Correlated and Uncorrelated Stocks:
A Story About Portfolio Structures

Why mixing different kinds of stocks changes risk

What are we trying to understand?

- Imagine you have some money and you want to invest in the stock market.
- You rarely buy just one stock; you buy a **portfolio** – a mix of several stocks.
- Big question (in simple words):

Should I put my money in many stocks that move together,
or in stocks that behave differently from each other?

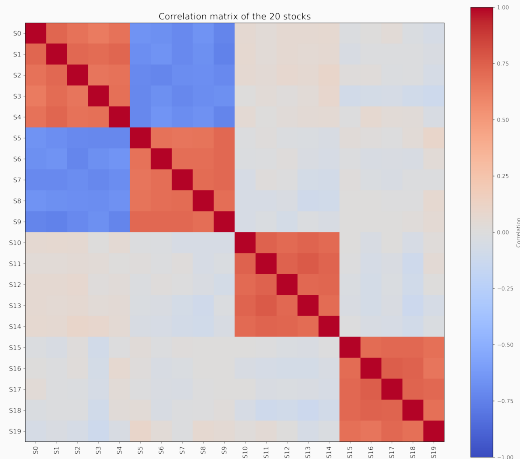- This talk uses a small simulation with **20 imaginary stocks** to tell that story.

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The basic ingredients

We create:

- **20 stocks** with daily returns over about one year.
- They are grouped into four families:
    - **Group 1:** Stocks S0–S4, move together.
    - **Group 2:** Stocks S5–S9, move together, but often *opposite* to Group 1.
    - **Group 3:** Stocks S10–S14, move together but mostly independent of 1 and 2.
    - **Group 4:** Stocks S15–S19, also independent from the others.
- Think of each group as a "sector" or "theme".

Correlation in one picture

- To see how similar the stocks are, we compute a **correlation matrix**.

- Correlation is a number between -1 and $+1$:
  - $+1$: move together perfectly.
  - 0: no clear link.
  - -1: move exactly opposite.

- Red areas: strong positive relationship. Blue areas: strong negative relationship.

Correlation matrix of the 20 stocks

(red = positive, blue = negative).

- The **red blocks along the diagonal**:
    - Each block corresponds to one group of 5 stocks.
    - Inside each block, stocks are strongly positively correlated.
    - They move *together*.
- The **blue rectangle** between Group 1 and Group 2:
    - When Group 1 tends to go up, Group 2 tends to go down.
    - They are **negatively correlated**.
- The area involving Groups 3 and 4:
    - Mostly pale colours near zero.
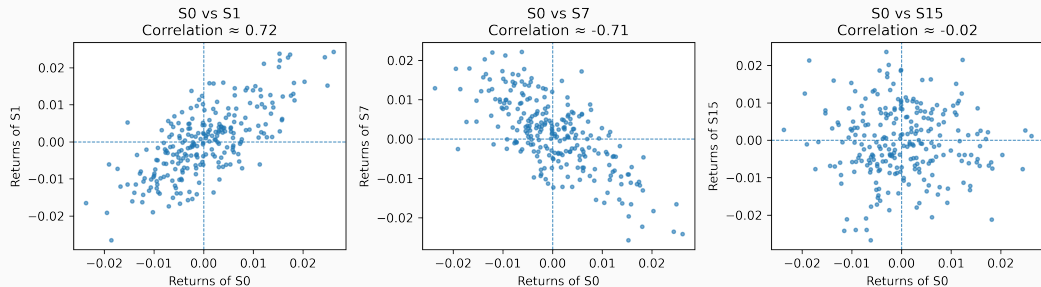   - These groups are **weakly correlated** with the others.

Figure 1: Daily return scatter plots for three pairs of stocks.

Explaining the scatter plots

Each dot is one day: x-axis is Stock A's return, y-axis is Stock B's return.

- **Left panel: S0 vs S1 (same group, positive).**
    - Cloud leans upwards.
    - When S0 has a good day, S1 usually does too.
- **Middle panel: S0 vs S7 (Group 1 vs Group 2, negative).**
    - Cloud leans downwards.
    - When S0 is up, S7 is often down.
- **Right panel: S0 vs S15 (almost uncorrelated).**
    - Cloud is more round, no clear slope.
    - Knowing S0's return tells you little about S15's return.

Four portfolios built from these stocks

We construct four portfolios, each starting with 100 units of money:

A: Group 1 only
- Equal weight in S0–S4.
- All stocks move together $\Rightarrow$ very concentrated bet.

B: Balanced opposite groups
- Equal weight in Groups 1 and 2 (S0–S9).
- One group can partly cancel the other.

C: Uncorrelated mix
- Equal weight in Groups 3 and 4 (S10–S19).
- Bets based on mostly independent stories.

D: Unbalanced opposite groups
- Same stocks as B (Groups 1 and 2), **but** 75% of the money in Group 2.
- Negatively correlated, but tilted strongly to one side.

How we simulate portfolio values

- Each day, every stock has a small random return:

$$\text{New value} = \text{Old value} \times (1 + \text{return}).$$

- A portfolio's daily return is the **weighted average** of its stocks' returns.

- Starting from 100, we apply these daily returns day by day to get a **value path**:

$$V_{t+1} = V_t \cdot (1 + r_{t,\text{portfolio}}).$$

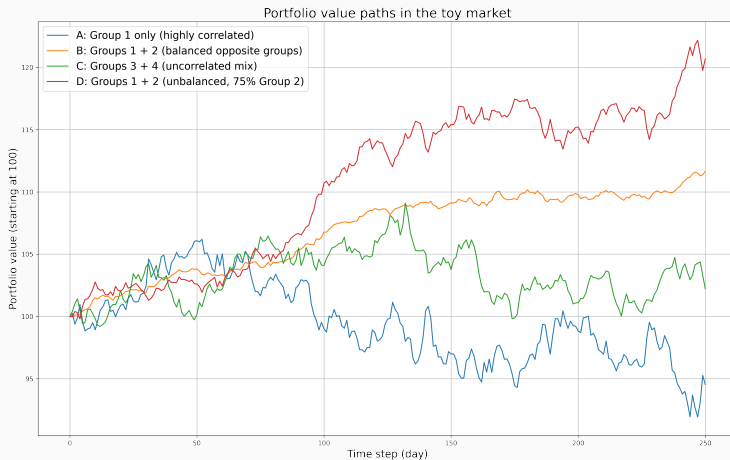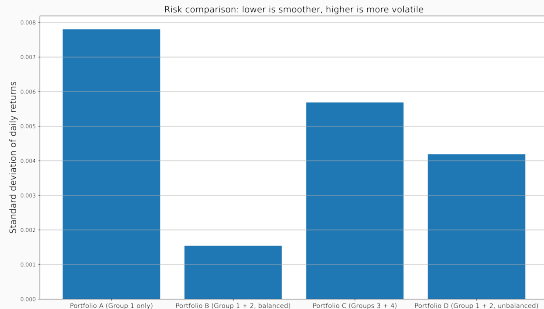- This creates four lines: A, B, C, D, one for each portfolio.

Figure 2: Value of 100 invested in each portfolio over time.

Reading the portfolio value plot

- **Portfolio A (blue):**
    - Built from one group that moves together.
    - The line is quite wiggly and can swing strongly up or down.
- **Portfolio B (orange):**
    - Combines two opposite groups with equal weight.
    - When one group hurts, the other may help.
    - The path is often smoother than A, yet still growing.
- **Portfolio C (green):**
    - Mix of two largely independent groups.
    - Moves come from different sources; bumps can cancel.
- **Portfolio D (red):**
    - Also uses opposite groups, but with 75% in Group 2.
    - Negative correlation reduces some swings, but because we bet more on the side that happens to do better in this run, it can shoot up – in other runs it might drift down.
    - The key point: correlation shapes the path, but weights and performance still decide who wins.

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- One simple measure of risk is the **standard deviation** of daily returns.

- Bigger standard deviation $\Rightarrow$ bigger typical day-to-day moves (a bumpier ride).

- We computed this number for each of the four portfolios to compare how "noisy" they are.

Standard deviation of daily returns

for the four portfolios.

What the risk bars tell us

- **Portfolio A** has the **highest** risk:
  - Many highly correlated stocks $\approx$ one big stock.
- **Portfolios B and C** tend to show **lower** risk:
  - B uses **negatively correlated** groups that offset each other.
  - C uses **uncorrelated** groups that behave independently.
- **Portfolio D** can have risk in between, but its *direction* (up or down) depends on which group is overweighted.
  - You can have "clever" correlation structure and still lose money.

Key lessons in plain language

1. **Correlation is about moving together.**
   - Positive: gain and lose together.
   - Negative: one up when the other is down.
   - Near zero: they do their own thing.
2. **Owning many similar stocks is not real diversification.**
   - Portfolio A shows that a "crowd of twins" still behaves like one bet.
3. **Mixing different kinds of stocks can smooth the ride.**
   - Portfolios B and C are examples where bumps partly cancel.
4. **Negative correlation is helpful, not magic.**
   - Portfolio D reminds us: if you put most money in the side that does badly, correlation won't save you.

Final remarks

- This simulation is deliberately simple and uses made-up numbers.
- Real markets are more complex, with many factors, costs and human behaviour.
- But the core ideas carry over:
    - Diversification works best when assets are not all moving together.
    - Correlation, expected return, and how you **weight** each asset all matter.
- This is an educational story, not investment advice – but it should help your intuition the next time you hear the word *portfolio*.