ACTL30003 Contingencies
Project - Semester 2, 2016
Reverse Mortgage

Group 5
Yi-Wei Ho (722076)
Elaine Ng (777930)
Jing Sheng Lim (713489)
Chuping Liu (691858)
Jie Sheng Lim (670512)

Introduction

The key objective of this ACTL30003 Contingencies Project 2016 is to compute the value of a reverse mortgage portfolio and its profitability over a one-year period. This project consists of three parts.

Part 1 discusses the background of reverse mortgages including the various types, the motives for people obtaining a reverse mortgage, the risks involved, its history, and the various ways to value a reverse mortgage.

Part 2 involves fitting the Lee-Carter model of mortality rates onto the historical life tables for Australian males and females and finding the correlation between the residuals $Z(t)$ for males and females. Then, we determine the parameters of the multivariate normal distribution which models the annualized log-returns of the property indices of eight capital cities in Australia.

Part 3 is where we determine the formulae for the values of single and joint life reverse mortgages and use these to valuate the portfolio of reverse mortgages at 1/7/2016 and 1/7/2017. We find the total "death strain at risk" in respect of the portfolio at 1/7/2016. The total profit or loss to the company for the year is found and attributed across experiences in mortality and changes in values of properties.

Part 1

What is a reverse mortgage and the various types
A reverse mortgage is a loan for the elderly that allows them to use the equity in their homes as a form of income. They are referred to as "reverse mortgages" as they are opposite to traditional mortgages in that the borrower receives payments from the lender (Redstone, 2010.). Typically no repayment of the mortgage is required until the borrower dies or the home is sold. The repayment amount is the minimum of the accumulated loan and the property price at the policy end time (negative equity protection). Reverse mortgages can be of term form, where the borrower is provided with income for a specific period of time; or tenure form, where income is provided for as long as the borrower continues to occupy the property.

The 3 types of reverse mortgages are the single purpose reverse mortgages, federally-insured reverse mortgages (Home Equity Conversion Mortgage (HECM) in U.S.), and proprietary reverse mortgages.("Are there different types of reverse mortgages?", 2016.) Single purpose reverse mortgages are regulated and the money can only be used for a single purpose determined by the lender. The HECM is regulated by the U.S. Department of Housing and Urban Development and the Federal Housing Administration insures it (Redstone, 2010). There are requirements the borrower needs to meet before taking out a HECM. The borrower may choose the method of payment of loan proceeds as well as what interest rates to pay (fixed or variable). Proprietary reverse mortgages are underwritten by private lenders and tend to be more expensive and have higher interest rates. These are typically designed for borrowers with higher home values. ("Are there different types of reverse mortgages?", 2016). Proprietary lenders may be more accommodating for borrowers with special needs and/or not eligible for HECMs.("Reverse Mortgages: 3 Different Types," 2008)

Motives for obtaining a reverse mortgage
Reverse mortgages are obtained to fund retirement, cover unexpected expenses, healthcare costs or to buy a new house. Some people use single purpose reverse mortgages to pay property taxes or for home repairs. For many elderly homeowners, the equity in their homes represents their largest assets and the reverse mortgage acts as a vehicle for those who want to turn their home equity into a form of income (Redstone,2010).

Risk to lenders and borrowers and how they can be managed
Reverse mortgages expose lenders to mortality risk, house price risk, interest rate risk and collateral risk. Mortality risk arises when the borrower dies later than expected, leading to a loss to the lender due to negative equity protection. It is managed via the purchase of an annuity, this shifts the risk to the insurance company (Redstone, 2010). Economic and real estate cycle risks can be managed by strategic adjustments based on the level of market activity anticipated during an approaching cycle phase (Tappan, 1992). In the US, collateral risk is shifted to the federal government under the FHA program.

Reverse mortgages expose borrowers to interest rate risk, reducing the estate value for immediate family members. Reverse mortgage contracts are usually complicated and difficult to understand. There is also a high cost involved, including mortgage insurance premium, third party charges, loan origination fee and servicing fee.(Folger, 2016) There has been an increase in scams involving reverse mortgages, putting consumers at risk. The eligibility of a pension might also be affected. Experts advise people to think carefully before taking out a reverse mortgage and also consider other types of loans.

History of reverse mortgages
The first reverse mortgage originated in 1961 in Maine, United States. It was written by Nelson Haynes of Deering Savings Loan to Nellie Young, a widow. It provided her with an income and at the same time allowed her to stay in her home despite the loss of her husbands' income. Throughout the 1970s, many other private banks offered products similar to the reverse mortgage but adoption rates never took off. It was only in late 1987 that the FHA insured these loans following the Housing and Community Development Act and the market for reverse mortgages improved.("History of reverse mortgages," 2015)

Market for reverse mortgages in America
America has an aging population and there will be an increase in people aged 65 or older with no mortgage debt, representing over $4 trillion in home equity. Borrowers of HECMs must be at least 62 years old with the homes that they borrow against being their primary residence that fulfills FHA standards. Borrowers have to go for counselling sessions conducted by approved HECM counsellors and pass financial assessments to ensure that they have sufficient residual income and good credit history to keep up with payments of property taxes and insurance. Under the new guidelines revised by FHA on 25 April 2014, spouses of borrowers who are younger than 62 years old are allowed to enter into the contract which allows them to continue staying in their homes after the borrowers' deaths as long as they continue paying property taxes and homeowners insurance while being able to maintain their homes in satisfactory conditions.

Market for reverse mortgages in Europe
Before the exit of Britain from the European Union, a rising trend in home ownership rate and home prices was observed. However, the exact market size for European reverse mortgage is hard to be accurately estimated as there is no central collection of reverse mortgage statistics and the markets interact in complicated ways within each country. (Doling & Overton, 2010) European countries are having zero to negative population growth these years. The number of elderly who collects pensions grow much faster than the number of young people who are paying taxes. More outflows than inflows into the government fund implies that the government may no longer be able to pay the large sum of pension benefits for retirees to finance their rising costs of living. Henceforth, there would be more senior residents taking up reverse mortgages as their stable source of income. (Huan & Mahoney, 2002))

In Europe, reverse mortgage applications are mostly open to seniors aged at least 62 and reside in homes with satisfactory appraised values. However, in some European countries, borrowers must be 1 or 2 years older than 62 years old. ("About the European banking and reverse mortgage guide," n.d.)

Various approaches to valuing a reverse mortgage
Some of the frameworks that could be used to simulate reverse mortgage cash flows and analyse lender's net

financial position for valuation of reverse mortgages include:

1. Multi-State Markov Termination Model: There are 4 ways that reverse mortgages can be terminated: prepayment, refinancing, death and move-out to nursery home for long-term care. Hence, the probability of termination could be modelled with a multi-state Markov termination model accounting for these 4 reasons.

2. Vector Autoregressive-Based Economic Scenario Generation / Derive Stochastic DFs: Vector autoregressive method could be used to jointly model house prices, interest rates and other relevant economic variables besides projecting economic scenarios and deriving stochastic discount factors. (Cho & Hanewald Sherris, 2013)

Other relevant aspects
Reverse mortgages were introduced in Australia in the early 1980s. The growth of the Australian reverse mortgage market can be attributed to the increasing commercial viability of products and a change in attitudes towards inheritance and the desire to supplement income and fund a more comfortable retirement (Bridge, Mathews, Phibbs and Adams, 2009). The 'Senior Australians Equity Release Association of Lenders (SEQUAL)' was established in 2004 to protect borrowers by having accreditation standards and a code of conduct. Lenders and brokers have higher disclosure requirements following the official regulation of reverse mortgages in 2011 as part of the National Consumer Credit Protection code. In 2012, statutory 'negative equity protection' was introduced on all new reverse mortgage contracts.

Despite these developments, reverse mortgages are seen as risky and unpopular in Australia. There is now a focus to better handle Australia's aging population and convert the equity to income for retirees. Some proposals made by AustralianSuper include: developing new financial products, such as the home equity release product and improvements in quality and design of available annuities and pooled-risk products. (Rose, 2016).

Part 2

2(a) The Lee-Carter model is $\log m_x(t) = a_x + b_x t + X(t)$, where $X(0) = 0$, and $X(t) = X(t-1) + \gamma Z_t$ where $Z_t \sim N(0,1)$. Approximate $m_x(t)$ for both males and females across all ages from 1921 ($t = 0$) to 2011, where $m_x = \frac{d_x}{L_x} = \frac{l_x - l_{x+1}}{\int_0^1 l_{x+t}dt} \approx \frac{l_x - l_{x+1}}{l_{x+\frac{1}{2}}} \approx \frac{l_x - l_{x+1}}{\frac{1}{2}(l_x + l_{x+1})}$ assuming Uniform Distribution of Deaths so l_x is a linear function. We ignored the m_x when it was undefined under the assumption, which occured for older ages where there were too few people alive to determine an accurate mortality rate. Parameters a_x are determined by finding $\log m_x(0)$.

As $\Delta \log m_x(t) = \log m_x(t) - \log m_x(t-1) = b_x + X(t) - X(t-1) = b_x + \gamma Z_t$, therefore
$b_x = \mathbf{E}\left(\Delta \log m_x(t)\right) \approx \frac{1}{T}\sum_t \Delta \log m_x(t)$. We do not include observations where $\log m_x(t)$ is undefined. From Figures 1 and 2 in the Appendix, a_x and b_x show an increasing trend for both males and females.

From the given formula $X(t) = X(t-1) + \gamma Z_t$, we get $\gamma^2 Var(Z_t) = Var\left(X(t) - X(t-1)\right)$
$\gamma^2 = Var\left(X(t) - X(t-1)\right)$ as $Z_t \sim N(0,1)$
$\gamma^2 = Var\left(\Delta \log m_x(t) - b_x\right)$
$\therefore \gamma^2 = E\left(\Delta \log m_x(t) - b_x\right)^2$ (We have excluded undefined terms for greater accuracy of the parameter)

γ is estimated by taking the square root of the average of the squared deviations of $\Delta\, log\ m_x(t) - b_x$ over all x and t. Finally $Z(t) = \frac{X(t) - X(t-1)}{\gamma}$ where we take the average over all x at each time t and ρ is computed as the correlation between $Z^{male}(t)$ and $Z^{female}(t)$ with Excel CORREL function. Refer to Table 1 for male parameters, Table 2 for female, and Table 3 for $Z(t)$ and correlation.

2(b) Assuming the quarterly log-returns of the property index are normally distributed, $\log\left(\frac{S(t+\frac{1}{4})}{S(t)}\right) \sim N(\frac{1}{4}\mu, \frac{1}{4}\sigma^2)$ Therefore, $\log\left(S(t+\frac{1}{4})\right) = \log\left(S(t)\right) + \frac{1}{4}\mu + \frac{1}{2}\sigma Z$ where $Z \sim N(0,1)$.

The mean for annualised log returns is calculated by the average of the log-return and multiplying by 4, and the standard deviation is calculated by the square root of variance, which is found by the Excel built in function of VAR.S and multiplying by 4. We chose to work with the sample variance as a more conservative estimate as our sample is small. These values are shown in Table 4.
To find the variance-covariance matrix: in each entry (i,j), we have $Cov(i,j) = \rho_{i,j}\sigma_i\sigma_j$ as the covariance between city i and j. In order to find the correlation factor, we can use the built in function of CORREL. These values can be located in Table 5.

Part 3

We define the value of the portfolio as the expected present value of all possible future cash flows. All formulae derivations are given in the Appendix.

3(a) Single Life Case Suppose we are at time t and a life dies at time $\tau > t$. The accumulated loan value at time τ is $K(\tau)$, where, $K(\tau) = L\exp(r_\ell(\tau - t_0))$

From our model in 2b), the price of the property at time τ is: $S(\tau) = S(t)\exp(\mu(\tau-t) + \sigma\sqrt{(\tau-t)} \times Z)$, where $Z \sim N(0,1)$, μ and σ is the mean and standard deviation of the annual log-returns of the property index with respect to the property location.
Repayment of the reverse mortgage is $\min\left\{K(\tau), S(\tau)\right\}$ which is undetermined at time t, so we calculate $E(\min\left\{K(\tau), S(\tau)\right\}) = S(t)\exp(\mu(\tau-t) + \frac{1}{2}\sigma^2(\tau-t))N(z^* - \sigma\sqrt{(\tau-t)} + K(1-N(z^*))$

Where $z^* = \frac{\log K(\tau) - \log S(t) - \mu\times(\tau-t)}{\sigma\sqrt{\tau-t}}$.
To turn τ stochastic, we make some simplifying assumptions.
1. Assume an age definition of age nearest birthday at 1st July. (Allows us to round age to nearest integer)
2. Assume if lives die, they die in the middle of age interval. (i.e. 31st December when payment is due)
3. Assume $\gamma = 0$ when projecting mortality rates with the Lee-Carter model.

The formula for single life reverse mortgage is therefore:
$V^{(RM)}(t) = \sum_{\tau=t}^{\infty} \exp(-r_d(\tau + \frac{1}{2} - t))\Pr(K_{x+t} = \tau - t) \times E\left(\min\left\{K(\tau+\frac{1}{2}), S(\tau+\frac{1}{2})\right\}\right)$
$V^{(RM)}(t) = \sum_{\tau=t}^{\infty} \exp(-r_d(\tau + \frac{1}{2} - t))_{\tau-t}p_{x+t}(s)q_{x+\tau}(s+\tau-t) \times E\left(\min\left\{K(\tau+\frac{1}{2}), S(\tau+\frac{1}{2})\right\}\right)$
Where ${}_{\tau-t}p_{x+t}(s) = \exp(-\sum_{j=0}^{\tau-t-1} m_{x+t+j}(s+j))$ and $q_{x+\tau}(s+\tau-t) = 1 - \exp(-m_{x+\tau}(s+\tau-t))$

3(b) Joint Life Case The repayment of loan occurs on the death of the last survivor of borrowers. The valuation is similar to the single life case, the only difference is the calculation of the relevant probabilities as it is now regarded as a last survivor status.

Assuming that the lives x and y are independent, the last survivor status probabilities are calculated as follows:

$$
\begin{aligned}
{}_{\tau-t}p_{\overline{x+t:y+t}}(s,s)q_{\overline{x+\tau:y+\tau}}(s+\tau-t, s+\tau-t) =& {}_{\tau-t}\, p_{x+t}(s)q_{x+\tau}(s+\tau-t) + {}_{\tau-t}\, p_{y+t}(s)q_{y+\tau}(s+\tau-t) - \\
. \quad & {}_{\tau-t}p_{x+t}(s)\,{}_{\tau-t}p_{y+t}(s) \times \left(1 - p_{x+\tau}(s+\tau-t)p_{y+\tau}(s+\tau-t)\right)
\end{aligned}
$$

The formula for joint life reverse mortgage is therefore:

$$V^{(RM)}(t) = \sum_{\tau=t}^{\infty} \exp(-r_d(\tau+\tfrac{1}{2}-t))\,{}_{\tau-t}p_{\overline{x+t:y+t}}(s,s)\,q_{\overline{x+\tau:y+\tau}}(s+\tau-t,s+\tau-t)\times E\left(\min\left\{K(\tau+\tfrac{1}{2}),S(\tau+\tfrac{1}{2})\right\}\right)$$

3(c) Applying the two formulas derived above, the total value of the portfolio at 1/7/2016 is **\$16,715,175.60**
To calculate DSAR, consider a life aged (x) at 2016, and the recursive formula: ${}_{2016}V \times \exp(0.07) = q_x S \exp(0.07*0.5) + p_x({}_{2017}V)$, where ${}_{2016}V$ is the provision and we take it to be the negative reverse mortgage value at 2016. S is the (negative) payment to the lender if the life dies during the year. We therefore calculate the expected provision at 2017 as:
${}_{2017}V = \frac{{}_{2016}V\times\exp(0.07)-q_x S\exp(0.07*0.5)}{p_x}$ Where $(-{}_{2017}V)$ is **Expected 2017 Value** of the reverse mortgage. Total $(-{}_{2017}V)$ was calculated to be \$17,955,762.53, or provision at 2017 of ${}_{2017}V = -\$17,955,762.53$
The DSAR = $S\exp(0.07*0.5) - {}_{2017}V$. We calculate DSAR for each life and take the sum to obtain a total DSAR for the year being: **\$1,772,612.10**.

3(d) In 2017, life status associated with a reverse mortgage may change. The three cases are:
Case 1. Life status is the same (Compute in the same way as 2016 value)
Case 2. Contract has terminated (Payment received, however it is not included in the value of the portfolio.)
Case 3. Joint life where during the year, one life passes away and the other is alive. This is treated as a single life reverse mortgage in the 2017 portfolio.
Applying the formulas to these three cases (with projected mortality rates from 2017 onwards), the portfolio value at 1/7/2017 is calculated to be: **\$16,208,811.40**
The profit/loss for the year is (Total actual 2017 value − Total expected 2017 value) = −**\$75,948.80** . Where Total Actual value = Portfolio value + Payments from terminated contracts accumulated to 1/7/2017
Total Expected value= Total $(-{}_{2017}V)$ from the recursive formula above. We may attribute this to experiences in mortality and changes in property values (as forecasted probabilities are linear and not stochastic)
Actual - Expected 2017 value for **Case 1.** = Profit/loss due to change in property value =**\$11,339.65**
Actual - Expected 2017 value for **Case 2.** = Profit/loss due to mortality experiences = −**\$62,458.94**
Actual - Expected 2017 value for **Case 3.** = −**\$24,829.54**, however this value can be due to both mortality experience and changes in property value.
Mortality Profit = EDS-ADS. Where EDS $= \sum q_x \times DSAR$ over all policy holders and ADS $= \sum DSAR$ for contracts that have been terminated. So Mortality Profit for the year is $= EDS - ADS = \$28,600.0 - \$62,458.9 = -$ **\$33,859.0**

Conclusion

We fitted the Lee-Carter model on the mortality rates and found similar increasing a_x and b_x trends for both genders. After fitting the multivariate normal model on property log-returns, and projecting mortality rates, we obtained portfolio values and a positive DSAR. This is expected as borrowing rates are much higher than lending rates, so it is more beneficial for lives to live longer. Over the year, there was a loss of \$75,948.80. The figure emphasises the mortality risk and house price risk to the lender. The loss is mainly due to mortality experiences, and with a mortality profit being negative, this suggests that mortality rates may not be accurate. These figures need to be taken with caution, as the assumptions made to compute the value do not reflect reality. Our fitted Lee-Carter model shows positive gamma values and high correlation between residuals which contradict our assumptions, however this is for the purpose of obtaining an approximate valuation.

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Appendix

Figure 1. a_x and b_x for males

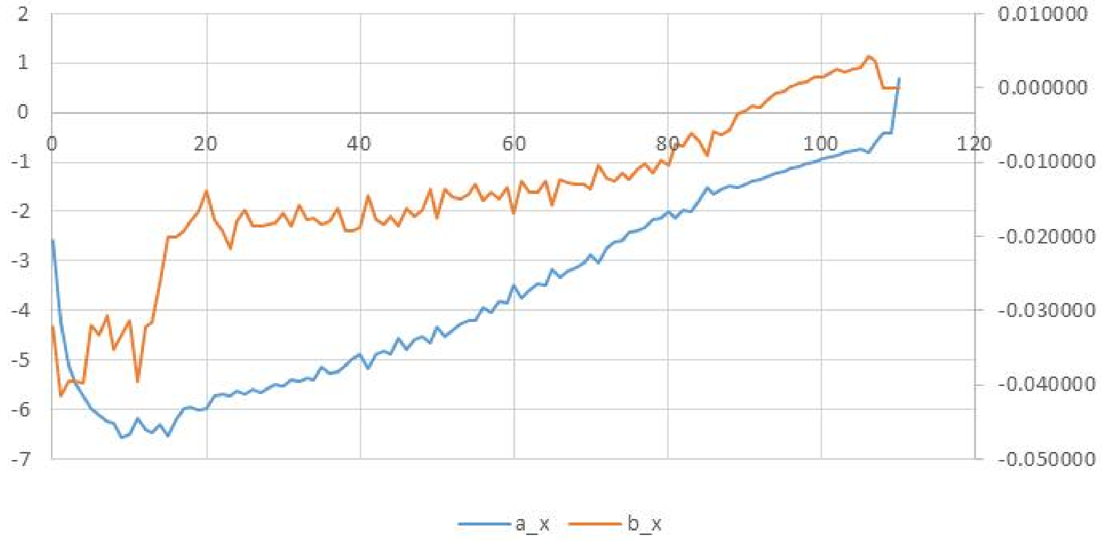

Figure 2. a_x and b_x for females

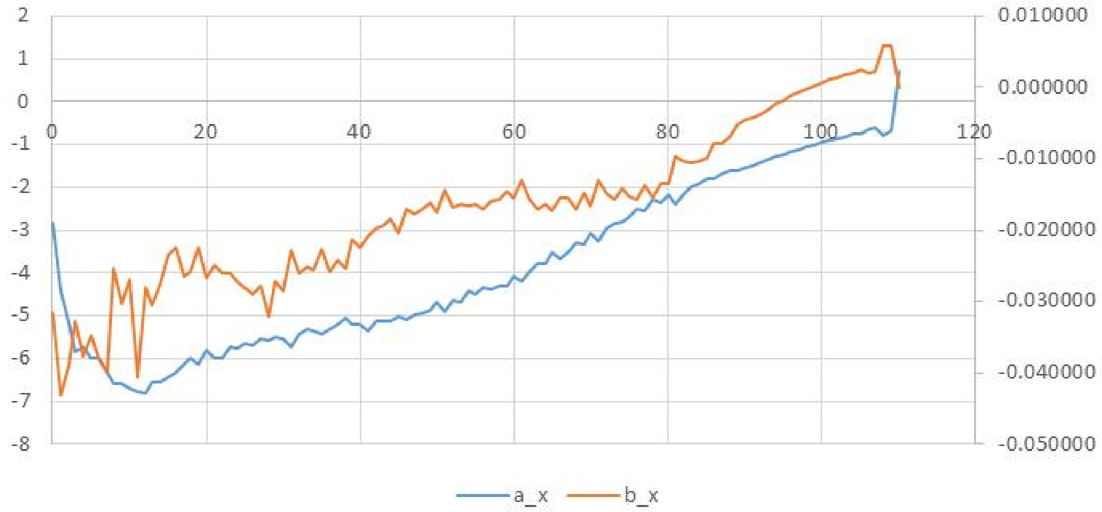

Table 1. Male a_x, b_x and gamma

age	ax	bx	age	ax	bx	age	ax	bx	gamma_m
0	-2.58896	-0.0321	37	-5.22783	-0.01621	74	-2.57042	-0.01151	0.1395751
1	-4.2445	-0.04153	38	-5.11405	-0.01934	75	-2.42277	-0.01238	
2	-5.1087	-0.03963	39	-4.97895	-0.01932	76	-2.3913	-0.01109	
3	-5.47251	-0.03945	40	-4.88547	-0.01893	77	-2.321	-0.01031	
4	-5.728	-0.03981	41	-5.15936	-0.01454	78	-2.15068	-0.01157	
5	-5.98816	-0.03201	42	-4.88045	-0.01785	79	-2.14641	-0.00986	
6	-6.11303	-0.0333	43	-4.8049	-0.01838	80	-1.99645	-0.01039	
7	-6.25097	-0.0308	44	-4.8866	-0.01738	81	-2.12869	-0.00757	
8	-6.26068	-0.03519	45	-4.56805	-0.01867	82	-1.9801	-0.00782	
9	-6.55649	-0.03339	46	-4.77089	-0.01616	83	-2.00907	-0.00607	
10	-6.49394	-0.03129	47	-4.57713	-0.01729	84	-1.79076	-0.00722	
11	-6.1663	-0.03944	48	-4.51271	-0.01653	85	-1.51187	-0.00903	
12	-6.41193	-0.0322	49	-4.64911	-0.01375	86	-1.63966	-0.00581	
13	-6.47396	-0.03151	50	-4.33365	-0.01748	87	-1.53593	-0.00641	
14	-6.3171	-0.02619	51	-4.52012	-0.01364	88	-1.46754	-0.00557	
15	-6.5231	-0.02006	52	-4.39354	-0.01468	89	-1.50876	-0.00348	
16	-6.2015	-0.0201	53	-4.26406	-0.015	90	-1.45263	-0.00318	
17	-5.98846	-0.01926	54	-4.21207	-0.01435	91	-1.39755	-0.00238	
18	-5.96355	-0.01789	55	-4.18839	-0.01308	92	-1.34279	-0.00259	
19	-6.025	-0.01678	56	-3.95426	-0.01518	93	-1.28823	-0.00163	
20	-5.99464	-0.01386	57	-4.03243	-0.01406	94	-1.23616	-0.00065	
21	-5.72376	-0.01767	58	-3.82654	-0.01503	95	-1.18599	-0.00039	
22	-5.69962	-0.01918	59	-3.85062	-0.01354	96	-1.13286	5.16E-05	
23	-5.73487	-0.02155	60	-3.48347	-0.01678	97	-1.08727	0.000525	
24	-5.61047	-0.01794	61	-3.75338	-0.01248	98	-1.0343	0.000873	
25	-5.69975	-0.01649	62	-3.57855	-0.01399	99	-1.0026	0.001426	
26	-5.59085	-0.01862	63	-3.45564	-0.01417	100	-0.93156	0.001452	
27	-5.6418	-0.01871	64	-3.4805	-0.01267	101	-0.89382	0.001837	
28	-5.56187	-0.0183	65	-3.15203	-0.01582	102	-0.88239	0.002466	
29	-5.48421	-0.01814	66	-3.33024	-0.01242	103	-0.79113	0.002137	
30	-5.53866	-0.01698	67	-3.20329	-0.01272	104	-0.75911	0.002539	
31	-5.38891	-0.01863	68	-3.12275	-0.01301	105	-0.72824	0.002664	
32	-5.42155	-0.01589	69	-3.03481	-0.01293	106	-0.81093	0.004206	
33	-5.36693	-0.01785	70	-2.88417	-0.0136	107	-0.60614	0.003671	
34	-5.41244	-0.01757	71	-3.03683	-0.01042	108	-0.40547	-8.4E-18	
35	-5.14077	-0.01845	72	-2.74493	-0.01205	109	-0.40547	-3E-17	
36	-5.27266	-0.01805	73	-2.631	-0.01265	110	0.693147	0	

Table 2: Female a_x, b_x and gamma

female									
age	ax	bx	age	ax	bx	age	ax	bx	gamma_f
0	-2.8302	-0.0317	37	-5.1928	-0.0242	74	-2.7918	-0.0142	0.157483
1	-4.4179	-0.0430	38	-5.0375	-0.0254	75	-2.6866	-0.0152	
2	-5.1530	-0.0391	39	-5.2135	-0.0213	76	-2.5170	-0.0157	
3	-5.8474	-0.0328	40	-5.1839	-0.0223	77	-2.5583	-0.0137	
4	-5.7278	-0.0376	41	-5.3328	-0.0208	78	-2.2842	-0.0154	
5	-5.9863	-0.0347	42	-5.1372	-0.0198	79	-2.3740	-0.0134	
6	-5.9968	-0.0381	43	-5.1376	-0.0194	80	-2.1673	-0.0135	
7	-6.3120	-0.0399	44	-5.1129	-0.0184	81	-2.4127	-0.0097	
8	-6.5782	-0.0255	45	-5.0010	-0.0203	82	-2.1707	-0.0103	
9	-6.5927	-0.0302	46	-5.0878	-0.0171	83	-1.9926	-0.0106	
10	-6.6923	-0.0269	47	-4.9843	-0.0177	84	-1.9283	-0.0103	
11	-6.7551	-0.0406	48	-4.9375	-0.0171	85	-1.7978	-0.0099	
12	-6.7924	-0.0280	49	-4.8816	-0.0162	86	-1.8168	-0.0079	
13	-6.5563	-0.0306	50	-4.6968	-0.0175	87	-1.6899	-0.0079	
14	-6.5471	-0.0275	51	-4.9099	-0.0144	88	-1.6314	-0.0070	
15	-6.4364	-0.0236	52	-4.6234	-0.0168	89	-1.6165	-0.0052	
16	-6.3116	-0.0224	53	-4.6905	-0.0164	90	-1.5492	-0.0045	
17	-6.1365	-0.0264	54	-4.4326	-0.0167	91	-1.4829	-0.0042	
18	-6.0000	-0.0259	55	-4.4791	-0.0164	92	-1.4187	-0.0038	
19	-6.1216	-0.0224	56	-4.3336	-0.0170	93	-1.3553	-0.0032	
20	-5.8150	-0.0266	57	-4.3847	-0.0160	94	-1.2937	-0.0022	
21	-5.9745	-0.0248	58	-4.2870	-0.0158	95	-1.2324	-0.0018	
22	-5.9764	-0.0261	59	-4.2867	-0.0146	96	-1.1743	-0.0012	
23	-5.7276	-0.0260	60	-4.0841	-0.0155	97	-1.1167	-0.0007	
24	-5.7632	-0.0272	61	-4.1880	-0.0131	98	-1.0584	-0.0002	
25	-5.6477	-0.0281	62	-3.9516	-0.0156	99	-1.0014	0.0002	
26	-5.6769	-0.0290	63	-3.7882	-0.0171	100	-0.9520	0.0007	
27	-5.5281	-0.0279	64	-3.7749	-0.0164	101	-0.9111	0.0012	
28	-5.5653	-0.0322	65	-3.5348	-0.0173	102	-0.8473	0.0014	
29	-5.4975	-0.0272	66	-3.6585	-0.0154	103	-0.8109	0.0018	
30	-5.5336	-0.0285	67	-3.5069	-0.0155	104	-0.7538	0.0020	
31	-5.7229	-0.0230	68	-3.2903	-0.0171	105	-0.7419	0.0025	
32	-5.4208	-0.0261	69	-3.3434	-0.0149	106	-0.6419	0.0019	
33	-5.3208	-0.0252	70	-3.0783	-0.0166	107	-0.6061	0.0022	
34	-5.3501	-0.0256	71	-3.2511	-0.0131	108	-0.7732	0.0058	
35	-5.4307	-0.0227	72	-2.9743	-0.0149	109	-0.6931	0.0059	
36	-5.3020	-0.0259	73	-2.8439	-0.0156	110	0.6931	0.0000	

Table 3: Z(t) and Correlation

Z(t) (Male)				Z(t) (Female)				
t	Z(t)_m	t	Z(t)_m	t	Z(t)_f	t	Z(t)_f	CORREL
1	0.0022	46	-0.2509	1	-0.1400	46	-0.2319	0.760476
2	0.3833	47	0.5463	2	0.7000	47	0.5016	
3	-0.1068	48	-0.2749	3	-0.4295	48	-0.3366	
4	-0.1683	49	0.3384	4	-0.3377	49	0.4588	
5	0.3610	50	-0.1721	5	0.3249	50	-0.2688	
6	0.0074	51	-0.0758	6	0.2991	51	-0.0503	
7	-0.0011	52	-0.0649	7	0.2125	52	-0.1009	
8	0.4892	53	0.3718	8	0.1193	53	0.1881	
9	-0.8850	54	-0.3696	9	-0.7804	54	-0.3268	
10	0.1308	55	0.0231	10	0.2031	55	0.1152	
11	-0.0226	56	-0.2139	11	0.1032	56	-0.1898	
12	0.1356	57	-0.2543	12	0.1403	57	0.0055	
13	0.4949	58	-0.0174	13	0.2028	58	-0.2894	
14	0.0537	59	-0.1143	14	0.0395	59	-0.0871	
15	-0.0781	60	0.1213	15	-0.0088	60	-0.0090	
16	0.1256	61	0.2035	16	-0.1230	61	0.1978	
17	0.0264	62	-0.5975	17	0.2080	62	-0.2543	
18	0.4069	63	0.0922	18	0.1025	63	0.2215	
19	-0.3208	64	0.1923	19	-0.3758	64	0.1624	
20	0.0083	65	-0.3448	20	0.3161	65	-0.2664	
21	0.1301	66	0.1012	21	0.4402	66	0.0884	
22	-0.2356	67	-0.0293	22	-0.0234	67	0.0144	
23	-0.7911	68	0.3168	23	-0.6177	68	0.2141	
24	-0.0160	69	-0.5401	24	-0.1436	69	-0.3790	
25	0.7260	70	-0.2749	25	0.1256	70	-0.0639	
26	-0.1025	71	0.1599	26	-0.2968	71	0.1444	
27	0.1982	72	-0.1474	27	-0.0155	72	-0.1474	
28	-0.2373	73	0.3618	28	-0.3451	73	0.1790	
29	0.1900	74	-0.2075	29	0.3406	74	0.1225	
30	0.5050	75	0.1059	30	0.0859	75	-0.0326	
31	-0.3306	76	-0.1090	31	-0.1071	76	0.1146	
32	-0.0978	77	0.0247	32	-0.2057	77	-0.1122	
33	0.0645	78	-0.1281	33	-0.0749	78	-0.0133	
34	-0.0588	79	-0.0754	34	-0.1591	79	-0.0060	
35	0.2444	80	-0.2549	35	0.1695	80	-0.3797	
36	-0.0771	81	0.0199	36	-0.0356	81	0.1845	
37	-0.1689	82	-0.1963	37	-0.2489	82	0.0293	
38	0.2678	83	-0.0537	38	0.3149	83	-0.1465	
39	-0.1137	84	-0.2095	39	-0.0768	84	-0.2003	
40	0.0799	85	-0.0925	40	-0.1127	85	0.1576	
41	0.1406	86	0.1375	41	0.1575	86	0.1769	
42	0.0565	87	-0.0352	42	0.1206	87	-0.1751	
43	0.4079	88	-0.0543	43	0.3704	88	0.1403	
44	-0.0417	89	-0.1533	44	-0.0422	89	-0.1311	
45	0.1188	90	-0.0673	45	0.1568	90	0.2062	

Table 4. μ and σ of the cities

	Sydney	Melbourne	Brisbane	Adelaide	Perth	Hobart	Darwin	Canberra
Mu	0.0605	0.0725	0.0723	0.0664	0.0746	0.0776	0.0773	0.0644
Sigma	0.0548	0.0501	0.0554	0.0434	0.0649	0.0662	0.0477	0.0516

Table 5. Variance-Covariance Matrix

	Sydney	Melbourne	Brisbane	Adelaide	Perth	Hobart	Darwin	Canberra
Sydney	0.000750	0.000509	0.000401	0.000283	0.000082	0.000258	0.000001	0.000405
Melbourne	0.000509	0.000627	0.000469	0.000392	0.000219	0.000375	0.000075	0.000467
Brisbane	0.000401	0.000469	0.000768	0.000506	0.000313	0.000740	0.000159	0.000592
Adelaide	0.000283	0.000392	0.000506	0.000471	0.000213	0.000515	0.000148	0.000421
Perth	0.000082	0.000219	0.000313	0.000213	0.001052	0.000494	0.000468	0.000340
Hobart	0.000258	0.000375	0.000740	0.000515	0.000494	0.001095	0.000233	0.000610
Darwin	0.000001	0.000075	0.000159	0.000148	0.000468	0.000233	0.000569	0.000116
Canberra	0.000405	0.000467	0.000592	0.000421	0.000340	0.000610	0.000116	0.000665

Part 3 Derivation:
Suppose we are at time t. Assuming life aged $(x+t)$ dies at time τ . We can work out the payment $E(\min\left\{K(\tau),S(\tau)\right\})$ by writing
$S(\tau)=S(t)\times\exp(\mu\times(\tau-t)+\sigma\sqrt{\tau-t}\times Z)$ where $Z\sim N(0,1)$
Then we have $S(\tau)>K(\tau)$ when

$$S(t)\times\exp(\mu\times(\tau-t)+\sigma\sqrt{\tau-t}\times Z)>K(\tau)$$

i.e. when

$$Z>\frac{\log K-\log S(t)-\mu\times(\tau-t)}{\sigma\sqrt{\tau-t}}=z^*$$

Therefore $E(\min\left\{K(\tau),S(\tau)\right\})$ is:

$\int_{-\infty}^{\infty}\min\left\{K(\tau),S(\tau)\right\}\times\frac{1}{\sqrt{2\pi}}\exp(-\frac{1}{2}z^2)dz$

$$=\int_{-\infty}^{z^*}S(\tau)\frac{1}{\sqrt{2\pi}}\exp(-\frac{1}{2}z^2)dz+\int_{z^*}^{-\infty}K(\tau)\frac{1}{\sqrt{2\pi}}\exp(-\frac{1}{2}z^2)dz$$
$$=S(t)\exp\left(\mu\times(\tau-t)\right)\int_{-\infty}^{z^*}\exp(\sigma\sqrt{\tau-t}z)\frac{1}{\sqrt{2\pi}}\exp(-\frac{1}{2}z^2)dz+K(\tau)\times(1-N(z^*))\text{and by completing the square,}$$
$$=S(t)\exp\left(\mu\times(\tau-t)+\frac{1}{2}\sigma^2(\tau-t)\right)\int_{-\infty}^{z^*}\frac{1}{\sqrt{2\pi}}\exp(-\frac{1}{2}(z-\sigma\sqrt{\tau-t})^2)dz+K(\tau)\times(1-N(z^*))$$
$$=S(t)\exp\left(\mu\times(\tau-t)+\frac{1}{2}\sigma^2(\tau-t)\right)N(z^*-\sqrt{\tau-t})+K(\tau)\times(1-N(z^*)).$$

Under our assumptions, if death occurs between time τ and $\tau+1$, life will die at time $\tau+\frac{1}{2}$ and therefore the associated payment is: $E\left(\min\left\{K(\tau+\frac{1}{2}),S(\tau+\frac{1}{2})\right\}\right)$. Our situation here is almost identical to a whole life assurance policy with formula A_x, however instead of receiving 1 at the end of year of death, we are receiving: $E\left(\min\left\{K(\tau+\frac{1}{2}),S(\tau+\frac{1}{2})\right\}\right)$ at time $\tau+\frac{1}{2}$. The probability of this payment is thus the curtate lifetime probability: $Pr(K=\tau)$.

$$Pr(K=\tau) = {}_{\tau-t}\, p_{x+t}(s) q_{x+\tau}(s+\tau-t)$$

To obtain life probabilities in 2016, we used the Lee-Carter model fitted in Part 2 to project mortality rates from 2011. Assuming $\gamma=0$ we have $X(t)$ being the same for all times after 2011. We thus project mortality rates:

$$\begin{aligned}\log(m_x(s)) - \log(m_x(u)) &= b_x(s-u)\\ m_x(s) &= m_x(u)\exp(b_x(s-u))\end{aligned}$$

And compute survival and death probabilities

$$\begin{aligned}{}_{\tau-t}p_{x+t}(s) &\approx \exp\left(-\sum_{j=o}^{\tau-t-1} m_{x+t+j}(s+j)\right)\\ q_{x+\tau}(s+\tau-t) &\approx 1-\exp(-m_{x+\tau}(s+\tau-t))\end{aligned}$$

We therefore can compute the expected present value for a life aged $(x+t)$ at year (s)

$$V^{(RM)}(t) = \sum_{\tau=t}^{\infty} \exp(-r_d(\tau+\frac{1}{2}-t))_{\tau-t}p_{x+t}(s) q_{x+\tau}(s+\tau-t) \times E\left(\min\left\{K(\tau+\frac{1}{2}),S(\tau+\frac{1}{2})\right\}\right)$$

For the joint life case, where repayment of loan occurs on later of the deaths, which means it is paid on the failure of a last survivor status. This is similar to $A_{\overline{x:y}}$ where for life x and y

$$\begin{aligned}A_{\overline{x:y}} &= A_x + A_y - A_{xy}\\ &= \sum_{\tau=0} ({}_\tau p_x q_{x+\tau} + {}_\tau\, p_y q_{y+\tau} - {}_\tau\, p_{xy} q_{x+\tau:y+\tau}) v^{\tau+1}\\ &= \sum_{\tau=0} ({}_\tau p_x q_{x+\tau} + {}_\tau\, p_y q_{y+\tau} - {}_\tau\, p_{x\tau} p_y (1-p_{x+\tau} p_{y+\tau})) v^{\tau+1} \quad \text{assuming independent lives}\end{aligned}$$

Drawing from this, the only difference between the single and joint reverse mortgage is the probability. The valuation method remains the same.