ST495/590 – Assignment 4 – Due 2/10

(1) Assume the likelihood $Y|\lambda \sim \text{Poisson}(\lambda)$ and prior $p(\lambda) = 1$ for all $\lambda > 0$. Show that this is an improper prior, compute the posterior (for example, $\lambda|Y \sim \text{Beta}(3, Y + 1)$), and argue that the posterior is proper for any value of Y.

(2) Say that $Y \sim \text{Binomail}(N, 1/2)$ where $N \in \{0, 1, 2, ...\}$ is the unknown parameter of interest and has prior $N \sim \text{Poisson}(1)$. Given $Y = 5$, plot the posterior distribution of N. Give a real-life example where this might be a reasonable statistical analysis.

(3) Say that $Y \sim \text{Exponential}(\lambda)$ so that the likelihood is

$$p(y|\lambda) = \lambda \exp(-\lambda y).$$

Assuming the prior is $\lambda \sim \text{Gamma}(a, b)$, find the posterior of λ.

(4) A clinical trial is conducted to compare the effectiveness of three drugs. 100 patients are randomly assigned to each drug (300 total patients), and $Y_1 = 12$, $Y_2 = 18$, and $Y_3 = 10$ patients have successful outcomes in the three drug groups. Using uniform priors for the success probabilities of each drug, compute the posterior distribution of the success probability for each drug, and the posterior probability that drug 2 is the best drug.

You should turn in your responses to these questions in 1-2 pages (i.e., one piece of paper with text on both sides). Code and derivation can be on a separate sheet, stapled to the answer sheet. Be sure all plots are labeled and code is commented!