

## ST495/590 – Assignment 3 – Due 2/3

- (1) For this problem pretend we are dealing with a language with six-word dictionary

$\{\text{cat, hat, fat, cap, cup, hot}\}$ .

An extensive study of literature written in this language reveals that all words are equally likely except that “hot” is  $\alpha$  times as likely as the other words. Further study reveals that

- (a) Each keystroke is an error with probability  $\theta$
- (b) All letters are equally likely to produce errors
- (c) Given that a letter is typed incorrectly it is equally likely to be any other letter
- (d) Errors are independent across letters.

For example, the probability of correctly typing “cat” (or any other word) is  $(1 - \theta)^3$ , the probability of typing “sat” or “cah” when intending to type is “cat” is  $\theta(1 - \theta)^2$ , and the probability of typing “kit” or “hot” when intending to type “cat” is  $\theta^2(1 - \theta)$ .

Use Bayes rule to develop a simple spell checker for this language. For each of the typed words “hat”, “cip”, “abs”, give the probability that each word in the dictionary was the intended word, e.g., given they typed “cah” what is the probability they were trying to type “hot”? Perform this for (i)  $\alpha = 2$  and  $\theta = 0.1$ ;  $\alpha = 50$  and  $\theta = 0.1$ ; and (iii)  $\alpha = 2$  and  $\theta = 0.95$ . Briefly comment on the changes you observe in these three cases.

- (2) Assume the data is distributed  $Y \sim \text{Poisson}(\lambda)$  and the prior is uniform over the set  $\lambda \in \{0, 1, 2, \dots, 19, 20\}$ . Plot the prior, compute the prior mean and standard deviation, and find an interval so that  $\lambda$  is in the interval with prior probability 0.9. Now we observe  $Y = 2$ . Plot the posterior, compute the posterior mean and standard deviation, and find an interval so that  $\lambda$  is in the interval with posterior probability 0.9.

You should turn in your responses to these questions in 1-2 pages (i.e., one piece of paper with text on both sides). Be sure all plots are labeled and code is commented!