This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TVT.2015.2458015, IEEE Transactions on Vehicular Technology

Maximizing Mobile Coverage via Optimal Deployment of Base Station and Relays

Xu Li, Dongning Guo, John Grosspietsch, Huarui Yin, Guo Wei

***Abstract*—Deploying relays and/or base stations is a major means of extending the coverage of a wireless network. This paper presents models, analytical results, and algorithms to answer two related questions: The first is where to deploy relays in order to extend the reach from a base station to the maximum; the second is where to deploy a base station and how many relays are needed to reach any point in a given area. An important use case of the results is the Public Safety Broadband Network, in which deploying relays and base stations is often crucial to provide coverage to an incident scene.**

***Index Terms*—Maximum reach, mobile coverage, outage probability, Public Safety Broadband Network, relay.**

I. INTRODUCTION

Even with today's seemingly ubiquitous wireless access, many areas and corners are not fully covered by existing networks. There is often no cellular connection in the basement level of large buildings and in remote unpopulated areas. The existing cellular infrastructure may also be knocked out of service for periods of times in areas hit by disasters [1]–[5]. To extend the coverage of the Public Safety Broadband Network[1] with manageable cost, we have proposed that relays carried by smart drones or base stations carried by vehicles are sent to incident scenes in [6]–[9]. Generally, the Public Safety Broadband Network is dedicated for the public safety organizations such as the police and fire departments. If a cellular network is used as the platform of the Public Safety Broadband Network, relays could be deployed to extend the wireless coverage [10], [11]. If the cellular network is not sufficient or out of service, a base station carried by vehicles and relays could form an isolated or connected network to provide wireless coverage, where data may be processed locally. An important question is how to optimally deploy relays and the base station.

According to the deployment of relays, most literatures optimized the quality of service (QoS) when the distance between the source and the destination is known [12]–[15]. But for some circumstances, the location of the destination is unknown. Hence in this paper, the distance between the source and the destination is maximized subject to the QoS requirement. According to the deployment of the base station, to the best of our knowledge, this paper is the first to consider the circumstances with placement restrictions, such as not inside a building on fire.

The base station, relays and the destination form a multi-hop relay network to extend the wireless coverage [16]–[19]. Most literatures have studied the conditions where the communication range of all hops are independent. When each guard can monitor unlimited range but has no vision through the wall, reference [20] studied the deployment of the guards to cover an art gallery with n walls, i.e., an n-vertex polygon which is non-convex in general. When each sensor can monitor an arbitrary limited range, the optimal deployment patterns for full coverage and k-connectivity were studied in [21]–[23]. The numerical deployment algorithms with a minimum number of sensors to provide full coverage were discussed in [24]–[29]. When the transmit range of each device is limited by the allocated energy, average traffic data and lifetime requirement, the optimal positions of the base station and the relays for maximizing the system lifetime were considered in [30]–[32].

In a practical multi-hop relay network, the communication range of all hops are interrelated and mutually determined by the resource allocation scheme and the QoS requirement. In [33], numerical simulations were shown to evaluate the QoS performance with fixed relays, where the position of the relays may not be optimal. In [34], [35], the optimal power allocation with fixed relays were discussed for maximizing the data rate. With fixed distance between the source and destination, the optimal relays positions were analyzed for maximizing the data rate in [12] or minimizing the outage probability in [13]–[15]. With an outage probability requirement, the optimal position of a single relay was studied for maximizing the distance between the source and destination in [36]. However, the time resource was uniformly allocated to each hop. In this paper, we jointly optimize the relays deployment and resource allocation for maximizing the distance between the source and destination subject to an outage probability or a data rate requirement.

The remainder of this paper is organized as follows. Section II introduces the network models and channel model.

Section III studies the deployment of relays. Specifically, the relays are deployed between the base station and the destination. We analytically determine the optimal positions

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This work was supported in part by Motorola Center for Seamless Communications at Northwestern University, the National Science Foundation under Grant No. ECCS-1231828, the National Basic Research Program of China (973 Program) under Grant No. 2012CB316004, and the National Science Foundation of China under Grant No. 61171112.

[1]The Public Safety Broadband Network, conceived to be a single, connected, universal network for all public safety purposes, is currently being planned and tested in the U.S.

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of the relays, so that the reach to the destination is maximized subject to the outage probability or data rate requirement. To satisfy the outage probability requirement, the maximum reach to the destination increases with the number of relays. To satisfy the data rate requirement, the maximum reach to the destination increases with the number of relays. But beyond a certain number, deploying more relays does not provide further improvement (in fact, it decreases the maximum reach if the same data rate needs to be maintained).

Section IV studies the deployment of a base station. Specifically, the base station is deployed to cover an arbitrary polygon, which may or may not be convex. Both the general case where the base station can be located anywhere and the situation where the base station is constrained to be outside or on the boundary of the polygon are considered. We propose efficient algorithms to compute the optimal position of the base station, so that the minimum signal-to-noise ratio (SNR) of any point over the entire region is maximized. If the polygon can not be covered by the base station alone, relays are deployed to extend the coverage. The goal here is limited to reaching any point in the region, rather than covering the entire region at the same time.

Section V shows numerical results. Concluding remarks are given in Section VI.

II. Network Models

A. Network Model with Relays

Deploying relays is a major means of extending the coverage of a wireless network. An important question to consider is how far the coverage could be extended, especially for the cases when the location of the destination is unknown. For example, when an incident happens in a tunnel, the fireman does not known where the victims are and he needs to search as far as possible to save life and property.

The network model with relays is shown in Fig. 1, where $K-1$ relays are deployed between the base station and the destination. To extend the coverage of the base station to the maximum, relays are located along the line segment that connects the base station and the destination. In practice, a first responder may carry several small relay devices and deploy one relay at a time to keep in contact with the base as the responder probes the area. It is conceived that one or two relays should be sufficient under most circumstances. As far as the maximum reach is concerned, it is reasonable to assume that the relays are deployed on the line segment connecting the base station and the destination. If multiple responders deploy relays, the resource is assumed to be orthogonalized so that it suffices to study one user.[2] The $K+1$ devices are connected by K segments and the length of the k-th segment is d_k in meters for $k \in \{1, \dots, K\}$. The transmit power of the k-th forward and backward hop are p_k and q_k. The total bandwidth of available frequency resource is W Hz.

The resource is shared using time division scheme, where every hop uses the entire frequency band and all hops are orthogonal in time. This scheme is simple and guarantees the robustness much needed by the relay networks. Understanding of this simple scheme is also the first step toward more sophisticated solutions involving spectrum sharing.

[2]Sharing of relays is relegated to future work.

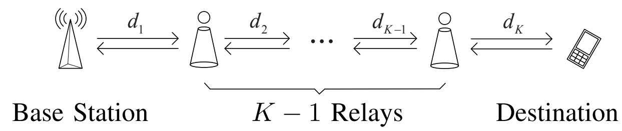

Fig. 1. Network model with relays.

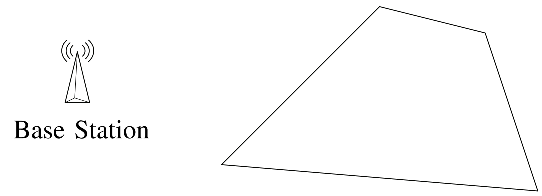

Fig. 2. Network model with a base station.

We study where the relays should be deployed, so that the reach to the destination is maximized subject to the outage probability or data rate requirement.

B. Network Model with a Base Station

In order to provide coverage for a target area, such as a building on fire, deploying base station is another option to provide seamless coverage for saving life and property. In some places, we still need the base station to be rapidly deployed, so that the base station could be carried by vehicles. The shape of most buildings are well modeled as polygons, and an area that appears to be irregular can be easily covered by an approximate polygon. In this paper, the target area is assumed to be an arbitrary polygon, which may or may not be convex. The position of the base station significantly influences the coverage which should be studied.

The network model with a base station is shown in Fig. 2. where the base station is deployed to cover the polygon. We first consider the case where the base station can be deployed anywhere. We then consider the case where the base station is constrained to be outside or on the boundary of the polygon, which is common at incident scenes.

In the case that only the base station is deployed, we study where it should be deployed, so that the minimum SNR of any point in the entire polygon is maximized. Due to the outage probability or data rate requirement, part of the polygon may be beyond the coverage of the base station. In that case, relays are deployed to extend the coverage, where the optimal positions of the base station and the relays need to be determined.

The study directly applies to the case of an isolated network formed by the base station and relays, which is dedicated for the public safety organizations with a central command at the base station. However, if connection to the core network is needed, the base station could be regarded as a relay. Then the network model with a base station becomes identical to the network model with relays, and the results apply straightforwardly.

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C. Channel Model

We use Rayleigh fading model for hop between nearby devices [37]. Flat fading is assumed for ease of discussion, while the formulation of the optimization problem and the solution are easy to generalize to the case of frequency selective fading. Let the channel power gain of a hop be:

$$h = Ad^{-\alpha}G \tag{1}$$

where A is a constant value that considers shadowing and antenna gain, d is the hop distance, α is the path loss exponent, and G denotes the power gain of the Rayleigh fading channel, which follows the exponential distribution with unit mean.

Let p be the transmit power, and σ^2 be the power spectral density of white Gaussian noise. At the receiver, in the absence of interference, the signal-to-noise ratio (SNR) is:

$$\text{SNR} = \frac{p\eta G}{d^{\alpha}} \tag{2}$$

where

$$\eta = \frac{A}{\sigma^2 W}. \tag{3}$$

Let γ be a SNR threshold, and the corresponding modulation and coding schemes are used at the transmitter [38]. If the SNR at the receiver is equal or greater than the threshold, the decoding error is negligible. Otherwise, an outage event is declared. The outage probability is [15]:

$$\mathcal{P} = \Pr\left(\text{SNR} < \gamma\right) = 1 - \exp\left(-\frac{\gamma d^{\alpha}}{p\eta}\right). \tag{4}$$

The outage capacity is [39]:

$$\mathcal{D} = W\log_2(1+\gamma)\Pr\left(\text{SNR} \geq \gamma\right) \tag{5}$$
$$= W\log_2(1+\gamma)\exp\left(-\frac{\gamma d^{\alpha}}{p\eta}\right) \tag{6}$$

which is the expected data rate for a time varying channel whose coherent time is larger than the coding block length. (This is the case in the setting of the Public Safety Broadband Network as the mobility is not much faster than walking speed).

III. The Deployment of Relays

In order to extend the coverage of the base station to the maximum, $K-1$ relays are deployed along the line segment that connects the base station and the destination. The optimal distance tuple $(d_1, \ldots, d_K)$ maximizes the reach to the destination subject to the QoS requirement. Once the optimal distance tuple is computed, the optimal positions of the relays are straightforward. In the amplify-and-forward (AF) network, each relay amplifies and forwards the signals from both directions, which has several advantages over decode-and-forward (DF) network in terms of complexity. However, the amplified noise in previous hop significantly degrades the system performance, especially for the cases with large number of hop. In this paper, we study the cases with multiple relays and DF scheme is utilized to maximize the reach to the destination.

A. The Maximum Reach with Outage Probability Requirement

Because decode and forward scheme is adopted throughout the paper, each relay either recovers the original message exactly or suffers outage (the outage probability is quantified). Moreover, all forward and backward channels are made orthogonal using time division. Therefore, the SNR of each hop only depends on its channel state information, and depends on no other hops. According to the multiple hops between the source and the destination, an outage event is declared whenever the SNR of any hop falls below the SNR threshold. Hence the outage probability of the forward and backward hops are [15]:

$$\mathcal{P}_f = \Pr[\min(\text{SNR}_{f,1}, \ldots, \text{SNR}_{f,K}) \leq \gamma] \tag{7}$$
$$= 1 - \prod_{k=1}^{K} \exp\left(-\frac{\gamma d_k^{\alpha}}{p_k \eta}\right) \tag{8}$$
$$\mathcal{P}_b = \Pr[\min(\text{SNR}_{b,1}, \ldots, \text{SNR}_{b,K}) \leq \tau] \tag{9}$$
$$= 1 - \prod_{k=1}^{K} \exp\left(-\frac{\tau d_k^{\alpha}}{q_k \eta}\right) \tag{10}$$

where $\text{SNR}_{f,k}$ and $\text{SNR}_{b,k}$ are the SNR of the k-th forward and backward hop, and γ and τ are the SNR threshold of the forward and backward hops. When the QoS metric is the outage probability, we assume that the SNR threshold of forward and backward hops are given values instead of variables. Otherwise, let the SNR threshold be zero, the outage probability is minimized to zero. However, the data rate is also zero which is meaningless.

Assuming $\mathcal{P} \in [0,1]$ and $\beta\mathcal{P} \in [0,1]$ are the outage probability requirement of the forward and backward hops, the optimization problem to maximize the reach to the destination is:

$$\underset{d_1,\ldots,d_K}{\text{maximize}} \quad \sum_{k=1}^{K} d_k \tag{11a}$$
$$\text{subject to} \quad 1 - \prod_{k=1}^{K} \exp\left(-\frac{\gamma d_k^{\alpha}}{p_k \eta}\right) \leq \mathcal{P} \tag{11b}$$
$$1 - \prod_{k=1}^{K} \exp\left(-\frac{\tau d_k^{\alpha}}{q_k \eta}\right) \leq \beta\mathcal{P} \tag{11c}$$

which can be rewritten as:

$$\underset{d_1,\ldots,d_K}{\text{maximize}} \quad \sum_{k=1}^{K} d_k \tag{12a}$$
$$\text{subject to} \quad \sum_{k=1}^{K} \frac{d_k^{\alpha}}{p_k} \leq b \tag{12b}$$
$$\sum_{k=1}^{K} \frac{d_k^{\alpha}}{q_k} \leq c \tag{12c}$$

where

$$b = -\frac{\eta}{\gamma}\log\left(1 - \mathcal{P}\right) \tag{13}$$
$$c = -\frac{\eta}{\tau}\log\left(1 - \beta\mathcal{P}\right). \tag{14}$$

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The optimization problem (12) is convex and the associated *Lagrangian* is [40]:

$$L(d_1, \ldots, d_K, \lambda, \nu) = -\sum_{k=1}^{K} d_k + \lambda \left(\sum_{k=1}^{K} \frac{d_k^\alpha}{p_k} - b \right) + \nu \left(\sum_{k=1}^{K} \frac{d_k^\alpha}{q_k} - c \right) \quad (15)$$

where λ and ν are the *Lagrangian* multipliers. Then using the *Karush-Kuhn-Tucker* (KKT) conditions, the optimal solutions could be computed efficiently, which is omitted due to space limitations.

In the following, we study the cases with identical parameters, which shed more light on the maximum reach to the destination as the number of relays increases. The identical parameters are $\beta = 1$, $\gamma = \tau$ and $p_k = q_k = p$ for $k \in \{1, \ldots, K\}$.

Proposition 1. *With identical parameters, the maximum reach to the destination increases with the number of relays subject to the outage probability requirement.*

Proof. With identical parameters, the optimal solutions are obtained when the inequality of the constraint (12b) is tight. The optimal distance of each segment and the maximum reach to the destination are:

$$d_1 =, \ldots, = d_K = \left(\frac{bp}{K} \right)^{\frac{1}{\alpha}} \quad (16)$$

$$\sum_{k=1}^{K} d_k = (bp)^{\frac{1}{\alpha}} K^{1-\frac{1}{\alpha}}. \quad (17)$$

Then the optimal distance of each segment decreases but the maximum reach to the destination increases with the number of relays subject to the outage probability requirement. Thus the proposition is proved, which also holds for the general cases with non-identical parameters. □

B. The Maximum Reach with Data Rate Requirement

In the following, we still consider the network model with relays, but take the data rate as the QoS metric. The SNR of each hop only depends on its channel state information. According to the multiple hops between the source and the destination, the data rate is dominated by the weakest hop. Hence using the data rate expression of each hop in [39], the data rate of the forward and backward hops are:

$$\mathcal{C}_f = \min_{k \in \{1,\ldots,K\}} t_k W \log_2(1+\gamma_k) \exp\left(-\frac{\gamma_k d_k^\alpha}{p_k \eta} \right) \quad (18)$$

$$\mathcal{C}_b = \min_{k \in \{1,\ldots,K\}} s_k W \log_2(1+\tau_k) \exp\left(-\frac{\tau_k d_k^\alpha}{q_k \eta} \right) \quad (19)$$

where t_k and s_k are the allocated fraction of time, and γ_k and τ_k are the SNR threshold of the k-th forward and backward hop.

Differing from previous literatures where resource is uniformly allocated to each hop, we optimally allocate the resource to each hop according to its channel state information. Moreover, the SNR threshold of each hop is also optimized, which means that the modulation of different relays varies. Assuming $\mathcal{D}$ and $\rho\mathcal{D}$ are the data rate requirement of the forward and backward hops, the optimization problem to maximize the reach to the destination is:

$$\underset{\substack{d_1,\ldots,d_K \\ t_1,\ldots,t_K, s_1,\ldots,s_K \\ \gamma_1,\ldots,\gamma_K,\tau_1,\ldots,\tau_K}}{\text{maximize}} \quad \sum_{k=1}^{K} d_k \quad (20a)$$

$$\text{subject to} \quad t_k W \log_2(1+\gamma_k) \exp\left(-\frac{\gamma_k d_k^\alpha}{p_k \eta} \right) \geq \mathcal{D}, \quad \forall k \in \{1, \ldots, K\} \quad (20b)$$

$$s_k W \log_2(1+\tau_k) \exp\left(-\frac{\tau_k d_k^\alpha}{q_k \eta} \right) \geq \rho\mathcal{D}, \quad \forall k \in \{1, \ldots, K\} \quad (20c)$$

$$\sum_{k=1}^{K} (t_k + s_k) = 1 \quad (20d)$$

which is in general non-convex, and has $5K$ variables. Assuming the exhaustive searching method quantizes each variable into N values, the complexity is the comparison of N^{5K} quantized tuples. For a large N, solving the optimization problem lay on exhaustive searching method is challenging.

Using the KKT conditions is another option to compute the optimal solutions, but it does not help too much to reduce the complexity. In the following, we study the structure of the optimization problem to develop a more efficient algorithm. Specifically, the first derivative of the amount of allocated time resource of each segment with respect to the segment distance is utilized to compute the optimal solutions.

With fixed distance tuple $(d_1, \ldots, d_K)$, the optimal γ_k, τ_k maximize the left side of (20b) and (20c), which are the unique solutions to:

$$(1+\gamma_k) \log(1+\gamma_k) = \frac{p_k \eta}{d_k^\alpha} \quad (21)$$

$$(1+\tau_k) \log(1+\tau_k) = \frac{q_k \eta}{d_k^\alpha}. \quad (22)$$

In other words, γ_k, τ_k are functions of d_k, which are represented by $\gamma_k(d_k)$, $\tau_k(d_k)$ in the following.

The data rate of each hop is linearly increasing in the amount of allocated time resource. To maximize the reach to the destination, the constraints (20b) and (20c) should be tight. Otherwise, we could reduce the amount of allocated time resource. Thus the amount of allocated time resource of the k-th forward and backward hop are:

$$t_k(d_k) = \frac{\mathcal{D}}{W \log(1+\gamma_k(d_k))} \exp\left(\frac{\gamma_k(d_k) d_k^\alpha}{p_k \eta} \right) \quad (23)$$

$$s_k(d_k) = \frac{\rho\mathcal{D}}{W \log(1+\tau_k(d_k))} \exp\left(\frac{\tau_k(d_k) d_k^\alpha}{q_k \eta} \right). \quad (24)$$

For the k-th segment which has two hop, the first derivative of the amount of allocated time resource with respect to the segment distance is:

$$\frac{\mathrm{d}\left(t_k(d_k) + s_k(d_k)\right)}{\mathrm{d}d_k} = \frac{\alpha d_k^{\alpha-1} \gamma_k(d_k) t_k(d_k)}{p_k \eta} + \frac{\alpha d_k^{\alpha-1} \tau_k(d_k) s_k(d_k)}{q_k \eta} \quad (25)$$

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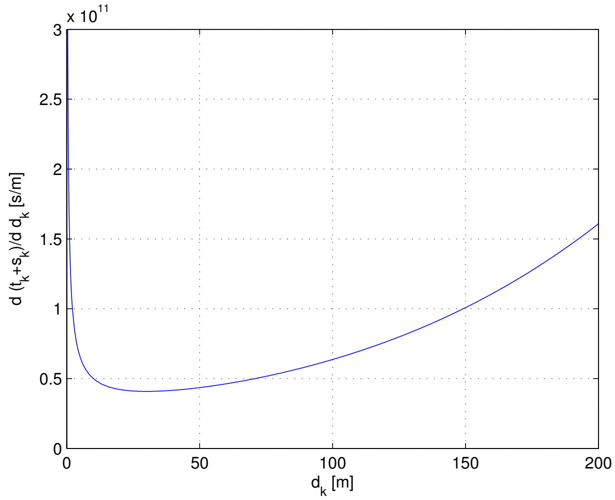


Fig. 3. The first derivative of the amount of allocated time resource of each segment with respect to the segment distance.

which is positive and convex with respect to d_k as shown in Fig. 3. The positive is easy to be noted and the convexity is proved in Appendix.

Proposition 2. *For the optimal solutions of the optimization problem (20), the first derivative given by (25) should be equal for different segments, i.e.,*

$$\frac{d\left(t_k(d_k)+s_k(d_k)\right)}{dd_k}=\frac{d\left(t_{k'}(d_{k'})+s_{k'}(d_{k'})\right)}{dd_{k'}},\quad \forall k,k'\in\{1,\ldots,K\}. \tag{26}$$

Proof. If there exist two different segment indexes k, k' such that $\frac{\mathrm{d}(t_k(d_k)+s_k(d_k))}{\mathrm{d}d_k} > \frac{\mathrm{d}(t_{k'}(d_{k'})+s_{k'}(d_{k'}))}{\mathrm{d}d_{k'}}$, we can shift some time resource from the k-th segment to the k'-th segment. The shifted time resource satisfy $\int_{d_k-\Delta_{d_k}}^{d_k}\frac{\mathrm{d}(t_k(d_k)+s_k(d_k))}{\mathrm{d}d_k}\mathrm{d}d_k = \int_{d_{k'}}^{d_{k'}+\Delta_{d_{k'}}}\frac{\mathrm{d}(t_{k'}(d_{k'})+s_{k'}(d_{k'}))}{\mathrm{d}d_{k'}}\mathrm{d}d_{k'}$. Since $\frac{\mathrm{d}(t_k(d_k)+s_k(d_k))}{\mathrm{d}d_k} > \frac{\mathrm{d}(t_{k'}(d_{k'})+s_{k'}(d_{k'}))}{\mathrm{d}d_{k'}}$, the increase of $d_{k'}$ is larger than the decrease of d_k. Then the maximum reach to the destination increases, which contradicts the assumption. Hence the proposition is proved. □

Using *Proposition 2*, we compute all the feasible distance tuples satisfying (20d) and (26), and the one with the maximum reach to the destination is the optimal solutions. Specifically, let L be the value of the first derivative given by (25), and $\mathcal{S}_k(L)$ be the set of d_k satisfying:

$$\mathcal{S}_k(L)=\left\{d_k\middle|\frac{\mathrm{d}\left(t_k(d_k)+s_k(d_k)\right)}{\mathrm{d}d_k}=L\right\} \tag{27}$$

which has two elements in general. Then each distance tuple $(d_1,\ldots,d_K)\in(\mathcal{S}_1(L),\ldots,\mathcal{S}_K(L))$ leads to a unique time resource tuple $(t_1(d_1)+s_1(d_1),\ldots,t_K(d_K)+s_K(d_K))$. For each quantized value of L, if the time resource tuple satisfies (20d), the distance tuple is a feasible one. Comparing all the feasible distance tuples, the one with the maximum reach to the destination is the optimal solutions.

The numerical method for computing the optimal distance tuple, resource allocation and SNR threshold for maximizing the reach to the destination subject to the data rate requirement is summarized as Algorithm 1, where Δ represents the quantization interval. In step 4, when the segment distance is zero or the maximum, the first derivative given by (25) is maximized due to its convex nature. Recall that the complexity of the exhaustive searching method is the comparison of N^{5K} quantized tuples. After studying the structure of the optimization problem, the complexity of Algorithm 1 is the comparison of $N2^K$ quantized tuples. Generally, L is a large number and the complexity of Algorithm 1 is much smaller than that of the exhaustive searching method.

Algorithm 1 Computing the optimal distance tuples, resource allocation and SNR threshold for maximizing the reach to the destination subject to the data rate requirement

1: **Input:** $\mathcal{D}$, ρ, W, A, α, σ^2 Δ, p_k, q_k for $k\in\{1,\ldots,K\}$.
2: **Output:** $d_k, t_k, s_k, \gamma_k, \tau_k$ for $k\in\{1,\ldots,K\}$.
3: Compute the maximum segment distance $d_{k,\max}$ when all time resource is allocated to the k-th segment for $k\in\{1,\ldots,K\}$.
4: Compute the upper bound of the first derivative as:

$$L_{\max}=\max\left\{\frac{\mathrm{d}\left(t_k(d_k)+s_k(d_k)\right)}{\mathrm{d}d_k}, d_k\in\{0,d_{k,\max}\}, k\in\{1,\ldots,K\}\right\}.$$

5: Let $N=\lceil\frac{L_{\max}}{\Delta}\rceil$.
6: **for all** $L\in\{\Delta,2\Delta,\ldots,N\Delta\}$ **do**
7: Compute $\mathcal{S}_k(L)=\left\{d_k|\frac{\mathrm{d}(t_k(d_k)+s_k(d_k))}{\mathrm{d}d_k}=L\right\}$ for $k\in\{1,\ldots,K\}$.
8: Find the feasible distance tuples that $(d_1,\ldots,d_K)\in(\mathcal{S}_1(L),\ldots,\mathcal{S}_K(L))$ and satisfy $\sum_{k=1}^{K}(t_k(d_k)+s_k(d_k))=1$.
9: **end for**
10: Compare all feasible distance tuples and **return** the one with the maximum reach to the destination. Moreover, the resource allocation and SNR threshold are computed by (21), (22), (23) and (24).

Similarly, in the following, we study the cases with identical parameters, which shed more light on the maximum reach to the destination as the number of relays increases. The identical parameters are $\rho=1$ and $p_k=q_k=p$ for $k\in\{1,\ldots,K\}$.

Proposition 3. *With identical parameters, there exists a certain number of relays, beyond which more relays do not provide further improvement in the maximum reach to the destination subject to the data rate requirement.*

Proof. As shown in Fig. 3, let d_k^* be the distance where the first derivative $\frac{\mathrm{d}(t_k(d_k)+s_k(d_k))}{\mathrm{d}d_k}$ is minimized. For the optimal distance tuple, no more than one segment satisfies $d_k\in(0,d_k^*)$. If there exist two segment indexes k, k', such that $d_k\in(0,d_k^*)$ and $d_{k'}\in(0,d_{k'}^*)$, then we can shift some time resource from the k-th segment to the k'-th segment and have $\int_{d_k-\Delta_{d_k}}^{d_k}\frac{\mathrm{d}(t_k(d_k)+s_k(d_k))}{\mathrm{d}d_k}\mathrm{d}d_k =$

$\int_{d_{k'}}^{d_{k'}+\Delta_{d_{k'}}} \frac{\mathrm{d}(t_{k'}(d_{k'})+s_{k'}(d_{k'}))}{\mathrm{d}d_{k'}}\mathrm{d}d_{k'}$. The second derivative $\frac{\mathrm{d}^2(t_k(d_k)+s_k(d_k))}{\mathrm{d}d_k^2}$ and $\frac{\mathrm{d}^2(t_{k'}(d_{k'})+s_{k'}(d_{k'}))}{\mathrm{d}d_{k'}^2}$ are negative, so that $\frac{\mathrm{d}(t_k(d_k)+s_k(d_k))}{\mathrm{d}d_k}$ and $\frac{\mathrm{d}(t_{k'}(d_{k'})+s_{k'}(d_{k'}))}{\mathrm{d}d_{k'}}$ are decreasing for $d_k \in (0, d_k^*)$ and $d_{k'} \in (0, d_{k'}^*)$, i.e.,

$$\begin{aligned}
&\frac{\mathrm{d}\left(t_k(d_k)+s_k(d_k)\right)}{\mathrm{d}d_k}\Delta_{d_k} \\
&\le \int_{d_k-\Delta_{d_k}}^{d_k} \frac{\mathrm{d}\left(t_k(d_k)+s_k(d_k)\right)}{\mathrm{d}d_k}\mathrm{d}d_k && (28)\\
&= \int_{d_{k'}}^{d_{k'}+\Delta_{d_{k'}}} \frac{\mathrm{d}\left(t_{k'}(d_{k'})+s_{k'}(d_{k'})\right)}{\mathrm{d}d_{k'}}\mathrm{d}d_{k'} && (29)\\
&\le \frac{\mathrm{d}\left(t_{k'}(d_{k'})+s_{k'}(d_{k'})\right)}{\mathrm{d}d_{k'}}\Delta_{d_{k'}}. && (30)
\end{aligned}$$

Using *Proposition 2* where $\frac{\mathrm{d}(t_k(d_k)+s_k(d_k))}{\mathrm{d}d_k} = \frac{\mathrm{d}(t_{k'}(d_{k'})+s_{k'}(d_{k'}))}{\mathrm{d}d_{k'}}$, we have $\Delta_{d_k} \le \Delta_{d_{k'}}$ which means that the decrease of d_k is smaller than the increase of $d_{k'}$. Then the maximum reach to the destination increases, which contradicts the assumption. Thus at least $K-1$ segments satisfy $d_k \in [d_k^*, d_{k,\max}]$.

The first derivative $\frac{\mathrm{d}(t_k(d_k)+s_k(d_k))}{\mathrm{d}d_k}$ is positive, so that the time resource allocated to each segment increases in the segment distance. For the $K-1$ segments satisfying $d_k \in [d_k^*, d_{k,\max}]$, the time resource allocated to each segment is no less than $t_k(d_k^*)+s_k(d_k^*)$. According to the resource constraint, we have $(K-1)\left(t_k(d_k^*)+s_k(d_k^*)\right) \le 1$. Then the number of relays is upper bounded by $K \le \frac{1}{t_k(d_k^*)+s_k(d_k^*)} + 1$, which indicates that more relays do not provide further improvement in the maximum reach to the destination subject to the data rate requirement. Hence the proposition is proved, which also holds for the general cases with non-identical parameters. □

IV. The Deployment of a Base Station

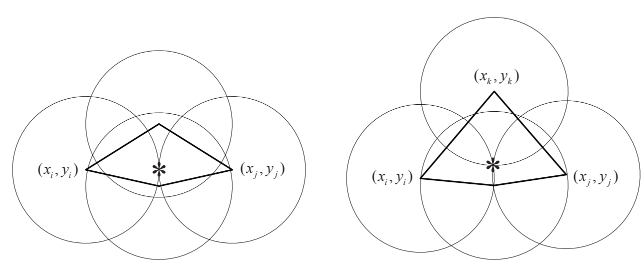
In order to provide temporary coverage for a target area, such as a burning building, a base station is deployed. The target area in general well modeled by a polygon, which may or may not be convex as discussed in Section II-B. The optimal position of the base station maximizes the minimum SNR of any point over the entire polygon. Both the general case where the base station can be located anywhere and the case where the base station is constrained to be outside or on the boundary of the polygon are considered. We develop numerical ways to compute the optimal position of the base station.

A. Without Placement Restrictions

For isotropic channel and omni-directional antennas, the coverage of the base station is a disk. The optimal position of the base station for maximizing the minimum SNR of any point over the entire polygon is the center of the minimum disk covering the polygon.

Proposition 4. *The optimal position of the base station for covering the polygon is identical to the optimal position of the base station for covering all vertices of the polygon.*

Proof. The minimum disk covering the polygon is defined as $\mathcal{V}$, which covers all vertices of the polygon. The minimum



(a) (b)

Fig. 4. The optimal position of the base station without placement restrictions.

disk covering all vertices of the polygon is defined as $\mathcal{U}$, which should be proved to cover the entire polygon. Since the vertices are covered by $\mathcal{U}$, the edges of the polygon connecting any two adjacent vertices are covered, and hence also the line segments connecting any two points on the sides. Then the entire polygon is covered, so that $\mathcal{V}$ and $\mathcal{U}$ are the same disk. Hence the proposition is proved. □

Following *Proposition 4*, we simplify the problem to find the center of the minimum disk covering all vertices of the polygon. Let the coordinate of the M vertices of the polygon be $(x_1, y_1), \ldots, (x_M, y_M)$. The optimization problem to find the optimal position of the base station is:

$$\underset{r,x,y}{\text{minimize}} \quad r^2 \tag{31a}$$

$$\text{subject to} \quad (x-x_i)^2 + (y-y_i)^2 \le r^2, \quad \forall i \in \{1 \ldots M\} \tag{31b}$$

where (x, y) is the optimal position of the base station, and r is the maximum distance from the base station to the vertices. Treating r^2 as one variable, the optimization problem (31) becomes convex. Using KKT conditions, the optimal solution is obtained by comparing $M!$ candidate positions. In the following, we study the structure of the optimal position of the base station to develop a much more efficient numerical method.

In order to satisfy the constraint (31b), (x, y) locates in a disk whose center is the vertex (x_i, y_i) and radius is r for $i \in \{1, \ldots, M\}$. If r is too small, the intersection of the M disks centered at $(x_1, y_1), \ldots, (x_M, y_M)$ is empty. Let r increase and at the first time there is one point in the intersection, the point is the optimal position of the base station. There are only two possible cases as far as the geometry of the problem is concerned, which are illustrated in Fig. 4. The coverage of the base station is a disk, which could be determined by a diameter in Fig. 4(a) or three points on the boundary in Fig. 4(b). The optimal position of the base station is marked by a star. We define $\mathcal{C}_i$ as the circle whose center is the vertex (x_i, y_i) and radius is r.

In Fig. 4(a), the optimal position of the base station (x, y) is the sole intersection point of two circles $\mathcal{C}_i$ and $\mathcal{C}_j$. The line segment connecting the vertices (x_i, y_i) and (x_j, y_j) is a diameter of the coverage of the base station. For two indexes

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$i, j \in \{1, \ldots, M\}$, the coordinate of the base station is:

$$x = \frac{x_i + x_j}{2} \tag{32a}$$

$$y = \frac{y_i + y_j}{2}. \tag{32b}$$

In Fig. 4(b), the optimal position of the base station is the sole intersection point of three circles $\mathcal{C}_i$, $\mathcal{C}_j$ and $\mathcal{C}_k$, which is also the center of the circumcircle of the triangle whose vertices are (x_i, y_i), (x_j, y_j) and (x_k, y_k). For three different indexes $i, j, k \in \{1, \ldots, M\}$, the coordinate of the base station is shown as (33) at the top of next page.

We find all candidate positions described by (32) and (33), and compute the maximum distance to all vertices for each candidate position as:

$$r = \max_{i \in \{1, \ldots, M\}} \sqrt{(x - x_i)^2 - (y - y_i)^2}. \tag{34}$$

The candidate position with the minimum maximum distance to all vertices is the optimal position of the base station.

Algorithm 2 Computing the optimal position of the base station without placement restrictions

Input: (x_i, y_i) for $i \in \{1, \ldots, M\}$.
Output: (x, y).
Find the candidate positions using (32), (33).
Compute the maximum distance to all vertices for each candidate position using (34).
Return (x, y) which is the coordinate of the candidate position with the minimum maximum distance to all vertices.

The numerical method for computing the optimal position of the base station without placement restrictions is summarized as Algorithm 2. Recall that the complexity of using KKT conditions is the comparison of $M!$ candidate positions. The complexity of Algorithm 2 is the comparison of no more than $\frac{1}{6}M^3 - \frac{1}{6}M$ candidate positions satisfying (32) or (33) where $M \geq 3$. For a large number of M, the complexity of Algorithm 2 is much smaller than that of using KKT conditions.

B. With placement restrictions

When the base station shall be deployed outside or on the boundary of the polygon, the optimization problem is changed to:

$$\underset{r,x,y}{\text{minimize}} \quad r^2 \tag{35a}$$

$$\text{subject to} \quad (x - x_i)^2 + (y - y_i)^2 \leq r^2, \quad \forall i \in \{1 \ldots M\} \tag{35b}$$

$$(x, y) \notin \mathcal{I} \tag{35c}$$

where $\mathcal{I}$ stands for the interior of the polygon.

Although the optimization problem (35) is in general non-convex, it is easy to solve for the three variables. Let r increases and at the first time the intersection area of the M disks centered at $(x_1, y_1), \ldots, (x_M, y_M)$ has one point outside or on the boundary of the polygon, the point is the optimal position of the base station. There are only four possible cases which

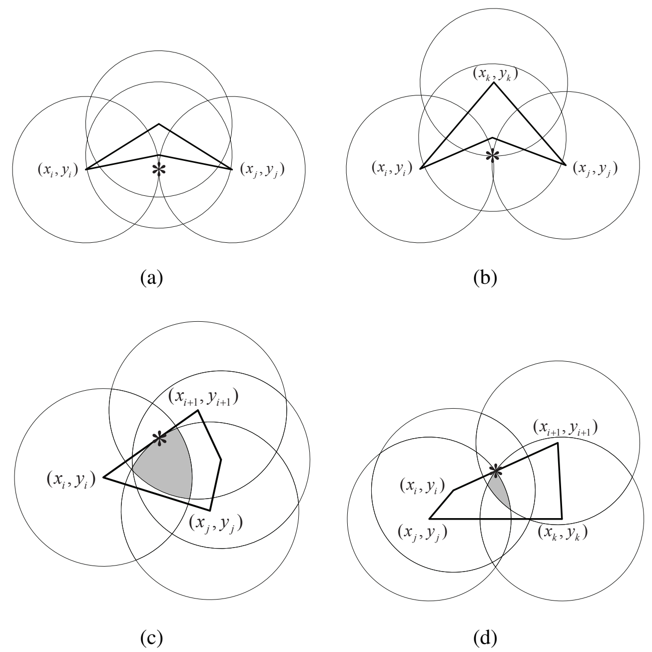

Fig. 5. The optimal position of the base station outside or on the boundary of the polygon.

are illustrated in Fig. 5. The intersection area is the shadowed part and the optimal position of the base station is marked by a star. In Fig. 5(a) and Fig. 5(b), the optimal position of the base station is outside the polygon. In Fig. 5(c) and Fig. 5(d), the optimal position of the base station is on the boundary of the polygon. We define $\mathcal{E}_i$ as the edge of the polygon connecting two adjacent vertices (x_i, y_i) and (x_{i+1}, y_{i+1}) for $i \in \{1, \ldots, M\}$ where $(x_{M+1}, y_{M+1}) = (x_1, y_1)$.

In Fig. 5(a), the optimal position of the base station (x, y) is the sole intersection point of two circles $\mathcal{C}_i$ and $\mathcal{C}_j$, and outside the polygon. For two non-adjacent indexes $i, j \in \{1, \ldots, M\}$, the coordinate of the base station is shown as (32).

In Fig. 5(b), the optimal position of the base station (x, y) is the sole intersection point of three circles $\mathcal{C}_i$, $\mathcal{C}_j$ and $\mathcal{C}_k$, and outside the polygon. For three different indexes $i, j, k \in \{1, \ldots, M\}$, the coordinate of the base station is shown as (33) at the top of next page.

In Fig. 5(c), the optimal position of the base station (x, y) is the intersection point of one circle $\mathcal{C}_j$ and one edge $\mathcal{E}_i$. The line segment connecting (x_j, y_j) and (x, y) is orthogonal to $\mathcal{E}_i$ and (x, y) is on $\mathcal{E}_i$. For three different indexes $i, i+1, j \in \{1, \ldots, M\}$, the coordinate of the base station is shown as (36) at the top of next page.

In Fig. 5(d), the optimal position of the base station (x, y) is the intersection point of two circles $\mathcal{C}_j$ and $\mathcal{C}_k$ and one edge $\mathcal{E}_i$. The distance from (x, y) to (x_j, y_j) and (x_k, y_k) are the same and (x, y) is on $\mathcal{E}_i$. For four indexes $i, i+1, j, k \in \{1, \ldots, M\}$ where $j \neq k$, the coordinate of the base station is shown as (37) at the top of next page.

The candidate position with the minimum maximum distance to all vertices is the optimal position of the base station.

The numerical method for computing the optimal position of the base station with placement restrictions is summarized

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$$x = \frac{(x_i^2 + y_i^2)(y_j - y_k) + (x_j^2 + y_j^2)(y_k - y_i) + (x_k^2 + y_k^2)(y_i - y_j)}{2x_i(y_j - y_k) + 2x_j(y_k - y_i) + 2x_k(y_i - y_j)} \tag{33a}$$

$$y = \frac{(x_i^2 + y_i^2)(x_k - x_j) + (x_j^2 + y_j^2)(x_i - x_k) + (x_k^2 + y_k^2)(x_j - x_i)}{2x_i(y_j - y_k) + 2x_j(y_k - y_i) + 2x_k(y_i - y_j)} \tag{33b}$$

$$x = \frac{x_j(x_{i+1} - x_i)^2 + (y_{i+1} - y_i)(x_i y_{i+1} + x_{i+1} y_j - x_i y_j - x_{i+1} y_i)}{(x_{i+1} - x_i)^2 + (y_{i+1} - y_i)^2} \tag{36a}$$

$$y = \frac{y_j(y_{i+1} - y_i)^2 + (x_{i+1} - x_i)(x_{i+1} y_i + x_j y_{i+1} - x_j y_i - x_i y_{i+1})}{(x_{i+1} - x_i)^2 + (y_{i+1} - y_i)^2} \tag{36b}$$

$$x = \frac{(x_{i+1} - x_i)(x_k^2 - x_j^2 + y_k^2 - y_j^2) + 2(y_k - y_j)(x_i y_{i+1} - x_{i+1} y_i)}{2(y_k - y_j)(y_{i+1} - y_i) + 2(x_k - x_j)(x_{i+1} - x_i)} \tag{37a}$$

$$y = \frac{(y_{i+1} - y_i)(x_k^2 - x_j^2 + y_k^2 - y_j^2) + 2(x_k - x_j)(x_{i+1} y_i - x_i y_{i+1})}{2(y_k - y_j)(y_{i+1} - y_i) + 2(x_k - x_j)(x_{i+1} - x_i)} \tag{37b}$$

Algorithm 3 Computing the optimal position of the base station with placement restrictions

1: **Input:** (x_i, y_i) for $i \in \{1, \ldots, M\}$.
2: **Output:** (x, y).
3: Find the candidate positions using (32), (33) and delete the ones inside the polygon.
4: Find the candidate positions using (36), (37) and delete the ones outside the polygon.
5: Compute the maximum distance to all vertices for each candidate position using (34).
6: **Return** (x, y) which is the coordinate of the candidate position with the minimum maximum distance to all vertices.

as Algorithm 3. The complexity is the comparison of no more than $\frac{2}{3}M^3 + \frac{1}{2}M^2 - \frac{19}{6}M$ candidate positions satisfying (32), (33), (36) or (37) where $M \geq 3$.

C. Relay-Assisted Coverage

The optimal position of the base station maximizes the minimum SNR of any point over the entire region. But due to the outage probability or data rate requirement, part of the polygon may be beyond the coverage of the base station. When the destination locates at the area beyond the coverage of the base station, the minimum number of relays are deployed along the line segment connecting the base station and the destination.

We first compute the optimal positions of the relays for maximizing the reach to the destination as the number of relays increases. Once the maximum reach is larger than or equal to the distance between the base station and the destination, the number of relays is the minimum one and the relays are deployed at the optimal positions. We caution that, with relays, the goal here is limited to reaching any point in the region, rather than covering the entire region at the same time.

D. Discussions

For the non-convex polygon, without placement restrictions, the optimal position of the base station could be inside, outside or on the boundary of the polygon. With placement restrictions, the optimal position of the base station could be outside or on the boundary of the polygon.

Proposition 5. *For the convex polygon, no matter with or without placement restrictions, the optimal position of the base station could not be outside the polygon.*

Proof. If the optimal position of the base station is outside the convex polygon, there exists at least one edge of the polygon, such that the base station and the polygon locate at the different sides of the edge. We draw an orthogonal line from the base station to the edge. The maximum distance to all vertices from the intersection point is smaller than that from the base station, so that the intersection point is a better position of the base station which contradicts the assumption. Therefore, the optimal position of the base station could not be outside the convex polygon. □

Therefore, for the convex polygon without placement restrictions, the optimal position of the base station could be inside or on the boundary of the polygon. With placement restrictions, the optimal position of the base station must be on the boundary of the polygon.

V. Numerical Results

To extend the mobile coverage, the optimal positions of the relays and base station are carried out by simulations. The main simulation parameters chosen according to LTE standards [41] are listed in TABLE I.

A. The Deployment of Relays

1) Outage Probability: When the QoS metric is the outage probability, the optimal distance tuple maximizes the reach to the destination subject to the outage probability requirement.

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TABLE I
MAIN SIMULATION PARAMETERS

Parameter	Value
Transmit power of the base station	30 dBm
Transmit power of the relays	27 dBm
Transmit power of the destination	24 dBm
Outage probability ratio β	1
Data rate ratio ρ	1
Path loss $Ad^{-\alpha}$	$-15.3-37.6\log_{10} d$ dB, d in meters
Power spectral density of Gaussian noise σ^2	-174 dBm/Hz

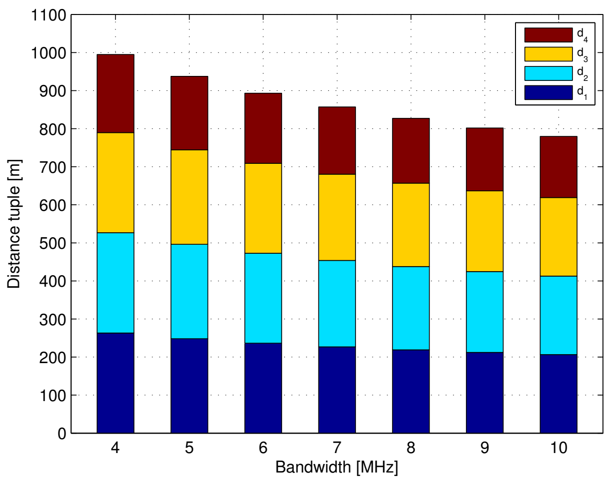

Fig. 6. The optimal distance tuple as the amount of bandwidth increases when the number of relays, SNR threshold and outage probability requirement of the forward and backward hop are 3, 10 dB and 5%.

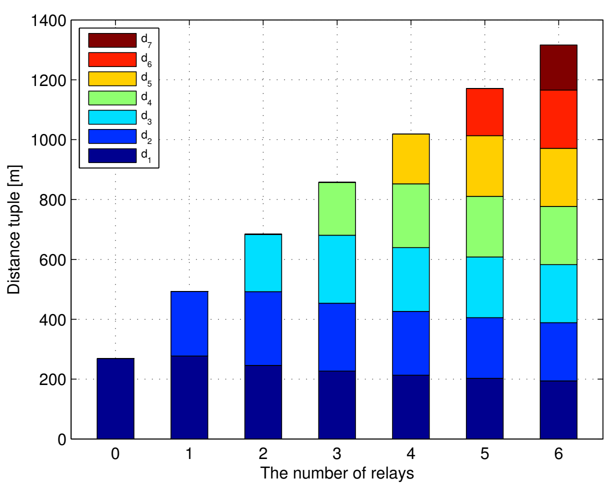

Fig. 7. The optimal distance tuple as the number of relays increases when the total bandwidth, SNR threshold and outage probability requirement of the forward and backward hop are 7 MHz, 10 dB and 5%.

When the number of relays, SNR threshold and outage probability requirement of the forward and backward hop are 3, 10 dB and 5%, the optimal distance tuple as the amount of bandwidth increases is shown in Fig. 6. It can be seen that the maximum reach decreases as the amount of bandwidth increases. This is because of that the SNR of each hop depends on the power of the transmitter and noise. Since the power of the noise is linearly increasing in the amount of bandwidth, with fixed power of the transmitter, the SNR decreases which increases the outage probability as the amount of the bandwidth increases. For the case with 4 MHz bandwidth, the maximum reach is 995 meters. For the case with 5 MHz bandwidth, the maximum reach decreases to 937 meters. For the cases with more bandwidth, the maximum reach further decreases.

When the total bandwidth, SNR threshold and outage probability requirement of the forward and backward hop are 7 MHz, 10 dB and 5%, the optimal distance tuple as the number of relays increases is shown in Fig. 7. As discussed in *Proposition 1*, the maximum reach increases with the number of relays. For the case without relay, the maximum reach is 269 meters. For the case with one relay, the maximum reach increases to 493 meters, where the optimal position of the relay is 277 meters from the base station. We note that due to the two-way outage probability requirement, the coverage of the base station not only depends on the transmit power of the base station, but also the transmit power of the relay or the destination. In the experiment, the transmit power of the relay is assumed to be higher than that of the destination. Hence the reach to the optimal position of the relay for the case with one relay could be further than that to the destination for the case without relay. For the cases with more relays, the maximum reach further increases.

2) Data Rate: When the QoS metric is the data rate, the optimal distance tuple maximizes the reach to the destination subject to the data rate requirement. Moreover, the SNR threshold and allocated resource of each hop are also optimized.

When the number of relays and data rate requirement of the forward and backward hop are 3 and 4 Mbps, the optimal distance tuple as the amount of bandwidth increases is shown in Fig. 8. It can be seen that the maximum reach increases as the amount of bandwidth increases. For the case with 4 MHz bandwidth, the maximum reach is 154 meters. For the case with 5 MHz bandwidth, the maximum reach increases to 206 meters. For the cases with more bandwidth, the maximum reach further increases.

When the total bandwidth and data rate requirement of the forward and backward hop are 7 MHz and 4 Mbps, the optimal distance tuple as the number of relays increases is shown in Fig. 9. As discussed in *Proposition 3*, there exists a certain number of relays, beyond which more relays do not provide further improvement in the maximum reach. For the case without relay, the maximum reach is 185 meters. For the case with one relay, the maximum reach is extended to 259 meters, where the optimal position of the relay is 150 meters from the base station. For the case with two or three relays, the maximum reach is extended to further. But beyond

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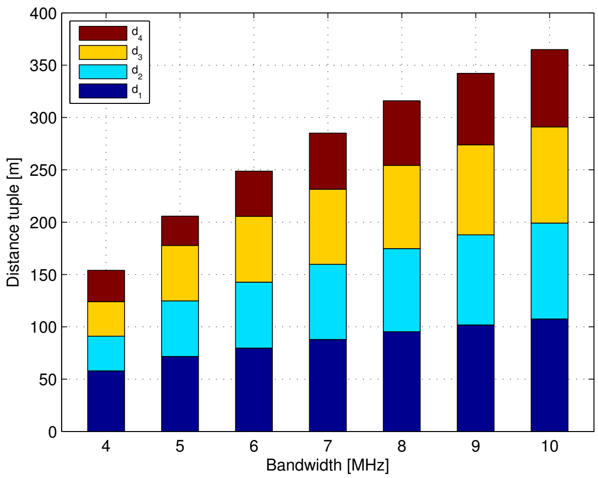

Fig. 8. The optimal distance tuple as the amount of bandwidth increases when the number of relays and data rate requirement of the forward and backward hop are 3 and 4 Mbps.

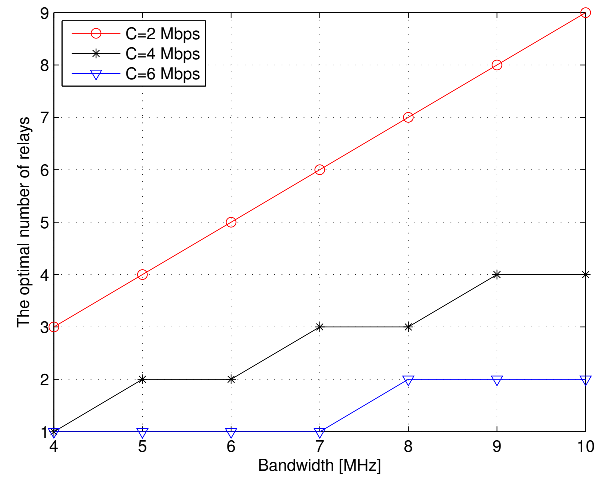

Fig. 10. The optimal number of relays as the amount of bandwidth increases when the data rate requirement of the forward and backward hop are 2, 4 and 6 Mbps.

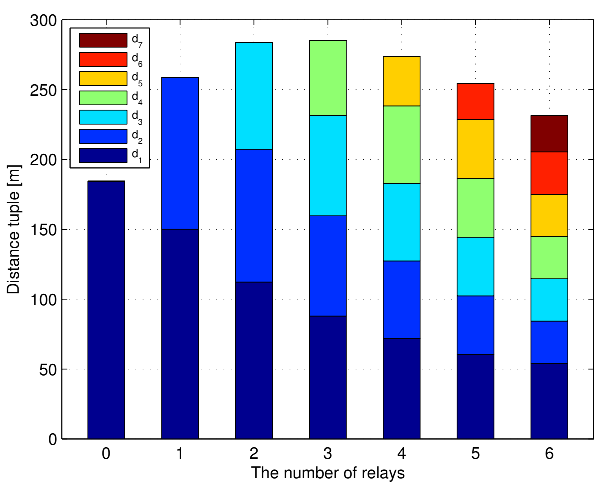

Fig. 9. The optimal distance tuple as the number of relays increases when the total bandwidth and data rate requirement of the forward and backward hop are 7 MHz and 4 Mbps.

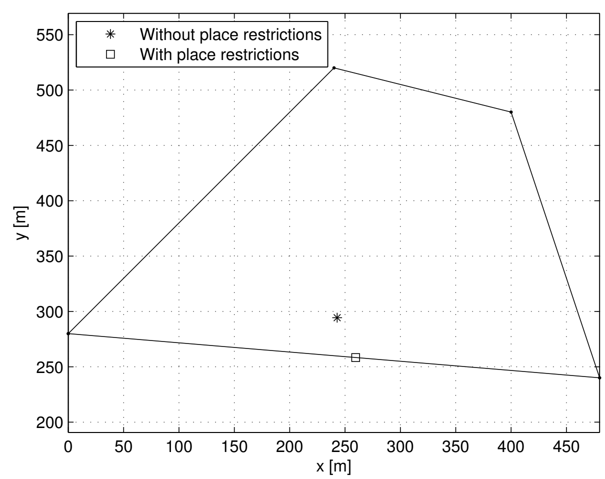

Fig. 11. The optimal position of the base station without and with placement restrictions.

three relays, more relays decrease the maximum reach. In other words, the optimal number of relays is three for maximizing the reach to the destination.

When the data rate requirement of the forward and backward hop are 2, 4 and 6 Mbps, the optimal number of relays as the amount of bandwidth increases is shown in Fig. 10. It can be note that the optimal number of relays increases as the amount of bandwidth increases and the data rate requirement decreases. The reason is that deploying more relays increases the number of hop, but also decreases the amount of allocated resource of each hop. Then there exists a tradeoff between the number of relays and the amount of resource, which should be considered during the deployment of relay networks.

B. The Deployment of the Base Station

In the case of the deployment of the base station, the polygon is a quadrilateral whose four vertices are $(0, 280)$, $(480, 240)$, $(400, 480)$, and $(240, 520)$ in meters. The optimal position of the base station maximizes the minimum SNR of any point over the entire region.

The optimal position of the base station without and with placement restrictions are shown in Fig. 11. Without placement restrictions where the base station could be deployed anywhere, the optimal position of the base station is $(294, 243)$ in meters as marked by a star. The distance from the base station to the vertices of the polygon are 243, 243, 243 and 226 meters. With placement restrictions where the base station shall be deployed outside or on the boundary of the polygon, the optimal position of the base station is $(258, 260)$ in meters as marked by a square. The distance from the base station to the vertices of the polygon are 260, 221, 262 and 262

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meters. The results could be utilized to efficiently deploy base station to provide temporary coverage for a target area, such as a building on fire whose geometry topology is usually a polygon, and an area that appears to be irregular can be easily covered by an approximate polygon. In order to satisfy the outage probability or data rate requirement, relays may be deployed to reach any point in the polygon. We first compute the optimal positions of the relays for maximizing the reach to the destination as the number of relays increases as shown in Section V-A. Once the maximum reach is larger than or equal to the distance between the base station and the destination, the number of relays is the minimum one and the relays are deployed at the optimal positions.

VI. Conclusion

In this paper, multiple relays and a base station are deployed for extending the coverage of a wireless network. For the deployment of multiple relays, the optimal positions of relays are studied for maximizing the reach to the destination subject to the QoS requirement. When the QoS requirement is the outage probability requirement, the relays deployment is optimized. The maximum reach increases with the number of relays. When the QoS requirement is the data rate requirement, the relays deployment, resource allocation and SNR threshold are jointly optimized. When the number of relays is small, the maximum reach increases with the number of relays. But beyond a certain number, more relays do not provide further improvement. For the deployment of a base station, the optimal position of the base station is analyzed for maximizing the minimum SNR of any point in a polygon without or with placement restrictions. Due to the outage probability or data rate requirement, relays may be deployed to reach any point in the polygon. Maximizing the reach to the wireless network using spectrum sharing scheme and providing full coverage to a given area are left for future work, where interference and scheduling are crucial to the wireless coverage.

Appendix A
Proof of the convexity of (25)

The first derivative given by (25) has two parts, and we only prove the convexity of the first part and the second part follows the same way. In order to simplify the expression, let other parameters be unit except the segment distance, path loss exponent, SNR threshold and allocated time resource, which does not affect the convexity of (25). Meanwhile, the subscript is omitted during the proof. Using (21) and (23), we have,

$$d^{-\alpha} = (1+\gamma)\log(1+\gamma) \tag{38}$$

$$t = \frac{1}{\log(1+\gamma)}\exp\left(\frac{\gamma}{(1+\gamma)\log(1+\gamma)}\right) \tag{39}$$

and need to prove that the third derivative $\frac{\mathrm{d}^3 t}{\mathrm{d}d^3}$ is positive.

When $\alpha \geq 3$, the third derivative $\frac{\mathrm{d}^3 t}{\mathrm{d}d^3}$ is:

$$\begin{aligned}
&\frac{\mathrm{d}^3 t}{\mathrm{d}d^3}\\
=&\frac{\mathrm{d}^3 t}{\mathrm{d}\gamma^3}\left(\frac{\mathrm{d}\gamma}{\mathrm{d}d}\right)^3 + 3\frac{\mathrm{d}^2 t}{\mathrm{d}\gamma^2}\frac{\mathrm{d}^2\gamma}{\mathrm{d}d^2}\frac{\mathrm{d}\gamma}{\mathrm{d}d} + \frac{\mathrm{d}t}{\mathrm{d}\gamma}\frac{\mathrm{d}^3\gamma}{\mathrm{d}d^3} \qquad (40)\\
=&\alpha d^{-3} t\left[\left(\alpha^2 - 3\alpha + \frac{3\alpha^2\gamma}{\log(1+\gamma)+1}\right)\times\right.\\
&\left(\frac{\gamma}{(1+\gamma)\log(1+\gamma)} - \frac{1}{\log(1+\gamma)+1}\right) +\\
&\frac{\gamma}{(1+\gamma)\log(1+\gamma)}\left(1 - \frac{\alpha\gamma}{(1+\gamma)\log(1+\gamma)}\right)\times\\
&\left.\left(2 - \frac{\alpha\gamma}{(1+\gamma)\log(1+\gamma)}\right) + \frac{\alpha^2\log(1+\gamma)}{(\log(1+\gamma)+1)^2}\right] \qquad (41)\\
\geq&\frac{\alpha d^{-3} t}{\log(1+\gamma)}\left[\frac{\gamma}{1+\gamma}\left(1 - \frac{\alpha\gamma}{(1+\gamma)\log(1+\gamma)}\right)\times\right.\\
&\left.\left(2 - \frac{\alpha\gamma}{(1+\gamma)\log(1+\gamma)}\right) + \frac{\alpha^2\log^2(1+\gamma)}{(\log(1+\gamma)+1)^2}\right] \qquad (42)
\end{aligned}$$

where (42) is because $\alpha \geq 3$ and

$$\frac{\gamma}{(1+\gamma)\log(1+\gamma)} - \frac{1}{\log(1+\gamma)+1} \geq 0. \tag{43}$$

If $\alpha\gamma/((1+\gamma)\log(1+\gamma))$ is either less than or equal to 1 or greater or equal to 2, then it is easy to see that (42) is positive. On the other hand, if $\alpha\gamma/((1+\gamma)\log(1+\gamma))$ is sandwiched between 1 and 2, we have:

$$2 \geq \frac{\alpha\gamma}{(1+\gamma)\log(1+\gamma)} \geq \frac{3\gamma}{(1+\gamma)\log(1+\gamma)} \tag{44}$$

so that,

$$\gamma \geq 1.397. \tag{45}$$

Then it can be obtained that,

$$\begin{aligned}
\frac{\mathrm{d}^3 t}{\mathrm{d}d^3} \geq& \frac{\alpha d^{-3} t}{\log(1+\gamma)}\left[\frac{\gamma}{1+\gamma}(1-1.5)(2-1.5) + \right.\\
&\left.\frac{\alpha^2\log^2(1+1.397)}{(\log(1+1.397)+1)^2}\right] \qquad (46)\\
\geq& \frac{\alpha d^{-3} t}{\log(1+\gamma)}\left[-0.25 + 1.96\right] \qquad (47)\\
\geq& 0 \qquad (48)
\end{aligned}$$

where (46) is because

$$\begin{aligned}
&\left(1 - \frac{\alpha\gamma}{(1+\gamma)\log(1+\gamma)}\right)\left(2 - \frac{\alpha\gamma}{(1+\gamma)\log(1+\gamma)}\right)\\
&\geq (1-1.5)(2-1.5)
\end{aligned} \tag{49}$$

and $\frac{\log^2(1+\gamma)}{(\log(1+\gamma)+1)^2}$ is increasing in γ, and (47) is because of that $\frac{\gamma}{1+\gamma} \leq 1$ and $\alpha \geq 3$. Consequently, $\frac{\mathrm{d}^3 t}{\mathrm{d}d^3}$ is positive and the proof is done.

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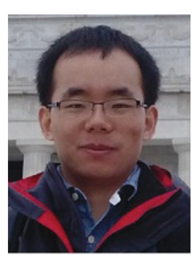

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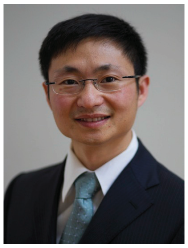
This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TVT.2015.2458015, IEEE Transactions on Vehicular Technology



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