Mixed Wind Farm Optimization

Dustan Kraus, Yuri Neves, and PJ Stanley

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Abstract

Many people have optimized the annual energy production (AEP) of a wind farm using either horizontal axis wind turbines (HAWTs) or vertical axis wind turbines (VAWTs). This paper focus on an optimization of a wind farm containing both types of wind turbines. An optimization of the exact ratio of HAWTs and VAWTs is outside the scope of this paper; however, we provide an optimization of a wind farm considering a set ratio of HAWTs and VAWTs.

1 Introduction

A large barrier in the wind energy industry is lack of efficiency and ability to create cheap energy that competes with traditional coal power plants. One way to increase the efficiency of wind farms is to optimize the layout of the farm before construction to avoid negative impacts from wake loss. These wind farms have always been designed for a single turbine type, usually horizontal axis wind turbines (HAWTs). These wind farms give good results, but there is a lot of empty space wasted between these large turbines that is not generating any power. If much of this area were filled with smaller vertical axis wind turbines (VAWTs), much more power could be generated.

We seek to optimize the layout of a wind farm with both HAWTs and VAWTs. This will be accomplished by analyzing the wake behind each turbine and then figuring out the optimum turbine layout. Wind farms have size limitations, and can only fit a certain number of HAWTs. Because VAWTs are significantly shorter than HAWTs; however, they can be added to a wind farm to increase the power output without increasing the amount of land required. The integration of wind turbine types into a single farm is an enormous step forward for renewable energy. This type of farm will use its land much more efficiently and create more energy.

2 Model Description

Wind passing turbines is slowed as some of its kinetic energy is taken by the turbine blade. This slowed portion is called a wake, and is created downwind of all turbines. When designing a wind farm, it is desirable to have turbines experiencing the highest wind speed possible, to generate the most energy. For this reason, farms are designed such that losses from wakes are minimized. To design with this in mind, it is necessary to simplify and model wake behavior behind wind turbines.

2.1 Jensen Wake Model

We used the Jensen wake model to simulate the wake behind the HAWTs. This model uses conservation of mass and momentum in its derivation, and assumes a constant spread angle of 10 degrees. We added a cosine curve to the wake model such that the wind speed near the center of the wake is slower than near the edge of the wake. This is so that there is a gradient and turbines in the center of the wake will be pushed out during the optimization.

2.2 VAWT Wake Model

Eric Tingey, a graduate student at Brigham Young University (BYU), has developed a parameterized VAWT wake model using CFD vorticity data [1]. The model he developed is valid for tip-speed ratios betwen 1.5 and 7.0, solidities between 0.15 and 1.0, and Reynolds numbers between 200,000 and 6,000,000. Using this model, he developed a simplified wake model with a best fit curve. We ultimately ended up using this simplified wake model in our optimization as it reduced the computational expense significantly.

We also assumed that the wind velocity at VAWTS (and therefore the VAWT power output) is only affected by HAWT towers (as discussed further in section 2.3) and other VAWTS. They are not affected by the HAWT wakes.

2.3 HAWT Tower Wake Model

As mentioned in the previous section, since the HAWT tower will have a significant affect on the VAWTs energy performance, we created a wake model of the HAWT towers that will overlap with the vertical turbines.

For the tower wake model, we assumed the base of the horizontal turbines to be a perfect circular cylinder. Because there is no formal wake model for a cylinder in free flow, we derived a simplified model using the conservation of mass and momentum. For a full derivation, see Appendix A.

We made several important assumptions that affect the accuracy of this model, some of which were simplifications of empirical data. For the actual wake, we assumed a spread angle of 10 degrees, as was assumed in the Jensen model. We also assumed a constant drag coefficient of 0.3 [2]. Drag coefficient is really a function of the Reynolds number, so it will change with changes in wind speed and tower diameter. However, because the Reynolds number remains in the same order of magnitude, a constant C_D is realistic.

2.4 Direction and Speed Frequencies

To calculate AEP, the frequency of wind from different directions is taken from empirical data of the Princess Amalia Wind Farm. This data is taken at discrete points in 5 degree intervals. To make this a continuous function, we linearly interpolated it.

The probability distribution of the wind speeds is approximated with a Weibull function, with a max wind speed of 30 m/s and a lower range of 2 m/s (any lower speeds did not satisfy the assumptions of the simplified VAWT model used).

3 Optimization

The optimization is defined by:

$$\begin{aligned} \underset{x}{\text{maximize}} \quad & AEP(x, y) \\ \text{subject to} \quad & d_{HAWT-HAWT} \geq 8 * r_{HAWT} \\ & d_{HAWT-VAWT} \geq 4 * r_{HAWT} \\ & d_{VAWT-VAWT} \geq 8 * r_{VAWT} \end{aligned} \tag{1}$$

Where AEP is the annual energy production of the wind farm, x and y are the x and y coordinates of each wind turbine. $d_{HAWT-HAWT}$, $d_{HAWT-VAWT}$, and $d_{VAWT-VAWT}$ are the distances between each HAWT and all other HAWTs, each HAWT and all other VAWTs, and each VAWT and all other VAWTs, respectively. r_{HAWT} and r_{VAWT} are the radii of the HAWTs and VAWTs.

We used three classes of constraints to constraint our problem. The first constrain, is that the position of the vertical turbines cannot be closer than eight radii apart. The second is that the horizontal turbines cannot, also, be closer than eight radii apart. The last class of constraint, is that each vertical turbine cannot be closer than four radii apart from another horizontal turbine. In all, each turbine has several specific constraints. We also included a simple bound constraint so that the turbines remain within a specific farm footprint.

We used six parameters to run our optimization: the number of vertical turbines (from this, the number of horizontal can also be determined), the radius of the horizontal turbines blades, the radius of the vertical turbine, the radius of the horizontal turbines tower, the direction of the wind, and the initial velocity of the wind. Each of these parameters can easily be changed to test different scenarios, since they are all passed collectively into the optimizer.

We used the commercial optimizer SNOPT [3].

4 Results

Initial AEP	Optimized AEP
294.2 GW-h	374.8 GW-h

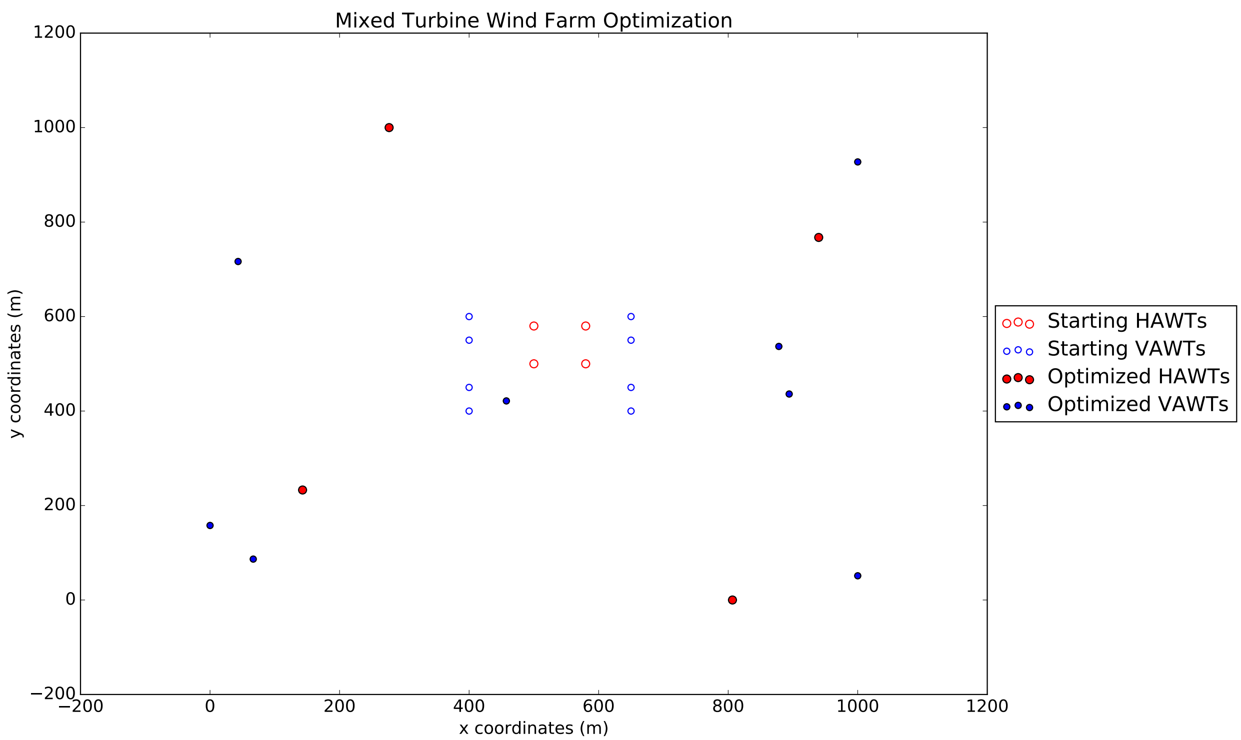

Figure 1: This plot shows the initial and optimal turbine locations for a mixed wind farm using SNOPT [3].

5 Conclusions and Future Work

This paper presented the results of an optimization for the layout of a mixed turbine wind farm. Wind farms have size limitations, and can only fit a certain number of HAWTs. Because VAWTs are significantly shorter than HAWTs; however, they can be added to a wind farm to increase the power output without increasing the amount of land required. This optimization did not focus on optimizing the number of HAWTs or VAWTs in a wind farm, but rather focused on optimizing a wind farm given the number of HAWTs and VAWTs.

In the future, a useful study would be to research the optimal proportion of HAWTs vs. VAWTs in a mixed turbine wind farm.

References

[1] Tingey, E. and Ning, A., "Parameterized Vertical-Axis Wind Turbine Wake Model Using CFD Vorticity Data," *Wind Energy Symposium (AIAA SciTech)*, San Diego, CA, jan 2016.

[2] NASA, "Drag of a Sphere," https://www.grc.nasa.gov/www/k-12/airplane/dragsphere.html.

[3] Perez, R. E., Jansen, P. W., and Martins, J. R. R. A., "pyOpt: A Python-Based Object-Oriented Framework for Nonlinear Constrained Optimization," *Structures and Multidisciplinary Optimization*, Vol. 45, No. 1, 2012, pp. 101–118.