\alpha = 0.05$

③ $t = \dfrac{\bar{x} - 13.5}{s/\sqrt{n}} = 1.5$ ②

② $0.2 < \text{p-value} < 0.1$

0.05

④ p-value > α

1 ⇒ do not rej. Ho

1 ⇒ 無証據 ~~證明平均值~~ 支持主张是对的.

$\mu > 13.5$

6

7. A recent study indicated that 86% of the people ages 18 to 29 own a smartphone. A researcher wishes to be 95% confident that her estimate of the true proportion of individual who own a smartphone is within 4% of the true proportion.

6

(a) How large a sample must you take to be 95% confident that the estimate is within ~~0.03~~ 0.04 of the true proportion of the people ages 18 to 29 who own a smartphone?

(b) If no estimate of the sample proportion is available, how large should the sample be?

(a) $\hat{p} = 0.86$

① 1.96

$e = z \sqrt{\hat{p}\hat{q}/n}$

$\Rightarrow n = \left(\frac{z}{e}\right)^2 \hat{p} \cdot \hat{q}$

① $= 289.08$

① 取 290

(b) ① $\hat{p} = 0.5$

① $n = 600.25$

① 取 $n = 601$

+2 10

8. A running coach wanted to see whether runners ran faster after eating spaghetti the night before. A group of six runners was randomly chosen for this study. Each ran a 5 kilometer race after having a normal dinner the night before, and then a week later, reran the same race after having a spaghetti dinner the night before. The times for their races are shown in the table below. Can the coach conclude that runners ran faster after eating spaghetti the night before?

	1	2	3	4	5	6
Regular Dinner	1009 s	994 s	1018 s	979 s	1025 s	1005 s
Spaghetti Dinner	1002 s	989 s	1015 s	978 s	1026 s	1008 s
D	7	5	3	1	-1	-3

10

$\alpha = 0.05$

$df = 5$

2 ① $\begin{cases} H_0: \mu_D \le 0 \\ H_1: \mu_D > 0 \end{cases}$

1 ② CV: $t_c = 2.571$

① $\bar{D} = 2$

② $S_D = \cancel{90}\ \cancel{14}$ 3.74

2 ③ $t = \frac{\bar{D}}{s/\sqrt{n}} = \frac{2}{3.74/\sqrt{6}} = 1.31$

1 $\Rightarrow$ do not rej. H_0

2 1 無證據顯示前晚吃義大利面会跑較快.

9. In a random sample of 100 Americans, 56 wished that they were rich. In a random sample of 80 Europeans, 34 wished that they were rich.
   (a) Find the 98% confidence interval for the true proportion of American who wish to be rich.
   (b) Find the 98% confidence interval for the difference in proportion of Americans who wish to be rich and the proportion of Europeans who wish to be rich.
   (c) At $\alpha = 0.03$, test whether the proportion of American who wish to be rich is higher than 50%. (p-value method)
   (d) At $\alpha = 0.02$, test whether the proportion of Americans who wish to be rich differs from that of Europeans. (critical value method)

-4

(a) 98% C.I. for p

$(\hat{p}_A = 0.56)$ $n=100$

1 $\hat{p} \pm \overset{2.33}{z} \cdot \sqrt{\hat{p}\hat{q}/n}$

2 $\Rightarrow (44.43\%, 67.57)$

(b) 98% C.I. for $(\mu_A - \mu_E)$

$\Rightarrow (\hat{p}_1 - \hat{p}_2) \pm \overset{2.33}{z} \cdot \sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2 \cdot \hat{q}_2}{n_2}}$

$\Rightarrow (0.56 - 0.425) \pm 2.33 \times \sqrt{\frac{0.56 \times 0.44}{100} + \frac{0.425 \times 0.575}{80}}$

$\Rightarrow (-0.038, 0.308)$

-0.04 0.31

$\boxed{\hat{p}_A = 0.56, \quad \hat{p}_E = 0.425, \quad \bar{p} = 0.5}$

8 (c). $H_0: p \le 50\%$
$H_1: p > 50\%$

② $\alpha = 0.03$ $\quad \hat{p} = 0.56$

③ $z = \frac{\hat{p} - p_0}{\sqrt{\hat{p}\hat{q}/n}} = 1.2$

2

$p\text{-value} = P(z > 1.2)$
$= 0.1151$

2

④ Do not rej. H_0

1

1

7 (d) $H_0: p_A = p_E$
2 $H_1: p_A \ne p_E$ (claim)

② $\alpha = 0.02 \Rightarrow CV: z_c = 2.33$

1 ③ $z = \frac{(\hat{p}_A - \hat{p}_E)}{\sqrt{\bar{p}\bar{q}(\frac{1}{n_1} + \frac{1}{n_2})}} = 1.8$

2

1 ④ Do not rej. H_0

1 無顯著差異

10. An instructor who taught an online statistics course and a classroom course. The final exam score for the two courses are shown.

(a) Find the 90% confidence interval of the variance online course. 4

(b) Whether the variance of the final exam score is less than 5? 7

(c) The instructor feels that the variance of the final exam scores for the students who took the online course is greater than that of the students who took the classroom course. At $\alpha = 0.05$, is there enough evidence to support the claim? 7

18

online course	classroom course
$S_1 = 3.2$	$S_2 = 2.8$
$n_1 = 11$	$n_2 = 16.$

(a) 90% CI for σ^2_{online} $df = 10.$

$$\Rightarrow \frac{(n-1)S^2}{\underline{\chi^2_R}\ 18.307} < \sigma^2 < \frac{(n-1)S^2}{\underline{\chi^2_L}\ 3.94}$$ ① ①

$\Rightarrow$ ② $5.59 < \sigma^2 < 25.99$ 2

7 (b) $\begin{cases} H_0: \sigma_c^2 \geq 5 \\ H_1: \sigma^2 < 5 \end{cases}$ ① 2 $df = 15$

1 ② $\alpha = 0.05,\ \chi^2_{CV.} = 7.261$

2 ③ $\chi^2 = \frac{(n-1)S^2}{5^2} = 4.704$

④ Decision & Summary 1 1

not reject

not enough.

7. (c) $\begin{cases} H_0: \sigma^2_{online} \leq \sigma^2_c \\ H_1: \sigma^2_{online} > \sigma^2_c \end{cases}$ 2 ①

1 ② C.V. $F^{0.05}_{(10,15)} = 2.54$

2 ③ $F = \frac{S^2_{online}}{S^2_c} = \frac{3.2^2}{2.8^2} = 1.355.$

④

1 do not reject.

1 not enough.

8 11. Find the critical value and test value for testing the difference between two variance.

b. Sample 1: $s_1^2 = 37,\ n_1 = 14$
Sample 2: $s_2^2 = 89,\ n_2 = 25$
Right-tailed, $\alpha =$ ~~0.01~~ 0.05

CV: F

2 test value $= \frac{S^2_大}{S^2_小} = \frac{89}{37}$

$= 2.41$

2 CV: $F_{(24,\,13)} = 2.46$

c. Sample 1: $s_1^2 = 232,\ n_1 = 30$
Sample 2: $s_2^2 = 387,\ n_2 = 46$
Two-tailed, $\alpha = 0.05$

CV:

2 test value $= \frac{S^2_大}{S^2_小} = \frac{387}{232}$

$= 1.67$

2 CV: $F(45, 29) = 2.12$