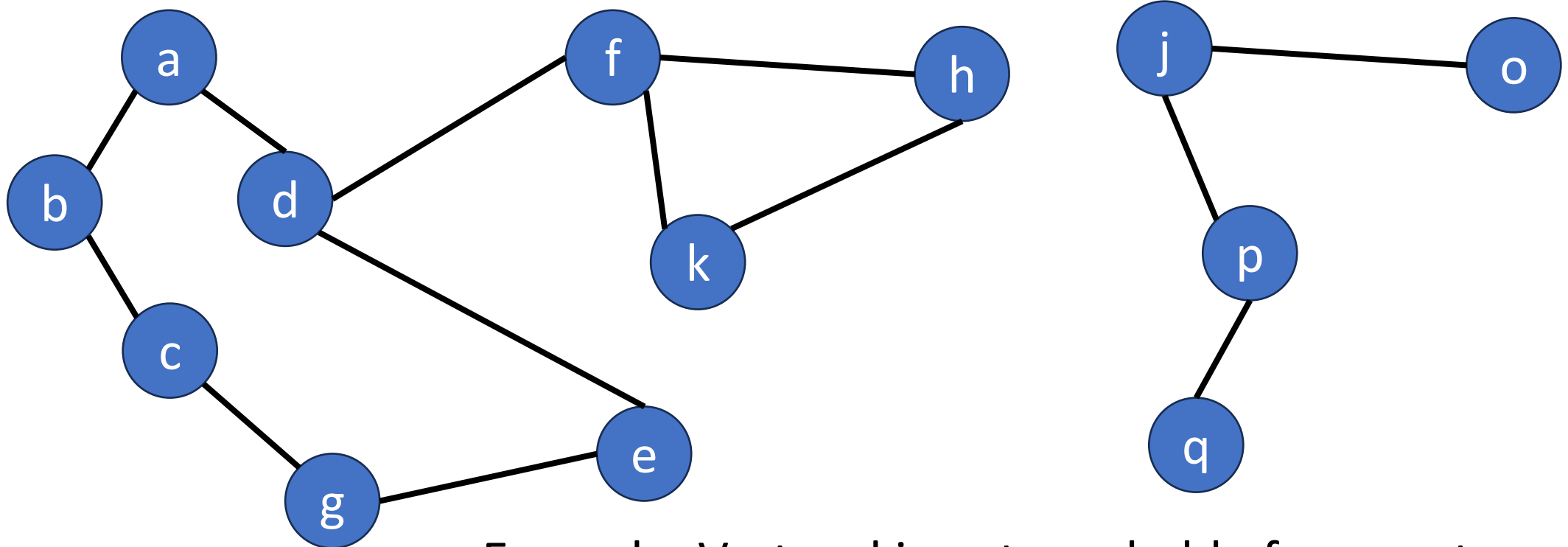


Graph search methods

Ch. 6.2

Graph search methods

- A vertex u is **reachable** from vertex v iff there is a path from v to u .
- A search method starts at a given vertex v and visits/labels/marks every vertex that is **reachable** from v .



Example: Vertex d is not reachable from vertex p.

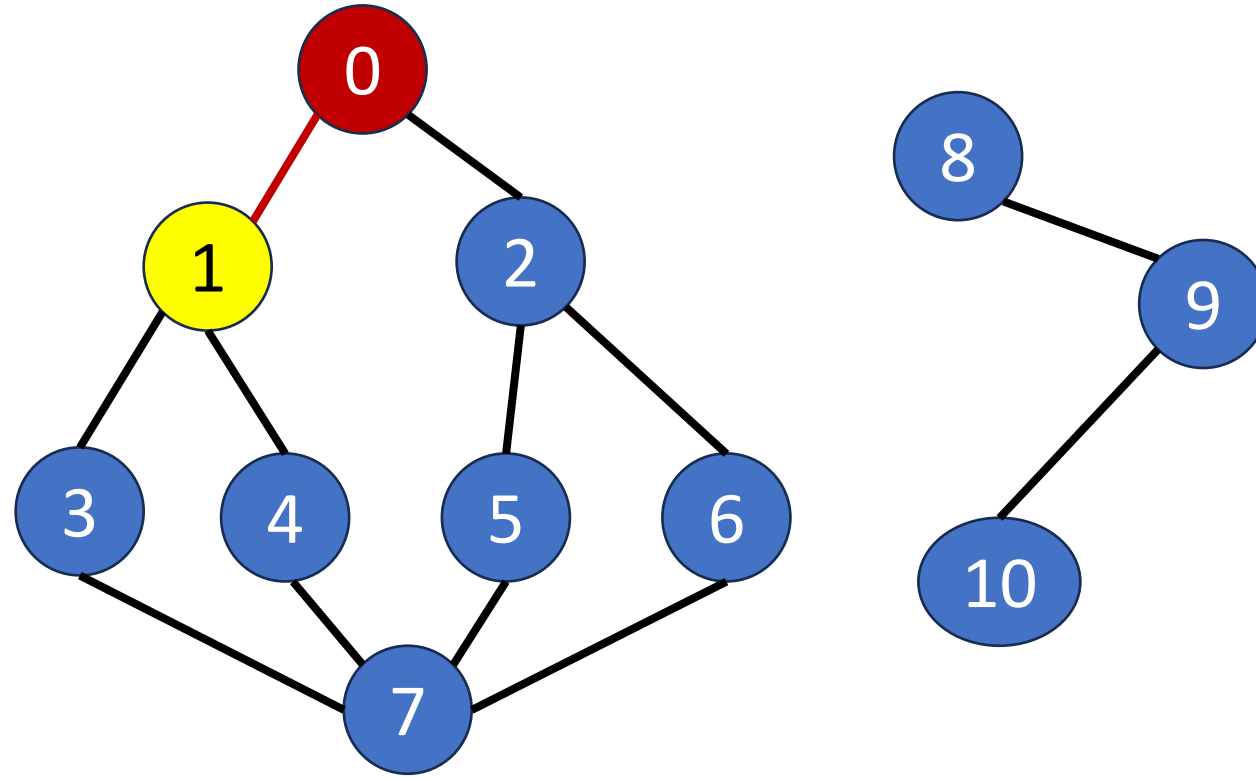
Graph search methods

- Commonly used search methods
 - **Depth** First Search
 - Similar to a preorder tree traversal
 - **Breath** First Search
 - Similar to a level-order tree traversal

Depth first search: method

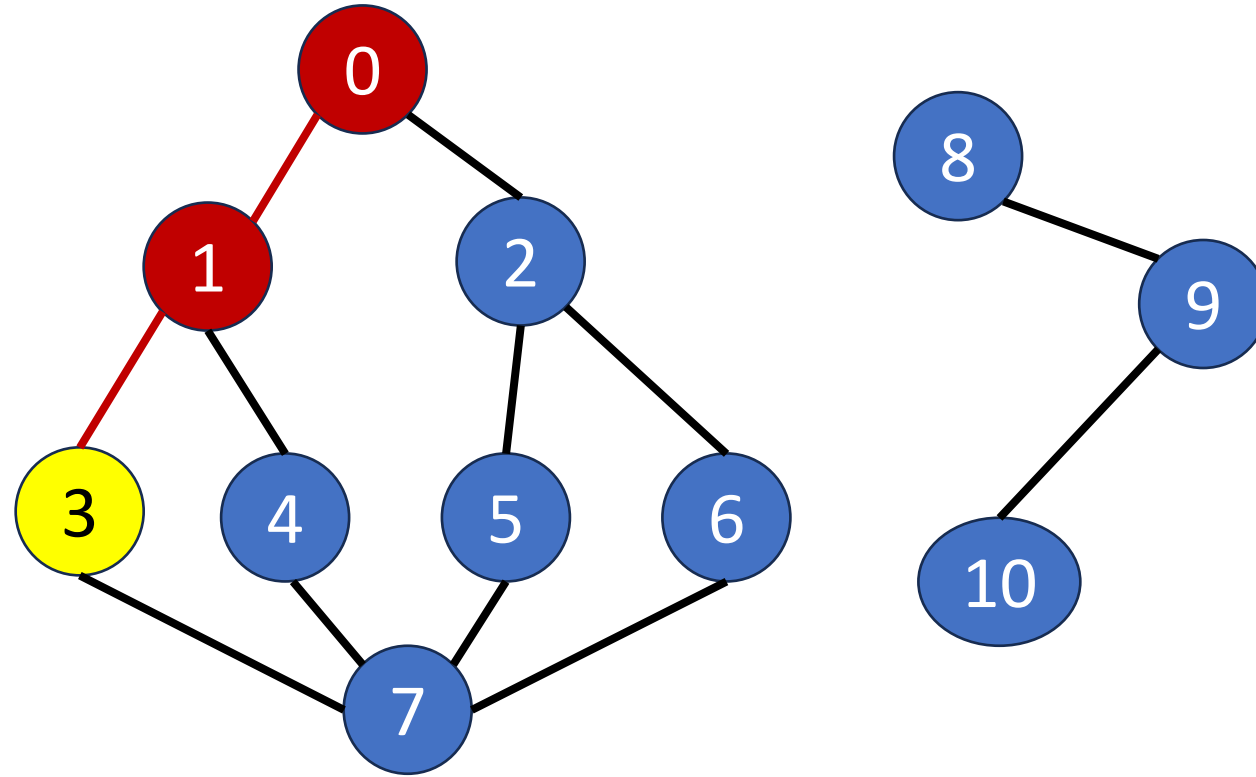
```
short int visited[MAX_VERTICES];  
/*Depth first search of a graph beginning at v.*/  
dfs(v)  
{  
    /*Label vertex v as reached.*/  
    visited[v] = TRUE;  
    /*Explore the adjacent unvisited vertices.*/  
    for (each unreached vertex u adjacent from v)  
        dfs(u);  
}
```

Depth first search: example



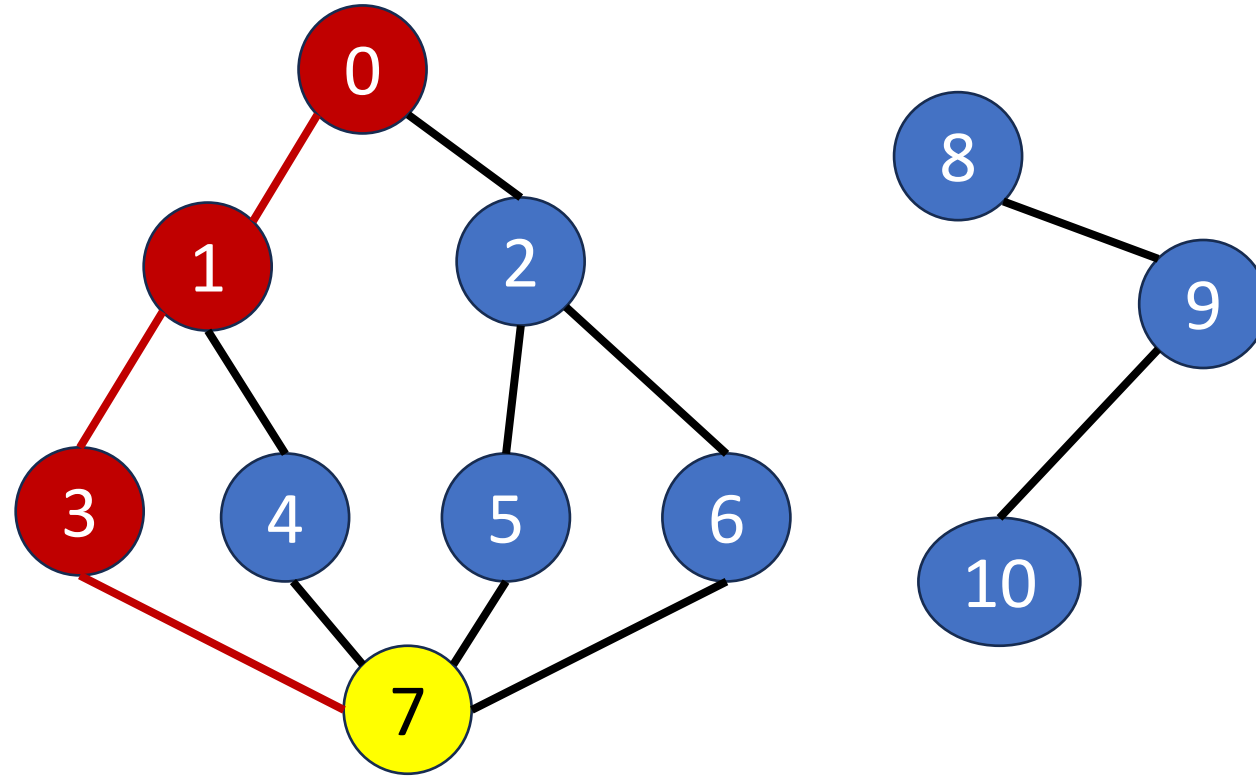
- $\text{dfs}(v_0)$: $[v_0]$
 - Start search at vertex 0.
 - Label vertex 0 and do a dfs from either vertex 1 or vertex 2.
 - In this example, vertex 1 is selected. Do $\text{dfs}(v_1)$.

Depth first search: example



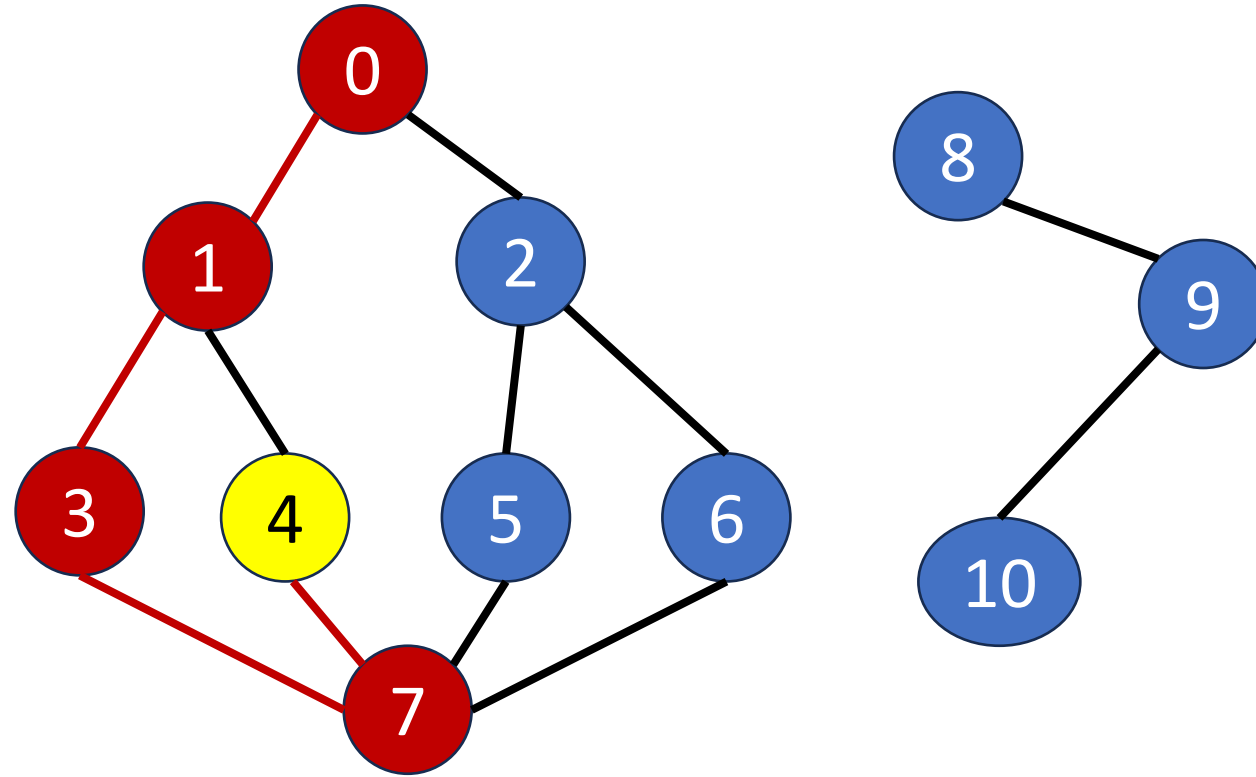
- $\text{dfs}(v_1)$: $[v_0, v_1]$
 - Start search at vertex 1.
 - Label vertex 1 and do a dfs from either vertex 3 or vertex 4.
 - In this example, vertex 5 is selected. Do $\text{dfs}(v_3)$.

Depth first search: example



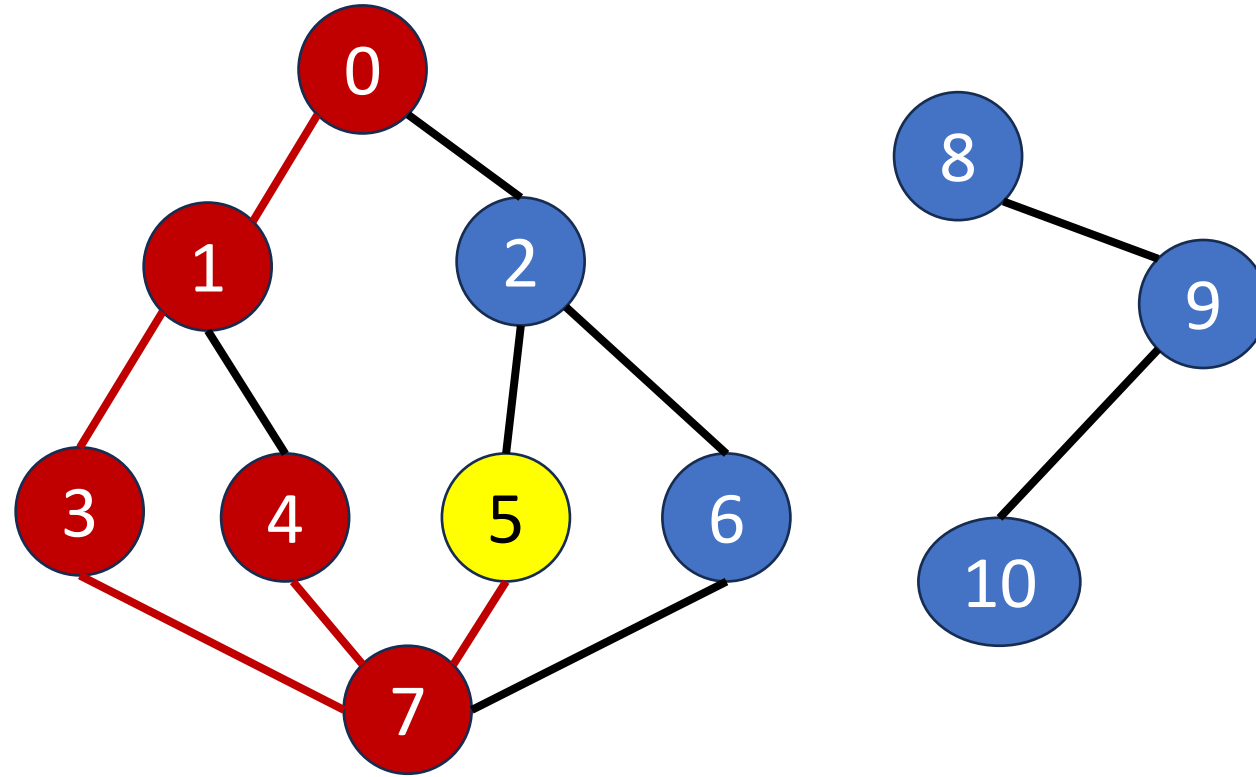
- $\text{dfs}(v_3)$: $[v_0, v_1, v_3]$
 - Start search at vertex 3.
 - Label vertex 3 and do a dfs from vertex 7.
 - Do $\text{dfs}(v_7)$.

Depth first search: example



- $\text{dfs}(v_7)$:
[v_0, v_1, v_3, v_7]
 - Start search at vertex 7.
 - Label vertex 7 and do a dfs from vertices 4, 5, or 6.
 - In this example, vertex 4 is selected. Do $\text{dfs}(v_4)$.

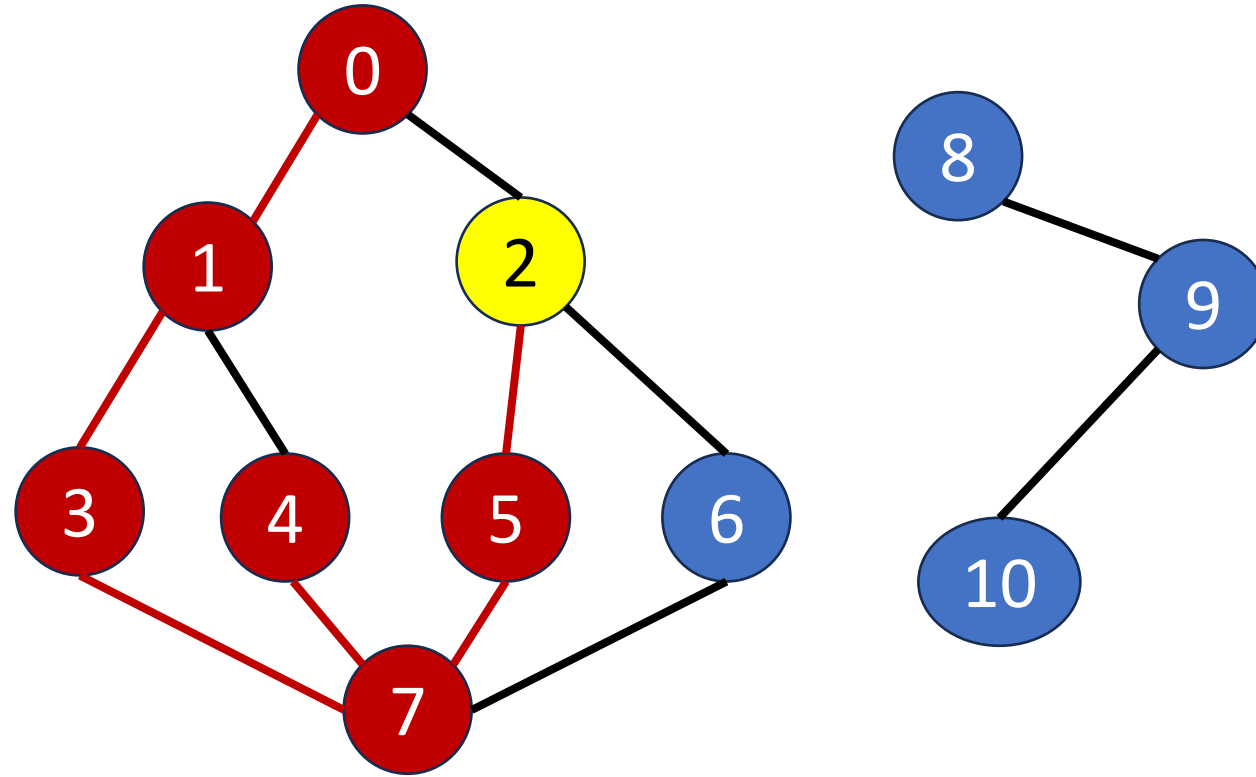
Depth first search: example



- $\text{dfs}(v_4)$:
• Start search at vertex 4.
• No unvisited adjacent vertex. Return to vertex 7.
- do a dfs from vertices 5 or 6.
- In this example, vertex 5 is selected. Do $\text{dfs}(v_5)$.

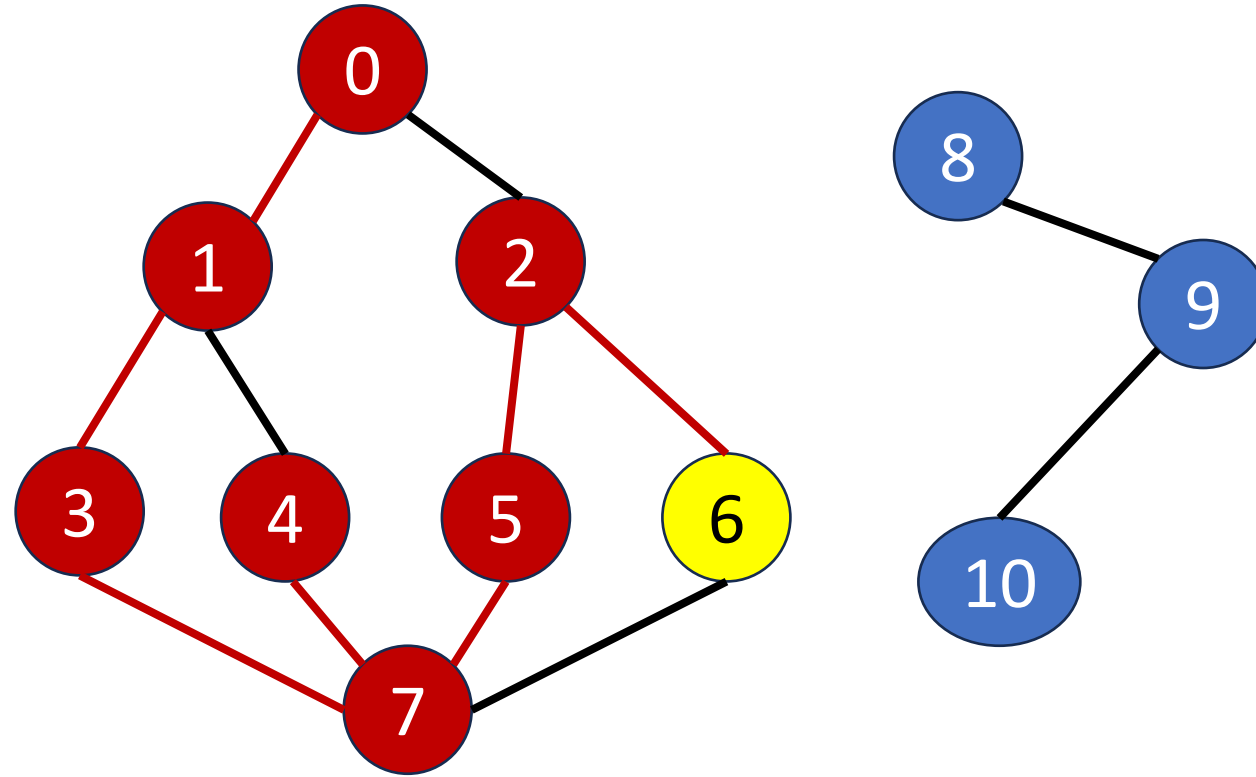
$[v_0, v_1, v_3, v_7, v_4]$

Depth first search: example



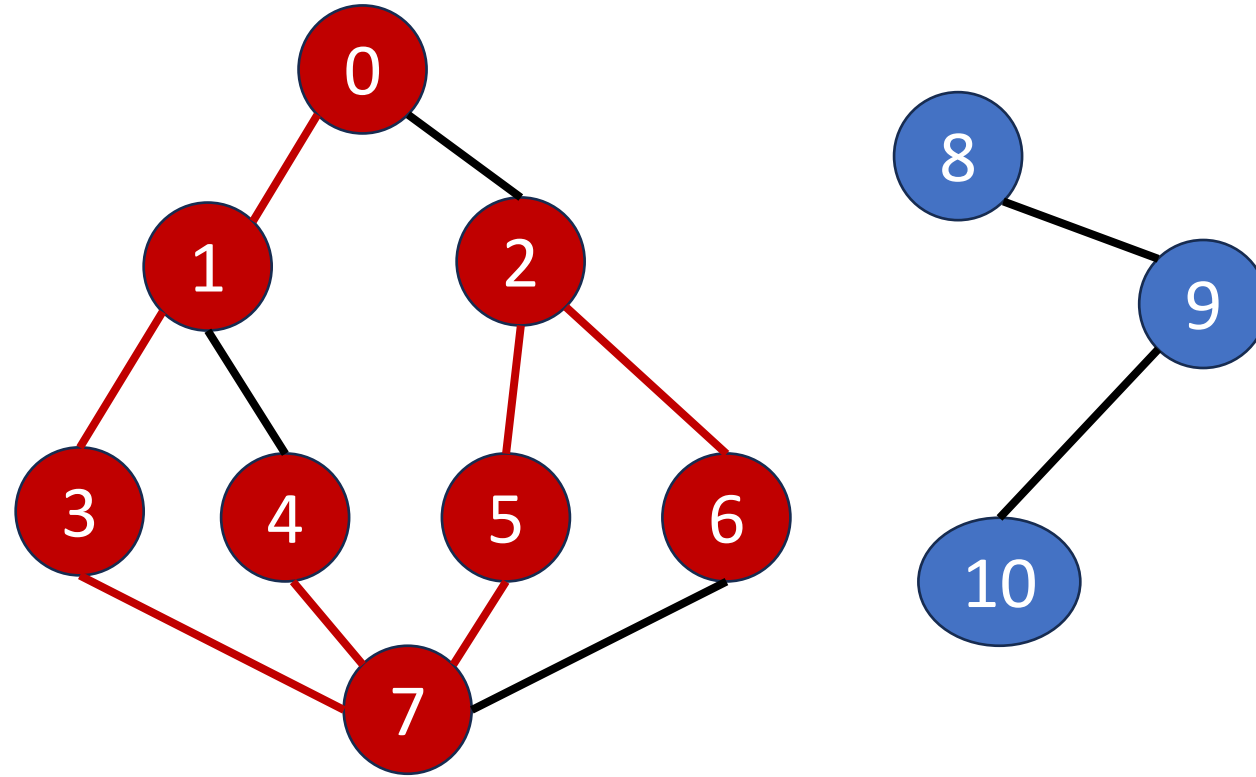
- $\text{dfs}(v_5)$: $[v_0, v_1, v_3, v_7, v_4, v_5]$
 - Start search at vertex 5.
 - Label vertex 5 and do a dfs from vertex 2.
 - Do $\text{dfs}(v_2)$.

Depth first search: example



- $\text{dfs}(v_2)$: $[v_0, v_1, v_3, v_7, v_4, v_5, v_2]$
 - Start search at vertex 2.
 - Label vertex 2 and do a dfs from vertex 6.
 - Do $\text{dfs}(v_6)$.

Depth first search: example



- $\text{dfs}(v_6)$: $[v_0, v_1, v_3, v_7, v_4, v_5, v_2, v_6]$
 - Start search at vertex 6.
 - Label vertex 6. No unvisited adjacent vertex.
- Return to vertices 2, 5, 7, 3, 1, 0 sequentially.

Time complexity

- When adjacency matrix is used

$$O(n^2)$$

- For a node, search the corresponding row to find adjacent vertices.
→ $O(n)$
- Visit at most n nodes. → $O(n \times n)$

- When adjacency lists are used

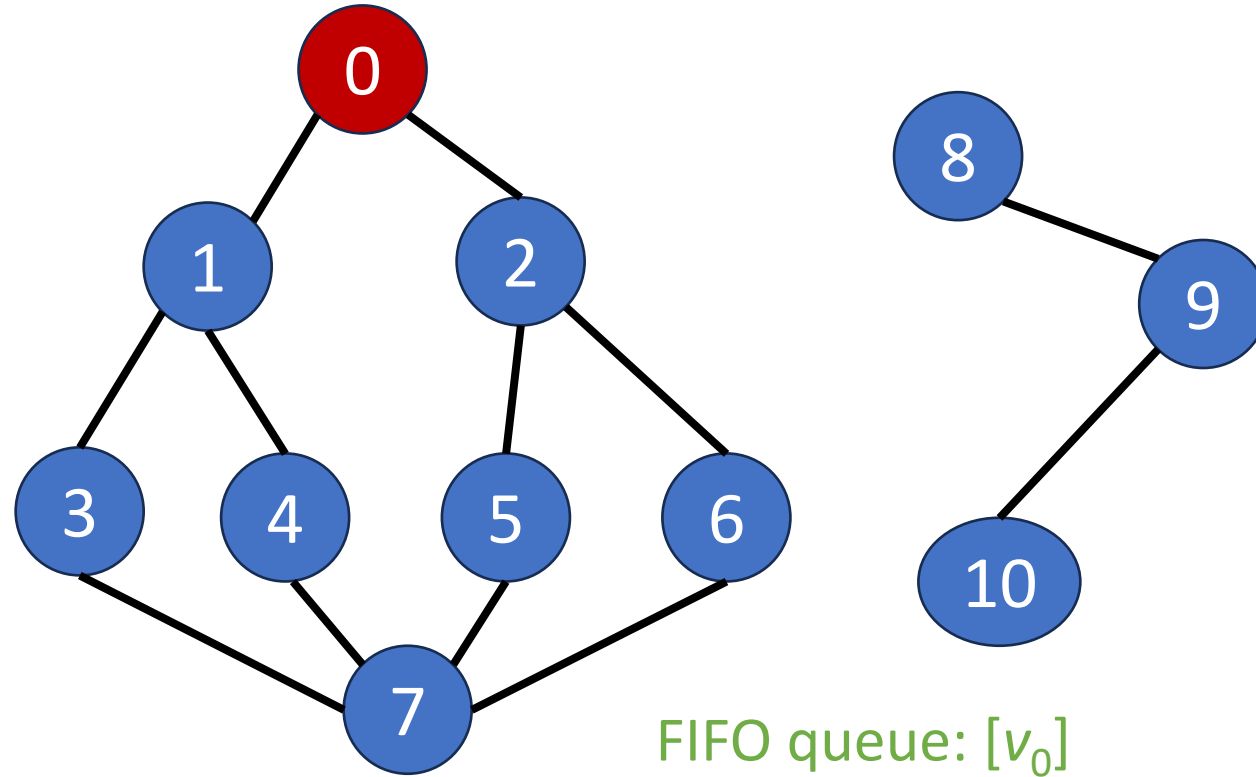
$$O(n+e)$$

- Search at most e edges and n nodes.

Breadth first search: method

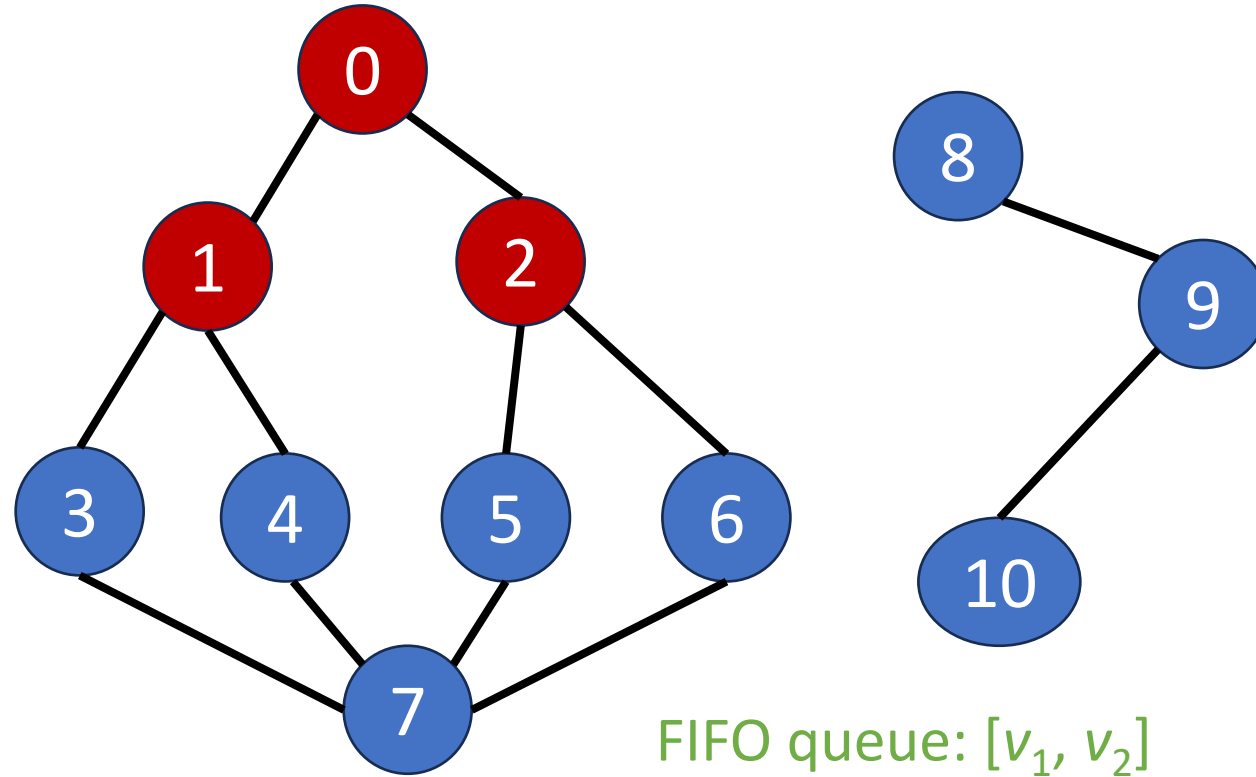
- Visit start vertex and put into a FIFO queue.
- Repeatedly remove a vertex from the queue, visit its **unvisited** adjacent vertices, put newly visited vertices into the queue.

Breadth first search: example



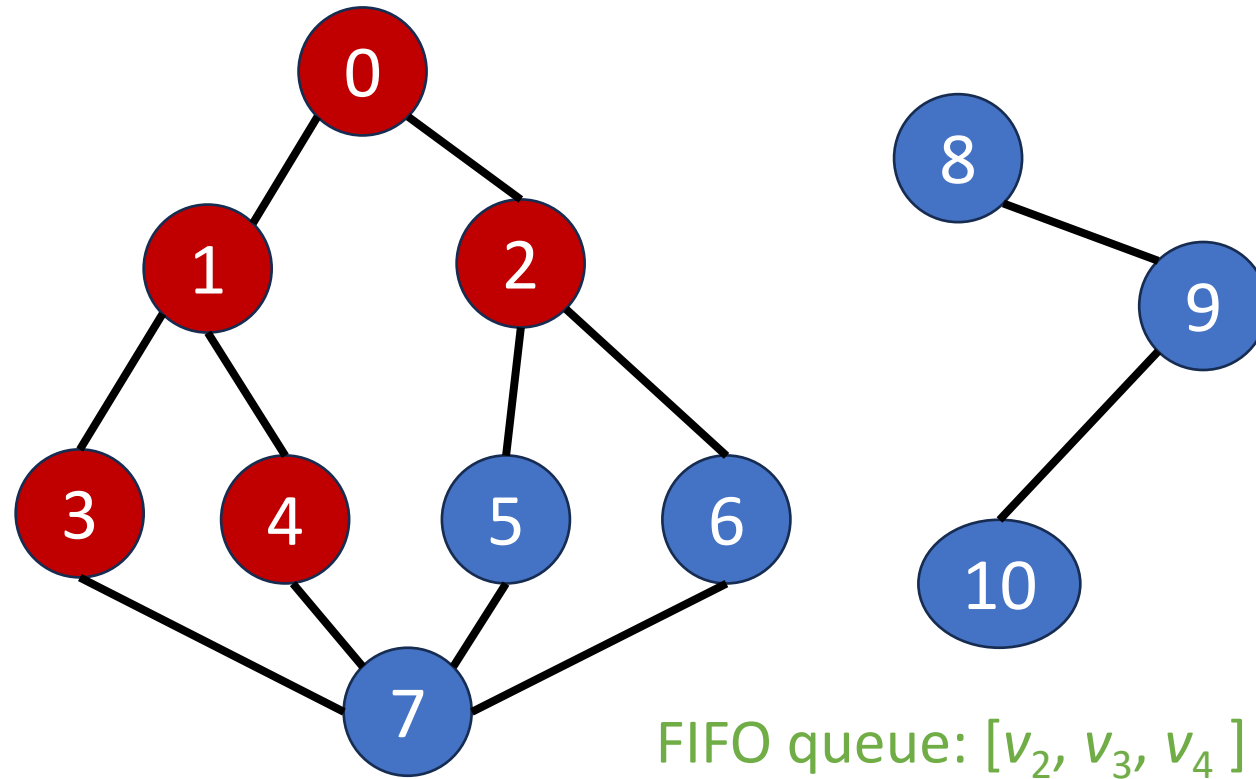
- Start search at vertex 0.
- Label vertex 0 and add vertex 0 to a FIFO queue.

Breadth first search: example



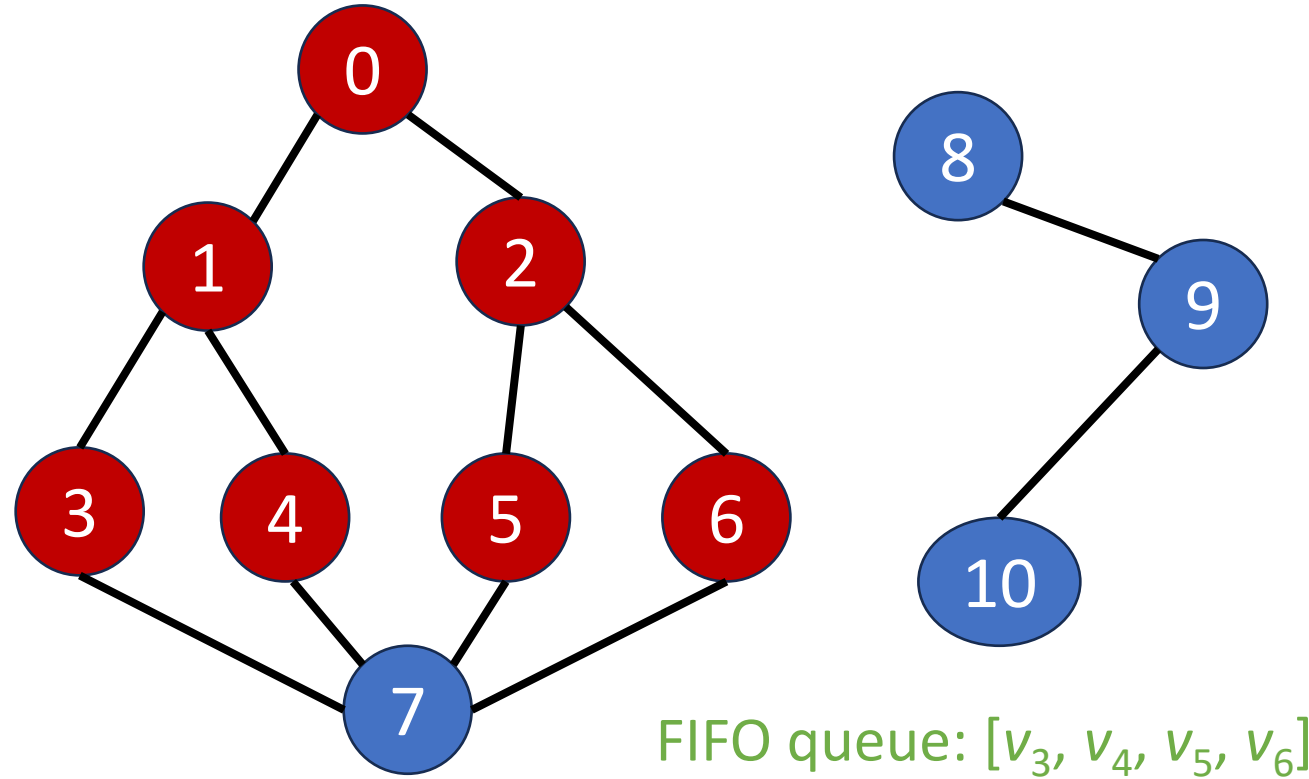
- Remove vertex **0** from queue.
- Visit adjacent unvisited vertices **1** and **2** and put into **FIFO queue**.

Breadth first search: example



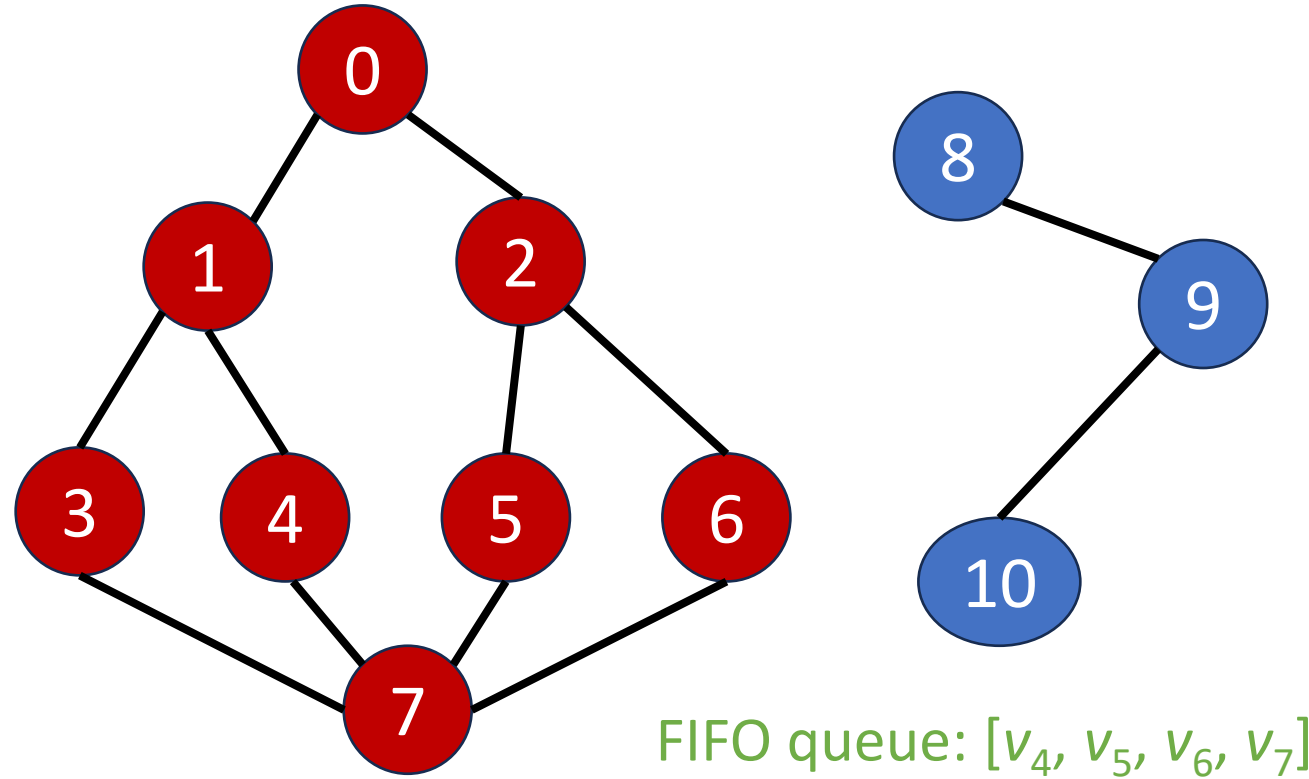
- Remove vertex **1** from queue.
- Visit adjacent unvisited vertices **3** and **4** and put into **FIFO queue**.

Breadth first search: example



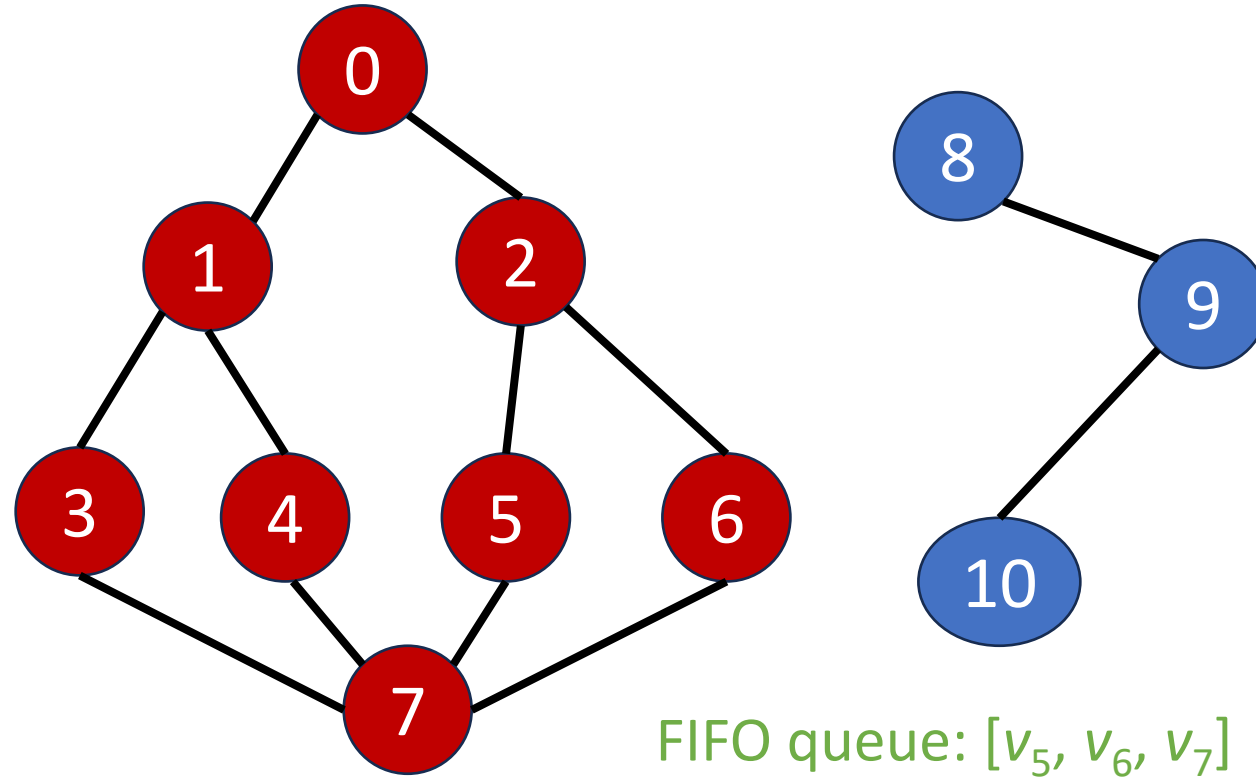
- Remove vertex **2** from queue.
- Visit adjacent unvisited vertices **5** and **6** and put into **FIFO queue**.

Breadth first search: example



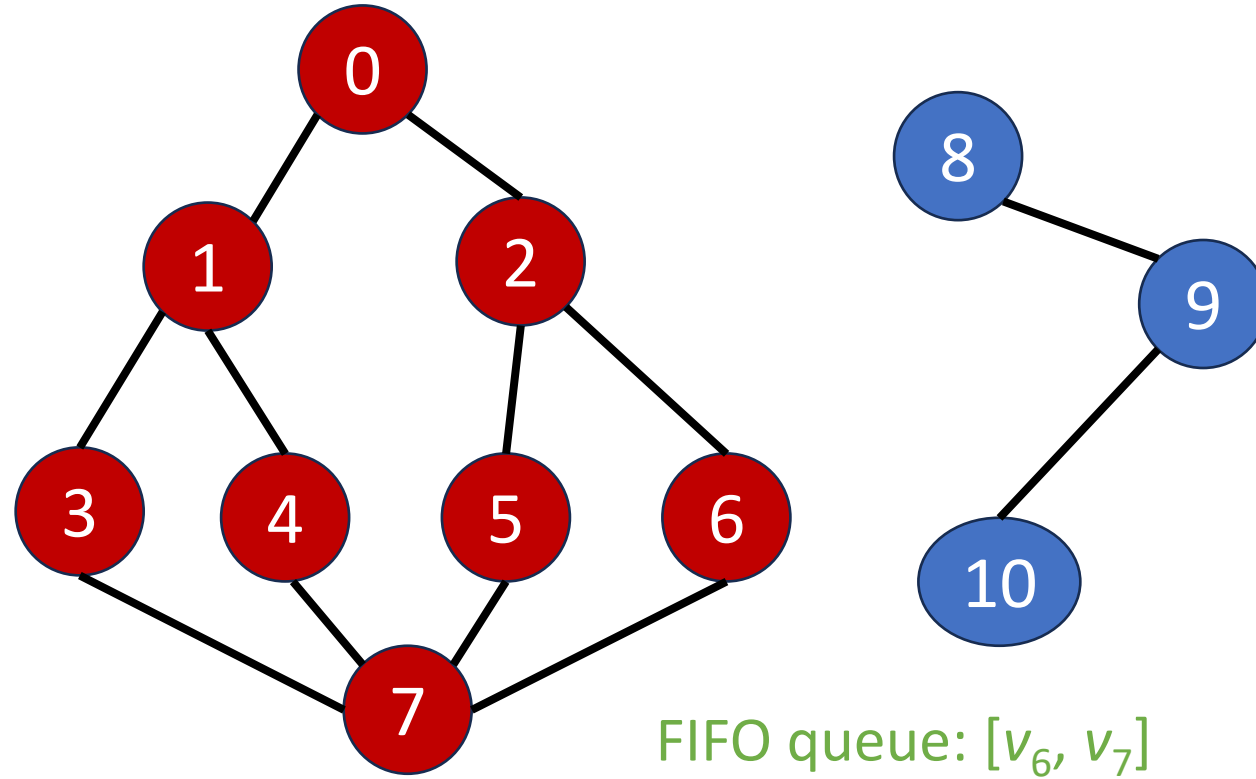
- Remove vertex **3** from queue.
- Visit adjacent unvisited vertex **7** and put into **FIFO queue**.

Breadth first search: example



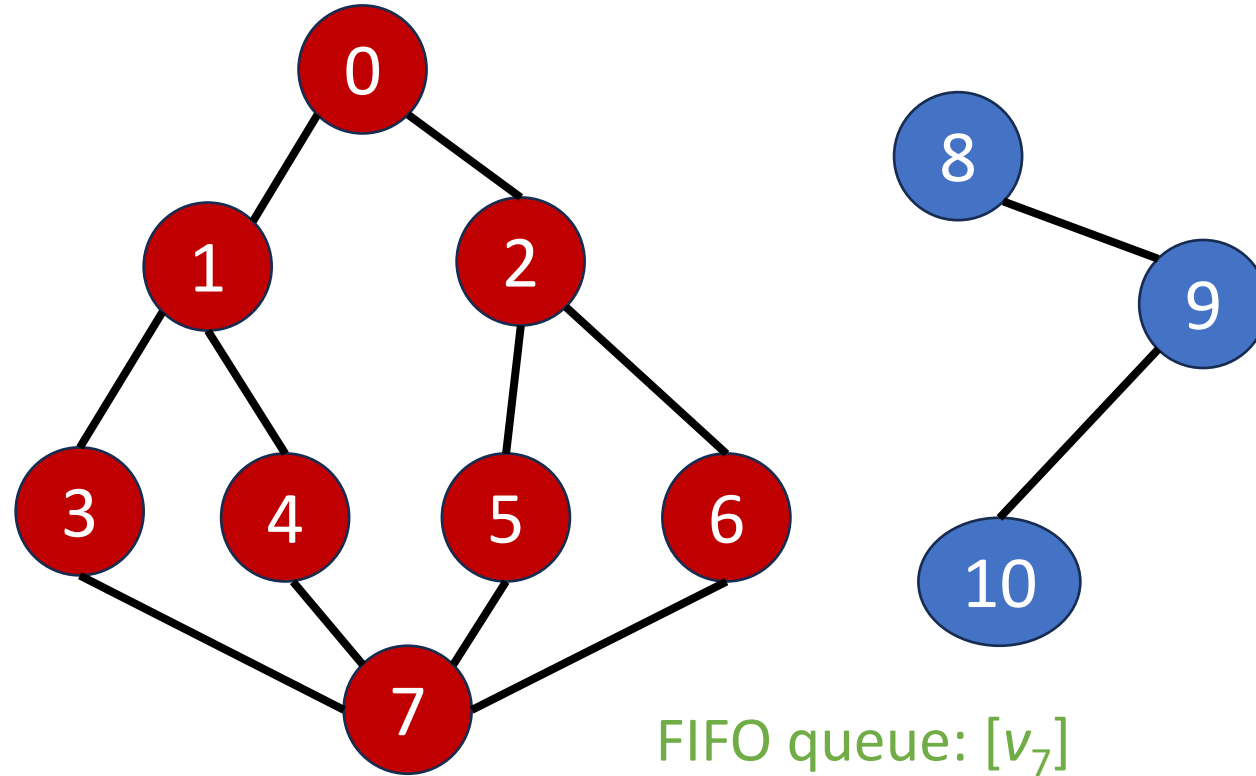
- Remove vertex **4** from queue.
- Visit adjacent unvisited vertex and put into **FIFO queue**.

Breadth first search: example



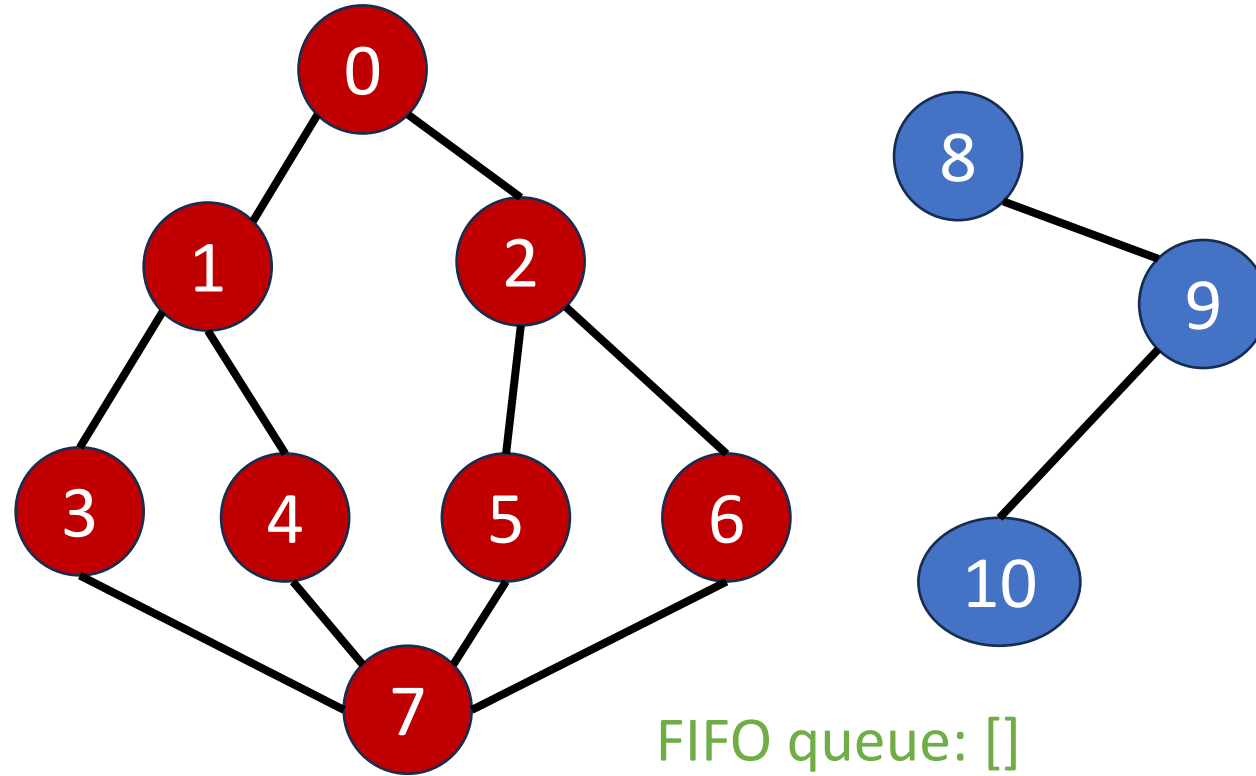
- Remove vertex **5** from queue.
- Visit adjacent unvisited vertex and put into **FIFO queue**.

Breadth first search: example



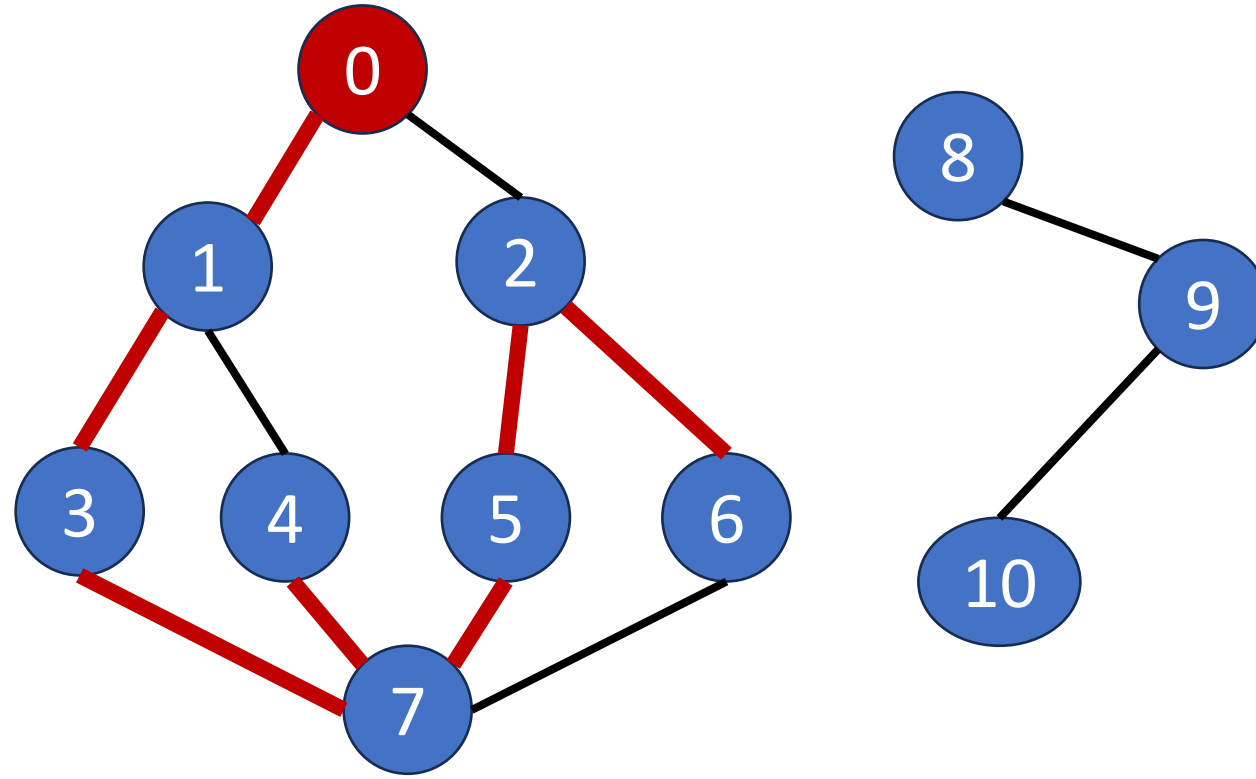
- Remove vertex **6** from queue.
- Visit adjacent unvisited vertex and put into **FIFO queue**.

Breadth first search: example



- Remove vertex **7** from queue.
- Visit adjacent unvisited vertex and put into **FIFO queue**.

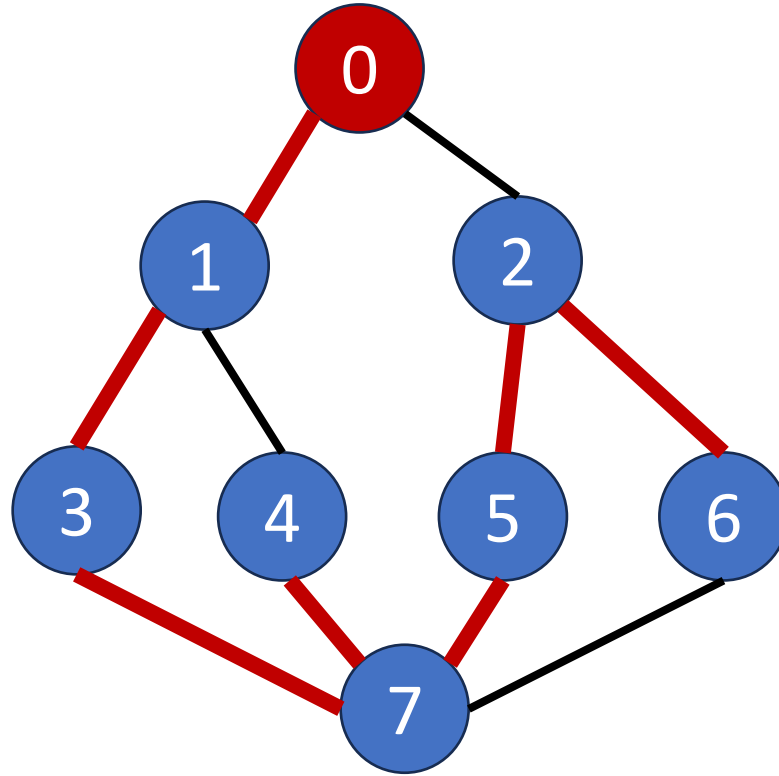
Applications: connected components



Depth-first search (or breadth-first search) from vertex 0.

Vertices 8, 9, and 10 are unvisited. ➔ The graph is not connected.

Applications: spanning tree



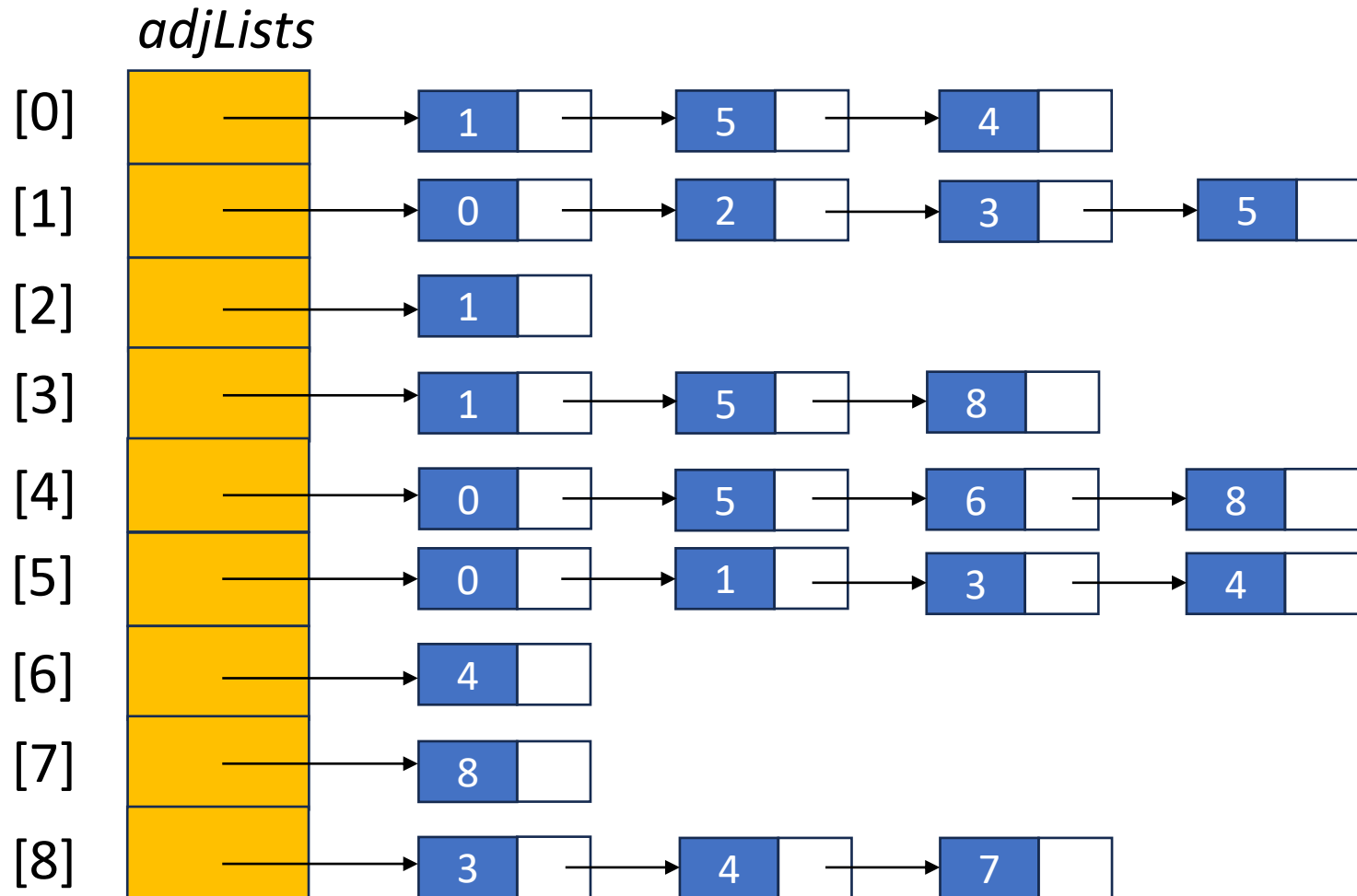
Depth-first search from vertex 0.
Generating a depth-first spanning tree.

Exercise

Please reply your answers of Q10-Q11 via the following link:



Group members: 1~3 people



- Q10: Write out the results of DFS starting at vertex **0**.
- Q11: Write out the results of BFS starting at vertex **0**.

Summary

- Depth first search
- Breadth first search