[STAT242] Homework 2

1. Provide an argument to justify that the probability distribution of the binomial random variable $X \sim B(n, \theta)$ is given by

$$P(X = x) = \binom{n}{x} \theta^x (1-\theta)^{n-x}, \qquad x = 0, 1, 2, \cdots, n.$$

2. In the Tasting-Tea-Lady example, suppose she identifies correct orders for 9 cups out of 12.

   (a) Compute the p-value for testing

$$H_0 : \theta = 1/2, \qquad H_1 : \theta > 1/2.$$

   (b) What is your conclusion under the significance level $\alpha = 0.05$?

   (c) What if $\alpha = 0.01$?

3. Suppose we have a sample proportion $p = X/n = 0.3$ when $X \sim B(n, \theta)$ with $n = 100$. Compute 95% confidence interval for θ based on the Table in p.19 (Chapter 2 slide).

4. Log-likelihood of Poisson random variables $(x_1, \cdots, x_n)$ is given by

$$L(\mu) = \sum_{i=1}^{n} (-\mu x_i + \log \mu - \log x_i!)$$

   Show that $L(\mu)$ is maximized when $\mu = \frac{1}{n} \sum_{i=1}^{n} x_i$. (Hint: The derivative of $L(\mu)$ should be 0 at the maximizer.)