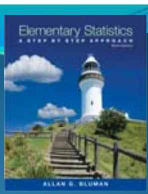
Chapter 8



Hypothesis Testing for One Population Parameter

- 假設檢定
  - 常態母體
    - 平均數
      - 一個母體
      - 二個母體(平均數差)
    - 變異數
      - 一個母體
      - 二個母體(變異數比)
  - 點二項母體
    - 一個母體比例
    - 二個母體比例差

單一母體平均數之假設檢定

- 假設檢定的基本概念
  - 兩個假設
  - 兩個決策
  - 兩個錯誤
  - 單尾檢定與雙尾檢定
  - 假設檢定的步驟
- 母體平均數的假設檢定-大樣本
  - 臨界值檢定
  - 信賴區間法
  - P值法
- 母體平均數的假設檢定-小樣本

Hypothesis Testing

Researchers are interested in answering many types of questions. For example,

- Is the earth warming up?
- Does a new medication lower blood pressure?
- Does the public prefer a certain color in a new fashion line?
- Is a new teaching technique better than a traditional one?
- Do seat belts reduce the severity of injuries?

These types of questions can be addressed through statistical hypothesis testing, which is a decision-making process for evaluating claims about a population.

Two hypothesis

- Two hypothesis: **Null & Alternative hypothesis**
- The **null hypothesis**, symbolized by H_o,
  - states that there is no difference between a parameter and a specific value, or that there is no difference between two parameters.
- The **alternative hypothesis**, symbolized by H_1 or H_a
  - states the existence of a difference between a parameter and a specific value, or states that there is a difference between two parameters.

Situation A

A medical researcher is interested in finding out **whether** a new medication will have any undesirable side effects. The researcher is particularly concerned with the pulse rate of the patients who take the medication. Will the pulse rate increase, decrease, or remain unchanged after a patient takes the medication? The researcher knows that the mean pulse rate for the population under study is 82 beats per minute.

The hypotheses for this situation are

Situation B

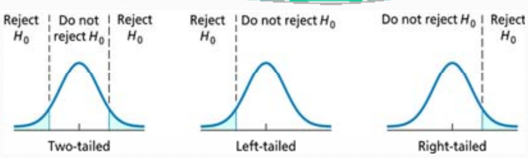
A chemist invents an additive **to increase** the life of an automobile battery. The mean lifetime of the automobile battery without the additive is 36 months.

In this book, the null hypothesis is always stated using the equals sign. The hypotheses for this situation are

Situation C

A contractor wishes **to lower** heating bills by using a special type of insulation in houses. If the average of the monthly heating bills is $78, what is her hypotheses about heating costs with the use of insulation?

The hypotheses for this situation are



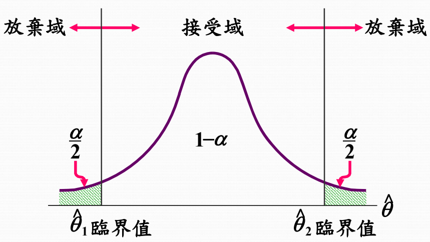
	Two-Tailed Test	Left-Tailed Test	Right-Tailed Test
Sign in H_0	=	≥	≤
Sign in H_1	≠	<	>
Rejection region	In both tails	In the left tail	In the right tail

Table 8–1 Hypothesis-Testing Common Phrases

>	<
Is greater than	Is less than
Is above	Is below
Is higher than	Is lower than
Is longer than	Is shorter than
Is bigger than	Is smaller than
Is increased	Is decreased or reduced from
≥	**≤**
Is greater than or equal to	Is less than or equal to
Is at least	Is at most
Is not less than	Is not more than
=	**≠**
Is equal to	Is not equal to
Is exactly the same as	Is different from
Has not changed from	Has changed from
Is the same as	Is not the same as

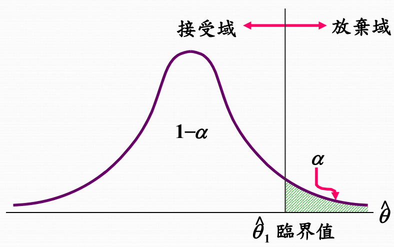
統計假設檢定概念

- 假設檢定模式的建立：
  - 雙尾檢定：



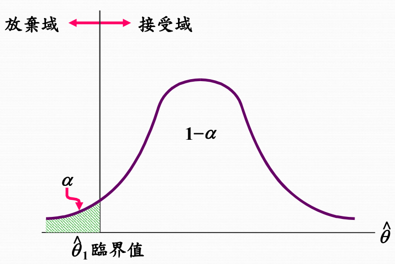
統計假設檢定概念

- 假設檢定模式的建立：
  - 右尾檢定：



統計假設檢定概念

- 假設檢定模式的建立：
  - 左尾檢定：



- In the hypothesis-testing situation, there are four possible outcomes.

	H_0 true	H_0 false
Reject H_0	Error Type I	Correct decision
Do not reject H_0	Correct decision	Error Type II

流感篩檢為例：
H_0: A君未患流感
H_1: A君已患流感

Hypothesis Testing

- In reality, the null hypothesis may or may not be true, and a decision is made to reject or not to reject it on the basis of the data obtained from a sample.

Two types of errors

Type I and Type II Errors

Type I error: Rejecting the null hypothesis when it is in fact true.

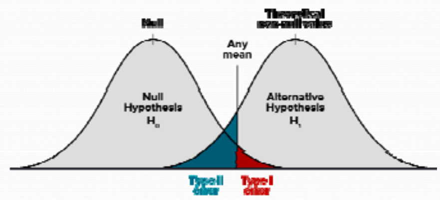
Type II error: Not rejecting the null hypothesis when it is in fact false.

Hypothesis Testing

- **Level of significance**
  - The maximum probability of committing a type I error.
  - This probability is symbolized by α.

  α = P (type I error) = P (H_o is rejected | H_o is true).

  β = P (type II error) = P (H_o is not rejected | H_o is false).



Hypothesis Testing

Three methods used to test hypotheses:

1. The critical value method
2. The confidence interval method
3. The P-value method

Procedure Table

Solving Hypothesis-Testing Problems
(critical value Method)

Step 1 State the hypotheses and identify the claim.

Step 2 Find the critical value(s) from the appropriate table in Appendix.

Step 3 Compute the test value.

Step 4 Make the decision to reject or not reject the null hypothesis.

Step 5 Summarize the results.

z Test for a Mean

The **z test** is a statistical test for the **mean of a population**. It can be used when **$n \geq 30$, or when the population is normally distributed and σ is known**.

The formula for the *z test is*

$$z = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}}$$

where

$\bar{X}$ = sample mean

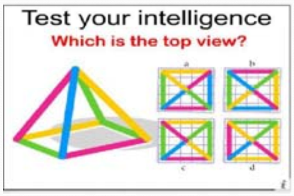
μ = hypothesized population mean

σ = population standard deviation

n = sample size

Example 8-3: Intelligence Tests

In a survey, the average IQ score of high school students is 101.5. The variable is normally distributed, and the population standard deviation is 15. A school superintendent claims that the students in her school district have an IQ higher than the average of 101.5. She selects a random sample of 30 students and finds the mean of the test scores is 106.4. Test the claim at $\alpha = 0.05$.



Example 8-5: Cost of Rehabilitation

The Medical Rehabilitation Education Foundation reports that the average cost of rehabilitation for stroke victims is $24,672. To see if the average cost of rehabilitation is different at a particular hospital, a researcher selects a random sample of 35 stroke victims at the hospital and finds that the average cost of their rehabilitation is $25,226. The standard deviation of the population is $3251. At α = 0.01, can it be concluded that the average cost of stroke rehabilitation at a particular hospital is different from $24,672?

Hypothesis Testing

- The traditional method for solving hypothesis-testing problems compares **z-values**:
  - critical value
  - test value
- The *P*-value method for solving hypothesis-testing problems compares **areas**:
  - alpha
  - *P*-value

Procedure Table

Solving Hypothesis-Testing Problems
(*P*-Value Method)

Step 1 State the hypotheses and identify the claim.

Step 2 Compute the test value.

Step 3 Find the *P*-value.

Step 4 Make the decision.

Step 5 Summarize the results.

Example 8: Cost of College Tuition

A researcher wishes to test the claim that the average cost of tuition and fees at a four-year public college is greater than $5700. She selects a random sample of 36 four-year public colleges and finds the mean to be $5950. The population standard deviation is $659. Is there evidence to support the claim at a 0.05? Use the *P*-value method.

Example 9: Wind Speed

A researcher claims that the average wind speed in a certain city is 8 miles per hour. A sample of 32 days has an average wind speed of 8.2 miles per hour. The standard deviation of the population is 0.6 mile per hour. At $\alpha = 0.05$, is there enough evidence to reject the claim? Use the *P*-value method.

One-Mean *z*-Test (*P*-Value Approach)

Purpose To perform a hypothesis test for a population mean, μ

Assumptions

1. Simple random sample
2. Normal population or large sample
3. σ known

STEP 1 The null hypothesis is H_0: $\mu = \mu_0$, and the alternative hypothesis is

H_a: $\mu \neq \mu_0$ (Two tailed) or H_a: $\mu < \mu_0$ (Left tailed) or H_a: $\mu > \mu_0$ (Right tailed)

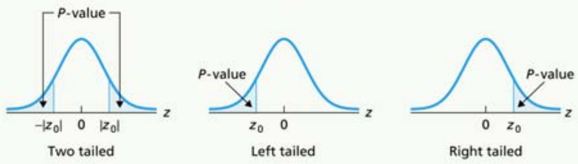
STEP 2 Decide on the significance level, α.

STEP 3 Compute the value of the test statistic

$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$$

and denote that value z_0.

STEP 4 Use Table II to obtain the *P*-value.



STEP 5 $\boxed{\text{If } P \leq \alpha \text{, reject } H_0\text{; otherwise, do not reject } H_0.}$

STEP 6 Interpret the results of the hypothesis test.

The hypothesis test is exact for normal populations and is approximately correct for large samples from nonnormal populations.

CRITICAL-VALUE APPROACH	or P-VALUE APPROACH
STEP 1 State the null and alternative hypotheses.	STEP 1 State the null and alternative hypotheses.
STEP 2 Decide on the significance level, α.	STEP 2 Decide on the significance level, α.
STEP 3 Compute the value of the test statistic.	STEP 3 Compute the value of the test statistic.
STEP 4 Determine the critical value(s).	STEP 4 Determine the *P*-value.
STEP 5 If the value of the test statistic falls in the rejection region, reject H_0; otherwise, do not reject H_0.	STEP 5 If $P \leq \alpha$, reject H_0; otherwise, do not reject H_0.
STEP 6 Interpret the result of the hypothesis test.	STEP 6 Interpret the result of the hypothesis test.

Guidelines for *P*-Values With No α

- If *P*-value ≤ 0.01, reject the null hypothesis. The difference is highly significant.
- If P-value > 0.01 but P-value ≤ 0.05, reject the null hypothesis. The difference is significant.
- If P-value > 0.05 but P-value ≤ 0.10, consider the consequences of type I error before rejecting the null hypothesis.
- If P-value > 0.10, do not reject the null hypothesis. The difference is not significant.

Table 9.12

P-value	Evidence against H_0
$P > 0.10$	Weak or none
$0.05 < P \leq 0.10$	Moderate
$0.01 < P \leq 0.05$	Strong
$P \leq 0.01$	Very strong

Significance

- The researcher should distinguish between **statistical significance** and **practical significance**.
- When the null hypothesis is rejected at a specific significance level, it can be concluded that the difference is probably not due to chance and thus is statistically significant. However, the results may not have any practical significance.
- It is up to the researcher to use common sense when interpreting the results of a statistical test.

Example I

- A產品平均壽命μ=1100小時，金製造商宣稱新型產品壽命已提升，自改善後產品中隨機抽取一組n=36之樣本進行測試，得知平均壽命為1125小時，標準差為300小時，則：
  (1)在α=0.05下，檢定改善後之產品壽命是否已提升？
  (2)求(1)中之p-value
  (3)求在μ=1225小時之type II error機率
  (4)求(1)之檢定力函數，

Example II

- $X \sim N(50, 6^2)$, 欲檢定 H0: μ ≦50, H1: μ>50
- 取樣n=16, 且取棄卻域 CR={X-bar|X-bar>53}, 則:

(1)求此檢定的顯著水準α
(2)求當μ=55時，犯型II誤差的機率
(3)求當X-bar=54.5時之p-value，並利用此值檢定此假設

t Test for a Mean

t Test for a Mean

The *t* test is a statistical test for the mean of a population and is used when the population is normally or approximately normally distributed, σ is unknown.

The formula for the *t test is*

$$t = \frac{\overline{X} - \mu}{s/\sqrt{n}}$$

The degrees of freedom are d.f. = $n - 1$.

Example 10: Hospital Infections

A medical investigation claims that the average number of infections per week at a hospital in southwestern Pennsylvania **is** 16.3. A random sample of 10 weeks had a mean number of 17.7 infections. The sample standard deviation is 1.8. Is there enough evidence to reject the investigator's claim at $\alpha = 0.05$?

Example 11: Substitute Salaries

An educator claims that the average salary of substitute teachers in school districts in Allegheny County, Pennsylvania, is less than $60 per day. A random sample of eight school districts is selected, and the daily salaries (in dollars) are shown. Is there enough evidence to support the educator's claim at $\alpha = 0.10$?

60 56 60 55 70 55 60 55 ➔ X-bar = 59.8

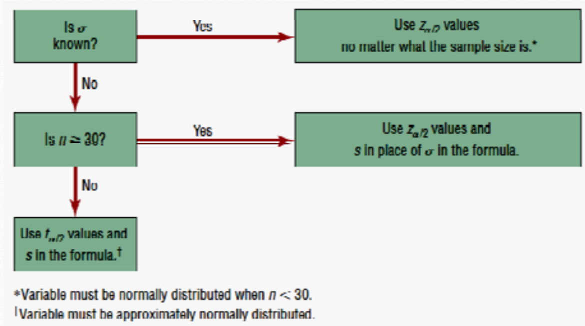
Example 12: Jogger's Oxygen Intake

A physician claims that joggers' maximal volume oxygen uptake is greater than the average of all adults. A sample of 15 joggers has a mean of 40.6 milliliters per kilogram (ml/kg) and a standard deviation of 6 ml/kg. If the average of all adults is 36.7 ml/kg, is there enough evidence to support the physician's claim at $\alpha = 0.05$?

小樣本下單一母體之母體平均數的檢定 (t檢定)

統計假設的配置法	棄卻域	檢定統計量
$H_0: \mu = \mu_0$，$H_1: \mu \neq \mu_0$ (雙尾檢定)	$C = \{\lvert T\rvert > t_{\frac{\alpha}{2}}(n-1)\}$	$T = \dfrac{\overline{X} - \mu_0}{S/\sqrt{n}}$
$H_0: \mu \geq \mu_0$，$H_1: \mu < \mu_0$ (左尾檢定)	$C = \{T < -t_{\alpha}(n-1)\}$	$T = \dfrac{\overline{X} - \mu_0}{S/\sqrt{n}}$
$H_0: \mu \leq \mu_0$，$H_1: \mu > \mu_0$ (右尾檢定)	$C = \{T > t_{\alpha}(n-1)\}$	$T = \dfrac{\overline{X} - \mu_0}{S/\sqrt{n}}$

Whether to use z or t



Confidence intervals and hypothesis testing.

Confidence intervals and hypothesis testing

- There is a relationship between confidence intervals and hypothesis testing.
- When the null hypothesis is rejected in a hypothesis-testing situation, the confidence interval for the mean using the same level of significance *will not* contain the hypothesized mean.
- Likewise, when the null hypothesis is not rejected, the confidence interval computed using the same level of significance *will* contain the hypothesized mean.

Example 13: Sugar Production

Sugar is packed in 5-pound bags. An inspector suspects the bags may not contain 5 pounds. A sample of 50 bags produces a mean of 4.6 pounds and a standard deviation of 0.7 pound. Is there enough evidence to conclude that the bags do not contain 5 pounds as stated at $\alpha = 0.05$? Also, find the 95% confidence interval of the true mean.

Example 13: Sugar Production

The 95% confidence interval for the mean is

$$\bar{X} - z_{\alpha/2}\frac{s}{\sqrt{n}} < \mu < \bar{X} + z_{\alpha/2}\frac{s}{\sqrt{n}}$$

$$4.6 - (1.96)\left(\frac{0.7}{\sqrt{50}}\right) < \mu < 4.6 + (1.96)\left(\frac{0.7}{\sqrt{50}}\right)$$

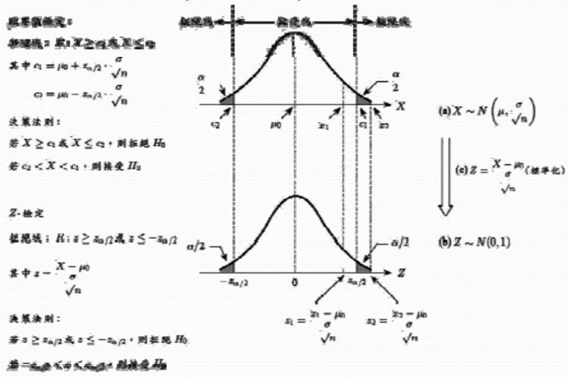
$$4.4 < \mu < 4.8$$

Notice that the 95% confidence interval of m does *not* contain the hypothesized value $\mu = 5$.

Hence, there is agreement between the hypothesis test and the confidence interval.

Bluman, Chapter 8

Z-檢定或t-檢定--1/3



Source: 方世榮，基礎統計學，華泰書局， 臨界值檢定與Z–檢定之比較的圖例

臨界值檢定與Z檢定(或t檢定)之比較(續)--2/3

(a)Z—檢定的情況	臨界值檢定 檢定統計量：$\bar{X}$	Z—檢定 檢定統計量：$z = \frac{\bar{X}-\mu_0}{\sigma/\sqrt{n}}$ (大樣本，σ未知，以s取代)
(1)雙尾檢定：$\begin{cases} H_0: \mu = \mu_0 \\ H_1: \mu \neq \mu_0 \end{cases}$	拒絕域：$\bar{X} \geq c_1$或$\bar{X} \leq c_2$	拒絕域：$z \geq z_{\alpha/2}$或 $z \leq -z_{\alpha/2}$
(2)左尾檢定：$\begin{cases} H_0: \mu \geq \mu_0 \\ H_1: \mu < \mu_0 \end{cases}$	拒絕域：$\bar{X} \leq c_1$ $c = \mu_0 - z_\alpha \frac{\sigma}{\sqrt{n}}$	拒絕域：$z \leq -z_\alpha$
(3)右尾檢定：$\begin{cases} H_0: \mu \leq \mu_0 \\ H_1: \mu > \mu_0 \end{cases}$	拒絕域：$\bar{X} > c$ $c = \mu_0 + z_\alpha \frac{\sigma}{\sqrt{n}}$	拒絕域：$z \geq z_\alpha$

Source: 方世榮，基礎統計學，華泰書局， 臨界值檢定與Z–檢定之比較的圖例

臨界值檢定與Z檢定(或t檢定)之比較(續)--3/3

(b)t—檢定的情況	臨界值檢定 檢定統計量：$\bar{X}$	t—檢定 檢定統計量：$t = \frac{\bar{X}-\mu_0}{S/\sqrt{n}}$
(4)雙尾檢定：$\begin{cases} H_0: \mu = \mu_0 \\ H_1: \mu \neq \mu_0 \end{cases}$	拒絕域：$\bar{X} \geq c_1$或$\bar{X} \leq c_2$	拒絕域：$t \geq t_{\alpha/2}(n-1)$或 $t \leq -t_{\alpha/2}(n-1)$
(5)左尾檢定：$\begin{cases} H_0: \mu \geq \mu_0 \\ H_1: \mu < \mu_0 \end{cases}$	拒絕域：$\bar{X} \leq c_1$ $c = \mu_0 - t_\alpha(n-1)\frac{S}{\sqrt{n}}$	拒絕域：$t \leq -t_\alpha(n-1)$
(6)右尾檢定：$\begin{cases} H_0: \mu \leq \mu_0 \\ H_1: \mu > \mu_0 \end{cases}$	拒絕域：$\bar{X} > c$ $c = \mu_0 + t_\alpha(n-1)\frac{S}{\sqrt{n}}$	拒絕域：$t \geq t_\alpha(n-1)$

Source: 方世榮，基礎統計學，華泰書局， 臨界值檢定與Z–檢定之比較的圖例

信賴區間檢定 1/4

- 在雙尾檢定中，H_0:$\mu=\mu_0$；H_1:$\mu\neq\mu_0$，如果母體平均數μ的(1–α)100%之信賴區間包含μ_0，則樣本平均數 $\bar{X}$ 的觀察值會落於接受域，此時將做出接受H_0的結論。

Source: 方世榮，基礎統計學，華泰書局

信賴區間檢定 2/4

- 臨界值檢定

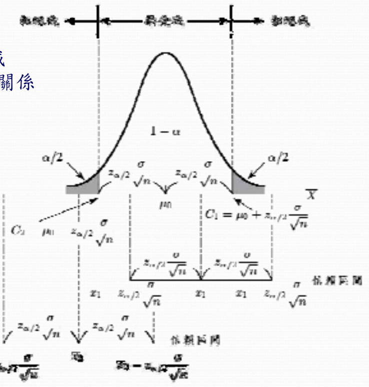
  計算臨界值，求出接受域(或拒絕域)，若檢定統計量的觀察值落於接受域，則接受H_0；反之，則拒絕H_0。
- 信賴區間檢定

  計算母體參數的信賴區間，若此區間包含H_0成立時的假想值(如μ_0)，則接受H_0；反之，則拒絕H_0。

Source: 方世榮，基礎統計學，華泰書局

信賴區間檢定 3/4

雙尾檢定的接受域
與μ之信賴區間的關係



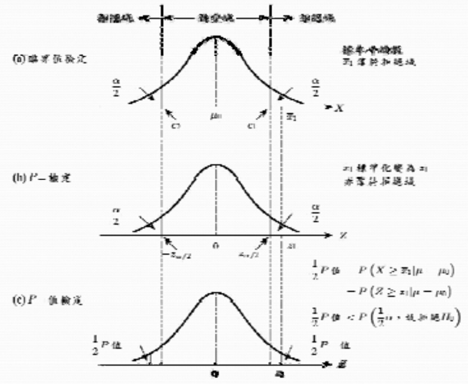
Source: 方世榮，基礎統計學，華泰書局

信賴區間檢定 4/4

	$\bar{X}$ 抽樣分配屬於常態 (若 σ 未知，可以 s 取代)	$\bar{X}$ 抽樣分配屬於 t 分配
(1)雙尾檢定 H_0:$\mu=\mu_0$ H_1:$\mu\neq\mu_0$	$\left(\bar{x}-z_{\alpha/2}\cdot\frac{\sigma}{\sqrt{n}}, \bar{x}+z_{\alpha/2}\cdot\frac{\sigma}{\sqrt{n}}\right)$	$\left(\bar{x}-t_{\alpha/2}\cdot\frac{s}{\sqrt{n}}, \bar{x}+t_{\alpha/2}\cdot\frac{s}{\sqrt{n}}\right)$
(2)左尾檢定 H_0:$\mu\geq\mu_0$ H_1:$\mu<\mu_0$	$\left(-\infty, \bar{x}+z_{\alpha}\cdot\frac{\sigma}{\sqrt{n}}\right)$	$\left(-\infty, \bar{x}+t_{\alpha}\cdot\frac{s}{\sqrt{n}}\right)$
(2)右尾檢定 H_0:$\mu\leq\mu_0$ H_1:$\mu>\mu_0$	$\left(\bar{x}-z_{\alpha}\cdot\frac{\sigma}{\sqrt{n}}, \infty\right)$	$\left(\bar{x}-t_{\alpha}\cdot\frac{s}{\sqrt{n}}, \infty\right)$

Source: 方世榮，基礎統計學，華泰書局

臨界值檢定、Z-檢定與P-值檢定三者之間的關係



Source: 方世榮，基礎統計學，華泰書局， 臨界值檢定與Z–檢定之比較的圖例

Hypothesis Testing for a Mean (μ)

H_0: μ = value H_1: $\mu \neq$ value Reject H_0 if: $\lvert Z\rvert > Z_{\alpha/2}$ $\lvert t\rvert > t_{\alpha/2,\ n-1}$	H_0: $\mu \geq$ value H_1: $\mu <$ value Reject H_0 if: $Z < -Z_{\alpha}$ $t < -t_{\alpha,\ n-1}$	H_0: $\mu \leq$ value H_1: $\mu >$ value Reject H_0 if: $Z > Z_{\alpha}$ $t > t_{\alpha,\ n-1}$

$\boxed{t=\frac{\bar{x}-\mu}{s/\sqrt{n}}}$ $\boxed{z=\frac{\bar{x}-\mu}{\sigma/\sqrt{n}}}$

Section 8-4

z Test for a Proportion

z Test for a Proportion

Since a normal distribution can be used to approximate the binomial distribution when $np \geq 5$ and $nq \geq 5$, the standard normal distribution can be used to test hypotheses for proportions.

The formula for the z test for a proportion is

$$z = \frac{\hat{p} - p}{\sqrt{pq/n}}, \text{ where } \hat{p} = \frac{X}{n} \text{ (sample proportion)}$$

$p =$ population proportion

$n =$ sample size

C.I. for a true proportion

$$\hat{p} - Z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}} < p < \hat{p} + Z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

Example 8-17: Avoiding Trans Fats

A dietitian claims that 60% of people are trying to avoid trans fats in their diets. She randomly selected 200 people and found that 128 people stated that they were trying to avoid trans fats in their diets. At $\alpha = 0.05$, is there enough evidence to reject the dietitian's claim?

Example: Call-Waiting Service

A telephone company representative estimates that less than 23% of its customers have call-waiting service. To test this hypothesis, she selected a sample of 100 customers and found that 11% had call waiting. At $\alpha = 0.01$, is there enough evidence to reject the claim?

Example: Call-Waiting Service

A telephone company representative estimates that less than 23% of its customers have call-waiting service. To test this hypothesis, she selected a sample of 100 customers and found that 11% had call waiting. At $\alpha = 0.01$, is there enough evidence to reject the claim? **Use p-value method**.

One-Proportion z-Interval Procedure

Purpose To find a confidence interval for a population proportion, p

Assumptions

1. Simple random sample
2. The number of successes, x, and the number of failures, $n - x$, are both 5 or greater.

STEP 1 For a confidence level of $1 - \alpha$, use Table II to find $z_{\alpha/2}$.

STEP 2 The confidence interval for p is from

$$\hat{p} - z_{\alpha/2} \cdot \sqrt{\hat{p}(1-\hat{p})/n} \quad \text{to} \quad \hat{p} + z_{\alpha/2} \cdot \sqrt{\hat{p}(1-\hat{p})/n}$$

where $z_{\alpha/2}$ is found in Step 1, n is the sample size, and $\hat{p} = x/n$ is the sample proportion.

STEP 3 Interpret the confidence interval.

Procedure

One-Proportion z-Test

Purpose To perform a hypothesis test for a population proportion, p

Assumptions

1. Simple random sample
2. Both np_0 and $n(1 - p_0)$ are 5 or greater

STEP 1 The null hypothesis is H_0: $p = p_0$, and the alternative hypothesis is

H_a: $p \neq p_0$ (Two tailed) or H_a: $p < p_0$ (Left tailed) or H_a: $p > p_0$ (Right tailed)

STEP 2 Decide on the significance level, α.

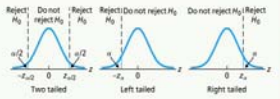
STEP 3 Compute the value of the test statistic

$$z = \frac{\hat{p} - p_0}{\sqrt{p_0(1-p_0)/n}}$$

and denote that value z_0.

Procedure (cont.)

CRITICAL-VALUE APPROACH	or	P-VALUE APPROACH
STEP 4 The critical value(s) are $\pm z_{\alpha/2}$ (Two tailed) or $-z_\alpha$ (Left tailed) or z_α (Right tailed). Use Table II to find the critical value(s).		**STEP 4** Use Table II to obtain the P-value.



STEP 5 If the value of the test statistic falls in the rejection region, reject H_0; otherwise, do not reject H_0.

STEP 5 If $P \le \alpha$, reject H_0; otherwise, do not reject H_0.

STEP 6 Interpret the results of the hypothesis test.

母體比例的檢定-- 臨界值檢定法

	臨界值	決策法則
(1)雙尾檢定：$\begin{cases} H_0: P = P_0 \\ H_1: P \neq P_0 \end{cases}$	$c_1 = P_0 + z_{\alpha/2} \cdot \sigma_{\bar{P}}$ $c_1 = P_0 - z_{\alpha/2} \cdot \sigma_{\bar{P}}$	若$\bar{P} \geq c_1$或$\bar{P} \leq c_2$，則拒絕H_0; 若$c_2 < \bar{P} < c_2$, 則接受H_0。
(2)左尾檢定：$\begin{cases} H_0: P \geq P_0 \\ H_1: P < P_0 \end{cases}$	$c = P_0 - z_\alpha \cdot \sigma_{\bar{P}}$	若$\bar{P} \leq c$，則拒絕H_0; 若$\bar{P} > c$，則接受H_0。
(3)右尾檢定：$\begin{cases} H_0: P \leq P_0 \\ H_1: P > P_0 \end{cases}$	$c = P_0 + z_\alpha \cdot \sigma_{\bar{P}}$	若$\bar{P} \geq c$, 則拒絕H_0; 若$\bar{P} < c$, 則接受H_0。

註：$\sigma_P = \sqrt{\dfrac{P_0(1-P_0)}{n}}$

母體比例的檢定- z值檢定法

	檢定統計量	決策法則
(1)雙尾檢定 H_0:P=P_0 H_1:P≠P_0	$Z = \dfrac{\bar{P} - P_0}{\sqrt{\dfrac{P_0(1-P_0)}{n}}}$	若$z \geq z_{\alpha/2}$或$z \leq -z_{\alpha/2}$, 則拒絕H_0; 若$-z_{\alpha/2} < z < z_{\alpha/2}$, 則接受$H_0$。
(2)左尾檢定 H_0:P≥P_0 H_1:P<P_0	同上	若$z \leq -z_\alpha$，則拒絕H_0; 若$z > -z_\alpha$，則接受H_0。
(3)右尾檢定 H_0:P≤P_0 H_1:P>P_0	同上	若$z \geq z_\alpha$，則拒絕H_0; 若$z < z_\alpha$，則接受H_0。

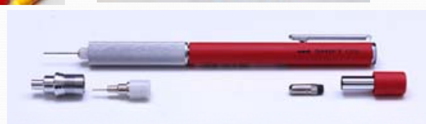
母體比例的檢定–信賴區間檢定法

	母體比例之信賴區間	決策法則
(1)雙尾檢定 H_0:P=P_0 H_1:P≠P_0	$\left(\bar{P} - z_{\alpha/2} \cdot \sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}, \bar{P} + z_{\alpha/2} \cdot \sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}\right)$	若區間包含P_0，則接受H_0，反之，則拒絕H_0。
(2)左尾檢定 H_0:P≥P_0 H_1:P<P_0	$\left(-\infty, \bar{P} + z_\alpha \cdot \sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}\right)$	同上
(3)右尾檢定 H_0:P≤P_0 H_1:P>P_0	$\left(\bar{P} - z \cdot \sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}, \infty\right)$	同上

Section 8-5

Test for a Variance or Standard Deviation

Introduction

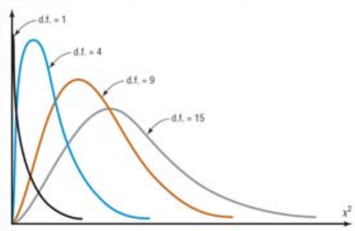


Chi-Square Distributions

- The chi-square distribution must be used to calculate confidence intervals for one population variance and standard deviation.
- The chi-square variable is similar to the *t* variable in that its distribution is a family of curves based on the number of degrees of freedom.
- The symbol for chi-square is χ^2 (Greek letter chi, pronounced "ki").
- A chi-square variable cannot be negative, and the distributions are skewed to the right.

Chi-Square Distributions

- At about 100 degrees of freedom, the chi-square distribution becomes somewhat symmetric.
- The area under each chi-square distribution is equal to 1.00, or 100%.



Example 8-21: Table G

Find the critical chi-square value for 15 degrees of freedom when $\alpha = 0.05$ and the test is right-tailed.

Example 8-22: Table G

Find the critical chi-square value for 10 degrees of freedom when $\alpha = 0.05$ and the test is left-tailed.

Example 8-23: Table G

Find the critical chi-square value for 22 degrees of freedom when $\alpha = 0.05$ and a two-tailed test is conducted.

Right-tailed test

- If a researcher believes the variance of a population to be greater than some specific value, say, 225, then the researcher states the hypotheses as

$$H_0: \sigma^2 \le 225 \quad \text{and} \quad H_1: \sigma^2 > 225$$

Left-tailed test

- If the researcher believes the variance of a population to be less than 225, then the researcher states the hypotheses as

$$H_0: \sigma^2 \ge 225 \quad \text{and} \quad H_1: \sigma^2 < 225$$

Two-tailed test

- If a researcher does not wish to specify a direction, he or she states the hypotheses as

$$H_0\colon \sigma^2 = 225 \qquad \text{and} \qquad H_1\colon \sigma^2 \neq 225$$

8-5 χ^2 Test for σ² or σ

The chi-square distribution is also used to test a claim about a single variance or standard deviation.

The formula for the chi-square test for a variance is

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2}$$

with degrees of freedom d.f. = $n - 1$ and

n = sample size

s^2 = sample variance

σ^2 = population variance

The Assumptions

The Assumptions for the χ^2 test for σ^2 or σ

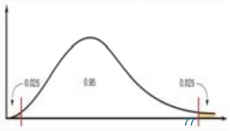
1. The sample must be randomly selected from the population.
2. The population must be normally distributed for the variable under study.
3. The observations must be independent of one another.

Example 8-24: Math SAT Test

The standard deviation for the Math SAT test is 100. The variance is 10,000. An instructor wishes to see if the variance of the 23 randomly selected students in her school is less than 10,000. The variance for the 23 test scores is 7225. Is there enough evidence to support the claim that the variance of the students in her school is less than 10,000 at a = 0.05? Assume that the scores are normally distributed.

Example 8-26: Nicotine Content

A cigarette manufacturer wishes to test the claim that the variance of the nicotine content of its cigarettes is 0.644. Nicotine content is measured in milligrams, and assume that it is normally distributed. A sample of 20 cigarettes has a standard deviation of 1.00 milligram. At $\alpha = 0.05$, is there enough evidence to reject the manufacturer's claim?



8.6 Additional Topics Regarding Hypothesis Testing

- There is a relationship between confidence intervals and hypothesis testing.
- When the null hypothesis is rejected in a hypothesis-testing situation, the confidence interval for the mean using the same level of significance *will not* contain the hypothesized mean.
- Likewise, when the null hypothesis is not rejected, the confidence interval computed using the same level of significance *will* contain the hypothesized mean.

Example 8-30: Sugar Production

Sugar is packed in 5-pound bags. An inspector suspects the bags may not contain 5 pounds. A sample of 50 bags produces a mean of 4.6 pounds and a standard deviation of 0.7 pound. Is there enough evidence to conclude that the bags do not contain 5 pounds as stated at $\alpha = 0.05$? Also, find the 95% confidence interval of the true mean.

Example 8-30: Sugar Production

The 95% confidence interval for the mean is

Power of a Statistical Test

The **power of a test** measures the sensitivity of the test to detect a real difference in parameters if one actually exists. The higher the power, the more sensitive the test. The power is $1 - \beta$.

	H_0 true	H_0 false
Reject H_0	**Type I** error α	Correct decision $1-\beta$
Do not reject H_0	Correct decision $1-\alpha$	**Type II** error β

Example I

- A產品平均壽命μ=1100小時，金製造商宣稱新型產品壽命已提升，自改善後產品中隨機抽取一組n=36之樣本進行測試，得知平均壽命為1125小時，標準差為300小時，則：

  (1)在α=0.05下，檢定改善後之產品壽命是否已提升？

  (2)求(1)中之p-value

  (3)求在μ=1225小時之type II error機率

  (4)求(1)之檢定力函數，

Example II

- $X \sim N(50, 6^2)$, 欲檢定 H0: μ ≦50, H1: μ>50
- 取樣n=16, 且取棄卻域 CR={X-bar|X-bar>53}, 則:

(1)求此檢定的顯著水準α

(2)求當μ=55時，犯型II誤差的機率

(3)求當X-bar=54.5時之p-value，並利用此值檢定此假設