Solutions to Homework 2

1. Define the events

$$A_i = \{\text{success on the } i\text{th trial}\},\ i = 1,\ 2,\ ....,\ n$$

We have a random variable X that counts the total number of successes in n independent trials with two possible outcomes. The event $\{X = x\}$ happens when exactly x of A_is happen and the rest $(n-x)$ do not, and its probability is $p^x(1-p)^{n-x}$ (i.e., x successes and $(n-x)$ failures). The total number of such configuration (with x successes out of n) is $\binom{n}{x}$ and this yields

$$P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}.$$

2. (a)

$$\begin{aligned} p - \text{value} &= P(X \geq 9|\theta = \frac{1}{2}) \\ &= P(X = 9|\theta = \frac{1}{2}) + P(X = 10|\theta = \frac{1}{2}) + P(X = 11|\theta = \frac{1}{2}) + P(X = 12|\theta = \frac{1}{2}) \\ &= 0.07299 \end{aligned}$$

(b)

$$p - \text{value} = 0.07299 > 0.05$$

therefore, H_0 can't be rejected.

(c)

$$p - \text{value} = 0.07299 > 0.01$$

therefore, H_0 can't be rejected.

3.

- c_1 = 0.3 $\Rightarrow$ θ = 0.39 (= d_1 (0.3))
- c_2 = 0.3 $\Rightarrow$ θ = 0.22 (= d_2 (0.3))

therefore, the 95% confidence interval for θ = 0.5 is [0.22 , 0.39]

4.

$$\begin{aligned} L(\mu) &= \sum_{i=1}^{n}(-\mu + x_i \log \mu - \log x_i!) \\ &= -n\mu + \log \mu \sum_{i=1}^{n} x_i - \sum_{i=1}^{n} \log x_i! \end{aligned}$$

$$\begin{aligned} \frac{\partial L(\mu)}{\partial \mu} &= -n + \frac{\sum_{i=1}^{n} x_i}{\mu} \stackrel{\text{set}}{=} 0 \\ \mu &= \frac{\sum_{i=1}^{n} x_i}{n} = \bar{x}_i \end{aligned}$$

$$\frac{\partial^2 L(\mu)}{\partial \mu^2} = -\frac{\sum_{i=1}^{n} x_i}{\mu^2} < 0$$

therefore, $L(\mu)$ is maximized when $\mu = \frac{\sum_{i=1}^{n} x_i}{n} = \bar{x}_i$