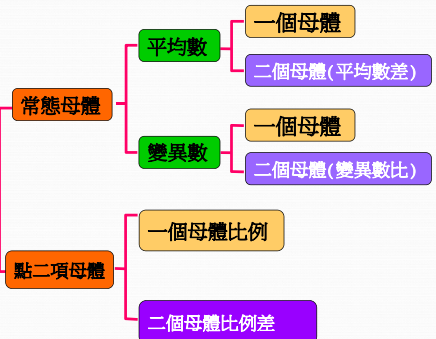


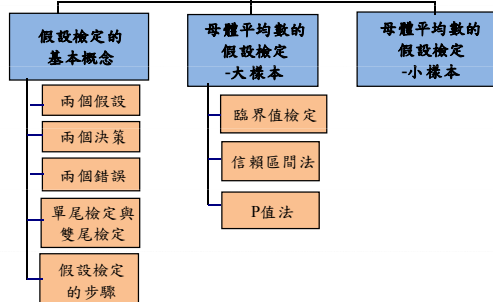
# Chapter 8

## Hypothesis Testing for One Population Parameter

### 假設檢定



### 單一母體平均數之假設檢定



## Hypothesis Testing

Researchers are interested in answering many types of questions. For example,

- Is the earth warming up?
- Does a new medication lower blood pressure?
- Does the public prefer a certain color in a new fashion line?
- Is a new teaching technique better than a traditional one?
- Do seat belts reduce the severity of injuries?

These types of questions can be addressed through statistical hypothesis testing, which is a decision-making process for evaluating claims about a population.

## Two hypothesis

- Two hypothesis: Null & Alternative hypothesis
- The **null hypothesis**, symbolized by  $H_0$ ,
  - states that there is no difference between a parameter and a specific value, or that there is no difference between two parameters.
- The **alternative hypothesis**, symbolized by  $H_1$  or  $H_a$ 
  - states the existence of a difference between a parameter and a specific value, or states that there is a difference between two parameters.

## Situation A



A medical researcher is interested in finding out **whether** a new medication will have any undesirable side effects. The researcher is particularly concerned with the pulse rate of the patients who take the medication. Will the pulse rate increase, decrease, or remain unchanged after a patient takes the medication? The researcher knows that the mean pulse rate for the population under study is 82 beats per minute.

The hypotheses for this situation are

## Situation B

A chemist invents an additive **to increase** the life of an automobile battery. The mean lifetime of the automobile battery without the additive is 36 months.

In this book, the null hypothesis is always stated using the equals sign. The hypotheses for this situation are

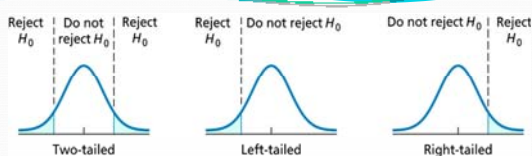


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## Situation C

A contractor wishes **to lower** heating bills by using a special type of insulation in houses. If the average of the monthly heating bills is \$78, what is her hypotheses about heating costs with the use of insulation?

The hypotheses for this situation are



	Two-Tailed Test	Left-Tailed Test	Right-Tailed Test
Sign in $H_0$	=	$\geq$	$\leq$
Sign in $H_1$	$\neq$	<	>
Rejection region	In both tails	In the left tail	In the right tail

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Table 8-1 Hypothesis-Testing Common Phrases

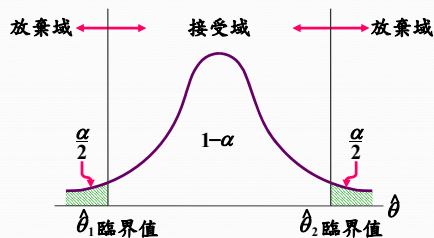
$>$	$<$
Is greater than	Is less than
Is above	Is below
Is higher than	Is lower than
Is longer than	Is shorter than
Is bigger than	Is smaller than
Is increased	Is decreased or reduced from
$\geq$	$\leq$
Is greater than or equal to	Is less than or equal to
Is at least	Is at most
Is not less than	Is not more than
$=$	$\neq$
Is equal to	Is not equal to
Is exactly the same as	Is different from
Has not changed from	Has changed from
Is the same as	Is not the same as

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## 統計假設檢定概念

### ● 假設檢定模式的建立：

#### ✦ 雙尾檢定：

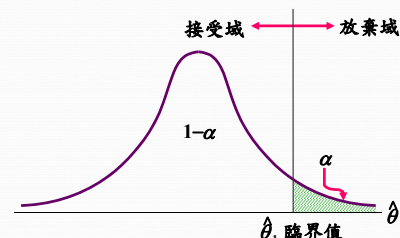


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## 統計假設檢定概念

### ● 假設檢定模式的建立：

#### ✦ 右尾檢定：

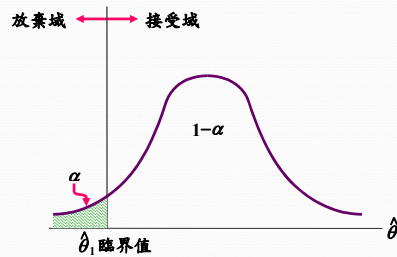


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## 統計假設檢定概念

### ● 假設檢定模式的建立：

#### ✚ 左尾檢定：



◆ In the hypothesis-testing situation, there are four possible outcomes.

	$H_0$ true	$H_0$ false
Reject $H_0$	<b>Error Type I</b>	Correct decision
Do not reject $H_0$	Correct decision	<b>Error Type II</b>

流感篩檢為例：  
 $H_0$ : A君未患流感  
 $H_1$ : A君已患流感

## Hypothesis Testing

- In reality, the null hypothesis may or may not be true, and a decision is made to reject or not to reject it on the basis of the data obtained from a sample.

### Two types of errors

#### Type I and Type II Errors

Type I error: Rejecting the null hypothesis when it is in fact true.

Type II error: Not rejecting the null hypothesis when it is in fact false.

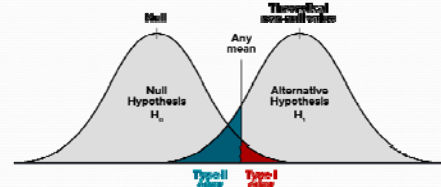
## Hypothesis Testing

### ● Level of significance

- The maximum probability of committing a type I error.
- This probability is symbolized by  $\alpha$ .

$$\alpha = P(\text{type I error}) = P(H_0 \text{ is rejected} \mid H_0 \text{ is true}).$$

$$\beta = P(\text{type II error}) = P(H_0 \text{ is not rejected} \mid H_0 \text{ is false}).$$



## Hypothesis Testing

Three methods used to test hypotheses:

1. The critical value method
2. The confidence interval method
3. The  $P$ -value method

## Procedure Table

Solving Hypothesis-Testing Problems  
(critical value Method)

- Step 1** State the hypotheses and identify the claim.
- Step 2** Find the critical value(s) from the appropriate table in Appendix.
- Step 3** Compute the test value.
- Step 4** Make the decision to reject or not reject the null hypothesis.
- Step 5** Summarize the results.



## z Test for a Mean

The **z test** is a statistical test for the **mean of a population**. It can be used when  $n \geq 30$ , or when the population is normally distributed and  $\sigma$  is known.

The formula for the *z test* is

$$z = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}}$$

where

$\bar{X}$  = sample mean

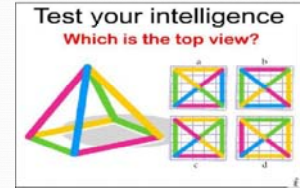
$\mu$  = hypothesized population mean

$\sigma$  = population standard deviation

$n$  = sample size

## Example 8-3: Intelligence Tests

In a survey, the average IQ score of high school students is 101.5. The variable is normally distributed, and the population standard deviation is 15. A school superintendent claims that the students in her school district have an IQ higher than the average of 101.5. She selects a random sample of 30 students and finds the mean of the test scores is 106.4. Test the claim at  $\alpha = 0.05$ .



## Example 8-5: Cost of Rehabilitation

The Medical Rehabilitation Education Foundation reports that the average cost of rehabilitation for stroke victims is \$24,672. To see if the average cost



of rehabilitation **is different** at a particular hospital, a researcher selects a random **sample of 35** stroke victims at the hospital and finds that the average cost of their rehabilitation is \$25,226. The standard deviation of the population is \$3251. At  $\alpha = 0.01$ , can it be concluded that the average cost of stroke rehabilitation at a particular hospital is different from \$24,672?

## Hypothesis Testing

- The traditional method for solving hypothesis-testing problems compares **z-values**:
  - critical value
  - test value
- The *P*-value method for solving hypothesis-testing problems compares **areas**:
  - alpha
  - *P*-value

## Procedure Table

### Solving Hypothesis-Testing Problems (*P*-Value Method)

- Step 1** State the hypotheses and identify the claim.
- Step 2** Compute the test value.
- Step 3** Find the *P*-value.
- Step 4** Make the decision.
- Step 5** Summarize the results.

## Example 8: Cost of College Tuition

A researcher wishes to test the claim that the average cost of tuition and fees at a four-year public college is **greater than** \$5700. She selects a random sample of 36 four-year public colleges and finds the mean to be \$5950. The population standard deviation is \$659. Is there evidence to support the claim at a 0.05? Use the *P*-value method.

### Example 9: Wind Speed

A researcher claims that the average wind speed in a certain city is 8 miles per hour. A sample of 32 days has an average wind speed of 8.2 miles per hour. The standard deviation of the population is 0.6 mile per hour. At  $\alpha = 0.05$ , is there enough evidence to reject the claim? Use the  $P$ -value method.

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### One-Mean $z$ -Test ( $P$ -Value Approach)

**Purpose** To perform a hypothesis test for a population mean,  $\mu$

**Assumptions**

1. Simple random sample
2. Normal population or large sample
3.  $\sigma$  known

**STEP 1** The null hypothesis is  $H_0: \mu = \mu_0$ , and the alternative hypothesis is

$$H_a: \mu \neq \mu_0 \quad \text{or} \quad H_a: \mu < \mu_0 \quad \text{or} \quad H_a: \mu > \mu_0$$

(Two tailed)      (Left tailed)      (Right tailed)

**STEP 2** Decide on the significance level,  $\alpha$ .

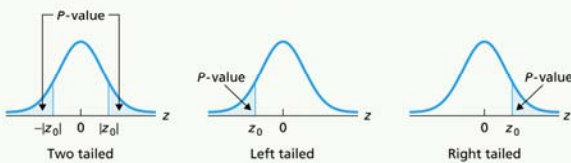
**STEP 3** Compute the value of the test statistic

$$z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$$

and denote that value  $z_0$ .

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**STEP 4** Use Table II to obtain the  $P$ -value.



**STEP 5** If  $P \leq \alpha$ , reject  $H_0$ ; otherwise, do not reject  $H_0$ .

**STEP 6** Interpret the results of the hypothesis test.

The hypothesis test is exact for normal populations and is approximately correct for large samples from nonnormal populations.

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#### CRITICAL-VALUE APPROACH

or

#### P-VALUE APPROACH

**STEP 1** State the null and alternative hypotheses.

**STEP 2** Decide on the significance level,  $\alpha$ .

**STEP 3** Compute the value of the test statistic.

**STEP 4** Determine the critical value(s).

**STEP 5** If the value of the test statistic falls in the rejection region, reject  $H_0$ ; otherwise, do not reject  $H_0$ .

**STEP 6** Interpret the result of the hypothesis test.

**STEP 1** State the null and alternative hypotheses.

**STEP 2** Decide on the significance level,  $\alpha$ .

**STEP 3** Compute the value of the test statistic.

**STEP 4** Determine the  $P$ -value.

**STEP 5** If  $P \leq \alpha$ , reject  $H_0$ ; otherwise, do not reject  $H_0$ .

**STEP 6** Interpret the result of the hypothesis test.

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### Guidelines for $P$ -Values With No $\alpha$

- If  $P\text{-value} \leq 0.01$ , reject the null hypothesis. The difference is highly significant.
- If  $P\text{-value} > 0.01$  but  $P\text{-value} \leq 0.05$ , reject the null hypothesis. The difference is significant.
- If  $P\text{-value} > 0.05$  but  $P\text{-value} \leq 0.10$ , consider the consequences of type I error before rejecting the null hypothesis.
- If  $P\text{-value} > 0.10$ , do not reject the null hypothesis. The difference is not significant.

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**Table 9.12**

$P$ -value	Evidence against $H_0$
$P > 0.10$	Weak or none
$0.05 < P \leq 0.10$	Moderate
$0.01 < P \leq 0.05$	Strong
$P \leq 0.01$	Very strong

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## Significance

- The researcher should distinguish between **statistical significance** and **practical significance**.
- When the null hypothesis is rejected at a specific significance level, it can be concluded that the difference is probably not due to chance and thus is statistically significant. However, the results may not have any practical significance.
- It is up to the researcher to use common sense when interpreting the results of a statistical test.

## Example I

- A產品平均壽命 $\mu=1100$ 小時，金製造商宣稱新型產品壽命已提升，自改善後產品中隨機抽取一組 $n=36$ 之樣本進行測試，得知平均壽命為1125小時，標準差為300小時，則：
  - 在 $\alpha=0.05$ 下，檢定改善後之產品壽命是否已提升？
  - 求(1)中之p-value
  - 求在 $\mu=1225$ 小時之type II error機率
  - 求(1)之檢定力函數，

## Example II

- $X \sim N(50, 6^2)$ , 欲檢定  $H_0: \mu \leq 50$ ,  $H_1: \mu > 50$
- 取樣 $n=16$ , 且取棄卻域  $CR = \{X\text{-bar} | X\text{-bar} > 53\}$ , 則:
  - 求此檢定的顯著水準 $\alpha$
  - 求當 $\mu=55$ 時，犯型II誤差的機率
  - 求當 $X\text{-bar}=54.5$ 時之p-value，並利用此值檢定此假設

## t Test for a Mean

### t Test for a Mean

The  $t$  test is a statistical test for the mean of a population and is used when the population is normally or approximately normally distributed,  $\sigma$  is unknown.

The formula for the  $t$  test is

$$t = \frac{\bar{X} - \mu}{s/\sqrt{n}}$$

The degrees of freedom are d.f. =  $n - 1$ .

### Example 10: Hospital Infections

A medical investigation claims that the average number of infections per week at a hospital in southwestern Pennsylvania is 16.3. A random sample of 10 weeks had a mean number of 17.7 infections. The sample standard deviation is 1.8. Is there enough evidence to reject the investigator's claim at  $\alpha = 0.05$ ?



### Example 11: Substitute Salaries

An educator claims that the average salary of substitute teachers in school districts in Allegheny County, Pennsylvania, is **less than** \$60 per day. A random sample of eight school districts is selected, and the daily salaries (in dollars) are shown. Is there enough evidence to support the educator's claim at  $\alpha = 0.10$ ?

60 56 60 55 70 55 60 55 →  $\bar{X} = 59.8$

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### Example 12: Jogger's Oxygen Intake

A physician claims that joggers' maximal volume oxygen uptake is **greater than** the average of all adults. A sample of 15 joggers has a mean of 40.6 milliliters per kilogram (ml/kg) and a standard deviation of 6 ml/kg. If the average of all adults is 36.7 ml/kg, is there enough evidence to support the physician's claim at  $\alpha = 0.05$ ?

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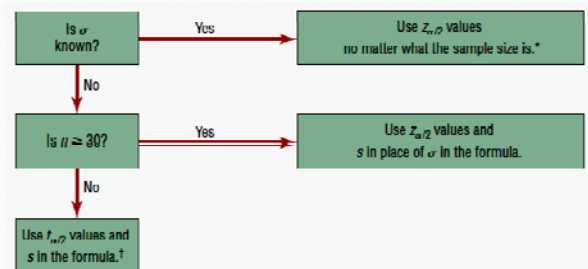
### 小樣本下單一母體之母體平均數的檢定 (t 檢定)

統計假設的配置法	棄卻域	檢定統計量
$H_0: \mu = \mu_0, H_1: \mu \neq \mu_0$ (雙尾檢定)	$C = \{ T  > t_{\frac{\alpha}{2}}(n-1)\}$	$T = \frac{\bar{X} - \mu_0}{S/\sqrt{n}}$
$H_0: \mu \geq \mu_0, H_1: \mu < \mu_0$ (左尾檢定)	$C = \{T < -t_{\alpha}(n-1)\}$	$T = \frac{\bar{X} - \mu_0}{S/\sqrt{n}}$
$H_0: \mu \leq \mu_0, H_1: \mu > \mu_0$ (右尾檢定)	$C = \{T > t_{\alpha}(n-1)\}$	$T = \frac{\bar{X} - \mu_0}{S/\sqrt{n}}$

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### Whether to use z or t



\*Variable must be normally distributed when  $n < 30$ .  
†Variable must be approximately normally distributed.

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## Confidence intervals and hypothesis testing.

### Confidence intervals and hypothesis testing

- There is a relationship between confidence intervals and hypothesis testing.
- When the null hypothesis is rejected in a hypothesis-testing situation, the confidence interval for the mean using the same level of significance *will not* contain the hypothesized mean.
- Likewise, when the null hypothesis is not rejected, the confidence interval computed using the same level of significance *will* contain the hypothesized mean.

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### Example 13: Sugar Production

Sugar is packed in 5-pound bags. An inspector suspects the bags may **not contain 5 pounds**. A sample of 50 bags produces a mean of 4.6 pounds and a standard deviation of 0.7 pound. Is there enough evidence to conclude that the bags do not contain 5 pounds as stated at  $\alpha = 0.05$ ? Also, find the 95% confidence interval of the true mean.

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### Example 13: Sugar Production

The 95% confidence interval for the mean is

$$\bar{X} - z_{\alpha/2} \frac{s}{\sqrt{n}} < \mu < \bar{X} + z_{\alpha/2} \frac{s}{\sqrt{n}}$$

$$4.6 - (1.96) \left( \frac{0.7}{\sqrt{50}} \right) < \mu < 4.6 + (1.96) \left( \frac{0.7}{\sqrt{50}} \right)$$

$$4.4 < \mu < 4.8$$

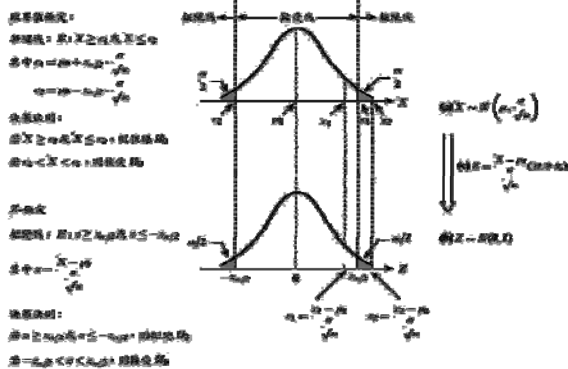
Notice that the 95% confidence interval of  $\mu$  does *not* contain the hypothesized value  $\mu = 5$ .

Hence, there is agreement between the hypothesis test and the confidence interval.

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### Z-檢定或t-檢定--1/3



Source: 方世榮, 基礎統計學, 華泰書局, 臨界值檢定與Z-檢定之比較的圖例

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### 臨界值檢定與Z檢定(或t檢定)之比較(續)--2/3

(a) Z-檢定的情況	臨界值檢定 檢定統計量: $\bar{X}$	Z-檢定 檢定統計量: $z = \frac{\bar{X} - \mu_0}{\sigma / \sqrt{n}}$ (大樣本, $\sigma$ 未知, 以 $s$ 取代)
(1) 雙尾檢定: $\begin{cases} H_0: \mu = \mu_0 \\ H_1: \mu \neq \mu_0 \end{cases}$	拒絕域: $\bar{X} \geq c_1$ 或 $\bar{X} \leq c_2$	拒絕域: $z \geq z_{\alpha/2}$ 或 $z \leq -z_{\alpha/2}$
(2) 左尾檢定: $\begin{cases} H_0: \mu \geq \mu_0 \\ H_1: \mu < \mu_0 \end{cases}$	拒絕域: $\bar{X} \leq c_1$ $c = \mu_0 - z_{\alpha} \frac{\sigma}{\sqrt{n}}$	拒絕域: $z \leq -z_{\alpha}$
(3) 右尾檢定: $\begin{cases} H_0: \mu \leq \mu_0 \\ H_1: \mu > \mu_0 \end{cases}$	拒絕域: $\bar{X} > c$ $c = \mu_0 + z_{\alpha} \frac{\sigma}{\sqrt{n}}$	拒絕域: $z \geq z_{\alpha}$

Source: 方世榮, 基礎統計學, 華泰書局, 臨界值檢定與Z-檢定之比較的圖例

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### 臨界值檢定與Z檢定(或t檢定)之比較(續)--3/3

(b) t-檢定的情況	臨界值檢定 檢定統計量: $\bar{X}$	t-檢定 檢定統計量: $t = \frac{\bar{X} - \mu_0}{s / \sqrt{n}}$
(4) 雙尾檢定: $\begin{cases} H_0: \mu = \mu_0 \\ H_1: \mu \neq \mu_0 \end{cases}$	拒絕域: $\bar{X} \geq c_1$ 或 $\bar{X} \leq c_2$	拒絕域: $t \geq t_{\alpha/2}(n-1)$ 或 $t \leq -t_{\alpha/2}(n-1)$
(5) 左尾檢定: $\begin{cases} H_0: \mu \geq \mu_0 \\ H_1: \mu < \mu_0 \end{cases}$	拒絕域: $\bar{X} \leq c_1$ $c = \mu_0 - t_{\alpha}(n-1) \frac{s}{\sqrt{n}}$	拒絕域: $t \leq -t_{\alpha}(n-1)$
(6) 右尾檢定: $\begin{cases} H_0: \mu \leq \mu_0 \\ H_1: \mu > \mu_0 \end{cases}$	拒絕域: $\bar{X} > c$ $c = \mu_0 + t_{\alpha}(n-1) \frac{s}{\sqrt{n}}$	拒絕域: $t \geq t_{\alpha}(n-1)$

Source: 方世榮, 基礎統計學, 華泰書局, 臨界值檢定與Z-檢定之比較的圖例

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### 信賴區間檢定 1/4

- 在雙尾檢定中,  $H_0: \mu = \mu_0$ ;  $H_1: \mu \neq \mu_0$ , 如果母體平均數  $\mu$  的  $(1-\alpha)100\%$  之信賴區間包含  $\mu_0$ , 則樣本平均數  $\bar{X}$  的觀察值會落於接受域, 此時將做出接受  $H_0$  的結論。

Source: 方世榮, 基礎統計學, 華泰書局

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## 信賴區間檢定 2/4

### ● 臨界值檢定

計算臨界值，求出接受域(或拒絕域)，若檢定統計量的觀察值落於接受域，則接受 $H_0$ ；反之，則拒絕 $H_0$ 。

### ● 信賴區間檢定

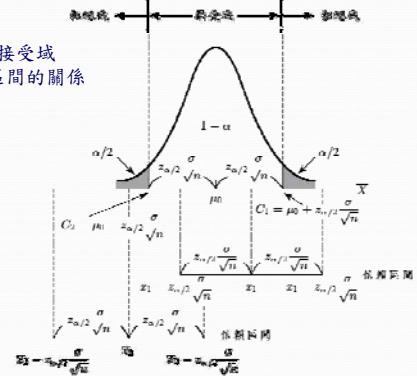
計算母體參數的信賴區間，若此區間包含 $H_0$ 成立時的假想值(如 $\mu_0$ )，則接受 $H_0$ ；反之，則拒絕 $H_0$ 。

Source: 方世榮, 基礎統計學, 華泰書局

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## 信賴區間檢定 3/4

雙尾檢定的接受域  
與 $\mu$ 之信賴區間的關係



Source: 方世榮, 基礎統計學, 華泰書局

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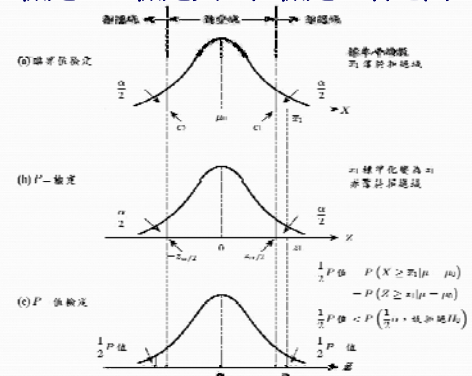
## 信賴區間檢定 4/4

	$\bar{X}$ 抽樣分配屬於常態 (若 $\sigma$ 未知, 可以 $s$ 取代)	$\bar{X}$ 抽樣分配屬於 $t$ 分配
(1) 雙尾檢定 $H_0: \mu = \mu_0$ $H_1: \mu \neq \mu_0$	$\left( \bar{x} - z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}, \bar{x} + z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \right)$	$\left( \bar{x} - t_{\alpha/2} \cdot \frac{s}{\sqrt{n}}, \bar{x} + t_{\alpha/2} \cdot \frac{s}{\sqrt{n}} \right)$
(2) 左尾檢定 $H_0: \mu \geq \mu_0$ $H_1: \mu < \mu_0$	$\left( -\infty, \bar{x} + z_{\alpha} \cdot \frac{\sigma}{\sqrt{n}} \right)$	$\left( -\infty, \bar{x} + t_{\alpha} \cdot \frac{s}{\sqrt{n}} \right)$
(2) 右尾檢定 $H_0: \mu \leq \mu_0$ $H_1: \mu > \mu_0$	$\left( \bar{x} - z_{\alpha} \cdot \frac{\sigma}{\sqrt{n}}, \infty \right)$	$\left( \bar{x} - t_{\alpha} \cdot \frac{s}{\sqrt{n}}, \infty \right)$

Source: 方世榮, 基礎統計學, 華泰書局

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## 臨界值檢定、Z-檢定與P-值檢定三者之間的關係



Source: 方世榮, 基礎統計學, 華泰書局, 臨界值檢定與Z-檢定之比較圖例

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## Hypothesis Testing for a Mean ( $\mu$ )

$H_0: \mu = \text{value}$   
 $H_1: \mu \neq \text{value}$

Reject  $H_0$  if:  
 $|Z| > Z_{\alpha/2}$   
 $|t| > t_{\alpha/2, n-1}$

$H_0: \mu \geq \text{value}$   
 $H_1: \mu < \text{value}$

Reject  $H_0$  if:  
 $Z < -Z_{\alpha}$   
 $t < -t_{\alpha, n-1}$

$H_0: \mu \leq \text{value}$   
 $H_1: \mu > \text{value}$

Reject  $H_0$  if:  
 $Z > Z_{\alpha}$   
 $t > t_{\alpha, n-1}$

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

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