Greedy Signature Processing with Arbitrary Location Distributions: A Divisible Load Framework

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The optimal partition of a huge, linear (flat) file among processing nodes in a network to minimize the time to search for signatures of interest in the file is considered. First, an expression is developed for the expected time of finding the kth signature (including the last signature) of K signatures for a uniform distribution of signatures in the file. Secondly, for a single signature we propose processing data in the file in order from that with the most probability mass (i.e., data with the most a priori likelihood of containing the signature) to that with the least probability mass in a "greedy" manner to speed processing time. Applications of this work include radar, sensors, image processing, and search.

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I. INTRODUCTION

A "signature" is a data pattern of interest in a large data file. Signature searching involves finding such signatures in the voluminous amount of data that can be produced with aerospace technology. This process is complicated by the possible presence of noise in the data and because sometimes ideal signatures may only be approximate in actual instances of data. There are natural radar and sensor applications for signature searching. However, as examples, three representative applications involving image processing follow.

1) An automated lander for Mars may process views of the surface for the signatures of good landing locations (e.g., flat areas, no boulders or trenches, etc.…).

2) Archaeologists may process satellite imagery of jungle areas for signatures of overgrown ancient structures.

3) Astronomers may search through images from a space based telescope, such as the Hubble, automatically cataloging distinctive astronomical features (e.g., spiral galaxies).

In this paper divisible load theory is applied to the problem of signature search time evaluation in flat file databases. That is, one can view the entire data set as a huge, linear (flat) file that is to be optimally partitioned among processing nodes in a network so that processing time is minimal. Flat (linear) files are a natural choice for early database implementations though more sophisticated database models are often later used [1].

In this paper after establishing the model and notation (Section II), the expected time for finding multiple signatures in a flat file is developed (Section III). In doing so, a uniform distribution of the signatures within the file is first assumed. An expression is developed for the expected time for finding the kth signature out of K signatures, including the last signature. This work extends the earlier work of Ko and one of the authors [1]. That work considered single signatures with a uniform distribution of signature location as well as finding the last signature of a number of uniformly distributed multiple signatures.

However signatures may not always be uniformly distributed. For instance a satellite may record images of the ocean in a search and rescue operation. Based on elapsed time and known ocean currents it may be possible to associate a probability density function to the current location of a lost individual. As a second example, probability density functions may also be associated with the positions of a vessel or a vehicle one has lost contact with. Here one would also take into account the potential speed of the vessel/vehicle.

In this paper we propose for such situations processing data in order from that with the most

probability mass (i.e., data with the most a priori likelihood to find a signature) to that with the least probability mass in a "greedy" manner to speed the processing time. Naturally for a small data set on a single computer, time may not be an issue. We are thus considering voluminous data sets, with significant processing and transmission time, that can be processed on a number of computers (such as a cluster of computers). We use the term "greedy" in the spirit of greedy combinatorial optimization algorithms which choose, each algorithm iteration, the combinatorial choice that yields the largest possible improvement possible in maximizing or minimizing an optimization (objective) function.

In Section IV distributing load for greedy processing for the case of a single signature is examined. Greedy processing for arbitrary location densities for a single signature is considered in Section V. An example involving a truncated normal probability density function appears in Section VI. This is followed by conclusions and future work in Section VII.

A. Divisible Load Theory

In this paper we use divisible load analysis because of its tractability and appropriateness for the model considered. Divisible load theory was introduced by [2] and [3] and, also independently in [4] as "large-grained parallelism." Divisble load modeling is concerned with massive computational and communication loads without any precedence relationships that must be optimally scheduled/allocated to processors and links. Here optimality is defined as processing the load in a minimal amount of time for a given scheduling policy and interconnection network topology.

Divisible load theory is a theory of proportions. If one has two processors and a very fast connection between them, with one processor twice as fast as the other, it makes sense that two-thirds of the load should be assigned to the faster processor and one-third of the load to the slower processor to process the load in a minimal amount of time. Given many processors interconnected by channel speed limited links in some sort of interconnection network one has a more complex problem of proportions but one that can still be solved through linear equations, mathematical programming or, in some cases, by simple algebraic recursions.

Ever since its introduction, divisible load theory has served as an effective modeling tool for data-intensive applications [5–10] and a large amount of work has been published for various network structures including linear networks [11], bus networks [12], single and multi level tree networks [13–15], mesh networks [16, 17], hypercubes [18] and arbitrary graphs [19] with different constraints

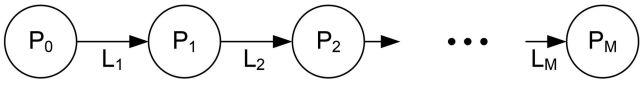

Fig. 1. Model of linear daisy chain network with communication links.

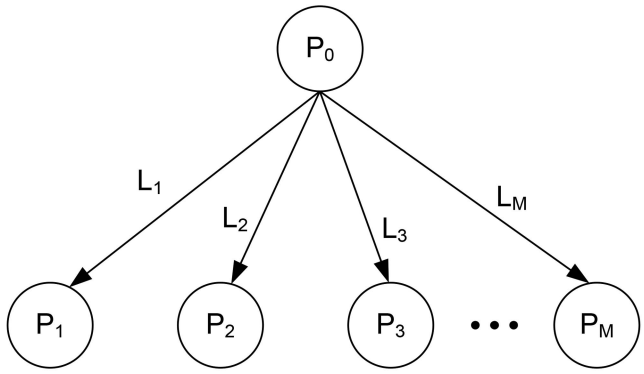

Fig. 2. Model of single level tree network with communication links.

such as different release times, buffer constraints [20], communication start-up costs and time-varying network capacity. Also, various distribution strategies have been studied including multi-installment of load distribution [21]. In [22], [23] and [24], scheduling strategies without knowledge of network resources are examined. In [25] signature searching is investigated on bus networks experimentally. In [1] a uniform distribution of the signature in the dataset is assumed as it is a feasible model of the distribution of signatures in large databases as discussed in [26].

II. SYSTEM MODEL

A. Network Model

We consider signature search time on two network models using divisible load theory. Figure 1 describes a linear daisy chain model and, in Fig. 2, a single level tree network model is described. In this second network model, one root processor, P_0, is connected to the rest of the processors via links. In both models, the load is originated from P_0. Communication links connected to P_j are shown as L_j. The notation used in this paper is summarized in Table I. Here α_i is the optimal fraction of load distributed to processors and links, calculated with divisible load theory using the rest of system parameters, w_i, z_i, T_{cp}, and T_{cm} which are given as constant values. For the scheduling policy for a single level tree network, we consider only the single-installment case, where communication of the load to each processor takes place only once for each processor. We also assume that the originating node also computes the load. It is also assumed that a child can only begin processing its load fraction after it receives it in it's entirety ("staggered start"). Furthermore, each processor is equipped with a front-end processor for off-loading communication,

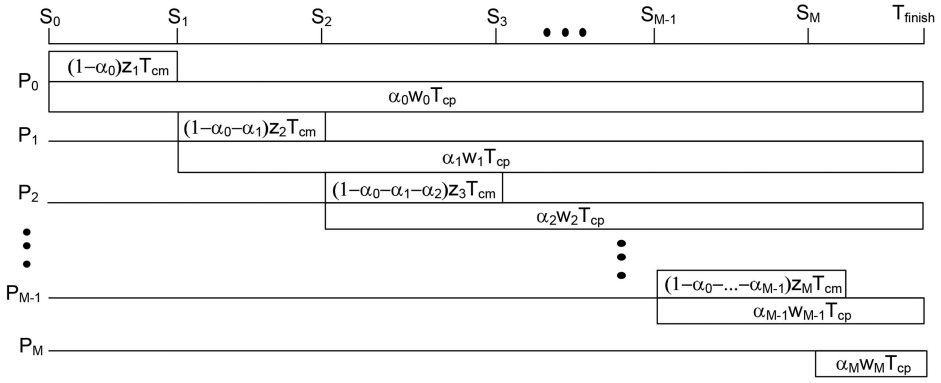

Fig. 3. Timing diagram for load distribution on linear daisy chain network.

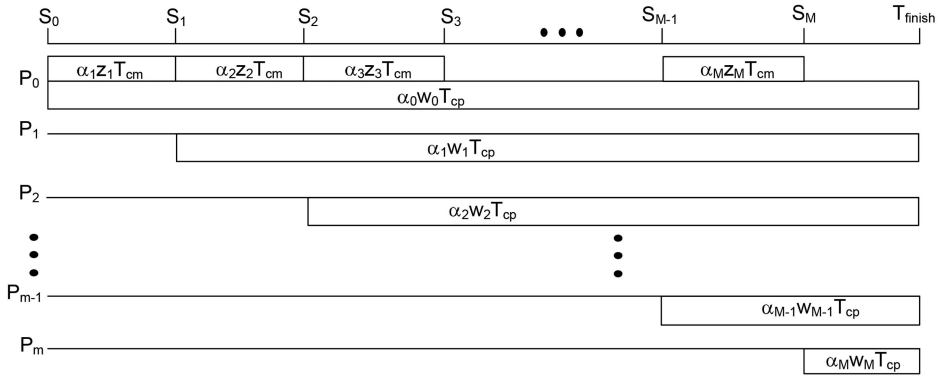

Fig. 4. Timing diagram for load distribution on single level tree network with single installment.

TABLE I
Summary of Notation for Divisible Load Theory

K	The number of signatures in a data file when there are multiple signatures
M	The number of processors besides the load originating processor (root)
B	The total number of bins
α_i	Fraction of entire processing load assigned to ith processor
w_i	Constant that is inversely proportional to the computation speed of the ith processor
z_i	Constant that is inversely proportional to the communication speed of the ith link
T_{cp}	Computation intensity: time taken to process a unit load on the ith processor when $w_i = 1$
T_{cm}	Communication intensity: time taken to communicate a unit load over link l_i when $z_i = 1$
S_i	Processing start time of the ith processor
β^i	Fraction of load that is processed during S_i and S_{i+1}
$\lvert\beta^i\rvert$	The size of β^i
$l(\beta^i), u(\beta^i)$	The lower and the upper boundaries of β_i
α_j^i	The fraction of load distributed to P_i taken from β^i
$\lvert\alpha_i^i\rvert$	The size of α_j^i
$l(\alpha_i^j), u(\alpha_i^j)$	The lower and the upper boundaries of α_i^j
Y	Random variable describing the amount of time to find a single signature contained in a data file
Y_k	Random variable describing the amount of time to find the kth of K signatures in a data file
A_m	Event when kth of K signatures is found in processor m

so that communication and computation can take place concurrently. A closed-from solution for the optimal load distribution fractions (α_i) and an expression for the finish time (makespan) appear in [1] and [5] and are reproduced in the next subsection.

B. Divisible Load Modeling

The solutions for the optimal load fraction to distribute to each processor and the starting time and the finish time of processing the distributed load by each processor is obtained using recursive equations in [5]. Here optimality is defined as the processing of the load in a minimal amount of time for a given interconnection network and given scheduling policy.

Figure 3 and Fig. 4 present timing diagrams of load distribution for a linear daisy chain network model and a single level tree network model, respectively. It is shown that the processing time of the ith processor is given as $\alpha_i w_i T_{cp}$ and the load communication time for the ith link is the amount of load to transfer multiplied by constant factor $z_i T_{cm}$. For example, in Fig. 4 the amount of time to transfer α_1 to P_1 is given as $\alpha_1 z_1 T_{cm}$. As the optimality criterion of divisible load theory states, the computation time has to be finished at the same instant by all processors T_{finish}. This criterion is intuitively reasoned in [5]—while distributing arbitrarily divisible loads, one should keep all the processors utilized until the last moment. If all processors do not stop at the same

TABLE II
Time to Start Searching

Network Model	S_0	S_m $(m = 1,\ldots,M)$
Linear Daisy Chain	0	$\sum_{j=0}^{m-1}\left[\left(1-\sum_{k=0}^{j}\alpha_k\right)\cdot z_{j+1}T_{cm}\right]$
Single Level Tree	0	$\sum_{j=1}^{m}\alpha_j\cdot z_j T_{cm}$

time, certainly the load can be transferred from busy processors to idle processor to improve the solution. A rigorous proof of this optimality criterion for various network models is presented in [5]. With this intuition concerning the nature of an optimal solution, M recursive equations can be written for the two network models of this paper as

$$\alpha_i w_i T_{cp} = \left(1-\sum_{j=0}^{i}\alpha_j\right) z_{i+1}T_{cm} + \alpha_{i+1}w_{i+1}T_{cp}, \quad i = 0,1,\ldots,M-1 \tag{1}$$

for the linear daisy chain network model and

$$\alpha_i w_i T_{cp} = \alpha_{i+1} z_{i+1} T_{cm} + \alpha_{i+1}w_{i+1}T_{cp}, \quad i = 0,1,\ldots,M-1 \tag{2}$$

for the single level tree network model. Each set of recursive equations, along with a normalization equation, form a system of $(M + 1)$ linear equations with $(M + 1)$ unknowns, $\{\alpha_0,\alpha_1,\ldots,\alpha_M\}$. The normalization equation is given as,

$$\sum_{i=0}^{M}\alpha_i = 1 \tag{3}$$

for both network models. The starting time S_i of ith processor P_i is determined by communication time of the load to the previous processors, from P_0 to P_{i-1} and P_i itself. The starting time for each network model is given in [1] and is reproduced in Table II.

III. EXPECTED SIGNATURE SEARCHING TIME FOR UNIFORM DISTRIBUTION

A. Problem Description

We define a set of random variables $\{\mathbf{X}_i\}$ that describes the positions of K signatures in the dataset with the normalized size 1. We assume that the positions of the signatures have a certain distribution,

$$\{\mathbf{X}_i\} \sim \mathbf{F} \tag{4}$$

where $\mathbf{F}$ is joint cumulative density function (cdf) defined on $[0,1]^K$. We denote $\mathbf{Y}_k$ as a random variable describing the amount of time to find the kth signature. The objective of this section of this paper is to find the expectation value of $\mathbf{Y}_k$.

B. Uniform Distribution with Single Installment

Let $\{\mathbf{X}_i\}$ be independent and identically distributed random variables with uniform distribution, $\mathbf{X}_i \sim U(0,1)$, $i \in [1,K]$. The position of the kth signature is given as $\mathbf{X}_{(k)}$, the kth order statistics of $\{\mathbf{X}_i\}$. The $E[\mathbf{Y}_k]$ can be expressed as

$$E[\mathbf{Y}_k] = \sum_{m=0}^{M} E[\mathbf{Y}_k \mid A_m]\Pr(A_m) \tag{5}$$

where A_m denotes the event when the kth signature is found on P_m. For the single installment case, we have

$$\Pr(A_0) = \Pr(0 \le \mathbf{X}_{(k)} \le \alpha_0)$$
$$\Pr(A_m) = \Pr\left(\sum_{j=0}^{m-1}\alpha_j \le \mathbf{X}_{(k)} \le \sum_{j=0}^{m}\alpha_j\right), \quad m \in [1,M]. \tag{6}$$

Given that the kth signature is found on P_m,

$$\mathbf{Y}_k \mid A_m = g_m(\mathbf{X}_{(k)} \mid A_m) = (\mathbf{X}_{(k)} \mid A_m - \sum_{i=1}^{m-1}\alpha_i)w_m T_{cp} + S_m \tag{7}$$

where $g_m(\cdot)$ is the transformation function presented in [1], which takes the position of a signature as an argument and gives the signature search time depending on which processor the signature is found. The search times for the processors are partially overlapping and the start times are generally increasing in signature position in the file. Plotted as a graph search time versus position exhibits a characteristic saw tooth type shape. Here, S_m denotes the starting time of P_m as described in Table II and $(\mathbf{X}_{(k)} \mid A_m - \sum_{i=1}^{m-1}\alpha_i)$ is the offset of the position of the kth signature from the beginning of load fraction distributed to P_m.

The kth order statistics of the uniform distribution follows the well-known beta distribution (see (19)) of applied mathematics,

$$\mathbf{X}_{(k)} \sim \mathbf{B}(k, K+1-k). \tag{8}$$

If we denote $f_{k,K}(x)$ as the probability density function of the kth signature of K signatures, given that $\mathbf{X}_{(k)}$ is on P_m, the conditional distribution of $\mathbf{X}_{(k)}$ is given as

$$\mathbf{X}_{(k)} \mid A_m \sim \frac{f_{k,K}(x)}{\int_{\gamma_{m-1}}^{\gamma_m} f_{k,K}(x)dx} = \frac{f_{k,K}(x)}{I_{\gamma_m}(k) - I_{\gamma_{m-1}}(k)}, \quad \gamma_{m-1} \le \gamma_m \tag{9}$$

where the limits of the integration γ_{m-1} and γ_m are $\sum_{j=0}^{m-1}\alpha_j$ and $\sum_{j=0}^{m}\alpha_j$, respectively. We set $\gamma_{-1} = 0$ for convenience. $I_x(k)$ is the shorthand notation for

$I_x(k, K+1-k)$, which is the cumulative distribution function of the beta distribution, $f_{k,K}(x)$.

$$I_x(k, K+1-k) = \int_0^x f_{k,K}(\zeta)d\zeta. \tag{10}$$

It follows $I_{\gamma_{-1}} = I_0 = 0$. Taking the expectation of (7),

$$E[\mathbf{Y}_k \mid A_m] = \left(E[\mathbf{X}_{(k)} \mid A_m] - \sum_{i=0}^{m-1}\alpha_i\right) w_m T_{cp} + S_m. \tag{11}$$

From (9),

$$E[\mathbf{X}_{(k)} \mid A_m] = \frac{\int_{\gamma_{m-1}}^{\gamma_m} x f_{k,K}(x)dx}{I_{\gamma_m}(k) - I_{\gamma_{m-1}}(k)}. \tag{12}$$

From (6),

$$\begin{aligned}\Pr(A_m) &= \int_{\gamma_{m-1}}^{\gamma_m} f_{k,K}(x)dx \\ &= I_{\gamma_m}(k) - I_{\gamma_{m-1}}(k).\end{aligned} \tag{13}$$

Using these results in (5), we have an expression for the expected time for finding the kth signature out of K signatures:

$$\begin{aligned}E[\mathbf{Y}_k] = \sum_{m=0}^{M}\left(\left(\frac{\int_{\gamma_{m-1}}^{\gamma_m} x f_{k,K}(x)dx}{I_{\gamma_m}(k) - I_{\gamma_{m-1}}(k)} - \gamma_{m-1}\right) w_m T_{cp} + S_m\right) \\ * (I_{\gamma_m}(k) - I_{\gamma_{m-1}}(k)).\end{aligned} \tag{14}$$

Single Signature Case: When there is only one signature, we have $K = k = 1$. We use the identity,

$$I_x(a,b) = \sum_{j=a}^{a+b-1} \frac{(a+b-1)!}{j!(a+b-1-j)!} x^j (1-x)^{a+b-1-j}. \tag{15}$$

When $a = k = 1$ and $b = K+1-k = 1$,

$$I_{\gamma_m}(1,1) = \gamma_m = \sum_{j=1}^{m}\alpha_j. \tag{16}$$

Then from (13) $\Pr(A_m) = I_{\gamma_m}(k) - I_{\gamma_{m-1}}(k) = \sum_{j=0}^{m}\alpha_j - \sum_{j=0}^{m-1}\alpha_j = \alpha_m$. Since $\mathbf{X}_{(1)}$ is uniformly distributed, when $K = 1$, $E[\mathbf{X}_{(1)} \mid A_m] = (\gamma_m + \gamma_{m-1})/2$. Also, $(\gamma_m + \gamma_{m-1})/2 - \gamma_{m-1} = (\gamma_m - \gamma_{m-1})/2 = \alpha_m/2$. Substituting these results into (5),

$$\begin{aligned}E[\mathbf{Y}_1] &= \sum_{m=0}^{M}\alpha_m\left(\frac{\alpha_m w_m T_{cp}}{2} + S_m\right) \\ &= \sum_{m=0}^{M}\alpha_m\left(\frac{\alpha_m w_m T_{cp} + 2S_m}{2}\right) \\ &= \sum_{m=0}^{M}\alpha_m\left(\frac{T_{\text{finish}} + S_m}{2}\right)\end{aligned} \tag{17}$$

where T_{finish} denotes the time when the computation finishes and is given as $T_{\text{finish}} = \alpha_m w_m T_{cp} + S_m$. The expected time of single signature search time is found as the weighted average of midpoints of the processing times of each processor. This special case confirms the result presented in [1].

Time to find the last signature: If the file contains multiple, uniformly distributed, signatures then the distribution of the position of the last signature is given as

$$\mathbf{X}_{(K)} \sim \mathbf{B}(K, 1). \tag{18}$$

The standard form of the probability density function of the beta distribution is given as,

$$f(x;a,b) = \frac{1}{B(a,b)}x^{a-1}(1-x)^{b-1} \tag{19}$$

where $B(a,b)$ is the beta function with the parameters a and b. That is, $B(a,b) = \int_0^1 t^{a-1}(1-t)^{b-1}dt$. When $k = K$, $a = K$, $b = K+1-k = 1$, $f_{K,K}(x) = f(x;K,1) = (1/B(K,1))x^{K-1}$. Also, from (15),

$$I_x(K,1) = x^K. \tag{20}$$

Using these with (12), with m being processor number,

$$\begin{aligned}E[\mathbf{X}_{(K)} \mid A_m] &= \int_{\gamma_{m-1}}^{\gamma_m} x\frac{f(x;K,1)}{\gamma_m^K - \gamma_{m-1}^K}dx \\ &= \int_{\gamma_{m-1}}^{\gamma_m} x\frac{x^{K-1}}{B(K,1)}\frac{1}{\gamma_m^K - \gamma_{m-1}^K}dx \\ &= \int_{\gamma_{m-1}}^{\gamma_m} \frac{x^K}{B(K,1)}\frac{1}{\gamma_m^K - \gamma_{m-1}^K}dx \\ &= \frac{K}{K+1}\frac{\gamma_m^{K+1} - \gamma_{m-1}^{K+1}}{\gamma_m^K - \gamma_{m-1}^K}\end{aligned} \tag{21}$$

where we use $B(K,1) = 1/K$ from $B(a,b) = \int_0^1 t^{a-1}(1-t)^{b-1}dt$. With (5), (11), (13), and (20),

$$\begin{aligned}E[\mathbf{Y}_K] = \sum_{m=0}^{M}\left(\left(\frac{K}{K+1}\frac{\gamma_m^{K+1} - \gamma_{m-1}^{K+1}}{\gamma_m^K - \gamma_{m-1}^K} - \gamma_{m-1}\right) w_m T_{cp} + S_m\right) \\ * (\gamma_m^K - \gamma_{m-1}^K).\end{aligned} \tag{22}$$

In [1] the expected search time of the last signature is derived using the concept of an equivalent processor for the uniformly distributed case. Here, a closed-form solution using the beta distribution is derived.

Figure 5 shows the search time for the last signature when there are K signatures. The dotted line at the top is the finish time when the processors do not stop processing after the last signature is found. It can be seen, as expected, search time can be decreased by adding processors up to a saturation limit. However, as shown in the plot, as more processors are added the expected signature search time is slightly increasing after the search time is saturated (i.e., saturates at four processors). While the search time through parallel processing is not decreased because of the communication overhead,

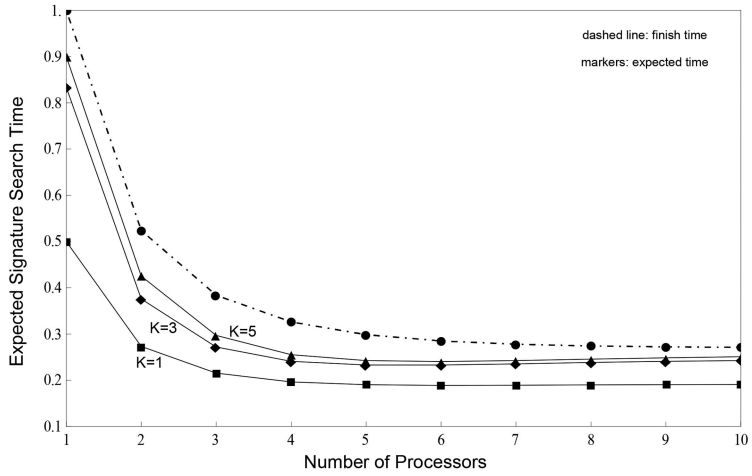

Fig. 5. Linear daisy chain network: time to find last signature versus number of processors, K signatures, $w = 1$, $z = 0.2$, $T_{cp} = 1$, $T_{cm} = 1$.

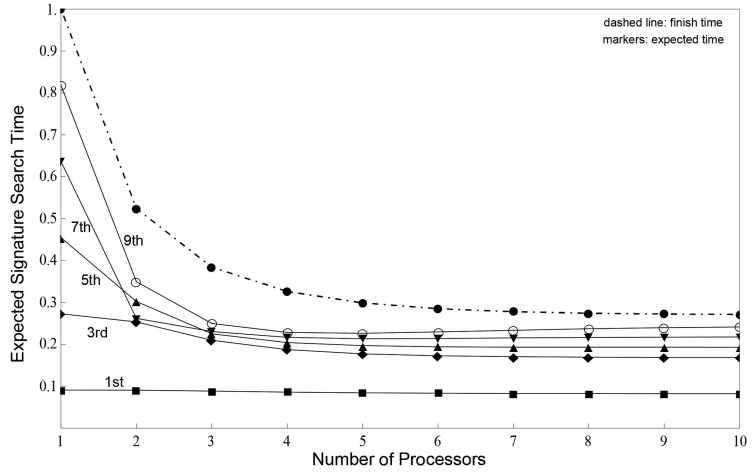

Fig. 6. Linear daisy chain network: time to find kth signature versus number of processors, 10 signatures, $w_i = 1$, $z_i = 0.2$, $T_{cp} = 1$, $T_{cm} = 1$.

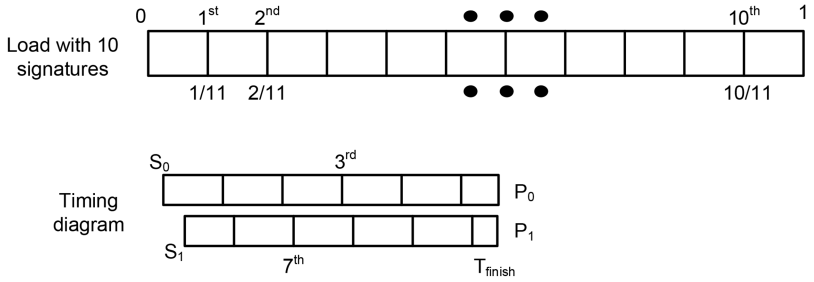

Fig. 7. Load with 10 uniformly distributed signatures and mini timing diagram for two processors, $w = 1$, $z = 0.2$, $T_{cp} = 1$, $T_{cm} = 1$.

when calculating the expected value of the signature search time, the residual probability mass of finding signatures on the processors that are added after the saturation contributes to decreasing the expectation value albeit in a negligible way.

Figure 6 shows the search time for the kth signature when the number of signatures is fixed to $K = 10$. It can be observed that before the speedup saturates, the order of expected times to find the signature changes as shown in the figure.

The reason for this can be explained with Fig. 7. In the figure the upper part describes the normalized load with size 1 with 10 uniformly distributed signatures of those expected locations shown as vertical lines. With the same system parameters for generating the plot for Fig. 6, when there are two processors the load distribution is given as $\{0.52381, 0.47619\}$. The bottom part shows a mini timing diagram when the load is distributed with this proportion and the expected location of signatures.

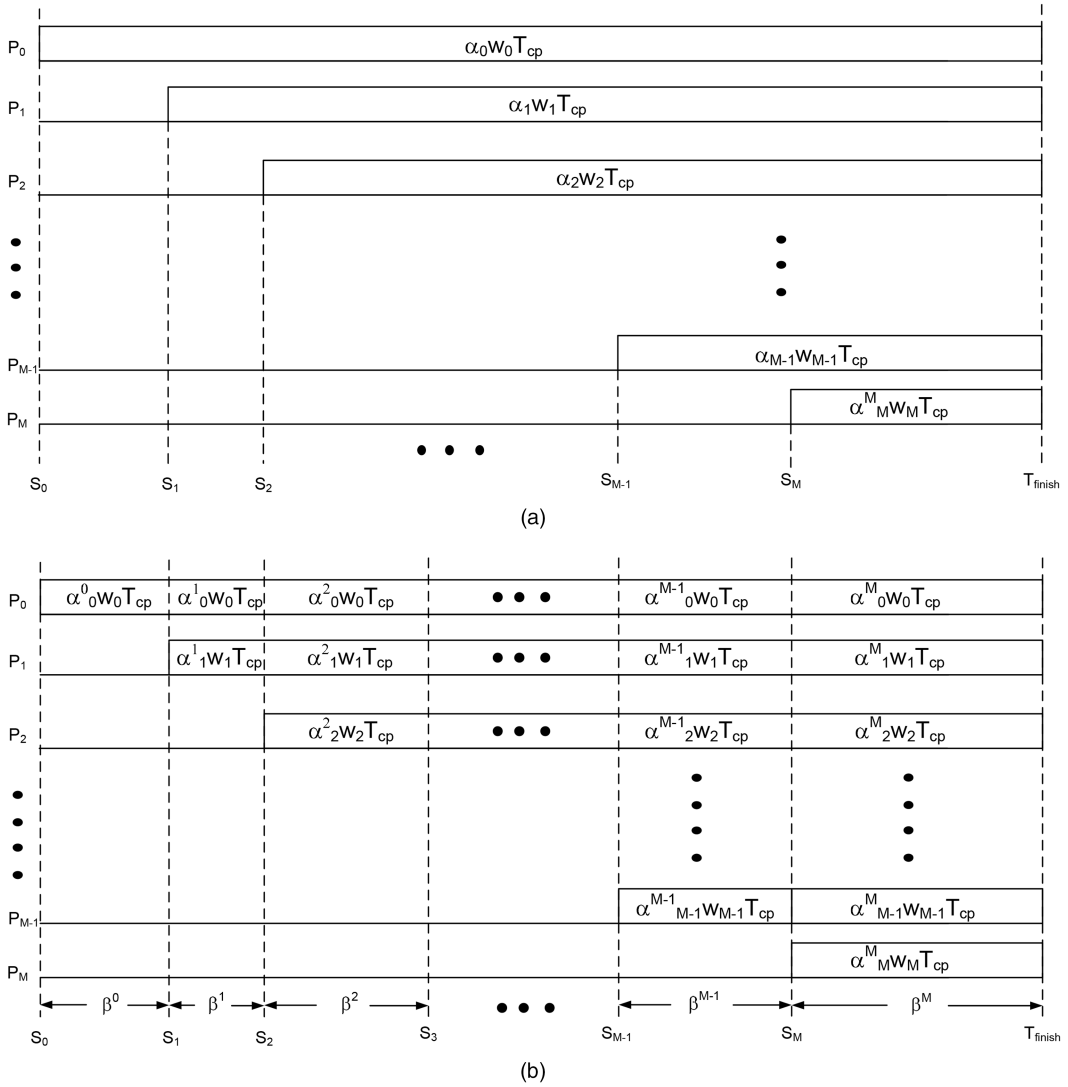

Fig. 8. Comparison of load distribution schemes. (a) Sequential load distribution. (b) Load distribution based on starting time.

As shown in the timing diagram, the expected time to find the 7th signature is smaller than the expected time of the 3rd signature because of parallel processing. Although this diagram is not exact as it uses the expected positions, it explains why the order of expected search time may be switched as shown in Fig. 6. A similar effect is also pointed out in [1].

This all assumes a load is mapped continuously and sequentially into each load fraction for each processor. The signatures are not necessarily detected in the original order they appear in the original flat data file. In the case of signature searching, if the original flat file is indexed by time, there may be some concern that signatures spanning two sides of a load partition boundary are correctly detected. This can be handled by some overlapping of data fragments near partition boundaries. If this can be handled, then the original load could conceivably be multiplexed into each processing node sequentially and repetitively so that signatures are more likely to be detected closer to their original order if that is important.

IV. GREEDY PROCESSING. DISTRIBUTING LOAD

A. Load Classification based on Starting Time

Consider a search of a massive flat file for a single signature. Figure 8(a) shows the timing diagram of the computation time of distributed load for a single level tree network (in fact since the figure only involves computation it is also appropriate for a linear daisy chain network but for this discussion the figure can be thought of as representing a single level tree network). The load is distributed in sequential order as it has been assumed in the literature of divisible load theory, where the position of the fraction has not been considered. In sequential load distribution, once α_i, the fraction for P_i is determined, P_0 gets the first part of load and P_1 receives the next part of load. The ranges of the fractions inside the total load are $[0, \alpha_0]$ and $[\alpha_0, \alpha_0 + \alpha_1]$, respectively, for P_0 and P_1.

However, when the likelihood of finding a signature is not uniform, by greedily computing earlier the fractions of load with a higher likelihood of finding the signatures, the expected time of finding the signature will be shown to be reduced. Figure 8(b) shows the classification of load based on their starting time of computation. Here β^i denotes the fraction of load which is computed between S_i and S_{i+1}. Since β^0 is processed at the earliest time, it should contain the fraction of load that has highest probability of containing signatures. The fraction β^1 having the highest probability excluding β^0 is processed between S_1 and S_2. Since both P_0 and P_1 process their fraction during that time period, β^1 is separated into two fractions, α_0^1 and α_1^1, where α_j^i denotes the fraction of load distributed to P_j from β^i. We assume that β_i is from one contiguous region of load and later this assumption will be relaxed. Note that β_i is divided into $i+1$ fractions because there are $i+1$ processors computing their fraction between S_i and S_{i+1}. Also, each processor P_j receives the load from $M-j$ different β^i. Note that there are events $S_0, S_1, \ldots, S_M, T_{\text{finish}}$. Thus there are $(M+1)\beta_i$, hence each processor P_j receives load from $M+1-j$ different β_i. For example, P_0 computes its fraction from S_0 to T_{finish} so it receives the fractions from all β^i.

In the divisible load theory literature, α_i usually means the size of the fraction of the load and its position in load is not considered. Since β_i and α_j^i are from different parts of total load, their position needs to be taken account as well as their size.

For the size of β_i and α_j^i, we use $|\beta_i|$ and $|\alpha_j^i|$ and $l(\cdot)$ and $u(\cdot)$ for the boundaries of the range of fractions. For notational consistency, we also use $|\alpha_i|$ to denote the size of fraction of load distributed using the usual sequential distribution although α_i means the same quantity. For summary, our notation β_i and α_j^i consist of pairs of values, $(|\beta_i|, l(\beta_i))$ and $(|\alpha_j^i|, l(\alpha_j^i))$, respectively, and $u(\cdot) = l(\cdot) + |\cdot|$.

The values of $\{|\alpha_j^i|\}$ and $|\beta^i|$ can be obtained from $|\alpha_i|$ which can be obtained from well-known solutions of the literature of divisible load theory and can be calculated using recursive equations presented in Section II.

From $\{|\alpha_i|\}$, $\{|\alpha_j^i|\}$ is calculated with the following equations

$$|\alpha_j| = \sum_{i=j}^{M} |\alpha_j^i|, \qquad j = 0, \ldots, M \tag{23}$$

with a constraint,

$$|\alpha_0^i| w_0 T_{cp} = \cdots = |\alpha_j^i| w_i T_{cp}, \qquad i = 1, \ldots, M, \quad j \le i \tag{24}$$

and

$$|\beta^0| = |\alpha_0^0| \quad |\beta^i| = \sum_{j=1}^{i} |\alpha_j^i|, \qquad i = 1, \ldots, M. \tag{25}$$

From (23),

$$\begin{aligned}
\begin{pmatrix} |\alpha_0| \\ |\alpha_1| \\ \vdots \\ |\alpha_{M-1}| \\ |\alpha_M| \end{pmatrix}
&= \begin{pmatrix} |\alpha_0^0| & |\alpha_0^1| & \cdots & |\alpha_0^{M-1}| & |\alpha_0^M| \\ & |\alpha_1^1| & \cdots & |\alpha_1^{M-1}| & |\alpha_1^M| \\ & & \vdots & \vdots & \vdots \\ & & & |\alpha_{M-1}^{M-1}| & |\alpha_{M-1}^M| \\ & & & & |\alpha_M^M| \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \\ 1 \end{pmatrix} \\
&= \begin{pmatrix} |\alpha_0^0| & |\alpha_0^1| & \cdots & |\alpha_0^{M-1}| & |\alpha_0^M| \\ & \frac{w_0}{w_1}|\alpha_0^1| & \cdots & \frac{w_0}{w_{M-1}}|\alpha_0^{M-1}| & \frac{w_0}{w_M}|\alpha_0^M| \\ & & \vdots & \vdots & \vdots \\ & & & \frac{w_0}{w_{M-1}}|\alpha_0^{M-1}| & \frac{w_0}{w_M}|\alpha_0^M| \\ & & & & \frac{w_0}{w_M}|\alpha_0^M| \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix} \\
&= \begin{pmatrix} 1 & 1 & \cdots & 1 & 1 \\ & \frac{w_0}{w_1} & \cdots & \frac{w_0}{w_{M-1}} & \frac{w_0}{w_M} \\ & & \vdots & \vdots & \vdots \\ & & & \frac{w_0}{w_{M-1}} & \frac{w_0}{w_M} \\ & & & & \frac{w_0}{w_M} \end{pmatrix} \begin{pmatrix} |\alpha_0^0| \\ |\alpha_0^1| \\ \vdots \\ |\alpha_0^M| \end{pmatrix}.
\end{aligned} \tag{26}$$

Here, the second equality is from (24).

Taking the inverse of the $(M+1)*(M+1)$ matrix,

$$\begin{pmatrix} |\alpha_0^0| \\ |\alpha_0^1| \\ \vdots \\ |\alpha_0^M| \end{pmatrix} = \begin{pmatrix} 1 & 1 & \cdots & 1 & 1 \\ & \frac{w_0}{w_1} & \cdots & \frac{w_0}{w_{M-1}} & \frac{w_0}{w_M} \\ & & \vdots & \vdots & \vdots \\ & & & \frac{w_0}{w_{M-1}} & \frac{w_0}{w_M} \\ & & & & \frac{w_0}{w_M} \end{pmatrix}^{-1} \begin{pmatrix} |\alpha_0| \\ |\alpha_1| \\ \vdots \\ |\alpha_M| \end{pmatrix}. \tag{27}$$

The matrix form is given in (27) for exposition purposes. The αs may be solved more efficiently using standard divisible load theory algebraic recursions in the αs. Once $\{|\alpha_0^0|, |\alpha_0^1|, \ldots, |\alpha_0^m|\}$ is found, other $\{|\alpha_j^i|\}, j \le i$ and $|\beta^i|$ follow from (24) and (25).

$$|\alpha_j^i| = |\alpha_0^i| \frac{w_0}{w_j}. \tag{28}$$

The values of the β^i can be determined from (25).

When the probability distribution function for the location of signatures is monotonically increasing

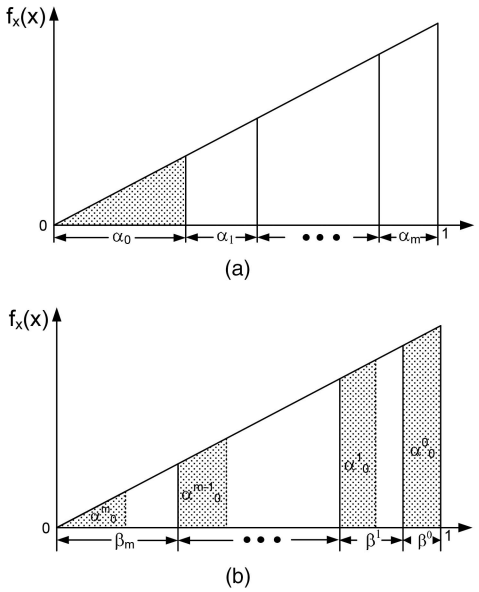

Fig. 9. Comparison between load distribution schemes. (a) Sequential distribution. (b) Distribution based on probability mass.

or decreasing as shown in Fig. 9, for all β_i, one contiguous fraction suffices to give the optimal range. In the next subsection, we give a motivational example with the probability distribution shown in the figure and discuss the case of arbitrary shapes of distributions in the following section.

B. Motivational Example

In this example, a linearly increasing probability density for the location of a single signature is considered. We describe a more efficient processing of the load based on the shape. Consider Fig. 9. It shows that the probability mass of the position of the signature linearly increases toward the end of the load. In the usual sequential load distribution, the load is partitioned in sequence as shown in Fig. 9(a). The load distributed to α_0 is chosen from the beginning of the whole load of which computation begins at S_0. Fig. 9(b) is a distribution scheme based on the timing diagram of Fig. 8(b), where the fraction of the load containing the larger mass of probability of finding the signature is distributed to P_0 and processed between S_0 and S_1, during the earliest time. The grayed area of the figures identify the portion of load distributed to P_0. In the figure, multiple fractions of load contribute to α_0, the load distributed to P_0. Note that, as probability mass monotonically increases, the range of β^i is continuous and each β^i, $i > 0$ is partitioned for multiple processors.

The probability distribution function of the signature position shown in the figure is given as

$$f_x(x) = 2x, \qquad 0 \le x \le 1. \tag{29}$$

For a linear daisy chain network with 5 processors, the $\{|\alpha_m|\}$ and $\{S_m\}$ is calculated using recursive equations presented in Section II and Table II. The calculated values are shown in Table III.

TABLE III
α_m and S_m for Linear Daisy Chain Network with $w = 1$, $z = 0.1$, $T_{cm} = T_{cp} = 1$

m	0	1	2	3	4
$\lvert\alpha_m\rvert$	0.299	0.229	0.181	0.152	0.139
S_m	0	0.0701	0.117	0.146	0.160

TABLE IV
α_j^i, Linear Daisy Chain Network with $w = 1$, $z = 0.1$, $T_{cm} = T_{cp} = 1$

i	0	1	2	3	4
$\lvert\alpha_0^i\rvert$	0.0701154	0.0472423	0.0290935	0.0138541	0.138541
$\lvert\alpha_1^i\rvert$		0.0472423	0.0290935	0.0138541	0.138541
$\lvert\alpha_2^i\rvert$			0.0290935	0.0138541	0.138541
$\lvert\alpha_3^i\rvert$				0.0138541	0.138541
$\lvert\alpha_4^i\rvert$					0.138541
$\lvert\beta^i\rvert$	0.0701154	0.0944847	0.0872806	0.0554162	0.692703

In Table IV, the size of the fraction of the load distributed to P_j from β^i, $|\alpha_j^i|$ is calculated using (27) and (28). Although the sizes are determined for α_j^i and β_i, with the aid of the divisible load theory solution, the range of the fractions of load still needs to be determined. In Fig. 9(b) the shaded area indicates the fractions of load that need to be distributed to P_0. Here, the loads distributed to P_0 are from multiple parts of the load based on their their probability mass. Now α_0^0 which is processed between S_0 and S_1 is taken from the part of the load with highest probability mass. The fraction of load processed during the period between S_M and T_{finish}, β_M, is taken from the beginning part (leftmost area in Fig. 9) of the original load because of its small probability mass. Since all the processors process the fraction the load during that period, β_M is partitioned and distributed to all of the processors. As the ranges of fractions are dispersed in the load, the position of each fraction needs to be obtained. To begin, the range of β_i needs to be obtained and then the positions of all α_j^i for $j = 1 \ldots i$ are calculated.

In this example with a monotonically increasing distribution, the range of β_i can be obtained in a straightforward manner with known $|\beta_i|$ as we can take β_i from the end of the load toward the beginning sequentially as shown in Fig. 9(b) and the lower boundaries of β_j^i are calculated and shown in Table V. The table shows the lower boundary of β_i, but the upper boundary can be easily derived with $|\beta_i|$: $u(\beta_i) = l(\beta_i) + |\beta_i|$, where $u(\cdot)$ denotes the upper boundary of the load fraction.

The lower boundary of $\{\alpha_m^i\}$ is shown in Table VI. Inside the range of β_i, we partition the β_i in the order of the processor index for simplicity. Therefore, $l(\alpha_0^1) = l(\beta_1)$ and $l(\alpha_1^1) = l(\beta_1) + |\alpha_0^1|$ and other lower

TABLE V
Lower Boundary of β^i with Linear Daisy Chain Network with $w = 1$, $z = 0.1$, $T_{cm} = T_{cp} = 1$, $u(\beta^i) = l(\beta^i) + |\beta^i|$

$l(\beta^4)$	$l(\beta^3)$	$l(\beta^2)$	$l(\beta^1)$	$l(\beta^0)$
0	0.692703	0.748119	0.8354	0.929885

TABLE VI
Lower Boundary of α_m^i, $l(\alpha_m^i)$ with Linear Daisy Chain Network with $w = 1$, $z = 0.1$, $T_{cm} = T_{cp} = 1$, $u(\alpha_m^i) = l(\alpha_m^i) + |\alpha_m^i|$

i	0	1	2	3	4
$l(\alpha_0^i)$	0.929885	0.8354	0.748119	0.692703	0
$l(\alpha_1^i)$		0.882642	0.777213	0.706557	0.138541
$l(\alpha_2^i)$			0.806306	0.720411	0.277082
$l(\alpha_3^i)$				0.734265	0.415623
$l(\alpha_4^i)$					0.554164

boundaries are obtained similarly. Other ways of partitioning β^i are possible but are not considered in this paper.

In order to find the expected signature search time for this distribution scheme, (5) can be rewritten as

$$E[\mathbf{Y}] = \sum_{m=1\ldots M, j\le m} E[\mathbf{Y} \mid A_m^j]\Pr(A_m^j) \tag{30}$$

where A_m^i denotes an event when the signature is found in α_m^i. Similarly to (7),

$$\begin{aligned}\mathbf{Y} \mid A_m^i &= g_m((\mathbf{X}) \mid A_m^i)\\ &= (\mathbf{X} \mid A_m^i - l(\alpha_m^i))w_m T_{cp} + S_m\end{aligned} \tag{31}$$

where $l(\alpha_m^i)$ denotes the starting position of α_m^i in load.

Taking the expectation of $\mathbf{Y} \mid A_m^i$ gives

$$E[\mathbf{Y} \mid A_m^i] = (E[\mathbf{X} \mid A_m^i] - l(\alpha_m^i))w_m T_{cp} + S_m. \tag{32}$$

The conditional expectation of the location of the signature given that it is distributed to P_m and from β^i, is

$$E[\mathbf{X} \mid A_m^i] = \frac{\int_{l(\alpha_m^i)}^{u(\alpha_m^i)} x f_x(\zeta)d\zeta}{\int_{l(\alpha_m^i)}^{u(\alpha_m^i)} f_x(\zeta)d\zeta}. \tag{33}$$

Also, the probability of the event A_m^i is

$$\Pr(A_m^i) = \int_{l(\alpha_m^i)}^{u(\alpha_m^i)} f_x(\zeta)d\zeta. \tag{34}$$

Finally, substituting the above equations into (30),

$$\begin{aligned}E[\mathbf{Y}] = \sum_{(m,i)} &\left(\left(\frac{\int_{l(\alpha_m^i)}^{u(\alpha_m^i)} x f_x(\zeta)d\zeta}{\int_{l(\alpha_m^i)}^{u(\alpha_m^i)} f_x(\zeta)d\zeta} - l(\alpha_m^i)\right) w_m T_{cp} + S_m\right)\\ &* \int_{l(\alpha_m^i)}^{u(\alpha_m^i)} f_x(\zeta)d\zeta.\end{aligned} \tag{35}$$

When probability distribution is given as (29), we have,

$$\begin{aligned}\int_{l(\alpha_m^i)}^{u(\alpha_m^i)} x f_x(\zeta)d\zeta &= \int_{l(\alpha_m^i)}^{l(\alpha_m^i)+|\alpha_m^i|} 2x^2 dx\\ &= \frac{6|\alpha_m^i|^2 l(\alpha_m^i) + 6|\alpha_m^i| l^2(\alpha_m^i) + 2|\alpha_m^i|^3}{3}\end{aligned} \tag{36}$$

and

$$\begin{aligned}\int_{l(\alpha_m^i)}^{u(\alpha_m^i)} f_x(x)dx &= \int_{l(\alpha_m^i)}^{l(\alpha_m^i)+|\alpha_m^i|} 2x dx\\ &= 2|\alpha_m^i| l(\alpha_m^i) + |\alpha_m^i|^2.\end{aligned} \tag{37}$$

By using the calculated values from the above tables, we plot the expected search time using the usual sequential distribution in the divisible load theory literature and when the load is classified by probability mass in Fig. 10. As shown in the figure, when the load is classified by the probability mass and scheduled with consideration of the starting time of each processor, the expected time to find signatures is faster. The finish time of computation is shown in the plot for comparison.

Similar curves for a linear daisy chain of processors as in Fig. 10 could be produced using (23)–(37).

V. GREEDY PROCESSING. ARBITRARY DENSITIES

A. Greedy Load Rearrangement Procedure

In this section we present a greedy procedure to distribute load based on the distribution of a single signature as introduced in the previous section. We assume that the probability distribution of the location of a single signature is known. In this approach the load is rearranged first in order of the likelihood of finding the signature. Figure 11 describes the procedure. As shown in Fig. 11(a) and Fig. 11(b), the load is sliced into equal sized bins. Practically, bins can be rearranged by a server that feeds load to a number of parallel processors according to an a priori known pattern (distribution) of signature location(s). The choice of bin size should take into account the number of processors available, the desired solution time and the saturating nature of the search time curves. These bins are arranged in decreasing order of probability mass (see Fig. 11(c)). A further approximation step (Fig. 11(d)) is described in the next subsection. Note that once we get as far as the steps of Fig. 11(c) and 11(d) then the results of the previous section apply.

To be specific,we denote B the total number of bins and, therefore, the normalized size of each bin is $1/B$. In the next step, shown in Fig. 11(b), the probability mass of each bin is calculated as

$$F_n = \int_{b_{n-1}}^{b_n} f_x(x)dx \tag{38}$$

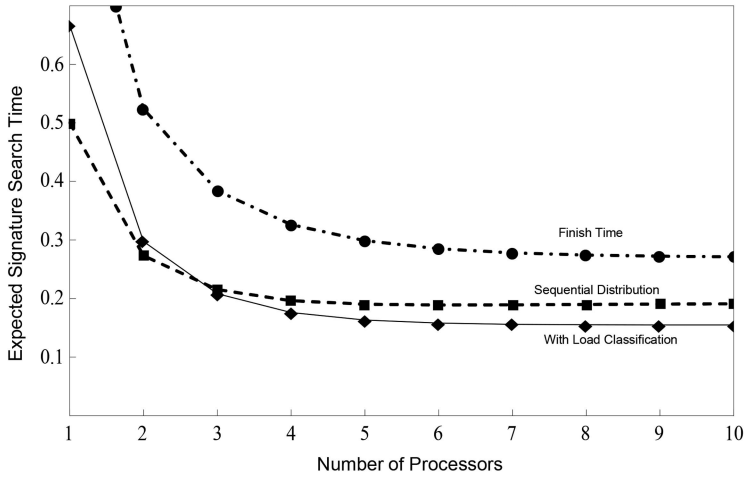

Fig. 10. Comparison of expected time to find single signature between sequential distribution and distribution based on load classification based on the probability distribution on single level tree network, $w_i = 1$, $z_i = 0.2$, $T_{cp} = 1$, $T_{cm} = 1$.

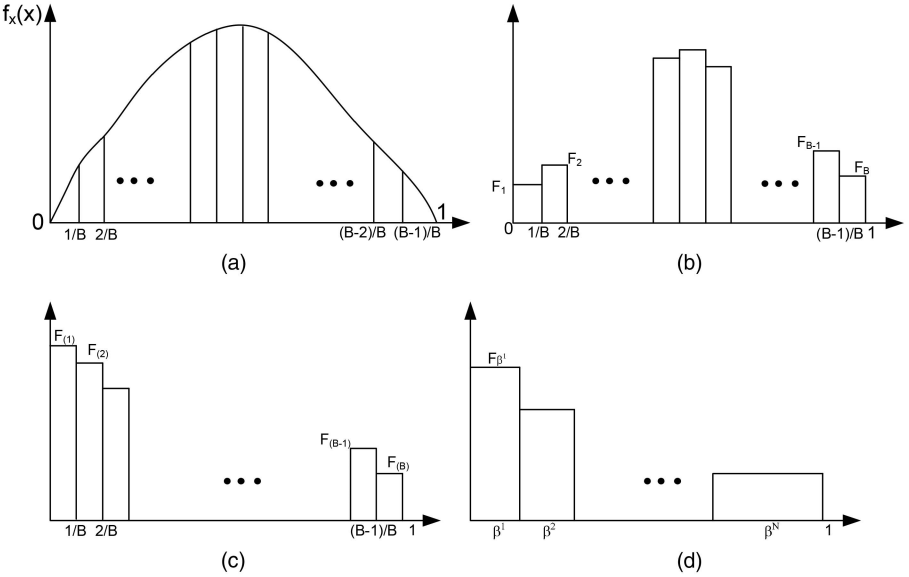

Fig. 11. Greedy rearrangement of loads based on ranking. (a) Sequential load distribution. (b) Load partitioned into B bins. (c) Bins arranged in order of contained probability mass. (d) Probability mass approximation for β_i.

where F_n denotes the probability mass of the nth bin and $b_n = n * 1/B$. Once the mass of each bin is calculated the load is sliced and rearranged in the order of the decreasing magnitude of the probability mass as shown in Fig. 11(c). We call this the sorted load.

Finally, the load is distributed according to precalculated $\{\beta^i\}$. Once the load is sorted according to the probability mass in the previous step, the range and the size of $\{\beta_i\}$ does not vary for a given network model with the same system parameters independent from the probability distribution of signatures. For example, the range and size of $\{\beta^i\}$ can be calculated for the linear daisy network with $w = 1$, $z = 0.1$, and $T_{cm} = T_{cp} = 1$ as shown in Table VII. The sizes of β_i are obtained using (23), (24), and (25). The range of β_i is taken from the beginning of the sorted load. Therefore, the upper boundaries shown in Table VII are the values in the sorted load, not in the original load.

TABLE VII
$|\beta^i|$ and Upper Boundaries of β^i with Linear Daisy Chain Network for Scaled Normal Distribution with $w = 1$, $z = 0.1$, $T_{cm} = T_{cp} = 1$

	β^0	β^1	β^2	β^3	β^4
$\lvert\beta^i\rvert$	0.0701154	0.0944847	0.0872806	0.0554162	0.692703
$u(\beta^i)$	0.0701154	0.1646	0.251881	0.307297	1

B. Approximation of Expected Signature Search Time

Figure 11(d) illustrates the approximated probability mass assigned to the portion of load with the same ranking. Since the exact distribution for a portion of the load with the same ranking is not

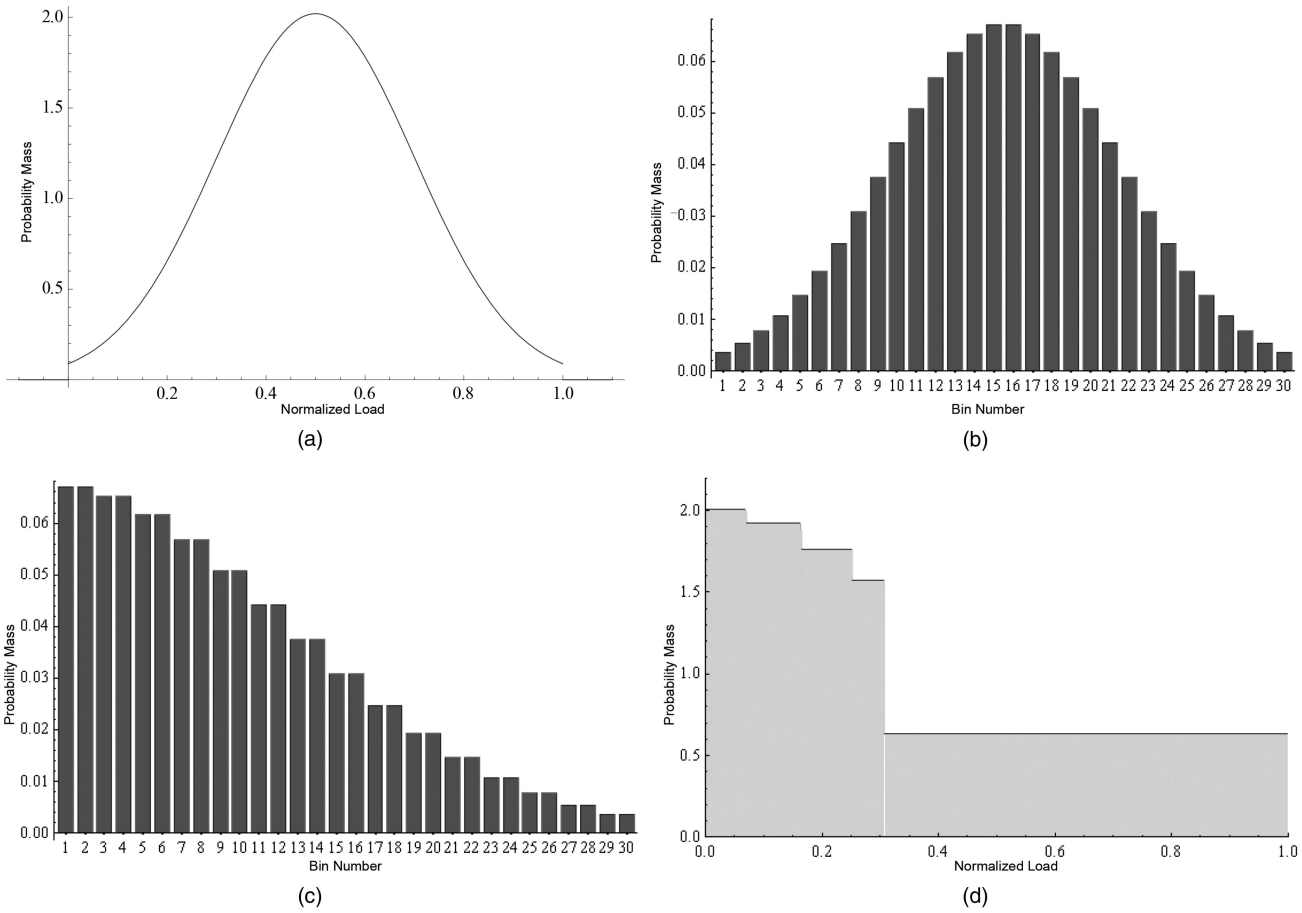

Fig. 12. Greedy rearrangement of loads based on ranking for truncated normal distribution. (a) Scaled normal distribution. (b) Partitioned load with B bins. (c) Rearranged bins based in order of their probability mass. (d) Approximated probability mass $\hat{F}$.

specified, the actual calculation of the expected search time can only be approximated. The approximated value of the expected time of signature search time is given as

$$\hat{E}[X] = \sum_{i=1}^{N} \frac{S_{i-1} + S_i}{2} * \hat{F}_{\beta^i} \tag{39}$$

where $\hat{F}_i$ denotes the approximated probability mass calculated for $\{\beta^i\}$.

$$\hat{F}_{\beta^i} = F_{(k-1)} \frac{(b_k - l(\beta^i))}{1/B} + \sum_{j=k}^{p} F_{(k)} + F_{(p+1)} \frac{(u(\beta^i) - b_p)}{1/B}. \tag{40}$$

Here $b_{k-1} \le l(\beta^i) \le b_k$ and $b_p < u(\beta^i) \le b_{p+1}$. Here the bs are the integration limits of (38). Here $F_{(k)}$ is the probability mass of the kth bin after sorting.

Intuitively, this equation shows that the approximated expected signature search time is the weighted sum of the mid points of processing time of $\{\beta_i\}$ with their approximated probability masses.

VI. EVALUATION AND ANALYSIS

As an example, assume that the distribution of the location of signature time is given as a truncated normal distribution on the range $[0,1]$ centered at the 1/2. The distribution can be written as

$$f(x;\mu,\sigma,a,b) = \frac{\frac{1}{\sigma}\phi\left(\frac{x-\mu}{\sigma}\right)}{\Phi\left(\frac{b-\mu}{\sigma}\right) - \Phi\left(\frac{a-\mu}{\sigma}\right)}. \tag{41}$$

Here $\phi(\cdot)$ is the probability density function of the standard normal distribution, and $\Phi(\cdot)$ is its cumulative distribution function. Also, a and b denote the limits of the range of truncation, and μ denotes the average and σ is the standard deviation value. With $a = 0$, $b = 1$ and $\mu = 1/2$,

$$f(x;1/2,\sigma,0,1) = \frac{\frac{1}{\sigma}\phi\left(\frac{x-1/2}{\sigma}\right)}{\Phi\left(\frac{1}{2\sigma}\right) - \Phi\left(-\frac{1}{2\sigma}\right)}. \tag{42}$$

This distribution is shown in Fig. 12(a). Following the procedure presented in the previous section, the load is sliced into bins as shown in Fig. 12(b) and rearranged based on the probability mass of the bins as shown in Fig. 12(c). The figures are based on 30 bins being used. Finally, the approximated probability mass of β^i is plotted in Fig. 12(d).

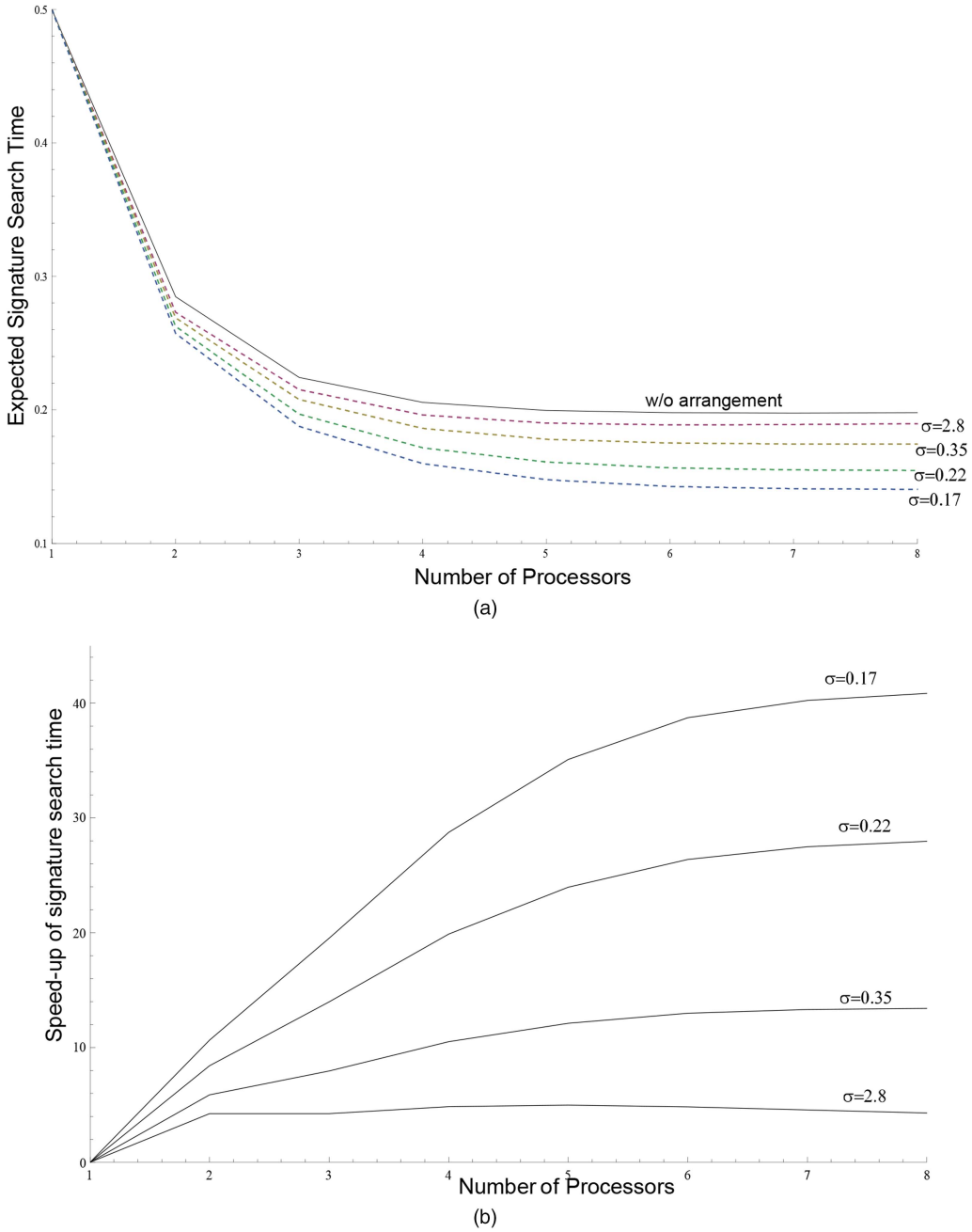

Fig. 13. Performance improvement (%) through greedy load rearrangement for various σ. (a) Expected signature search time. (b) Speedup of signature search time.

Figure 13(a) shows the expected signature search time with and without load arrangement procedure with various standard deviations, σ. The figure was computed using (39) and (40) and the definition of speedup. Figure 13(b) shows the performance improvement in terms of percentage, where the speedup is defined as the ratio of signature search time on one processor to signature search time on N processors. Naturally speedup is greater than one but is often less then N because of inefficiencies in parallel processing. "Speedup" is a common parallel processing advantage performance metric. As shown in the figures, as the probability of finding a signature is concentrated in a smaller area (lower σ value) the performance gain increases.

VII. CONCLUSION AND FUTURE WORK

In the first part of this work, closed-form solutions of the expected search time of the kth signature of K signatures, with a uniform distribution of signature locations, are derived. In the latter part, with a prior knowledge of the signature distribution, it is shown that the expected time of finding a single signature can be improved and a greedy procedure to speedup the signature search time and a simulation result are

presented. With highly concentrated distributions, the improvement in the speed of finding a signature is shown to be significant. As extensions of this work one can examine how database operations can be mapped to divisible load scheduling and investigate how a knowledge of input data can be used to create distribution strategies that improve performance.

The trend in data processing is the use of parallel processing to speed execution even to the level of a single machine (i.e., multicore architectures). Thus for radar, sensor, and image data processing, the use of divisible load theory leads to a better quantitative understanding of processing performance and processing options. Thus the result in this paper should be of interest for quite some time.

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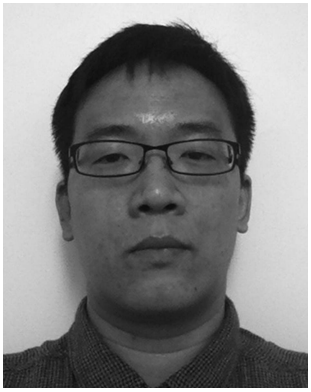

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