ST495/590 – Assignment 2 – Due 1/27

(1) Assume the Bayesian model with likelihood $Y|\theta \sim \text{Binomial}(n, \theta)$ and prior $\theta \sim \text{Beta}(a, b)$. Write a function that uses Monte Carlo sampling to estimate the posterior mean and standard deviation of θ given we observe $Y = y$. The function should take inputs y, n, a, and b. Given these inputs, the function should generate $1,000,000$ samples of (θ, Y) (by first drawing θ from a beta distribution and then $Y|\theta$ from a binomial distribution), extract the samples with $Y = y$, and return the mean and standard deviation of θ for these samples. Include code for this function in your write-up.

(2) Use the code from (1) with $n = 10$ and $a = b = 1$ to compute the posterior mean and standard deviation for θ for all $y = 0, 1, ..., n$ and plot the posterior mean and standard deviation as a function of y.

(3) Use the code from (1) with $n = 10$ and $a = b = 10$ to compute the posterior mean and standard deviation for θ for all $y = 0, 1, ..., n$ and plot the posterior mean and standard deviation as a function of y.

(4) Comment on the differences between the plots with $a = b = 1$ versus $a = b = 10$.

You should turn in your responses to these questions in 1-2 pages (i.e., one piece of paper with text on both sides). Be sure all plots are labeled and code is commented!