[STAT242] Homework 3

1. Suppose we are given a set of data with 12 observations (realizations of a random variable X):

$$40,\ 46,\ 49,\ 52,\ 60,\ 61,\ 64,\ 65,\ 67,\ 67,\ 76,\ 80.$$

Consider testing $H_0 : \theta = 65$ vs $H_1 : \theta < 65$, where θ denotes the center of distribution of data such that $P(X \leq \theta) = P(X \geq \theta) = 1/2$.

   (a) Under a significance level $\alpha = 0.05$, conduct a significant test based on $S = \sum_{i=1}^{n} I(65 - x_i)$ where $I(x)$ denotes an indicator function that takes 1 if $x > 0$ and 0 otherwise.

   (b) Conduct a significance test based on $\bar{x}_n = \sum_{i=1}^{n} x_i/n$ under the symmetric assumption on the distribution of X. We assume that the standard deviation of X, σ_0 is known to be 9. In order to calculate p-value you may use either the *z-table* (from anywhere you can find) or `pnorm` function in `R`.

   (c) Compute the p-value based on $\bar{x} = \sum_{i=1}^{n} x_i/n$ under the normality assumption on X. Compare it to the p-value obtained in (b) above and state their difference.

2. Based on the same data in Problem 1, answer the following questions.

   (a) Let $X_{(i)}$ denote the ith order statistic (ith smallest value in the data). Provide arguments to justify

$$P(X_{(i)} \leq \theta) = \sum_{k=i}^{n} \binom{n}{k} \left(\frac{1}{2}\right)^n$$

   (b) One can suggest the following confidence interval of θ:

$$\left[X_{(2)}, X_{(11)}\right]$$

   Compute the CI based on the given data, and provide its confidence level.

   (c) Alternatively, please compute the 95% confidence interval of θ based on $\bar{x}_n$ using Central Limit Theorem (CLT).

3. Suppose we have a sample proportion $p = X/n = 0.3$ when $X \sim B(n, \theta)$ with $n = 100$.

   (a) Compute 95% confidence interval of θ by applying CLT to the sample proportion p.

   (b) Compare the answer in (a) to the one obtained in Problem 3 in the last HW 2. Explain their difference.

   (c) Compute the p-value for testing $H_0 : \theta = 0.25$ vs $H_1 : \theta > 0.25$ based on the test statistic based on p.