ST495/590 Assignment 5 - Solutions

(1) *Explain in words the steps of a Gibbs sampler.*

To get the joint posterior of $(\lambda_1, \cdots, \lambda_n, \gamma)$, the main idea is to break the problem of sampling from the high-dimensional joint distribution into a series of samples from low-dimensional conditional distributions. The steps are as follows:

(i) drawing from $\lambda_i | \lambda_1, \cdots, \lambda_{i-1}, \lambda_{i+1}, \cdots, \lambda_n, \gamma, \boldsymbol{Y}$, for $i = 1, \cdots, n$;
(ii) drawing from $\gamma | \lambda_1, \cdots, \lambda_n, \boldsymbol{Y}$.

(2) *Full conditionals.*

$$
\begin{aligned}
& \left(\prod_{i=1}^{n} \pi(Y_i|\lambda_i)\pi(\lambda_i|\gamma)\right)\pi(\gamma) \\
= & \left(\prod_{i=1}^{n} \frac{e^{-\lambda_i}\lambda_i^{Y_i}}{Y_i!}\frac{\gamma^a}{\Gamma(a)}\lambda_i^{a-1}e^{-\gamma\lambda_i}\right)\frac{0.1^{0.1}}{\Gamma(0.1)}\gamma^{0.1-1}e^{-0.1\gamma} \\
\propto & \left(\prod_{i=1}^{n} e^{-\lambda_i}\lambda_i^{Y_i}\lambda_i^{a-1}e^{-\gamma\lambda_i}\right)\gamma^{na}\gamma^{0.1-1}e^{-0.1\gamma}
\end{aligned}
$$

So the full conditionals are

$$\pi(\lambda_i|\lambda_1, \cdots, \lambda_{i-1}, \lambda_{i+1}, \cdots, \lambda_n, \gamma, \boldsymbol{Y}) \propto \lambda_i^{Y_i+a-1}e^{-(1+\gamma)\lambda_i} \sim \text{Gamma}(Y_i + a, 1 + \gamma),$$

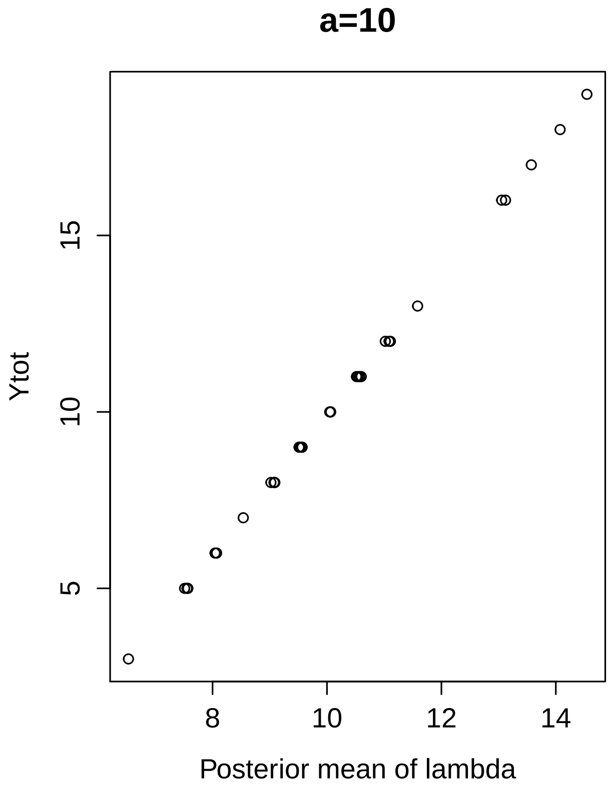
$$\pi(\gamma|\lambda_1, \cdots, \lambda_n, \boldsymbol{Y}) \propto \gamma^{na+0.1-1}e^{-(0.1+\sum\limits_{i=1}^{n}\lambda_i)\gamma} \sim \text{Gamma}(na + 0.1, 0.1 + \sum_{i=1}^{n}\lambda_i).$$

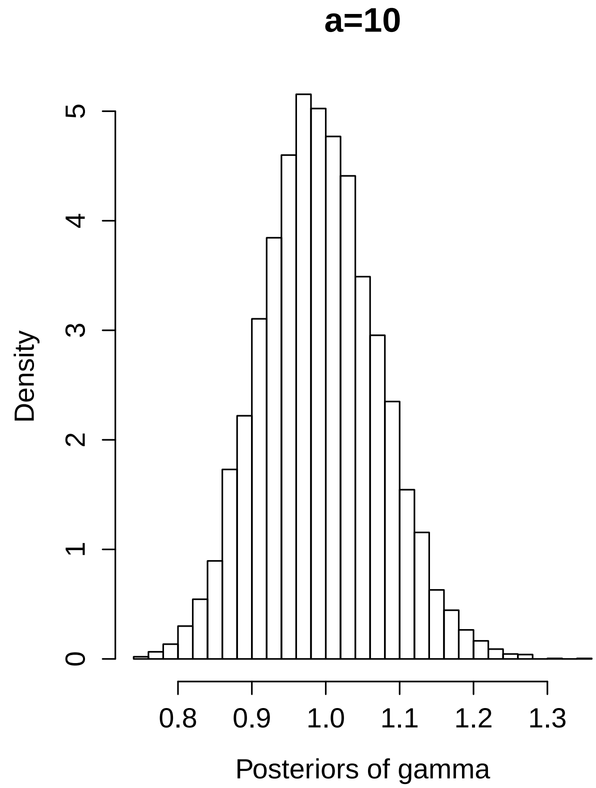
(2) *R coding.*

(3) *JAGS coding.* The results from R and JAGS are similar.

(4) *$a = 1$ and $a = 10$.*

$a = 10$ leads to larger posterior mean of gamma, which is due to the posterior mean of γ is $\frac{na+0.1}{0.1+\sum_{i=1}^{n}\lambda_i}$. When $a = 1$, the agreement of posterior mean of λ_i's is better than when $a = 10$. This is also because as a goes larger, the effect of observations Y_i's will be less, thus less agreement.


a=10
Ytot
5
10
15
8
10
12
14
Posterior mean of lambda


a=10
Density
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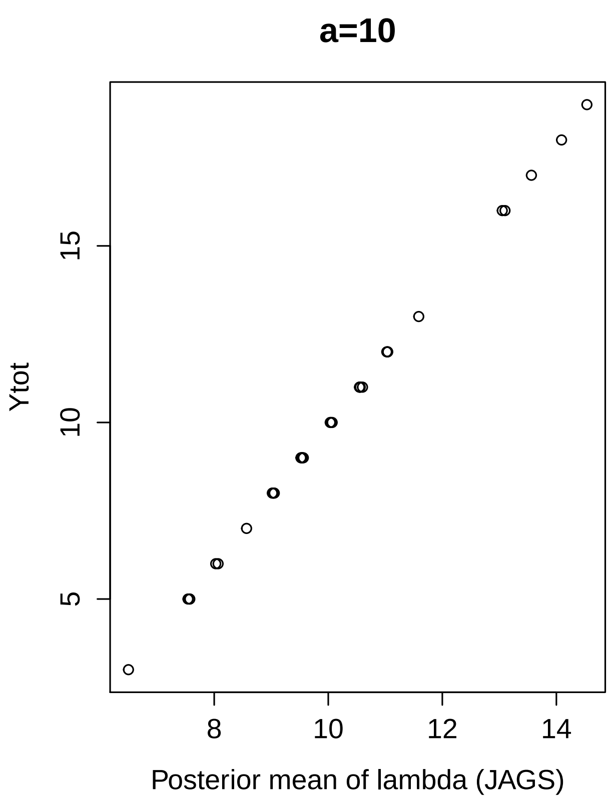
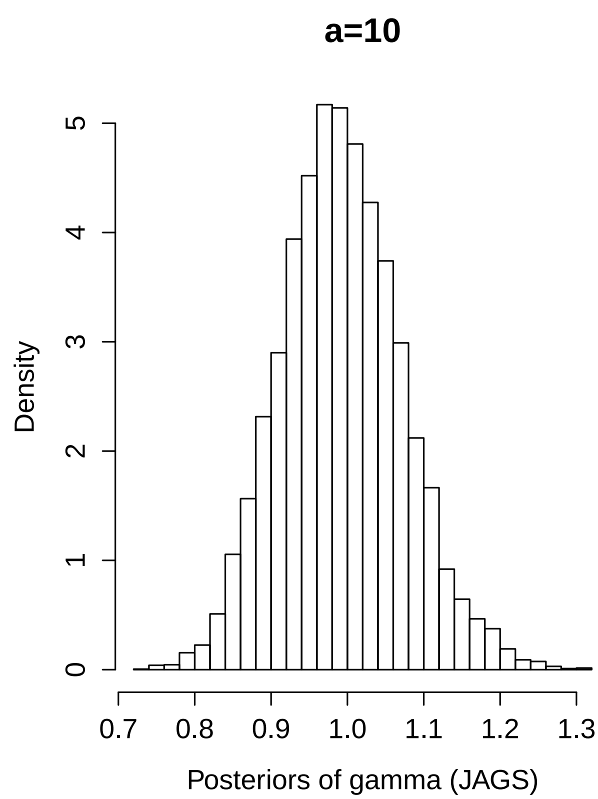
1
2
3
4
5
0.8
0.9
1.0
1.1
1.2
1.3
Posteriors of gamma


a=10
Ytot
5
10
15
8
10
12
14
Posterior mean of lambda (JAGS)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Density
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3
4
5
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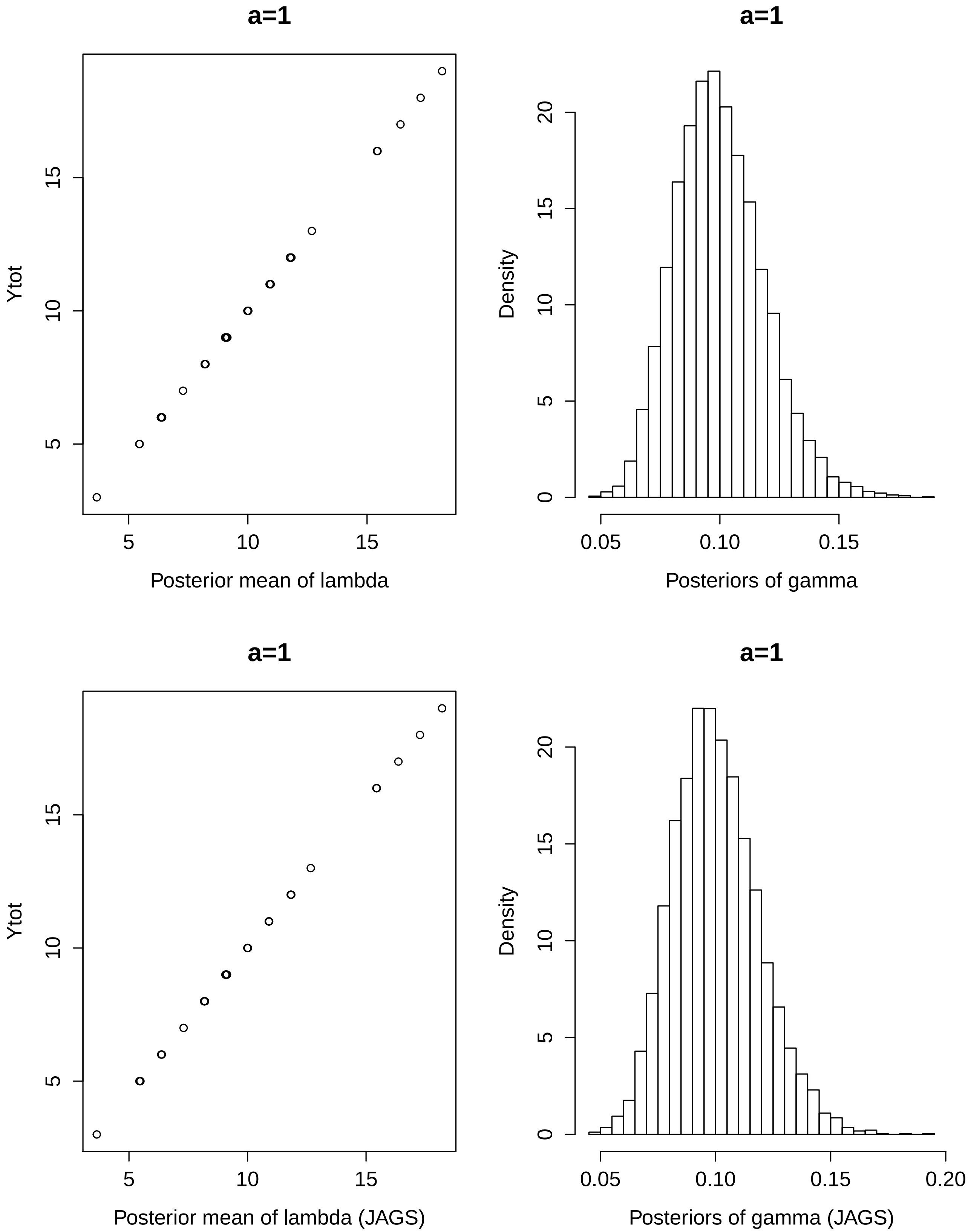
0.8
0.9
1.0
1.1
1.2
1.3
Posteriors of gamma (JAGS)


a=1
Ytot
Posterior mean of lambda
a=1
Density
Posteriors of gamma
a=1
Ytot
Posterior mean of lambda (JAGS)
a=1
Density
Posteriors of gamma (JAGS)