

Risk Sharing, Equity, and Why Shares Exist

A Gentle, Quantitative Story

Motivation: Why Did Shares Appear?

Big projects have always needed more money than a single person could safely put in:

- Ships for long-distance trade
- Railroads, factories, large tech firms

Problems without shares:

- One or two rich people fund everything \Rightarrow huge personal risk
- Partnerships with unlimited liability: if the project fails, you may lose *everything*

Idea of shares:

- Break the company into many tiny ownership pieces (shares)
- Many people can each put a small amount of money
- Each person's loss is limited to what they put in

A Simple Risk Story: Mean and Variance

Think of the company as one risky project with random payoff X :

- **Mean** $\mathbb{E}[X]$ (average): what you would get on average if you could repeat the project many times
- **Variance** $\text{Var}(X)$:
 - Measures how much the payoff wiggles around the mean
 - High variance: results are very spread out (big surprises)
 - Low variance: results are tightly clustered (more predictable)

Intuition:

- Mean is the “typical outcome”
- Variance is “how nervous you feel about the outcome”

Risk Sharing by Splitting Ownership

Suppose the project payoff is X .

- One person owns 100% of the company:
 - Their payoff is X .
 - They bear *all* of the wiggle.
- Now split the company among N equal shareholders:
 - Each owns fraction $1/N$.
 - Each person's payoff is X/N .

In variance terms:

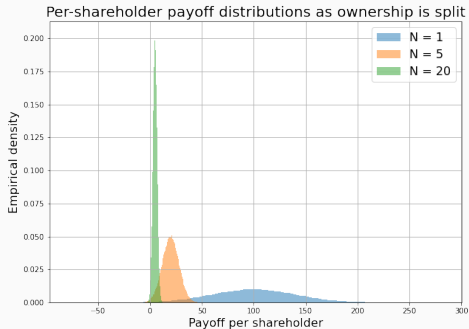
$$\text{Var}\left(\frac{X}{N}\right) = \frac{1}{N^2} \text{Var}(X).$$

Interpretation for non-statisticians:

- If the project typically jumps around by “40 units”,
- Your personal payoff jumps around by roughly $40/N$ units.

The project's total risk is unchanged, but it is *sliced* into smaller personal risks.

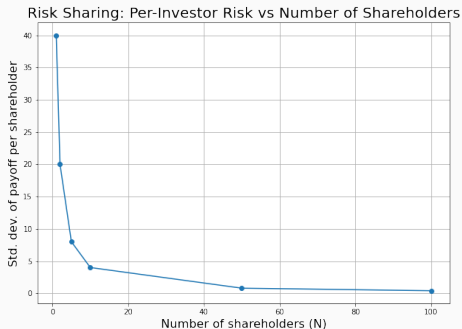
Example: Per-Shareholder Payoff Distributions



- Same project payoff X (same mean, same variance).
- Blue: one owner ($N = 1$) \Rightarrow very wide distribution.
- Orange: $N = 5$ owners.
- Green: $N = 20$ owners \Rightarrow very narrow distribution per person.

The project hasn't changed, but each investor's personal risk is much smaller when more people share it.

Example: Risk per Investor vs Number of Shareholders



- Vertical axis: standard deviation of payoff per investor (a measure of typical fluctuation).
- Horizontal axis: number of shareholders N .
- As N grows, the per-investor standard deviation falls quickly.

Key message: more shareholders \Rightarrow same total risk, but each person carries a smaller, more tolerable piece of it.

What Is Equity?

A company can get money in two main ways:

Debt Borrowed money

- Must pay interest and repay principal
- Lenders are not owners

Equity Owners' money

- People give money in exchange for shares
- They become part-owners of the company
- They get paid only if the company does well

Balance-sheet view (very simplified):

$$\text{Assets} = \text{Debt} + \text{Equity}.$$

Equity is the leftover: whatever remains for owners after paying the debt.

How Can a Company Increase Its Equity?

Main channels:

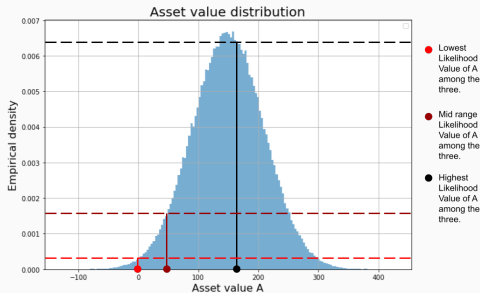
- **Initial investment:** founders and early investors put in cash \Rightarrow receive shares.
- **Retained earnings:** company makes profits and keeps them instead of paying everything out as dividends.
- **Issuing new shares:**
 - Private placements (to a few investors)
 - IPO (Initial Public Offering) or later public offerings

All of these add to the equity cushion on the right-hand side of

$$\text{Assets} = \text{Debt} + \text{Equity},$$

either by increasing assets, reducing debt, or both.

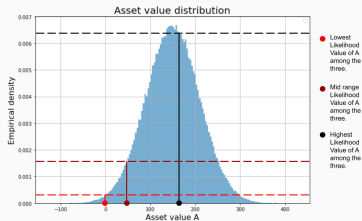
Reading the Bell-Shaped Picture



How to read this if you have never seen such a plot:

- The **horizontal axis** is all the possible future values of the company (its assets). Far left = company ends up very small, far right = company ends up very large.
- The blue bars form a *bell shape*: we simulated many futures and counted how often each value appeared.
 - Tall bars in the middle: those outcomes happen very often.
 - Very short bars at the far left/right: those are rare extremes.
- The red and black dots at the bottom and right mark some particular points on the axis and the chance of ending up below them (we will use these to talk about default risk on the next slide).

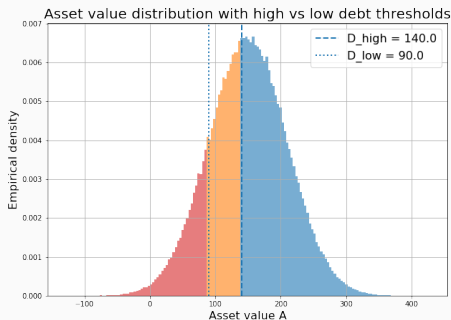
From the Picture to Default Risk



Now connect the picture to the idea of “too little money to pay the debt”:

- Pick a **debt level D** : the company must at least reach this length on the horizontal axis to be able to pay everything back.
 - Draw a vertical line at D .
 - Everything *to the left* of that line is “company ended up too small, cannot fully pay its debt” (default).
- The **chance of default** is simply “how much blue area” lies to the left of that line:
 - A *lower* debt level (line further to the left) cuts off only a small, thin slice of blue area \Rightarrow low chance of default.
 - A *higher* debt level (line closer to the centre) covers a much thicker chunk of blue area \Rightarrow higher chance of default.
- Increasing **equity** (owners’ money) lets the company rely on *less* debt, so the line moves left and the red “default slice” shrinks. That is genuine *risk reduction for the company*, not just risk sharing.

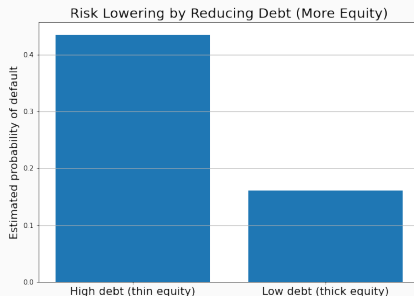
Asset Distribution and Default Regions



- Histogram: simulated distribution of future asset value A .
- Vertical lines:
 - D_{low} (low debt, thick equity cushion)
 - D_{high} (high debt, thin equity cushion)
- Default region is the part of the distribution to the left of the line.

Area left of D_{high} is larger than area left of $D_{\text{low}} \Rightarrow$ higher probability of default in the high-debt case.

Risk Lowering: Default Probability vs. Capital Structure



- Same asset distribution A in both cases.
- **High debt:** larger D ; firm defaults whenever $A < D_{\text{high}}$.
- **Low debt:** smaller D ; firm defaults only when $A < D_{\text{low}}$.
- Empirical result from the simulation:

$$\mathbb{P}(A < D_{\text{low}}) \leq \mathbb{P}(A < D_{\text{high}}).$$

This is *true risk reduction for the company*: the firm itself is less likely to be insolvent when it has more equity and less debt.

Many Shareholders vs. More Equity

Two different ideas that are easy to mix up:

More shareholders Same project, same total equity, same assets:

- We just slice ownership into more pieces.
- Per-person risk goes down (as in the payoff distribution plots).
- The project's total risk and the firm's default probability do *not* change.

More equity Company raises fresh capital and reduces debt:

- The threshold D moves left relative to the asset distribution.
- The probability $\mathbb{P}(A < D)$ genuinely decreases.
- This is a real reduction in firm-level risk.

Putting It All Together

- Shares allow many investors to pool money for large projects and *share* the risk.
- Mean and variance give a simple language:
 - Mean: typical payoff.
 - Variance: how wild the outcomes are.
- Equity is owners' money; it forms a buffer between assets and debt.
- More shareholders:
 - Good for *risk sharing* at the personal level.
- More equity capital:
 - Good for *risk lowering* at the firm level (lower default probability).