

FFT's

Fast Fourier Transform
Algorithms

What is a Fourier Transform

Well What is a Transform?

In this specific case it maps a function from its original function space into another function space via integration, where some of the properties of the original function might be more easily characterized and manipulated than in the original function space.

So what is a Fourier Transform?

A transform that decomposes a function into its constituent sinusoidal frequencies. Given the fact any function can be reproduced by adding sinusoidal functions together this can be useful for making sense of complex functions.

A great example of this is interpreting the waveform (signal or amplitude over time) of a musical chord and expressing it instead in terms of the volumes and frequencies of its constituent notes.

$$F(w) := F\{f(t)\} := \int_{-\infty}^{\infty} e^{-iwt} f(t) dt$$

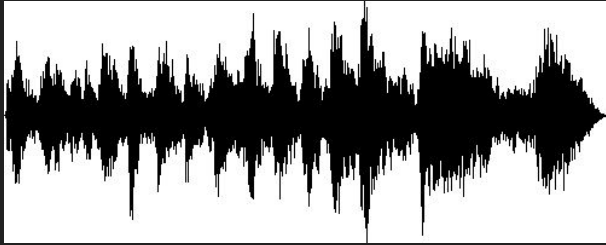
The function

$$F^{-1}\{F(w)\} := \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{iwx} F(w) dw$$

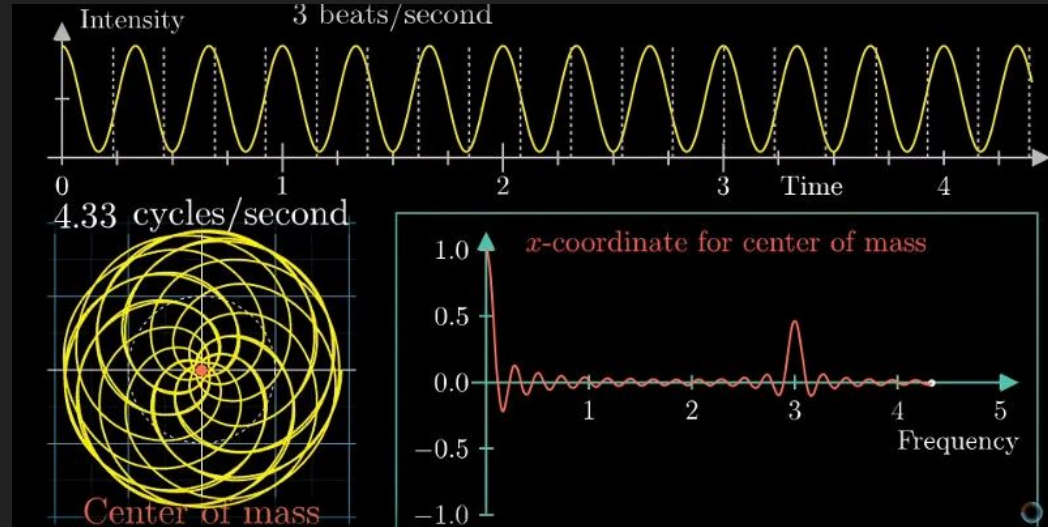
Is called an inverse Fourier transform of $F(w)$.

Some clarification...

Up top and to the right here we have a simple sine wave.
The sound we hear is generally made up of many of these
layered together such that it results in something a little
like this below.



This can be hard to make sense of and interpret but if we
go back to the simple sine wave and apply a fourier
transform to it you can see off to our right a visible spike
highlighting the frequency of the sine wave above. You
apply this a more complex signal and what you get is a
series of spikes that indicate the volume of all the
contributing sinusoidal frequencies that make up the
complete sound.



Applications

FFT's get used in digital recording, sampling, additive synthesis and pitch correction software

The FFT's importance derives from the fact that it has made working in the frequency domain equally computationally feasible as working in the temporal or spatial domain. Some of the important applications of the FFT include

Fast large-integer and polynomial multiplication

Efficient matrix-vector multiplication for Toeplitz, circulant and other structured matrices

Filtering algorithms (see overlap-add and overlap-save methods)

Fast algorithms for discrete cosine or sine transforms (e.g. fast DCT used for JPEG and MPEG/MP3 encoding and decoding)

Fast Chebyshev approximation

Solving difference equations

Computation of isotopic distributions.

So how do you pull this off as an algorithm

algorithm iterative-fft is

input: Array a of n complex values where n is a power of 2.

output: Array A the DFT of a .

bit-reverse-copy(a , A)

$n \leftarrow a.length$

for $s = 1$ **to** $\log(n)$ **do**

$m \leftarrow 2^s$

$\omega_m \leftarrow \exp(-2\pi i/m)$

for $k = 0$ **to** $n-1$ **by** m **do**

$\omega \leftarrow 1$

for $j = 0$ **to** $m/2 - 1$ **do**

$t \leftarrow \omega A[k + j + m/2]$

$u \leftarrow A[k + j]$

$A[k + j] \leftarrow u + t$

$A[k + j + m/2] \leftarrow u - t$

$\omega \leftarrow \omega \omega_m$

return A

algorithm bit-reverse-copy(a, A) is

input: Array a of n complex values where n is a power of 2.

output: Array A of size n .

$n \leftarrow a.length$

for $k = 0$ **to** $n - 1$ **do**

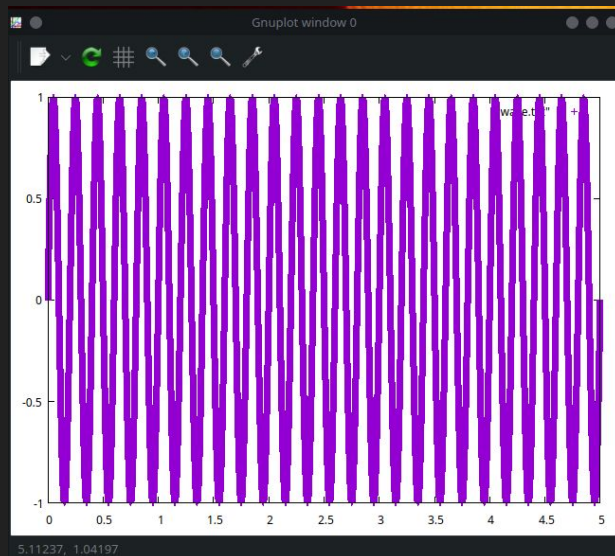
$A[\text{rev}(k)] := a[k]$

At their simplest a Fast Fourier Transform creates a discrete Fourier transform of the input by using a recursive divide and conquer approach

This approach generally results in $O(N \log N)$. There exists FFT algorithms that avoid recursion such as the iterative radix-2 variant listed to the left. This is the variation I decided to try and make parallel as it's design allows for better optimization than some of the simpler FFT's.

Use Case Demonstration

```
jkl@funRuiner:~/Documents/final_proj/src$ make
g++ -g -Wall -o serial_fft serial_fft.cpp
g++ -g -Wall -fopenmp -o omp_fft omp_fft.cpp -lm
jkl@funRuiner:~/Documents/final_proj/src$
```



```
jkl@funRuiner:~/Documents/final_proj/src$ ./serial_fft -sine 5 5
FFT performed on 5 second sample of 5 cycles per second in 0.119381412 seconds.
jkl@funRuiner:~/Documents/final_proj/src$ gnuplot
```

```
GNU PLOT
Version 5.4 patchlevel 1 last modified 2020-12-01

Copyright (C) 1986-1993, 1998, 2004, 2007-2020
Thomas Williams, Colin Kelley and many others

gnuplot home: http://www.gnuplot.info
faq, bugs, etc: type "help FAQ"
immediate help: type "help" (plot window: hit 'h')
```

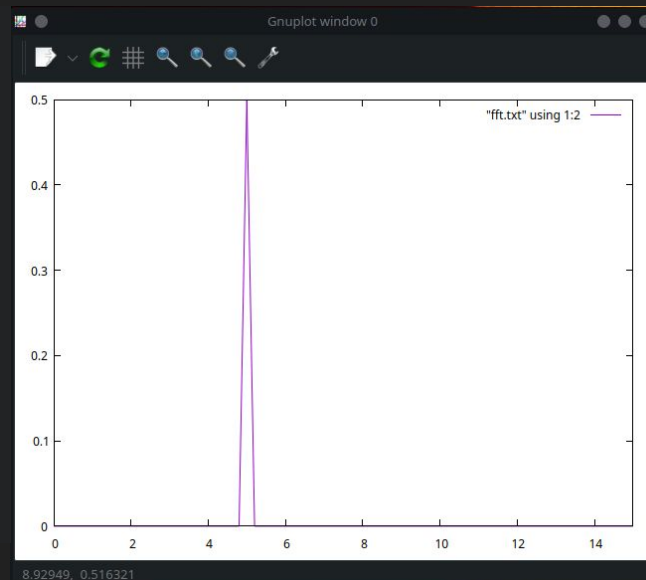
```
Terminal type is now 'qt'
gnuplot> plot "wave.txt"
gnuplot> quit
jkl@funRuiner:~/Documents/final_proj/src$ gnuplot
```

```
GNU PLOT
Version 5.4 patchlevel 1 last modified 2020-12-01

Copyright (C) 1986-1993, 1998, 2004, 2007-2020
Thomas Williams, Colin Kelley and many others

gnuplot home: http://www.gnuplot.info
faq, bugs, etc: type "help FAQ"
immediate help: type "help" (plot window: hit 'h')
```

```
Terminal type is now 'qt'
gnuplot> set xrange [0:15]
gnuplot> plot "fft.txt" using 1:2 with lines linestyle 1
gnuplot>
```

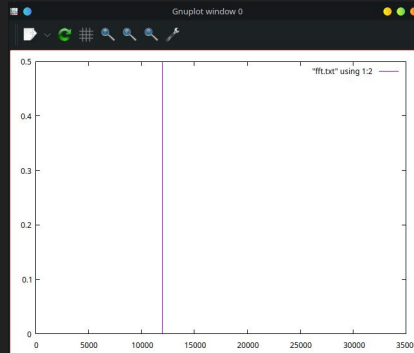
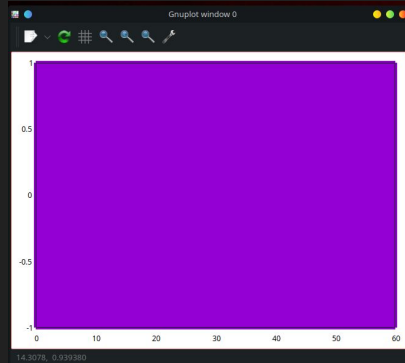


Use Case Elaborated Pt 1

```
jkl@funRuiner:~/Documents/final_proj/src$ ./serial_fft -sine 60 12000
FFT performed on 60 second sample of 12000 cycles per second in 3.627292067 seconds.
jkl@funRuiner:~/Documents/final_proj/src$ ./serial_fft -sine 60 12000
FFT performed on 60 second sample of 12000 cycles per second in 3.912764727 seconds.
jkl@funRuiner:~/Documents/final_proj/src$ ./serial_fft -sine 60 12000
FFT performed on 60 second sample of 12000 cycles per second in 3.622689726 seconds.
jkl@funRuiner:~/Documents/final_proj/src$ ./serial_fft -sine 60 12000
FFT performed on 60 second sample of 12000 cycles per second in 3.826525657 seconds.
jkl@funRuiner:~/Documents/final_proj/src$ ./serial_fft -sine 60 12000
FFT performed on 60 second sample of 12000 cycles per second in 3.858961935 seconds.
jkl@funRuiner:~/Documents/final_proj/src$ ./serial_fft -octaves 60 10000
FFT performed on 60 second sample of 10000 cycles per second octaves in 3.684357854 seconds.
jkl@funRuiner:~/Documents/final_proj/src$ ./serial_fft -octaves 60 10000
FFT performed on 60 second sample of 10000 cycles per second octaves in 3.781128308 seconds.
jkl@funRuiner:~/Documents/final_proj/src$ ./serial_fft -octaves 60 10000
FFT performed on 60 second sample of 10000 cycles per second octaves in 3.912142807 seconds.
jkl@funRuiner:~/Documents/final_proj/src$ ./serial_fft -octaves 60 10000
FFT performed on 60 second sample of 10000 cycles per second octaves in 4.77342253 seconds.
jkl@funRuiner:~/Documents/final_proj/src$ ./serial_fft -octaves 60 10000
FFT performed on 60 second sample of 10000 cycles per second octaves in 3.655579133 seconds.
jkl@funRuiner:~/Documents/final_proj/src$ █
```

No way to tell the frequency by looking at raw waveform when the cps gets too high.

Easy to see it's 12000 cycles per second now after taking the FFT



We can even identify octaves in noise!

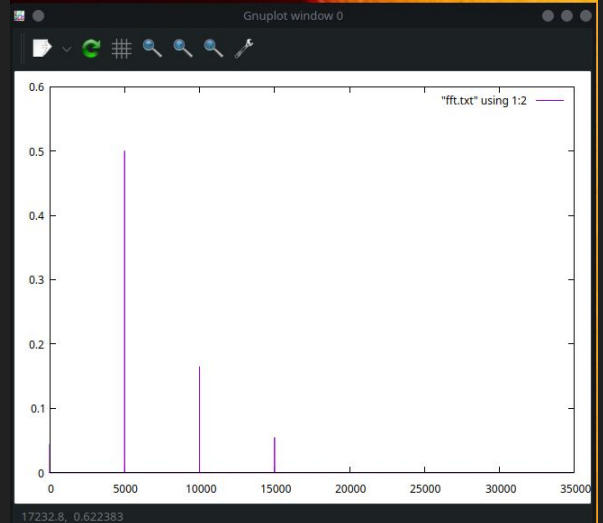
```
jkl@funRuiner:~/Documents/final_proj/src$ ./serial_fft -noisy 60 5000
FFT performed on 60 second sample of 5000 cycles per second octaves burried in noise in 3.849277698 seconds.
jkl@funRuiner:~/Documents/final_proj/src$ ./serial_fft -noisy 60 5000
FFT performed on 60 second sample of 5000 cycles per second octaves burried in noise in 3.828664707 seconds.
jkl@funRuiner:~/Documents/final_proj/src$ ./serial_fft -noisy 60 5000
FFT performed on 60 second sample of 5000 cycles per second octaves burried in noise in 3.664780936 seconds.
jkl@funRuiner:~/Documents/final_proj/src$ ./serial_fft -noisy 60 5000
FFT performed on 60 second sample of 5000 cycles per second octaves burried in noise in 3.847364550 seconds.
jkl@funRuiner:~/Documents/final_proj/src$ ./serial_fft -noisy 60 5000
FFT performed on 60 second sample of 5000 cycles per second octaves burried in noise in 3.735587265 seconds.
jkl@funRuiner:~/Documents/final_proj/src$ gnuplot

G N U P L O T
Version 5.4 patchlevel 1    last modified 2020-12-01

Copyright (c) 1986-1993, 1996, 2004, 2007-2020
Thomas Williams, Colin Kelley and many others

gnuplot home:      http://www.gnuplot.info
faq, bugs, etc:   type "help FAQ"
immediate help:    type "help" (plot window: hit 'h')

Terminal type is now 'qt'
gnuplot> plot "fft.txt" using 1:2 with lines linestyle 1
gnuplot> █
```



Let's speed it up!

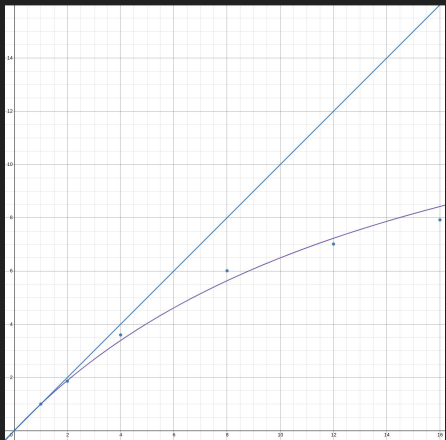
Speed Up: A Closer Look

$S(n) = 1 / ((1-P) + P/n)$							
		Action Time	Actual Speed up	85% Assumed	Amhdahl's Law Speed UP at		
					90% Assumed	95% Assumed	
	1	18.867	1	1	1	1	1
	2	10.133	1.861	1.74	1.82	1.9	
	4	5.245	3.597	2.76	3.08	3.48	
	8	3.14	6.008	3.9	4.71	5.93	
	12	2.69	7.013	4.53	5.71	7.74	
	16	2.382	7.92	4.92	6.4	9.14	
240 seconds(4min song)		1 Thread	2 Threads	4 Threads	8 Threads	8 Cores + SMT 12 Threads	8 Cores + SMT 16 Threads
	19.506	9.78	5.178	3.158	2.718	2.387	
	19.513	10.12	5.349	3.114	2.69	2.382	
	18.23	9.77	5.141	3.162	2.693	2.364	
	19.216	9.951	5.233	3.114	2.782	2.364	
	17.782	10.573	5.248	3.114	2.734	2.39	
	18.874	10.49	5.135	3.119	2.633	2.42	
	18.261	9.998	5.388	3.112	2.642	2.364	
	19.554	10.383	5.28	3.231	2.701	2.392	
AVG OF 8 RUNS		18.867 seconds	10.133 seconds	5.245 seconds	3.1405 seconds	2.69 Seconds	2.382 Seconds

AMD SMT == INTEL HYPERTHREADING. First Generation AMD Ryzen Processors 1300/1500/1700 took a small but measurable single threaded hit with SMT enabled. Any time we push over 8 threads there is potentially risk for single thread performance degradation. Not sure if this will come into play but making notes of it.

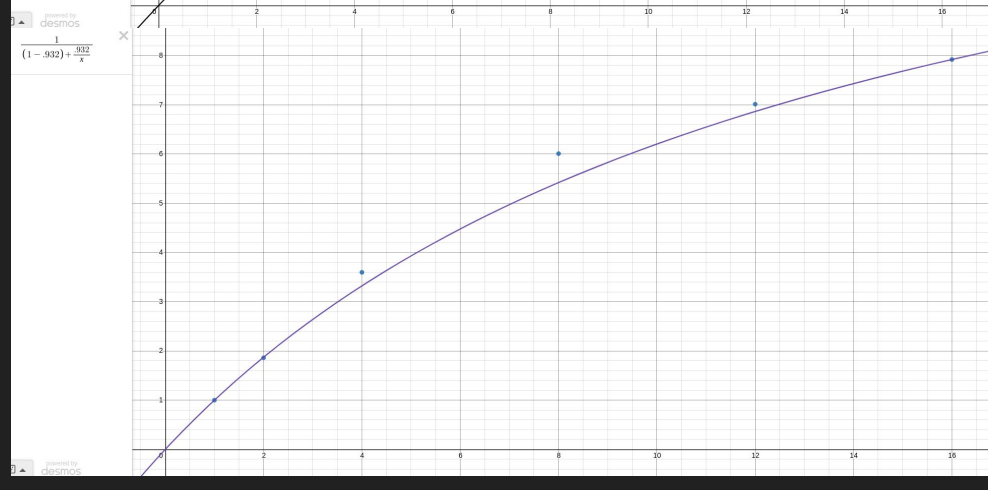
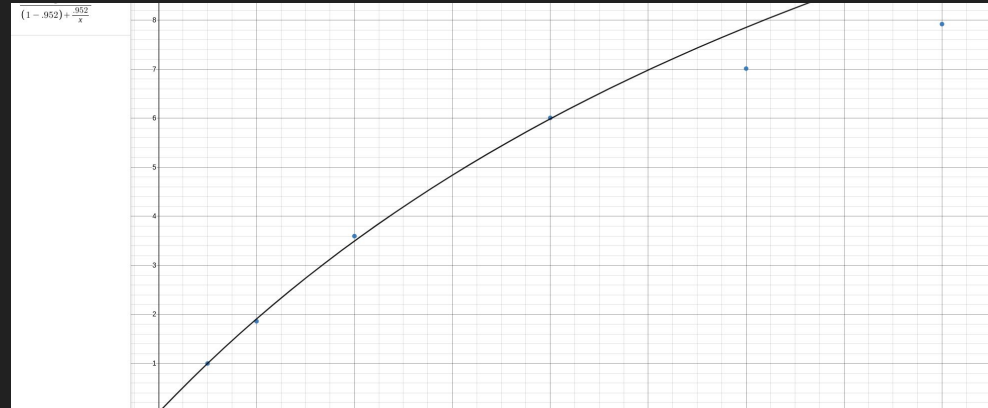
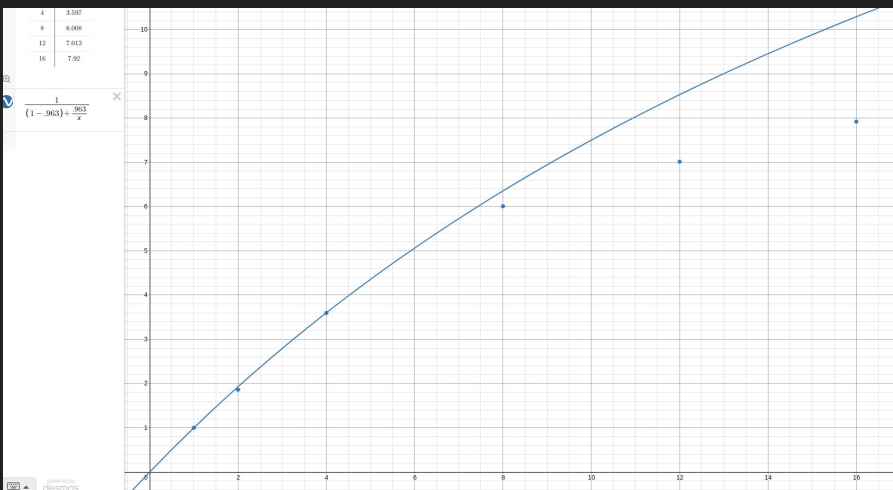
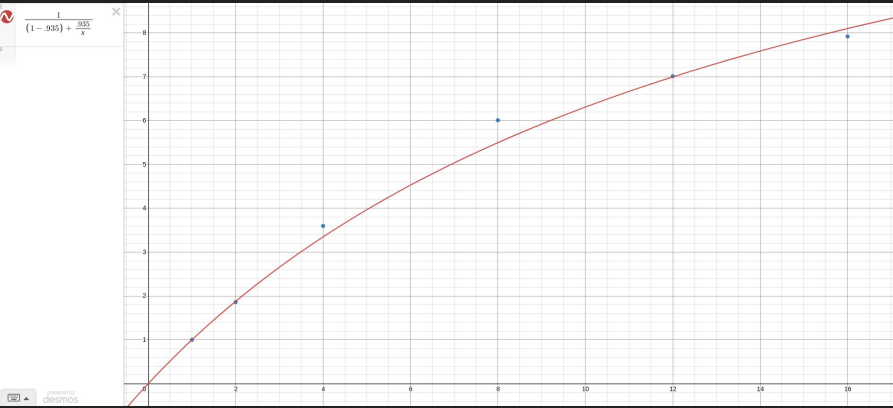
Notes on Data Selection

After doing 8 consecutive runs per thread allotment I did another set of runs for outlier replacement. Selectively replacing any obvious high point outliers from each category. The idea is there is unlikely to be serious error of measurement in the negative direction but competing resources on my machine and the scheduler could skew the data high. I didn't bother doing a Standard deviation check, just intuitively removed obvious outliers.



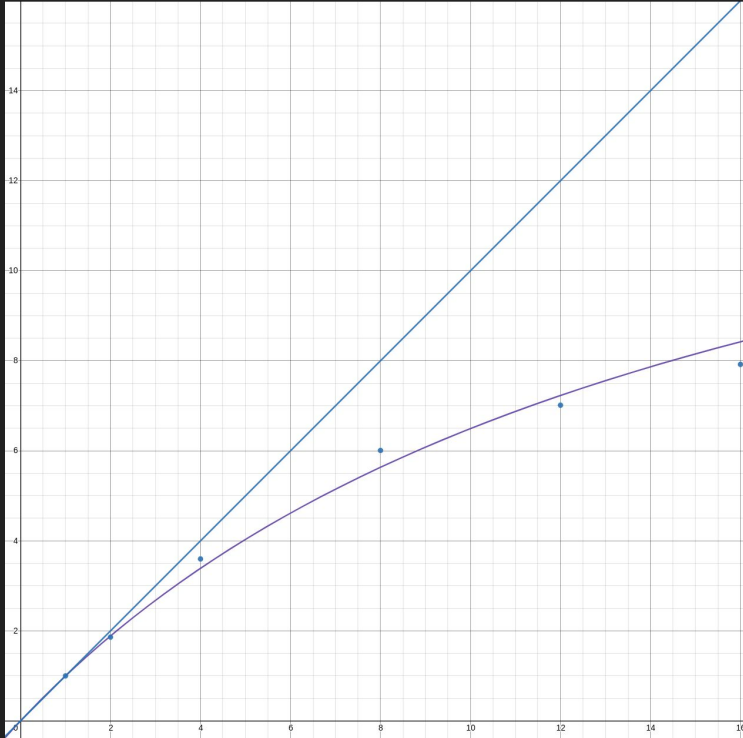
I used a table to estimate parallelization against my real world data and kept adjusting P until I reached the best fit I could. There was no perfect P that fit all data points perfectly but 94% seems to be the sweet spot. It's notable performance curve worsens as thread count gets closer to the total cores available. There is still improvement but less and less as we go further.

Matching Each Point to an Efficiency Curve



Comparative Speed Analysis.

My 94% parallelization vs 100%



My 94% parallelization vs theoretical best case scenario for FFTS of 96.7% according to <https://lirias.kuleuven.be/retrieve/200966>

